

ESSAYS ON MONETARY ECONOMICS

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by
JIAN SONG
Dr. Chao Gu, Dissertation Supervisor

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The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled

ESSAYS ON MONETARY ECONOMICS

presented by Jian Song,

a candidate for the degree of doctor of philosophy,

and hereby certify that, in their opinion, it is worthy of acceptance.

Professor Chao Gu

Professor Joseph Haslag

Professor Aaron Hedlund

Professor Rui Yao

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ABSTRACT

This dissertation consists of two essays on monetary economics. The first essay studies a monetary model in which the lack of repayment enforcement causes the agent's credit limit to bind. When the credit constraint binds, the allocation is usually not efficient. I show that government bonds are candidates to alleviate this problem. I also show that liquid bonds, as a perfect substitute to fiat money, can only change the equilibrium price of good markets but cannot change the allocation. On the other hand, illiquid bonds can help improve social welfare.

The second essay studies a pure credit model with productivity shocks. When record keeping and information are perfect, the existence of defaulters has no effect on other agents' consumption and output. But, if access to the agent's trading history is limited, the existence of defaulters can change other agents' allocation because sellers will charge all the buyers a default premium. The optimal punishment in a narrow class of credit contracts is also calculated.

Chapter 1

Limited Commitment and Illiquid Bonds

1.1 Introduction

Previous research has shown that loan markets aggregating consumption risk and reallocating money across agents can lower the holding cost of money and improve the allocation away from the Friedman rule¹ (see Berentsen et al. 2007). However, a common concern in the loan market is that the commitment to the future repayment is limited, which implies a finite credit limit. When the credit constraint binds, the allocation is usually inefficient.² Given this inefficiency, how can we recapture the benefit of reallocating currency across different types of agents?

To address this question, I introduce government bonds into a New Monetarist model.³ For analyzing the role of government bonds, it is helpful to distinguish

¹The Friedman rule is optimal in a wide class of models. But, in the real world, zero interest rate policies are not common. Many studies have been done to explain this phenomenon. It could be because of the existence of the maintenance cost of cash, theft issues, government's limited coercive power on tax, and so on. See Bhattacharya et al. (2009), He et al. (2008), Sanches and Williamson (2010), Andolfatto (2013), and Hu et al. (2009). In this paper, I assume a zero interest rate is not attainable.

²Kocherlakota (1998) shows that limited commitment implies less intertemporal exchange than what would occur in its absence.

³Recent surveys on this literature include Williamson and Wright (2010), Nosal and Rocheteau (2011)

illiquid bonds from liquid bonds. Illiquid bonds cannot be used in a good market as a means of payment, while liquid bonds can be used to purchase goods directly like fiat money. I show that liquid bonds, as a perfect substitute to fiat money, are redundant: they can only change the nominal price in a good market but cannot change the allocation. On the other hand, illiquid bonds are less flexible than liquid bonds. But it also means agents can trade illiquid bonds for money when they need to adjust their liquidity level. Moreover, bonds trade is *quid pro quo*, so it is not subject to the limited commitment problem. Because of these features, illiquid bonds provide us another channel to reallocate currency besides the loan market, which in turn improves social welfare.

This result is consistent with Kocherlakota (2003), where he studies the essential role of nominal bonds to enable agents to engage in intertemporal exchanges of money. But, different from Kocherlakota (2003), I show that illiquid bonds become essential only when an agent's borrowing constraint binds. A similar result can be found in Woodford (1990), where he argues that in an economy with borrowing-constrained agents, the government should issue a positive number of public debts to generate an efficient outcome.

There is a rich body of literature addressing the social benefits of government bonds. Diamond (1965) examines the effects of government debt on the long-run equilibrium in a growth model. Aiyagari et al. (1996) introduce indivisible money and bonds into a search model to explain why their coexistence can provide social benefits. Kocherlakota (2003), Boel and Camera (2006), Sun (2007), Shi (2008), and Andolfatto (2011) study the efficient rule of bonds when agents are subject to idiosyncratic shocks.

The baseline model used in this paper is inspired from the work of Berentsen et al. (2014).

al. (2007), which builds on Lagos and Wright (2005). A key feature of my model is that people meet bilaterally in a financial market where they trade bonds, borrow, and lend. In Berentsen et al. (2007), liquidity is reallocated by banks, which are perfectly competitive with free entry. So, when a borrower's borrowing constraint binds, banks can only adjust the interest rate to make sure borrowers will repay their debt. However, if the borrowing constraint is so tight that even under a zero nominal interest rate borrowers prefer to default, then the whole banking sector has to shut down. In other words, the banking sector is only active for a limited range of parameters. In my model, as the meetings in the financial market are bilateral, even if the borrowing constraint is tight, lenders can lend out part of their money, and equilibrium exists.

The rest of this paper is organized as follows. In Section 1.2, I set up the environment of the benchmark model. Section 1.3 studies the equilibrium with an exogenous debt limit. Section 1.4 endogenizes this debt limit. Section 1.5 shows the essential role of nominal bonds when borrowing constraint binds, and Section 1.6 concludes.

1.2 The Environment

The basic framework used in this paper is developed in Berentsen et al. (2007) and Nosal and Rocheteau (2011), where the role of the financial market is to reallocate currency between different types of agents.

There is a $[0, 1]$ continuum of agents living forever in discrete time with a discount factor $\beta \in (0, 1)$ between every time period t . In each period, agents meet sequentially in a financial market (FM), a decentralized good market (DM), and a centralized good market (CM). Good market DM and CM are assumed to be perfectly competitive. Perishable and perfectly divisible consumption goods are only produced in each good market; i.e., DM and CM.

At the very beginning of each period, agents get a preference shock that they can either consume or produce in the DM with equal probability. We refer to the agents who can only produce as sellers, denoted by s , and the rest who can only consume as buyers, denoted by b . Buyers can get utility $u(q)$ from consuming q units of good in the DM, where $u'(q) > 0$, $u''(q) \leq 0$, $u'(0) = +\infty$ and $u'(+\infty) = 0$. Sellers can produce q units of output with a cost function $c(q)$, where $c'(q) > 0$, $c''(q) \geq 0$ and $c(0) = u(0) = 0$. The DM trade is anonymous, so credit is not feasible. Consequently, a tangible medium of exchange is essential. Usually, fiat money serves this role.

In the CM, all agents consume and produce the same good. They can get utility $v(x)$ from x consumption, where $v'(x) > 0$, $v''(x) < 0$, $v'(0) = +\infty$ and $v'(+\infty) = 0$. Following Lagos and Wright (2005), x is produced one-for-one using labor H , and the CM utility function is $v(x) - H$. This quasi-linear utility function is commonly used in this kind of model because it is helpful to eliminate wealth effects and get analytic results. The period utility functions of buyers $\mathcal{U}^b(\cdot)$, and sellers $\mathcal{U}^s(\cdot)$ are, respectively

$$\mathcal{U}^b(q, x, H) = u(q) + v(x) - H$$

and

$$\mathcal{U}^s(q, x, H) = -c(q) + v(x) - H.$$

There is a government, which is the sole issuer of fiat money. The growth rate of the money stock γ is defined as $M_t = \gamma M_{t-1}$, where M_t and M_{t-1} are the total money stocks in period t and period $t - 1$, respectively. I also assume $\gamma > \beta$, which means the money supply is always above the Friedman rule. This assumption is necessary for the existence of a monetary steady state.

A financial market opens at the beginning of each period after agents' preference shocks are realized and it closes before agents go to the DM. In the FM, agents can buy and sell loans. Specifically, buyers and sellers meet bilaterally and every agent can get matched with probability one, which means there is no search friction in the

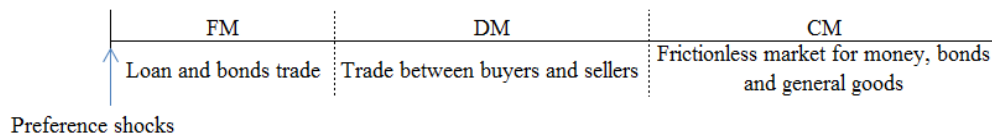


Figure 1-1: Timing

FM.⁴ Buyers borrow money and promise to repay in the subsequent CM. However, borrowers cannot be forced to repay their debt. So, lenders receive repayment in the CM only if borrowers are willing to do so by themselves.

Different from the DM, there is a public record keeping technology with zero cost in the FM. With this technology, we can punish the borrowers if they default on their debt. As in Kehoe and Levine (1993), the nature of punishment determines the limits on debts. In this paper, the punishment to defaulters is assumed to be banishing them from the financial market forever. The timing of the three markets and the corresponding events in a representative period is illustrated in Figure 1-1.

1.3 Equilibrium with Exogenous Debt Limits

I first study the equilibrium with an exogenous debt limit. The debt limit is endogenized in Section 1.4. To solve this model, I work backward from CM to FM. I focus on stationary equilibrium, so the value functions and real balances are time invariant. Variables corresponding to the next period and to the previous period are indexed by $+1$ and -1 , respectively.

⁴This setting can be extended to include search friction.

1.3.1 The CM problem

Let $W(m, \ell)$ be the value function for a representative agent at the beginning of the CM with m units of money and ℓ units of loan. If $\ell < 0$, the agent is a creditor. Otherwise, if $\ell > 0$, the agent is a debtor.⁵ For a representative agent, the CM problem is

$$W(m, \ell) = \max_{m_{+1}, x, H} [v(x) - H + \beta U(m_{+1})] \quad (1.1)$$

$$\text{s.t. } x + \phi m_{+1} = H + T + \phi [m - (1 + i)\ell] \quad (1.2)$$

where $U(\cdot)$ is the expected value function at the beginning of the next period before the preference shock is realized, i is the nominal interest rate on the loan, m is the money taken into the CM, m_{+1} is the money taken out of the CM, T is a lump-sum transfer if $T > 0$, or a lump-sum tax if $T < 0$, and ϕ is the real price of money measured in CM good. Substituting the budget constraint (1.2) into value function (1.1) to get rid of H yields

$$W(m, \ell) = \max_{m_{+1}, x} [v(x) - x - \phi m_{+1} + \beta U(m_{+1})] + T + \phi [m - (1 + i)\ell]. \quad (1.3)$$

The first order conditions with respect to x and m_{+1} are, respectively

$$v'(x) = 1 \quad (1.4)$$

and

$$\phi = \beta' U(m_{+1}). \quad (1.5)$$

From (1.4) we have the optimal quantity of CM consumption x^* , defined by $v'(x^*) = 1$. The left-hand side of (1.5) is the marginal cost of collecting one extra unit of money in the CM during this period, which is measured in CM good. The

⁵As will be shown in Section 1.3.3, buyers are liquidity constrained so they would like to borrow some money from sellers who cannot spend their money in the DM. So, buyers will become debtors and sellers will become creditors in the financial market.

right-hand side of (1.5) is the marginal utility of taking one unit of money to the next period. The envelope conditions are

$$\frac{\partial W(m, \ell)}{\partial m} = \phi \quad (1.6)$$

and

$$\frac{\partial W(m, \ell)}{\partial \ell} = -\phi(1 + i). \quad (1.7)$$

The envelope conditions imply that $W(\cdot)$ is linear in both m and ℓ with slopes ϕ and $-\phi(1 + i)$, respectively.

1.3.2 The DM problem

Consider now the DM. Let $V^b(\cdot)$ and $V^s(\cdot)$ be the buyer's and seller's DM value functions, respectively. A seller's choice problem in the DM is given by

$$V^s(m_\ell, \ell) = \max_{q_s} [-c(q_s) + W(m_\ell + pq_s, \ell)]$$

where $m_\ell = m + \ell$ is the amount of money taken out of the FM, p is the nominal market price of DM good which is taken as given, pq_s is the amount of money sellers can collect in the DM by selling q_s units of DM good, and $c(q_s)$ is the cost of producing q_s units of DM good.

The first order condition implies

$$-c'(q_s) + p \frac{\partial W(m, \ell)}{\partial m} = 0.$$

Using the envelope condition (1.6), it can be rewritten as

$$c'(q_s) = \phi p. \quad (1.8)$$

The left-hand side of (1.8) is the marginal cost of producing one additional unit of DM good, and the right-hand side of (1.8) is the marginal benefit they can get in the

subsequent CM by producing one additional unit of good in the DM. A buyer's DM problem is

$$V^b(m_\ell, \ell) = \max_{q_b} [u(q_b) + W(m_\ell - pq_b, \ell)] \quad (1.9)$$

$$\text{s.t. } pq_b \leq m_\ell \quad (1.10)$$

where q_b is the amount of DM good a buyer consumes in the DM. Suppose that the liquidity constraint (1.10) does not bind, the first order condition is

$$u'(q_b) - p \frac{\partial W(m, \ell)}{\partial m} = 0.$$

Combining with (1.6), (1.8), and good market clear condition $q_b = q_s$, we have the unconstrained efficient quantity q^* defined by $u'(q^*) = c'(q^*)$. Otherwise, if the buyer's liquidity constraint (1.10) binds, we have $q_b = m_\ell/p$. To summarize,

$$q_b = \begin{cases} q^* & \text{if } m_\ell \geq pq^* \\ m_\ell/p & \text{if } m_\ell < pq^*. \end{cases}$$

1.3.3 The FM problem

In the FM, the loan contract is assumed to be determined by the Kalai bargaining solution in which the buyer has bargaining power θ and the seller has bargaining power $1 - \theta$. If no agreement is reached in the FM, both buyer and seller enter the DM with m units of money. Then, in the DM, the buyer purchases q^m units of DM good where

$$q^m = \begin{cases} q^* & \text{if } m \geq pq^* \\ m/p & \text{if } m < pq^*. \end{cases}$$

The seller produces q_s units of good regardless of whether the loan is lent out or not. Therefore, the threat points for buyer and seller are, respectively, given by

$$V_0^b(m, 0) = u(q^m) + W(m - pq^m, 0)$$

and

$$V_0^s(m, 0) = -c(q_s) + W(m + pq_s, 0).$$

The total surplus from the financial market is $S^\ell = u(q_b) - u(q^m)$ and each agent receives a constant share of this surplus; i.e., $S_b = \theta S^\ell$ and $S_s = (1 - \theta)S^\ell$, where S_b and S_s denote buyer's and seller's surplus, respectively. Buyers receive the surplus by consuming more good in the DM net the cost of the loan. On the other hand, sellers receive their surplus in terms of interest payment from buyers. So, it is easy to show $S_b = u(q_b) - u(q^m) - i\phi\ell$ and $S_s = i\phi\ell$. The bargaining problem solves

$$(\ell, i) = \arg \max_{i, \ell \leq \min\{m, \bar{\ell}\}} [u(q_b) - u(q^m)] \quad (1.11)$$

$$\text{s.t. } i\phi\ell = (1 - \theta)[u(q_b) - u(q^m)] \quad (1.12)$$

where $\bar{\ell}$ is the limit of the loan, which is taken as given for now. In Section 1.4, I will show that it is determined endogenously in equilibrium. The expected utility of a representative agent with m units of money at the beginning of the FM before the preference shock is realized is

$$U(m) = \frac{1}{2} [u(q_b) + W(m_\ell - pq_b, \ell)] + \frac{1}{2} [-c(q_s) + W(m_\ell + pq_s, \ell)]. \quad (1.13)$$

Given the trading mechanism in the financial market, we can show that:

Lemma 1 *In a stationary monetary equilibrium, if $\gamma > \beta$, then the buyer's liquidity constraint (1.10) binds.*

Proof. Lagging (1.13) one period and plugging it into (1.3), a representative agent's CM problem becomes

$$\begin{aligned} W(m, l) = \max_{m_{+1}, x} \beta \left\{ \frac{1}{2} [u(q_b) - c(q_s)] + \frac{1}{2} i\phi_{+1}(\ell^s - \ell^b) + W(m_{+1}, 0) \right\} \\ + v(x) - x - \phi m_{+1} + T + \phi [m - (1 + i)\ell] \end{aligned} \quad (1.14)$$

where $\ell^s \geq 0$, $\ell^b \geq 0$ are the seller's and buyer's amounts of the loan in the FM, respectively. Also, note that q_b and ℓ^s are both functions of m_{+1} , since for buyers m_{+1} can be used to purchase DM good q_b , and for sellers, m_{+1} can be lent out to gain interest. The CM problem (1.14) can be reduced to

$$\max_{m_{+1}} (\beta\phi_{+1} - \phi)m_{+1} + \beta \left\{ \frac{1}{2}[u(q_b) - c(q_s)] + \frac{1}{2}i\phi_{+1}(\ell^s - \ell^b) + W(0, 0) \right\}. \quad (1.15)$$

The first order condition with respect to m_{+1} is

$$(\beta\phi_{+1} - \phi) + \beta \left[\frac{1}{2}u'(q_b)\frac{\partial q_b}{\partial m_{+1}} + \frac{1}{2}\frac{\partial i\phi_{+1}\ell^s}{\partial m_{+1}} \right] = 0. \quad (1.16)$$

I prove Lemma 1 by contradiction. Suppose the liquidity constraint $pq_b \leq m_\ell$ does not bind, then $q_b = q^*$, so $\frac{\partial q_b}{\partial m_{+1}} = 0$. From (1.12) we know $i\phi\ell = (1 - \theta)[u(q^*) - u(q^m)]$, where $u(q^*)$ is constant, and q^m is independent of the seller's money holding, so $\frac{\partial i\phi_{+1}\ell^s}{\partial m_{+1}} = 0$. Then (1.16) implies $\beta\phi_{+1} - \phi = 0$, which is a contradiction to the assumption $\beta\phi_{+1} < \phi$. So, in a stationary monetary equilibrium, if $\gamma > \beta$, the buyer's liquidity constraint must bind. ■

1.3.4 Equilibrium with unbinding borrowing constraint

Given Lemma 1, we know that the gain of the financial market is monotonically increasing with ℓ , so $\ell = \min(m, \bar{\ell})$. For now, let us first consider the case where the borrower's borrowing constraint does not bind, which means $\ell = m$. And I will discuss the case $\ell = \bar{\ell}$ later. Once we know $\ell = m$, it is easy to show

$$2q^m = q_b. \quad (1.17)$$

For notational simplicity in what follows, let q denote the quantity of DM good for both sellers and buyers, since the market clear condition $q_b = q_s$ always satisfies. From the Kalai bargaining solution (1.12), the nominal interest rate can be expressed

as⁶

$$i = \frac{(1 - \theta)[u(q) - u(q^m)]}{qc'(q)/2}. \quad (1.18)$$

From (1.13) we have the marginal value of money collected in the CM

$$U'(m) = \phi \left[\frac{u'(q)}{2c'(q)} + \frac{1 + i}{2} \right]. \quad (1.19)$$

That is, with probability 0.5, an agent becomes a buyer in the DM, so the extra one unit of money collected in the CM can be used to buy consumption good in the DM. And with probability 0.5, an agent becomes a seller; he cannot spend the money in the DM, but he can lend it out in the FM to gain interest payment. Lagging (1.5) one period, plugging it into (1.19), and substituting i with (1.18), we have

$$\frac{\gamma - \beta}{\beta} = \frac{1}{2} \left[\frac{u'(q)}{c'(q)} - 1 \right] + \frac{(1 - \theta)[u(q) - u(q^m)]}{qc'(q)}. \quad (1.20)$$

Definition 1 *Given an exogenous debt limit $\bar{\ell}$, an equilibrium with an unbinding borrowing constraint ($\bar{\ell} \geq m$) is a CM allocation (x, H) , a DM outcome (p, q, q^m) , a FM outcome (i, ℓ) , and the real money balance ϕM such that:*

1. (p, q, q^m) solves (1.8), (1.17), (1.20), and $2\phi M = qc'(q)$;
2. i solves (1.18) and $\ell = m$;
3. (x, H) solves (1.2) and (1.4) with $m = M$ for all agents.

Denote the solution to (1.20) by q^l , which tells us a buyer's DM consumption if he has $2m$ units of money taken out of the FM. Notice that this equilibrium exists if and only if $\phi\bar{\ell} \geq \frac{1}{2}q^l c'(q^l)$. So, once we have q^l , we can go back to check whether this borrowing constraint $\bar{\ell}$ binds or not. From Lagos and Wright (2005), we know that without the financial market

$$\frac{\gamma - \beta}{\beta} = \frac{1}{2} \left[\frac{u'(\tilde{q})}{c'(\tilde{q})} - 1 \right] \quad (1.21)$$

⁶Note that $\phi pq = \phi m + \phi \ell$; plugging (1.8) into the left-hand side, we have $\phi \ell = qc'(q)/2$ for $\ell = m$.

where \tilde{q} is the amount of DM consumption if there is no financial market. Equation (1.21) tells us that only buyers can use their money in the DM, but sellers will keep their money idle. Comparing (1.20) and (1.21), we can find that as long as $\gamma > \beta$ and $\theta \neq 1$,⁷ then $q^l > \tilde{q}$, which means the financial market improves social welfare.

The role of the financial market here is similar to the role of banking in Diamond and Dybvig (1983), where banks provide insurance against the need for liquidity. Here, the financial market helps reallocate liquidity efficiently. As a result, it improves social welfare. The financial market makes it as if agents knew their preference shock before last period's CM closes, in the sense that if they are buyers, they will acquire money, while if they are sellers, they will not collect any money.

1.3.5 Equilibrium with binding borrowing constraint

So far, I have assumed that the debtor's borrowing constraint does not bind; i.e., $\bar{\ell} \geq m$. In this section, I will investigate the situation when the debtor's borrowing constraint binds; i.e., $\bar{\ell} < m$. Because the debtor's borrowing constraint binds, they can borrow no more than $\bar{\ell}$; i.e., $\ell = \bar{\ell} < m$. Therefore, creditors cannot lend out all of their money, and they still keep some money when they go out of the FM.

Now let us denote $\bar{\ell} = (1 - \mu)m$, which means every creditor can only lend out $(1 - \mu)$ of their money. So if $\bar{\ell}$ binds, the marginal utility of money becomes

$$U'(m) = \phi \left[\frac{u'(q)}{2c'(q)} + \frac{1 + (1 - \mu)i}{2} \right]. \quad (1.22)$$

Lagging (1.5) one period, plugging it into (1.22), we have

$$\frac{\gamma - \beta}{\beta} = \frac{1}{2} \left[\frac{u'(q)}{c'(q)} - 1 \right] + \frac{(1 - \mu)i}{2}. \quad (1.23)$$

⁷Note that $\theta = 1$ means borrowers make a take-it-or-leave-it offer to lenders in FM, then we have the interest rate $i = 0$.

The Kalai bargaining solution implies

$$i\phi\ell = (1 - \theta)[u(q) - u(\bar{q}^m)] \quad (1.24)$$

where \bar{q}^m is the amount of DM good a buyer consumes if he does not borrow in the FM. From Lemma 1, we know the buyer's liquidity constraint always binds, so $\bar{q}^m = m/p$. If the buyer's borrowing constraint binds, then we have the expression of nominal interest rate i

$$i = \frac{(2 - \mu)(1 - \theta)[u(q) - u(\bar{q}^m)]}{(1 - \mu)qc'(q)}. \quad (1.25)$$

Plugging (1.25) into (1.23), we have

$$\frac{\gamma - \beta}{\beta} = \frac{1}{2} \left[\frac{u'(q)}{c'(q)} - 1 \right] + \frac{(2 - \mu)(1 - \theta)[u(q) - u(\bar{q}^m)]}{2qc'(q)}. \quad (1.26)$$

Definition 2 *Given an exogenous debt limit $\bar{\ell}$, an equilibrium with a binding borrowing constraint ($\ell = \bar{\ell}$) is a CM allocation (x, H) , a DM outcome (p, q, \bar{q}^m) , an FM outcome (i, ℓ) , and the real money balance ϕM such that:*

1. (p, q) solves (1.8), (1.26) with $\bar{q}^m = q/(2 - \mu)$, and $(2 - \mu)\phi M = qc'(q)$ where $1 - \mu = \phi\bar{\ell}/[qc'(q) - \phi\bar{\ell}]$;
2. i solves (1.25) and $l = \bar{\ell}$;
3. (x, H) solves (1.2) and (1.4) with $m = M$ for all agents.

Denote the solution to (1.25) and (1.26) by \bar{i} and \bar{q} , which are the nominal interest rate and DM consumption, respectively, when borrowing constraint binds. Comparing (1.20) and (1.26), we know that $\bar{q} < q^l$. Also, when the debtor's borrowing constraint binds, the gain from the financial market increases with the debt limit.

To summarize, given an exogenous debt limit $\bar{\ell}$, there are two possible outcomes:

1. If $\phi\bar{\ell} \geq \frac{1}{2}q^l c'(q^l)$, then borrowers face an unbinding borrowing constraint, $\ell = m$ and $q = q^l$.

2. If $\phi\bar{\ell} < \frac{1}{2}q^l c'(q^l)$, then borrowers face a binding borrowing constraint, $\ell = \bar{\ell}$ and $q = \bar{q}$.

1.4 Endogenous Debt Limits

Because there is no repayment enforcement, the borrowing contract must be self-enforcing, which means the borrower's expected lifetime utility must be higher if he repays the debt than that if he defaults. Let us denote $\widehat{W}(m, \ell)$ the value function if a borrower defaults on his debt. Then self-enforcing contract requires

$$W(m, \ell) \geq \widehat{W}(m, \ell). \quad (1.27)$$

Recall that in the CM a representative agent's lifetime utility is

$$W(m, \ell) = v(x^*) - x^* - \phi m_{+1} + \beta U(m_{+1}) + T + \phi[m - (1+i)\ell]. \quad (1.28)$$

Consider now defaulters; like other agents, they produce and consume in the CM but do not repay their debt. It is easy to show their CM problem is

$$\widehat{W}(m, \ell) = \max_{\widehat{m}_{+1}, \widehat{x}} [v(\widehat{x}) - \widehat{x} - \phi \widehat{m}_{+1} + \beta \widehat{U}(\widehat{m}_{+1})] + T + \phi m \quad (1.29)$$

where hat denotes the defaulter's choice. It is straightforward to show that $\widehat{x} = x = x^*$. Plugging (1.28) and (1.29) into (1.27), the self-enforcing contract requires

$$\phi(m_{+1} - \widehat{m}_{+1}) + \phi(1+i)\ell \leq \beta[U(m_{+1}) - \widehat{U}(\widehat{m}_{+1})]. \quad (1.30)$$

The LHS is the current gain from deviating, and the RHS is the current value of the loss of being banished from the financial market in the future. In the steady state, the non-defaulter's and defaulter's expected utility functions at the beginning of the FM are, respectively⁸

$$U(m_{+1}) = \frac{1}{1-\beta} \left\{ \frac{1}{2}[u(q) - c(q)] + \frac{1}{2}\phi(m - \ell) + v(x^*) - x^* - \phi m_{+1} + T \right\} \quad (1.31)$$

⁸If defaulters become sellers in the future, they will choose the same q as non-defaulters.

and

$$\widehat{U}(\widehat{m}_{+1}) = \frac{1}{1-\beta} \left\{ \frac{1}{2}[u(\widehat{q}) - c(q)] + \frac{1}{2}\phi\widehat{m} + v(x^*) - x^* - \phi\widehat{m}_{+1} + T \right\}. \quad (1.32)$$

Plugging (1.31) and (1.32) into (1.30),⁹ the debtor's borrowing constraint satisfies

$$\phi\ell \leq \frac{\beta}{(1-\beta)(1+i)} \left\{ \frac{1}{2}[u(q) - u(\widehat{q})] + \phi \left(\frac{1}{2} - \frac{\gamma}{\beta} \right) (m - \widehat{m}) - \frac{1}{2}\phi\ell \right\}. \quad (1.33)$$

It is straightforward to show that \widehat{q} satisfies (1.21). If the debtor's borrowing constraint does not bind, then he will borrow $\ell = pq^\ell/2$. With the market clear condition, we have $m = pq^\ell/2$, $\widehat{m} = p\widehat{q}$ and $\ell = m$. If the borrowing constraint binds, then $m = p\bar{q} - \bar{\ell}$. Now we can use the method in Gu et al. (2016) to endogenize the debt limit $\bar{\ell}$. First, pick an arbitrary $\bar{\ell}$. Depending on whether this debt limit binds or not, (1.33) can be written as

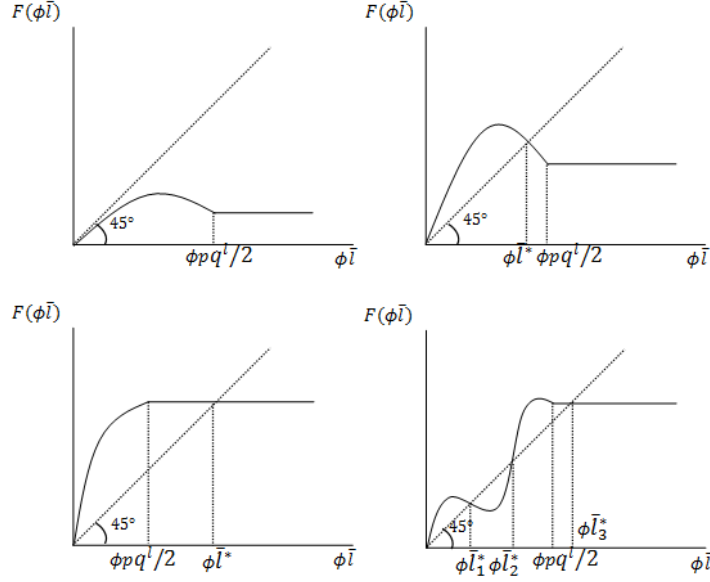
$$F(\phi\bar{\ell}) = \begin{cases} \frac{\xi}{1+i} \left\{ \frac{u(\bar{q}) - u(\widehat{q})}{2} + \frac{\beta - 2\gamma}{2\beta} c'(\bar{q})(\bar{q} - \widehat{q}) - \frac{\beta - \gamma}{\beta} \phi\bar{\ell} \right\} & \text{if } \bar{\ell} \leq pq^\ell/2 \\ \frac{\xi}{1+i} \left\{ \frac{u(q^\ell) - u(\widehat{q})}{2} + c'(\bar{q}) \left[\frac{\widehat{q}}{2} + \frac{\gamma}{\beta} \left(\frac{q^\ell}{2} - \widehat{q} \right) \right] \right\} & \text{if } \bar{\ell} > pq^\ell/2 \end{cases} \quad (1.34)$$

where $\xi = 1/r$ and $\beta = 1/(1+r)$. The first branch corresponds to the case where the borrowing constraint binds, and $F(\phi\bar{\ell})$ changes in $\phi\bar{\ell}$. The second branch corresponds to the case where the borrowing constraint does not bind. In this case, $F(\phi\bar{\ell})$ is constant.

An endogenous debt limit is a nonnegative fixed point $F(\phi\bar{\ell}^*) = \phi\bar{\ell}^*$. Obviously, $F(0) = 0$ is a fixed point:¹⁰ if there is no credit in the future, nobody will honor his obligation, because regardless of repaying the debt or not, debtors cannot get any loan in the future.

⁹Note that I use the fact $m_{+1} - \widehat{m}_{+1} = \gamma(m - \widehat{m})$.

¹⁰If $\bar{\ell} = 0$ then $\bar{q} = \widehat{q}$; from the first branch of (1.34), we can easily get $F(0) = 0$.

Figure 1-2: $F(\phi\bar{\ell})$

Now I want to investigate whether there are other fixed points. Figure 1-2 shows several possible cases: in the first panel, $F(0) = 0$ is the only fixed point; the second panel has another fixed point $\phi\bar{\ell}^* < \phi pq^l/2$, so the debtor's borrowing constraint binds; the third panel has a fixed point $\phi\bar{\ell}^* > \phi pq^l/2$, which means the borrowing constraint does not bind; note that the first branch of $F(\phi\bar{\ell})$ is not necessarily concave, so multiple positive fixed points is also possible, and the fourth panel shows this case.

We are interested in the cases in which the loan market is active, so we want to find out when a positive equilibrium $\phi\bar{\ell}^*$ exists. The sufficient condition for the existence of a positive fixed point is $F'(\phi\bar{\ell} = 0) > 1$. Let $\Omega \equiv [u(\bar{q}) - u(\hat{q})]/2 + (\beta - 2\gamma)c'(\bar{q})(\bar{q} - \hat{q})/2\beta - (\beta - \gamma)\phi\bar{\ell}/\beta$, then from the first branch of (1.34) we have

$$F'(\phi\bar{\ell}) = \xi \left\{ \frac{-\Omega}{(1+\bar{i})^2} \frac{\partial(1+\bar{i})}{\partial\bar{\ell}} + \frac{1}{1+\bar{i}} \left[\frac{u(\bar{q})}{2} \frac{\partial\bar{q}}{\partial\bar{\ell}} + \frac{\beta - 2\gamma}{2\beta} c'(q) \frac{\partial\bar{q}}{\partial\bar{\ell}} - 1 + \frac{\gamma}{\beta} \right] \right\}. \quad (1.35)$$

When $\bar{\ell} = 0$, $\bar{q} = \hat{q}$, so $\Omega = 0$. From (1.25) we know $\lim_{\mu \rightarrow 1} \bar{i} = (1 - \theta)u'(\bar{q})/c'(q)$.

Plugging it into (1.35), we have¹¹

$$F'(\phi\bar{\ell} = 0) = \frac{\beta(\gamma - \beta)}{(1 - \beta)[(1 - \theta)(2\gamma - \beta) + \beta]}. \quad (1.36)$$

For $\gamma > \beta$, (1.36) indicates that $F'(\phi\bar{\ell} = 0) > 1$, if and only if

$$\beta > \frac{2(1 - \theta)}{1 + 2(1 - \theta)} \quad (1.37)$$

and

$$\gamma > \frac{\beta[1 - (1 - \theta)(1 - \beta)]}{\beta - 2(1 - \theta)(1 - \beta)}. \quad (1.38)$$

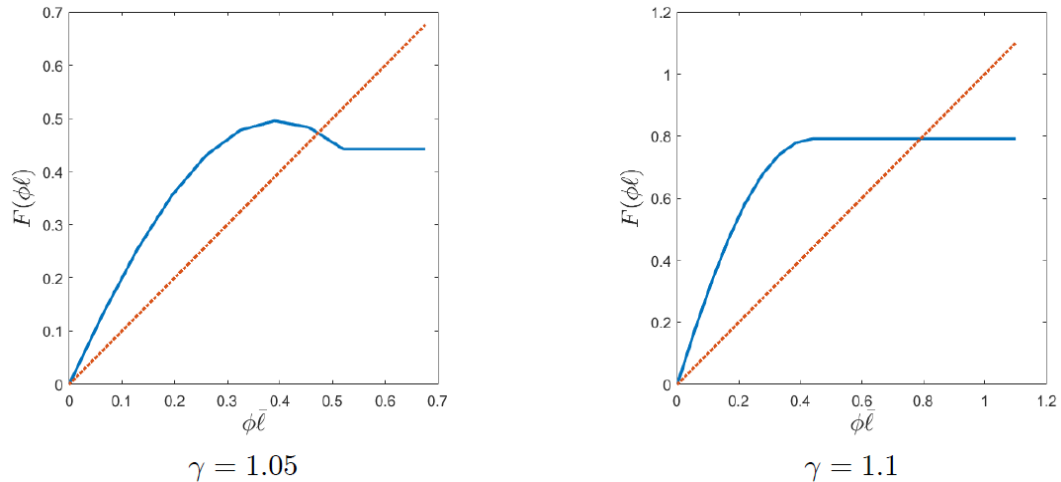
Inequality (1.37) says the agents need to be patient enough, so they will value the future consumption. Inequality (1.38) says that the money growth rate needs to be sufficiently big, so the gain from participating in the financial market is higher than the deviation payoff. This is because if γ decreases, the opportunity cost of holding money decreases. And the cost becomes 0 when $\gamma = \beta$, which is the Friedman rule. In this case, agents can self-insure against the preference shock by holding enough money at no cost. In other words, the value of the financial market vanishes if γ is sufficiently low.

The two conditions (1.37) and (1.38) guarantee the existence of positive fixed points. Now we want to know if the fixed point is located on the first or the second branch of (1.34). If $\phi\bar{\ell}^* > \phi pq^\ell/2$, this fixed point is located on the horizontal line. In other words, this endogenous debt limit does not bind. On the other hand, if $\phi\bar{\ell}^* \leq \phi pq^\ell/2$, then this debt limit binds. Now I summarize the results as follows:

Proposition 1 *Suppose conditions (1.37) and (1.38) hold, there exists a monetary equilibrium with a positive endogenous debt limit $\phi\bar{\ell}^*$, which satisfies $F(\phi\bar{\ell}^*) = \phi\bar{\ell}^*$, and*

1. *If $\phi\bar{\ell}^* > \phi pq^\ell/2$, the debt limit does not bind;*

¹¹Note that when $\bar{\ell} = 0$, $\bar{q} = \hat{q}$, and from (1.21) we know $u'(\hat{q})/c'(q) = (2\gamma - \beta)/\beta$.

Figure 1-3: $F(\phi\bar{\ell})$

2. If $\phi\bar{\ell}^* \leq \phi p q^\ell / 2$, the debt limit binds.

Figure 1-3 shows two numeric examples. The first panel plots $F(\phi\bar{\ell})$ against $\phi\bar{\ell}$ with utility function $u(q) = 2q^{0.5}$, cost function $c(q) = q$, discount factor $\beta = 0.95$, bargaining power $\theta = 0.8$, and money growth rate $\gamma = 1.05$. The second panel uses the same functions and parameters except that $\gamma = 1.1$. In the first graph, the 45° line crosses $F(\phi\bar{\ell})$ from the first branch; i.e., this endogenous debt limit $\phi\bar{\ell}$ binds and $\phi\bar{\ell} = 0.4929$. In the second graph, the 45° line crosses $F(\phi\bar{\ell})$ from the second branch; i.e., this endogenous debt limit $\phi\bar{\ell}$ does not bind and $\phi\bar{\ell} = 0.7914$. We have a higher debt limit when the inflation rate is high. That is because with a higher inflation rate, the cost of deviation (i.e., being banished from the financial market) is higher. So, borrowers have more incentive to keep the access to future loans.

1.5 Equilibrium with Nominal Bonds

Consider an economy where a one-period government bond is issued in the CM and it can be redeemed for one unit of fiat money in the next CM. Government bonds are

perfectly divisible, payable to the bearer, and default free. In every period, the total supply of government bonds is denoted by B . The price of the newly issued bonds in terms of CM good is ω . If $\omega < \phi$, the newly issued bonds are sold at a discount for money, and the government finances the interest payment on bonds by lump-sum tax in the CM. Government bonds can also be traded in the FM at price ψ in terms of CM good. But, trading bonds in the financial market incurs a fixed cost κ . I assume that bonds can be counterfeited at zero cost and agents are not able to distinguish genuine bonds from counterfeit bonds in the FM by themselves.¹² So, if they want to trade bonds in the financial market, they need to pay a fixed cost to the third party to verify the government bonds. And this cost is assumed to be shared by the buyer and seller according to their bargaining power; i.e., $\theta\kappa$ and $(1 - \theta)\kappa$, respectively.

1.5.1 Illiquid bonds

Illiquid bond means that it cannot be used to purchase good in the DM where fiat money is the only means of payment. Now a representative agent's CM problem is

$$W(m, \ell, b) = \max_{m_{+1}, b_{+1}, x, H} [v(x) - H + \beta U(m_{+1}, b_{+1})] \quad (1.39)$$

$$\text{s.t. } x + \phi m_{+1} + \omega b_{+1} = H + T + \phi[m + b - (1 + i)\ell] \quad (1.40)$$

where b and b_{+1} are the amounts of nominal bonds taken into and out of the CM, respectively. The first order conditions with respect to x , m_{+1} and b_{+1} are, respectively

$$v'(x) = 1, \quad (1.41)$$

$$\phi = \beta \frac{\partial U(m_{+1}, b_{+1})}{\partial m_{+1}} \quad (1.42)$$

and

$$\omega = \beta \frac{\partial U(m_{+1}, b_{+1})}{\partial b_{+1}}. \quad (1.43)$$

¹²Different from bonds, fiat money cannot be counterfeited. The government can distinguish genuine bonds from counterfeit bonds in the CM with zero cost.

The envelope conditions are

$$\frac{\partial W(m, \ell, b)}{\partial m} = \phi, \quad (1.44)$$

$$\frac{\partial W(m, \ell, b)}{\partial \ell} = -\phi(1 + i) \quad (1.45)$$

and

$$\frac{\partial W(m, \ell, b)}{\partial m} = \phi. \quad (1.46)$$

Equations (1.41)-(1.46) are similar to equations (1.4)-(1.7), except that we include bonds here. The buyer's DM problem is

$$V^b(m_\ell, \ell, b - a) = \max_q [u(q) + W(m_\ell - pq, \ell, b - a)] \quad (1.47)$$

$$\text{s.t. } pq \leq m_\ell \quad (1.48)$$

where $a > 0$ is the amount of bonds sold in the financial market and $m_\ell = m + \ell + a\psi/\phi$ is the total money holding taken out of the FM. Different from the benchmark model, here agents bargain over a portfolio (ℓ, i, a, ψ) . If they reach an agreement in the FM, the buyer's and seller's DM value functions are, respectively

$$V^b(m_\ell, \ell, b - a) = u(q) + W(m_\ell - pq, \ell, b - a)$$

and

$$V^s(m_\ell, \ell, b + a) = -c(q) + W(m_\ell + pq, \ell, b + a).$$

If they do not reach an agreement, the threat points are

$$V_0^b(m, 0, b) = u(q^m) + W(m - pq^m, 0, b)$$

and

$$V_0^s(m, 0, b) = -c(q) + W(m + pq, 0, b).$$

If bonds are traded in the FM, then the total surplus from the financial market is $S^{\ell b} = u(q) - u(q^m) - \kappa$. Buyers and sellers get a constant share of this surplus:

$S_b = u(q_b) - u(q^m) - i\phi\ell - (\phi - \psi)a - \theta\kappa$ and $S_s = i\phi\ell + (\phi - \psi)a - (1 - \theta)\kappa$. The amount of loan ℓ , the interest rate i , the amount of bonds traded in the FM a , and its price ψ satisfy

$$(\ell, i, a, \psi) = \arg \max_{\ell \leq \bar{\ell}, a \leq b, i, \psi} [u(q_b) - u(q^m)] - \kappa I_{(a>0)} \quad (1.49)$$

$$\text{s.t. } i\phi\ell + (\phi - \psi)a = (1 - \theta)[u(q) - u(q^m)] \quad (1.50)$$

$$\frac{\psi}{\phi}a + \ell \leq m \quad (1.51)$$

where $I_{(a>0)}$ is an indicator function, $I_{(a>0)} = 1$ if $a > 0$, and $I_{(a>0)} = 0$ if $a = 0$. When $\gamma > \beta$, the buyer's liquidity constraint binds. So, the seller's feasibility constraint (1.51) must hold at equality. Now let us assume that the creditor will lend out $(1 - \mu)m$ units of their money, where $(1 - \mu)m \leq \bar{\ell}$. If b is large enough, constraint $a \leq b$ will not bind. So, they can spend the rest μm units of money purchasing a units of bonds.¹³ That is, in the financial market, buyers can either borrow from sellers and promise to repay in the subsequent CM with a positive interest rate i , or they can trade their bonds for money at a discounted price ψ . Plugging $\ell = (1 - \mu)m$ and $a\psi/\phi = \mu m$ into (1.50), the nominal interest rate i and FM bond price ψ are, respectively

$$i = \frac{(1 - \theta)[u(q) - u(q^m)]}{\phi m} \quad (1.52)$$

and

$$\frac{\phi - \psi}{\psi} = \frac{(1 - \theta)[u(q) - u(q^m)]}{\phi m}. \quad (1.53)$$

From (1.52) and (1.53) we know that sellers can get the same investment return either from buying discounted government bonds from buyers, or from lending their money to them. At the beginning of every period with probability 0.5, an agent becomes a seller; while with probability 0.5, he becomes a buyer. The expected

¹³Note that μ is endogenous, but as I hope to make clear, it is indeterminate.

utility of an agent is

$$U(m, b) = \frac{1}{2}[u(q) + W(m_\ell - pq, \ell, b - a)] + \frac{1}{2}[-c(q) + W(m_\ell + pq, \ell, b + a)] \quad (1.54)$$

The marginal value of money and bonds collected in the CM are, respectively

$$\frac{\partial U(m, b)}{\partial m} = \phi \left\{ \frac{u'(q)}{2c'(q)} + \frac{1}{2} \left[(1 - \mu)(1 + i) + \mu \frac{\phi}{\psi} \right] \right\} \quad (1.55)$$

and

$$\frac{\partial U(m, b)}{\partial b} = \phi. \quad (1.56)$$

The two terms in the brackets of (1.55) tells us that if the borrower's borrowing constraint binds, then the seller can both lend out their money to earn interest and buy government bonds at a discounted price in the financial market. Lagging (1.43) one period plugging into (1.56), we have

$$\omega_{-1} = \beta\phi. \quad (1.57)$$

As long as $\beta < 1$, we have $\omega_{-1} < \phi$, which means government bonds are sold at a discounted price. Plugging (1.42), (1.52), and (1.53) into (1.55), we have

$$\frac{\gamma - \beta}{\beta} = \frac{1}{2} \left[\frac{u'(q)}{c'(q)} - 1 \right] + \frac{(1 - \theta)[u(q) - u(q^m)]}{qc'(q)}. \quad (1.58)$$

Definition 3 *An equilibrium with government bonds traded in the financial market is a CM allocation (ω, x, H) , a DM outcome (p, q, q^m) , an FM outcome (i, ℓ, a, ψ) , and the real money balance ϕM such that:*

1. (p, q) solves (1.8), (1.58) with $q^m = q/2$, and $2\phi M = qc'(q)$;
2. (i, ψ) solves (1.52), (1.53) with $\ell = (1 - \mu)m$, and $a\psi/\phi = \mu m$;
3. (ω, x, H) solves (1.40), (1.41) and (1.57) with $m = M$, and $b = B$ for all agents.

Notice that there exists a fixed cost κ , so if agents want to trade bonds in the financial market when the debtor's borrowing constraint binds,¹⁴ they need to compare the extra surplus of trading bonds with its cost. If agents only borrow and lend in the FM, from Section 1.3.5, we have the total surplus $\bar{S} = [u(\bar{q}) - u(\bar{q}^m)]$. Comparing S^{lb} and \bar{S} , when the borrower's borrowing constraint binds, the necessary condition for the agents to trade bonds in the FM is

$$\kappa \leq [u(q^{lb}) - u(q^m)] - [u(\bar{q}) - u(\bar{q}^m)] \quad (1.59)$$

where q^{lb} solves (1.58). Comparing (1.20) and (1.58) we know that $q^{lb} = q^l$. And q^m is the same as in Definition 1. That means if κ is small enough, even though the agent's borrowing constraint binds, we can still achieve efficient allocation as if it did not bind. That is because, besides lending and borrowing money, agents can also adjust their portfolio by trading illiquid bonds in the FM. In other words, the existence of illiquid bonds provides us an additional channel to readjust the portfolio in the FM. A special case is $\kappa = 0$, which means there is no additional cost to trade bonds. In this case, if agents have an abundant amount of bonds, they can readjust their portfolio in the FM by trading bonds only. And the loan market becomes redundant.

I show illiquid bonds' welfare-enhancing role with a numeric example using the same parameters and functions as in the first case of Figure 1-3, where $u(q) = 2q^{0.5}$, $c(q) = q$, $\beta = 0.95$, $\theta = 0.8$, and $\gamma = 1.05$. In this economy, the buyer's borrowing constraint binds. If there is no bond, the buyer can only consume $\bar{q} = 1.0153$. However, if they can trade bonds in the FM, then they can consume $q^{lb} = 1.0397$. The corresponding total surplus of the FM changes from $\bar{S} = 0.5697$ to $S^{lb} = 0.5973 - \kappa$. So, if $\kappa \leq 0.0276$, agents will trade bonds in the FM.

¹⁴Obviously, if borrowing constraint does not bind, buyers will not trade bonds, because doing so incurs a fixed cost κ . If they reallocate their money only by a loan contract, however, they can get rid of this cost.

1.5.2 Liquid bonds

Different from illiquid bonds, liquid bonds can be used to purchase DM good directly, just like fiat money. In this section, I show that liquid bonds are inessential. They can only change the good market equilibrium price but cannot change the allocation. From the non-arbitrage condition, we know that if bonds can be used to purchase good, they must have the same real price as fiat money. Otherwise, there will be arbitrage opportunities. Specifically, if bonds are sold at a discount (i.e., $\omega < \phi$), then from (1.42) and (1.43) we know that the marginal cost of holding bonds is lower than that of money. Then, money would not be valued. However, this cannot be an equilibrium outcome because bonds are redeemed by money. Therefore, in equilibrium, bonds must be a perfect substitute to fiat money (i.e., $\omega = \phi$). This result is consistent with the so-called rate of return dominance puzzle. Once we know bonds and money are perfect substitutes, it is straightforward to show that they must have the same price in the FM (i.e., $\phi = \psi$). Otherwise, the financial market cannot clear. So, (1.58) becomes

$$\frac{\gamma - \beta}{\beta} = \frac{1}{2} \left[\frac{u'(q)}{c'(q)} - 1 \right] + \frac{(1 - \mu)i}{2}$$

which is identical to (1.23). The real price of money changes from $\phi = qc'(q)/[(2 - \mu)M]$ to $\phi = qc'(q)/[(2 - \mu)(M + B)]$. But otherwise, nothing else is changed. So, liquid bonds cannot help recapture the gain of reallocating currency across different types of agents.

1.5.3 Intuition

Why can illiquid bonds help deal with the inefficiency problem? To answer this question, we would better first think about the source of the inefficiency. This inefficiency comes from two frictions. First, the preference shock is realized after agents choose

their money holdings. The existence of the financial market that allows for readjusting the portfolio after uncertainty is realized helps reduce this inefficiency.

The second source of inefficiency is the limited repayment commitment, which gives the agents a binding borrowing constraint. So, in the FM, they cannot borrow enough money to finance their DM consumption. In order to engage in more intertemporal exchange of consumption, they need to find another way to increase their liquidity level before entering the DM. Illiquid bonds have the potential to achieve this goal because they have a different liquidity property with money. This feature breaks down the non-arbitrage condition and gives agents another channel to readjust their portfolio in the FM. However, from the non-arbitrage condition, we know that liquid bonds and money are perfect substitutes. That means introducing liquid bonds into a monetary economy can only change the price of money but cannot change its allocation.

1.5.4 Other real assets

Now it seems worth asking: Can we replace bonds with other kinds of real assets like a Lucas tree? So, I now consider the Lucas tree as an alternative to government bonds.

The timing and trading mechanism is the same as before. The only difference is that the government no longer issues nominal bonds, but the economy is endowed with Lucas trees, in fixed supply of Z . At the beginning of each CM, each unit of Lucas tree generates d units of fruit as dividend, measured in terms of CM good, which belongs to the agent who owns the tree at the beginning of the CM. Lucas trees can be traded in the FM and CM but cannot be used as a means of payment in the DM. If they are traded in the FM, it also incurs the same fixed cost κ . So, the value function of a representative agent entering the CM with a portfolio of m units

of money, ℓ units of loan, and z units of Lucas trees is

$$W(m, \ell, z) = \max_{m_{+1}, z_{+1}, x, H} [v(x) - H + \beta U(m_{+1}, z_{+1})]$$

$$\text{s.t. } x + \phi m_{+1} + \rho z_{+1} = H + T + \phi[m - (1 + i)\ell] + (\rho + d)z$$

where ρ is the price of the Lucas trees if they are sold in the CM. The first order conditions with respect to x , m_{+1} and z_{+1} are, respectively

$$v'(x) = 1, \tag{1.60}$$

$$\phi = \beta \frac{\partial U(m_{+1}, z_{+1})}{\partial m_{+1}} \tag{1.61}$$

and

$$\rho = \beta \frac{\partial U(m_{+1}, z_{+1})}{\partial z_{+1}}. \tag{1.62}$$

The buyers' and sellers' DM value functions are, respectively

$$V^b(m_\ell, \ell, z - e) = u(q) + W(m_\ell - pq, \ell, z - e)$$

and

$$V^s(m_\ell, \ell, z + e) = -c(q) + W(m_\ell + pq, \ell, z + e)$$

where $e \geq 0$ is the amount of Lucas trees traded in the FM and δ is their price in terms of CM good and $m_\ell = m + \ell + e\delta/\phi$. With the Kalai bargaining solution, here we have

$$i\phi\ell + (\rho - \delta)e = (1 - \theta)[u(q) - u(q^m)].$$

Assume the borrower's borrowing constraint binds,¹⁵ the gain of trading Lucas trees net of the fixed cost κ is positive, and there are abundant Lucas trees (i.e.,

¹⁵Still, if the borrowing constraint does not bind, then agents can get efficient allocation through a loan contract, and Lucas trees become irrelevant.

$e \leq z$ does not bind), then the interest rate and the price of Lucas trees can be expressed as

$$i = \frac{\rho - \delta}{\delta} = \frac{(1 - \theta)[u(q) - u(q^m)]}{\phi m}. \quad (1.63)$$

The expected value function at the beginning of every period is

$$U(m, z) = \frac{1}{2}[u(q) + W(m_\ell - pq, \ell, z - e)] + \frac{1}{2}[-c(q) + W(m_\ell + pq, \ell, z + e)].$$

The marginal value of money and Lucas trees collected in the CM are, respectively

$$\frac{\partial U(m, z)}{\partial m} = \phi \left\{ \frac{u'(q)}{2c'(q)} + \frac{1}{2} \left[(1 - \mu)(1 + i) + \mu \frac{\rho + d}{\delta} \right] \right\} \quad (1.64)$$

and

$$\frac{\partial U(m, z)}{\partial z} = \rho + d. \quad (1.65)$$

Lagging (1.61) and (1.62) one period, plugging them into (1.64) and (1.65) respectively, we have

$$\frac{\gamma - \beta}{\beta} = \frac{1}{2} \left[\frac{u'(q)}{c'(q)} - 1 \right] + \frac{1}{2} \left[(1 - \mu)i + \mu \frac{\rho - \delta}{\delta} \right] \quad (1.66)$$

and

$$\rho_{-1} = \beta(\rho + d). \quad (1.67)$$

Denote the solution to (1.66) by q^{lz} , which is the amount of DM consumption of a buyer with $m + \ell + z\delta/\phi$ units of money. From (1.67) we know that Lucas trees are sold at the fundamental price in the CM. Since Lucas trees cannot be used to purchase good in the DM, their CM price does not include liquidity value. Plugging (1.63) into (1.66), and comparing it with (1.58), we can find that $q^{lz} = q^{lb}$.

However, if the amount of Lucas trees is scarce (i.e., $e \leq z$ binds), buyers will get $\bar{\ell}$ units of loan and sell all z units of Lucas trees in the FM. Sellers still have $m - \bar{\ell} - z\delta/\phi$ units of money left when they get out of the FM. Denote $m - \bar{\ell} - z\delta/\phi = (1 - \epsilon)m$, then with the similar method as in Section 1.3.5, we can find that when the amount

of Lucas tree is scarce, the quantity of DM consumption \bar{q}^{lz} , the interest rate i , and the price of Lucas trees δ solves

$$\frac{\gamma - \beta}{\beta} = \frac{1}{2} \left[\frac{u'(\bar{q}^{lz})}{c'(\bar{q}^{lz})} - 1 \right] + \frac{1}{2}(1 - \epsilon) \left[(1 - \mu)i + \mu \frac{\rho - \delta}{\delta} \right]$$

and

$$i = \frac{\rho - \delta}{\delta} = \frac{(2 - \epsilon)(1 - \theta)[u(\bar{q}^{lz}) - u(\bar{q}^m)]}{(1 - \epsilon)\bar{q}^{lz}c'(\bar{q}^{lz})}$$

where $\bar{q}^m = q/(2 - \epsilon)$. It is easy to show that $\bar{q}^{lz} < q^{lz}$. So, if the amount of Lucas trees is scarce, it can only recapture part of the FM surplus.

In summary, if there is an abundant amount of Lucas trees in the economy, the government bonds can be replaced by the Lucas trees. Otherwise, if Lucas trees are scarce, they cannot replace government bonds. In the real world, we usually have the freedom to choose the amount of government bonds. However, the amount of Lucas trees is in fixed supply.

1.6 Conclusion

This paper studies the functions of assets with different liquidity properties in the financial market. Specifically, illiquid bonds and private loans both can mitigate the welfare loss due to preference shocks. Without limited commitment, private loans can help achieve the second best. Without verification cost, illiquid bonds can help achieve the second best. They are coessential when both of them are subject to the frictions.

The result that illiquid bonds can improve social welfare, but liquid bonds cannot, may seem counterintuitive at first glance. Indeed, illiquid bonds are less flexible than liquid bonds. But, this feature also breaks down the non-arbitrage condition and gives agents another channel to readjust their portfolio in the financial market, which in turn improves social welfare.

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Chapter 2

Credit Arrangement under Default Risk

2.1 Introduction

Nowadays, bankruptcy is pretty common around the world. In the United States, 819,240 people filed for bankruptcy in 2015, as reported by the American Bankruptcy Institute. Under the U.S. Bankruptcy Code, borrowers can default on their loans by filing for bankruptcy. Then, their assets will be seized and the debt will be waived. A bankruptcy flag will stay on the defaulter's credit report for up to 10 years, which makes it hard for the defaulter to get new (unsecured) loans for 10 years.

Following Kehoe and Levine (1993), the literature on unsecured credit under limited commitment (e.g., Kocherlakota 1996, Gu et al. 2013) models borrowers' repayment decisions as disciplined by default punishment. In these papers, however, default never happens in equilibrium. No default in equilibrium obviously limits the ability of models to explain observations and to discuss whether the bankruptcy code is sufficient to maintain the health of the credit system. The goal of this paper is twofold. First, I model default in equilibrium in the Kehoe-Levine framework by introducing private information. Second, I examine the optimal punishment scheme under default risk.

Following Kehoe and Levine (1993), agents face consumption asynchrony in the

model. Borrowers get consumption good from lenders and pay back at a later time. However, some borrowers are not able to produce when they need to repay their debt, so they have to default. A key friction in this paper is that whether the borrower can produce is private information. Thus, even though borrowers default only because of their bad luck, punishment is still necessary to make sure they will repay their debt when they can produce.

The key contribution of this paper is twofold. First, I study a credit model that allows for default in equilibrium. I show that when record keeping and information on trading history are both perfect, the existence of defaulters has no effect on the other agents' consumption and output. If lenders can distinguish defaulters from non-defaulters, they will only trade with non-defaulters. But if the access to the agent's trading history is limited, lenders cannot tell the difference between defaulters and non-defaulters. They will charge all the borrowers a default premium. In this case, non-defaulters have to pay a higher price on their consumption and the quantity traded will be lowered. This result is similar to Carapella and Williamson (2015). The key difference between this paper and Carapella and Williamson (2015) is that, in their paper, default is a self-fulfilling phenomenon. However, they do not explain why agents default in the initial period. In my model, the borrowers' productivity determines their repayment.

Second, when default does happen in equilibrium, it is worth asking how harsh the default punishment should be, or what the optimal punishment is. As a benchmark, I first study a model with permanent productivity shocks; i.e., the same group of borrowers default in every period. In this case, permanent autarky punishment is optimal. The reason is the same as in Kehoe and Levine (1993): the nature of punishment determines the debt limit, harsher punishment means higher debt limit. And, typically, higher debt limit provides better allocation. Then, I modify the model

to include temporary productivity shocks. In this case, we need to take punishment more seriously, because severe punishment lowers the frequency of trade. As a first attempt to address this issue, I focus on a narrow class of credit arrangements. I find that the optimal default punishment should be intermediate, which can support a sufficient debt limit but does not lower the frequency of trade too much.

This paper is closely related to several strands of literature on unsecured debt. First, starting with Kehoe and Levine (1993), Kocherlakota (1996), Kehoe and Levine (2001), and more recently, Sanches and Williamson (2010), Gu et al. (2013), and Gu et al. (2016) study environments where unsecured credit is supported by the threat of default punishment. In these models, since agents' repayment constraint is always satisfied, default never happens in equilibrium. In this paper, however, a loan contract specifies the borrowers' obligation regardless of any future event, and gives borrowers the option to default.

This paper is also related to papers that study bankruptcy with incomplete markets, such as Zame (1993), Zhang (1997), Athreya (2002), and Chatterjee et al. (2007). These studies show that giving borrowers the option to default may help generate efficient outcomes.

Lastly, to the best of my knowledge, two papers address the optimal default punishment problem. Diamond (1984) uses a principal-agent model and finds that the default punishment should be maintained at an intermediate level to minimize the cost of monitoring information. Dubey et al. (2005) extend the incomplete markets model by treating assets as pools. They suggest an intermediate punishment to balance the trade-off between enlarging the asset span and the deadweight loss from default punishment.

The rest of this paper is organized as follows. Section 2.2 describes the environment of the benchmark model. Section 2.3 studies the equilibrium with an exogenous debt

limit. Section 2.4 endogenizes the debt limit. Section 2.5 discusses the optimal default punishment. Section 2.6 considers extension. Section 2.7 concludes.

2.2 Environment

There is a $[0, 2]$ continuum of agents living forever in discrete time with a discount factor $\beta \in (0, 1)$ between every time period t . Each period has two subperiods. In the first subperiod, half of the agents can only consume and the other half can only produce. I refer to these two types of agents as buyers and sellers, respectively. Buyers and sellers trade bilaterally, meeting randomly in the first subperiod with probability 1. The buyer makes a take-it-or-leave-it offer to the seller and promises to repay d units of good in the second subperiod. Buyers can get utility $u(q)$ from consuming q units of good in the first subperiod, where $u(0) = 0$, $u'(q) > 0$, $u''(q) \leq 0$, $u'(+\infty) = 0$, and $-qu''(q)/u'(q) < 1$. Sellers can produce q units of output with a cost function $c(q) = q$.

In the second subperiod, a fraction α of buyers (Type I buyers) can produce with a linear cost function $C(x) = x$, so they are able to repay their debt. But a fraction $1 - \alpha$ of buyers (Type II buyers) are not able to produce, so they have to default on their debt. As a benchmark, I start with the case where the buyer's type is permanent; namely, the same group of buyers cannot produce in any period. Starting from Section 2.5, the buyer's type is no longer permanent, but determined by an idiosyncratic shock realized at the beginning of the second subperiod. Sellers can only consume in the second subperiod; they get utility $U(x) = x$ from consumption.

A fraction ρ of the meetings in the first subperiod are limited-record-access meetings (LM), where the seller does not have access to the buyer's trading history. So, they do not know whether the buyer can repay the debt in the second subperiod. The remaining $1 - \rho$ of the meetings are full-record-access meetings (FM), where the

seller has access to the public record of the buyer's trading history. That means the seller can tell the buyer's type from his trading history. It should be clear from this setup that, in the FM, sellers will only trade with Type I buyers. Assuming all the agents trade in the LM, the probabilities of trade for Type I buyers, Type II buyers and sellers are, respectively, 1, ρ , and $\rho + \alpha(1 - \rho)$. The record keeping is perfect in both LM and FM. Also, α and ρ are both common knowledge to all the agents.

2.3 Equilibrium

I first study the equilibrium with an exogenous debt limit, which is similar to Carapella and Williamson (2015). Since Type II buyers cannot repay their debt, they get zero consumption in the FM. In the LM, they always mimic Type I buyers but renege on their debt in the second subperiod. The seller's choice is simple – they accept the offer if and only if the expected repayment covers the production cost. Therefore, I focus on the Type I buyer's problem.

In the first subperiod, if the meeting is full-record-access, sellers know the buyer's type from the past trading history. If a seller meets a buyer with no default history, he will trade with him, because he knows that the buyer is Type I and is in good standing. The buyer will not default in equilibrium as the repayment constraint, as will be shown below, will make sure he pays back. Meanwhile, sellers will not trade with Type II buyers since they cannot repay the debt. If the meeting is limited-record-access, sellers believe that α of the offer is from Type I buyers, and $1 - \alpha$ of the offer is from Type II buyers. Given sellers' beliefs, Type I buyers need to include a default premium in their offer to compensate sellers' potential loss of trading with Type II buyers. Obviously, Type II buyers make the same offer as Type I buyers in the LM, but they do not repay their debt. They consume the same amount of good as Type I buyers in the LM, and consume zero in the FM. In any period t , let V be

the Type I buyer's value function. A representative Type I buyer's choice problem is

$$V = \max_{q_L, d_L} \rho[u(q_L) - d_L] + \max_{q_F, d_F} (1 - \rho)[u(q_F) - d_F] + \beta V_{+1} \quad (2.1)$$

$$\text{s.t. } \alpha d_L \geq q_L \quad (2.2)$$

$$d_F \geq q_F \quad (2.3)$$

$$d_L \leq D \quad (2.4)$$

$$d_F \leq D \quad (2.5)$$

where subscript +1 indicates the next period value, and D is the buyer's debt limit, which is treated as exogenously given for now. They cannot borrow more than D units of loan. Subscripts L and F denote LM and FM amounts, respectively. Equation (2.1) says that in each meeting, LM or FM, a Type I buyer chooses an offer (q_L, d_L) or (q_F, d_F) to maximize his utility, subject to the seller's participation constraints (2.2) and (2.3), and his debt limits (2.4) and (2.5). Because the buyer makes a take-it-or-leave-it offer to the seller, (2.2) and (2.3) must hold at equality. Otherwise, the buyer can increase his surplus by decreasing the payment amount, and the seller will still accept this offer. Depending on whether the debt constraints (2.4) and (2.5) bind or not, there are three possible cases.

Case 1 *Neither debt constraint binds*

First, suppose neither (2.4) nor (2.5) binds. It is easy to show that $q_F = q^*$ and $q_L = q_\alpha$, where q^* and q_α solve $u'(q^*) = 1$ and $u'(q_\alpha) = 1/\alpha$, respectively. We refer to q^* as the market efficient quantity. As long as $\alpha < 1$, we have $q_\alpha < q^*$. Because of the existence of defaulters, non-defaulters consume less than the efficient quantity q^* in the LM, even though the debt constraint does not bind. And the condition for unbinding constraints is $D \geq q^*$. Also, note that because $-qu''(q)/u'(q) < 1$, $q_\alpha/\alpha < q^*$ for $\alpha < 1$, which means constraint (2.5) is tighter than constraint (2.4) in equilibrium.

Case 2 *LM debt constraint does not bind, FM debt constraint binds*

Now suppose constraint (2.4) does not bind, but constraint (2.5) binds. In this case, Type I buyers can still consume $q_L = q_\alpha$ units of good in the LM. But because of the constrained debt limit, Type I buyers can only consume $q_F = D$ units of good in the FM. In this case, $q_\alpha/\alpha \leq D < q^*$.

Case 3 *Both debt constraints bind*

If both constraints (2.4) and (2.5) bind, Type I buyers will exhaust their credit in both LM and FM; i.e., $d_L = d_F = D$. In the FM, they can consume $q_F = D$, but in the LM they can only consume $q_L = \alpha D$. In this case, $D < q_\alpha/\alpha$. I summarize these three cases as follows.

Proposition 2 *Given an exogenous debt limit D , there are three possible cases:*

1. *If $D < q_\alpha/\alpha$, then $q_L = \alpha D$, $q_F = D$, and $d_L = d_F = D$.*
2. *If $q_\alpha/\alpha \leq D < q^*$, then $q_L = q_\alpha$, $q_F = D$, and $d_L = q_\alpha/\alpha$, $d_F = D$.*
3. *If $D \geq q^*$, then $q_L = q_\alpha$, $q_F = q^*$, and $d_L = q_\alpha/\alpha$, $d_F = q^*$.*

2.4 Endogenous Debt Limit

In the previous section, I assume the debt limit D is given exogenously. Here, I show that with given default punishment, the debt limit is determined endogenously in equilibrium. The default punishment is assumed to be permanent autarky. Therefore, Type I buyers will honor their debt obligation only if the gain of repaying their debt and maintaining the access to future credit exceeds the gain of defaulting. Let \widehat{V} denote the defaulter's value function. In the steady state, it can be expressed as

$$\widehat{V} = \frac{\rho u(q_L)}{1 - \beta} \tag{2.6}$$

and the non-defaulter's steady state value functions is

$$V = \frac{\rho[u(q_L) - q_L/\alpha] + (1 - \rho)[u(q_F) - q_F]}{1 - \beta}. \quad (2.7)$$

The self-enforcing contract requires

$$-d + \beta V \geq \beta \widehat{V}. \quad (2.8)$$

Plugging (2.6) and (2.7) into (2.8), the Type I buyer's debt limit satisfies

$$d \leq \frac{\beta}{1 - \beta} \{-\rho q_L/\alpha + (1 - \rho)[u(q_F) - q_F]\}. \quad (2.9)$$

From Proposition 2, we know that depending on whether the debt limit D binds or not in the LM and FM, the right-hand side of (2.9) can be expressed as

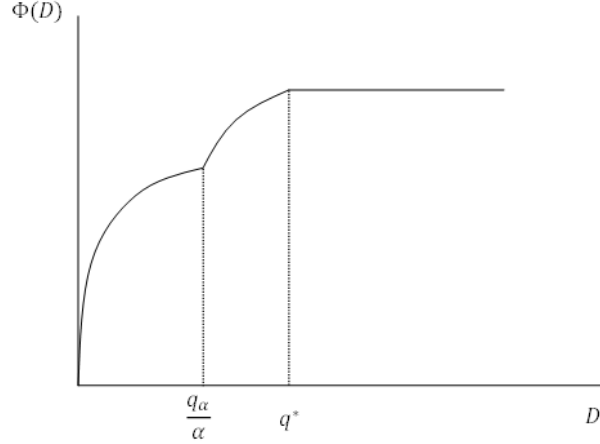
$$\Phi(D) \equiv \begin{cases} \beta[(1 - \rho)u(D) - D]/(1 - \beta) & \text{if } D < q_\alpha/\alpha \\ \beta\{-\rho q_\alpha/\alpha + (1 - \rho)[u(D) - D]\}/(1 - \beta) & \text{if } q_\alpha/\alpha \leq D < q^* \\ \beta\{-\rho q_\alpha/\alpha + (1 - \rho)[u(q^*) - q^*]\}/(1 - \beta) & \text{if } D \geq q^*. \end{cases} \quad (2.10)$$

The three branches correspond to the three cases in Proposition 2, respectively.

Following the method in Gu et al. (2016), we have the following definition:

Definition 4 *An endogenous debt limit is a nonnegative fixed point $\Phi(D^*) = D^*$.*

Figure 2-1 plots $\Phi(D)$ against D . Obviously, $\Phi(0) = 0$ is a fixed point: if the debt limit is zero, buyers have no incentive to repay their debt, since doing so will not give them access to future credit. Sellers know that they can never get repayment from buyers, so they will not lend anything to buyers. Notice $\Phi(D)$ is continuous, the third branch of (2.10) is constant, and it is easy to show that $\Phi'(0) = +\infty$. Thus, besides $\Phi(0) = 0$, there exists at least one positive fixed point such that $\Phi(D^*) = D^*$. Note that each branch of $\Phi(D)$ is concave, but $\Phi(D)$ is not concave on $[0, \infty)$, so we cannot guarantee the uniqueness of positive D^* .

Figure 2-1: $\Phi(D)$

If $\rho \rightarrow 0$, sellers can tell the buyer's type in all the meetings. They trade with non-defaulters only and buyers do not need to include a default premium in their offer. In this case, if agents are patient enough, namely β is sufficiently large, we can achieve the efficient quantity q^* in the FM. On the other hand, if $\rho \rightarrow 1$, the third branch of (2.10) is negative even though agents are sufficiently patient; i.e., $\beta \rightarrow 1$. In other words, if the probability of full-record-access meeting is small, the credit limit always binds. As ρ increases, the risk premium increases and the expected FM trading surplus decreases. As a result, borrowers are only willing to repay a very small amount of debt. The necessary condition for the existence of a stationary equilibrium with an unbinding debt limit is

$$\rho < \frac{u(q^*) - q^*}{u(q^*) - q^* + q_\alpha/\alpha}.$$

2.5 Optimal Default Punishment

The previous section considers permanent autarky as punishment since it is the severest punishment for unsecured credit and can support the highest debt limit. However,

it is worth asking how harsh the punishment should be when default is inevitable.

Different from previous sections, now assume that the buyer's type is no longer permanent. Instead, in every period, a productivity shock is realized at the beginning of the second subperiod that with probability $1 - \alpha$ the buyer cannot produce, which means he has to default on the debt in that period. Moreover, this productivity shock is private information to the buyer. Therefore, when default happens, the seller cannot tell whether it is because the buyer is unable to produce or it is because the buyer does not want to produce, which are sometimes referred to as "involuntary default" and "voluntary default," respectively (see Chatterjee et al. 2007). Due to private information, a contract contingent on shocks is not feasible here.¹ Thus, any buyers who fail to repay their debt will be treated as defaulters regardless of whether the default is voluntary or involuntary.

I also assume that any agents who default in period t will be marked as defaulters for n periods. Thus, n measures how harsh the punishment is. If $n = 0$, the defaulters will not be punished; and if $n = \infty$, the defaulters will be excluded from trade forever. The main purpose of this section is to find the social welfare maximizing punishment level n , given the exogenous default rate $1 - \alpha$. To simplify the model, from now on I assume $\rho = 0$, which means all the meetings are full-record-access meetings. The non-defaulter's steady state value function is

$$V = u(q) + \alpha(-q/\alpha + \beta V) + (1 - \alpha)\beta\widehat{V} \quad (2.11)$$

where \widehat{V} is the defaulter's value function. Value function (2.11) says, as a non-defaulter, he can consume q units of good in the first subperiod; with probability α , he repays the debt $d = q/\alpha$ in the second subperiod and is treated as a non-defaulter in the next period; while with probability $1 - \alpha$, he cannot repay the debt and will

¹Townsend (1982), Atkeson and Lucas (1995), and Aiyagari and Williamson (1999) discuss the effect of private information on credit arrangement.

be treated as a defaulter in the next period. The defaulter's value function is

$$\widehat{V} = \beta^n V. \quad (2.12)$$

Value function (2.12) says a defaulter will stay in autarky for n periods, and after that, he becomes a non-defaulter again. Plugging (2.11) and (2.12) into (2.8), the non-defaulter's debt limit is

$$d \leq \frac{\beta(1 - \beta^n)}{1 - \alpha\beta - (1 - \alpha)\beta^{n+1}} [u(q) - q]. \quad (2.13)$$

Depending on the value of D , we rewrite the right-hand side of (2.13) as

$$\Phi(D) \equiv \begin{cases} \beta(1 - \beta^n)[u(\alpha D) - \alpha D]/[1 - \alpha\beta - (1 - \alpha)\beta^{n+1}] & \text{if } D \leq q^*/\alpha \\ \beta(1 - \beta^n)[u(q^*) - q^*]/[1 - \alpha\beta - (1 - \alpha)\beta^{n+1}] & \text{if } D > q^*/\alpha. \end{cases} \quad (2.14)$$

The endogenous debt limit D^* solves $\Phi(D) = D$, which means $D = D(n)$ is a function of n . The total trading surplus S is given by

$$(1 - \beta)S = \theta[u(q) - q]$$

where $\theta = 1/[1 + (1 - \alpha)n]$ is the total measure of non-defaulters in the steady state.² It is easy to show that θ is monotonically decreasing with n . The harsher the punishment, the less active buyers in the economy. Buyers make a take-it-or-leave-it offer, so the total trading surplus in the steady state is the buyer's expected lifetime utility. The optimal length of default punishment n^* solves

$$n^* = \arg \max_n \theta[u(q) - q] \quad (2.15)$$

where $q = D(n)$ is also a function of n , if the buyer's debt limit binds. Otherwise $q = q^*$ is constant. Thus, depending on whether the debt limit D binds or not, the

²In the steady state $(1 - \alpha)\theta = (1 - \theta)/n$. The LHS is the measure of new defaulters, and the RHS is the measure of buyers who finished the n -period penalty. Solving for θ , we have $\theta = 1/[1 + (1 - \alpha)n]$.

right-hand side of (2.15) can be expressed as

$$G(n) \equiv \begin{cases} \theta[u(\alpha D) - \alpha D] & \text{if } D \leq q^*/\alpha \\ \theta[u(q^*) - q^*] & \text{if } D > q^*/\alpha. \end{cases} \quad (2.16)$$

In the first branch, $\theta = \theta(n)$ and $D = D(n)$ are both functions of n . In the second branch, $G(n)$ is independent of D and is monotonically decreasing with n . So n^* cannot be located on this branch. Intuitively, when the buyer's debt constraint does not bind, further relaxing D by increasing n does not change q , but it reduces the measure of active buyers.

It is clear that n should be neither too large nor too small. In particular, as $n \rightarrow 0$, $D \rightarrow 0$, which means $q \rightarrow 0$. The economy becomes an autarky, so the total social welfare is minimized. On the other hand, as $n \rightarrow \infty$, the debt limit D will increase, but in the steady state, the measure of non-defaulter is $\theta \rightarrow 0$, so the total social welfare is also minimized. Intuitively, although severer punishment can support a higher debt limit, it also means more buyers are excluded from the market, which decreases the frequency of trade. If the punishment is too mild, however, borrowers do not lose much from defaulting, which leads to a low debt limit. Thus, the optimal default punishment should be intermediate, which is consistent with Diamond (1984) and Dubey et al. (2005).

Given the nonlinearity of this model, it is hard to characterize, in general, the relationship among n , D , and S . So, I illustrate the results with examples using utility function

$$u(q) = \frac{q^{1-\gamma}}{1-\gamma}.$$

Example 1. Let $\alpha = 0.8$, $\beta = 0.95$, and $\gamma = 0.3$. The left panel of Figure 2-2 plots $G(n)$ against n . At the origin, $G(0) = 0$, if there is no default punishment, no buyer repays the debt, and sellers will lend nothing to buyers. So the total trading surplus is zero when $n = 0$. Increasing n from 0 to 2 raises the debt limit to 0.4627,

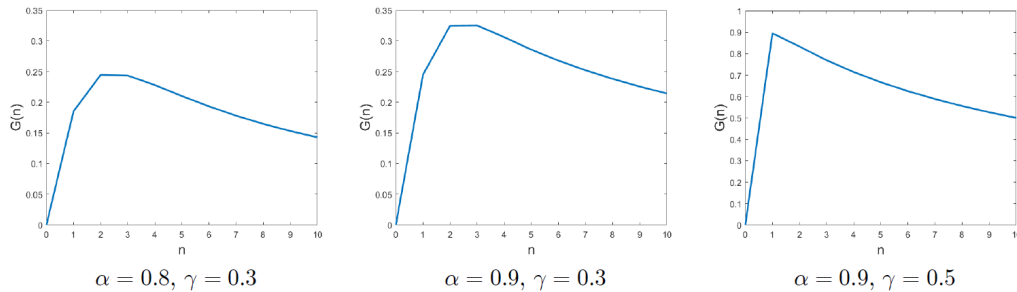
and reduces the measure of active buyers to 0.7143. The effect of enlarging the debt limit D dominates the effect of reducing the measure of active buyers. And $G(n)$ increases from 0 to 0.2445. A further increase in n can still raise the debt limit, but the effect of reducing the measure of active buyers dominates in this case. So $G(n)$ is maximized when $n = 2$ and $G(2) = 0.2445$.³

Example 2. The middle panel of Figure 2-2 uses the same utility function and parameters except that $\alpha = 0.9$. In this case, $G(n)$ is maximized when $n = 3$ and $G(3) = 0.3252$. The measure of active buyers is $\theta = 0.7692$ and the debt limit is $D = 0.9013$, which binds. The optimal n is larger than the previous case, because when the probability of default is low, a harsher punishment can support a higher debt limit, which in turn increases the quantity of good traded. And, it does not decrease the frequency of trade too much. Indeed, a special case is $\alpha = 1$, as in the strand of literature starting with Kehoe and Levine (1993). If default never happens in equilibrium, the total welfare is, in general, maximized when $n \rightarrow \infty$.

Example 3. The right panel of Figure 2-2 uses the same parameters as in Example 2 except that $\gamma = 0.5$. Now $G(n)$ is maximized when $n = 1$ and $G(1) = 0.8953$. The measure of active buyers is $\theta = 0.9091$ and the debt limit is $D = 0.8544$, which binds. As the elasticity of intertemporal substitution decreases, buyers have more incentive to stay in the credit economy. Thus, the optimal n is smaller.

I should point out that even though we have the optimal n under this circumstance, this is only a local optimal point. It is not the global optimal default punishment, because I restrict the punishment to be "autarky for n periods." A better credit arrangement specifies an agent's credit limit in every period based on the entire repayment history. In the following section, I consider a two-step default punishment

³When $n \leq 8$, $G(n)$ is located on the first branch. When $n > 8$, it is located on the second branch, where $G(n)$ is monotonically decreasing with n . So $n = 2$ is the maximum point.

Figure 2-2: $G(n)$

scheme as a first attempt toward understanding such credit arrangement.

2.6 Extension: Two-Step Default Punishment

I now consider a different kind of default punishment: buyers do not go to autarky immediately after defaulting. If a buyer defaults on debt for the first time, he is called a one-time defaulter and receives a different debt limit \widehat{D} for the next period. Then, if he defaults again next period, he will be marked as a two-time defaulter and stay in autarky for the next n periods. If the one-time defaulter repays both current debt and the debt from last period, he will be treated as a non-defaulter again from the next period and get debt limit D .⁴ With probability $1 - \alpha$, a buyer cannot produce in the second subperiod. So, in the steady state, the measure of non-defaulters, one-time defaulters, and two-time defaulters are, respectively, θ , $(1 - \alpha)\theta$, and $1 - (2 - \alpha)\theta$, where $\theta = 1/[(1 - \alpha)^2 n - \alpha + 2]$.⁵

⁴A one-time defaulter becomes a non-defaulter only if he pays both debts from two periods. If he only pays one debt, he will still be marked as a two-time defaulter from next period.

⁵In the steady state, $(1 - \alpha)^2 \theta = [1 - \theta - (1 - \alpha)\theta]/n$. The LHS is the measure of new two-time defaulters, and the RHS is the measure of two-time defaulters who finished the n -period punishment. Solving for θ , we have $\theta = 1/[(1 - \alpha)^2 n - \alpha + 2]$.

If the non-defaulter rolls over his current debt d , he needs to repay λd , where $\lambda \geq 0$, to a common fund next period, and all the sellers who do not receive repayment will share this fund.⁶ In general, this setup allows for a contract contingent on the entire trading history. But here I restrict my attention to the two-step scheme. If $\lambda > 1$, the buyer pays extra interest on the debt, and if $\lambda < 1$, he can have some amount of debt waived. Let \widehat{V} be the one-time defaulter's value function. Their problem is

$$\widehat{V} = \max_{\widehat{q}, \widehat{d}} u(\widehat{q}) + \alpha(-\widehat{d} - \lambda d) + \beta[\alpha V + (1 - \alpha)\widetilde{V}] \quad (2.17)$$

$$\text{s.t. } \alpha \widehat{d} \geq \widehat{q} \quad (2.18)$$

$$\widehat{d} \leq \widehat{D} \quad (2.19)$$

where \widehat{d} denotes the one-time defaulter's debt, and $\widetilde{V} = \beta^n V$ is the two-time defaulter's value function. Equation (2.17) says that given the accumulated debt λd , the one-time defaulter chooses an offer $(\widehat{q}, \widehat{d})$ to maximize his utility, subject to the seller's participation constraint (2.18) and his debt limit (2.19). Let V be the non-defaulter's value function. Given λ , their problem is

$$V = \max_{q, d} u(q) - \alpha d + \beta[\alpha V + (1 - \alpha)\widehat{V}] \quad (2.20)$$

$$\text{s.t. } \alpha d + (1 - \alpha)\alpha\beta\lambda d \geq q \quad (2.21)$$

$$d \leq D. \quad (2.22)$$

Note that because of the linearity issue, if we treat λ as a choice variable, there will be a continuum of equilibrium D and λ . So here we treat λ as a policy variable, which specifies the interest rate for the delayed repayment. In the steady state, V can be expressed as

$$V = \frac{u(q) - [\alpha + (1 - \alpha)\alpha\beta\lambda]d + (1 - \alpha)\beta[u(\widehat{q}) - \alpha\widehat{d}]}{1 - \alpha\beta - (1 - \alpha)\beta[\alpha\beta + (1 - \alpha)\beta^{n+1}]} \quad (2.23)$$

⁶Note that the buyer and the seller meet only for one period, so the one-time defaulter cannot pay the seller in the second period directly.

It is easy to show that the non-defaulter's endogenous debt limit satisfies

$$d \leq \beta(V - \widehat{V}) \quad (2.24)$$

and the one-time defaulter's endogenous debt limit satisfies

$$\widehat{d} \leq \beta(\widehat{V} - \widetilde{V}) - \lambda d. \quad (2.25)$$

Inequality (2.24) says the non-defaulter will repay the debt instead of rolling it over if he can produce now. Inequality (2.25) says that the one-time defaulter prefers repaying two periods' debts to defaulting on them. Plugging (2.17) and $\widetilde{V} = \beta^n V$ into (2.24) and (2.25), we have

$$d \leq \beta \left\{ [1 - \alpha\beta - (1 - \alpha)\beta^{n+1}] V - \left[u(\widehat{q}) - \widehat{q} - \frac{\alpha\lambda q}{\alpha + (1 - \alpha)\alpha\beta\lambda} \right] \right\} \quad (2.26)$$

and

$$\widehat{d} \leq \beta(1 - \beta^n)V - \lambda d \quad (2.27)$$

where V is given by (2.23). Depending on the value of \widehat{D} , D , and q^* , there are four possible cases:

1. If $D < q^*/[\alpha + (1 - \alpha)\alpha\beta\lambda]$ and $\widehat{D} < q^*/\alpha$, both the non-defaulter's and one-time defaulter's debt limits bind, so $\widehat{d} = \widehat{D}$, $d = D$, $\widehat{q} = \alpha\widehat{D}$, and $q = [\alpha + (1 - \alpha)\alpha\beta\lambda]D$.
2. If $D \geq q^*/[\alpha + (1 - \alpha)\alpha\beta\lambda]$ and $\widehat{D} < q^*/\alpha$, only the one-time defaulter's debt limit binds, so $\widehat{d} = \widehat{D}$, $d = q^*/[\alpha + (1 - \alpha)\alpha\beta\lambda]$, $\widehat{q} = \alpha\widehat{D}$, and $q = q^*$.
3. If $D < q^*/[\alpha + (1 - \alpha)\alpha\beta\lambda]$ and $\widehat{D} \geq q^*/\alpha$, only the non-defaulter's debt limit binds, so $\widehat{d} = q^*/\alpha$, $d = D$, $\widehat{q} = q^*$, and $q = [\alpha + (1 - \alpha)\alpha\beta\lambda]D$.
4. If $D \geq q^*/[\alpha + (1 - \alpha)\alpha\beta\lambda]$ and $\widehat{D} \geq q^*/\alpha$, neither debt limit binds, so $\widehat{d} = q^*/\alpha$, $d = q^*/[\alpha + (1 - \alpha)\alpha\beta\lambda]$ and $\widehat{q} = q = q^*$.

Based on these four possible cases, the right-hand side of (2.26) and (2.27) can be expressed as, respectively,

$$\Phi(D) \equiv \left\{ \begin{array}{ll} \xi\{u(\kappa D) - \kappa D + (1 - \alpha)\beta[u(\alpha\widehat{D}) - \alpha\widehat{D}] - \beta[u(\alpha\widehat{D}) - \alpha\widehat{D} - \alpha\lambda D]\} & \text{if } \begin{array}{l} D < q^*/\kappa \\ \widehat{D} < q^*/\alpha \end{array} \\ \xi\{u(q^*) - q^* + (1 - \alpha)\beta[u(\alpha\widehat{D}) - \alpha\widehat{D}] - \beta[u(\alpha\widehat{D}) - \alpha\widehat{D} - \alpha\lambda q^*/\kappa]\} & \text{if } \begin{array}{l} D \geq q^*/\kappa \\ \widehat{D} < q^*/\alpha \end{array} \\ \xi\{u(\kappa D) - \kappa D + (1 - \alpha)\beta[u(q^*) - q^*] - \beta[u(q^*) - q^* - \alpha\lambda D]\} & \text{if } \begin{array}{l} D < q^*/\kappa \\ \widehat{D} \geq q^*/\alpha \end{array} \\ \xi\{u(q^*) - q^* + (1 - \alpha)\beta[u(q^*) - q^*] - \beta[u(q^*) - q^* - \alpha\lambda q^*/\kappa]\} & \text{if } \begin{array}{l} D \geq q^*/\kappa \\ \widehat{D} \geq q^*/\alpha \end{array} \end{array} \right.$$

and

$$\widehat{\Phi}(\widehat{D}) \equiv \left\{ \begin{array}{ll} \beta(1 - \beta^n)\{u(\kappa D) - \kappa D + (1 - \alpha)\beta[u(\alpha\widehat{D}) - \alpha\widehat{D}]\} / \{1 - \alpha\beta - (1 - \alpha)\beta[\alpha\beta + (1 - \alpha)\beta^{n+1}]\} - \lambda D & \text{if } \begin{array}{l} D < q^*/\kappa \\ \widehat{D} < q^*/\alpha \end{array} \\ \beta(1 - \beta^n)\{u(q^*) - q^* + (1 - \alpha)\beta[u(\alpha\widehat{D}) - \alpha\widehat{D}]\} / \{1 - \alpha\beta - (1 - \alpha)\beta[\alpha\beta + (1 - \alpha)\beta^{n+1}]\} - \lambda q^*/\kappa & \text{if } \begin{array}{l} D \geq q^*/\kappa \\ \widehat{D} < q^*/\alpha \end{array} \\ \beta(1 - \beta^n)\{u(\kappa D) - \kappa D + (1 - \alpha)\beta[u(q^*) - q^*]\} / \{1 - \alpha\beta - (1 - \alpha)\beta[\alpha\beta + (1 - \alpha)\beta^{n+1}]\} - \lambda D & \text{if } \begin{array}{l} D < q^*/\kappa \\ \widehat{D} \geq q^*/\alpha \end{array} \\ \beta(1 - \beta^n)\{u(q^*) - q^* + (1 - \alpha)\beta[u(q^*) - q^*]\} / \{1 - \alpha\beta - (1 - \alpha)\beta[\alpha\beta + (1 - \alpha)\beta^{n+1}]\} - \lambda q^*/\kappa & \text{if } \begin{array}{l} D \geq q^*/\kappa \\ \widehat{D} \geq q^*/\alpha \end{array} \end{array} \right.$$

where $\xi = \beta[1 - \alpha\beta - (1 - \alpha)\beta^{n+1}] / \{1 - \alpha\beta - (1 - \alpha)\beta[\alpha\beta + (1 - \alpha)\beta^{n+1}]\}$ and $\kappa = \alpha + (1 - \alpha)\alpha\beta\lambda$. Given λ and n , the endogenous debt limits D and \widehat{D} solve the

system of equations $\Phi(D) = D$ and $\widehat{\Phi}(\widehat{D}) = \widehat{D}$. The corresponding trading surplus $G(\lambda, n)$ is

$$G(\lambda, n) \equiv \begin{cases} \theta[u(\kappa D) - \kappa D] + (1 - \alpha)\theta[u(\alpha\widehat{D}) - \alpha\widehat{D}] & \text{if } \begin{array}{l} D < q^*/\kappa \\ \widehat{D} < q^*/\alpha \end{array} \\ \theta[u(q^*) - q^*] + (1 - \alpha)\theta[u(\alpha\widehat{D}) - \alpha\widehat{D}] & \text{if } \begin{array}{l} D \geq q^*/\kappa \\ \widehat{D} < q^*/\alpha \end{array} \\ \theta[u(\kappa D) - \kappa D] + (1 - \alpha)\theta[u(q^*) - q^*] & \text{if } \begin{array}{l} D < q^*/\kappa \\ \widehat{D} \geq q^*/\alpha \end{array} \\ \theta[u(q^*) - q^*] + (1 - \alpha)\theta[u(q^*) - q^*] & \text{if } \begin{array}{l} D \geq q^*/\kappa \\ \widehat{D} \geq q^*/\alpha. \end{array} \end{cases}$$

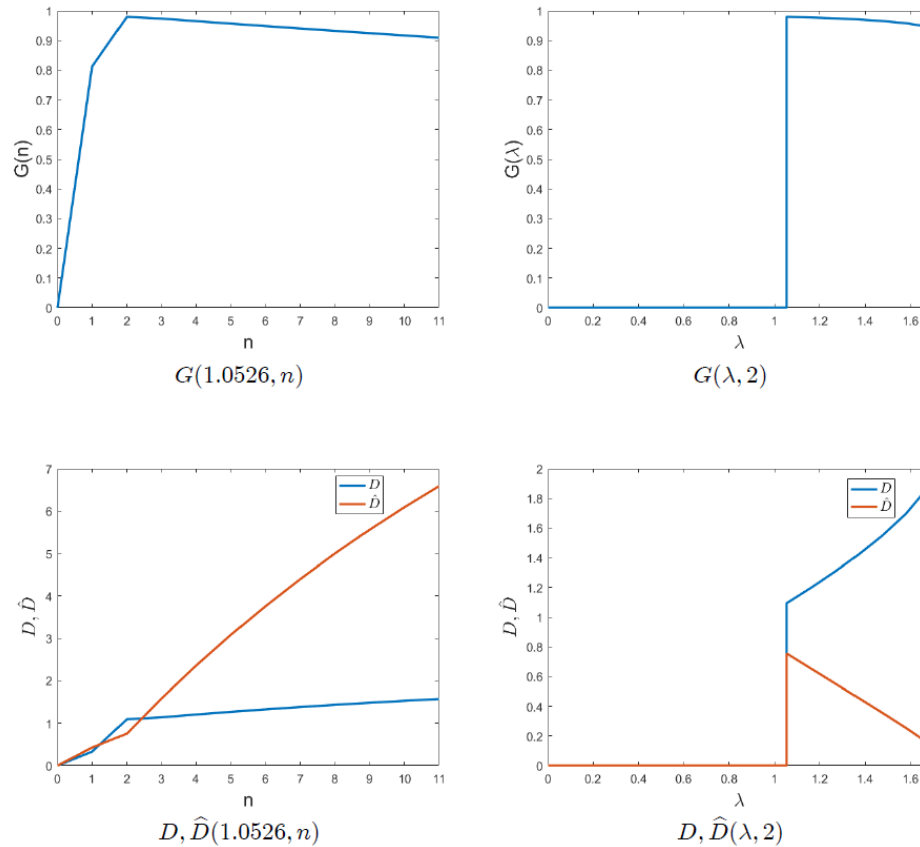
Again, I illustrate the results with examples using the same utility function as in the previous section.

Example 4. Continuing Example 3, $G(\lambda, n)$ is maximized when $\lambda = 1.0526$ and $n = 2$. In this case, the measure of non-defaulters is $\theta = 0.8929$ and the measure of one-time defaulters is $(1 - \alpha)\theta = 0.0893$, which means only 0.0178 of buyers are excluded from trade. The non-defaulter's debt limit is $D = 1.0936$, which does not bind, and the one-time defaulter's debt limit is $\widehat{D} = 0.7551$, which binds. The maximized trading surplus is $G(1.0526, 2) = 0.9794$. Recall that in Example 3, the maximized surplus is $G(1) = 0.8953$, which is smaller than 0.9794. And the measure of inactive buyers was 0.0909, which is much higher than 0.0178. That means the two-step punishment scheme is a better credit arrangement than the one we considered in the previous section as it provides more flexibility. If we fix λ , $G(n)$ becomes a function of n . Similarly, if we fix n , then we can plot $G(\lambda)$ against λ . Figure 2-3

shows these two cases.

In the upper left panel of Figure 2-3, I fix $\lambda = 1.0526$ and plot $G(n)$ against n . Similar to Figure 2-2, $G(n)$ starts from point $(0,0)$. When $n = 1$, both the non-defaulter's and one-time defaulter's debt limits bind. When $n = 2$, only the one-time defaulter's debt limit binds, and $G(n)$ is maximized. When $n \geq 3$, neither debt limit binds and $G(n)$ is monotonically decreasing with n . The lower left panel of Figure 2-3 plots D and \hat{D} against n . Harsher punishment increases D and \hat{D} simultaneously. Note that \hat{D} is not necessarily lower than D . Intuitively, the two-time defaulter will be punished to autarky immediately, whereas the one-time defaulter has one more chance to repay the debt so the punishment to the first time default is not so severe.

In the upper right panel of Figure 2-3, I fix $n = 2$ and plot $G(\lambda)$ against λ . When $\lambda < 1.0526$, $G(\lambda) = 0$. When $\lambda > 1.0526$, $G(\lambda)$ is decreasing with λ . The lower right panel of Figure 2-3 plots D and \hat{D} against λ . When $\lambda < 1.0526$, $D = 0$, and $\hat{D} = 0$, the economy is an autarky. Intuitively, for a non-defaulter, rolling over the debt has some cost: first, if he gets a productivity shock in the following period, he will be excluded from trade for n periods; second, he pays interest on the debt. If $\lambda < 1/\beta$, the second one actually becomes a benefit. When this benefit exceeds the potential loss of getting a productivity shock, no one pays the debt in the first period, so we have $D = 0$. When $\lambda > 1.0526$, D is increasing with λ and \hat{D} is decreasing with λ . When $\lambda \geq 1.6842$, $\hat{D} = 0$. That is because one-time defaulters have already accumulated a large amount of debt. The default punishment can no longer support any debt. So, one-time defaulters are actually treated the same as two-time defaulters. On the other hand, non-defaulters will only repay the debt in the first period. If they roll over the debt because of the productivity shock, they will renege on it in the second period because the cost of repaying debt λd exceeds the cost of being excluded from trade for two periods. In this case, the credit scheme reduces to the one we studied

Figure 2-3: G, D and \hat{D}

in the previous section.

These results have several significant implications. From a theoretical perspective, these results provide us an initial insight for understanding more complicated credit arrangements. From a policy perspective, when we have more policy tools (i.e., n and λ) to implement the credit arrangement, it usually provides us a better allocation. But, we should be careful when picking these tools. An inappropriate combination can result in huge welfare loss. From a market perspective, if we treat the two-step punishment as a two-period contract, then a long-term contract is better than a short-term contract.

2.7 Conclusion

In this paper, I study a credit model in which default happens in equilibrium. In terms of result, first, I found that when record keeping and information are perfect, we can achieve the efficient quantity, which is the same as in the equilibrium without defaulters. However, if the access to the agent's trading history is limited, the quantity traded will be lower than the efficient level even though borrowers have unlimited debt limit. The existence of defaulters creates a wedge between the buyer's marginal utility and the seller's marginal cost.

Second, when the agents are subject to idiosyncratic temporary productivity shocks, the optimal punishment should be maintained at an intermediate level. Indeed, harsher default punishment can support a higher debt limit, but it also excludes more agents from trading, which decreases the frequency of trade.

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VITA

Jian Song was born on November 16, 1987 in Hangzhou, Zhejiang, China. He attended Hangzhou Xuejun High School. He received dual bachelor's degrees in Communication and Economics in University of International Relations, Beijing, China in 2010. He enrolled in Ph.D. Program in department of economics, University of Missouri-Columbia in 2010. He received his Ph.D. in Economics in 2016.