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GROWTH AND DEVELOPMENT

With Special Reference to Domestic Animals

VII. Equivalence of Age During the Self-Inhibiting Phase
of Growth.

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GROWTH AND DEVELOPMENT

With Special Reference to Domestic Animals

VII. Equivalence of Age During the Self-Inhibiting Phase of Growth.

SAMUEL BRODY

ABSTRACT.—1. A method is described for determining equivalence of age with respect to growth in weight during the phase of growth following puberty. 2. Thirty-eight equivalence charts prepared by this method are presented in this bulletin, and their significance is interpreted. 3. The following are the most striking new ideas indicated by these charts: (a) There are enormous differences in the curves of different animals with respect to the ratios of the segment following the inflection (puberty) to the segment preceding the inflection. (b) There are enormous differences with respect to the rapidity of approach to the mature weight, and with respect to gains in absolute weight of animals of the same species. This suggests the need of investigating the effect of genetic and environmental factors on growth rates. (c) Of all the animals examined, the pigeon (and dove) is the most rapidly maturing species, and its mature weight is highest in relation to the time required to reach it. (d) Growth curves of plants, and of populations of organisms have the same general form as growth curves of animals. Equivalence charts between growth curves of plants, populations, and animals are also presented in this bulletin.

I. INTRODUCTION

In the third and sixth bulletins of this series (Research Bulletins 97 and 101) it was explained that following the major inflection (puberty) in the growth curve, the time-rate, or velocity, of growth declines by a constant fraction, k , per unit time; or what is the same, the velocity of growth bears a constant ratio, k , to the growth yet to be made to reach the mature weight, A . It was explained that such a curve may be represented by the equation

$$W = A - Be^{-kt} \quad (1)$$

in which W is the weight of the animal at the age of t , k is the fractional decline in the velocity of growth, and B is an age-parameter employed to correct for the fact that while age is counted from birth or conception this equation fits the data during the phase of growth following the inflection only.

The constant B in equation (1) may be dispensed with by beginning to count not from birth or conception, but from age t^* when the curve of the equation meets the age-axis. By this change equation (1) takes the form

$$W = A [1 - e^{-k(t-t^*)}] \quad (2)$$

or

$$\frac{W}{A} = 1 - e^{-k(t-t^*)} \quad (3)$$

II. EQUIVALENCE OF AGE

The constant k in equations (1) to (3) has a very definite meaning; it represents, as already explained, the fractional decline in the velocity of growth with age. The greater the fractional decline, k , the more rapidly will the limiting, or mature, value A be approached. Indeed, the rapidity of approach to the mature weight, A , is directly proportional to the numerical value of k , as shown in Fig. 1, and the relative duration of the periods of growth of two animals is, therefore, inversely proportional to the numerical values of their k 's. These facts give a basis for computing the equivalence of growth age in different animals.

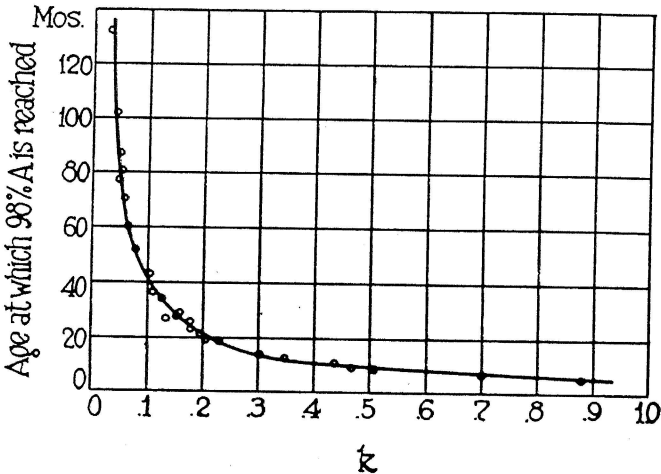


Fig. 1.—The relation between the numerical values of the velocity constants of growth, k , and the time required to reach 98 per cent of the nature value, A , in different animals.

The lettering in this and in the following charts was done by Mr. Raymond Hase, a student in the University of Missouri.

Thus, from Table I of the preceding paper, the numerical value of k of the Jersey cow is .054; of the female white rat it is .644. Therefore, 1 month in the rat is equivalent (during the phase of growth following the point of inflection) to $1 \times \frac{.644}{.054}$ or 11.9 months in the cow; or 1 month in the cow is equivalent to $1 \times \frac{.054}{.664}$ or .08 months in the rat. A few such equivalent values referred to 1 month in the Jersey cow and 1 month in

the white rat are given in Table I. Growth in weight-equivalence may be similarly computed by comparing the numerical values of the mature weights A . The value of A for the Jersey cow is 420 kilos; of the female

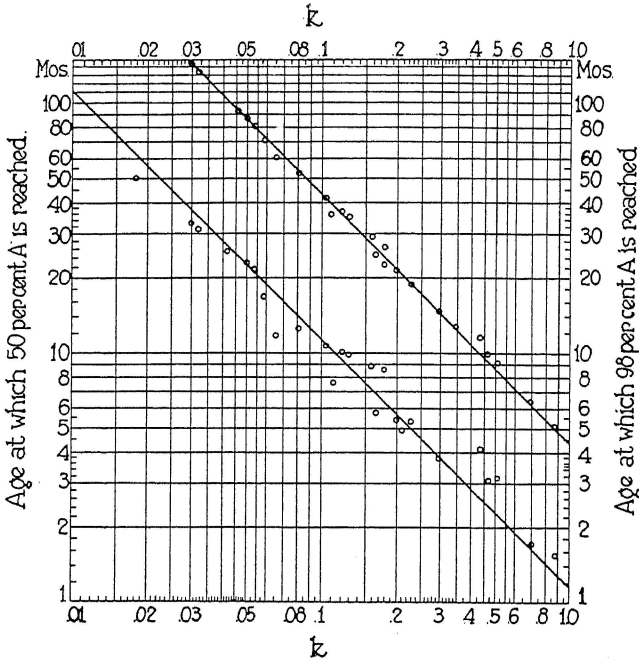


Fig. 1a.—The relation between the numerical value of the growth constant, k , in different animals and the time required to reach 98 per cent of the mature weight (upper line) and 50 per cent of the mature weight (lower line) plotted on logarithmic coordinate paper. The linear distribution of the data on this paper indicates that the time, t , required to reach a given fraction of the mature weight is inversely proportional to k . That is

$$t = \frac{c}{k}$$

c is a constant. c has a value of 4.35 when t represents the time required to reach 98 per cent A ; $c = 1.12$ when t represents the time required to reach 50 per cent A . The data on the 50 per cent line are less evenly distributed than on the 98 per cent line, due to the fact that at the lower levels the period of growth preceding the point of inflection, with which equation (1) is not concerned, becomes a factor of relatively increasing importance.

rat it is .203 kilos. Therefore, 1 gm. in the rat corresponds to 2.07 kilos in the cow. Such equivalent values on the basis of the Jersey cows are given in Table I (page 8).

TABLE 1.—EQUIVALENCE OF AGE FOLLOWING THE POINT OF INFLECTION; ALSO EQUIVALENCE OF WEIGHT

	Age equivalence			Weight equivalence	<i>A</i> (Mature weight)	<i>k</i>
	Referred to 1 unit in the Jersey cow as a standard.	Referred to 1 unit in the male guinea pig as a standard	Referred to 1 unit in the unmated white rat as a standard.	Referred to 1 unit in the Jersey cow as a standard		
Jersey cow (Eckles).....	1.000	4.26	11.9	1.000	kg. 420	.054
Ayrshire cow (Eckles).....	1.080	4.60	12.9	1.100	460	.050
Holstein cow (Eckles).....	1.174	5.00	14.1	1.300	550	.046
Duroc-Jersey swine, females (F. B. Mumford)870	3.70	10.4	.480	200	.062
Suffolk sheep, females (Murray)....	.292	1.24	3.48	.190	80	.185
Shropshire-Merino sheep, females (Murray)287	1.22	3.43	.119	50	.188
Guinea pig, male (Wright).....	.235	1.00	2.80	.002	.825	.230
White rat, unmated females (Donaldson, Dunn, and Watson)084	.357	1.00	.00048	.203	.644
White rat, males (Donaldson, Dunn, and Watson)135	.575	1.61	.00067	.280	.400
White rat, males (Greenman and Duhring).....	.154	.657	1.84	.00083	.350	.350
White mouse, males (Robertson)....	.087	.371	1.04	.000065	.0275	.620
White mouse, females (Robertson) ..	.066	.280	.785	.000056	.0235	.820

III. CONSTRUCTING EQUIVALENCE CHARTS WITH THE AID OF THE NUMERICAL VALUES OF k AND A

Having elaborated a method for determining equivalence of growth age, we may proceed to compare growth curves of animals in order to learn something of their distinctive peculiarities and, incidentally, to test our method of determining equivalence.

The results of these comparisons are shown in Figs. 2 to 13. In most of the graphs, the Jersey cow is taken for the standard of comparison.

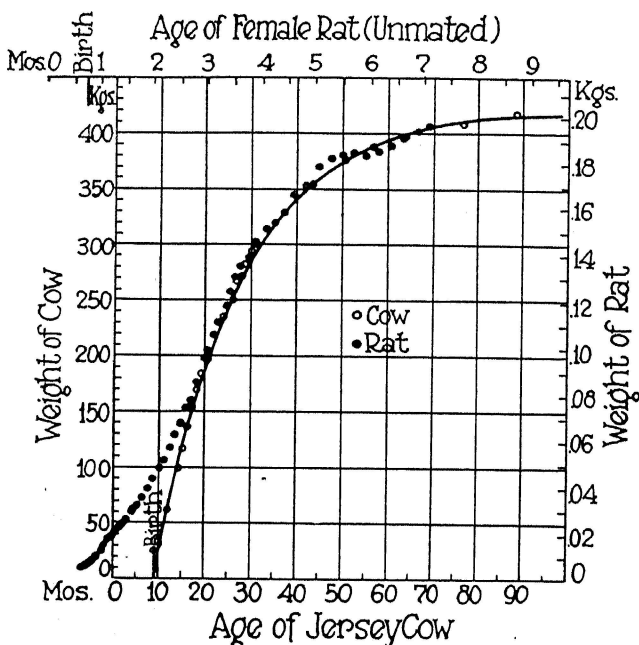


Fig. 2.—Growth-equivalence of Jersey cow (Eckles) and unmated female white rat (Donaldson). After the point of inflection, 1 month in the rat is equivalent to 11.9 months in the cow, and 1 gm. of weight in the rat is equivalent to 2.068 kilos in the cow. The period of growth preceding the point of inflection is relatively longer in the rat than in the cow.

son. The observed values are represented by circles, the values computed from equation (1) are represented by smooth curves. The observed values of the animals under comparison agree satisfactorily except at the very early stages of growth with which, as pointed out, equation (1) is not concerned. The agreements between the observed values and the values computed from equation (1) are also satisfactory for the phase of

growth following the points of inflection, which is the only phase under consideration.

The equivalence charts are self-explanatory and they make more interesting reading than any comments that we may offer in their behalf. It only remains to illustrate this method of preparing an equivalence chart.

We may take, for purposes of illustration, the steps involved in preparing an equivalence chart for the Jersey cow and the female white rat (Fig. 2). We have found, as already explained in the preceding section, that

1 month in the cow = .084 months in the rat, or
 1 month in the rat = 11.91 months in the cow.

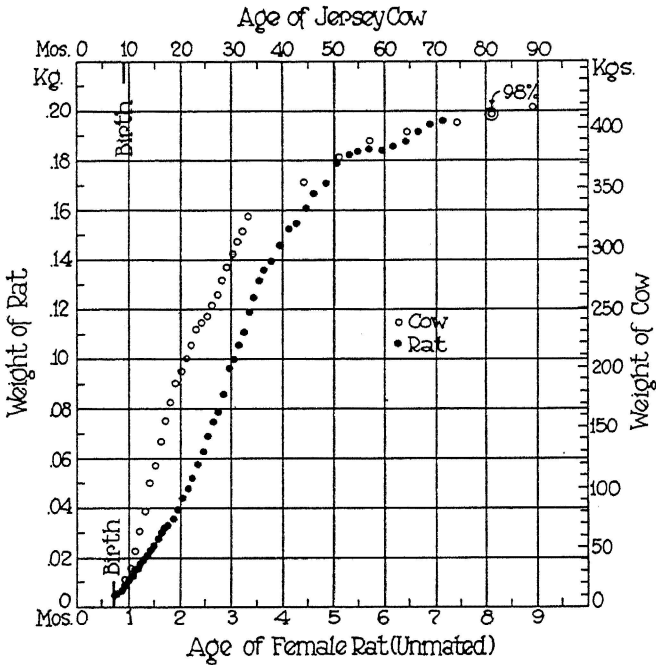


Fig. 2a.—Growth-equivalence of cow (Eckles) and unmated female white rat (Donaldson) obtained on the assumption that the age when 98 per cent of the mature weight is reached is equivalent in the two animals. Compare with Fig. 2.

Now this statement does not imply that a 1 month old rat is physiologically as old as an 11.9 months old cow; for, as pointed out, this equivalence was obtained on the basis of the numerical values of the k 's in equation (1) for the cow and rat, and equation (1) represents only the phase of growth following the point of inflection in the smoothed growth

curve. The above relation, which was found to hold true for the phase of growth following the point of inflection, may or may not hold true for the phase of growth preceding the point of inflection. As a matter of fact, this relation does not hold true for the rat and the cow for the phase

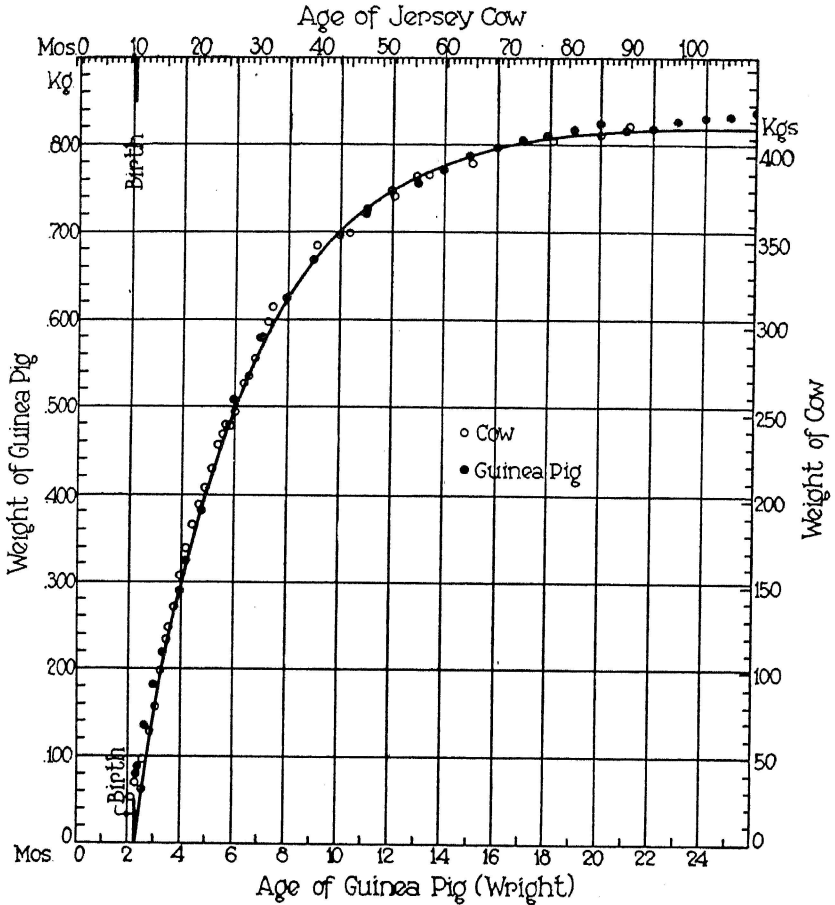


Fig. 3.—Growth-equivalence of Jersey cow (Eckles) and male guinea pig (Wright). 1 month in the guinea pig is equivalent to 4.26 months in the cow and 1 gm. in the guinea pig is equivalent to 509.1 gm. in the cow. Unlike the case of the rat, growth in the guinea pig appears to follow the same course as growth in the cow from conception.

of growth preceding inflection (cf. Figs. 2 and 2a). For the cow and guinea pig, on the other hand, the relation between their k 's also applies for the phase of growth preceding inflection (cf. Figs. 3 and 3a).

All this is equivalent to saying that conception cannot be taken as a point of reference in preparing equivalence charts which should represent equivalence of growth in weight following the point of inflection. Instead of taking conception, we take for the points of reference the age when the curve of equation (1) meets the age-axis. As already explained in detail in the third bulletin of this series, this age is designated by t^* . It is only after the age designated by t^* that 1 month in the rat is equivalent to 11.9 months in the cow.

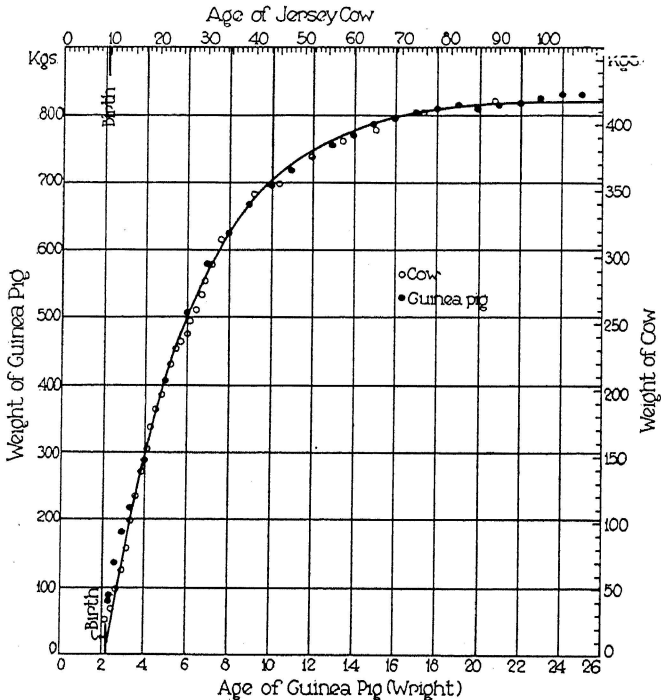


Fig. 3a.—Growth-equivalence of cow (Eckles) and guinea pig (Wright) obtained on the assumption that the age when 98 per cent of the mature weight is reached is equivalent in the two animals. Compare to Figs. 2, 2a, and 3.

The numerical value of t^* may either be read from the graph as previously explained, or it may be computed as follows: When $t = t^*$, $W = 0$ by hypothesis, and equation (1) becomes

$$\begin{aligned}
 0 &= A - Be^{-kt^*} \\
 A &= Be^{-kt^*} \\
 \ln A &= \ln B - kt^* \\
 t^* &= \frac{\ln B - \ln A}{k} \quad (4)
 \end{aligned}$$

The work is now laid out as follows:

	A	k	t^*	
	kg		mos.	
Unmated female white rat	.203	.644	2.03	Age factor = $\frac{.644}{.054} = 11.9$
Jersey cow-----	420.	.054	8.9	

The values of t^* in the cow and rat, namely 2.03 and 8.9 months, constitute by hypothesis one pair of corresponding points. Another pair of corresponding values may be found as follows: Take 8 months from conception in the rat, or $8.00 - 2.03 = 5.97$ months from t^* in the rat as another point of reference. 5.97 months from t^* in the rat is equivalent to $5.97 \times 11.91 = 71.1$ months from t^* in the cow or $71.1 + 8.9 = 80.0$ months from conception in the cow. 8 months in the rat and 80 months in the cow (counted from conception) thus constitute the second pair of corresponding points in the ages of the cow and rat.

For weight equivalence, zero in the rat corresponds to zero in the cow and .203 kilos in the rat corresponds to 420 kilos in the cow. It remains only to graduate uniformly the axes between the points of reference, and to extrapolate the graduations if desired. Figs. 2 to 14 were prepared by this method.

IV. CONSTRUCTING EQUIVALENCE CHARTS WITH THE AID OF EQUIVALENT TIME SCALES, $k(t-t^*)$

Equation (3) suggests a simpler technique for constructing equivalence charts. It is evident from equation (3) that for a given value of $k(t-t^*)$, there is one, and only one, value of $\frac{W}{A}$. If, therefore, the val-

ues of $\frac{W}{A}$ of two or more curves are plotted against the corresponding values of $k(t-t^*)$, the curves must necessarily coincide, that is, within the limits of the fit of the equations to the corresponding sets of data.

After the data are plotted on the $k(t-t^*)$ grid, the absolute age-scales are laid off, and the chart is completed. One such chart, having the data for the mouse, rat, guinea pig, fowl, rabbit, sheep, swine, cow, and man has been presented in the third bulletin of this series (Res. Bul. 97). Figs. 15 to 26 in this bulletin are additional charts of the same type. The significance of each chart is indicated in its legend.

Aside from any theoretical interest, the principal practical value of charts in which the data are thus plotted on $k(t-t^*)$ axes is that they enable one to evaluate, in a simple manner, equivalence of age with

respect to growth in weight. Thus supposing that it is desired to determine the age of the albino mouse corresponding to the age of 92 months in the Jersey cow. From Fig. 16, 92 months in the Jersey cow corresponds to the value 5 on the $k(t-t^*)$ scale, and from Fig. 23, the value 5 on the $k(t-t^*)$ scale corresponds to 6.4 months for the female albino mouse. Hence 6.4 months in the mouse is equivalent to 92 months in the cow. These values, of course, hold true only during the phase of growth following the major inflection, and only with respect to growth in weight.

To facilitate plotting the curve of equation (3), Table II is presented in which values of $\frac{W}{A}$ are given for corresponding values of $k(t-t^*)$. In the preparation of these charts, coordinate paper was used having 20 squares between 0 and 100. The values in the column headed by $20 \frac{W}{A}$ are given for plotting on such paper.

TABLE II.—TABLE OF VALUES FOR THE $k(t-t^*)$ SCALE

$$\frac{W}{A} = 1 - e^{-k(t-t^*)}$$

$k(t-t^*)$	$e^{-k(t-t^*)}$	$\frac{W}{A}$	$\frac{20W}{A}$	$\frac{W}{A}$	$e^{-k(t-t^*)}$	$k(t-t^*)$
0	1.0000	.0000	0.000	.00	1.00	.000
.1	.9048	.0952	1.904	.50	.95	.051
.2	.8187	.1813	3.626	.10	.90	.105
.3	.7408	.2592	5.184	.15	.85	.162
.4	.6703	.3297	6.594	.20	.80	.223
.5	.6065	.3935	7.870	.25	.75	.288
.6	.5488	.4512	9.024	.30	.70	.357
.7	.4966	.5034	10.068	.35	.65	.431
.8	.4493	.5507	11.014	.40	.60	.511
.9	.4066	.5943	11.868	.45	.55	.598
1.0	.3679	.6321	12.642	.50	.50	.693
1.2	.3012	.6988	13.976	.55	.45	.799
1.4	.2466	.7534	15.068	.60	.40	.917
1.6	.2019	.7981	15.962	.65	.35	1.050
1.8	.1653	.8347	16.694	.70	.30	1.204
2.0	.1353	.8647	17.294	.75	.25	1.386
2.5	.0821	.9197	18.358	.80	.20	1.609
3.0	.0498	.9502	19.004	.85	.15	1.897
3.5	.0302	.9698	19.396	.90	.10	2.302
4.0	.0183	.9817	19.634	.95	.05	2.995
4.5	.0111	.9889	19.778	.96	.04	3.220
5.0	.0067	.9933	19.866	.97	.03	3.500
5.5	.0041	.9959	19.918	.98	.02	3.910
6.0	.0025	.9975	19.950	.99	.01	4.600
7.0	.0009	.9991	19.982	---	---	---
8.0	.0003	.9997	19.994	---	---	-----

V. CONSTRUCTING EQUIVALENCE CHARTS ON THE BASIS OF CONCEPTION AND AGE WHEN 98 PER CENT OF THE MATURE WEIGHT IS REACHED AS POINTS OF REFERENCE

An attempt has been made to obtain equivalent age values by taking conception and some convenient fraction of the mature weight A as points of reference. This method is similar in principle to the method employed by Pearl and co-workers for comparing mortality curves of man, *Drosophila*, and *Proales decipiens*. This method of determining equivalence would be entirely satisfactory if the ratios between the segments of the growth curve preceding the point of inflection to the segment following it were the same in the animals under investigation. This is the case for the Jersey cow and male guinea pig. The agreement between the curves of these two animals is as good by the use of this method (Fig. 3a) as it is by the method of comparing the k 's and t^* 's (Fig. 3). But the agreement between the curves of the cow and the rat (Fig. 2a) is very poor by this method throughout the whole curve, while by the use of the preceding method (Fig. 2) it is good at least after the point of inflection. Both methods are useful, depending on the viewpoint of the investigators; but, of course, neither method is entirely satisfactory if the several phases of growth do not occupy proportional parts of the curve in the animals under comparison.

It should be noted in this connection that in 1906 Donaldson published a method for determining equivalence of age between man and rat. It was based on the assumption that the rat at the age of 3 years is comparable in age to man at 90 years, and that the same proportional relations hold for fractions of the life period. The objection against employing the duration of life as a basis for estimating equivalence of age consists, as Donaldson had pointed out, in the incomplete information at our disposal concerning the onset of old age and duration of life. There is, of course, also the possibility that the same proportional relation may not hold during the whole of the life span of the organisms under comparison.

VI. A TABLE OF EQUIVALENCE OF AGE

It is often desirable to know the ages when given weights, or given fractions of the mature weights, are reached. These may be determined from equation (1).

$$\begin{aligned}
 A - W &= B e^{-kt} \\
 \ln(A - W) &= \ln B - kt \\
 kt &= \ln B - \ln(A - W) \\
 t &= \frac{\ln B - \ln(A - W)}{k} \quad (5)
 \end{aligned}$$

Assuming that it is desired to determine the age when some fraction such as 90 per cent, of the mature weight is reached, W is replaced by $.9A$ and equation (5) is changed to

$$t = \frac{\ln B - \ln(A - .9A)}{k}$$

$$= \frac{\ln B - \ln .1A}{k}$$

The age t , when a given fraction of the mature weight is reached may be computed in similar fashion from equation (3)

$$\frac{W}{A} = 1 - e^{-k(t-t^*)}$$

$$1 - \frac{W}{A} = e^{-k(t-t^*)}$$

$$\ln\left(1 - \frac{W}{A}\right) = -k(t - t^*)$$

$$t = t^* - \frac{\ln\left(1 - \frac{W}{A}\right)}{k} \quad (7)$$

TABLE III.—TABLE TO FACILITATE ESTIMATING AGES AT WHICH DIFFERENT FRACTIONS OF THE MATURE WEIGHT ARE REACHED

Fraction of Mature Weight $\left(\frac{W}{A}\right)$	$\ln\left(1 - \frac{W}{A}\right)$
.25	—0.28768
.30	—0.35667
.35	—0.43078
.40	—0.51083
.45	—0.59784
.50	—0.69315
.55	—0.79851
.60	—0.91629
.65	—1.0498
.70	—1.2040
.75	—1.3863
.80	—1.6094
.85	—1.8971
.90	—2.3026
.95	—2.9957
.98	—3.9120
.99	—4.6052
.999	—6.9078

TABLE IV.—EQUIVALENCE OF GROWTH AGE

Ages in months counted from conception when different fractions of the mature weight, A , are reached. The upper rows represent ages as interpolated from growth curves in which the observed values were connected by straight lines; the lower rows represent ages as computed from formula (3).

Percentage of mature weight.....	10	20	30	40	50	60	70	80	90	95	98	Mature weight
												kg.
Holstein cow (Eckles) -----	10.5	13.5	16.0	19.5	23.8	28.7	34.0	43.0	59.0			
	10.6	13.1	16.0	19.4	23.4	28.2	34.5	43.3	58.4	73.4	93.2	550
Ayrshire cow (Eckles) -----	10.8	13.4	16.0	19.4	23.0	27.4	32.8	41.4	55.5			
	11.2	13.6	16.2	19.3	23.0	27.4	33.2	41.3	55.1	69.0	87.3	460
Jersey cow (Eckles) -----	10.9	13.6	15.8	18.4	22.0	26.4	30.5	28.3	53.0			
	10.9	13.1	15.5	18.4	21.8	25.9	31.2	38.7	51.6	64.3	81.4	420
Duroc-Jersey sow (Mumford)-----	6.7	9.2	10.7	12.7	15.8	19.5	24.2	31.0	43.0	54.0	70.5	200
	5.9	7.9	10.1	12.7	15.7	19.5	24.3	31.0	42.4	53.9	67.3	
Suffolk ewe (Murray)-----	5.1	6.0	6.8	7.6	8.5	9.8	11.3	13.6	17.2			80
	5.3	6.0	6.7	7.6	8.6	9.8	11.4	13.7	17.5	21.4	26.5	
Guinea pig, male (Wright) -----	2.7	3.2	3.7	4.3	5.2	6.1	7.2	8.8	12.0	14.4	18.8	.825
	2.8	3.4	3.9	4.5	5.3	6.3	7.5	9.3	12.3	15.4	18.9	
White rat, unmated female (Donaldson <i>et al.</i>)-----	1.4	2.0	2.4	2.8	3.1	3.4	3.9	4.5	5.4	6.8		.203
	2.2	2.4	2.6	2.8	3.1	3.5	3.9	4.5	5.6	6.7	8.1	
White rat, male (Donaldson <i>et al.</i>) -----	1.6	2.4	2.8	3.1	3.6	4.1	4.9	5.7	7.9			.280
	2.1	2.4	2.8	3.1	3.6	4.1	4.9	5.9	7.6	7.4	11.7	
White rat, male (Greenman and Duh-ring)-----	1.6	2.3	2.8	3.3	4.0	4.4	5.0	6.3	8.4	10.1	13.1	.350
	2.0	2.4	2.8	3.2	3.4	4.4	5.2	6.3	8.4	10.0	12.9	
White mouse, female (Robertson) ----	.85	1.2	1.4	1.5	1.9	2.2	2.6	3.3	3.9	5.3	6.7	.0235
	.92	1.1	1.3	1.6	1.9	2.2	2.7	3.3	4.4	5.5	7.0	
White mouse, male (Robertson)-----	.95	1.1	1.3	1.5	1.8	2.0	2.3	2.7	3.9	4.8	5.5	.0275
	1.1	1.2	1.3	1.5	1.8	2.0	2.3	2.8	3.7	4.6	5.6	

Values for $\ln \left(1 - \frac{W}{A} \right)$ are given in Table III to facilitate computation by this method.

A series of ages when different fractions of the mature weight are reached as computed from equation (5) and also ages when different fractions of mature weight are reached as obtained by direct interpolation from the smoothed weight-age curves are shown in Table IV. The agreement between these two sets of values is satisfactory for the later stages of growth. The deviation between these two sets of values increases, however, with decreasing age on account of the fact that the

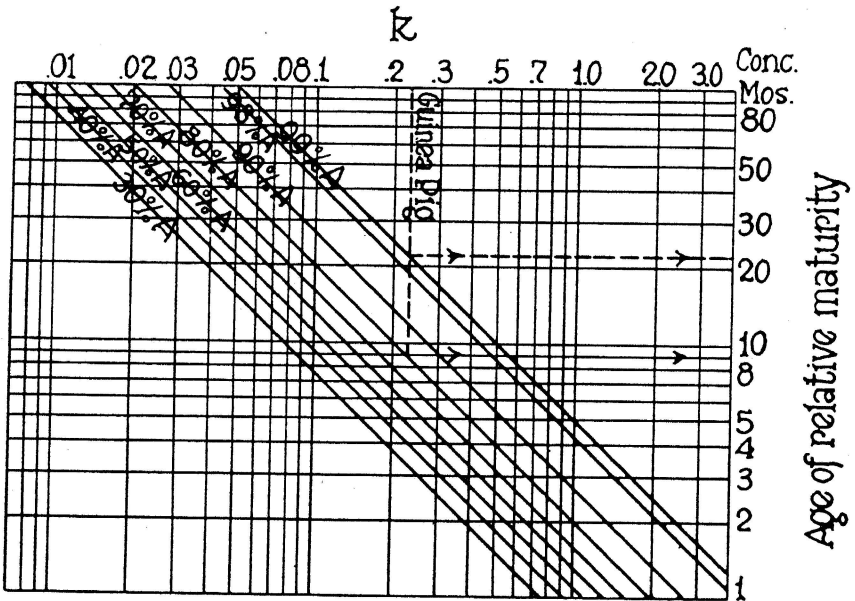


Fig. 4. (Supplementing Table IV).—The relation between the numerical value of k and the ages when different fractions of the mature weight are reached. Thus, the value of k for the guinea pig is .23 and from the chart it is easy to read that the guinea pig reaches 99 per cent A at 22 months, 80 per cent A at 9 months (from conception), and so on.

differences in the values of t^* make themselves felt in an increasing degree with decrease in age, since equation (1) does not represent the course of growth during the phase of growth preceding the inflection. These agreements and deviations are also shown graphically in Fig. 1a. Table IV is supplemented by Fig. 4 from which different fractions of the mature weight may be easily interpolated if the numerical value of k is known. (For errors involved *cf.* Fig. 1a.)

VII. SUMMARY AND CONCLUSIONS

The numerical values of the growth constants given in the preceding bulletin of this series are utilized in the present bulletin for determining equivalence of growth age and growth weight in several classes of domestic animals. For the sake of completeness, equivalence charts are also presented for growth curves of plants, and for populations of organisms.

Aside from the inherent interest attached to the method of determining equivalence of age described in the bulletin, the charts presented herewith bring out several facts which have considerable scientific interest and practical importance. One fact is that different breeds of the same species (e. g. rabbits) differing in mature weight by over 100 per cent may reach maturity at exactly the same age. A second fact is that different groups of animals of the same breed (e. g. albino rats) having nearly the same mature weight may differ by over 100 per cent with respect to the time at which the mature weight is reached. From the agricultural point of view next in importance to learning how to grow two blades of grass where one grew before, is to learn how to grow two blades of grass in the time formerly required to grow one blade. From Fig. 19, the Flemish rabbit produces over twice the weight of tissue during a given age interval that the polish rabbit produces. From Fig. 22, Hoskins' rats produced a given weight of tissue in about half the time of the other rats. These differences may be due to environmental or genetic factors. The whole problem of the relation between the mature weight, and the age at which a given fraction of the mature weight is reached in members of a given species needs to be investigated from both the genetic and nutritional points of view.

Another arresting fact brought out by these equivalence charts is that animals differ enormously with respect to the position of the major inflection (puberty) in the growth curve. In the pigeon, for example, the inflection occurs relatively very early in life—much earlier than in the rat. The position of the inflection in the curve of a given species is probably related to the position of the species in the scale of evolution. This idea will be substantiated by growth curves of man, which will be presented in a forthcoming bulletin.

Similar differences are observed with respect to the relative rapidities of approach to the mature weight. Considering the mature weight of the pigeon, it approaches the mature weight more rapidly than any other animal examined in this bulletin.

The equivalence charts for plants and animals (Fig. 25) and for populations (Fig. 26) serve to call attention in a concrete manner to the

essential similarity in the shape of growth curves of multicellular animals and plants and of populations of organisms. That there should be such a close similarity between the growth curves of animals, plants, and populations of plants and animals, is not surprising in view of the "atomic" nature of the growth process in all these cases, as explained in section IV of Research Bulletin 97 of this series.

SOURCES OF DATA AND BIBLIOGRAPHY

The equivalence charts presented in this bulletin have been constructed with the aid of Tables I and IA given in the preceding bulletin of this series (Missouri Agricultural Experiment Station Research Bulletin 101). The sources of data are given in these tables. The numerical data on growth made use of in this bulletin may be found either in the first bulletin of this series (Missouri Agricultural Experiment Station Research Bulletin 96) or in the references cited in the bibliography of that bulletin.

Figures 5 to 26.

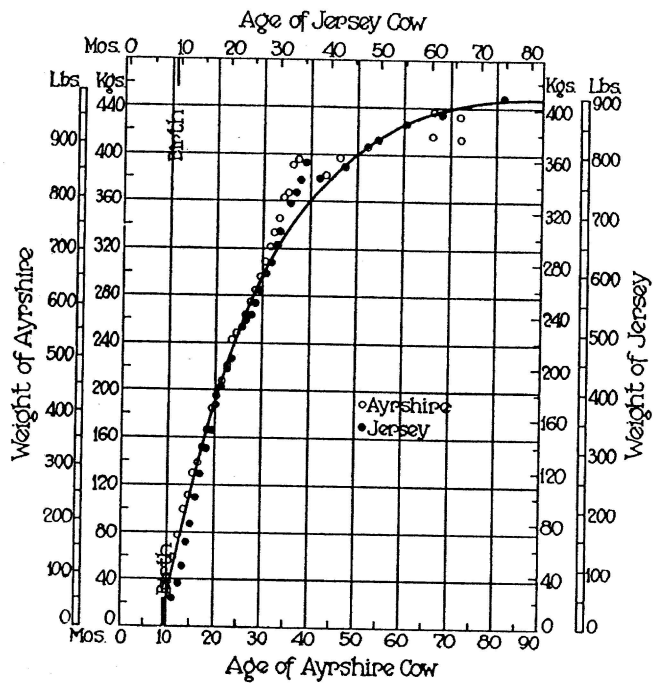


Fig. 5a.—Growth-equivalence of Jersey cow and Ayrshire cow. 1 month in the Jersey cow is equivalent to 1.08 months in the Ayrshire cow; and 1 kilo in the Jersey cow is equivalent to 1.1 kilos in the Ayrshire cow.

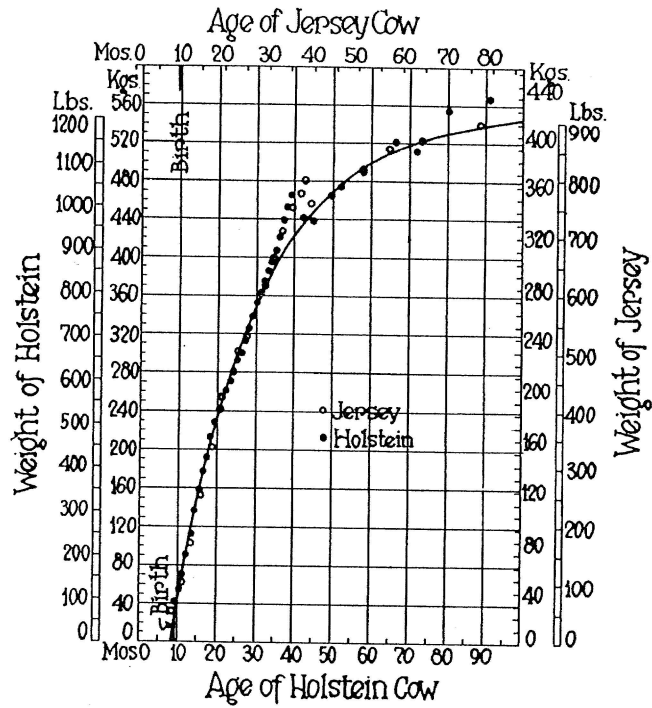


Fig. 5b.—Growth-equivalence of Jersey cow and Holstein cow. 1 month in the Jersey cow is equivalent to 1.17 months in the Holstein cow and 1 kilo in the Jersey cow is equivalent to 1.3 kilos in the Holstein cow.

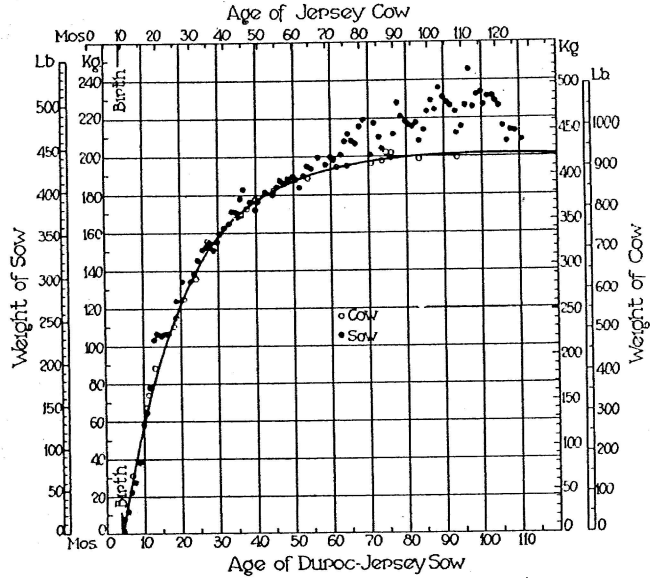


Fig. 6.—Growth-equivalence of Jersey cow (Eckles) and Duroc-Jersey sow (Mumford). 1 month in the sow is equivalent (after the point of inflection) to 1.15 months in the cow, and 1 kilo in the sow is equivalent to 2.1 kilos in the cow. The period before the point of inflection appears to be longer in the cow than in the sow. Since the data for swine were obtained on animals bred at unusually early ages, it is probable that growth was retarded, i. e., mature weight was approached less rapidly than it would under average conditions of management.

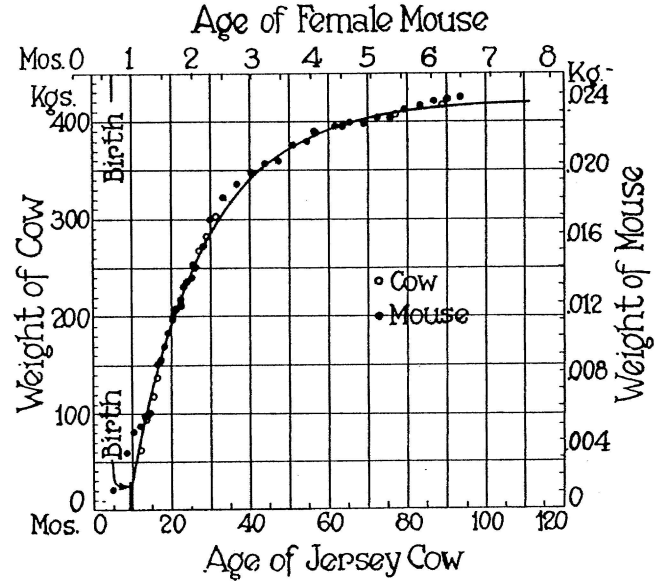


Fig. 7.—Growth-equivalence of Jersey cow and female white mouse. One month in the mouse is equivalent (after the point of inflection) to 15.29 months in the cow. One gm. in the mouse is equivalent to 2.076 gm. in the cow. The period of growth preceding the point of inflection is relatively longer in the mouse than in the cow.

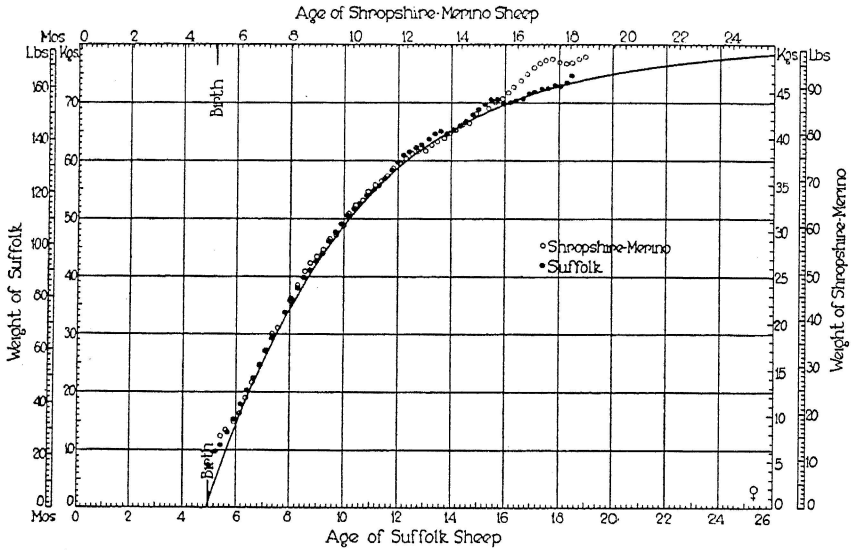


Fig. 8.—Growth-equivalence of Suffolk sheep and Shropshire-Merino sheep. A unit weight in the Shropshire-Merino sheep is equivalent to 1.6 units weight in the Suffolk sheep.

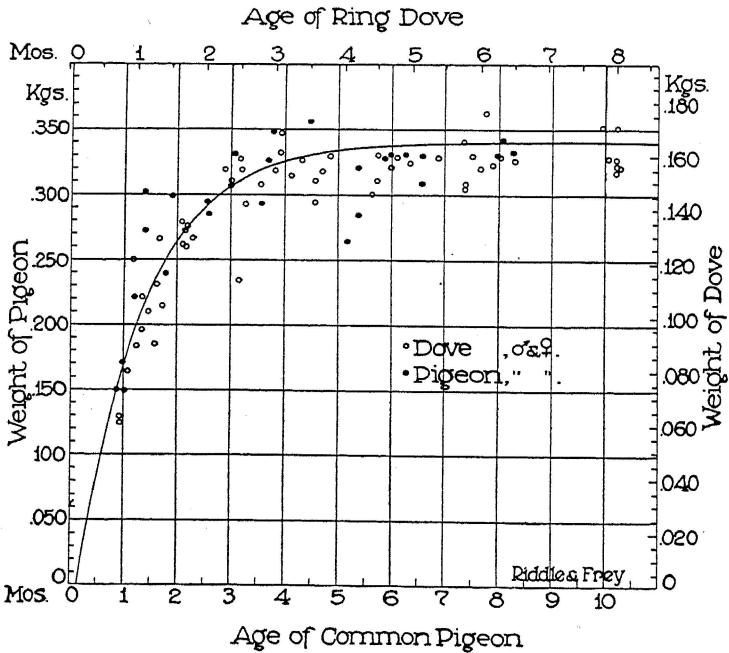


Fig. 9.—Growth equivalence between pigeon and dove.

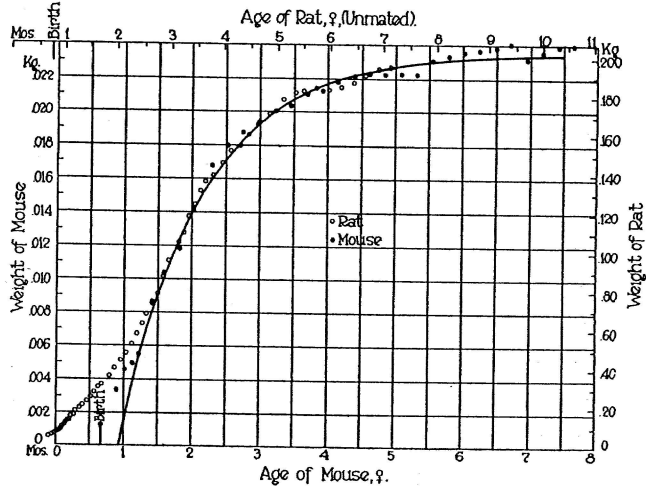


Fig. 10.—Growth-equivalence of unmated female white rat (Donaldson) and female mouse (Robertson). 1 month in the mouse is equivalent (after the point of inflection) to 1.27 months in the rat. 1 gm. in the mouse is equivalent to 8.64 gm. in the rat. The period preceding the point of inflection appears to be much longer in the rat than in the mouse, *i. e.*, the infantile and juvenile periods appear to be relatively longer in the rat than in the mouse.

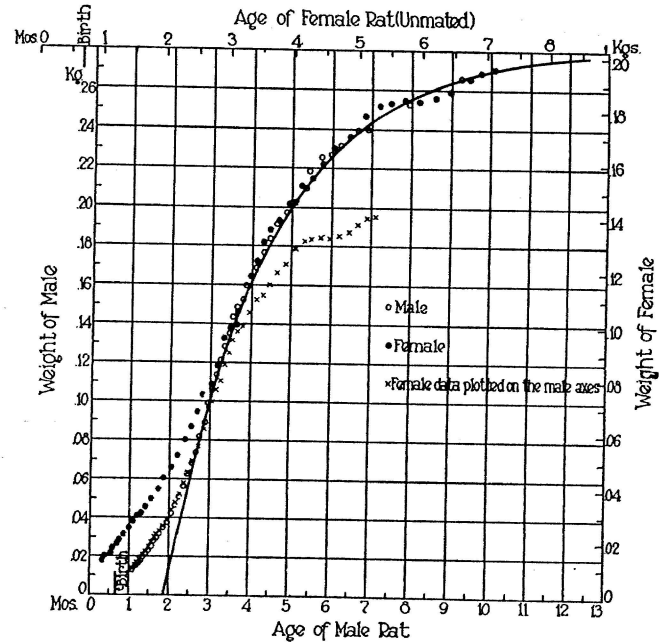


Fig. 11.—Growth-equivalence of male and female white rat of Donaldson, Dunn and Watson. After the point of inflection, 1 month in the female is equivalent to 1.61 months in the male. 1 gm. in the female is equivalent to 1.38 gm. in the male. The period of growth following the point of inflection is evidently longer in the male than in the female.

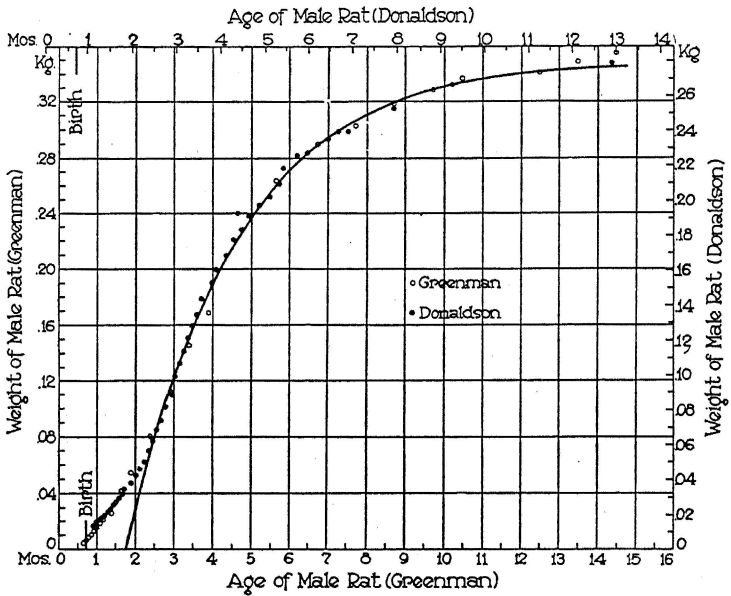


Fig. 12.—Growth-equivalence between the “specially well cared for and fed” albino male rats of Greenman and Duhring, and the male rat of Donaldson, Dunn and Watson. 1 month in the rat of Donaldson, Dunn and Watson is equivalent to 1.14 months in the rat of Greenman and Duhring; that is, the rat of Greenman and Duhring had a longer period of growth. 1 gm. in the rat of Donaldson, Dunn, and Watson is equivalent to 1.25 gm. in the rat of Greenman and Duhring.

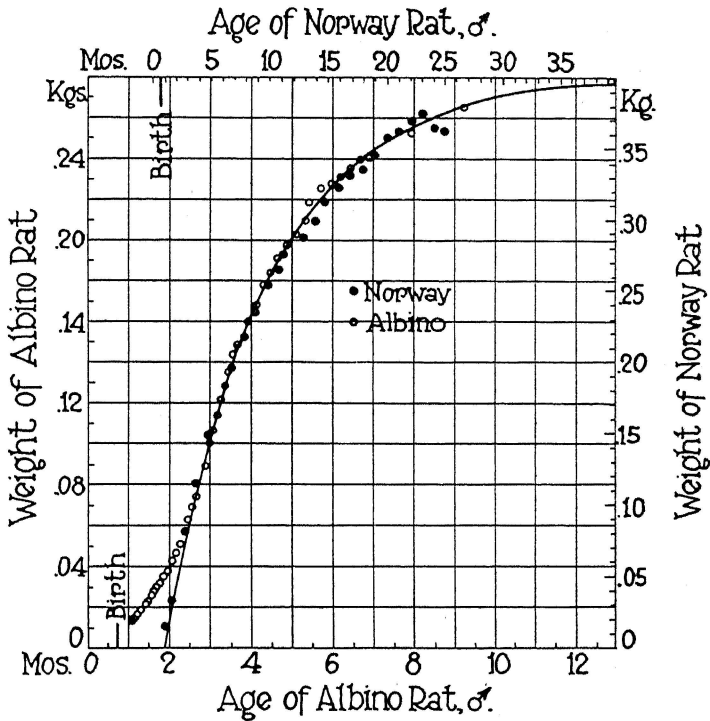


Fig. 13.—Growth-equivalence of male Norway rat (King) and male albino rat of Donaldson, Dunn, and Watson.

One month in the albino rat is equivalent (after the point of inflection) to 3.3 months in the Norway rat. This is a remarkable fact.

The question naturally suggests itself, what would be the numerical values of A and k in crosses between Norway and albino rats, or, more generally, how do the characters A and k behave in genetic operations; also, what is the relation between A and k —can these two characters be varied independently? The conception of k should give the geneticist a quantitative measure in the study of inheritance of the character rate of growth, and help him in developing rapidly growing animals of large size—if such development is physiologically possible.

Dr. Helen Dean King, who is investigating the change in shape of growth curves of Norway rats in successive generations grown under laboratory conditions, informs us that the curve of the Norway rat is approaching more nearly with each generation, the shape of the curve of the albino rat. This indicates that the shape of the curve is closely related to some environmental factors.

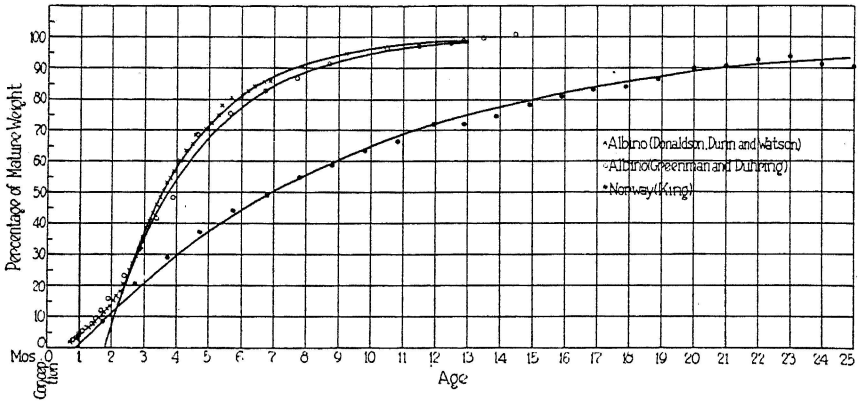


Fig. 14.—Growth curves of the two sets of male albino rats and male Norway rat. The body weights are represented as percentages of the mature weight. This chart is intended to indicate the possible effect of environment and domestication on the course of growth and on the shape of the growth curve of the rat.

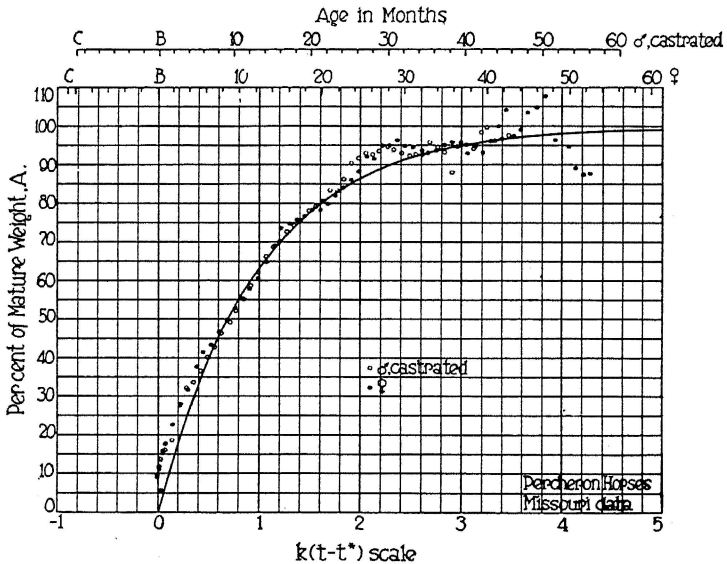


Fig. 15.—The growth of the horse plotted on a $k(t-t^*)$ grid. It appears that castration did not change the speed of approach to the mature weight. (Compare with Fig. 4 of the preceding bulletin—Research Bulletin 101).

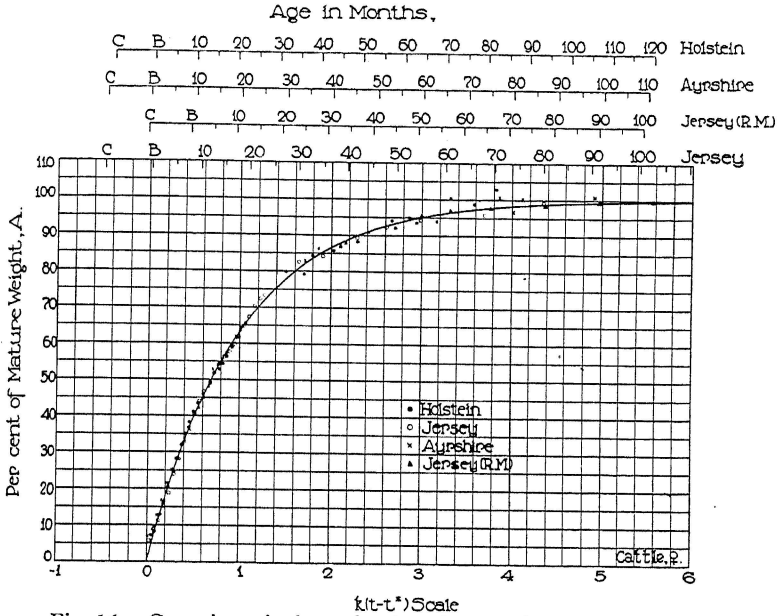


Fig. 16.—Growth-equivalence for three breeds of dairy cattle constructed with the aid of equivalent-time scales.

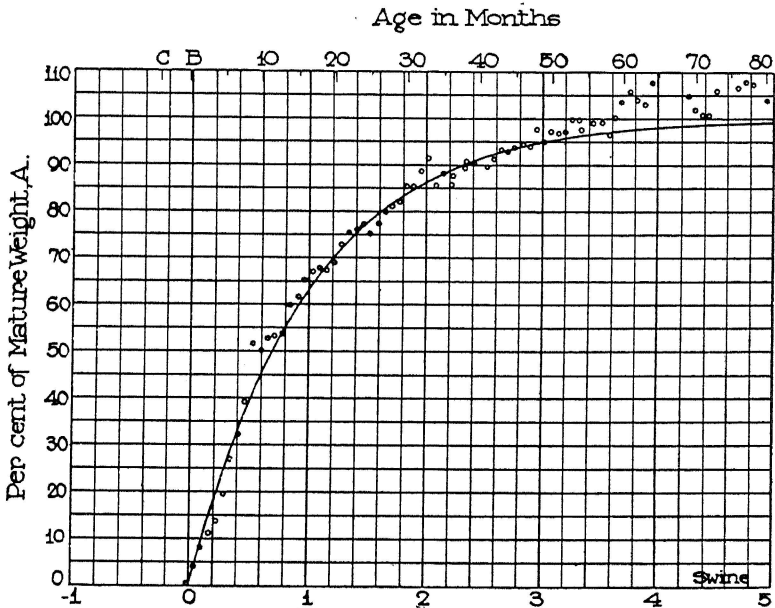


Fig. 17.—The growth curve of Duroc-Jersey swine plotted on an equivalent-age scale grid. It is necessary to recall the fact, pointed out in the preceding bulletin, that these data represent the averages of sows used in an investigation by Dean Mumford to determine the effect on growth of breeding immature animals, and that it is possible that animals kept under different conditions would approach the mature weight at a different rate.

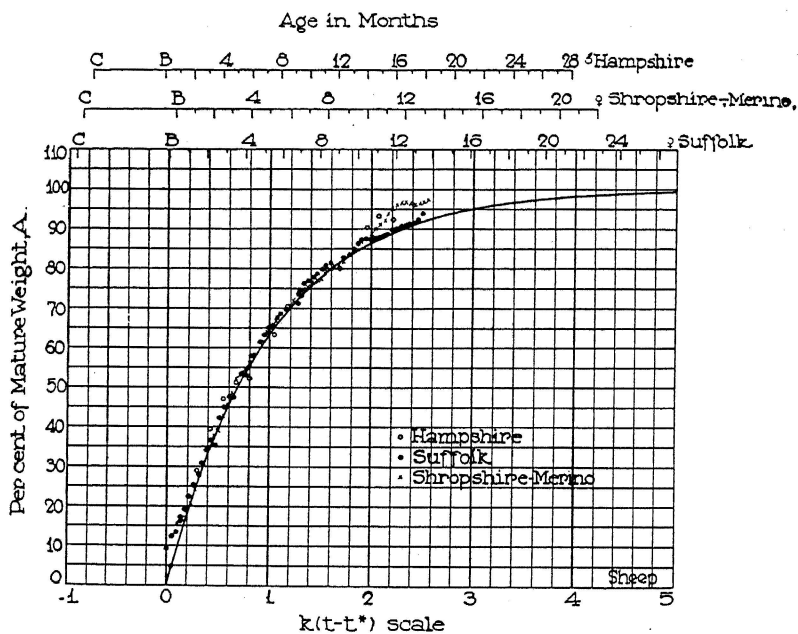


Fig. 18.—Growth curves of sheep on an age-equivalence grid. Hampshire males (Trowbridge et al, Missouri data) appear to approach the mature weight less rapidly than the Suffolk or Shropshire-Merino females (Murray, England).

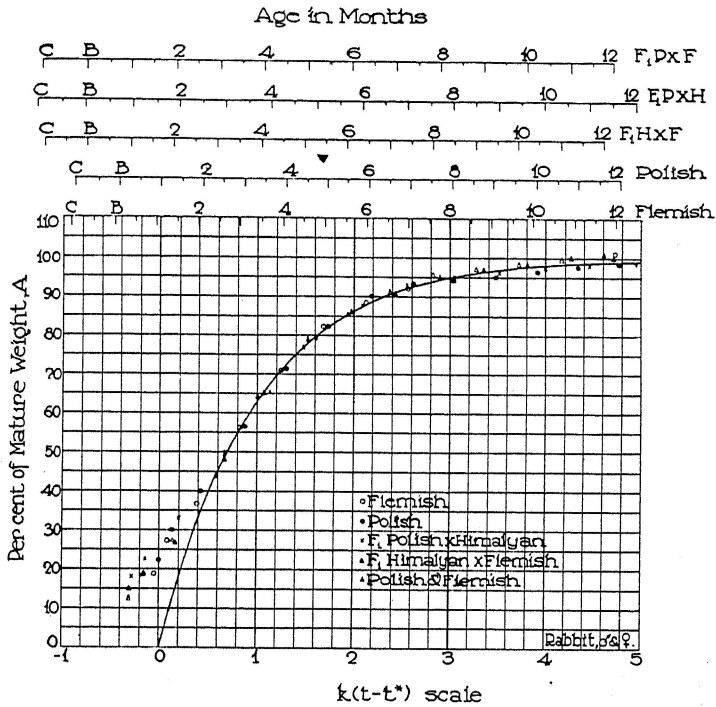


Fig. 19a.—Growth curves of the rabbit on an age-equivalence grid. All four breeds appear to approach the mature weight at the same rate. This is remarkable considering the difference in mature weight. The difference with respect to mature weight of Polish and Flemish rabbits is indicated in Fig. 19b. (Castle.)

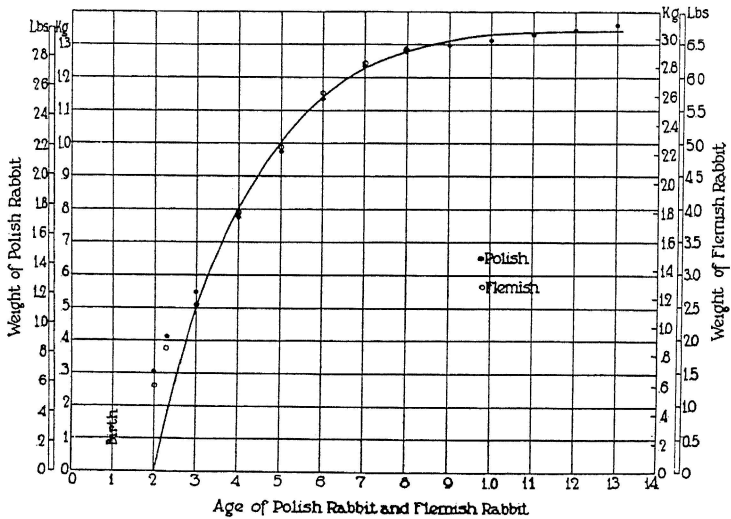


Fig. 19b.—Compare to Fig. 19a. Growth-equivalence between the Polish and Flemish rabbits. While the mature Flemish rabbit is over twice the size of the mature Polish rabbit, they both, nevertheless, reach mature weight at the same time. This is evidently a fact of considerable economic importance. In a given period of time the Flemish rabbit gains twice the weight of the Polish rabbit, and consequently, at least a portion of the cost of maintenance is saved in the production of a unit of weight in the Flemish rabbit as compared to that in the Polish rabbit. (Castle.)

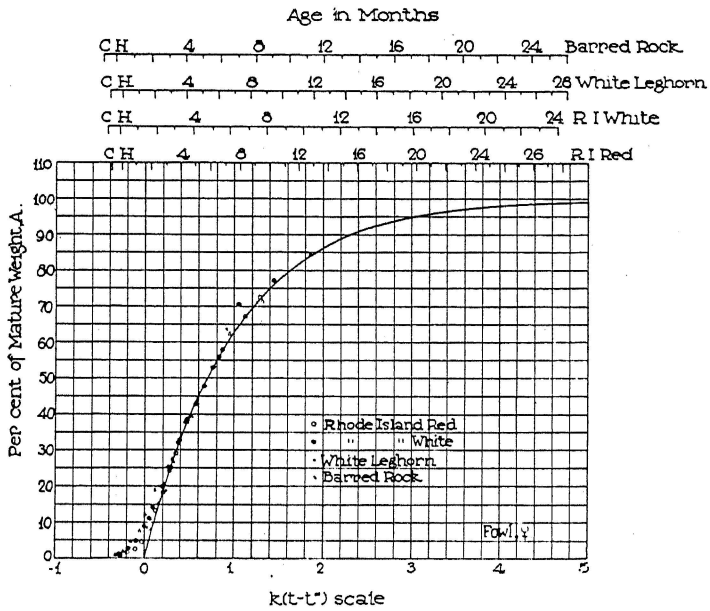


Fig. 20a.—Growth-curves of the domestic fowl on an age-equivalence grid. Females.

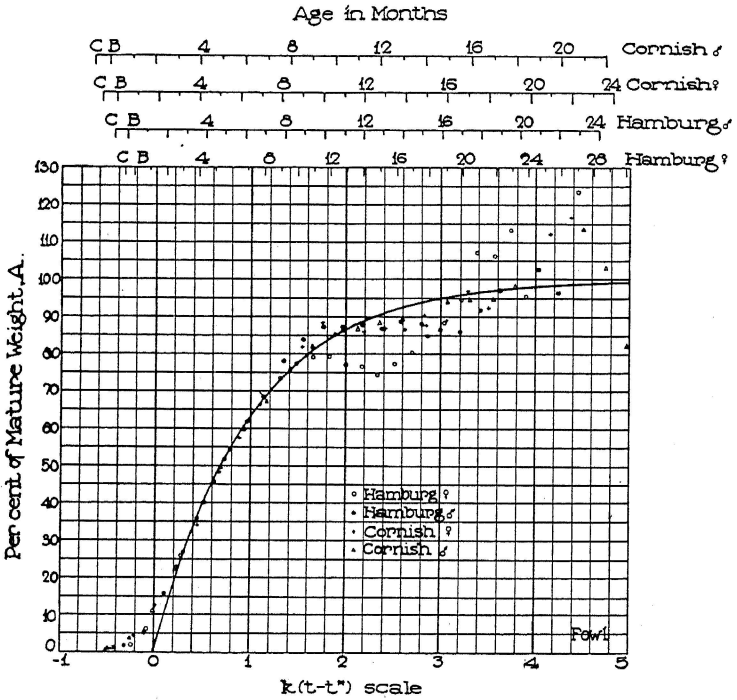


Fig. 20b.—Additional curves for the domestic fowl, males and females. It appears from this chart that females have a longer growing period than males. This may be an expression of the retarding effect of ovulation on the growth of the female. (May.)

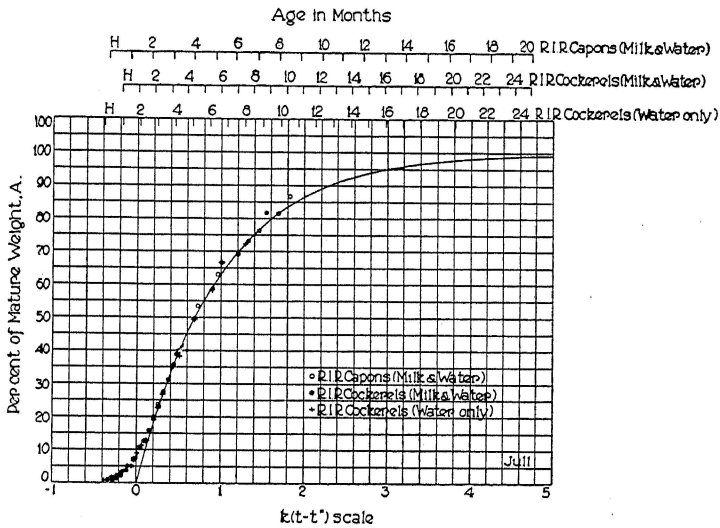


Fig. 20c.—Curves for the domestic fowl, males and castrated males (capons). This chart appears to indicate that capons have a shorter period of growth than normal males. This fact, if a fact, is of considerable physiological significance. Under the given conditions the addition of milk to the ration did not appear to accelerate the approach to the mature weight.

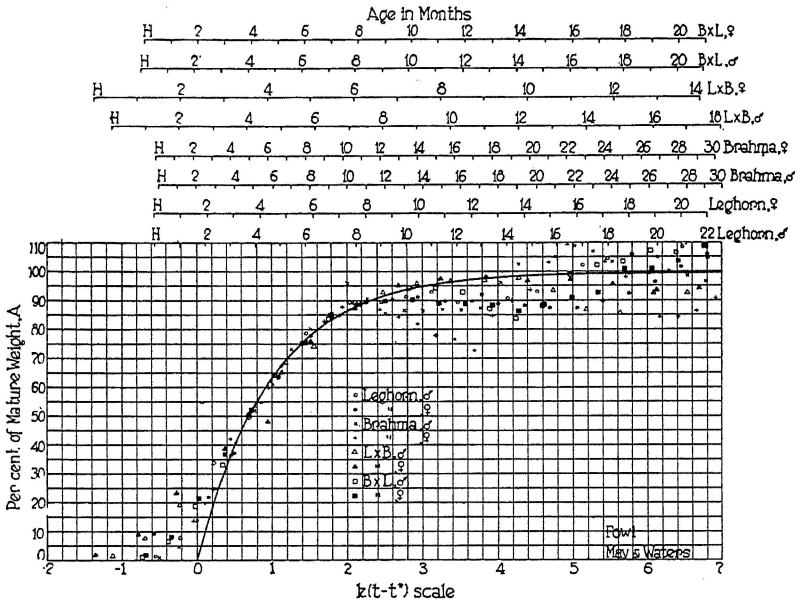


Fig. 20d.—Additional curves for the domestic fowl including curves for Brahmas, for Leghorns, and for first-generation weight crosses of these two breeds. The small Leghorn (mature weight 2.2 kilograms) evidently approaches the mature weight more rapidly than the larger Brahma (mature weight 4.1 kilograms). The offspring from the Leghorn males appear to approach the mature weight more rapidly than the offspring from Brahma males. (The mature weights of B X L females, is 2,450 grams; of L X B females, is 3280 grams; of B X L males, is 2140 grams, and of LXB males, 2270 grams.) (May and Waters.)

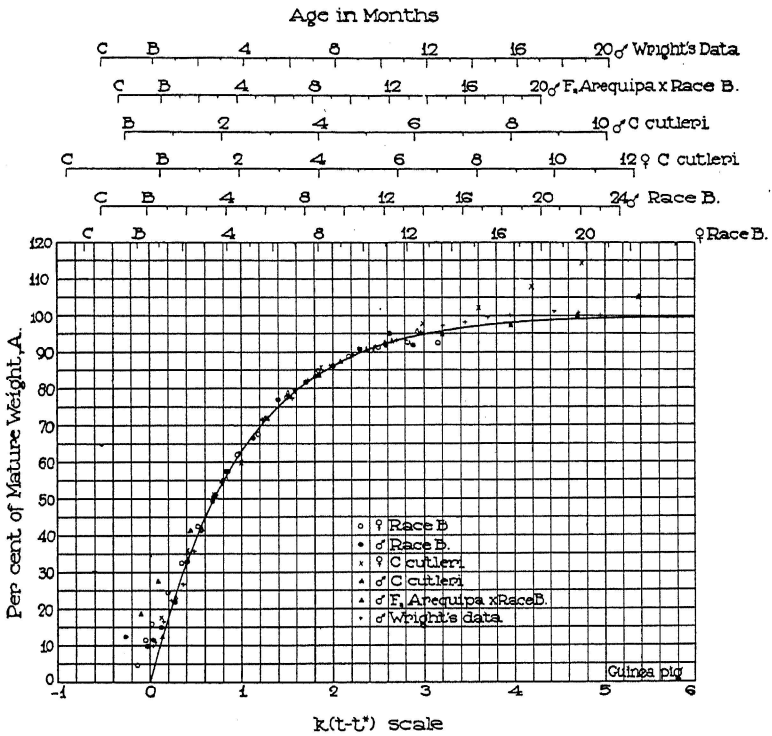


Fig. 21a.—Growth curves of various breeds and crosses of guinea pigs plotted on an age-equivalence grid are of considerable interest in indicating the enormous differences with respect to the time required to reach maturity. Without exception, the larger the animal the longer the time required to reach a given fraction of the mature weight. The question suggests itself, "is there a functional relation between the mature weight and the time required to reach a given fraction of the mature weight?" (Castle.)

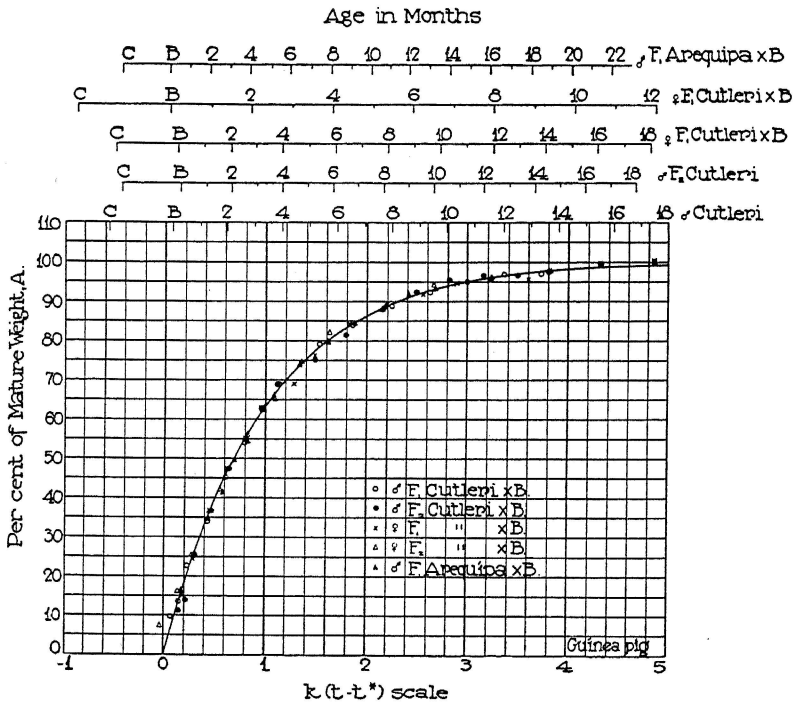


Fig. 21b.—Additional curves for the guinea pig. *C. Cutleri* approaches the mature weight twice as rapidly as Race B, or Wright's animals. There is relatively little difference between the male and female series. In the case of Race B the males have a somewhat longer growing period. In the case of *Cutleri*, the females appear to have a longer growing period. But their differences may be within the limits of computation errors. (Castle.)

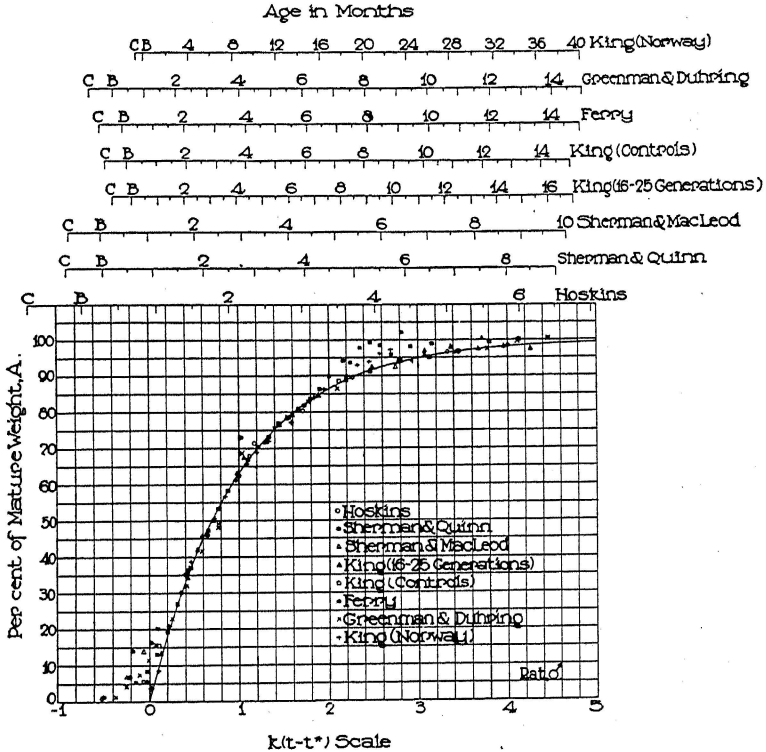


Fig. 22a.—Growth curves of the rat on an age-equivalence grid, males. The most important fact to note relates to the relative times required to reach a given fraction of the mature weight in the different groups of animals. Thus, 6 months in the rat of Hoskins corresponds to 13 months in the rat of King, Ferry, or Greenman and Duhring; and to 36 months for the F_1 Norway rat! Are these differences due to genetic or environmental factors? These are questions of the greatest theoretical interest and practical importance.

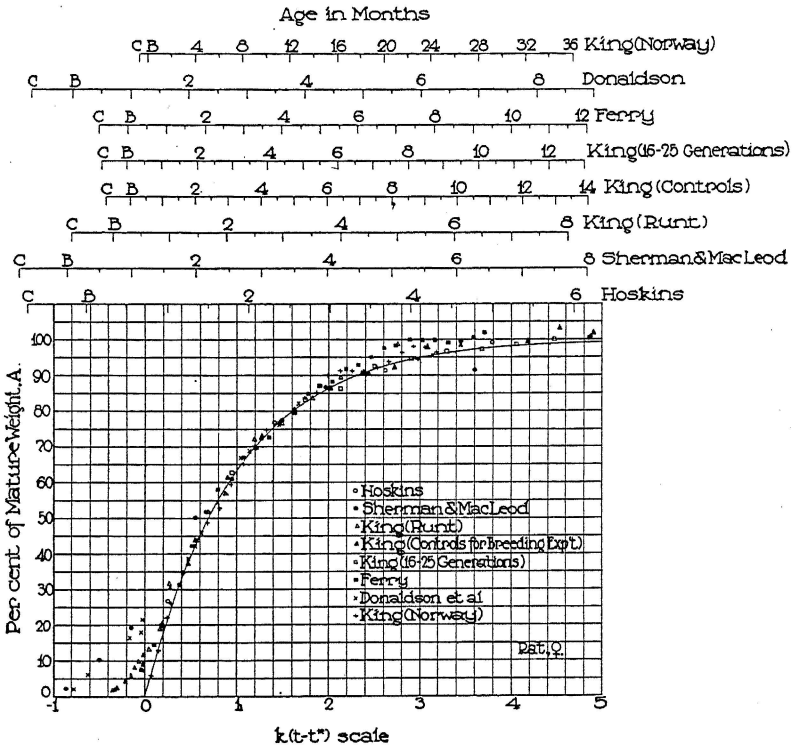


Fig. 22b.—Growth of the rat, females. The differences in time required to reach a given fraction of the mature weight are as great in this as in the preceding chart.

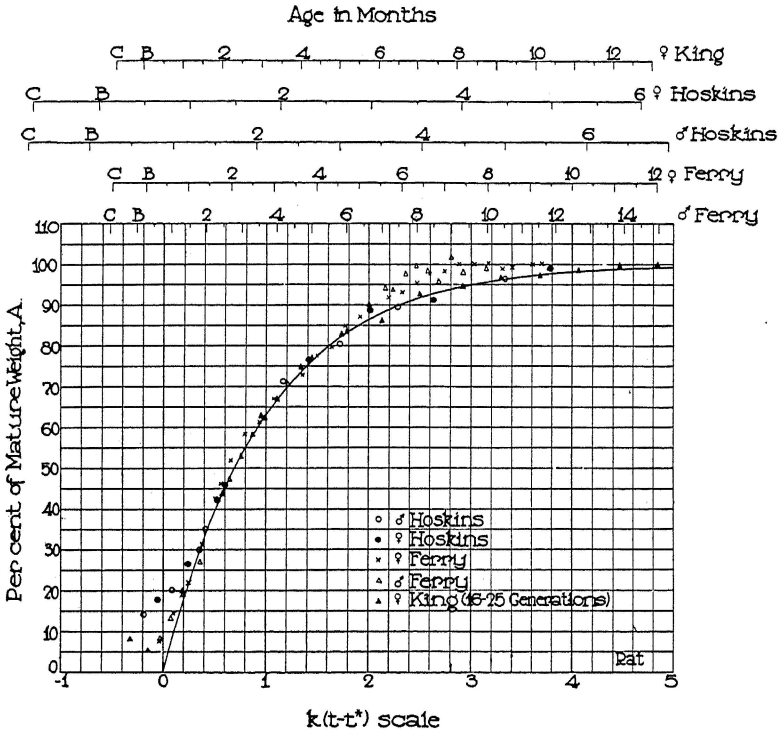


Fig. 22c. Growth of the rat, male and female. The period of growth is probably longer in the male (see also Fig. 11).

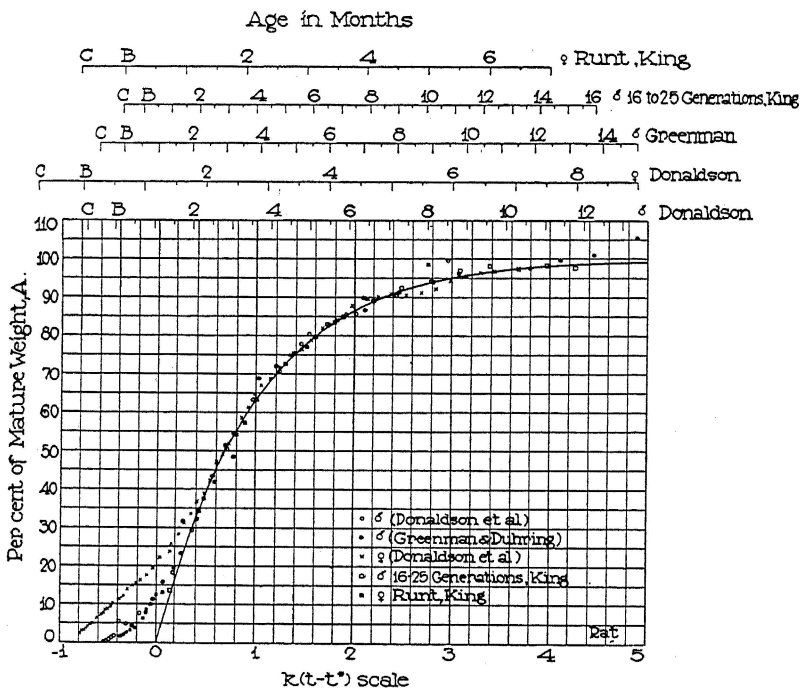


Fig. 22d.—A comparison of the growth curves of the rats of Donaldson Dunn, and Watson; of the specially well cared for rats of Greenman and Duhring; of the rats of King inbred for 16-25 generations and of a "runt".

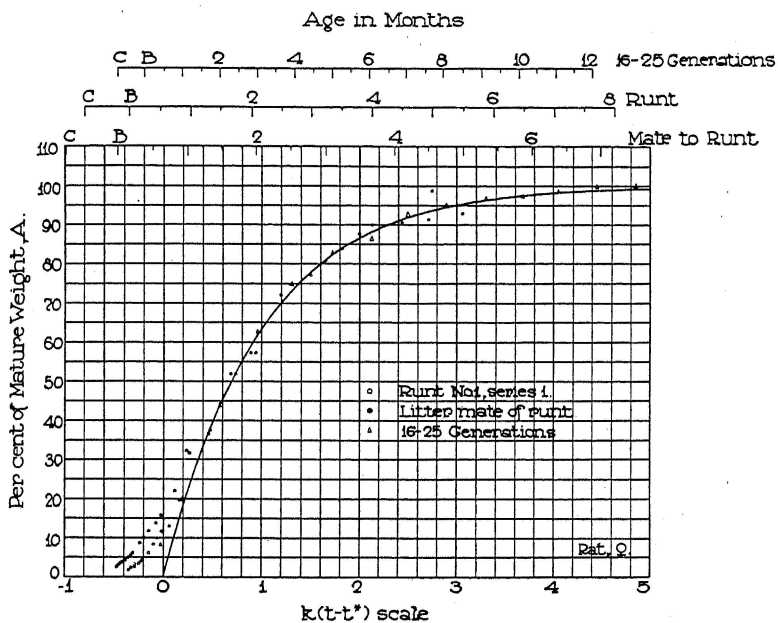


Fig. 22e.—Growth of the rat, normal and “runt” animals. The “runt” and its mate appear to approach the mature weight at nearly the same rate. (King.)

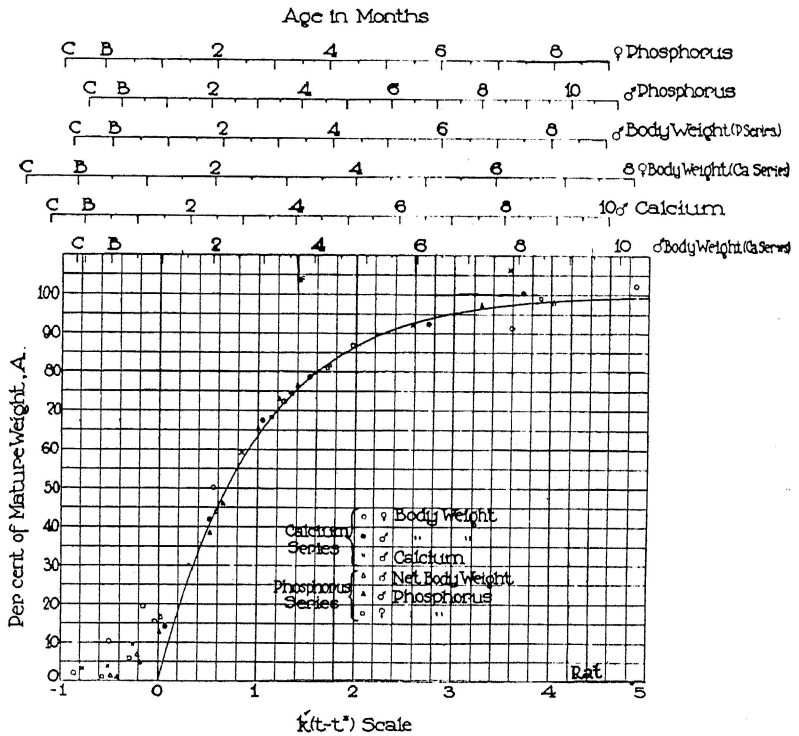


Fig. 22f.—Growth with respect to the body weight and with respect to calcium and phosphorus. The females evidently reach the maximum with respect to phosphorus sooner than the males. It requires a longer interval to reach a given fraction with respect to the maximum net body weight than with respect to the phosphorus. The calcium and body weight, however, appear to approach the mature values at the same rate. (Sherman.)

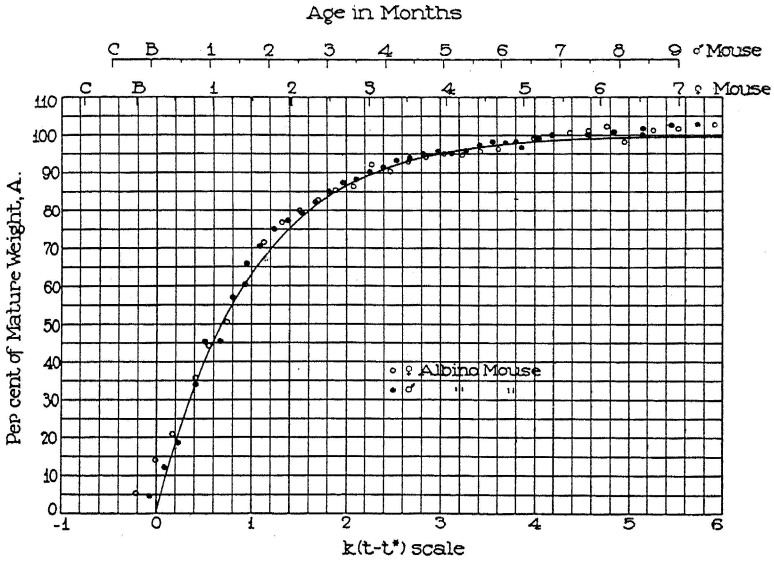


Fig. 23.—Growth of the albino mouse plotted on equivalent-time scales. The male appears to have a longer period of growth than the female. (Robertson.)

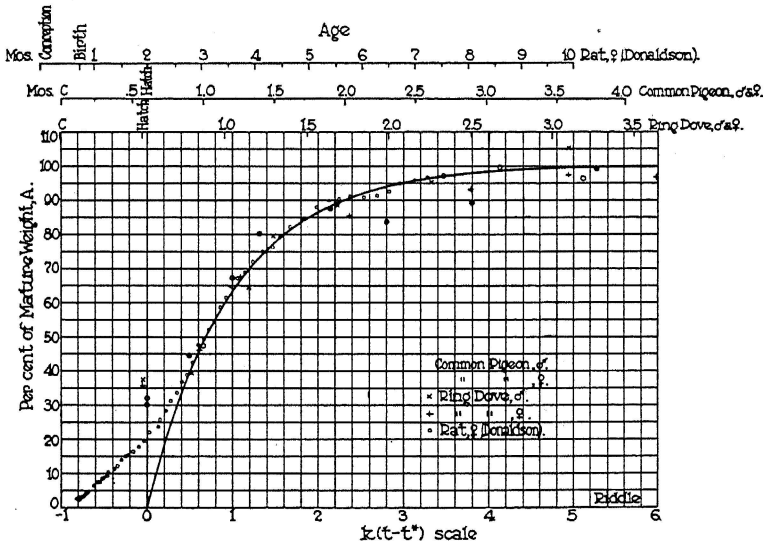


Fig. 24.—Growth of the pigeon and dove plotted on equivalent-time scales.

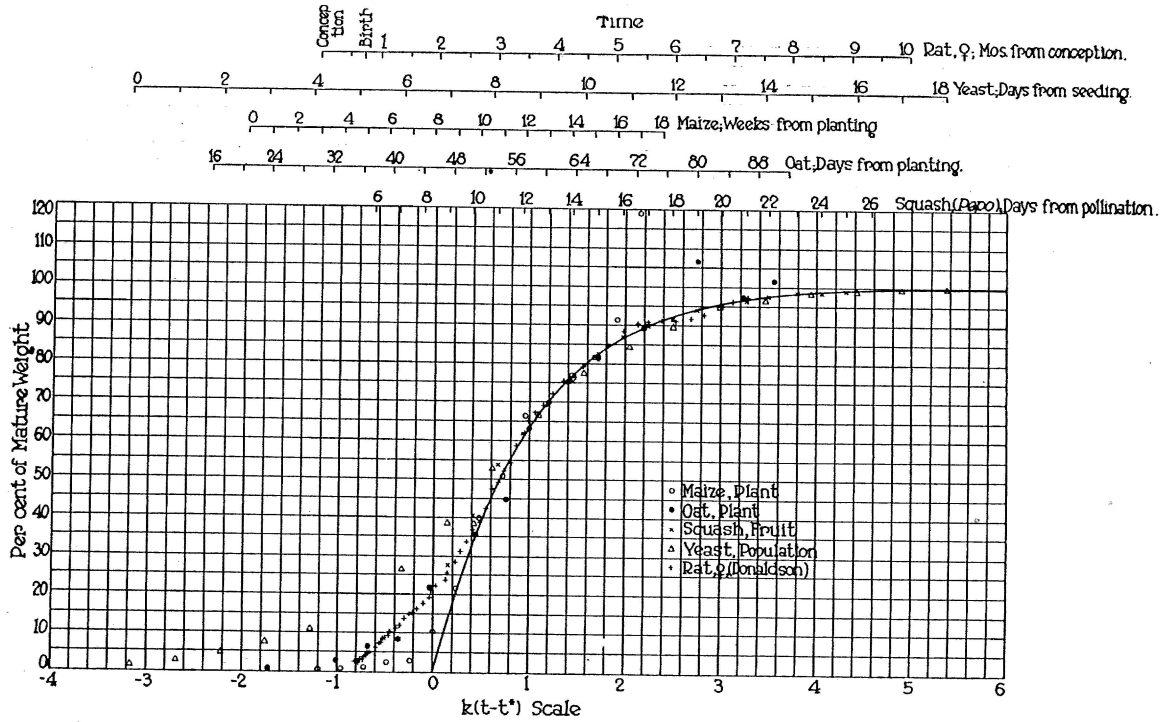


Fig. 25.—Growth data for plants plotted on equivalent-time scales. For the purpose of comparison, the growth curve of an animal (rat), and of a population (yeast), is also included.

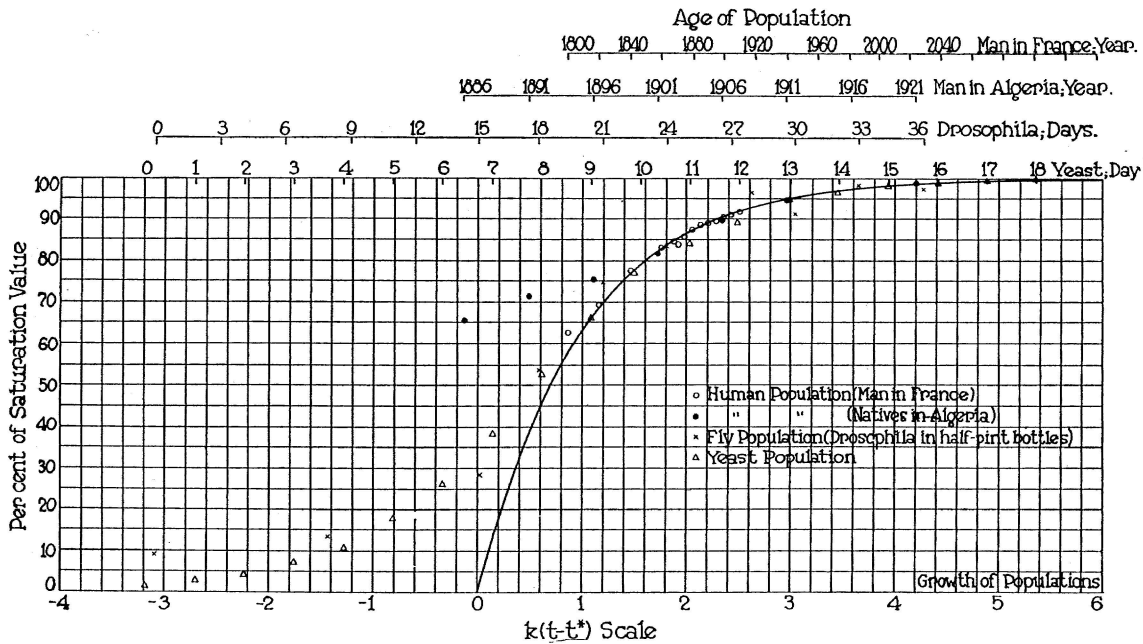


Fig. 26.—Growth of several types of populations represented on a growth-equivalence grid.