PCK DEVELOPMENT OF BEGINNING SECONDARY MATHEMATICS TEACHERS

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by

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The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled

PCK DEVELOPMENT OF
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presented by Rebecca Bruton,

a candidate for the degree of Doctor of Philosophy,

and hereby certify that, in their opinion, it is worthy of acceptance.

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To everyone who has been affected by mental health or mood disorders.

To everyone to has supported someone during a difficult time.

To my husband and children. You are the reason I am here.

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ABSTRACT

In this study I examined the development of pedagogical content knowledge of three beginning secondary mathematics teachers. My analysis included the development of the teachers' knowledge of curriculum and their knowledge of student understanding during their first two years of teaching. The participants began the study with little or no experience or education regarding the teaching of mathematics. During the first year of the study, all three participants were enrolled in graduate level mathematics methods courses and were under the guidance of a mentor teacher.

Using existing research, I analyzed pre and post interviews as well as 12 interviews conducted during 4 observation cycles. Beyond participant interviews, I analyzed interviews with the mentor teachers and researcher notes regarding their observation of the participants' PCK.

Analysis revealed that participants demonstrated little knowledge of curriculum at the beginning of the study. Throughout the study, their knowledge of the curriculum developed differently as they approached teaching in different ways (seen through their goals for instruction) and engaged with their curriculum materials differently. Two of the participants developed detailed knowledge of their students' understanding, which they gained through use of their standards-based curriculum materials and their use of assessment, while the third participant was only able to speak to student understanding at the whole-class level.

CHAPTER 1: STATEMENT OF THE PROBLEM AND BACKGROUND

Among the opportunities afforded to students through schooling is the opportunity to learn. While many factors play a role in student learning, teachers have the most influence as they have a significant amount of contact with students, can pique students' curiosities and interest in learning, and create and enact tasks that guide students to develop their knowledge. In order for teachers to do those things well, they must have the knowledge to do so. Teachers bring a wealth of knowledge to their classroom including content knowledge and pedagogy, but more important is the specialized kind of knowledge that integrates their pedagogical knowledge with their content.

An increasing number of K-12 classrooms are led by teachers who have entered education through alternative certification programs (Humphrey, 2007). These programs differ from bachelor's programs in teacher education and vary considerably among each other (Zeichner & Schulte, 2001). As we look for the best ways to educate future teachers, we must consider alternative certification programs as one pathway for teachers who enter the profession. While much research has been done surrounding teacher learning through traditional four year programs, we lack information about knowledge development of alternatively certified teachers. As standards for teacher education programs continue to evolve, more information about how and what teachers learn through their education experiences will help us make more informed decisions.

Statement of the Problem

In their synthesis of the research, the authors of *Adding It Up* (National Research Council, 2001) found that student "opportunity to learn" is the most important predictor of student achievement. According to the NRC (2001), student opportunity to learn is directly influenced by the teacher because, "What is learned depends on what is taught. Choosing the content, deciding how to present it, and determining how much time to allocate to it are ways in which learning is affected by how the teacher interacts with the content," (p.333). Additionally, in a study among schools in New York city that compared high and low achieving schools with similar student characteristics, Armour-Thomas, Clay, Domanico, Bruno, and Allen found that 90% of the variation in student achievement in reading and mathematics could be accounted for by the differences in teacher qualifications (Armour-Thomas et al., 1989).

Qualifications for teachers have evolved over time and suggest that teachers need more than just a background in content knowledge. In order to explain the complexities of teaching and shed light on the type of knowledge necessary for teaching, the National Academy of Education Committee on Teacher Education (2005) stated that,

> On a daily basis, teachers confront complex decisions that rely on many different kinds of knowledge and judgment and that can involve high-stakes outcomes for students' futures. To make good decisions, teachers must be aware of the many ways in which student learning can unfold in the context of development, learning differences, language and cultural influences, and individual temperaments, interests, and approaches to learning. In addition to foundational knowledge about these areas of learning and performance, teachers need to know how to take the steps necessary to gather additional information that will allow them to make more grounded judgments about what is going on and what strategies may be helpful. (Darling-Hammond & Bransford, 2005, p.2)

By describing the multitude of processes required to make teaching decisions, it is apparent that teachers must call on knowledge beyond content alone. Working with students requires teachers to use a variety of strategies and make complicated decisions.

While research suggests that teacher education and in-service teacher opportunities can lead to the development of teacher knowledge, we lack a characterization of the development of this knowledge. This information is essential so that we can effectively design and implement initial preparation and professional development for teachers.

With further research connecting alternative certification programs to teacher learning, we can maximize teacher learning through these programs by understanding how this knowledge develops. Because alternatively certified teachers spend considerably less time engaging in coursework in preparation for teaching, it especially important to understand how their knowledge develops in order to effectively provide instruction and support.

In the following sections, I further detail the theoretical underpinnings of mathematical knowledge for teaching, as this theoretical lens will guide my data collection and analysis. I then discuss the issues surrounding the specific subset of teachers addressed in this study.

Mathematical Knowledge for Teaching

Darling-Hammond and Bransford (2005) described the complex role encountered by teachers. As a mathematics teacher, one must have knowledge beyond operational mathematics and have specialized knowledge of how to teach mathematics. To better understand what is meant by "mathematical knowledge for teaching," I define the key components of this knowledge and the terms used to describe the specialized knowledge of mathematics teachers. I also briefly explain current research and questions we have about this knowledge and how it develops.

In a push to reform mathematics teacher education, Lee Shulman asked, "How does the successful college student transform his or her expertise in the subject matter into a form that high school students can comprehend?" (Shulman, 1986, p.8). Shulman explained that in order to become effective, teachers develop their pedagogical content knowledge (PCK). This term describes knowledge that combines teacher pedagogical teachers with their knowledge of content. It goes beyond knowing how or what to teach and is an intertwined knowledge of practice and content.

Since Shulman's conception in 1986, researchers have reorganized and restructured the PCK construct. The conceptualized PCK include different components of teacher knowledge and provide descriptions of how teachers' beliefs and epistemology make up one's knowledge for teaching. Deborah Ball and her colleagues describe mathematical knowledge for teaching (MKT) (Ball & Bass, 2003; Hill, Shilling, & Ball, 2004; Hill, Ball, & Shilling, 2008) as including two major categories: subject matter knowledge and pedagogical content knowledge (Hill, et al., 2008). Based on theories about subject matter knowledge and pedagogical knowledge, Hill Ball, and Rowan (2005) found that teachers' mathematical content knowledge for teaching was a significant predictor of student learning gains.

At the same time that researchers were examining the importance of PCK, they were also analyzing how PCK develops, what learning opportunities encourage this development, and how teachers use PCK in their classrooms (Horn, 2005; Kinach, 2002; Magnusson, et al., 1999). Many perspectives exist about what makes a good teacher and what knowledge those teachers should have, but we have little insight into how this knowledge develops (Lannin et al., 2013). Questions remain about the development of PCK, including how much of the knowledge is gained through teacher education coursework or the beginning teaching experiences.

The Council for the Accreditation of Educator Preparation [CAEP] (2013) calls for teacher certification programs to develop PCK for pre-service teachers. However, with little evidence of how teachers actually develop PCK, this may be unproductive. Ball (2000) questioned how this could be included in preservice teacher education, "What would it take to bring the study of content closer to practice and to prepare teachers to know and be able to use subject matter knowledge effectively in their work as teachers?" (Ball, 2000, p.244). This study is designed to further our understanding by examining the sources of knowledge for beginning teachers as they complete their education coursework during their first years of teaching.

Factors Influencing PCK

In order to deepen our understanding of beginning teachers' PCK and how it develops, we need to examine their educational experiences. As new teachers engage in discussion and reflection with others and participate in learning opportunities throughout their pathway to teacher certification, they develop knowledge of instructional strategies, curriculum, students, and assessment. However, we are unsure what specific experiences provide teachers with opportunities to develop PCK.

Studies have shown that this knowledge develops differently among teachers (Lannin, et al., 2013) and that the quantity and quality of PCK varies. Little has been done, however, to determine what factors impact the development of this type of knowledge. In this section, I describe some of the key elements in teacher education and how these components provide an opportunity for future teachers to develop PCK. I begin by discussing the common components of teacher education programs. I then describe the differences among teacher education programs and the pathways in which mathematics teachers can become certified. These similarities and differences provide insight into the sources that contribute to teachers' PCK and how the specific teachers in this study may have different opportunities for PCK development versus and traditionally certified teachers.

Teacher Education's Impact on PCK

Researchers (Darling-Hammond, 2000; Darling-Hammond, Holtzman, Gatlin, & Heilig, 2005; Wilson, Floden, & Ferrini-Mundy, 2002) have shown that teacher education matters. Teacher education programs can provide a rich social setting for beginning teachers and experienced teachers to interact. In a review of the literature on teacher education, Linda Darling-Hammond wrote, "Reviews of research over the past 30 years have concluded that even with the shortcomings of current teacher education and licensing, fully prepared and certified teachers are generally better rated and more successful with students than teachers without this preparation (Ashton & Crocker, 1986; Evertson, Hawley, & Zlotnik, 1985; Greenberg, 1983; Haberman, 1984; Olsen, 1985)" (Darling-Hammond, 2000). Darling-Hammond synthesized the research which demonstrates that higher student gains are correlated with having more teachers who are fully certified and have increased subject-matter and teaching knowledge.

Teacher education can provide an opportunity for teachers to develop PCK (Lowery, 2002; Kinach, 2002). A case study by Lowery (2002) attributed the development of PCK to learning venues in teacher education programs that focus on the interactions among methods courses, the school based context, and collaborative learning environment which included relevant and authentic interactions for the pre-service teachers. Teacher education programs have an opportunity to provide future teachers with a setting that can develop PCK. Many programs include course work in both content and teacher preparation along with time spent in classrooms, engaging with students and experienced teachers. In these settings, pre-service teachers can create mathematical tasks with others, develop content-specific assessments, teach, and learn from their teaching. It is through these social interactions in their programs that they translate these tasks into knowledge.

While we expect teacher education programs provide future teachers with rich learning opportunities, we must realize that not all programs are created equal. Traditional teacher certification programs across the United States include a range of learning opportunities focused on varying components of teacher knowledge. For example, Howey and Zimper (1989) examined the students, faculty, and curricula across various institutions that prepare teachers. They found that the differences among the faculty and programs shaped beginning teachers' beliefs about teaching and contributed to differences in teacher preparation. One aspect that varies among teacher preparation programs is the extent to which students study mathematics and how it is incorporated into teacher development. Mathematical knowledge is a key component of the knowledge needed for teaching and some critics claim that further focus on mathematics content knowledge is necessary to strengthen teachers' knowledge while others say that content knowledge is necessary, but not sufficient. One perspective can be found in The Conference Board of the Mathematical Sciences, American Mathematical Society, and Mathematical Association of America's 2001 report that calls for prospective high school mathematics teachers to complete the equivalent of an undergraduate degree in mathematics, including a 6-hour course connecting their college mathematics courses with high school mathematics. However, Ball (1990) studied prospective teachers' mathematical understanding and found that a background as a mathematics major does not necessarily lead to an explicit understanding of the concepts, principles, or meanings necessary for successful teaching of mathematics. She suggests that in addition to a deep understating of mathematics, teachers need "knowledge about mathematics."

Components of Teacher Education Programs

Adding to the complexity of teacher preparation are the different pathways through which teachers enter the teaching profession through (Darling-Hammond, Chung, & Frelow, 2002; Humphrey, Wechsler, & Hough, 2008). Often, teachers complete a four-year degree program that prepares them to be a classroom teacher. However, because of a need for more mathematics teachers, a variety of alternative certification programs have recruited individuals with content specific degrees. During the course of a traditional teacher certification program preservice teachers often engage in activities or interactions that relate to these components of knowledge. Many alternatively certified teachers, however, have not engaged in these types of experiences prior to entering their alternative certification program. Once enrolled in their program, much of their work focuses on general pedagogy. Because they have already completed the necessary content specific education to become a teacher, their programs include instruction centered on good teaching practices, often disconnected from subject-specific knowledge for teaching.

Another noticeable difference in teacher education programs, both traditional and alternative, are the variety of field experience designs that are utilized. While many teachers read about how students' learning develops and discuss ways in which to support their thinking, some teacher development programs include prospective teacher observing and working with a mentor teacher and students throughout their learning. Darling-Hammond (2006) argued that we have learned that effective teacher preparation, "… require students to spend extensive time in the field throughout the entire program, examining and applying the concepts and strategies they are simultaneously learning about in their courses alongside teachers who can show them how to teach in ways that are responsive to learners" (p. 8). These interactions provide opportunities for future teachers to develop new knowledge through their interactions with that mathematics and the classrooms in which it is taught.

While numerous differences exist in these pathways and each program has its own strengths, these differences fuel debate about the knowledge that is necessary to teach effectively. As we work to improve teacher education, it is important to understand how that knowledge develops through these different pathways. We need to better understand how components of teacher preparation programs contribute to teacher knowledge and to what extent.

Purpose of the Study

As the United States continues to look for more ways to recruit highly qualified mathematics teachers, various alternative certification programs continue to emerge. Since more teachers enter the profession through alternative routes, we must look at their knowledge and examine how it develops through their certification programs and into their teaching. Studies have found beginning teachers lack PCK (Appleton, 2003; Carpenter, Fennema, Petersen, & Carey, 1988; Feiman-Nemser & Parker, 1990). In order to understand beginning teachers' knowledge and how their PCK develops throughout their first two years of teaching, this study characterizes the development of beginning teachers PCK. Specifically, I characterize teachers' knowledge at different points throughout the two years by describing which strands of PCK the participants have demonstrated growth and what that knowledge looks like.

Research Question

In order to characterize the development of beginning teachers' knowledge throughout their certification program and teaching experience, my analysis focused on the following questions:

1. How does the PCK of middle and high school mathematics teachers develop through the first two years of their career?
    a. How does the knowledge of curriculum of middle and high school mathematics teachers develop?
    b. How does the knowledge of students' understanding of middle and high school mathematics teachers develop?

Theoretical Considerations

In this section I describe the components of mathematical knowledge for teaching that will guide this study and describe where they fit within the larger body of teacher knowledge. Furthermore, to frame this study I explain the constructivist lens I used to understand the development of this knowledge and propose an initial framework for the sources of teacher knowledge.

Theoretical Perspective

Learning, including the development of teacher knowledge, takes place in many different situations and through a variety of experiences. While these experiences influence one's learning, the construction of knowledge happens within the individual. It is a function of the brain that takes in conversations and new experiences and makes connections between these things and existing knowledge to create new knowledge. This process applies the new information to your previous experiences, beliefs, and views on how the world works. Built on Piaget's theory of cognitive development, Ernest von Glasersfeld (1996) explains that, "Knowledge is not passively received either through the sense or by way of communication; knowledge is actively built up by the cognizing subject," (p.51). Based on our different experiences upon which our knowledge is built, this web of knowledge is individual and unique. For this study, each participant was analyzed as a separate case and interviews were analyzed in order to study the knowledge development of the individual.

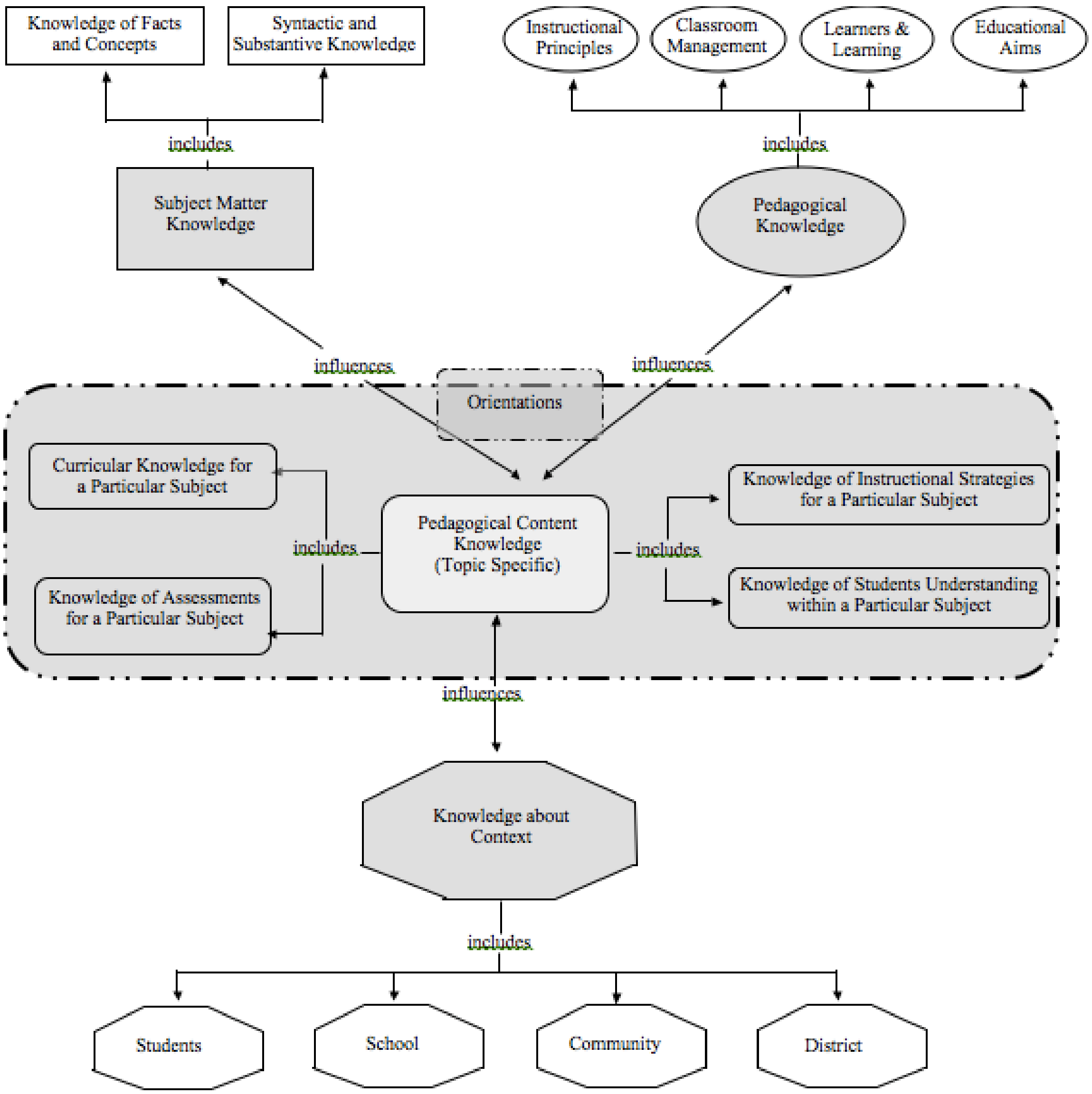
While the main tenants of constructivism include that knowledge belongs to the individual and cannot be merely passed from person to person, our engagement in social activity directly influence our knowledge development. When describing how they saw this knowledge development through social activity, Cobb, Wood, and Yackel (1989) argued that

> …opportunities to construct mathematical knowledge arose from attempts to resolve conflicting points of view (Perret-Clermont, 1980), attempts to reconstruct and verbalize an interpretation of solution (Levina, 1981), attempts to distance the self from ongoing activity in order to understand an alternative interpretation or solution (Sigel, 1981), and, more general, attempts to mutually construct a consensual domain for mathematical activity and discourse with others (Barnes & Todd, 1977). (p.92-93)

However, these social interactions go beyond language. Discrepant events seen through the use of technology or engagement with curriculum materials can facilitate opportunities to resolve conflicts within and reflect on one's own thinking, providing a chance to develop new knowledge. It is important to note that while knowledge is developed through social activity, it is not socially formed and even though knowledge is built in a contextual setting, it does become ours and we can take it and apply it elsewhere. The teachers in this study developed their own knowledge for teaching mathematics and this learning was influenced by a variety of sources.

Cognitive Components of Mathematical Knowledge for Teaching

In order to observe the development of knowledge of mathematics teachers, I used the framework in *figure 1.1*. This framework was developed by the Researching Science and Mathematics Teacher Learning in Alternative Certification Models (ReSMAR^{2}T) project to study the development of alternatively certified mathematics and science teachers. By drawing on the work of Grossman (1990) and Magnusson, Krajcik, & Borko, (1999) this framework provides a representation of the different kinds of knowledge that mathematics teachers develop and how they are related.



From The Re-SMAR2T Project at the University of Missouri-Columbia, Sandra K. Abell, PI, 1/08 based on:

Grossman, P. (1990). *The making of a teacher: Teacher knowledge and teacher education.* New York: Teachers College Press.

Magnusson, S., Krajcik, J. & Borko, H. (1999). Nature, sources, and development of PCK for science teaching. In J. Gess-Newsome & N.G. Lederman (Eds.), *Examining pedagogical Content Knowledge* (pp. 95-132). Dordrecht: Kluwer Academic Publishers.

Figure 1.1 A model of teacher knowledge

Specifically, I studied how teachers develop the four components of PCK. This includes 1)knowledge of curriculum for a particular subject which encompasses knowing what pre-requisite knowledge students need to understand a specific learning objective, how it fits with their prior knowledge, justification for learning a specific topic, goals for instruction, and what materials or tools could benefit their learning of the specific objective, 2)knowledge of instructional strategies which entails knowing what tasks and classroom activities lead to student understanding as well as what questions to ask or homework to assign, 3)knowledge of assessments for a particular subject which includes knowing what to assess and the assessment strategies that could be used for assessing that knowledge, and 4)knowledge of student understanding which requires the teacher to know students' typical learning progressions, common misconceptions about the learning objectives, and where students may struggle. Based on the participants in this study, I chose to analyze the development knowledge of curriculum and knowledge of student understanding.

Beyond these four strands on knowledge, PCK is an interconnection of our understanding of mathematics, teaching, and context and is not developed without regard to our orientations toward teaching. Knowledge and experiences interact to influence one's development of PCK (Van Driel, J. H., Verloop, N., & de Vos, W., 1998). As teachers participate in the cultural activities of our K-12 educational system and teacher preparation programs, they make connections among these categories of knowledge, creating a web of information about teaching mathematics. The internalization of this knowledge can be traced to interactions with others and engagement with social sources.

In this study, teachers engaged in both course-work and a yearlong internship that provided numerous opportunities for knowledge development. In order to look at what social components of their alternative certification program led to their PCK, I developed an initial framework to demonstrate the hypothesized connections among learning opportunities in this specific teacher development program and knowledge for teaching mathematics.

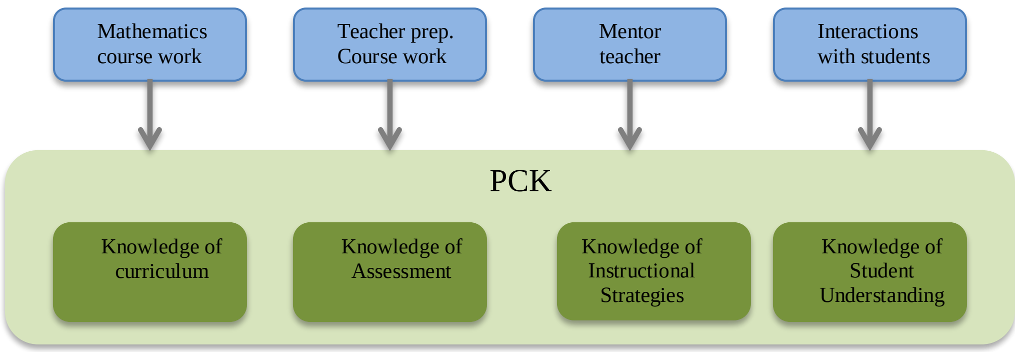


Figure 1.2. Sources of PCK development

The four blue boxes represent factors that have been shown to influence the development of teacher knowledge while the dark green boxes represent the four strands of pedagogical content knowledge described in *figure 1.1*. I propose that teacher interactions related to these four categories lead to the development of PCK.

In this chapter I introduced the problem of practice as a lack of information about the development of mathematics teachers' PCK. I explained the importance of teacher knowledge as knowledge beyond knowledge of the content and pedagogy as separate components and the complexity of the integration of knowledge. I described knowledge development as a process of individual construction and I provided sources that research has shown to contribute to teacher knowledge.

In the following chapters I provide a review of the current literature on teacher knowledge and, more specifically, the pedagogical content knowledge specific to mathematics teachers. I also characterize the issues surrounding alternative certification programs and the opportunities they provide to their students. In the methods chapter, I describe my methodology and provide the coding framework which was used to analyze the two themes described in the findings. Last, I discuss the findings and implications.

CHAPTER 2: REVIEW OF RELATED RESEARCH

To inform my framework that guides my study, I reviewed the literature surrounding teacher knowledge, how it develops, and what we know about alternatively certified teachers specifically. First, I discuss teacher learning and the sources of learning identified from previous research. Then, to better understand teacher learning in an alternative certification program specifically, I define alternative certification and highlight previous findings concerning PCK and teacher preparation in an alternative certification program. I also explain the conceptualization of PCK and share findings from existing literature that describe the development of teachers' PCK. This information deepens the description of the teacher knowledge addressed in my framework and supports my understanding of how to code for PCK in teacher interviews.

Teacher Learning

Teacher education programs provide teachers with the tools they need to understand how students learn, plan and enact lessons, incorporate technology, and tend to the changing demands of the teachers' role. However, teacher education programs are varied and complex and many programs incorporate a variety of learning opportunities. Below, I discuss the importance of teacher education in regards to student learning, followed by three areas of teacher education that have the opportunity to influence teacher learning: a) coursework, b) teaching experience, and c) reflection and collaboration, including working with a mentor teacher.

Under current education reform that include the implementation of the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), we expect K-12 students to learn rigorous mathematics content, develop a knowledge of facts and the underlying concepts, make connections, problem solve, and apply their knowledge in an ever changing world. Considering the complexity of building a classroom that instills all of these components, we must wonder if mathematics teachers have been adequately prepared to meet these demands. This has led to further examination of how we educate teachers, making this issue more critical than ever before.

Teacher education, while greatly varied, can contribute to the use of effective teaching practices. Studies on student performance have shown that teachers who complete a teacher preparation program and are fully certified demonstrate higher student achievement (Ferguson, 1991; Fuller, 1999; Fetler, 1999). However, more than certification is necessary for improving student learning. In her review on teacher quality and student achievement, Darling-Hammond (1999) concluded that "…teacher quality variables appear to be more strongly related to student achievement than class sizes, overall spending levels, teacher salaries (at least when unadjusted for cost of living differentials), or such factors as the statewide proportion of staff who are teachers," (p.38). She elaborates further that, "Among variables assessing teacher "quality," the percentage of teachers with full certification and a major in the field is a more powerful predictor of student achievement than teachers' education levels (e.g., master's degrees)," (Darling-Hammond, p.38, 1999).

Disagreement remains regarding what coursework should look like or what should be included in teacher preparation. However, we know that we need to expand teacher knowledge that leads to increases in student learning. In order to identify the strengths and weaknesses in teacher education, I examine the components of teacher preparation programs that have been shown to support teacher learning.

Coursework. In 1985, the National Education Association surveyed a random sample of 2,530 NEA member teachers. The survey required teachers to respond to 14 sources of learning identified by the NEA. Related directly to teacher education programs, four sources were included: 1. Undergraduate education courses, 2. Undergraduate courses in field of specialization, 3. Graduate college courses in education, and 4. Graduate courses in field of specialization. In his 1989 report of the findings of this survey, Smylie explained that in response to insight into what sources of learning were effective with respect to knowledge and skills needed in teaching situations, graduate courses in field of specialization were among the top responses from high school teachers. He described that this may be a result of the demands of high school teaching and perceptions that dictate the importance of the subject matter (p.547).

Other research also supports importance of subject-matter education. Lowery (2002) observed how preservice teachers developed teacher knowledge and PCK and examined the components that provided students the opportunities to develop this knowledge. He concluded that content-specific methods courses, limited to mathematics and science, provided an important learning venue for teachers (p.76).

Schifter (1998) claimed that as teachers engage in mathematical learning, it is reflected in their teaching, as they are better able to foresee difficulties and connections and anticipate students' thinking. Also, Kennedy (1998), in an analysis of in-service programs, found that programs that focused on subject matter knowledge and knowledge of students were likely to "have a greater impact on student learning than are programs that focus on teaching behavior" (p.10) (Wilson & Berne, 1999).

Beyond courses that focus solely on mathematics content, teacher education programs include coursework in child development, learning theory, and pedagogy. Kinach's (2002) study of pre-service teachers included participants enrolled in a mathematics methods course. The teaching experiment in which the participants engaged in consisted of, "…three tasks designed to elicit, assess, challenge, and develop prospective teachers' instructional explanations (i.e., their PCK) …" (p.56). Kinach demonstrated how these tasks developed a deeper level of subject understanding by demonstrating changes in both their PCK and subject matter knowledge.

Teaching Experience. Smylie (1989) surveyed teachers and identified "direct experience as a teacher" as the most effective source of teacher learning. This is similar to other studies that report that the time spent teaching has the biggest impact on teacher development. Ball and Cohen (1999) explain how knowledge about content, children, student learning, and pedagogy cannot fully prepare teachers for the unpredictable work or interacting with students and curriculum. For this reason, many teacher development programs include classroom experiences or internships as part of their program. As such, beginning teachers interact with students to gain insight into how students learn mathematics.

Franke and Kazemi (2001) begin by describing their work with Cognitively Guided Instruction(CGI). Their previous work with CGI included sharing frameworks of students' mathematical thinking with mathematics teachers. In their study, teachers made sense of children's' thinking, evaluated their own understanding, and figured out how to make use of this knowledge in practice. As Franke and Kazemi observed what happened as a result of sharing this information with teachers and they noticed teachers eliciting students' mathematical thinking and listening to their strategies, they described that the teachers engaged in generative growth. During their four year, longitudinal study, teachers continued to sustain themselves and grow professionally (p.106). Their work began as a cognitive approach but as a result of these observations, they suggest in this article, a more situated approach. They observed teachers continued growth of their understanding of students thinking by learning alongside their students and other teachers.

Similarly, Park and Oliver (2008) concluded that teachers produce knowledge for teaching through their experiences in practice and reflection on its use. Describing what they call "knowledge-in-action", an active process of knowledge development during teaching (p.268) and "knowledge-on-action", a static process of knowledge that has been elaborated through reflection after the teaching process (p.269) they say

> This interrelationship implies that PCK development encompasses knowledge acquisition and knowledge use. It is unlikely that teachers acquire PCK first, and then enact it. Rather, knowledge acquisition and knowledge use are interwoven within the context of instructional practices. (Park & Oliver, 2008, p.278)

Specifically, Park and Oliver address the idea that interactions with students through teaching impacts teachers' PCK development. Through their analysis they identified three interactions with students that led to opportunities for knowledge development. The first way is when students pose challenging questions to teachers. The second way is through teachers' assessment of the students' participation in class. Finally, students' creative and critical ideas led teachers to create innovative instruction for future classes (p.274). They go on to identify opportunities where their participants expanded their PCK as a result of students' misconceptions. They point out that without these key interactions with students, the teachers in their study may not have developed the knowledge they observed them gain.

While course work, internships, and teaching experience are important venues for developing knowledge, research on teacher development programs have found that the integration of field experiences and coursework offers optimal learning experiences (Darling-Hammond, 2006; Lowery, 2002; Koerner, Rust, & Baumgartner, 2002). Students can make sense of the ideas they are learning and apply their coursework in classroom situations.

Teacher Learning Through Curriculum Materials. Remillard and Bryans (2004) conducted a study of eight teachers and their use of standards-based curriculum materials. Through their analysis they determined that the use of the materials offered teachers opportunities to learn by: a) expanding one's repertoire of activities, b) providing insights into student thinking, c) explorations of mathematics, and d) constructing the teacher's role in orchestrating student learning (p.34). These categories parallel with some of the strands of PCK allowing teachers to develop that specific knowledge. In their study, they point out that teacher's orientations towards and views of the curriculum materials affect how the teachers use and engage with the materials. While two teachers who used the materials only minimally did not create opportunities for their own growth, other teachers who had similar opportunities to learn followed different learning trajectories even while using the same materials.

Doerr and Chandler-Olcott (2009) also write about teacher learning through curriculum planning professional development focused on the demands of their standards-based curriculum materials. They found that through they work teachers gained curricular vision as they understood the mathematical ideas of the investigations in their curriculum as well as how it aligned with the development of students' mathematical writing (p.299) This study provides "…evidence for a form of curricular knowledge that involves knowing the links between the development of communicative practices and the development of mathematical content" (p.300).

This is similar to the work of Roth McDuddie and Mather (2009) who also found that teachers working with the same standards-based curriculum exhibited curricular vision. They further defined this as, "the idea that teachers understand where curriculum materials are relative to the mathematical ideas, and what students are learning" (p.316).

Opportunities to collaborate and reflect. Some studies have looked at specific activities that have led to teacher learning. For example, Schifter (1998) described the teacher learning that occurred in an in-service teacher seminar. She found that one helpful learning opportunity for teachers is to analyze transcript of students' words to gain insight into their understandings and misunderstandings. Wood, Cobb, and Yackel (1991) examined how the process of watching and reflecting on classroom video can help teachers develop knowledge about both teaching and student learning. Describing a variety of social settings in which teachers have developed teaching knowledge, Putnam and Borko (2000) noted that through discourse communities, "…community members can draw upon and incorporate each other's expertise to create rich conversations and new insights into teaching and learning" (p.8).

Another key feature of a classroom-based internship is the time spent working with a mentor teacher. Interacting with a mentor teacher can provide beginning teachers with immediate feedback and prompt them to reflect on their practice. In their exploration of the literature surrounding teacher education, Hagger and McIntyre (2000) concluded that, "...effective initial teacher education will require the kind of individual attention to learner teachers' thinking, practice and learning which can ideally be provided through the close ongoing one-to-one working relationships that they should have with their mentors" (p.490).

In 2001, the state of California implemented legislation that provided funding to support beginning teachers. This program included beginning teachers working with a mentor teacher. Fletcher, Strong, and Villar (2008) measured the effects of student achievement in schools that participated in this teacher mentoring. Their analysis suggests that supporting new teachers with a mentor can have positive effects on student achievement. Specifically, they found that new teachers in the school districts involved in their study were more often assigned to low achieving classes, yet their classes had greater achievement gains (p. 2284).

From this research on teacher learning, some important aspects to note are that teacher certification matters and specific components of the certification process have been specifically identified as contributing to teacher knowledge. Content-specific knowledge and knowledge of content that is connected with appropriate teaching methods can have a positive impact on the development of teacher knowledge, along with an opportunity to have hands on classroom experience. Also, working with a mentor can provide teachers with interactions that can develop teacher knowledge.

Alternative Certification

While much research has been conducted concerning teacher knowledge, many studies do not focus on or differentiate the data for alternatively certified teachers. It is important to look at the differences in their experiences and possible sources of learning in alternative certification settings and how that may or may not lead to different PCK.

Alternative certification (Zeichner & Schulte, 2001) is described as an alternative to the traditional undergraduate teacher education program. This is a broad description that includes numerous routes to becoming a teacher. In this paper, I refer to "alternative certification programs" as programs that enroll non-certified individuals who have at least a bachelor's degree (Adelman, 1986).

Research has provided mixed results in terms of teacher knowledge developed and the instructional effectiveness of graduates of alternative certification programs. Presumably, these results emanate from the considerable variation in these programs. Studies have shown that students in alternative certification develop PCK (Brown, Abell, & Friedrichsen, 2008; Lannin et al, 2013), and in their review of teacher preparation and student achievement, Grissom and Vandas (2010) found that, "…studies of specific alternative certification programs generally find few differences between alternatively certified teachers and traditionally prepared teachers with similar in-service experiences (Goebel, Ronacher, & Sanchez, 1989; Hutton, Lutz, & Williamson, 1990; Miller, McKenna, & McKenna, 1998; Raymond, Fletcher, & Luque, 2001)" (p.8). However, many alternative certification graduates claim that they do not feel prepared to teach and the high attrition of graduates is comparable to the teaching field as a whole (Darling-Hammond, 2000, p.168).

Although alternatively certified teachers have been shown to develop PCK, because of the mixed results in terms of effectiveness of teachers who completed an alternative certification program and their claims that these programs have fallen short in terms of preparing them for the job of teaching, we are left wondering more about their PCK and how their program can support this development.

Pedagogical Content Knowledge

After Shulman's introduction of PCK as a theoretical construct in 1986, mathematics education researchers developed instruments to assess PCK, assessed PCK of pre-service and in-service teachers, created their own frameworks of this type of teacher knowledge, and tried to further our understanding of what teacher knowledge actually looks like. However, little research has been done to understand how PCK develops or to connect teachers' PCK with specific sources of development.

In his introduction, Shulman (1987) wrote that PCK included "an understanding of how particular topics, problems, or issues are organized, presented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (p. 8). He went on to say that, "The key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (p. 15).

Using this idea of PCK as the foundation for her framework, Grossman (1990) offered four categories of teacher knowledge which are the four areas of the conceptual framework for this study: General pedagogical knowledge, subject matter knowledge, pedagogical knowledge, and knowledge of context.

In recent years, Hill, Rowan, and Ball (2005) delineated what they call mathematical knowledge for teaching.

> By "mathematical knowledge for teaching," we mean the mathematical knowledge used to carry out the *work of teaching mathematics*. Examples of this

> "work of teaching" include explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs. (p.373)

This model consists of two major categories: subject matter knowledge and pedagogical content knowledge (Hill, Ball, & Shilling, 2008). In this model, PCK includes: knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum.

The conceptual framework for this study includes four strands of PCK adapted from Magnusson, Krajcik, and Borko (1999). They define pedagogical content knowledge as, "…a teacher's understanding of how to help students understand specific subject matter. It includes knowledge of how particular subject matter topics, problems, and issues can be organized, represented, and adapted to the diverse interest and abilities of learners, and then presented for instruction," (p.96). While curricular knowledge, knowledge of instructional strategies, and knowledge of student understanding can be correlated to the three strands in the model presented by Hill, Ball, and Shilling (2008), it also includes knowledge of assessment.

Lloyd (1999) conceptualized how teachers' views of mathematics content, their students, their pedagogical knowledge, and the context of the department in which they worked impacted changes in teacher learning. This shows us that teacher learning can be prompted by aspects of their current knowledge and the work they are doing which may result in teachers' knowledge developing differently based on their background or setting in which they work. Lannin et al. (2013) also described how PCK developed differently for two teachers in an alternative certification program. By attending to specific aspects of their teaching, the teachers' knowledge development focused on different strands of

PCK.

Lee, Brown, Luft, & Roehrig (2007) conducted a study of beginning science teachers. The study took place during the participants' first year of teaching. Over the course of the study, they found that the teachers had very limited PCK despite their science backgrounds. They did observe some growth of their PCK between the pre and post-tests, with significant growth in their knowledge of student learning.

This synthesis provides the background for the four strands of PCK included in my framework along with support for the four hypothesized sources of learning that I included. We know that learning my look different from teacher to teacher and for those enrolled in an alternative certification program. Little research exists regarding the development of PCK of alternatively certified teachers or how their programs create opportunities for such development.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

Research has shown that teachers develop specialized knowledge for teaching (Horn, 2005; Kinach, 2002; Lannin et al., 2013; Magnusson, et al., 1999). Most of this research surrounding PCK has focused on what that knowledge looks like in a specific content area; some (scholars/researchers) have also studied PCK development through a specific learning opportunities such as a video study, but little research has studied how PCK develops over time. In this study, I analyzed three teachers' PCK development throughout their first two years of teaching. Through analysis of extended interviews about teachers' lesson plans and lesson enactments, I describe how teachers' PCK developed. Specifically, I describe how knowledge of curriculum and student understanding developed for three beginning teachers with different experiences. In this chapter I begin by detailing the case study methodology I used for this study. I then characterize the participants in my study as well as the context of the larger study and participant selection. Finally, I provide information regarding the data sources and data collection process and my analysis procedures.

Research Methodology

This study was qualitative in nature as I sought to obtain a detailed understanding of teachers' knowledge. Creswell (2007) describes qualitative research characteristics as taking place in the natural setting where the participant's experience the issue under study, the researcher is the key instrument who collects data and interview participants, the researchers gathers multiple sources of data, and the analysis is an inductive process of working back and forth between the data and patter, categories, or themes (p.38).

For this study, I used a case study methodology (Yin, 2018) to understand the knowledge development of three individual teachers, focusing on how their specific knowledge for teaching mathematics developed over two years. This case study is explanatory as I explain the phenomenon by tracing the operational processes (i.e. development of PCK) of the teachers over time (Yin, 2018, p.10). To understand the development of teachers' knowledge, the data gathered for this study were collected over many hours in the field between June 2006 and June 2011 and provides insight into individuals' stories.

More specifically, this study is a holistic, multiple-case design (Yin, 2018, p.61) where each of my three cases, the individual teachers, fit within their own context.

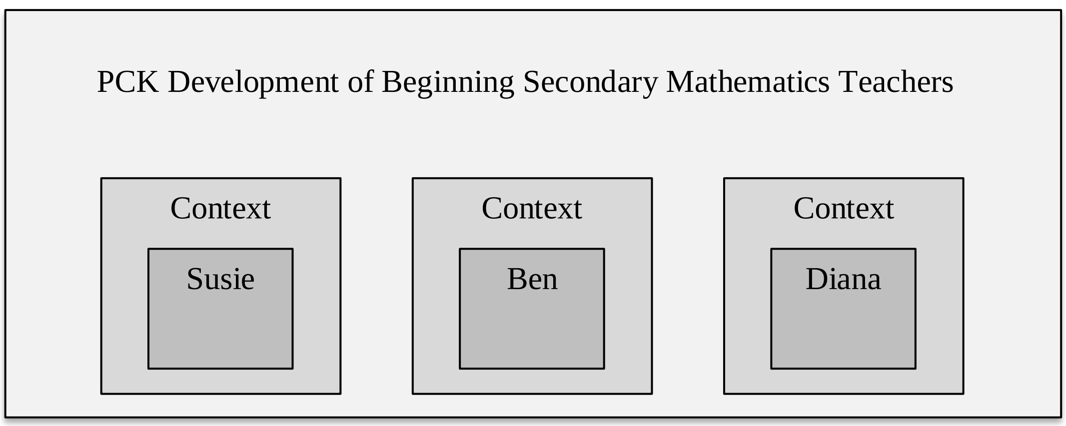


Figure 3.1. Multiple case study

The teachers in this study entered their teaching careers with different backgrounds and worked in different school contexts which led to different opportunities for learning. The boundaries of these cases include data collection over a two-year time span and during six distinct data collection cycles.

In this study, I investigated the following questions:

1. How does the PCK of middle and high school mathematics teachers develop through the first two years of their career?
    a. How does the knowledge of curriculum of middle and high school mathematics teachers develop?
    b. How does the knowledge of students' understanding of middle and high school mathematics teachers develop?

Case Study Methodology. These questions follow the principles of a case study by asking "how" about a contemporary event over which the researcher has little or no control. Yin (2018) defines a case study as an empirical inquiry that (1) investigates a contemporary phenomenon (p.15). In this study, the contemporary phenomenon I sought to understand was how PCK develops for beginning teachers. According to the US Department of Education (2014) the number of new teachers in the United States has fluctuated around 6% of the total teaching staff annually for more than two decades. Currently, there are more than 200,000 new teachers in our classrooms each year. As we strive to better prepare and support new teachers for the work of teaching, the development of their specific knowledge for teaching their content is a very real concern. Also, a case study (2) is conducted within its real-life context (p.15). The real-life context in which this study was conducted included the every-day work of teachers in the teachers' classrooms and, through data collection, sought to understand what knowledge the teacher had surrounding their specific lessons and students. Finally, Yin defines a case study when (3) the boundaries between phenomenon and context are not clearly evident (p.15). In this study, it is unclear if teachers' knowledge develops similarly because of their lack of experience and common coursework and internships or if their different school context, curriculum materials, mentors, and personal experiences lead to different knowledge development.

Unit of Analysis

The unit of analysis is at the individual level (Figure 3.1). Each of these cases were chosen as a theoretical replication (Yin, 2018, p.177), aimed at producing contrasting results due to predictable reasons. I predicted differences in the development of PCK among the three cases, predictably due to the inherent differences among individuals, differences in their school context, and factors beyond the scope of this study. From the above research which study specific opportunities for teacher learning and the knowledge they demonstrate (Fletcher, Strong, & Villar, 2008; Kinach's, 2002; Lloyd, 1999; Putnam & Borko, 2000; Schifter, 1998), the venues in which teacher learning can occur (e.g., coursework, teaching experience, collaboration with other teachers, work with a mentor) were all aspects of each case but may have looked different, leading to differences in learning opportunities.

Below, I describe these three cases and the participant selection process. I also detail how my data collection and analyses fit within the definition of a case study methodology and provide insight into the development beginning teachers' PCK.

Context of the Study

The data for this study drew on a subset of data from a larger study. Funded by the National Science Foundation in 2006, the Researching Science and Mathematics Teacher Learning in Alternative Certification Models ($ReSMAR^2T$) team consisted of the following principal investigators in both mathematics and science education: John K. Lannin, Kathryn Chval, Fran Arbaugh, Patricia Friedrichsen, Sandra Abell, and Mark J.

Volkman as well as numerous graduate student research assistants. I joined the team as a research assistant in 2010. The project examined teacher learning through an alternative certification program and into their first years of teaching. The certification program was a 15-month program that prepares individuals with college degrees that include strong mathematics backgrounds to become teachers. These individuals completed 32 credit hours of coursework that included general education and three content-specific mathematics methods courses, leading to a master's degree and teacher certification at the middle or secondary school level. The participants were also involved in a year-long internship in their assigned mentor teacher's classroom. These participants entered with a variety of mathematics related undergraduate degrees and work experience.

Data Sources

Four cohorts of beginning teachers participated in the larger study which included complete data for 15 teachers. Each of the teachers in the study taught middle or high school mathematics. Data collection began the summer before the participants started their alternative certification program, with the first cohort beginning in 2006. For their entry tasks, participants began by completing a lesson planning task (Van Der Valk & Broekman, 1999) that required designing two consecutive 50-minutes lessons for teaching a selected ninth grade mathematics concept. Immediately following the lesson-planning task, each participant engaged in a semi-structured interview (Patton, 2002) that was intended to gain insight into their lesson plan. To begin, interviewers asked participants to describe the process they used to design the two days of instruction. In addition, the research team asked questions specific to the four PCK categories in order to document their knowledge of students' understandings, instructional strategies, assessment, and curriculum. Follow up questions about where they developed their knowledge provided the participants with an opportunity to share the sources of their specific knowledge. The interviews lasted between 30 minutes and one hour and were audio recorded and transcribed verbatim for later analysis.

During their first school year in the program, participants (also) wrote a two-day lesson plan, as they normally would for the class they were teaching. Before implementation, they participated in a pre-observation interview. The interviews assessed their PCK, what resources they used for planning the lesson, and their expectations for the lesson. The researchers observed, videotaped, and took field notes for the two consecutive lessons taught to the same class and interviewed the participant following class each day in a stimulated recall interview (Pirie, 1996; Schempp, 1995). These two-day observation cycles occurred twice each year, one in the fall and one in the spring. The following school year, the same observation cycle data collection process was followed (Table 3.1). During this time, teachers were in their own classrooms rather than with a host teacher. After two years of study, each participant repeated the same entry tasks that they completed during their first summer. This allowed the researchers to observe knowledge of PCK pertaining to the same math content and lesson. I participated in the data collection process by filming and conducting interviews during teachers second year of teaching.

Table 3.1

Observation Cycles

	School Year 1		**School Year 2**		
Entry Task	**Observation Cycle 1**	**Observation Cycle 2**	**Observation Cycle 3**	**Observation Cycle 4**	**Exit Task**
Prior to first year teaching 1. Lesson	Fall 1. Pre-	Spring 1. Pre-	Fall 1. Pre-	Spring 1. Pre-	After second year of teaching

planning task 2. Interview	Observation interview 2. Observation 1 3. Stimulated recall interview 1 4. Observation 2 5. Stimulated recall interview 2	Observation interview 2. Observation 1 3. Stimulated recall interview 1 4. Observation 2 5. Stimulated recall interview 2	Observation interview 2. Observation 1 3. Stimulated recall interview 1 4. Observation 2 5. Stimulated recall interview 2	Observation interview 2. Observation 1 3. Stimulated recall interview 1 4. Observation 2 5. Stimulated recall interview 2	1. Lesson planning task 2. Interview

Beyond this data from the six observation cycles (Table 3.2), an observer from the research team also noted the teachers' actions during teaching and answered questions about the observance of PCK. Also, as a way to triangulate the data, the teachers' mentor teacher was interviewed during Observation Cycle 1 and 2. The mentor teacher participated in a semi-structured interview prompting them specifically for information about the new teachers' PCK.

Participants Selection

For this study, I chose to analyze the knowledge development for three beginning teachers. In this section I describe the process for choosing the participants and provide context for each of the three.

First, to understand each of the participant's contexts, I compared aspects of the teaching positions of each of the 15 participants with complete data from the Science and Mathematics Teacher Learning in Alternative Certification Models (ReSMAR^{2}T) project with complete data sets (Table 3.2). The selection criteria included the size of school in which they were teaching and whether or not they stayed in the same school for both years. The size of the school was a factor as teaching at a small school often means the teacher is teaching many different courses and only teaches each one once while teachers at a large school teach fewer different courses and have the opportunity to teach each course more than once. Teaching at a small school also limits access to other math teachers in the building with which to collaborate. I also considered the grade band at which they taught, the curriculum resources used, and access to a professional learning community within the school. In addition, I completed an initial analysis of PCK by analyzing responses to two interview questions in both the entry task and exit task. The questions I analyzed included 1) “Walk me through your plan” and 2) “During the two days of instruction, how will you know if your students are getting it or not getting it?” The first question was very open and allowed the participant to demonstrate any knowledge they used to create their plan. The second question was specifically seeking information about the participants’ knowledge of assessment but allowed for them to also connect this with other strands of PCK. I used this initial analysis to represent evidence of PCK at the beginning of the study and after completion of their certification program and two years of teaching.

To choose the participants for my study, I used theory-based purposeful sampling (Patton, 1990, p.177) in order to include participants who could provide information about the phenomenon being studied while also offering the different circumstances shown to influence PCK development. From this chart of information, I chose participants who showed evidence of PCK at the end of the two years. I included teachers who demonstrated knowledge in at least two of the four strands of PCK (instructional strategies, student understanding, curriculum, and assessment) to ensure the phenomenon (PCK development) could be studied. From this narrowed list, I chose participants with a variety of contexts to make a theoretical replication. For this study, I chose the following participants:

1. Susie – Susie was a middle and high school teacher who taught in the same private school both years. She worked with one to two other mathematics teachers in the small school with small class sizes. She taught with a traditional textbook and had the opportunity to teach eighth grade math through pre-calculus. Prior to the program Susie completed three courses related to education and mathematics education specifically.
2. Ben –Ben was a junior high teacher who taught in two different schools both of which were in the same district. He taught using both the Connected Mathematics Program and a traditional textbook series and participated in mathematics professional learning communities. Prior to the program, Ben was a math tutor. He tutored mathematics courses for four years, ranging from college algebra through calculus III. Before entering education, he worked as a banker.
3. Diana –Diana was a high school teacher. She is also taught in two schools in the same district using the Connected Mathematics Program and a traditional textbook as well as participated in mathematics professional learning communities. However, after just a few months into teaching, Diana began a full-time teaching position that required her to switch schools and teach her own classroom without a mentor. She was the only participant in the study to not share a classroom with their mentor during the entire first year of teaching. Diana has some background in education. She began taking college courses after high school and then took time off. Once she returned to college, she considered becoming a

middle school math teacher so she took an elementary mathematics methods course while completing her bachelor's degree.

Table 3.2

Participant Context Information

	Susie	Ben	Diana
Grade level taught	MS and HS	Junior High	High School
Number of students in school building(first year)	130	800	1800
Stayed in the same school both years	Yes	No (same district)	No (same district)
Textbook used	Single subject	CMP & single subject	Core Plus & single subject
Participated in PLC	No	Yes	Yes
Additional information			No mentor after November, Year 1
Evidence of PCK during Entry Task Q1	No	Yes	No
Evidence of PCK during Entry Task Q2	Yes	No	Yes
Evidence of PCK during Exit Task Q1	Yes	Yes	Yes
Evidence of PCK during Exit Task Q2	Yes	Yes	Yes

To protect the rights and welfare of these participants, this study and the research team members were approved by the Institutional Review Board (IRB) of the University of Missouri.

Analysis

In this section I describe the coding process I used to code for all four strands of PCK as well as how I developed the two themes discussed in my findings. My analysis included three levels, each described below. To remind the reader of my research questions, I describe how the PCK of middle and high school teachers developed through the first two years of their career.

All interviews with the three participants and their mentors as well as observation notes were transcribed and organized using the qualitative data analysis computer software, NVivo. My process for analysis was adapted from the diagram in *figure 3.2*, which Yin (2003) uses to pictorialize the multiple case study method, by first conducting each individual study, then writing individual case reports, and finally conducting a cross-case analysis. In this study, I refer to my Level 1 and Level 2 analysis as the first and second columns respectively in the middle section, which Yin refers to as "Prepare, Collect, & Analyze." During these analyses, I focused on the broader research question, including all four strands of PCK in my analysis. These two levels of analysis are further described below.

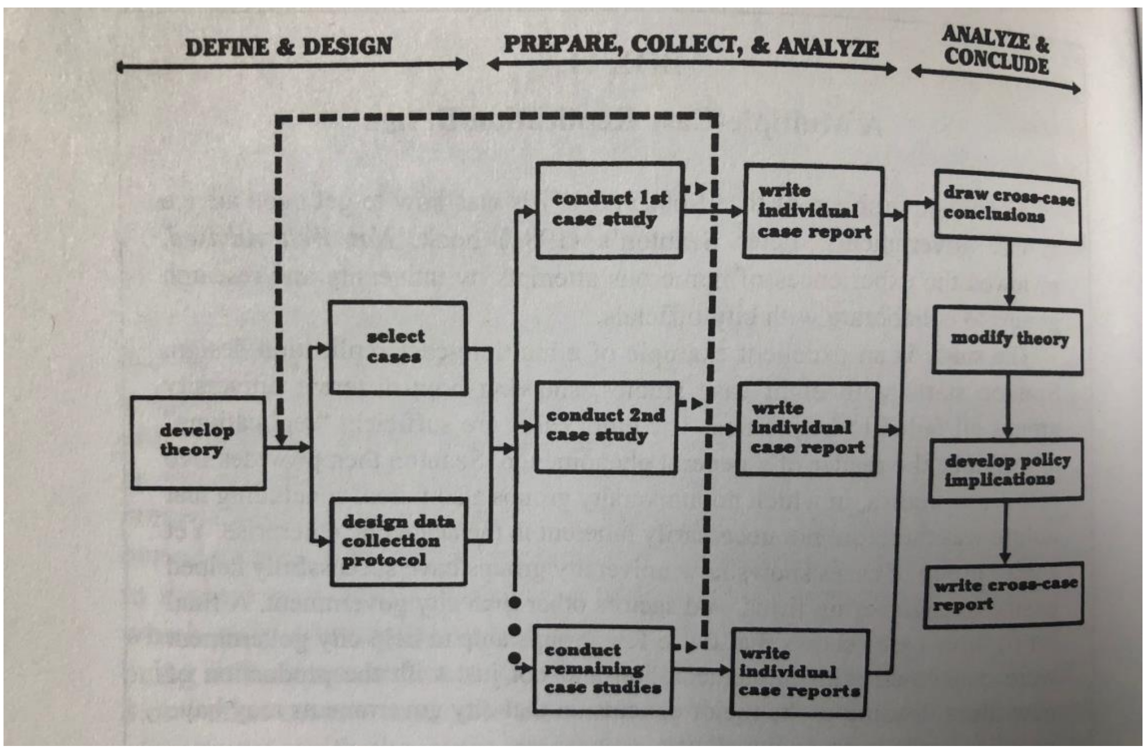


Figure 3.2. Case study method of COSMOS Corporation (Yin, 2003, p.50)

Level 1 Analysis

I completed an initial level of analysis by using deductive coding, with pre-determined codes (King, 2004), to code each participant interview (Table 3.3) for the four categories of PCK in the ReSMAR2T framework (*figure 1.1*): (a) knowledge of students' understanding, (b) knowledge of instructional strategies, (c) knowledge of assessment, and (d) knowledge of curriculum.

Table 3.3

Data Sources

	School Year 1		School Year 2		
Entry Task/	**Observation Cycle 1**	**Observation Cycle 2**	**Observation Cycle 3**	**Observation Cycle 4**	**Exit Task**
Lesson Plan interview	Pre-observation interview	Pre-observation interview	Pre-observation interview	Pre-observation interview	Lesson Plan interview
	2 Stimulated recall interviews	2 Stimulated recall interviews	2 Stimulated recall interviews	2 Stimulated recall interviews	
	Observer notes	Observer notes	Observer notes	Observer notes	
	Mentor interview	Mentor interview			

Further descriptions of the codes are found in *table 3.4*, which provides examples of the types of knowledge coded for each of the four strands of PCK. This coding dictionary (Table 3.4) was developed by the research team and used for this study. To ensure the accuracy of my coding, I coded two excerpts from each of my participants' entry and exit interviews for all four strands of PCK. This was also done by two other members of the research team. Disagreements in the coding were resolved through discussion and clarification was further noted in the dictionary. These codings were used throughout the remainder of my analysis.

Table 3.4

PCK Codes, Definitions, and Examples

PCK Component	Is evident when the teachers talk about...	Example

Table 3.4

Knowledge of Student Understandings	• Student misconceptions • Student difficulties • When students find certain concepts easy to understand • How students might approach a concept or problem • What strategies a student may use to solve a problem • Student prior knowledge	"I think that some of the struggling students have issues with what increments we use on our x and y-axis sometimes when you get ugly data. They want to go 1, 3, 7, because that's what they collected. How do you do the even intervals? I think they struggle sometimes with that; the lower end kids do." Erica [242-245] **(student difficulties)** "The phrase "a picture is worth a thousand words," I think it's really true that when kids can get a graphical representation of what is happening or see in the data how the values are changing. They seem to grasp onto that a lot quicker than they do just working with the equation in raw algebraic form." Diana [197-200] **(when students find certain concepts easy to understand)** "…I assumed they already knew something about linear relationships. They probably had graphed. They probably had written some linear formulas and equations. They probably made some tables." Leo [94-96] **(student prior knowledge)**
Knowledge of Instructional Strategies	• How to organize instruction • Specific actions that the teacher can take during instruction • Activities to use for specific mathematical content • What materials are needed for instruction • What representations are best for particular content	"…I assumed I'd have a SmartBoard and like I have a SmartCamera now so I would have taken their calculators and captured those pictures and put them up in SmartNotebook so as the class discussion we could click on those and look at them and talk about them so they had that visual." Marie [41-44] **(what materials are needed for instruction; how to organize instruction)** "We're predicting something in the future, something that hasn't been done yet. Can the pattern that you've collected be used accurately for a prediction tool for something we don't know yet?... Predict your height that thirteen M&Ms would give you. And then test it. And again this experiment is always one of my very first ones with a linear relationship because it is so accurate to predict." Myra [477-482] **(activities to use for specific mathematical content)** "In the group work [task] I was thinking of what manipulatives could I use. I looked at the manipulatives trying to think maybe I could have them do some activity…where they looked at [the relationship] and it was a constant rate of change because I've done something with like weights and adding it to a cup and then making a table and making a graph from that." Nicole [52-56] **(activities to use for specific mathematical content)**

Table 3.4

Knowledge of Curriculum	• Goals for instruction • Curricular resources • Content of textbooks (i.e., specific knowledge of things included in curricular materials) • Scope and sequencing of mathematical topics • Source of knowledge of curriculum • Knowledge of standards	"So, I was thinking okay in order to go symbolic, we want to make sure they've got a good understanding of the x and y-axis, independent and dependent variable and how they are related to each other." Craig [55-57] **(scope and sequencing of mathematical topics)** "First I thought about the objectives that I wanted. I wanted to have the kids focus on the relationships that they are going to develop between graphs, charts, equations, and situations." Russ [17-19] **(goals for instruction)** "I instantly go to Core Plus. That's my first and foremost. That's what I drew from my knowledge. I was trying hard to think back because it's been a while since I did one, but the thing is that the activities don't usually leave you very quickly or easily. As I'm teaching I'll even mention in the class to the kids, "Do you remember when you did the spaghetti breaking?" They'll immediately say, "Oh yeah, yeah." Putting the activity with the math content helps retain it." Edy [493-498] **(curricular resources)**
Knowledge of Assessment	• Why they assess students • How they assess students • What they do with the information gathered from assessment • Knowledge of assessment challenges • Knowledge of assessment strategies • Knowledge of potential teacher responses based on assessment results • Knowledge of assessment purposes • Knowledge of what to assess	"I think you have to decide then at that point, so okay obviously we need more practice with graphing. Do I need to come back and do more practice with graphing before we go on to the next piece or not? …I have to have a good graph with a good line of best fit drawn or day two…doesn't make much sense. So I think if it's something that impacts what follows later you're going to have to pause and go back and do some more practice and do some more checking, and fixing, and editing. If it's something where you really could go on…I could just give them the answer. I could say, 'okay let's just assume for a moment that we're going to use this equation' and I do that with you individually because your equation's crummy. Here, why don't you use this equation instead? We can still go ahead and finish out but just knowing additional practice is needed in the days that follow." Meg [556-567] **(knowledge of potential teacher responses based on assessment results)** "I use…the handheld whiteboards a lot because then I can say…what is your prediction…and I can make sure every kid has an answer and [is] involved. I can write an equation on the board [and say], "Write down the slope of this equation." Show me the boards and I know real quick, in a matter of a second, I don't have to take tests home and grade them, I know yep everybody but these two [students] know the component of that equation that represents the slope of a line." Myra [702-708] **(How they assess students) –note—dialog continues.**

Level 2 Analysis

To characterize the participants' PCK at each of the six points throughout the study, I created descriptions of their knowledge for each collection cycle, putting the data in chronological order. For each observation cycle, as well as the entry and exit tasks, I summarized what knowledge the participants demonstrated in each of the four strands of PCK. Yin (2018) refers to this type of general analytic analysis as "developing a case description" (p.171). This analysis served as a tool for the cross-case synthesis I completed as the Level 3 analysis.

The demonstration of PCK by the participants was triangulated by interviews with their mentor and observation notes by the data collection team during the same observation cycle. Both protocols prompted the mentor and observer for observations of participants' PCK. Through my analysis, evidence of the participants' knowledge of PCK was constantly compared to the other data sources in order to be consistent with a case-based analysis. The information provided in these data sources was also included in these chronological descriptions.

Level 3 Analysis

To compare and contrast the PCK of the three participants in my study, I completed a cross-case synthesis. For this analysis, I compared how each strand of PCK developed similarly or differently, chronologically, for each participant as well as analyzed the overall PCK development among the participants. Through this case-based (Yin, 2018) approach, I developed two themes of the participants' knowledge development. These two themes continued to be evident throughout the two years of data and provided insight into how knowledge developed similarly or differently among the participants. This analysis resulted in my two sub questions.

The first theme contrasts the development of the participants' knowledge of curriculum, focusing specifically on how their different goals for instruction led their overall knowledge of curriculum to develop differently. The second theme describes how all three participants developed knowledge of student understanding while their demonstration of their understanding occurred at different times through their planning and teaching process and was described at different levels of specificity. These themes are discussed in the findings.

Methodological Rigor

Lincoln and Guba (1985) define the concept of trustworthiness by including the criteria of credibility, transferability, dependability, and confirmability. To provide credibility, Lincoln and Guba suggest techniques such as extended time in the field, data collection triangulation, and persistent observation. It is worth noting that each participant in this study began with no classroom teaching experience and was observed over a period of two years with the same implementation of observation cycles conducted by members of the research team. Data was collected at six times throughout the study. The research team members had an in-depth knowledge of the framework for PCK upon which the interview protocols were built, the protocols were developed as a team, they met on a regular basis, and often conducted the interviews in pairs. Observation cycles were conducted in the same manor and the researchers used the same, interview protocols for all participant interviews and mentor interviews. For this study, three data sources were analyzed and coding was validated by members of the research team.

To protect the rights and welfare of these participants, this study and the research team members were approved by the Institutional Review Board (IRB) of the University of Missouri. Participation in this study was voluntary and informed consent was provided. Confidentiality and anonymity were met as participants' names were changed and no identifying information (such as the school district) were reported.

Limitations

The participants in this study taught in a town with more than one college, including an R1 state University. The connections with the university and local schools afforded opportunities for professional development and collaboration beyond what is available to many districts. Two of the participants were placed in mentor teacher classrooms who had received extensive training on the use of standards-based curriculum and were provided with those materials for teaching. Also, each of the participants began this study with a variety of background knowledge and college experiences.

Data collection was done by the research team. However, the semi-structured interviews were not all conducted by the same researchers and the researchers' observations of PCK were noted by a variety of researchers across the study.

This chapter provided specifics about my process of analysis in this multiple case study and described my participants and the selection process as well as the limitations of the data. In the next chapter, I provide the result of these analyses, organized into two sections: Knowledge of Curriculum and Knowledge of Student Understanding.

CHAPTER 4: ANAYLSIS OF THE DATA AND RESULTS

Knowledge of Curriculum

In this section, I provide my findings in relation to the components of knowledge of curriculum (i.e., knowledge of the content standards, scope and sequence of mathematical topics, curricular resources, and instructional goals), how the participants used that knowledge to develop lesson plans, and how that knowledge developed over time. Each participant attended to different components of knowledge of curriculum when developing a lesson and each path developed differently in terms of the order of the knowledge and the pace of development. I begin with Ben, who provided an unanticipated progression of knowledge of curriculum by focusing first on his goals for student thinking. I continue with Susie, who developed a broad sense of the scope and sequence over the two years and finally, I describe Diana's knowledge of curriculum, which remained focused on mathematical content units.

Ben

As Ben began his teaching career, his knowledge of curriculum could be seen through his goals for instruction that focused on engaging students in the learning process, giving students authority over the mathematics, encouraging group work and communication and attending to both general learning practices and mathematical processes. Ben drew little from the textbook or other resources, instead he focused on engaging his students. His task selection was based on getting students to think about and do mathematics. During his first year, Ben demonstrated his knowledge of scope and sequence by considering how the math concepts in his course would continue to be used in higher level mathematics. However, he demonstrated little knowledge of the content standards or how his lessons fit into the scope of the course. Over the course of two years Ben's knowledge of curriculum expanded to include knowledge of how the content in the lesson met the standards and how it fit into the larger content unit and course. While Ben used this knowledge to influence his instructional strategies, his goals for instruction continued to focus on learning practices and mathematical processes.

The processes for students' engagement in learning that Ben describes within his knowledge of curriculum are best portrayed using his own words for what he would like students to be able to do beyond only obtaining the mathematical content knowledge.

Below, I provide a description of Ben's knowledge development of curriculum, providing examples of Ben's focus on general and mathematical processes in both his goals for instruction and through his instructional strategies.

Year 1, Observation cycle 1 - Ben demonstrated little knowledge of the content standards or scope and sequence during his first year of teaching. As Ben entered his internship and began planning lessons with the guidance of his mentor teacher and with the help of the district curriculum and the textbook, he struggled to connect the mathematics with his instructional strategies. Ben's mentor teacher explained how she used the standards and the textbook in her classroom and shared how she communicated about these resources with Ben.

> Yes, I think he does (make connections to the standards) because I stressed that from the very beginning. That at the very beginning I talked to him about our district curriculum, which for us is just defined as measurable learner objectives which are built off the state standards which are GLE's, Grade Level Expectations and that we have a new text series. It's very traditional, and a very heavy book, and that there's no way nor was it ever intended to teach this lesson by lesson. And you are not going to be able to make connections, some of its repeat. I think the first three chapters are, traditionally in these textbooks, are last year's learning and that if you don't use the [state's learning standards] as your guide, it's a bad

> thing. You are going to be in trouble with me first of all. But no that's what you have, that has to be your guiding light. (Mrs. Martin, mentor, fall)

This created a couple of challenges for Ben. First, while his mentor expected Ben to plan lessons based on the standards, he was not confident in his understanding of the scope and sequence. He did understand that the current objective focused on symbolic understanding and said that was why his emphasis was on symbolic representations. He also explained how his current lesson fit with previous lessons that he taught. However, he said,

> My understanding of the reasoning for [current school districts'] curriculum is pretty limited. Some of it seems really bananas to me. For instance, [in the current course Ben is teaching] a week and a half ago we were supposed to be teaching them how to solve equations with fractions and decimals. In a month and a half, the students will be learning about fractions and decimals. That seems bizarrely out of order to me. (Ben, Observation cycle 1, pre-observation interview)

Also, Ben followed the practices of his mentor teacher who chose not to use the textbook as a primary resource and instead focused on the district curriculum standards. This left Ben with little guidance to choose tasks or problems. Ben explained that he often used tasks from the course textbook and adapted them by removing information or rehearsed steps and changed the numbers, but did not provide specific details related to choices such as how his tasks addressed student understanding or how they met the mathematics standards. Reflecting on his lesson during this observation, he said

> I felt like it went pretty well today. Like, the lesson plan we came up with was, I wasn't sure about because I didn't know how to approach inequalities and especially solving them. Both because I am not sure how familiar students are with inequalities and I didn't, I wanted to be able to do something that contextualized it so they would have to talk about inequality as a concept rather than just the alligator… (Ben, Observation cycle 1, day 2 interview)

In his planning, Ben sometimes implemented instructional strategies that he observed his host teacher enact on a regular basis without much regard to how these strategies were helping his students learn math. During his reflection, Ben questioned how these strategies align with his goals for instruction. This prompted him to further consider what mathematical processes he wants to emphasize and why.

> The other one is more, I think, important, and a lot of it has to do with Mrs. Martin. She likes teaching things with the context. I think I agree with her, but I don't know where I come down on that. I'm still trying to figure it out, but it's for trying to get the context. They like that a lot because I think a really important part of mathematics is being able to translate it into your everyday thinking or your everyday language. If you can do that then it becomes more widely applicable because it develops your reasoning ability generally, but also because the mathematics become tools that you can use because of the translational ability. I think giving them word problems and then asking them to figure out how to set up an equation to model that is explicitly about that translation and the discussions that we'll have during their class period will focus on that translation. How does the word problem get into the equation? (Ben, Observation cycle 1, pre-observation interview).

At this point, Ben did not use his knowledge of the scope and sequence of the mathematics or an understanding of the mathematics content to justify his choice of specific problems or tasks and continued to make instructional choices based on his goals for learning practices. *Figure 4.1* represents Ben's knowledge of curriculum and how he used it to plan individual lessons. At this point, his lessons were not connected or situated in the larger mathematics context. Ben's instructional strategies included strategies to meet his goals for instruction.

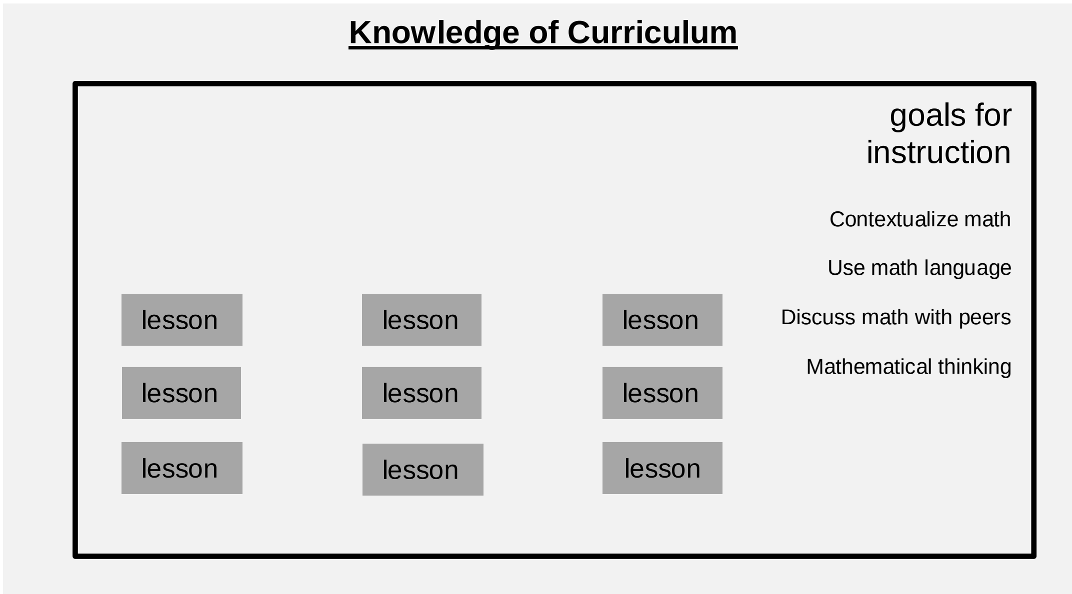


Figure 4.1. Ben's knowledge of curriculum during his first year of teaching.

Year 1, Observation Cycle 2 - Ben situated his goals for instruction within his growing knowledge of the content standards. As Ben progressed in his internship, he described how his lesson met his goals for instruction, focusing on learning processes and his knowledge of the content standards. While Ben's primary goals continued to focus on learning processes, his mathematical goals were still specific to each lesson. In this lesson, Ben mentioned how his lesson provided students with the opportunity to develop understanding of the properties of quadrilaterals. In his lesson, he situated the mathematical knowledge standard within his goals for instruction which included providing students with opportunities to discuss the mathematics, reflect, and attend to precision in their explanations:

> They had an opportunity to practice that process of making their explanation more precise and more clear given the time constraint. They had an opportunity to reflect on that process. And, in terms of math content, they had the opportunity to develop a better understanding of which properties hold for which quadrilaterals.

> Which is something they did through discussion and through the listening they did when they were doing the round robin activity. (Ben, Obs. 2, Stim. Recall- Day 1)

When asked if he felt that his lesson accomplished his goals for the day, he responded,

> Yeah, I would say so. It felt, it was kind of a mushy target. [laughs] If you throw something at a sponge, I guess. But it's, I would say so. I felt like they got out of it what I wanted them to get out of it. And it seemed like the structure, or like, my personal target is in a sense to test out that discussion structure. And it seemed to keep a really high percentage of students involved in the task without a lot of redirects or input from me. (Ben, Obs. 2, Stim. Recall- Day 1)

Again, his goal for the class focused on a new strategy for leading small group discussion rather than the mathematics content itself. These goals for instruction seem to fit with the goals of his mentor teacher. Ben had a different mentor teacher for the second half of his internship. Mr. Brand explained how he sees his role as a mathematics teacher. He said, "My role is to guide them in that struggle and help them through that rather than push them from one objective to the next, quote, unquote, completing the list of things we have to do…" (Mr. Brand, mentor, spring). He went on to explain that the district curriculum does not provide a deep explanation of student expectations and that the textbook does not provide you with a day-to-day guide for instruction.

During this observation, Ben's lessons were structured so that he observed students collaborate and looked for their ability to reason, construct arguments, and critique the reasoning of others. He showed that this is a priority by letting go of additional problems so that students could continue to work in their groups. He said,

> It still seemed like there was a lot of productivity happening. I know, from like this corner, there were pairs of students working furiously to finish presenting. They didn't quite get done and they were asking each other questions so that was cool. We kind of, like, let it ride to help those students. In the long run, I'm thinking there's a lot of squishy time at the end with the table. I didn't plan on talking about that today. I wanted them to go through the thought process and sort of mull it over. If they got it done in class, great. If they didn't, it would be work to do at home. (Ben, Obs. 2, Stim. Recall- Day 1)

The general learning processes remained the focus as he addressed students' questions. Ben continued to focus on implementing tasks that encouraged students to problem solve, make connections, and to work together. While Ben did focus on situating the content within the broader picture, and know which skills he would like for his students to address, it was not until his second year of teaching that he was able to explain his choices for specific mathematics problems.

Year 2, Observation Cycles 3 and 4 - Ben demonstrated growth of knowledge of curriculum as he began to apply his knowledge of the content standards to his lessons. As Ben entered his second year of teaching, now without the daily guidance of a mentor teacher, Ben was making all of the lesson decisions. However, the first observation cycle included content he was familiar with teaching because of his previous experience teaching Algebra during his internship. During this observation, Ben felt comfortable with where this content was leading and what the students needed to understand.

> In algebra, this is during my student teaching last year, the emphasis was to help students develop an understanding of slope and be able to understand that as a ratio change in *y* by a change in *x*, and the *y*-intercept is a shift of that. We looked at a lot of different forms; we looked at point-slope form, the standard form, as well as the slope-intercept form. Which meant that the discussion of the slopes and *y*-intercepts, when we were talking about those other forms, got deeper because we were looking at the slope form and deriving the point-slope equation from that. We went into a lot more depth and a lot more abstract depth in terms of slope and how it shows up in different representations. This year we're the arch and the difference is to be able to distinguish between linear and nonlinear relationships. Once we've distinguished between those linear relationships fail to tell the difference between different linear relationships. So, we're comparing and contrasting according to slope and y-intercepts; which, of course, is the same kind of stuff because it's all linear relationships, but it has a different feel because we're not going into detail (Ben, Obs. 3, Pre-observation interview).

This knowledge of the curriculum was also tied to knowledge of instructional strategies. Ben provided detailed information about content learning goals as well as his choices for specific tasks. He further explained how he adapted a textbook task to meet these knowledge goals. Ben's explanation of his plan included his knowledge of the content standards, as he explained how the lesson included specific ideas about slope and the *y*-intercept. At the same time, he continued to focus on learning processes, which included critiquing different strategies and constructing reasonable arguments. You can see this in his response below:

> Interviewer: Can you talk to me about how you plan to help students learn the important mathematical ideas you talked about earlier—about linear relationships tomorrow—so how are you going to help them learn that?
>
> Ben: Tomorrow we're going to be concentrating on a graph that they've already made an attempt at on their own. We'll start with what they've already attempted to do. I think there's a lot of different strategies in the room right now for how they're constructing the graph, so we'll discuss that, discuss those different strategies, then we'll hear arguments for which one they think is the most appropriate. After the class has come to an agreement about that we'll answer a few questions regarding the slope and how it shows up in that graph and on the table we analyzed today. We'll take a look and attach the definition of y-intercept to a spot on the graph, and with that under our belts, we'll have an opportunity to work through a couple of more question that will attach the equation to those things. I guess, my primary strategy is to encourage reasonable argument and ask questions that accomplish that. (Ben, Obs. 3, Pre-observation interview)

During this first year in his own classroom, Ben continued to develop PCK surrounding curriculum. He focused on understanding the learning objectives set forth by the district and was concerned with how to best address them in his lessons. When observing Ben later that year, he was asked where got the ideas for his lesson. He explained that he was pulling together information from multiple areas to construct the lesson.

> I'm pulling it out of my head. Our curriculum gives kind of a vague objective. It asks us to get kids prepared to make the right choice about how to represent data and we have seven different kinds of charts and graphs from which they'll be able to choose. So, they have to be able to choose the right thing to demonstrate it, which means they'll need to be able to compare and contrast, I think, what each of them do. And histograms are thrown into that curricular objective, but histograms are only briefly covered in our textbook, so to try to meet that objective, I've been trying to construct a lot of stuff myself. (Ben, Observation cycle 4, pre-observation interview)

Ben continued to develop tasks that met his goals for instruction, including creating problems that led students to discovering the learning objectives rather than him lecturing. During this observation, Ben admitted that he does not always know the best way to present the mathematical ideas or which examples to use. He had not taught this content previously and when asked about his plan he said,

> Yes, I'm still puzzling over it. I'm trying to make it fit in the right order. So, the main structure of class is going to be built around an activity that helps kids start to construct arguments as to whether or not the distribution is normal. I'm not sure what the right thing to do is right now in order to get them warmed up and ready to do that main activity. [Inaudible] it would probably start with a normal distribution and having kids write down descriptors of that. (Ben, Observation cycle 4, pre-observation interview)

Ben considered the mathematics and the students' understanding and how that should impact his plan, while still focusing on his goals for instruction—in this lesson his goals was again centered around students constructing arguments. Ben's planning practices shifted during this year. While he was still focusing on his goals for instruction and how he was encouraging them in his lesson and that they are sometimes still a basis for developing his plan, he moved to considering the content, curriculum and the students when designing his lesson.

During Ben's reflection of this lesson and how he might implement it in the future, his focus was on how to best teach the content:

Interviewer 1: I asked this at the end of yesterday and I said don't worry I'm going to ask it again today so given the opportunity to sit down with a colleague you truly trust what questions would you ask about this mathematical topic that you taught either yesterday or today?

Ben: I would ask about number three and I think that's much more clear to me having had

Interviewer 1: For B?

Ben: Yes, having opportunity to cover more depth today because like I said the five, six, seven things that I did was on the fly so I didn't have, I mean it became clear that I didn't have adequate plan for trying to teach that.

Interviewer 1: So how to address that mathematical concept that the mean is so much greater than the median?

Ben: Right in a skewed set and so I think when I go about teaching math next year I'll do more work on the front end to see if other folks have ideas about that and what I can do to try to cover. (Ben, Obs. 4, Stim. Recall- Day 2)

Exit Task - Ben's knowledge of curriculum, seen through his exit task. When Ben completed his exit task at the conclusion of his second year of teaching, he showed growing confidence in his lesson plan and the tasks he chose through his justifications for his choices, including his knowledge of curriculum. Ben previously taught the content that he was expected to design lessons around in this exit task. His knowledge developed in all areas of PCK and became more connected across the strands. Ben justified his lesson plan according to his knowledge of curriculum, instructional strategies, student understanding, and assessment, based on his experience teaching this content. He altered a previously used task according to his perception of what was successful and what was not successful in the lessons he enacted. At this point, Ben connected his understandings within his knowledge of curriculum. Ben used his knowledge of the content standards included in the lesson as well as how they fit within the unit and across mathematics to develop this lesson. Ben applied his goals for instruction as well as instructional strategies aimed at meeting these standards and goals to develop his lesson plans.

During his second year, Ben developed his lessons based on his goals for instruction. His goals included general learning practices such as constructing arguments and mathematics specific learning processes such as choosing appropriate mathematical representations. Ben also developed knowledge of the mathematics standards beyond each lesson. He understood how each lesson was situated within the unit and beyond.

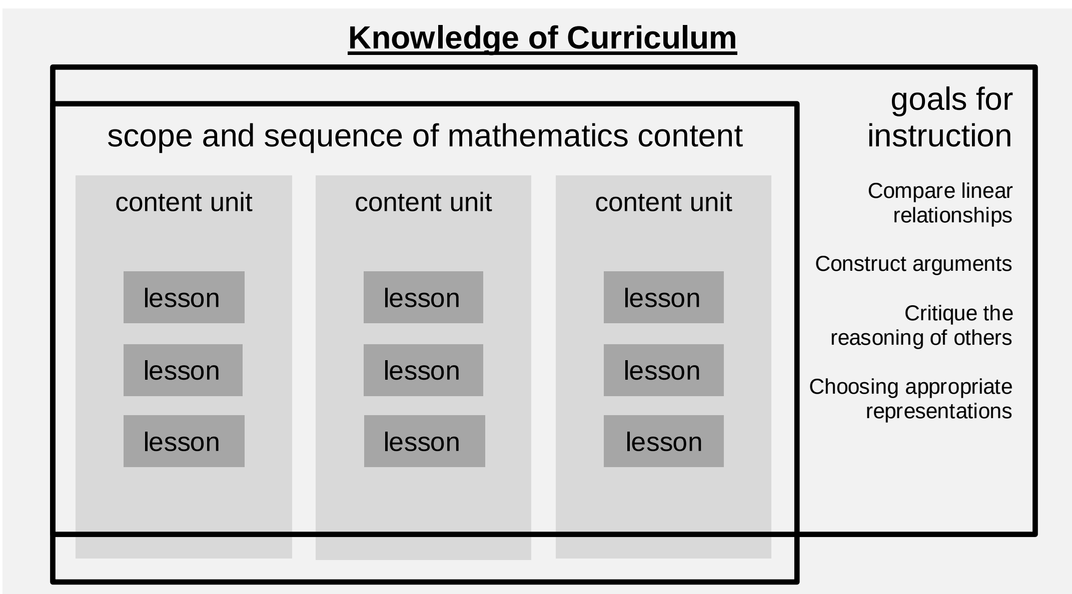


Figure 4.2. Ben’s knowledge of curriculum during his second year of teaching.

Ben explained that he had a lot of practice thinking through and designing a lesson. He chose not to use the textbook tasks as written, claiming that they often missed the critical reasoning piece. Since attending to his goals for instruction was a focus in Ben’s lessons, he felt the need to create his own tasks. He explained how this has been different than Diana's (fellow participant) experience.

> Interviewer: And what about your curriculum materials you use, so obviously this isn't going to be in your curriculum materials, how are you using textbooks or supplemental resources? What are you creating on your own? Give us a sense of that.
>
> Ben: I create almost everything on my own. I occasionally assign problems out of the textbook for homework but it's, they're just not that useful I think. There, I know some of my colleagues use them and they sort of approach it in a fashion that the textbook advises because they come with stacks and stacks of how to teach this stuff, which I mean I think they talk and they show and the kids practice. I know that happens. And it be useful if I did that but I don't, it doesn't, if you do that then all, you give me the action piece but not the critical reasoning piece. And so I don't really use them for a whole lot. I was actually thinking, like this project, this particular activity that you assigned us today because I know that Diana teaches Integrated and those curriculum resources are better at this kind of stuff and so you can pull something out of the Integrated book and do something that students can dig into more easily than you could pull something out of our, I can't remember the title.
>
> Interviewer: Whatever your typical [inaudible] book is?
>
> Ben: Yes, right. I was thinking 'well because of my constraint of my curriculum materials, I likely have a lot more practice with writing an activity start to finish from scratch. (Ben, exit task)

In his follow-up interview, Ben was asked, "What would you say are the two most important ideas that you want students to know about linear relationships?" He explained how he wanted his students to understand using linear relationships to model things in the real world and be proficient at critiquing its application. Both of these goals for instruction are included in the Common Core's Standards for Mathematical Practice (cite). It is unclear whether Ben based these goals off of personal beliefs about the teaching and learning of mathematics or if he drew these ideas from the Common Core.

Ben: I am not sure. In the long term, I would like for it to be like students, in the long term maybe outside of this lesson, maybe even outside of an Algebra 1 class, I want them to be able to understand that it's a useful model and developing that useful model takes some statistical work. That's probably just a stethoscope of just a linear relationships section in Algebra. In the linear relationship section, they have to understand that linear relationships represent some kind of constant rate of change. I guess those are the two.

Interviewer: Okay, very good. And why do you think those are important or the most important?

Ben: One, the first idea is important because it gets tossed around so often. It's sort of normal distributions or something like that. You see all of these relationships printed in US today or in Time or they're thrown up on the screen for two minutes on the news. While this stuff is shown, especially if they have line graphs for stocks or line graphs for president's approval ratings and they want to talk about trends, it's important to have an understanding of how those trends are identified and how predictions about the future are made based on the relationships that we can identify, so you can be critical of those claims because so many of them are malarkey or poorly researched or researched by the people who have an interest in the outcome of the research. That's why that's important because I want to live among citizens. (Ben, exit task)

Ben began this study with little PCK surrounding curriculum. We can see at the beginning of the study that Ben was able to articulate his instructional goals and because he had little other knowledge of mathematics instruction, he focused his lessons around them. He drew on mentor teachers who valued these practices, encouraged him to teach them, and provided an example for how to incorporate them into his instruction. As Ben's knowledge of curriculum developed, he used this knowledge to plan and design for instruction. His justification for tasks and classroom activities included a growing understanding of his knowledge of curriculum as well as a knowledge from the other strands of PCK.

Susie

Susie's knowledge of curriculum developed differently than Ben's. Early in the study, Susie's goals for instruction demonstrated her compartmentalized knowledge of the content for each specific lesson. As the study progressed, Susie demonstrated knowledge of the content in the context of the larger mathematics unit and mentioned goals for instruction focused on mathematical processes. By the end of the study, her knowledge of curriculum included a broad picture of scope and sequence and the mathematics content standards remained a priority in her lesson planning.

Prior to her first year of teaching, Susie's responses in her entry task included ideas about how students learn and understand mathematics. Susie demonstrated a knowledge of the content standards and instructional strategies by recreating a lesson plan she previously designed for a college course assignment. Her plan integrated this knowledge to include actions that led students to development of the mathematics content and the process goals. As Susie entered her first year of teaching (under the guidance of a mentor teacher) she abandoned some of these ideas about the teaching and learning of mathematics and designed her lesson around only the content goals for the particular lesson.

Year 1, Observation Cycle 1 - Susie's knowledge of curriculum focused on the content standards for the individual lesson. During the first observation cycle Susie implemented a lesson plan that followed the examples provided by the textbook. After enacting the lesson, the interviewer asked why she skipped over some of the problems in the lesson. After talking it over with the interviewer, she realized how the skipped problems fit into the lesson and led into the following day's lesson. In this example, we see how she had not considered the value of the strategy presented in the book.

> Interviewer: I did notice them using proportion quite a bit and I wondered, I even looked in your copies of the pages that you gave me in the lesson, how they set up. They set up some of these problems without a proportion but more of a simple equation, so they find the scale factor and then use an equation. But you chose not to pursue that. Talk about that, because a lot of times teachers say ‘oh this is what the book says, this is what I do.’
>
> Susie: Let us see. I guess that is definitely a good, maybe I will use that tomorrow, a good transition step between the using of the proportions and recognizing that the proportion, that the second part of the, that proportion is basically your scale factor…Yeah, I guess I did not really. I did not see that as a transition step, I thought that they could just see it, and they did not. So that is good, that is why they had that in there. (Susie, Obs. Cycle 1, stimulated recall day 2).

This interaction was followed by the following excerpt in which Susie explained the lesson goals for next class period, describing which section of the book she would teach.

> Interviewer: So it sounds to me, making sure I understand, that in the end of perhaps tomorrow, the goal will be to say ‘well if you are going from this shape to this shape, and we know the scale factor, then we just multiply the side by the scale factor and then we get the answer. We do not have to set up a proportion.’ That is the goal there?
>
> Susie: Right, the goal of tomorrow. I have one very small part of this second chapter, or the second unit, lesson I guess, section, that needs to be taught tomorrow, so we will have an opportunity to explore more of that and then continue on in the exercises in the section. (Susie, Obs. Cycle 1, stimulated recall day 2).

While Susie answered a question about where the knowledge of scale will be useful for students in the future, she did not tie the lessons within this unit together. At the start of

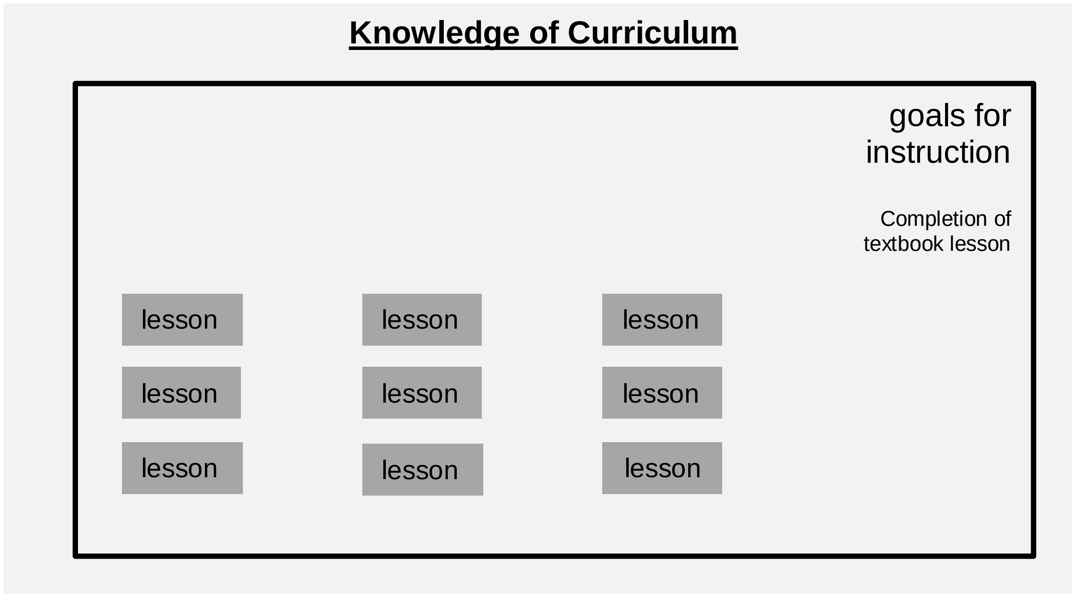


Figure 4.3. Susie's of knowledge of curriculum at the beginning of her first year of teaching.

her teaching, Susie developed her lessons as compartmentalized instructional units without consideration of the larger content unit or scope and sequence of mathematics. Her goals for instruction did not include goals beyond completion of the textbook pages.

Year 1, Observation Cycle 2 - Susie's knowledge of curriculum expanded. During the second observation cycle, Susie taught content that included scientific notation. During this cycle she mentioned her goals for the lesson as they mimicked the goals for the whole chapter. She displayed knowledge of the scope of the content unit beyond the lesson observed. She said,

> My goals for these two days are that students with scientific notation initially understand, well fluency with exponents in general is the continuing theme through the whole chapter, so scientific notation how it is helpful, they will be using it in science. I want them to be able to convert things into a simpler form and then simply be able to manipulate expressions with exponents I guess. (Susie, Obs. 2, pre-observation interview)

Susie's knowledge of curriculum also developed as she has started to think about the goals she had for her students beyond mastering the content standards. In the pre-observation interview she described that she is trying include opportunities for her students to engage in deeper thinking. She stated,

> Let's see I am really striving to incorporate in my teaching deeper understanding. I guess in mathematics, and I think a lot of that comes from working with and being forced to think and strategize and that kind of thing. I am trying to incorporate at least a couple things a week into my classes, so it's not every day, it depends on the section too. (Susie, Obs. 2, pre-observation interview)

Susie enacted this goal of having her students strategize in her lesson the following day. During the lesson Susie gave her students a task without prescribing a specific solution strategy. She was surprised by how organizing her lesson in this way provided an opportunity for a rich discussion about the mathematics

> Interviewer: Alright, well then you moved on to the planet activity. Tell me about what you thought about that. What was your purpose of doing this planet activity and do you feel like it was met? And what evidence do you have for that?
>
> Susie: My plan for that, I just wanted them to explore the idea of the magnitude of numbers of scientific notation, and what that exponent really does, in relation to the magnitude and everything? I think that it actually worked out, they got more out of it than I was expecting them too. So my goal is for them not to give any prior instructions at all, and I did that. And I gave it to them. And they came up with their own strategies. And some of them used one strategy and then realized there was a simpler way, so they modified their strategy and the discussion was really good. That was the best discussion I think we've had in that class. (Susie, Obs. 2, Stim-recall day 1)

As Susie gain experience in the classroom during the second half of the year, she demonstrated some knowledge of how the content was situated in the larger unit of mathematics. She also began to discuss goals beyond the mathematics content. However, she did not prioritize these goals surrounding learning practices. In *figure 4.4*, Susie's knowledge of curriculum appears very similar to Ben's knowledge of curriculum during his second year of teaching. One difference is the lack of specificity surrounding her goals and also how she uses this knowledge to plan her lessons. Susie's lessons remained focused on the content and she discusses engaging her students in mathematical practices as an additional piece or extra activity.

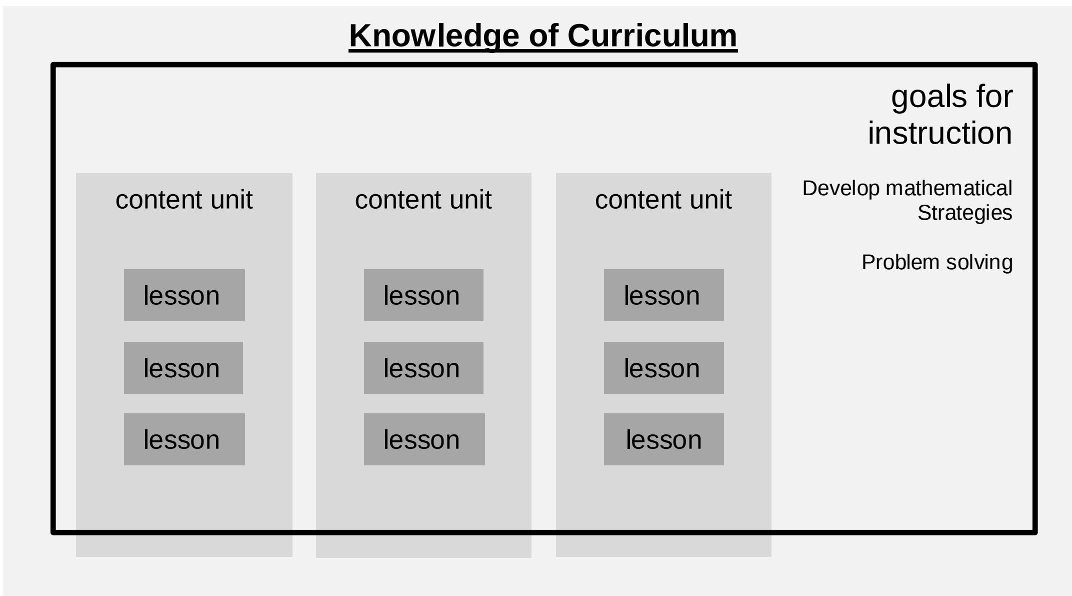


Figure 4.4. Susie's knowledge of curriculum at the end of her first year.

Year 2 - Susie's knowledge of curriculum grew and her content focus remains on her goals when planning lessons. It was not until the beginning of her second year teaching that Susie demonstrated a connected knowledge of the content across lessons. After teaching the same curriculum the previous year, she saw where the content led and how she might help make connections for the students.

> And also, that was something that is going to go directly into, after we finish this section on factoring trinomials, then we're going to go to the quadratic equation, the quadratic formula, which we're going to be using to solve trinomials that aren't factorable, so I can just refer back to this and say, "Remember there were three problems on there that you said didn't make any sense?" Here's what we

> can do to remedy that and you can solve all quadratic equations. (Susie, Obs. 3, Stim-recall day 1)

While Susie discussed the value of engaging students in tasks that embody goals for doing mathematics during the final observation cycle, Susie discussed how that work has a particular time and a place. It is unclear whether she felt that the seriousness of pre-calculus was the major factor in eliminating the inclusion of mathematical applications or if she still saw the learning value in engaging students in application in this course but felt pressure to move through the curriculum, limiting time for such work.

> Interviewer: So how come the difference there? In pre-calc they don't need applications? How come the other classes get it but this class doesn't?
>
> Susie: There's just so much in this book that most of the students in pre-calc are going to take calculus as well, and I feel like pre-calc is just a preparation for calculus almost, and there are so many tools that I really want them to know really well that I've spent more time on the algebra and working by hand and everything, because I don't think that they're going to be coming back to me next year, so I wouldn't be able to fit in those details for them. So I asked the principal that was the prior teacher. He actually teaches calculus. So I just said, "What do you want them to know for next year?" and he said, "I really want them to know their unit circle and I really want them to know this and that," and I'm like, "Okay, I'm going to make sure they know this and that." So it's a heavy course. (Susie, Obs. 4, Pre-observation interview)

In this example, Susie explains that she prioritizes coverage of the mathematics content over this mathematical practice. Susie describes her two days of lesson by detailing the mathematics content that she will teach and assess.

Exit Task – Susie includes both content goals and mathematical processes. Susie created a two-day lesson on linear equations per her exit task. During the first day, Susie planned to introduce the ideas of a linear relationship to her students through real world examples and by looking at different representations of the relationships. She said,

> Okay, day one, what I wanted the students to learn is actually I was waiting a day, day one is more intro to the linear relationship and what it means in a word problem and graphically speaking the table thing, I wanted them to be able to relate to all of those things and then day two is where I actually address the issue at hand. (Susie, exit task)

During the second day, Susie planned for students to graph linear relationships and explain their reasoning to the class as to why they think their relationship is linear. Susie continues to value a focus on the content goals as the real learning while also including mathematical practices such as reasoning, students justifying the mathematics, and real-world application.

Susie demonstrated her knowledge of the sequence of math content by suggesting what lesson would follow these two days and what prior knowledge students need for learning about linear relationships.

> Interviewer: Where would you take students after this lesson? After these two days what would you do?
>
> Susie: I would probably go onto two linear equations and have them find solutions to two linear equations.
>
> Interviewer: What do you think they need to know in order to be ready to do the lessons that you planned, what builds up to this set of lessons?
>
> Susie: They do need to know how to use equations, evaluate by plugging in values, they need to be able to know how to graph XY pairs, coordinate pairs, they need to be familiar, relatively comfortable with variables and setting up equations, they don't need to know anything about linear equations or how to graph lines. (Susie, Exit Task)

Over the course of her first two years of teaching, Susie's knowledge of curriculum grew from understanding individual lessons to gaining perspective of how each lesson fit within the broader scope of high school mathematics. Susie had more opportunities for growth in this area as she taught multiple courses and recognized how the content connected across the course. During her lesson planning, Susie focused mostly on content standards for that lesson. She did incorporate and explain the value of mathematical practices in some of her lessons.

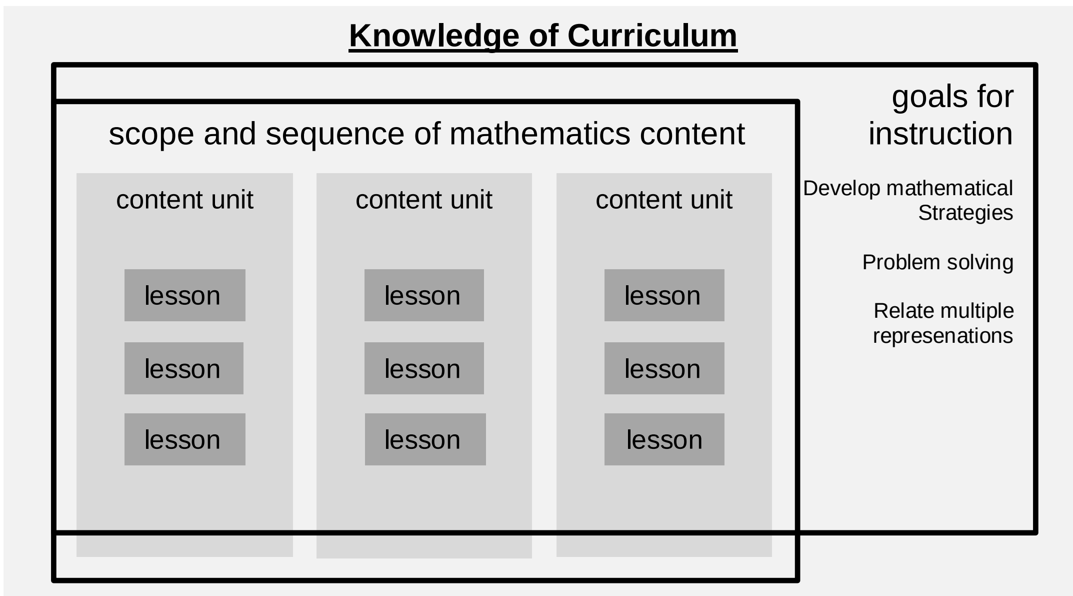


Figure 4.5. Susie's knowledge of curriculum during his second year of teaching.

Diana

As Diana began her teaching career, her knowledge of curriculum could be seen through her understanding of the content standards. During each observation, Diana explained the mathematics that she expected students to learn. Even from the start of her first year, Diana looked beyond one day of instruction and focused on how the mathematics of that day fit into the larger unit surrounding that topic. She admitted that she lacked the knowledge of the larger scope and sequence of the curriculum, beyond each content unit, and was curious how this content will be further developed in subsequent courses. Overall, Diana's knowledge of curriculum does not appear to change much throughout her first two years of teaching. Diana's goals were most focused on content. She did mention some habits of mind that she would like for her students to engage with but these are not mentioned in her goals for each class period or not the reason for particular task selections or instructional strategy choices.

Year 1, Observation Cycle 1 - Diana's knowledge of curriculum can be seen through her knowledge of the content units. During the first observation cycle, Diana explained her goals for the two days of observation.

> Interviewer: What are your goals for these two days?
>
> Diana: Well, certainly I want the kids to get a firm grasp of what a proportion is, where in real-life situations they'll be found and how to solve one. Seems like two days is a long time to be going over proportions but I feel like the kids need it. (Diana, Obs. 1, pre-observation interview).

In some circumstances, having students engage with "real-life situations" can be a venue for focusing on mathematical practices. However, in this lesson, Diana intended to provide students with real-life examples without encouraging the students to engage in any additional learning processes. Instead, students followed the steps provided by the teacher. During this observation, Diana focused mostly on the content goals. Diana articulated what her students were expected to know based on state and local standards. She understood that her current lesson (proportions) was situated in the larger unit of rational numbers. She explained,

> Well, I feel like the learning goal is not very specific. The objectives that are given are very vague and they don't specifically say, proportional reasoning. It says, working with rational numbers and being able to solve problems with rational numbers and I think proportions is just one small piece of rational numbers. In the guidelines that we are given, it's broken down into units and the unit objectives match the Missouri learning goals and then with this new book

> that they've gotten, the new curriculum, they've broken it down into what chapters and what sections of those chapters they expect to have taught in that unit. So matching proportion with one of the objectives that's sent to us from the math department at [school district], it's not real obvious but I know that they expect us to go over chapter 7.2 and that's what we're doing so I'm trying to make the best of what I have. (Diana, Obs.1, Pre-observation interview)

After teaching the lesson, Diana discussed the importance of students having a variety of strategies for solving problems. She had not planned for her students to use the method of cross multiplying and dividing to solve the problems in her lesson but once some of her students began using this method, she did not want to discourage them from using it. In the following example the interviewer asked her about the use of this solution strategy.

> Diana: I asked because I know they're going to be expected to show more than one way of solving and I don't know if I've said it in your company, but it is important for kids to know that there's more than one way of solving a problem, I've lost my train of thought, remind me of your initial question.
>
> Interviewer: Cross products and dividing
>
> Diana: Okay, I knew they would grab onto cross products because this class has the one student that always says cross products, but I wanted them to see that it could be solved as an equation and the equations that we gave them previously, the variable was always on the top, in the numerator and I knew that was going to confuse some of them, I kind of feel like I left them hanging on that one, I felt like I should have worked that through so they could see the difference, but I didn't want to do it all for them, so it was really a struggle with myself at that time, do I finish it or do I not finish it, how many am I, I don't just want to give them the information, I want to make them think about it, but how many of them are going to go home and actually going to think about it, I don't know. (Diana, Obs. 1, Stimulated recall-Day 1).

Diana also explained that she faced a tough, in-class, decision to either tell her students something or to let them struggle to figure it out, as a way to encourage them to think about the mathematics. In this excerpt, Diana considered how to encourage mathematical practices in her classroom, while they were not a specific goal of her lesson.

During Diana's first semester teaching, her mentor teacher acknowledged her curriculum knowledge growth. She said, "I think she has a little bit better understanding of how it all goes together. In realizing that things do not match and I probably has a good idea of what we are supposed to be covering in eighth grade and I think she will know where to go, where to look for what they need to know for the class that she is teaching," (Elaine, mentor interview, Obs. Cycle 1).

Year 1, Observation Cycle 2 – Diana moves to a new school and her own classroom. Diana admitted that being a new teacher leads to the disadvantage that she does not know where the curriculum is going. This was especially true in Diana's circumstances as she was unfamiliar with the integrated curriculum materials, which do follow the traditional Algebra- Geometry-Algebra II path. However, she considered her students' experiences beyond this course. During this first year of teaching, Diana used the Connected Mathematics Project (CMP) curriculum. During this observation cycle, she was in her first semester using the materials. She was able to show some knowledge of the content and scope but was aware that there was a lot she did not know. She said

> Well, unfortunately I wish that I had a class that was integrated one, integrated two, and integrated three so I could see what my integrated three kids have a foundation knowledge of and what my [integrated] two kids would have from integrated one. I know I had with the absolute value question, "Do I really want to address deeply what that means? Is that something that they're going to experience in integrated four? Is it really something that they've had before in integrated, or is something they've had in some other content course like chemistry?" I don't know, and I felt like that was a handicap. (Diana, Obs. 2, Stimulated recall-Day 1)

Diana worked to interpret the expectations set forth by the curriculum materials and what types of problems and representations were included.

> Well, they have in their homework they are seeing other things that are not necessarily functions as f(x) equals and there's some continuous function that can be graphed. There's this one that just has a like an arrow diagram of the input and the output, and there's this one that has just coordinate points. F(x) and then they have a set of coordinate points and they have to identify the inverse. There's one in their investigation where they have a graph of points and they're asked to find the inverse of those points and what they notice. Well, they're on the other side of y=x. SO I think that we'll be experiencing those kinds. (Diana, Obs. 2, Stimulated recall-Day 1)

She went on to explain that she was introduced to pieces of the content, that she had not previously considered, that should be included while meeting with her PLC.

Year 2 - During her second year of teaching, Diana continued to unpack the learning standards for each unit. In her second year of teaching Diana continued to be surprised by the order of the content and unsure what previous concepts her students had been exposed to. She said, "…as I was thinking this morning, and looking at what was coming next it was using equilateral and scalene triangles. That's one thing that I took for granted that the kids already knew and I was contemplating when I am going to really talk about scalene and equilateral triangles," (Diana, Obs. 3, Stimulated recall-Day 1). This was particularly difficult for Diana as she has not taught the same courses throughout her first two years. She mentioned a change in the standards, but not their textbooks. As a result, she tried to determine what had been taught as the order the school chose to teach some courses did not match the order of the content in the textbooks. Diana had an awareness of where the content was leading and knew how she expected her students to apply their learning later in the unit.

> But how does all this fit together? The last section of this unit is actually writing proof so I want the kids to be able to take a given piece of information like maybe

> a conditional statement and use that and then write their proof on their own based on the information that they have so having these properties of equality are necessary when they're trying to justify and write their own proof. (Diana, Obs. 4, pre-observation interview)

Also during her second year of teaching, Diana explained the habits of mind she valued. Although, her lesson plans were developed around the content and she does not explain how her instruction was designed to promote particular mathematical practices.

> Well we all decided that we would teach this before we taught similarity which makes sense to me, I think similarity is a huge concept in all of mathematics, trying to get the kids to reason more sophisticated, maybe that's not the word I mean. This is setting them up for proofs, I feel like when I want the kids to do something, I want them to be more mature about it I don't want them to say that's because that's the way it looks. No, I want them to really show me that is the way that they think it is and that's a huge idea in math in my opinion of being able to show that things are true and then lead into a general case that they're true all the time. (Diana, Obs. 3, pre-observation interview).

Diana valued abstract reasoning and knew that this type of thinking was important for proofs but did not apply this to particular instructional strategies.

Exit Task – Diana prioritizes mathematical practices. In her exit task interview, Diana explained that she wants students to make sense of problems and communicate their understanding.

> Ultimately for students to understand writing equations of a line, what the *y*-intercept is, what the slope is, and situational meaning, so if I've got this equation and then I understand that *x* equals three therefore *y* equals whatever that fits for the equation, I want them to be able to communicate what that means realistically not just mathematically. (Diana, exit task interview).

She went on to explain specifics about the content by listing big ideas of her lesson on linear relationships and what future things this would lead to but she is not including in this lesson.

> Those are the big ideas, slope, Y intercept, X intercept, equations… Yes, I don't think so, certainly linear, in the high school lessons that I've taught we just to add to that with linear programming and with systems of equations and graphing more

> than one line, so if we graphed more than one line I don't think that at an entry level lesson I certainly wouldn't teach that the intersection of those two lines is when they're equal to each other and we're not dealing with inequalities we're dealing with equations so having a line dotted if we're dealing with less than or greater than and not equal to, those are things that are not necessary to talk about yet, rational functions if we have a variable, okay I haven't talked about domain and range, I don't know if that's the most important thing to start with, maybe that's the next lesson, so entry level linear relationships I'm only thinking Y intercept, I'm not thinking domain and range, I'm not thinking systems, I'm not thinking inequalities. (Diana, Exit Task)

During her two years of teaching, Diana's knowledge of curriculum remained focused on the mathematical content units and she continued to question how the units all fit together. While she values her students engaging in mathematical practices, Diana did not

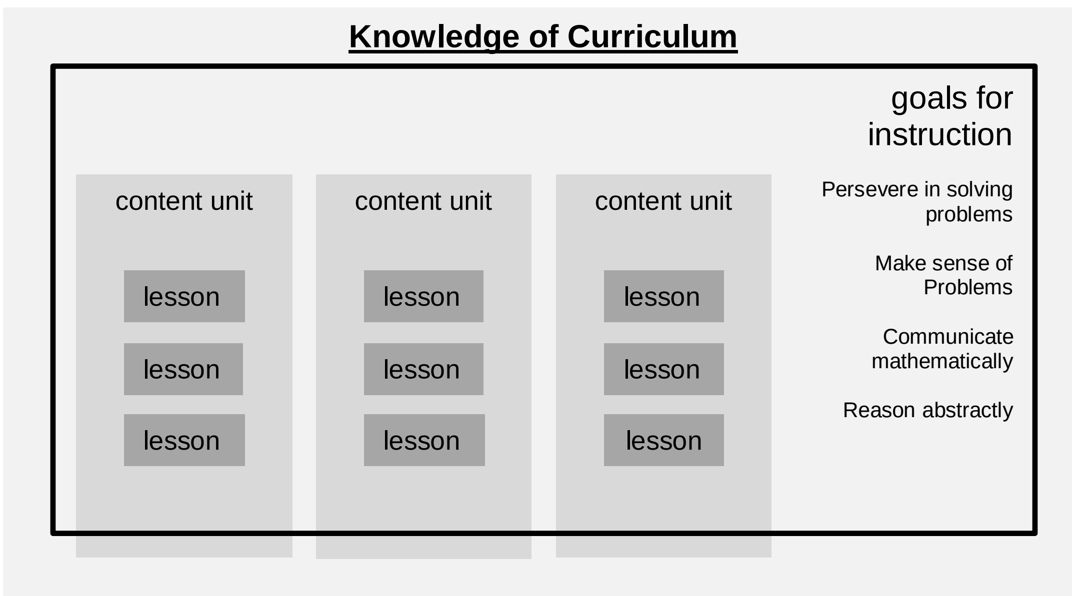


Figure 4.6. Diana's knowledge of curriculum throughout the study.

prioritize these learning practices as instructional goals or, as you can see in *figure 4.5*, include them as part of her lesson plans.

Diana taught in two different schools during her first year of teaching, which included two different approaches to the arrangement of the content. During her second year of teaching, Diana switched back to teaching in a single-subject mathematics series.

However, at that time, she was teaching at the high school level. By switching teaching assignments so often during her first two years of teaching Diana struggled to see the larger picture of the curriculum and further develop knowledge of the curriculum. Diana did not demonstrate knowledge of her textbook materials, instead focusing her instruction on the state learning standards. She gained some insight into how eighth grade math covers linear relationships and solving equations which provided her a glimpse into her students' previous knowledge but overall was left with many questions about where the content was leading in her own course and beyond. Diana's goals for instruction remained focused on the mathematics content, and she designed her lessons with learning standards as the driving force of her instruction.

Comparison of development of curriculum knowledge

Over the course of their first two years of teaching, Ben, Susie, and Diana's knowledge of curriculum developed differently. While each of these new teachers entered the field with different knowledge, they continued to develop PCK in different settings and while attending to different components of their planning and instruction. Ben focused on mathematical practices and including these learning processes as a focal point of his lessons. As he worked through the curriculum, his knowledge grew to include an understanding of the scope and sequence of the mathematics content. Ben also connected his knowledge of the curriculum to his knowledge of instructional strategies, describing how his lessons met his curricular goals. Susie initially focused on the mathematics content. Throughout the two years in the classroom, her knowledge of the content grew to include the larger scope of curriculum how the content connected. Susie's knowledge of curriculum also began to include goals for mathematical processes. However, her plans remained focused on the content. Diana did not develop an understanding of the scope and sequence of high school mathematics. She planned her lessons by looking at the broader mathematical unit but did not make connections beyond the unit. Diana talked about mathematical practices as goals for her classroom and students. However, she did not connect these to her teaching. There were differences in each of these participants' experiences afforded them or prevented them the opportunities to grow this knowledge as they varied in the variety of classes they taught, textbook series they used, and ideas shared through their interactions with their mentor teachers.

Knowledge of student understanding

Through my initial analysis of knowledge of student understanding, I noticed that in some instances, the participants described the student understanding they gained through teaching and described specific aspects of the students' thinking. At other times, the participants suggested students had a particular mathematical understanding without evidence to support their claim or said they were unsure how a student might approach a task or what misconceptions they might have. As I categorized the different types of knowledge regarding students' understanding, I noticed differences among participants in terms of the specifics and evidence they provided. In this section I characterize the knowledge of student understanding development for each participant.

Ben

Year 1 - When Ben began teaching, he predicted some student difficulties based on his experience as a mathematics tutor. Other times, he was unsure what to expect from his students. For example, Ben was unsure what strategies students used in Algebra I or how they would attempt a particular task. He said, "…say if I gave them a lot of data and had them try to predict what's next, then they could do a lot of – they could plot it if that's something that they thought of, or maybe somebody would draw a line, who knows? But they could be using a table…" (Ben, Entry task interview). This example came from Ben's entry task interview, prior to the start of his first year of teaching. Ben quickly learned strategies that his students used once he began teaching.

As Ben began teaching, he gained knowledge of how students approached problems and what misconceptions or confusion they had. Ben was curious about how his students were thinking about mathematics and engaged students in conversations to understand their thinking. During the first observation he explained in detail what the student was thinking

> But we were just talking through the problem and he just nailed it, he had the whole thing and he explained why he set up the equation the way he did and then the only thing that he didn't explain was why the two quantities were equal. He could tell me why this quantity represents the total number of miles driven for that person, why this quantity represents the total number of miles. And it is really tough question like why do you set those equal, how does that get you the right answer? So I asked him that question and he came up with a really good explanation. He was like 'well if they are, if they traveled the same distance then that means they are at the same point and if their miles add up to the same thing and they have, there is an hour where that happens then that is the point that they are going to be matched up.' (Ben, Obs. 1, Stim-recall day 1)

He went on to describe one student's strategy

> That is generally how tries to solve equations, like I think that, as far as I can tell that is how they were taught last year, so if they had the equation x minus 5 equals 8, then they have to think what number minus 5 equals 8. And then they do it in their head that way. And so we have been trying to get them to do it more rigorously so they can handle more bizarre numbers but he is stuck, he wants to use that other method all the time so that is what he is doing. Like he is thinking 'what number minus 6 divided by 5 equals ten?' and so he thinks 'a number divided by 5 has to equal ten, so I know that is 50 and so a number minus 6 has to be 50' and so he thinks. But then he ended up explaining his reasoning for how he came up with 56 and it was that 'well I knew that you had multiply 10 times 5 and then had to add 6 to it.' So he is doing the same mental operations to find the number. (Ben, Obs. 1, Stim-recall day 1)

This type of knowledge of student understanding is representative of Ben's knowledge as he spent considerable class time engaging in informal assessment where he asked questions to gain insight into student thinking.

Year 2 - After investing in getting to know the students and the curriculum, Ben described his knowledge of student understanding through lesson planning. He used common student strategies or misconceptions that he gained from student interaction and his colleagues to choose specific problems or design his lesson. In this example, after discussing the lesson with other teachers, Ben had some expectations for student difficulties although he does not know what instructional strategies to implement to help students. He said,

> They know that the median is if you line up the numbers least to greatest, it's the one in the middle. And they know the mean is if you add all the number up and divide by the number of numbers, that's the mean. I think they don't have a clear understanding of how they differ. Like, they know that both of them have to be in the range of data, so if they calculate an average and they misstep somewhere, they find that the average of a set of numbers between 0 and 10 is like 47, they know they've made a mistake, I think. And so, they have that understanding of the mean and the median, I just don't think they have a comparative understanding and I'm not actually sure what the good route is to get them to see that. Because I can see how these things, if the data's skewed to the left and you have this big tail, that tail's going to pull the average up but it's not going to move the median very much because that tail's really spread out. So I'm not sure. (Ben, Observation cycle 4, pre-observation interview)

During Ben's exit task interview, he looked back at student understanding over a period of time and connect how his lesson designs may have prevented students from having opportunities to compare different types of relationships. In his exit task lesson plan, he created more opportunity for this.

> The reason, I mentioned a little bit, the reason for including multiple representations or multiple kinds of relationships is that my experience teaching it the first time when we only gave exponential is that it didn't do enough to give

> kids something to attach their later ideas of exponential functions too, so that was one reason to give more. And I think that, you can identify features of a linear relationship but since this is the first relationship that, like the first functional relationship that kids interact with, they don't have anything to contrast it with. Like when you get to exponential relationships, most, like even if you are teaching a way that doesn't show exponential relationships at the beginning, you can say 'it's not linear, why is it not linear?' But when you are just starting out there's nothing to go on and it becomes this sort of one idea that is just floating in a fog and I don't think most kids are able to ask 'well what happens when the relationship somewhere else?' Very few kids will ask the question 'well what about a ball's flight in air, is that linear?' And it's not that they're not bright or anything else, it's just that they don't have anything to contrast it with. And so identifying features of a linear relationship, I think it's important to be able to contrast that with other relationships because I think that helps to clarify what's going on in a linear relationship. (Ben, exit task interview)

Throughout the two years of observation, the connectedness of Ben's knowledge of assessment and students understanding was evident. Ben consistently used informal assessment to measure his students' understanding and, as Ben's knowledge of student understanding grew, he used this knowledge to inform his assessment, recognizing what difficulties and misconceptions to draw out. Ben talked about the process of gaining knowledge of student understanding as that was a focus for him. Over time, he predicted his students' understanding prior to teaching a lesson. This afforded him the ability to tailor his tasks to target specific misconceptions.

Susie

Year 1 - Susie's knowledge of student understanding developed differently that Ben's. She often commented about whether her students understood or did not understand her lesson. However, rather than providing specifics about their understanding or misconceptions or how students understood the mathematics, Susie provided an overall assessment of how well her class understood the material. Many times Susie suggested possible areas where students might be confused or struggle but did not provide support from student thinking in relation to her claims.

In some cases, when Susie was asked about the students' understanding of the content, she based her response on how she generally hypothesized people thought about mathematics or referenced her own learning. In her entry task interview she said, "It's one thing if you are given the equation and I'm told that C is 14 then I can go plug that in and multiply and that is difficult for some students, too, I think that it is hard for people to see a connection between a scenario and an equation" (Susie, entry task interview). In the classroom, Susie gained perspective on the scope and sequence of the curriculum. She then suggested that students would know the math content based on the fact that they had seen it before. During her first observation, she said

> I think, day one at least, again I think they will feel like they've seen it before. You know what? I think they'll definitely recognize that they're adding quite a bit to it but I think it will sit pretty well with them. I know they've see what we did today; ratios and proportions; and I think they've experienced this probably in their early elementary school days. They might have had to predict like the size, I don't know for sure. (Susie, Obs. 1, pre-obs interview)

Again, Susie did not base her knowledge on the fact that she knows how or why students will approach the mathematics in any specific way, only that they will be ok with it because they have seen it before.

During her first year of teaching, Susie discussed how students' understood the lesson we observed after she taught the lesson. In her descriptions of her students' understanding Susie stated the she recognized when her students understood or not but did not support her conclusion with any evidence. She did not take the opportunity to better understand their thinking. In the following example she did not inquire about specific students' thinking. Instead she restated her initial instruction and assumed the student understood because she did not ask any more questions.

> Interviewer: So what was the issue then with the student being confused about 1.5 times 10 to the zero? Was there an issue there? I thought there was something where a student was uncertain, do you recall?
>
> Susie: Well, I'm not sure. I remember not investigating it too much, but I thought my understanding of what she was saying was why don't we leave it just 1.5 rather than 1.5 times 10 to the 0. And I don't know necessarily (that it) was absolutely the case that was the issue. When I explained I don't think she challenged me on it anymore, to my understanding, but I don't know. So I just described that standard form. (Susie, Obs. 2, stim-recall 1)

Susie continued to describe student understanding in this way. She was unsure how many of her students were thinking and described student understanding as her students' ability to repeat the process she demonstrated. She said

> There are a couple that I don't know about, the other ones I feel that they well Dillon was questioning and trying to follow along but he just needed to understand the process a little bit, but there are two students I don't really know what they think but the others I think got it. (Susie, Obs. 2, stim-recall 2)

Year 2 - During several interviews, Susie described students understanding based on the time it took students to complete a task. She used this as a measure for both the class as a whole and for individual students. During her second year of teaching she said

> I think, overall, I believe they are all able to factor from what I saw walking around the room and asking them questions. There was the one student in particular that was struggling with the foiling concept that I feel may have a little hesitation with this. He seemed to be taking a little longer to do things. (Susie, Obs. 3, stim-recall 1)

She continued to base her knowledge on superficial cues rather than specific student thinking. Also during her second year of teaching, she said the following after our two days of observing her.

Interviewer: Now, you said that you were able to identify some students you thought maybe weren't ready to go on. Can you tell me a little bit about how you figured that out? What was it and who was it?

Susie: Primarily facial expressions, I guess, I saw. And then I walked through the room quite a bit today and I noticed. The first problem we did together, the second problem I had them try on their own and I started walking around and they, either wrote down the problem or didn't write down the problem and were just staring at the board and trying to follow, back step my work from the previous example. So, they just looked a little overwhelmed, like they didn't follow at all. Sometimes I think when I'm excited or when students are getting it, at the same time if they're not getting it, sometimes I speed up my speech, which is possibly what happened and then they're trying to follow my words plus follow what's going on the board. So, that's something I'm definitely working on. Primarily their facial expressions clued me in and also the hesitation when they were working at the desks. They weren't ready to just turn away from the board. They were still staring at the board wondering what had just happened. (Susie, Obs. 3, stim-recall 2)

Throughout her interviews, Susie based her knowledge of student understanding of the mathematics content on cues such as speed, hesitation, facial expressions, and general mood. She also tended to discuss student understanding as an overall class understanding versus individual students, even though her class sizes were approximately half the size of the other participants'. This can be a result of her beliefs about the teacher and the students' role in the classroom and the instructional strategies she employed. Susie believed that she was the owner of knowledge and the students role was to recreate the strategies she demonstrated. Also, Susie did not demonstrate a development of knowledge of assessment throughout this study. Susie did not seek to understand student thinking or diagnose their confusion. While there were some cases where she described a students' incorrect assumption or strategy, her development of this knowledge is limited in regard to students' misconceptions and strategies.

Diana

Year 1 - Diana demonstrated development of student understanding by explaining what her students did or struggled with after teaching a lesson. She gained this knowledge through her experience working with students as she reflected on why they may have struggled. Diana often provided explanations for her in-the-moment instructional choices based on students' understanding but does not predict students' strategies or understanding prior to teaching the lesson.

During our observations, Diana spent considerable time talking with students one-on-one. Diana's classes were 90 minutes long and during each class she spent time considering student thinking. In this example, Diana considered where and why the student was struggling and what methods he was using to successfully interpret the data. She helped him this particular student by validating his method and explaining how he could continue to use it in the future.

> Well, yesterday I was working with one student, you know it is the end of the lesson and we are getting ready to quiz on it and at the beginning of the lesson I had them gauge or kind of I guess assess what they already know. So for the lesson objectives I had them rate themselves one to four, what is your understanding and so yesterday I had them go back through and say what is your understanding now and if there is any huge gaps we need to work on filling those before we move on. One student, it was the different representations of the same function. He could look at the graph and he could look at the arrow diagrams and say I totally understand why this is or isn't one-to-one or it doesn't have an inverse function. But when we just look at a table of values then there wasn't any connection there. It didn't make any sense to him. I kind of felt like that was kind of a success for me to be able to assess what the problem was for him and I made a suggestion to him and whether or not he will follow through with this or not, we had a table of values and he couldn't decipher whether or not it had an inverse function, so I said here is a piece of graph paper, lets graph these points and see what you see and then he graphed them and was like oh well yeah this doesn't have an inverse function because there are two x-values that give me the same y. Well okay but for some reason on the table he couldn't see that. So I suggested to him, when faced with something like this ask me for a piece of graph paper or I will give you a piece of graph paper. I am not sure how to get him over that hump but I would like to at least foster something that he does understand so he can make it through. (Diana, Obs. 2, Stim-recall 2)

Diana continued to consider students' thoughts beyond the class period and analyze why they might be confused. In this example she explained how she was not able to understand her students' comments until the following day.

> Diana: Something that I was understanding yesterday and some things that the kids had said the day before when we doing the coding and decoding, they were given an *x* squared as their coding function, and many of them said "Well I can decode that accurately because I'm not using negative numbers in my code."
>
> Interviewer: What do they mean by that?
>
> Diana: Because I was saying when we take the square root of something, how many solutions do we get? They'd say "Well, two, but I don't have to worry about the negative numbers because all of the numbers in my code are positive. So I know I'm not going to use the negative number."
>
> Interviewer: Meaning their inputs or their outputs?
>
> Diana: They were looking in their table of inputs and recognizing none of their inputs had negative numbers. So when I decode, if I get a negative number, then I'm not going to use it. I didn't make that connection until yesterday. It didn't dawn on me, "Oh, that's what they were talking about when they said I know I don't have to use this negative number." What I was trying to get them to think was more functionally, more mathematically, more theoretically than the actual situation which is one of my goals; to get them to understand situation math rather than just theoretical. Yeah I want them to know the theoretical, but I really want them to understand the situations and understand how it applies to the things they're going to do. (Diana, Obs. 2, Stim-recall 1)

Year 2 - During this study, Diana was often unsure of the students' prior knowledge or where they might struggle. She developed much of her knowledge of students' understanding through her teaching. During her second year of teaching, she was surprised when her students' understanding did align with her expectations of their knowledge.

> When we started with the Pythagorean theorem a lot of kids could say, a^2 + b^2 = c^2 but they had no idea what that meant really and as I go along I feel like I'm learning more and more what they should know and they don't know, we started with Pythagorean theorem and just doing some hands-on visual proofs, taking some triangles and cutting them and having the areas of the squares created by their sides fit into the largest square and we did a couple of algebraic proofs one was President Garfield's trapezoid proof and I thought that the kids should already know the area of a triangle and I found that a lot of them didn't, I thought the kids would already know the sum of the interior angles of a triangle is 180 and a lot of them didn't (Diana, Obs. 3, pre-obs interview)

During the final observation cycle, Diana's knowledge of student understanding grew through her previous interactions with her students. She used this knowledge to think about her unit prior to teaching it.

> …then I have some kids that they didn't do so hot on the geometric figures when we were going over lines and planes and points and intersection and bisection and all of those just basically vocabulary at the beginning of the year and so now it's coming back to them and they're like, what wait a minute I thought we were done with this? So that's going to be a challenge for some of them when we look at justifying geometric problems and solutions to geometric problems… (Diana, Obs. 4, pre-obs interview)

Diana demonstrated growth of her knowledge of student understanding as she engaged with students to analyze their thinking. She was able to describe the thinking of individual students and eventually use this knowledge to inform her instruction.

Comparison of knowledge of student understanding

Knowledge of student understanding developed differently for each participant. In particular, Diana was able to discuss students' understanding of particular content after teaching the lesson. While Ben provided this same evidence of his growth of knowledge, he also discussed student understanding as he connected to his lessons prior to teaching them. Diana and Ben are both able to describe, in detail, how their students are thinking about the mathematics. Susie, on the other hand, tended to generally state whether her students understood the math, without describing specific thinking.

Again, it is important to note that each of these participants were teaching in different settings which provided different curriculum resources and instructional expectations and the different types of tasks and instructional strategies used in each of the participants' classrooms offer different opportunities to learn about student understanding.

Summary of Findings

In this chapter, I described two themes that I identified through the analysis of the participants' development of PCK: the differences in the development of their knowledge of curriculum as they attended to different components in their planning and the differences in their knowledge of student understanding. While their development of PCK shared some similarities and differences, it is important to remember the similarities and differences among each of the participants' backgrounds and their experiences during their first year in the study (in a mentor teacher's classroom) and their second year in the study in their own mathematics classrooms. Through the interviews, I learned that each of the participants had some introduction to mathematics education prior to the start of teaching. Diana and Susie had both considered education as a career option and had taken at least one mathematics methods course in college. Ben had spent time as a mathematics tutor, working one-on-one with college students.

Ben and Diana began their first year of teaching in very similar classrooms. Both teachers were in a large school district where there were other mathematics teachers in their building with whom they collaborated on a regular basis. Both of them were placed in classrooms of under the guidance of veteran teachers described as having expertise in teaching mathematics and each of them had the opportunity to teach with both a traditional single-subject mathematics textbook series and a problems-based curriculum. While Diana spent her first semester of teaching under this mentorship, she moved into a different building with her own classroom mid-year and taught a variety of different high school courses in her first two years. Ben continued to teach in the same middle school both years of the study. Susie's taught in a small private school where she worked with little collaboration and taught from a tradition, single-subject textbook series during both years of the study.

Through these interactions with their mentor, students, curriculum materials, co-workers and school culture, as well as their previous experiences, each of these participants developed different PCK. Ben developed knowledge of curriculum that included knowledge of each lesson, the larger content unit, and scope and sequence of mathematics while remaining focused on curricular goals that reached beyond the content and focused on mathematical practices and general learning processes. He developed very specific knowledge of student understanding, describing how specific students approach problems or their misconceptions of specific content. In his interviews, Ben used this knowledge to predict his students' understanding, develop tasks, and plan lessons. At the end of the study, Susie had developed similar knowledge of curriculum but had focused on the specific mathematics content of each lesson during her planning and description of her goals. As she developed knowledge of student understanding, she tended to describe an assessment of the class as a whole rather than specifics of the understanding or each student, often discussing overall the effectiveness of her lesson. Diana focused both on her goals for learning processes and mathematical practices and the math content. She did not demonstrate an understanding of the larger scope and sequence of the high school curriculum. Instead, she focused on each content unit. Like Ben, Diana's knowledge of student understanding developed around specific students and content. She often considered this knowledge during the lesson and would reflect on it in the interviews. However, she did not often demonstrate this knowledge through predictions about student difficulties or her justification for task selection.

These three participants began this study and their teaching experience with little PCK for teaching mathematics and developed knowledge in each of these areas.

CHAPTER 5: DISCUSSION AND IMPLICATIONS

In this chapter I discuss my findings from the study I conducted on mathematics teachers' development of PCK, how these findings connect and extend the existing research, and the implications these findings hold for the field and future research.

Findings and Discussion

My study sought to answer the question: How does the PCK of middle and high school mathematics teachers develop through the first two years of their career? My findings resulted in two main themes that focused on the similarities and differences among the knowledge development of three beginning teachers. Each theme is summarized below along with a discussion of each theme.

Development of Knowledge of Curriculum

As each teachers' PCK developed, their knowledge of curriculum grew in different ways. Despite having a strong background in mathematics content, at the start of this study these beginning teachers lacked knowledge of how the high school curriculum develops or what mathematical connections can be made over time. Over the course of two years, Ben and Susie developed knowledge of the larger scope and sequence of the mathematics curriculum, while Diana provided little evidence of such growth. Similarly, Lee, Brown, Luft, & Roehrig (2007) point out the discrepancy in data surrounding the relationship between strong science content knowledge and PCK. In their study of first year science teachers they said, "Our data also reveals that most beginning secondary science teachers have a limited level of PCK despite their science backgrounds. Unlike previous studies in the field (Gess-Newsome, 1999; Hashweh, 1987; Smith, & Neale, 1989), the result of this study indicate that a strong science background does not guarantee a proficient level of PCK" (p. 57). Their findings are similar to this study in that content knowledge did not guarantee a depth of understanding of PCK of curriculum. The theoretical models of knowledge of curriculum in *figure 4.1* and *figure 4.3* depict the knowledge of Ben and Susie respectively. Both of these figures represent their knowledge during their first year of teaching. Neither of these participants demonstrated knowledge of how the mathematics content of each lesson connected in the larger scope of the curriculum. During their observations, they discussed each lesson as sort of individual islands.

Remillard and Bryans (2004) studied the implications for teacher learning by studying their orientations towards mathematics curriculum materials. Their study focused on the use of standards-based curricula similar to what Ben and Diana used as resources for some of the classes they taught. The authors found that through different use of the materials, teachers were afforded varying opportunities for their own knowledge development, including different opportunities to develop knowledge of instructional strategies, student understanding, and the scope and sequence of the mathematics content. They further explain how teacher's beliefs about mathematics teaching and learning and views of the curriculum materials shaped the opportunities for the teacher to learn (p. 383). This provides insight into the different learning that Ben, Susie, and Diana experienced as they too approached teaching in different ways (seen through their goals for instruction) and engaged with their curriculum materials differently. Susie relied heavily on her traditional textbooks as she planned for instruction. Traditional textbooks often provide daily topics to cover that can be viewed as compartmentalized and often do not provide insight for the students or the teachers as to how each section or chapter is connected or why they are placed in a particular order. In her first year of teaching, Susie was focused on each individual lesson and it was not until she had taught the entire course or range of courses that she demonstrated knowledge of the larger scope of the mathematics curriculum. Susie taught a wider range of courses, teaching one section of most of the high school mathematics offerings. In her first year of teaching, Susie had the opportunity to see how the curriculum developed through middle and high school and was able to use this knowledge during her second year of teaching as she taught the same courses for a second year. Understanding what should be covered in each course was a priority for her as she would see these students again the following year in the next course. This course load can be beneficial in providing opportunities to develop knowledge of curriculum. However, it may come with some disadvantages that should be weighed when considering course assignments for teachers. Ben, who had an opportunity to work with two different set of curriculum materials under the guidance of his mentors both followed the instructional vision set forth by the integrated materials and used the traditional materials only as a reference. Diana followed her materials but also deviated from the materials at times. This may have been a result of her orientations or beliefs about the curriculum materials and as a result she may not have allowed herself to gain knowledge of the curriculum that was potentially offered through her materials.

Development of Knowledge of Student Understanding

Ben and Diana developed a detailed knowledge of their students' understanding, solution methods, misconceptions. Ben began to use this knowledge to inform his future planning for instruction. Susie often talked about her students' understanding of the content but was unable to provide details of their thinking and lumped her class together rather than checking the mathematics understanding for individual students.

Ben and Diana worked with mentor teachers who spent many years receiving professional development of and teaching with a National Science Foundation funded curriculum materials, which included an investigative approach to learning mathematics. The instructional vision promoted by the curriculum supported students working together to discover the mathematics without initial teacher instruction. This classroom structure encourages teachers to consider student thinking as they work toward making sense of the mathematics. In Franke and Kazemi's (2001) article, *Learning to Teach Mathematics: Focus on Student Thinking,* they discuss their paradigm shift in regard to developing teachers' knowledge of student thinking. In their work surrounding Cognitively Guided Instruction (CGI) they had focused on educating teachers about research-based knowledge describing the development of children's mathematical thinking. They realized during the observations of their participants that

> Listening to students' mathematical thinking had another benefit. It transformed teachers into learners. They learned in the context of their practice about the teaching and learning of mathematics and became engaged in what Richardson (1990, 1994) terms "practical inquiry." (p.104)

They describe how teachers were able to continue to develop knowledge years after their professional development because their teaching experience afforded them the opportunity to develop learning with understanding by making rich connections. As a result, Franke and Kazemi suggest a situated approach where teachers are learning along with each other and their students (p.105). Both the collaborative approach to teacher support and the instructional vision shared in the communities in which Ben and Diana were teachers, promoted an atmosphere similar to that described by Franke and Kazemi.

Also, this role of understanding student thinking could be seen through Ben and Diana's developing knowledge of assessment. Both participants discussed the actions they took during teaching to draw out student thinking and work in small groups and one-on-one with students to participate with them in thinking about mathematics. The instructional strategies used by Susie and her lack of PCK of assessment did not afford her opportunities to see student thinking.

Further Discussion

The results of this study confirm the findings of similar studies (Brown, Abell, & Friedrichsen, 2008; Lannin et al, 2013) that alternatively certified teachers develop PCK. Each of these participants gained knowledge in a variety of strands of PCK as they engaged in the work of teaching. However, while the teachers in this study all developed PCK, this knowledge developed differently among them. This is similar to the findings discussed in Lannin et al. (2013) in which two beginning teachers focused on different components of teaching developed knowledge of different strands of PCK. Magnusson, Krajcik, & Borko (1999) provide some explanation for the difference in knowledge development. They stated,

> Teachers' knowledge and beliefs serve as filters through which they come to understand the components of pedagogical content knowledge. These understandings, in turn determine how specific components of pedagogical content knowledge are utilized in classroom teaching. Just as students' existing knowledge and beliefs serve as the starting point for their learning, teachers' knowledge and beliefs are important resources and constraints on change. (p.122)

Also attending to these differences in knowledge among teachers, Lloyd (1999) conceptualized how teachers' views of mathematics content, their students, their pedagogical knowledge, and the context of the department in which they worked impacted changes in teacher learning. Thus, teachers' knowledge may develop differently based on the setting in which they work. Ben and Diana both taught in schools with active professional learning communities. In their interviews, they mentioned meeting with other teachers to discuss instruction. While both participants explicitly said that they chose to sometimes do things different than the other teachers in their PLC's, they were engaged in conversations, hearing different approaches, and considering their practices further. The benefits of these types of interactions with colleagues were observed by Remillard and Bryans (2004) in their observation of the impact of the social setting on teacher learning. All three participating teachers engaged in work with a mentor teacher and mathematics methods coursework and experienced learning afforded to them through the work of teaching.

While further analysis is necessary to connect the knowledge development of the teachers in this study with the sources of their knowledge, Ben and Diana discussed the learning opportunities that working in PLC's and with their mentor afforded them.

Implications for Mathematics Teacher Education

Teacher Education

Guided Practice. The three participants in this study had strong backgrounds in mathematics with little teacher education coursework. While they gained PCK throughout the first two years of teaching, each demonstrated a comparatively small amount of PCK in their first two observation cycles, suggesting that teacher preparation should extend beyond content alone.

Beyond the implication that teacher preparation should include coursework with a focus on teacher knowledge, Ball and Cohen (1999) suggest that knowledge about content, children, student learning, and pedagogy cannot fully prepare teachers for the unpredictable work or interacting with students and curriculum. They recommend a foundation of professional education that includes incorporating teacher learning into the practice of teaching. They argue that the characteristics of teacher knowledge in which they value are situated in practice; they must be learned in practice. Magnusson et al. (1999) recommends a similar foundation for PCK growth where teachers engage in situated learning experiences in meaningful contexts (p.121). The idea that we focus on the areas that need improvement and practice in a way that recreates an exemplar model will lead to us closer to that desired model. It makes sense that, to know and use PCK, teachers need opportunities to engage in work that develops this knowledge. As a result, teacher education should continue to in-service teachers. This is not something that should be done separate from their everyday work. Instead, it should take place in within schools and districts and with teachers teaching the same curriculum, both the same course, and across courses.

Changes to Education Programs. To provide meaningful practice for teachers, education programs must consider what this will look like. Several researchers (e.g., Friedrichsen et al., 2009; Smylie, 1989; van Driel, Beijaard, & Verloop, 2002) suggest that teaching experience matters when considering the development of teacher knowledge, but how can we do this prior to entering the field and how? In order to provide developing teachers with opportunities for practice, we must move away from the traditional classroom setting where students sit and get the information. Instead, courses should be redesigned to include a workshop model where teachers engage in authentic practice that includes conversations about the content with learners and on the spot decision making about instructional practices. Also, having expert guidance and time for reflection and revision during this process can provide opportunities to develop PCK. Lee et al. (2007) point out that "field experience" is not enough.

While classroom experiences provide real-life students in a learning environment, which can be hard to recreate in a university setting, to gain the most from these experiences, we must be clear about our expectations for the role of the teacher and pre-service teacher in order for the pre-service teacher to have opportunities to gain PCK. The teachers in this study benefitted from opportunities to work with other teachers and have meaningful interactions with students through one-on-one conversations and whole class conversations. Currently, pre-service teachers do not get opportunities to see or be involved in the work of teachers, getting to see what decisions go into their lessons and how they make decisions and on the spot changes based on insights gained through their interactions with their students. Engaging pre-service teachers in this work requires changes to traditional field experiences which typically include observation of instruction only. Also, allowing for opportunities for discussion and reflection based on field experiences can also provide opportunities for learning. We should be designating time for this to be done with classroom teachers or college instructors and classmates. This requires an interactive relationship with colleges of education and school systems that allows for collaboration and access into the work of teachers. If we can foster a community of teacher learning in our k-12 classrooms, everyone stands to gain from this model.

Building on Knowledge of Student Understanding. The participants in this study frequently demonstrated their knowledge of student understanding. Beyond learning what their students know, how they attack problems, and where they struggle, knowledge of student understanding was often a basis for making connections among strands of PCK. Whether they were using their knowledge of other strands to gain a deeper knowledge of student understanding (i.e., creating tasks that elicited student thinking) or using their knowledge of student understanding to inform their knowledge of other strands, this could be viewed as a focus for both Ben and Diana and it promoted the type of learning environment that provided an opportunity for growth in their knowledge of student thinking. As we seek ways for our teacher preparation programs to better prepare teachers, providing opportunities for preservice teachers to focus on student understanding can provide a means for developing teachers' PCK.

The idea that teacher knowledge can be developed through engaging with students' thinking extends to the type of and use of curriculum materials and curricular goals. The teachers in the study who gained PCK surrounding student understanding implemented tasks or engaged in discussion that allowed for meaningful interactions with students and the mathematics. Teacher preparation that therefore must reach beyond analysis of student thinking and dissect what types of tasks or questions elicit meaningful student thinking.

Opportunities in Curriculum Development

In many cases, teaching materials marketed with curriculum materials provide additional information, strategies, or tasks for teachers to use. However, these materials fall short when considering how teachers can make connections among this information or how to apply it in their own classrooms. Remillard and Bryans (2004) said, "…in the current wave of curriculum development, some developers have taken up the task of designing curriculum materials that will not only provide teachers with guidance for classroom instruction, but will also foster teachers' learning as they use them," (p.4). In their study of a standards-based curriculum, one way that teachers' opportunity to learn was fostered by the curriculum was through the use of student tasks that elicit student thinking. They also explain that the potential for teacher learning is affected by how teachers' use the materials, often as a result of the orientations and beliefs. I suggest curriculum developers work to help teachers make the connections among the student tasks, suggested assessments, instructional strategies, and the scope and sequence of the content. By offering teachers readable information supporting the instructional vision of the curriculum and how teachers can support it with each of the strands of PCK, teachers may see the vision for the curriculum and fully invest in it, gaining this knowledge and these connections.

Limitations

As we consider the implications of this study, two important distinctions should be considered. The participants in this study completed a very specific alternative certification program which included a year-long internship. During their first year of teaching, the participants taught in a hand selected mentor teacher's classroom and engaged in coursework that included regular class meetings. This is different than the education opportunities and experiences of most alternative certification programs which do not include 15 months of coursework taught by University faculty and a year-long internship. It is also different than most teacher development programs which include coursework and internships at separate times in the program.

Also, I describe how teachers' PCK develops during the first two years of teaching. However, because of teaching assignments and schedules, I was not able to observe how teachers' developed PCK around a specific topic. For each observation cycle, teachers were generally focused on different mathematical content. As a result, the findings in this study do not demonstrate continued growth of PCK around specific topic. Rather, it represents continued overall PCK growth.

Further limitations include that data analysis was limited to interviews only providing limited perspective of the use of PCK during planning and teaching and little information about the context of their classwork and schools. We know little about each teachers' background and their experiences prior to this study which influenced their beliefs and knowledge development. This analysis did not include insight into the potential sources of the teachers' knowledge development (i.e., work with the mentor teacher and other teachers in their building, information about textbook and resources, student information, or details about their University courses).

This study had a large research team and not all interviews were conducted by the same researcher. With a semi-structured interview approach, this allows for some varying focus throughout the interviews. Also, I observed and participated in interviews for two of the three participants.

Implications for Future Research

Practicing Teaching

The implications in the previous section suggest that the benefits of providing future teachers with authentic teaching practice leave us to wonder what models of teacher preparation can offer rich opportunities for teachers to develop PCK. Further research is needed to understand how teacher development programs can make use of classroom or internship settings to offer these learning opportunities.

Sources of PCK Development for Teachers

The three participants in this study developed a substantial amount of PCK and the depth of their PCK grew as they were able to make connections among the strands. The analysis of the sources of their knowledge is therefore something that should be considered when supporting the work of current teachers. Throughout their interviews, the participants noted some sources of their knowledge. Future research is needed to connect the sources to content knowledge, PK, or PCK, and what better understand what and how specific sources provided teachers with opportunities to gain PCK. Further, it would be relevant to know how these sources interacted and at what times in their teacher development these opportunities should be provided.

Beliefs and Orientations in the Development of PCK

It was clear that each participant brought a different background and various ideas of teaching and learning with them as they began their first year of teaching. Further research is needed to determine how different beliefs about teaching and learning lead to development of specific strands of PCK or more PCK overall. These findings suggest what teacher educators should be attending to in terms of beliefs and orientations in order to maximize learning opportunities.

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APPENDIX A: ENTRY TASK INTERVIEW PROTOCOL

Talking Through the Plan

Say to the participant: The first part of the interview is about the plan you just wrote. We want to make sure that we understand your plan and what you intended for these two days.

Begin with these questions:

- You spent an hour writing this plan. Tell me about the process you used to create the plan.
- Have you ever taught this topic before? Describe your experiences with teaching this topic.

Then ask the participant to walk you through his/her plan by asking:

- Walk me through your plan. How did you start the first day? *Continue to ask clarifying questions; your task is to be able to really understand what the participant intended for each part of the plan. Possible clarifying prompts:*
    - o What did you mean when you wrote _______________?
    - o Could you clarify what the students are doing during this part? (Probe: Talk to me about why you decided to do that.)
    - o Could you clarify what you are doing during this part? (Probe: Talk to me about why you decided to do that.)
    - o Could you tell me why you decided to do that? (Probe: Talk to me about why you decided to do that.)

Probing Participant's Knowledge

Subject Matter Knowledge (SMK/CKT)

Say to the participant: One part of what a teacher needs to know is something that we call content knowledge. In your case, we mean your own understandings of the (mathematics/science) that you will be teaching. These next questions are designed to probe what you know about linear relationships. Again, there is no right or wrong answers. We are interested in what you know and how you think about linear relationships at this point.

1. What are the big ideas associated with this topic?
    - How are these big ideas related to each other?
2. What are the two most important ideas you want students to know about this topic?
    - Why do you think these ideas are the most important?
    - How does this topic relate to other topics in (mathematics/science) in middle or high school?
3. Talk to me about how your plan addresses these big ideas? *(Probe for specifics based on the plan).*
4. What else do you know about this topic that you wouldn't teach to middle school students? High school students?
5. What else do you need to learn about this topic before you teach it?
6. When did you learn about this topic?
7. Have you applied this topic in an out-of-school experience? If so, describe.
8. [NOTE TO INTERVIEWER: IF THE INTERN REPORTED THAT (S)HE HAD TAUGHT BEFORE, THEN ASK THIS QUESTION] What did you learn about this topic when you taught it before?

Knowledge of Learners
Say to the participant: The next questions are designed to probe what you know about how students learn linear relationships.

1. What do you think students already know about this topic?
    - Where do you think they may have learned this?
2. How do you expect students to perform on (a particular task from the lesson)?
    - What strategies might students use?
    - What responses might students provide?
3. What about this topic do you expect students to have difficulty learning?
    - Why do you think they will have difficulty with that?
4. How do you think students learn about this topic best?

Knowledge of Instructional Strategies (NEW)
Say to the participant: We want to know more about how you organized the instruction during these two days. The next questions will help us better understand your decisions about what and how to teach linear relationships. I want to go back through the major sections of your plan and probe a little more about decisions you made. [INTERVIEWER: THESE QUESTIONS ARE INTENDED FOR YOU TO PROBE ABOUT INSTRUCTIONAL APPROACH (E.G., LECTURE, LAB, INVESTIGATION, ACTIVITY, SMALL GROUP WORK, WHOLE CLASS DISCUSSION, DEMONSTRATION, SEAT WORK, USE OF REAL-WORLD PROBLEMS, USE OF DIFFERENT REPRESENTATIONS, USE OF ANALOGIES, …)

1. Let's look at how you began the first class. I see that you chose to *begin* the class with ___________. Talk to me about making that decision. [INTERVIEWER: YOU MAY NEED TO REPEAT THIS QUESTION, DEPENDING ON THE DIFFERENT WAYS THE INTERN CHOOSES TO ORGANIZE DIFFERENT PARTS OF INSTRUCTION, SEE d. BELOW]
    - Where did you learn about how to teach this way?
    - How do you think _________ helps students learn this topic?
    - In what other way(s) did you consider teaching this part of class? If you didn't think of other ways before, what ways can you think of now? Where did you learn these ways of teaching? [INTERVIEWER: MAKE THIS A SAFE PLACE FOR YOU TO PROBE ABOUT WHAT KNOWLEDGE THE INTERN HAS ABOUT DIFFERENT INSTRUCTIONAL STRATEGIES]
    - IF REPEAT OF QUESTION IS NECESSARY: I see that you then chose to change what you and the students are doing. Talk to me about that decision. (REPEAT A-C ABOVE)

2. I noticed that you used a picture (graph, equation, Punnett square, analogy…) in your plan. Tell me why you used that ____________ at that point in your plan.
    - How do you think this (picture, graph, equation, analogy) helps students learn about linear relationships?
    - Did you consider representing that idea another way?

Knowledge of Curriculum
Say to the participant: These next questions are designed for us to know something about where your ideas for these two days came from.

1. What is your goal for these two days of class? Why are those goals important?
2. Where did you get your ideas for these two days of class?
3. What specific resources would have helped you design the plan?

- What specific information is in those resources that would have helped?

4. Tell me about the materials (handouts, transparencies) you prepared.
    - Where did you get the ideas for these materials?
5. How do you think these materials will help or hinder achieving the purpose of your plan?
6. What would you teach just before this topic? Just after?
7. States typically have a set of “standards” that guide mathematics/science instruction. For example, in Missouri, we have the GLEs (Grade-level expectations). Tell me what you know about these types of documents.

Knowledge of Assessment

Say to the participant: The last area I want to ask you about is how you will know what students learn from these two days of class.

1. During the 2 days of instruction, how will you know if students are “getting it” or ”not getting it”
    - Can you give me a specific example? (e.g., of a test item or homework question)
    - Why would you do this?
    - What will you do with the information you gain from this?
2. What else could you do to determine what your students learned in class on these two days?

Is there anything else about your plans that you want us to know?
Thank you again for participating in this interview.

APPENDIX B: EXIT TASK INTERVIEW PROTOCOL

Talking Through the Plan

Say to the participant: The first part of the interview is about the plan you just wrote. We want to make sure that we understand your plan and what you intended for these two days.

Begin with these questions:

- You spent an hour writing this plan. Tell me about the process you used to create the plan.
- Have you ever taught this topic before? Describe your experiences with teaching this topic.

Then ask the participant to walk you through his/her plan by asking:

- Walk me through your plan. How did you start the first day? *Continue to ask clarifying questions; your task is to be able to really understand what the participant intended for each part of the plan. Possible clarifying prompts:*
    - o What did you mean when you wrote _______________?
    - o Could you clarify what the students are doing during this part? (Probe: Talk to me about why you decided to do that.)
    - o Could you clarify what you are doing during this part? (Probe: Talk to me about why you decided to do that.)
    - o Could you tell me why you decided to do that? (Probe: Talk to me about why you decided to do that.)

Probing Participant's Knowledge

Subject Matter Knowledge (SMK/CKT)
Say to the participant: One part of what a teacher needs to know is something that we call content knowledge. In your case, we mean your own understandings of the (mathematics/science) that you will be teaching. These next questions are designed to probe what you know about linear relationships. Again, there is no right or wrong answers. We are interested in what you know and how you think about linear relationships at this point.

9. What are the big ideas associated with this topic?
    - How are these big ideas related to each other?
10. What are the two most important ideas you want students to know about this topic?
    - Why do you think these ideas are the most important?
    - How does this topic relate to other topics in (mathematics/science) in middle or high school?
    - Why should students learn about linear relationships?
11. Talk to me about how your plan addresses these big ideas? *(Probe for specifics based on the plan).*
12. What else do you know about this topic that you wouldn't teach to middle school students? High school students?
13. What else do you need to learn about this topic before you teach it?
14. When did you learn about this topic?
15. Have you applied this topic in an out-of-school experience? If so, describe.
16. [NOTE TO INTERVIEWER: IF THE INTERN REPORTED THAT (S)HE HAD TAUGHT BEFORE, THEN ASK THIS QUESTION] What did you learn about this topic when you taught it before?
17. In what ways does your lesson plan reflect the field of mathematics?

Knowledge of Learners

Say to the participant: The next questions are designed to probe what you know about how students learn <u>linear relationships</u>.

5. What do you think students already know about this topic?
    - Where do you think they may have learned this?
6. How do you expect students to perform on (a particular task from the lesson)?
    - What strategies might students use?
    - What responses might students provide?
7. What about this topic do you expect students to have difficulty learning?
    - Why do you think they will have difficulty with that?
8. How do you think students learn about this topic best?

Knowledge of Instructional Strategies (NEW)

Say to the participant: We want to know more about how you organized the instruction during these two days. The next questions will help us better understand your decisions about what and how to teach <u>linear relationships.</u> I want to go back through the major sections of your plan and probe a little more about decisions you made. [INTERVIEWER: THESE QUESTIONS ARE INTENDED FOR YOU TO PROBE ABOUT INSTRUCTIONAL APPROACH (E.G., LECTURE, LAB, INVESTIGATION, ACTIVITY, SMALL GROUP WORK, WHOLE CLASS DISCUSSION, DEMONSTRATION, SEAT WORK, USE OF REAL-WORLD PROBLEMS, USE OF DIFFERENT REPRESENTATIONS, USE OF ANALOGIES, …)

3. Let's look at how you began the first class. I see that you chose to *begin* the class with ___________. Talk to me about making that decision. [INTERVIEWER: YOU MAY NEED TO REPEAT THIS QUESTION, DEPENDING ON THE DIFFERENT WAYS THE INTERN CHOOSES TO ORGANIZE DIFFERENT PARTS OF INSTRUCTION, SEE d. BELOW]
    - Where did you learn about how to teach this way?
    - How do you think _________ helps students learn this topic?
    - In what other way(s) did you consider teaching this part of class? If you didn't think of other ways before, what ways can you think of now? Where did you learn these ways of teaching? [INTERVIEWER: MAKE THIS A SAFE PLACE FOR YOU TO PROBE ABOUT WHAT KNOWLEDGE THE INTERN HAS ABOUT DIFFERENT INSTRUCTIONAL STRATEGIES]
    - IF REPEAT OF QUESTION IS NECESSARY: I see that you then chose to change what you and the students are doing. Talk to me about that decision. (REPEAT A-C ABOVE)
4. I noticed that you used a picture (graph, equation, Punnett square, analogy…) in your plan. Tell me why you used that ____________ at that point in your plan.
    - How do you think this (picture, graph, equation, analogy) helps students learn about <u>linear relationships</u>?
    - Did you consider representing that idea another way?

Knowledge of Curriculum

Say to the participant: These next questions are designed for us to know something about where your ideas for these two days came from.

8. What is your goal for these two days of class? Why are those goals important?
9. How do your goals for these two days reflect your overall goals for teaching mathematics/science?
10. Where did you get your ideas for these two days of class?
11. What specific resources would have helped you design the plan?

- What specific information is in those resources that would have helped?

12. Tell me about the materials (handouts, transparencies) you prepared.
    - Where did you get the ideas for these materials?
13. How do you think these materials will help or hinder achieving the purpose of your plan?
14. What would you teach just before this topic? Just after?
15. States typically have a set of "standards" that guide mathematics/science instruction. For example, in Missouri, we have the GLEs (Grade-level expectations). Tell me what you know about these types of documents.

Knowledge of Assessment

Say to the participant: The last area I want to ask you about is how you will know what students learn from these two days of class.

3. During the 2 days of instruction, how will you know if students are "getting it" or "not getting it"
    - Can you give me a specific example? (e.g., of a test item or homework question)
    - Why would you do this?
    - What will you do with the information you gain from this?
4. What else could you do to determine what your students learned in class on these two days?

Comparison to Entry Task

Here is the lesson plan you wrote 2 years ago upon entering the SMAR^{2}T program. Take a look at it and think about what you know now that you did not know then.

1. What's your reaction to what you wrote 2 years ago?
    a. How do you know that ___________ now? (PROBE FOR DIFFERENT KNOWLEDGE TYPES)
        i. Subject matter knowledge
        ii. Knowledge of learners
        iii. Knowledge of curriculum
        iv. Knowledge of instructional strategies
        v. Knowledge of assessment?

2. What do you know about ______ now that would have changed the way you wrote this plan 2 years ago? (Where did you learn this—if participant has not already answered this above)?
    i. Subject matter for this topic
    ii. How your students learn this topic
    iii. Curriculum for teaching this topic?
    iv. instructional strategies for teaching this topic
    v. assessment of student learning of this topic?

Final Reflection

1. Now that you have been teaching in your own classroom, if you could go back to the beginning of the SMAR^{2}T program, what do you wish you would have learned that you didn't?
2. What do you wish you had learned in your internship (or from your first year mentor [for ALTs]) that would have helped you teach math/science this past year?"

Is there anything else about your plans that you want us to know?
Thank you again for participating in this interview.

APPENDIX C: OBSERVATION CYCLE PROTOCOL

RE-SMAR^{2}T Observation Cycle Protocol

Submitted by the intern via email to the team 48 hours prior.

Written plan (*purpose: to provide a written guide for the observers*)

We would like to observe your teaching over two class periods. Please select a section of instruction within a unit. Please send us your plan for these two class periods and answer the following:

- What mathematics/science do you want the students to learn?
- Describe what will happen during the beginning, middle, and end of each class. What will you do? What will the students do?
- How will you know the students learned what you wanted them to learn?
- Describe what will be needed for these two class periods (e.g., resources, materials, equipment, etc.)
- Include copies of any handouts, overhead transparencies, assessments, etc. that you plan to use.

Prior to first observation:

Researcher role: Our role is to assume a stance of empathic neutrality. That is, we empathize with the participant and care about him/her. However, our role is to UNDERSTAND, not to Evaluate or Teach. Please keep these ideas in mind during your visit.

Pre-Observation Interview (*purpose: to clarify the plans and uncover the intern's CKT and PCK*)

Opening Questions

1. Update us about what is going to occur over the next 2 days we are observing.
    a. What will we see in Day 1? In Day 2?
    b. What will you be doing?
    c. What will the students be doing?
    d. What are your purposes and goals for these 2 days?
    e. How did you decide on these purposes and goals?
    f. Why are these purposes and goals important to you?
    g. Fast forward 2 days. Imagine that you've successfully taught these two lessons. What would success look like? How will you know you've been successful?

Subject Matter Knowledge (SMK/CKT)

Say to the participant: One area that we are interested in is what we call content knowledge. In your case, we mean your own understandings of the science/math that you will be teaching.

2. What are your previous experiences with (this topic)?
    a. How well do you think you know (this topic)?

b. Where did you learn about (this topic)?
c. Have you taught (this topic) previously?

3. What do you think is important for students to know about (this topic)?
   a. Why do you think that is important?
   b. Tell me about where and how you learned these things.
   c. What else do you know about (this topic) that students might not need to know?

4. How do the science/mathematical ideas in (this topic) relate to other science/mathematical ideas?

5. What science/mathematics content did you learn why preparing these two lessons?

Knowledge of Learners

Say to the participant: Another part of what a teacher knows has to do with how students think about mathematics/science. The next questions are designed to probe what you know about how students might think about (this topic).

5. Tell me about the students in this class, in terms of science/mathematics.
   a. Tell me more about your students' attitudes about science/mathematics.
   b. Tell me about your students' science/mathematical abilities.
   c. How do you think this particular group of students learn math/science best? Why do you think that?
   d. How have your experiences with these students influenced the way you teach?

6. In order for students to learn [this topic], what should they already know?
   a. How well do you think your students already know that information?
   b. What misunderstandings do you think students may have that would influence their learning of this topic?
   c. How did you come to know this?

7. How do you expect students to perform on (a particular task from the lesson)?
   - What strategies might students use?
   - What responses might students provide?
   - What, if anything, about this topic do you expect students to have difficulty with?
     a. Why do you think they will have difficulty with that?

   [Probe for SOURCES of Knowledge of Learners]

Knowledge of Instructional Strategies

Say to the participant: We want to know more about how you organized the instruction during these two days. The next questions will help us better understand your decisions about what and how to teach (this topic).

8. Talk to me about how you plan to help students learn the important mathematical/science ideas you talked about earlier *(Probe for specifics based on the plan; use prompts in #9 to help you probe about parts to the plan).*

[*Note to interviewer: the questions in #9 can be asked about any different stages of the plan. We are interested in how the intern organized the flow of the class and why]*

9. From your plan, it appears that you chose to start the class (continue class; end the class) with __________________ (i.e., warm-up, lecture, experiment, investigation). Talk to me about making that decision.
    a. Why did you choose to start this way?
    b. Where did you learn about this way to start (continue; end) a class?
    c. Did you consider starting (continuing; ending) the class in a different way? Why/why not?
    d. What other factors influenced your planning decisions?

10. I noticed that you used a picture (graph, equation, analogy…) in your plan. Tell me why you used that ____________ at that point in your plan.
    a. How do you think this (picture, graph, equation, analogy) helps students learn about (this topic)?
    b. Did you consider representing that idea another way?

Knowledge of Curriculum

Say to the participant: These next questions are designed for us to know something about where your ideas for these two days came from.

11. Where did you get your ideas for teaching (this topic)? *Probe for sources.*
    a. Tell me about the materials (lecture notes, handout, transparencies) you prepared. Where did the materials (lecture notes, activities, worksheets, etc.) come from? *Probe for sources of activities as necessary.*
    b. What modifications did you make to existing materials?
    c. How do you think these materials will help or hinder achieving the purpose of your plans?

12. I have some questions for you related to how these plans relate to other topics that you might teach.
    a. How do you see these 2 days of instruction as related?
    b. How do these 2 days of instruction fit into the unit you currently are teaching?
    c. How does that math/science fit into the bigger picture of what students learn in this class?
    d. How does (this topic) fit into the "big picture" of what students learn about math/science in middle school/high school?

Knowledge of Assessment

Say to the participant: The last area I want to ask you about is how you will know what students learn from these two days of class.

13. During the 2 days of instruction, what do you plan to assess? Why do you think it is important to assess this?

14. How do you plan to assess these (things)?
    a. Describe how you will find out if students learned what you intended? Are there other ways that you might know what your students learn in class on these two days?
    b. Where did you learn about those strategies for finding out about what students learned?
15. What will you do with the information you gain from the assessment?
16. What challenges do you foresee as you assess students?

Is there anything else about your plans that you want us to know?
Thank you again for participating in this interview.

During the Observation
The observer(s) will have selected 3-5 interesting instances to discuss. What constitutes an interesting instance?

Knowledge of Learners
Student making a profound comment and the teacher does or doesn't recognize it or misinterprets what the student says or does.
Student makes a comment that demonstrates confusion, and the teacher does or doesn't recognize or misinterprets why the student is confused?
Teacher explicitly recognizes potential student difficulties.

Knowledge of Instructional Strategies
The teacher makes an instructional decision that alters the flow of the classroom by asking a question or directing students to perform a particular task.
The teacher uses an example or analogy or representation to clarify an idea.

Knowledge of Curriculum
A particular task is chosen that may or may not elicit the student thinking that was intended.
The teacher modifies the plan "on the fly" based on what occurs in the classroom.
Teacher refers to math/science content in other parts of the course/curriculum (vertical or horizontal curriculum alignment).

Knowledge of Assessment
Teacher implements assessment to ascertain student prior knowledge.
The teacher recognizes that the students are having difficulty with a particular idea.
The teacher uses a low-level assessment strategy such as providing an "exit slip" that requires students to define rather than explain or synthesize.
The teacher acts on data collected during student assessment.

SMK
Teacher demonstrates particularly strong SMK.
Teacher demonstrates inaccurate SMK.

After each observation:
Stimulated recall interview (*purpose: to have the intern immediately reflect on the instruction as a window into CKT and PCK and connect to pre-interview).*

Stimulated Recall Interview

1. How do you think the lesson went? In what ways was the lesson I observed different than other periods you taught it? Different from your plans?
2. We have selected some parts of the instruction we found particularly interesting. We want to watch them together and ask you some questions about them.

   Let's watch this part (interviewer asks questions starting in one of the following categories based on the reason for selecting the specific interesting instance).

   a. What were you thinking when this was occurring? Tell me more about what was happening when you __________.
   b. **[K of Learners]** What do you think the student was thinking? Why do you think the student was having difficulty at that point? What knowledge about students did you use to make instructional decisions? In what ways, did students influence your teaching decisions today?
   c. **[K of Instructional Strategies]** Tell me about that (example/analogy/activity/lab)? Why did you decide to use that? How did this teaching strategy help you achieve your overall goals? How could you teach this topic in a different way? Where did you learn to teach it that way? [NOTE TO INTERVIEWER: questions about instructional strategies should probe all of the different ways that the participant might know to teach a particular topic. For this PCK component, we are interested in "mining" the participants' knowledge about all kinds of different instructional strategies. You should ask this series of questions many times during the interview.]
   d. **[K of Curriculum]** Did the activities achieve the purpose you intended? Why do you think that? How did your curriculum materials support or hinder you in implementing your plan?
   e. **[K of Assessment]** What do you think students got out of the lesson? How do you know? Tell me about how you found out about student learning. Why did you decide to do that? Where did that idea come from? How do you think it worked?
   f. **[SMK]** What science/mathematics content did you learn while teaching this lesson?
   g. **[SMK]** Given the opportunity to sit down with a colleague you trust, what questions would you ask about this mathematics/science topic?
   h. [**SMK**] What were the critical mathematical/science ideas in today's lesson?

3. Was there a time during the instruction when you changed your plan? Tell me about that.
4. Based on what happened today, what do you plan to do tomorrow? Will you change anything from your original plans?

***Ask the following questions only during the 2nd post-observational interview.**

5. [**Orientations**]. Imagine your best day of teaching science/mathematics.
   a. Describe what makes it a "best day" for you.
   b. How do these two lessons that you've taught compare to your "best day" description.

6. [**Orientations**]. Now consider a typical day of teaching for you.
   a. What is the teacher's role in a typical lesson?
   b. What is the students' role in a typical lesson?
   c. How do you prefer to teach?
   d. How does this compare to your mentor teacher?
   e. How does this compare to what you've learned in the SMART program?
   f. In what ways have your ideas about teaching changed since you entered the SMART program? Probe for sources of these changes.
   g. Now think of yourself as a math/science learner, how do you best learn math/science concepts?

NOTE: consider asking these questions at the end of Day 2 if you have time.

Right after the observation (submitted by email)

Post written analysis (*purpose: for the intern to reflect on practice after as a window into CKT and PCK*)

What do you know now that you did not know when you planned these 2 days?

1. What math or science did you learn while teaching these 2 days?
2. What did you learn about the students while teaching these 2 days?
3. What did you learn about your teaching while teaching these 2 days?
4. What did you see as the strengths and limitations of the curriculum materials you used for teaching?
5. How would you evaluate the strengths and weaknesses of your assessment strategies?
6. What constrained you in carrying out your plans?
7. What helped you in carrying out your plans?
8. The next time you teach this, what will you keep the same and what will you change? Why?

APPENDIX D: MENTOR TEACHER INTERVIEW PROTOCOL

Say to participant: Thank you again for participating today. We are interested in the knowledge development of beginning teachers as they move through the SMAR^{2}T program. In particular, we are interested in what SMAR2T students learn through their experiences in their (internship or teaching experience). In addition, we would like to better understand how their interactions with you impact the learning of [insert name]. During this interview, I will ask you questions about your goals for your SMAR^{2}T intern [insert name].

Start the audio-recorder.
Say to the participant: This is **_______** (Graduate Student), interviewing **_____________** on **_____________** (date). Do I have your permission to audio record this interview? (Wait for positive response)

Probing Instructions Views of Intern Learning

1. What are some things you think [insert name] has learned during his/her time in your classroom? (about the students, science/math teaching, other). Probe for specific examples.

2. What do you think [insert name] has learned <u>about how students learn mathematics/science</u> during his/her internship? (intentions)
   - Tell me about an example where you saw gains in [insert name]'s knowledge. (actions)
   - How has [insert name]'s knowledge of student <u>learning</u> changed since he/she began the internship? (outcome)

   - What else about learners have you shared with [insert name]? (Probe participants for all the goals they had for knowledge of learners using the sequence of probes above.)
   - If you were to explain to [insert name] how your students learn math/science, what would you say? Would your answer change for the different types of courses you teach? Probe: Why?
   - Compare your view of the role of students in a math/science class to [insert name's] view of the students' role? (orientations)

3. What do you think [insert name] has learned <u>about teaching methods</u> from your mentoring during his/her internship? (intentions)
   - Tell me about an example where you saw gains in [insert name]'s knowledge. (actions)
   - How has [insert name]'s knowledge of teaching methods changed since he/she began the internship? (outcome)

- What other teaching methods do you want [insert name] to learn? (Probe participants for all the goals they had for instructional strategies using the sequence of probes above.)
- Why do you think these methods are important? (orientations)
- How does your view of the teacher's role in a mathematics/science class compare to [insert name]'s view? (orientations)
- How does [insert name]'s view of the "best day" of teaching mathematics/science compare to your view of the "best day" teaching mathematics/science (orientations)?

4. What do you think [insert name] has learned about curriculum (for example, standards, scope and sequence curriculum materials) from your mentoring during his/her internship? (intentions)
    - Tell me about an example where you saw gains in [insert name]'s knowledge. (actions)
    - How has [insert name]'s knowledge of curriculum changed since he/she began the internship? (outcome)

    - What else about curriculum have you shared with [insert name]? (Probe participants for all the goals they had for curriculum using the sequence of probes above.)
    - How do you think curriculum materials help or hinder achieving your instructional purposes and goals? (orientations)
    - How does your view of why is it important for students to study mathematics/science in middle/high school compare to [insert name]'s view? (orientations)

5. What do you think [insert name] has learned about assessment from your mentoring during his/her internship? (intentions)
    - Tell me about an example where you saw gains in [insert name]'s knowledge. (actions)
    - How has [insert name]'s knowledge of assessment changed since he/she began the internship? (outcome)

    - What else about assessment have you shared with [insert name]? (Probe participants for all the goals they had for assessment using the sequence of probes above.)
    - How does your view of the importance of assessment compare to [insert name]'s view of the importance of assessment? (orientations)

6. What do you think [insert name] has learned about math/science subject matter from your mentoring during his/her internship? (intentions)
    - Tell me about an example where you saw gains in [insert name]'s math/science knowledge. (actions)

- How has [insert name]’s math/science subject matter knowledge changed since he/she began the internship? (outcome)
- How does your view of what mathematics/science is compare to [insert name]’s view? (orientations)

VITA

Rebecca (Markland) Bruton is the daughter of John Markland and Beverly Markland and was born September 27, 1982, in Fayette, Missouri. She graduated from Fayette High School in 2001. She received her Bachelor of Arts in Middle/High School Mathematics Education in 2006 and her Master of Arts in Curriculum and Instruction in 2007 both from the University of Missouri in Columbia, Missouri. She taught high school mathematics at Rock Bridge High School in Columbia, Missouri for four years before returning to the University to pursue her Doctor of Philosophy degree in Mathematics Education. She currently resides in Columbia with her husband, Jake, and her children, Noah, Jack, and Georgia.