INVESTIGATION OF DYNAMIC MATRIX CONTROL

A Thesis

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Master of Science in Chemical Engineering

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by

Zaki A. Shanaa

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Prof. R. H. Luecke   Thesis Supervisor

TO MOM AND DAD

ACKNOWLEDGMENTS

I would like to give recognition to my advisor, Dr. R. H. Luecke, for the hours of assistance and advisement throughout the course of this investigation. Many thanks are offered to Dr. D. S. Viswanath and Dr. R. McLaren for their encouragement and advise.

Furthermore, I wish to express my sincere appreciation to my family in Lebanon, especially my sister, Dr. Zuhrieh Shanaa, for their support, care, love, and sympathy.

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NOMENCLATURE

A	Matrix of dynamic coefficients
A^T	Transpose of matrix A.
D	Disturbance input
$\underline{E}$	Vector of projected errors in the output
G	Flow rate of inert gas stream (Gas Absorber), controller transfer function
G_d	Transfer function in the feedforward loop
H	Inert vapor holdup on each plate (Gas Absorber)
$\bar{H}$	Output observability matrix
I	Manipulated input
$\underline{I}$	Vector of manipulated moves
L	Flow rate of inert liquid absorbant (Gas Absorber)
O	System Output
P	Positive Definite matrix
T	Sampling period
a_i	Constants used in dynamic vector $\underline{a}$
$\underline{a}$	Dynamic vector (manipulated variable)
b_i	Constants used in dynamic vector $\underline{b}$
$\underline{b}$	Dynamic vector (Disturbance Input)
h	Inert liquid holdup on each plate (Gas Absorber)
m	Control moves on the manipulated input
n	Elmenets in projected error $\underline{E}$.
P	Number of outputs

q	Number of manipulated variables
t	Time
u	Control element
x_i	Composition of liquid leaving the i^{th} plate (Gas Absorber)
x_B	Bottoms mole fraction (Distillation Column)
x_D	Overhead mole fraction (Distillation Column)
y_i	Composition of vapor leaving the i^{th} plate (Gas Absorber)
α	Approximation parameter
β	Approximation parameter
ε	small positive number
ρ	Square root of residual sum of squares
Δ	Constant matrix defined by equation (4-9)
ϕ	Transition matrix

CHAPTER I

INTRODUCTION

Applying digital computers to process control has developed rapidly since the late 1950's. One way of using computers in process control is by direct digital control (DDC). In DDC the computer completely replaces the analog controller, utilizing the information about process variables to keep the process under direct control. The logical decisions made by the computer are based on a sequence of logical steps, called an algorithm, designed by the control engineer.

The Dynamic Matrix Control (DMC) Algorithm (Culter and Ramaker, 1979) is a direct digital control technology that has been used by Shell Oil Company since 1973. As it is implied by its name, the whole notion of the DMC Algorithm is based on representing the system dynamics by a set of numbers arranged in vectors and matrices. Including the system dynamics in the design of DMC has helped maintaining awareness of deadtime and other unusual dynamic behavior. Some complex control problems, which can not be treated satisfactorily with traditional proportional-integral-derivative (PID) techniques, can be solved using a digital computer with logic in the control loop.

The DMC Algorithm is discussed in this thesis. The discussion will interpret DMC in terms of a combination of feedforward and feedback essentially unconstrained control of multivariable systems with multiple delays or deadtimes. The application of DMC to two example systems, namely, a distillation column and a gas absorber is also included for the purpose of illustration. Both of the systems were used by other authors to test the capabilities of control algorithms.

CHAPTER II

GENERAL DEVELOPMENT

1. Process Variables

Computer control plays an important role in different aspects of process industry. Examples include petroleum, chemical, glass, paper, steel and cement industries. In general all of these processes have three important kinds of variables.

The process outputs are the variables to be controlled. There are two types of control problems, the regulator-type and the servo-type. In the regulator-type problem the outputs are to achieve a certain limit. For example, if a chemical reaction takes place at certain temperatures then it is desired to keep the temperature of the chemical reactor at certain levels. In the servo-type problem the outputs are to follow a prescribed path. The tracking of missiles and aircraft is a well-known example of the servo-type.

The second group of process variables consists of the manipulated inputs to the process. These variables usually can be measured and adjusted to control the process outputs. Examples include the inlet process flow rate or the process feed temperature to a distillation column.

The third group of process variables is called the disturbances. Although these inputs affect the outputs, and can sometimes be measured, they can not be used to control the outputs. Ambient air temperature is an example of the disturbance input to the system.

2. Applicability

The DMC algorithm is applied to systems that can be represented or at least approximated by a set of linear differential equations. Linear systems have two important properties which are at the basis of DMC algorithm. The first property is the preservation of scale which is illustrated in Figure (2-1). Curve 1 is the output response due to a step change in the input and curve 2 is the output response due to a two-unit step-change in the input. Curve 2 is similar to curve 1 but twice the magnitude.

The second property of linear systems is superposition which is illustrated in Figure (2-2). Curve 1 is the output reaction to a step change in input 1, curve 2 is the output reaction to a step change in input 2, and curve 3 is the output reaction to a step change in both inputs. Curve 3 is the algebraic sum of curve 1 and curve 2.

3. Mathematical Analysis

To explain the application of DMC algorithm, consider a "black box" system with one manipulated input I, one disturbance input D, and one output O as shown in

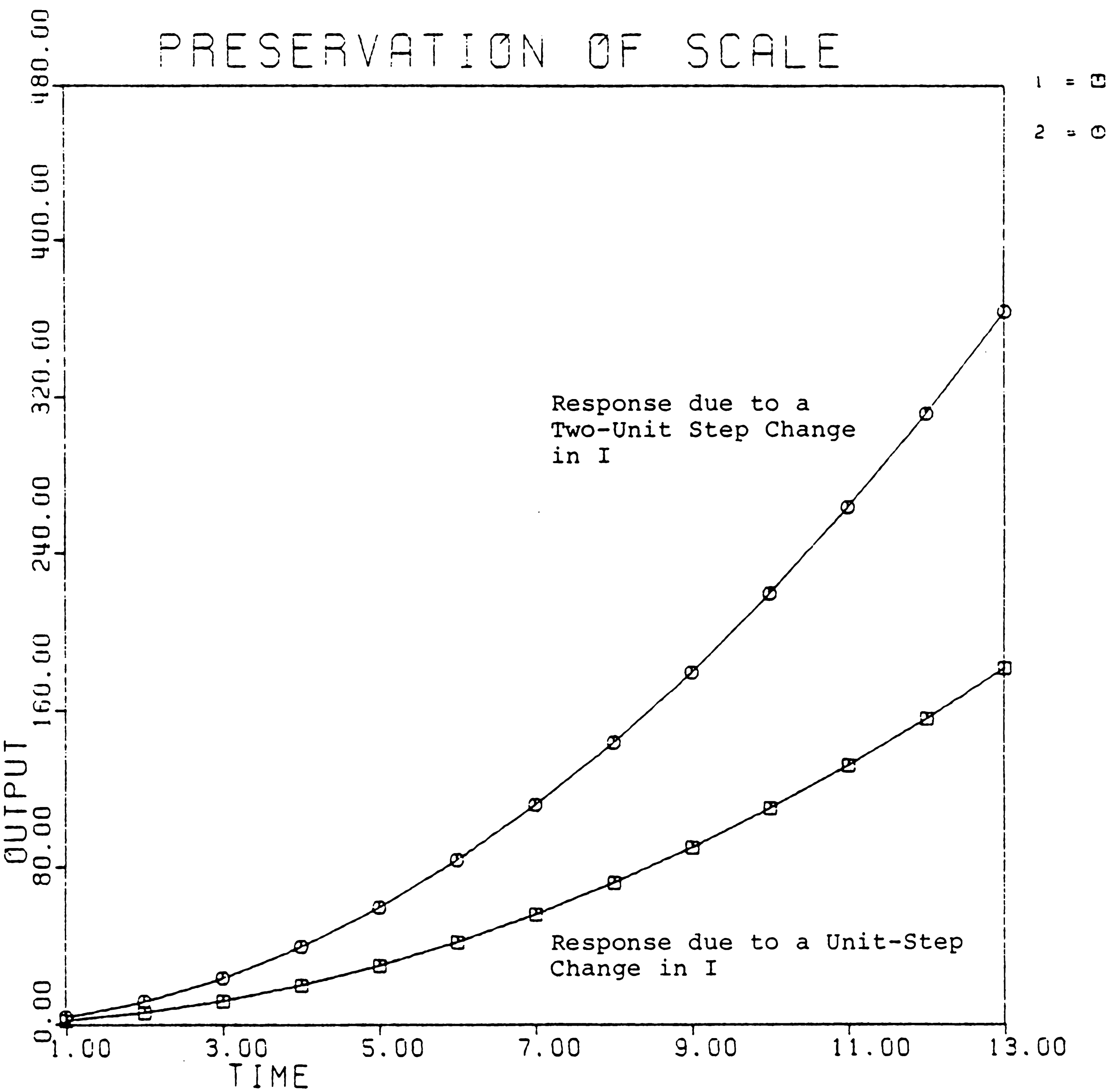

PRESERVATION OF SCALE
1 = □
2 = ○
OUTPUT
480.00
400.00
320.00
240.00
160.00
80.00
0.00
TIME
1.00
3.00
5.00
7.00
9.00
11.00
13.00
Response due to a Two-Unit Step Change in I
Response due to a Unit-Step Change in I

Figure (2-1).

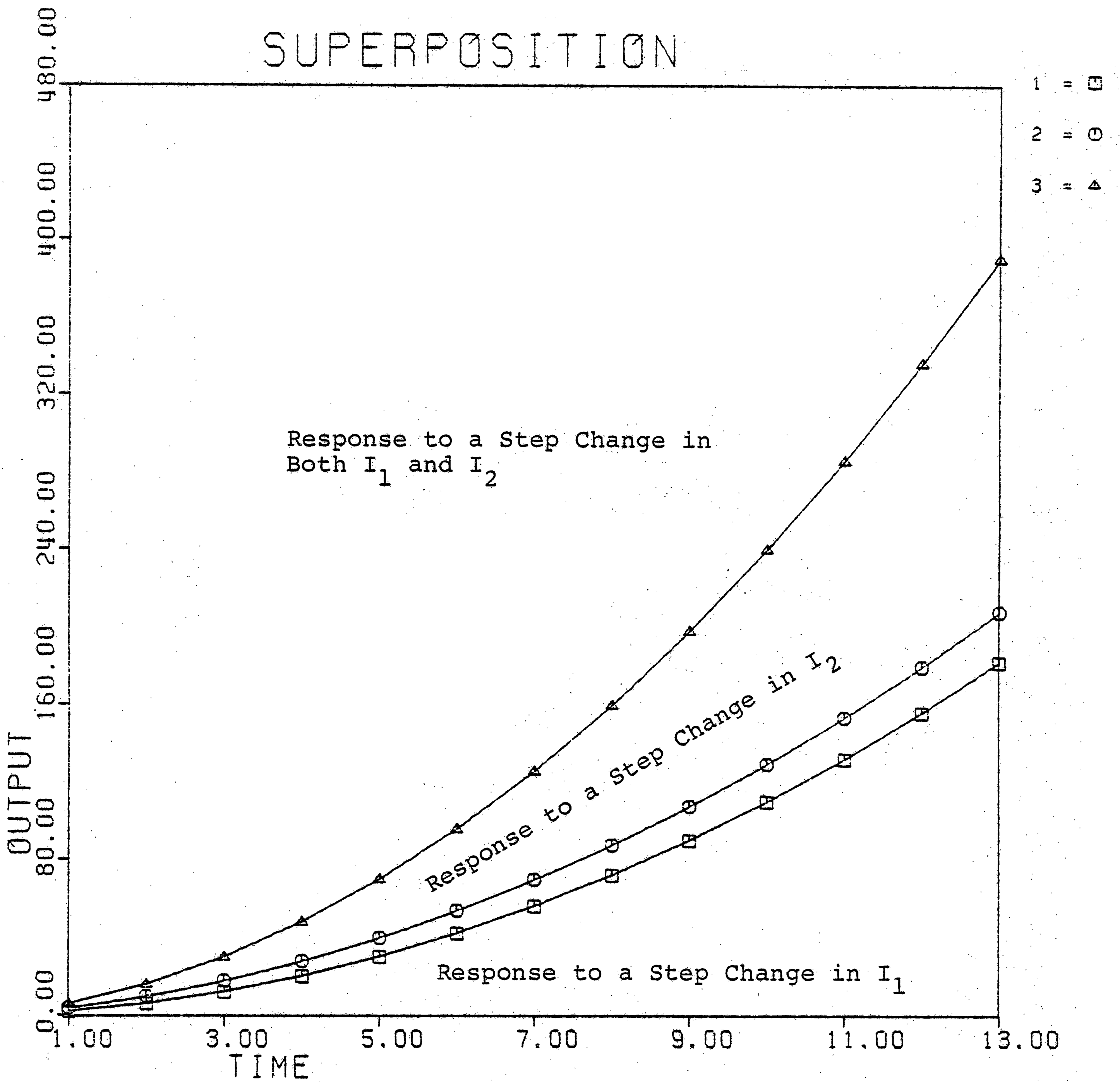

SUPERPOSITION
1 = □
2 = ○
3 = △
Response to a Step Change in Both I_1 and I_2
Response to a Step Change in I_2
Response to a Step Change in I_1
OUTPUT
TIME
0.00
80.00
160.00
240.00
320.00
400.00
480.00
1.00
3.00
5.00
7.00
9.00
11.00
13.00

Figure (2-2).

Figure (2-3). Assume that the desired output is zero, that is the final value of the output should go to zero.

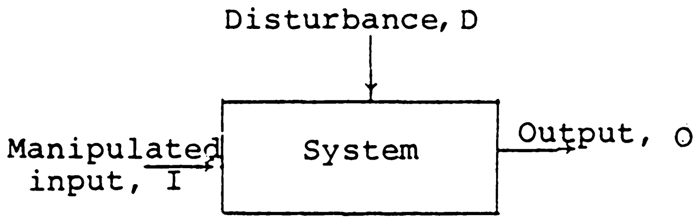


Figure (2-3). "Black Box" System.

A. Dynamic Representation

The dynamics of the system shown in Figure (2-3) can be characterized by showing how the system output O is affected by a step change in the manipulated input I and by a step change in the disturbance D. Suppose that curve 1 in Figure (2-4) represents the output response to a step-change in I, and curve 2 in the same figure depicts the output response to a step change in D. These curves are called process reaction curves. Since digital computers deal with numbers rather than equations, it is convenient to represent these curves by discrete numbers stored in vectors.

In Figure (2-5) the process reaction curve due to the manipulated input I is sampled and the values α_i are normalized by dividing by the highest value α_{max}. Then the response of 0 due to a unit step change in I is represented by vector $\underline{a}$ of normalized coefficients,

$$\underline{a} = [a_1 \; a_2 \; \cdots \; a_n]^T, \; a_i = \alpha_i/\alpha_{max}.$$

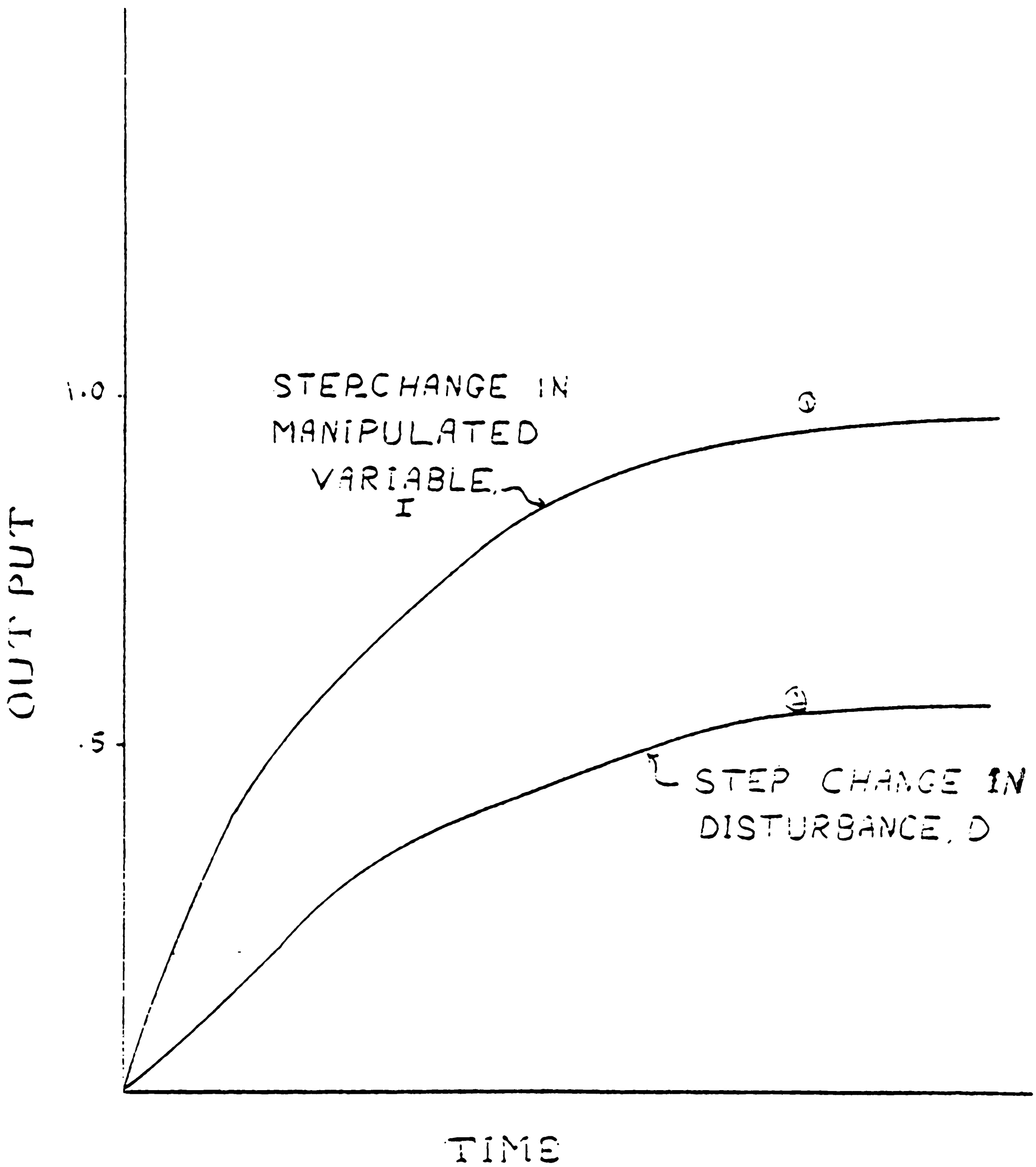


Figure (2-4). Dynamics of the black box system.

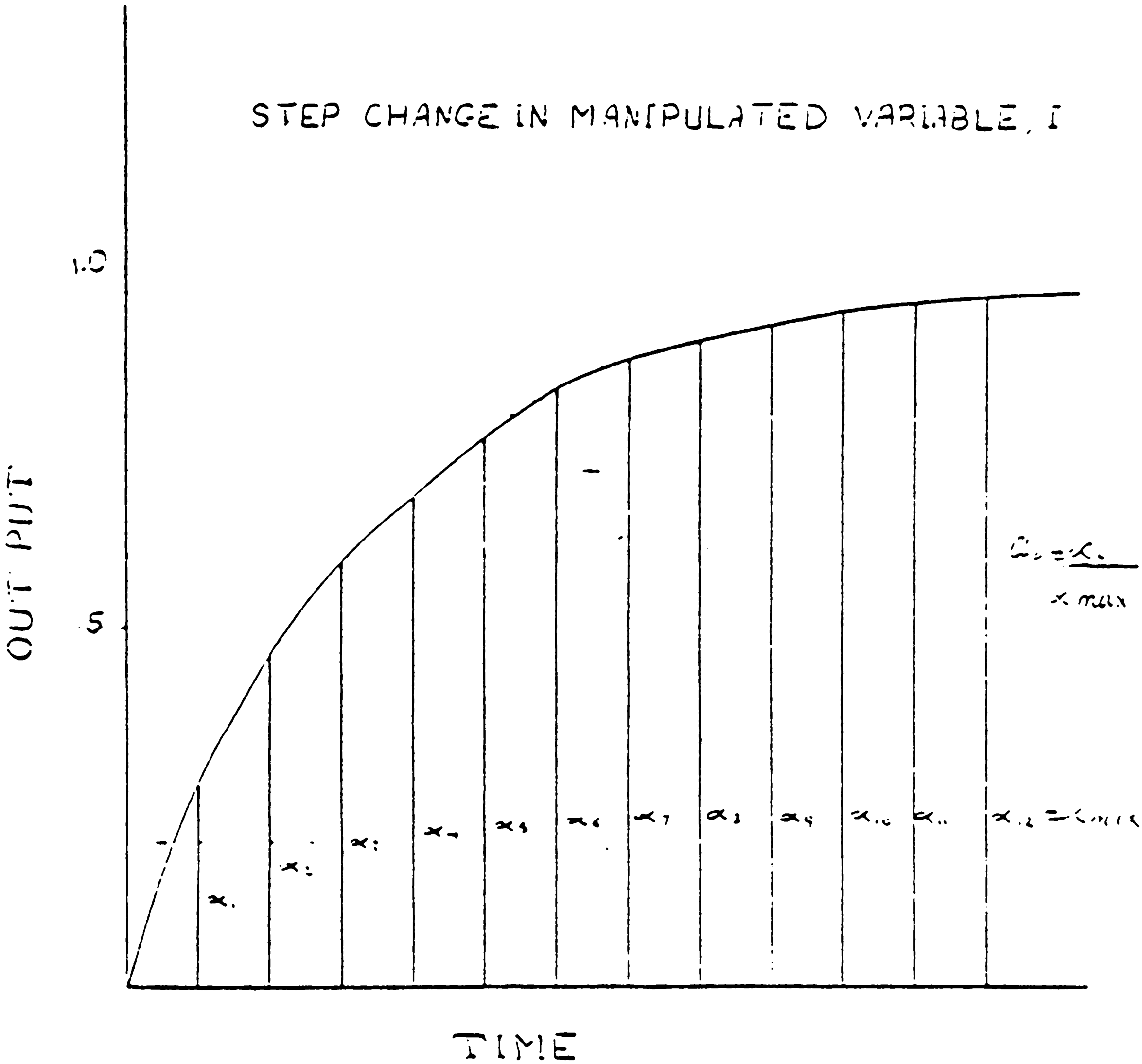


Figure (2-5). Matrix A.

In Figure (2-6) the process reaction curve due to the disturbance D is sampled and the coefficients b_i are obtained. The response of O to a unit step change in D is represented by vector $\underline{b}$,

$$\underline{b} = [b_1\ b_2\ \cdots\ b_n]^T.$$

<u>Control strategy</u>. The DMC mechanism for the system in Figure (2-3) is illustrated in Figure (2-7).

Sufficient information about the system at some instant of time t_o must be available to predict or calculate the system response to a given set of initial conditions. Suppose that curve 1 represents the output response O_i of the system due to the initial conditions.

Curve 2 represents the projected change in the output due to the disturbance D. This curve is obtained by measuring D and multiplying its magnitude by the scale vector $\underline{b}$ which is shown in Figure (2-5).

Using the superposition property of linear systems, curve 1 and curve 2 are algebraically added to give curve 3. Curve 3 represents the projected change in the output due to the disturbance and the set of initial conditions.

The final output should be taken to zero by making m consecutive step changes in the manipulated input I. To obtain the final value zero graphically, curve 3 is added to curve 4 which is symmetric to curve 3 with respect to

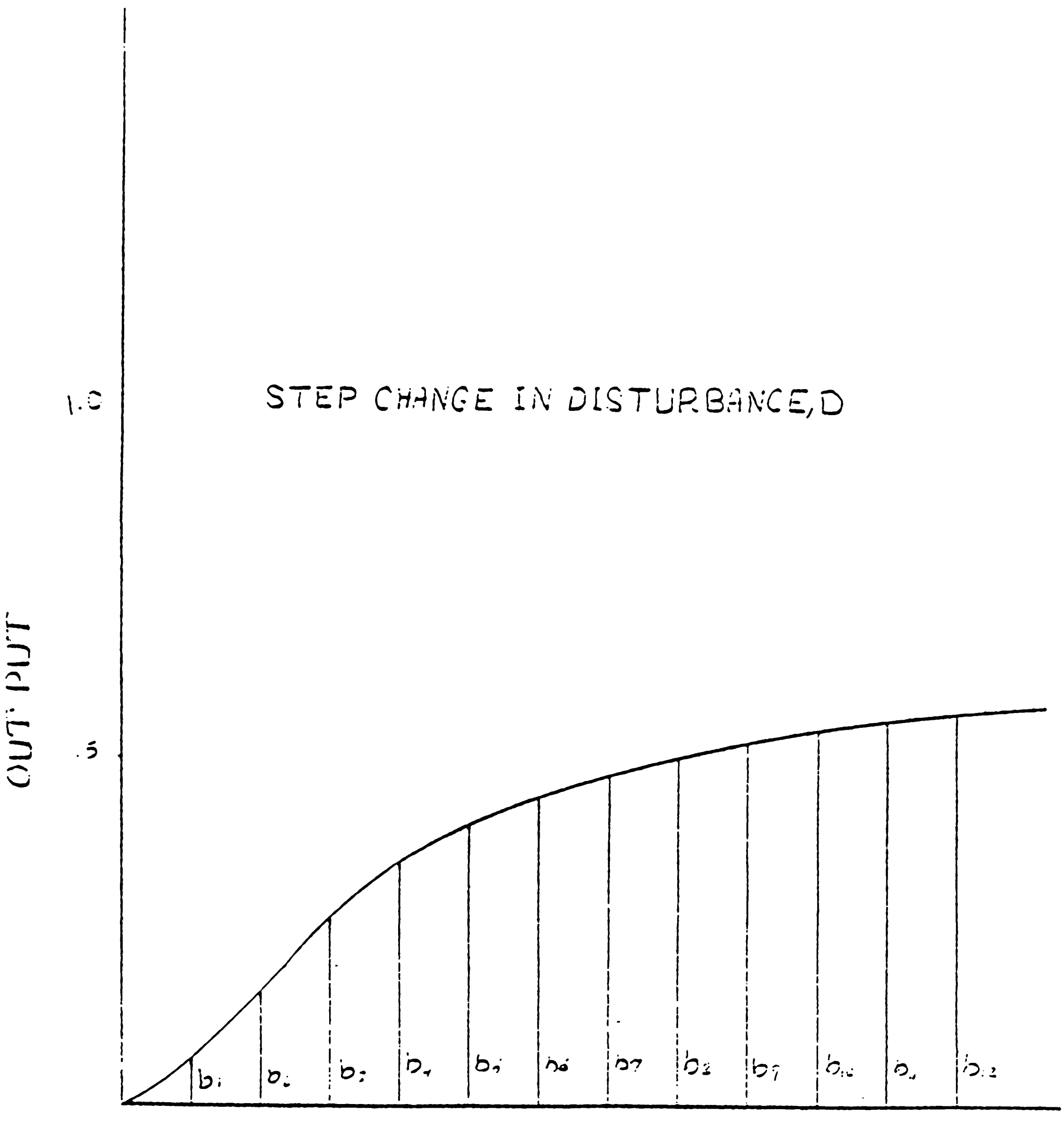


Figure (2-6). Vector $\underline{b}$.

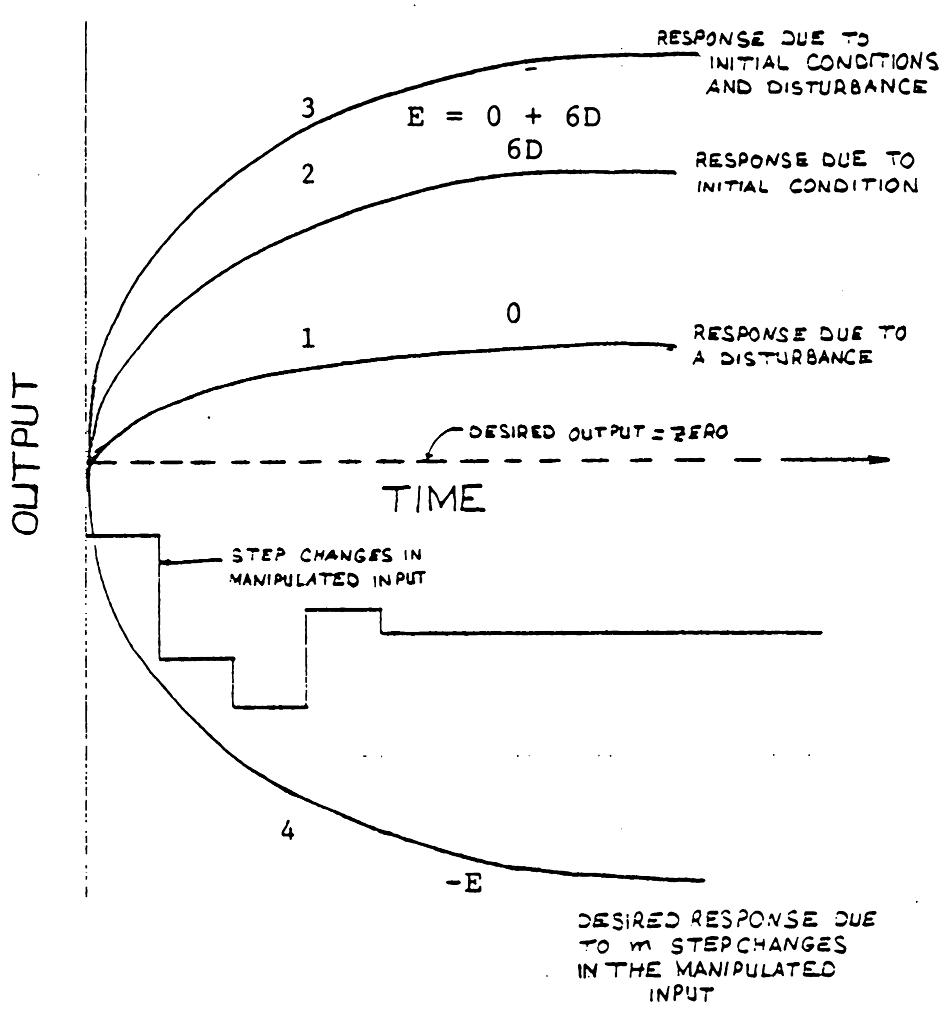


Figure (2-7). Mechanism of the DMC Algorithm.

the time axis. Therefore curve 4 represents the desired net effect of the m-step changes in I on O.

Mathematically, the equation is:

$$\underline{O} + \underline{b}D = -A \cdot \underline{I}$$

$$\text{or} \quad \underset{(n\times 1)}{\underline{E}} = \underset{(n\times m)}{-A} \cdot \underset{(m\times 1)}{\underline{I}} \qquad (2\text{-}1)$$

where $\underline{E} = \underline{O} + \underline{b}D$ is an $n\times 1$ vector of the projected error in the controlled variable, and A is an $n\times m$ matrix whose columns are coefficient vector $\underline{a}_i$ computed in Figure (2-5). The general structure of matrix A is shown in Figure (2-8).

In equation (2-1), the output response due to the initial conditions $\underline{O}$, the coefficient vectors $\underline{a}_i$ and $\underline{b}$, and the magnitude of the disturbance D are known. The

$$A = \begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ a_2 & a_1 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a_n & a_{n-1} & a_{n-2} & & a_{n-m+1} \end{bmatrix}$$

Figure (2-8). Matrix of Coefficients.

problem is to determine $\underline{I}$, the vector of projected moves on the manipulated variable by solving equation (2-1).

If A is a nonsingular* square matrix (m=n), the solution of equation (2-1) is given by:

$$\underline{I} = -A^{-1} \cdot \underline{E}. \qquad (2\text{-}2)$$

Equation (2-2) gives the exact values of I that will compensate for the projected error in the output. This solution may result in large manipulated moves which are not physically realizable. Even if such control moves could be applied physically, there might be large differences between successive input moves (ringing phenomena) which tends to wear the actuator quickly. Figure (2-9) illustrates what is meant by ringing phenomena.

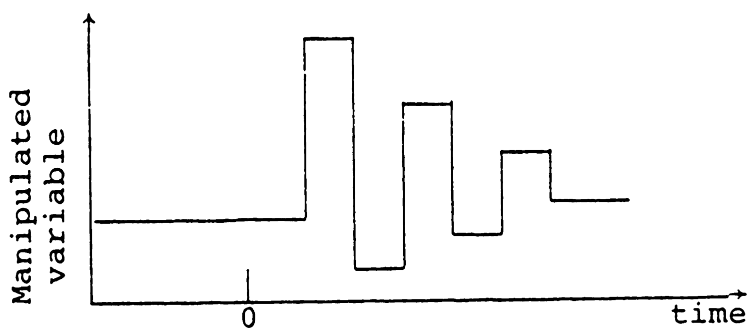


Figure (2-9). Ringing Phenomena.

*This is not restrictive since we can always construct pseudoinverses.

On the other hand, if A is a rectangular matrix (m<n), equation (2-1), with n elements in the projected error in the output $\underline{E}$, can be solved for m control moves on the manipulated variable $\underline{I}$ using the least square formulation. To obtain the least square solution of equation (2-1), first premultiply both sides of the equation by A^T, the transpose of A to get:

$$A^T \cdot \underline{E} = -A^T \cdot A \cdot \underline{I} \ . \qquad (2\text{-}3)$$

Then (A^TA) is a symmetric square matrix (of size m×m < n×n) and the solution of equation (2-3) is

$$\underline{I} = -(A^TA)^{-1} \cdot A^T \cdot \underline{E} \ . \qquad (2\text{-}4)$$

Solving equation (2-1) by the least square method is equivalent to minimizing the function

$$\rho^2(I) = ||\underline{E} - (-A)\underline{I}||_2^2 \qquad (2\text{-}5)$$

where $||\cdot||_2$ is the Euclidean norm (Nobel and Daniel, 1977). The values of $\underline{I}$ obtained by (2-4) will not compensate for the projected error in the output completely; the least square minimum will remain. This will tend to smooth the manipulated input curve while keeping the error $\rho^2(I)$ in equation (2-5) at minimum.

Further smoothing of the manipulated variable curve occurs if the diagonal of the square matrix (A^TA) in equation (2-3) is multiplied by a constant $(1+\varepsilon)$, where ε is a small positive number in the order of .01. This operation is equivalent to adding the diagonal elements of (A^TA) times ε, to the matrix (A^TA). If $\underline{I}_1$ is the solution of the modified equation, then

$$(A^TA + \varepsilon \cdot \mathrm{Diag}(A^TA)) \cdot \underline{I}_1 = -A^T \cdot \underline{E} . \qquad (2\text{-}6)$$

Premultiply both sides of equation (2-6) by $(A^TA)^{-1}$ and let $P = \varepsilon \cdot (A^TA)^{-1} \cdot \mathrm{Diag}(A^TA)$, then

$$(J+P)I_1 = -(A^TA) \cdot A^T \cdot \underline{E} \qquad (2\text{-}7)$$

where J is the $(m\times m)$ identity matrix. Compare equations (2-4) and (2-7) to conclude that

$$(J+P)I_1 = I. \qquad (2\text{-}8)$$

Since P is positive definite, the norm of the modified vector of manipulated moves $\underline{I}_1$, is less than that of $\underline{I}$. Hence multiplying the diagonal of (A^TA) by $(1+\varepsilon)$ will tend to smooth the input moves curve without increasing the least square error $\rho^2(I)$ in equation (2-5) by much since ε is small.

Up to this point, the DMC algorithm has been similar to the conventional feedforward control since only the measurement of D was needed to solve for the manipulated moves $\underline{I}$ that would compensate for the projected error in the output $\underline{E}$ totally or partially.

Vector $\underline{I}$, obtained from equation (2-1), provides m manipulated moves. The first move is implemented and the entire vector of predicted outputs is shifted forward one interval of time. The output O, due to first manipulated move is measured and the deviation of the measured value from the predicted value at that instant of time is used to adjust all the m values of predicted outputs proportionally. This adjustment in the prediction provides the feedback to compensate for the unmeasured disturbances and the errors in dynamic predictions. At the beginning of the second interval of time, the difference between the predicted values of $\underline{O}$ and the setpoint is used to calculate another m manipulated moves. The pattern is repeated for each successive interval of time.

The above discussion of the DMC algorithm can be expanded to incorporate the control of multivariable systems. The multivariable problem is solved in the same manner as illustrated for the example system in Figure (2-3). Consider a system with p output variables and q manipulated input variables, the matrix of coefficients is expanded to

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1q} \\ A_{21} & A_{22} & \cdots & A_{2q} \\ \vdots & \vdots & & \vdots \\ A_{p1} & A_{p2} & \cdots & A_{pq} \end{bmatrix}. \qquad (2\text{-}9)$$

Each A_{ij} of A (i = 1,2, ···, p; j = 1,2, ···, q) has the same dimensions and the same structure as that of the matrix given in Figure (2-8). A_{ij} represents the response of the i^{th} output to a step change in the j^{th} manipulated input. The dimensions of the elements in equation (2-1) will be:

$$\underset{(pn\times 1)}{\underline{E}} = \underset{(pn\times qm)}{-A} \cdot \underset{(qm\times 1)}{\underline{I}} \qquad (2\text{-}10)$$

where

$$\underline{E} = [\underline{E}_1^T\ \underline{E}_2^T \cdots \underline{E}_p^T] \text{ and } \underline{I} = [\underline{I}_1^T\ \underline{I}_2^T \cdots \underline{I}_q^T]^T. \qquad (2\text{-}11)$$

$\underline{E}_i$ is a p×1 vector which represents the projected error in the i^{th} output, and $\underline{I}_i$ is a q×1 vector which represents the projected moves in the i^{th} manipulated input.

Since the coefficients matrix A is constant, so are matrices (A^TA) and $(A^TA)^{-1}A^T$. Thus $(A^TA)^{-1}A^T$ may be computed offline . . . a distinct advantage of the dimensions of A are large.

CHAPTER III

GAS ABSORBER

The first linear system to be considered here, was presented by Lapidus and Luus (1967), and consists of a six-plate gas absorber controlled by inlet feed streams (Figure 3-1). Perfect mixing with complete vapor-liquid equilibrium is assumed for each stage and the holdup of noninteracting liquid and vapor in each stage is assumed to be constant. Material balances around the i^{th} plate yield:

$$H\frac{dy_i}{dt} + h\frac{dx_i}{dt} = L(x_{i-1} - x_i) + G(y_{i+1} - y_i) \quad (3\text{-}1)$$

for $i = 1,2, \cdots, 6$, where

x_i, y_i = compositions of liquid and vapor leaving the i^{th} plate (lb solute/lb inert),

h, H = inert liquid and vapor holdups on each plate (lb),

L = flow rate of inert liquid absorbent (lb/min),

G = flow rate of inert gas stream (lb/min),

t = time of operation (min).

A linear equilibrium relation between liquid and vapor at each plate of the form: $y_i = \alpha x_i + \beta$ is assumed. Since the liquid composition at each stage is a function of time, the dynamics of the system can be consolidated

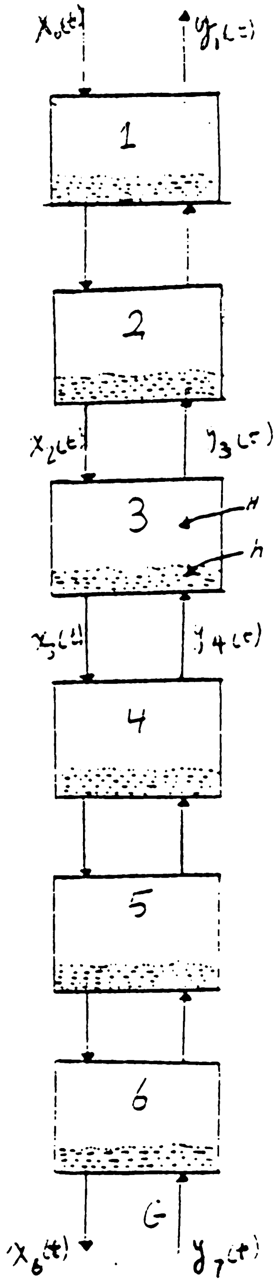


Figure (3-1). Gas absorber system.

into a six-dimensional state vector with a two-dimensional control vector of the form:

$$\underline{x}_{k+1} = \phi\underline{x}_k + \Delta\underline{u}_k.$$

The system was assumed to have an initial equilibrium state, $\underline{x}(0) = \underline{x}_0$ (prescribed). The elements of the control vector $\underline{u}_k$ were taken as the corresponding to the inlet liquid and vapor compositions:

$$u_1 = x_0$$

$$u_2 = (y_7-\beta)/\alpha.$$

The following set of convenient parameters was chosen: $\alpha = .72$, $\beta = 0$, L = 40.8 lb/min, G = 66.7 lb/min, G = 66.7 lb/min, H = 1.0 lb, and h = 75 lb. The values of ϕ, Δ and $\underline{x}_0$, computed with sampling period of one minute, are given in Table (3-1). The details of these computations are found in Lapidus and Luus (1967).

The DMC algorithm was implemented to control x_3 and x_6, the compositions of the liquid leaving the third and the sixth plates respectively. Choosing x_3 and x_6 as the controlled variables gives an idea about the behavior of the entire system. The output vector is related to the state vector by: $\underline{y}_k = \bar{H}\underline{x}_k$ where $\bar{H} = [0\ 0\ 1\ 0\ 0\ 1]$.

The outputs x_3 and x_6 are measured and controlled by the inlet feed streams u_1 and u_2. The problem is to find the proper moves on the manipulated inputs that would

Table 3-1. Computation with Sampling Period of One Minute.

$$\phi = \begin{bmatrix} 0.36538 & 0.21953 & 0.06771 & 0.01407 & 0.00221 & 0.00028 \\ 0.19060 & 0.32291 & 0.23149 & 0.06593 & 0.01431 & 0.00221 \\ 0.04892 & 0.19677 & 0.42452 & 0.23169 & 0.06958 & 0.01407 \\ 0.00864 & 0.05027 & 0.15693 & 0.32452 & 0.23169 & 0.06771 \\ 0.00115 & 0.00879 & 0.05027 & 0.19677 & 0.32293 & 0.21951 \\ 0.00012 & 0.00115 & 0.00864 & 0.04892 & 0.18660 & 0.36538 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.13000 & 0.00003 \\ 0.07253 & 0.00031 \\ 0.01165 & 0.00290 \\ 0.00146 & 0.01957 \\ 0.00015 & 0.10042 \\ 0.00001 & 0.33110 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} -0.0306 \\ -0.0569 \\ -0.0738 \\ -0.0977 \\ -0.1138 \\ -0.1271 \end{bmatrix}$$

change the compositions from the initial values (x_3 = -.0788 and x_6 = -.1273) to the desired final value of zero. The responses of x_3 and x_6 was considered for ten intervals of time and the movement on each manipulated variable was considered for four intervals. The problem has the following dimensions:

p = 2; two interacting outputs (x_3 and x_6)

q = 2; two manipulated variables (u_1 and u_2)

n = 10; ten elements in the error projection for each output

m = 4; four control moves on each manipulated input.

For this example equation (2-10) is:

$$\underset{(20\times 1)}{\underline{E}} = \underset{(20\times 8)}{-A} \cdot \underset{(8\times 1)}{\underline{I}} .$$

The coefficient matrix A is given by equation (2-9):

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} .$$

A_{11} and A_{12} describe the reaction of x_6 to a step change in u_1 and u_2 respectively. A_{21} and A_{22} represent the corresponding reactions of x_3. The coefficient vectors needed to construct the matrices A_{ij} of A are shown in Figures (3-2 to 3-5). Matrix A is given in Table (3-2). The projected

Table 3-2.

FOR A GAS ABSCRBER SYSTEM:

WITH TWO OUTPUTS AND TWO INPUTS.

THE DYNAMIC MATRIX A IS

0.0001	0.0	0.0	0.0	0.4245	0.0	0.0	0.0
0.0040	0.0001	0.0	0.0	0.6010	0.4245	0.0	0.0
0.0205	0.0040	0.0001	0.0	0.6956	0.6010	0.4245	0.0
0.0545	0.0252	0.0040	0.0001	0.7554	0.6956	0.6010	0.4245
0.1047	0.0545	0.0252	0.0040	0.7972	0.7554	0.6956	0.6010
0.1658	0.1047	0.0545	0.0252	0.8284	0.7972	0.7554	0.6956
0.2326	0.1658	0.1047	0.0545	0.8528	0.8284	0.7972	0.7554
0.3007	0.2326	0.1658	0.1047	0.8726	0.8528	0.8284	0.7972
0.3671	0.3007	0.2326	0.1658	0.8889	0.8726	0.8528	0.8284
0.4301	0.3671	0.3007	0.2326	0.9027	0.8889	0.8726	0.8528
0.0116	0.0	0.0	0.0	0.0028	0.0	0.0	0.0
0.0474	0.0116	0.0	0.0	0.0209	0.0028	0.0	0.0
0.0904	0.0474	0.0116	0.0	0.0544	0.0209	0.0028	0.0
0.1314	0.0904	0.0474	0.0116	0.0960	0.0544	0.0209	0.0028
0.1680	0.1314	0.0904	0.0474	0.1397	0.0960	0.0544	0.0209
0.1998	0.1680	0.1314	0.0904	0.1824	0.1397	0.0960	0.0544
0.2275	0.1998	0.1680	0.1314	0.2225	0.1824	0.1397	0.0960
0.2516	0.2275	0.1998	0.1680	0.2594	0.2225	0.1824	0.1397
0.2726	0.2516	0.2275	0.1998	0.2929	0.2594	0.2225	0.1824
0.2910	0.2726	0.2516	0.2275	0.3231	0.2929	0.2594	0.2225

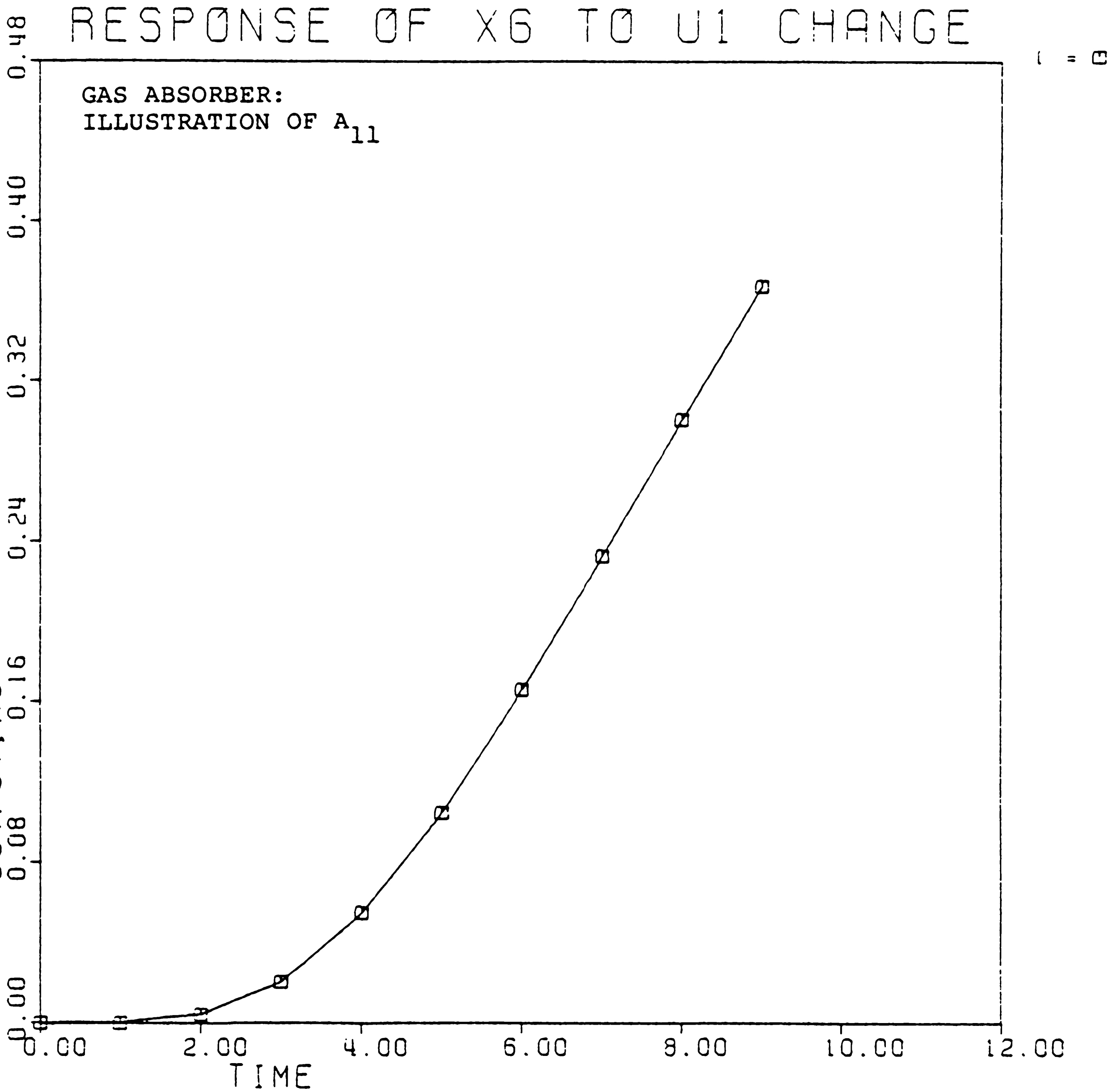

RESPONSE ØF X6 TØ U1 CHANGE
GAS ABSORBER:
ILLUSTRATION OF A11
0.48
0.40
0.32
0.24
0.16
0.08
0.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
TIME

Figure (3-2).

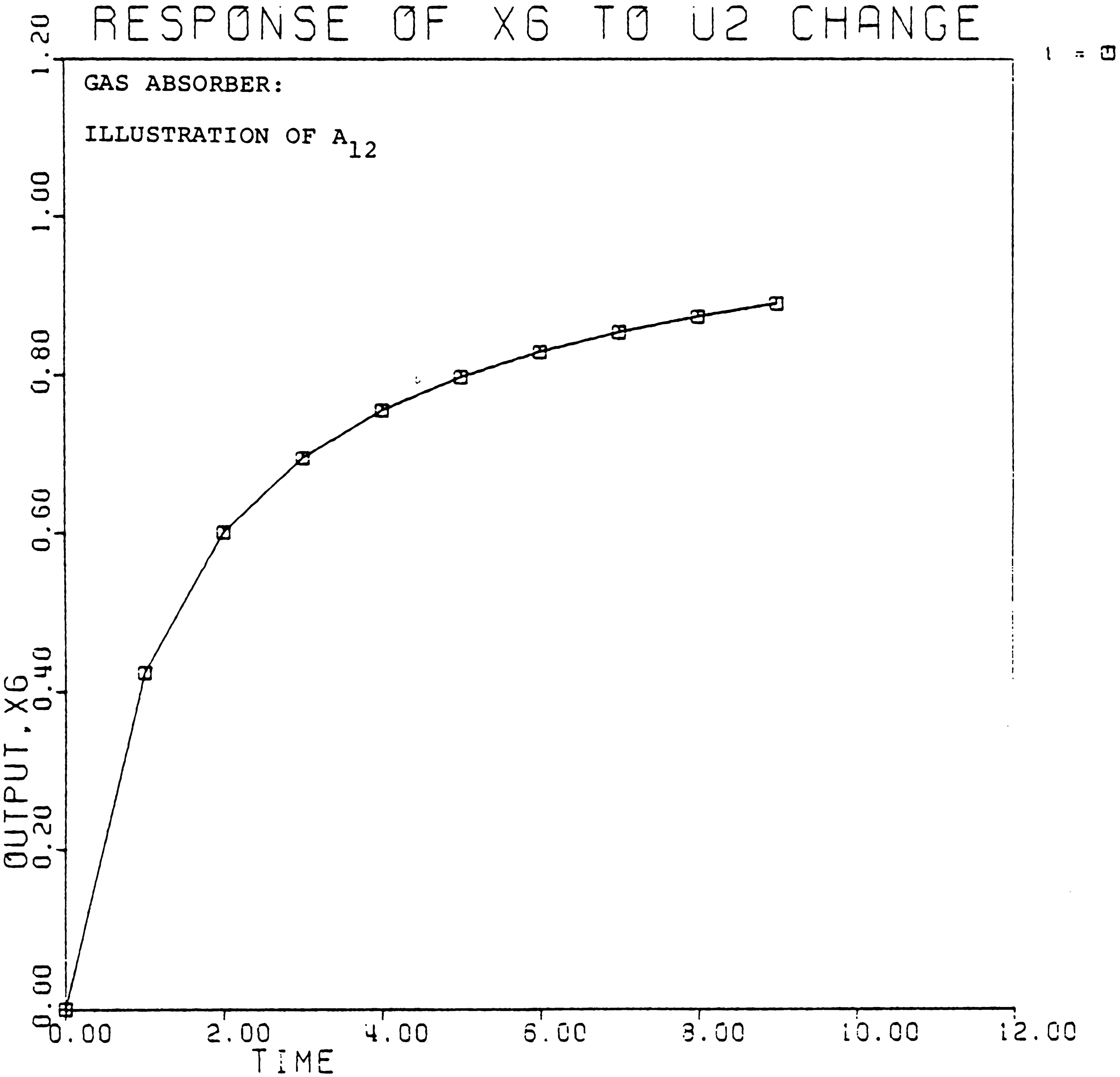

RESPONSE OF X6 TO U2 CHANGE
GAS ABSORBER:
ILLUSTRATION OF A_{12}
OUTPUT, X6
TIME
0.00
0.20
0.40
0.60
0.80
1.00
1.20
0.00
2.00
4.00
6.00
8.00
10.00
12.00

Figure (3-3).

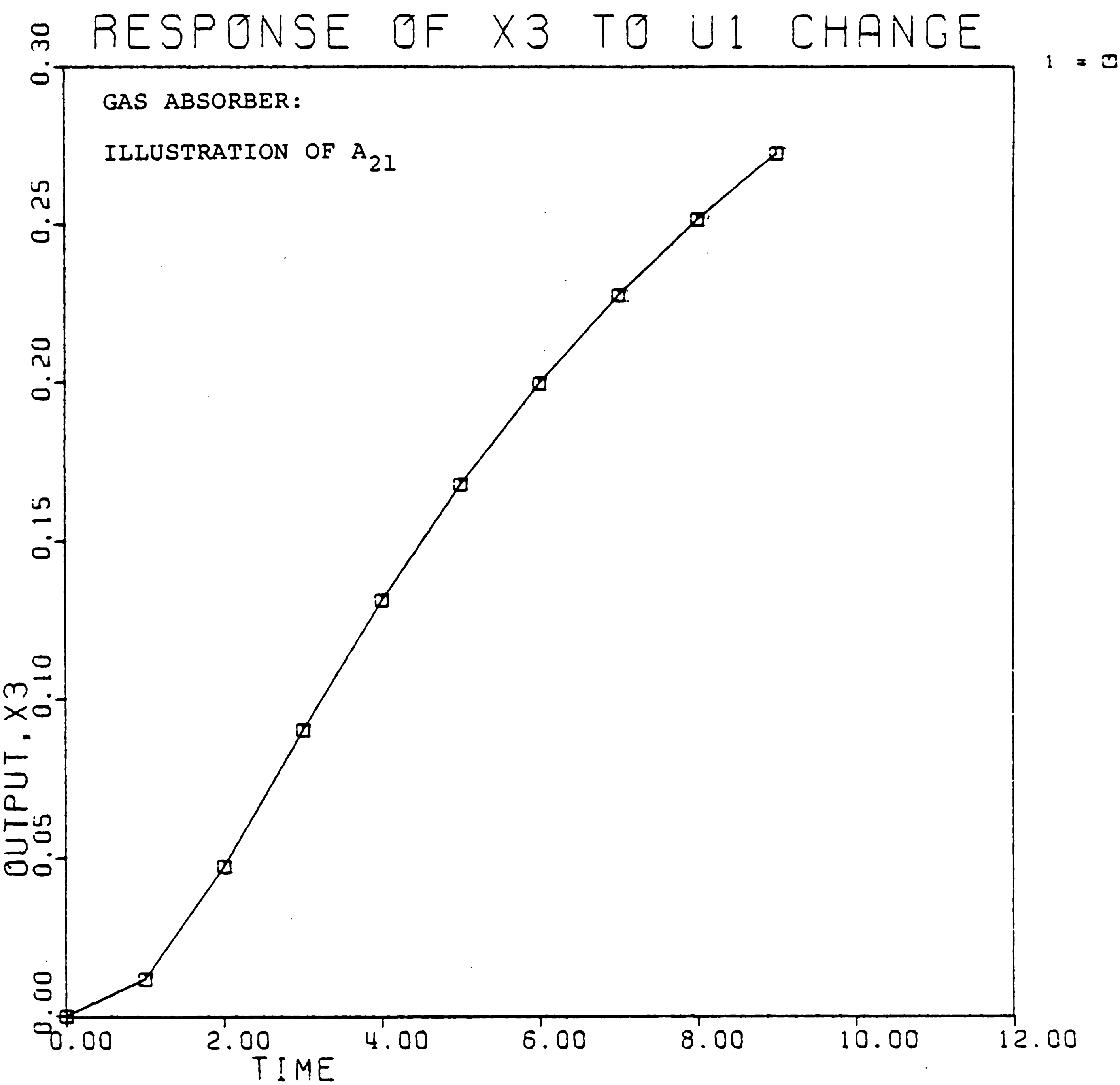

RESPONSE OF X3 TO U1 CHANGE
GAS ABSORBER:
ILLUSTRATION OF A21
1 = ⊡
OUTPUT, X3
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.00
2.00
4.00
6.00
8.00
10.00
12.00
TIME

Figure (3-4).

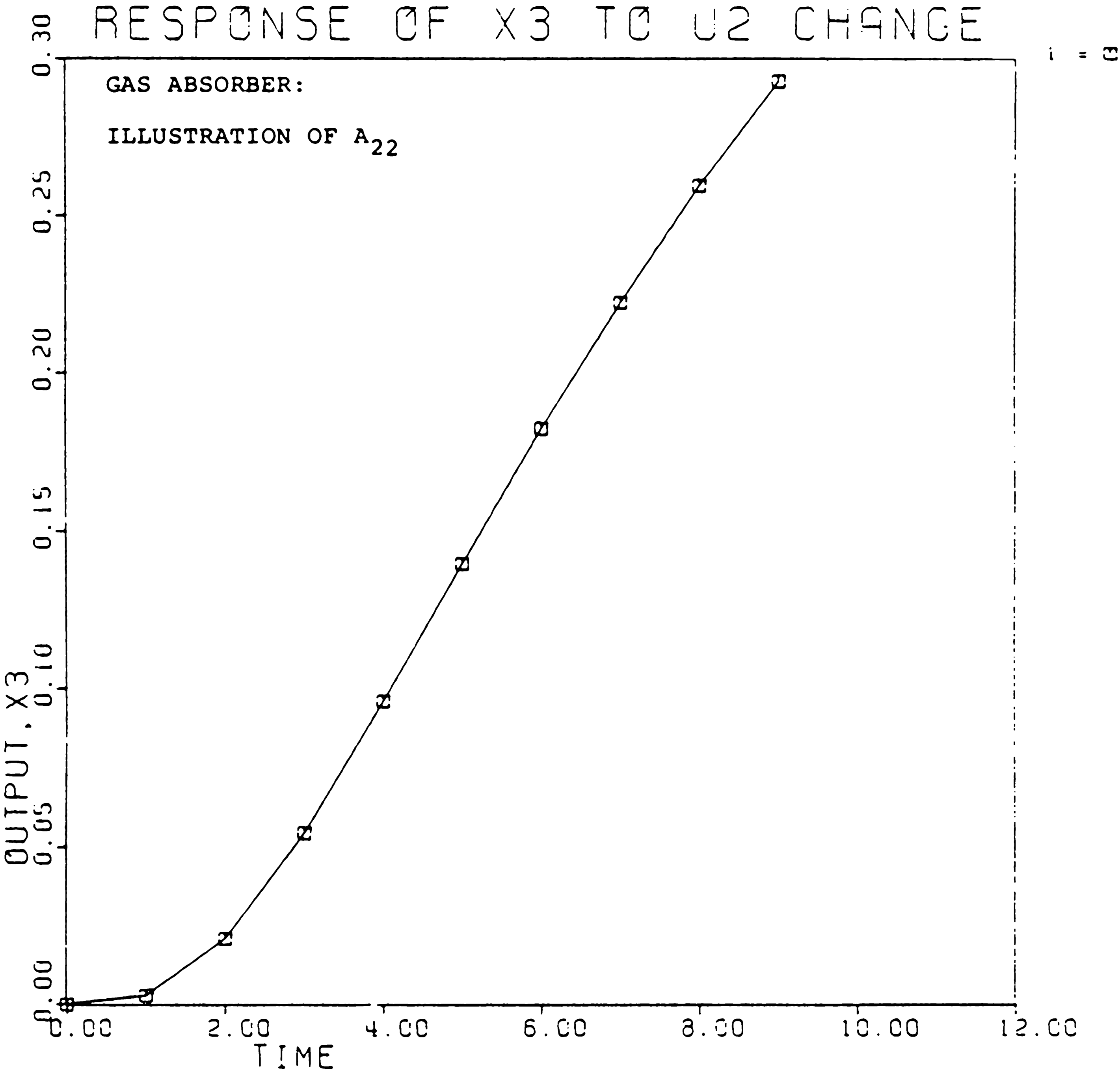

RESPONSE OF X3 TO U2 CHANGE
GAS ABSORBER:
ILLUSTRATION OF A22
OUTPUT, X3
0.00
0.05
0.10
0.15
0.20
0.25
0.30
TIME
0.00
2.00
4.00
6.00
8.00
10.00
12.00

Figure (3-5).

error in the output $\underline{E} = [E_1^T \; E_2^T]^T$. E_1 and E_2 are 10×1 vectors which represent the uncontrolled response of x_6 and x_3 due to the initial conditions.

Knowing $\underline{E}$ and A, equation (2-10) can be solved for I. The results of the application of DMC Algorithm to the gas absorber are presented and discussed in Chapter V.

CHAPTER IV

BINARY DISTILLATION COLUMN

The second numerical example is a binary distillation column, sketched in Figure (4-1), studied by Wood and Berry (1973) and Ognnaike and Ray (1979). The example was chosen to investigate the applicability of DMC algorithm to a multivariate system with multiple delays or dead times.

The column service was methanol-water separation. The description of the column and its associated instrumentation are found in Wood and Berry (1973). The column was modelled by the transfer function model:

$$\underline{y}(s) = G(s)\underline{u}(s) + G_d(s)\underline{d}(s) \qquad (4\text{-}1)$$

where in terms of deviation from steady state,

y_1 = overhead mole fraction methanol,

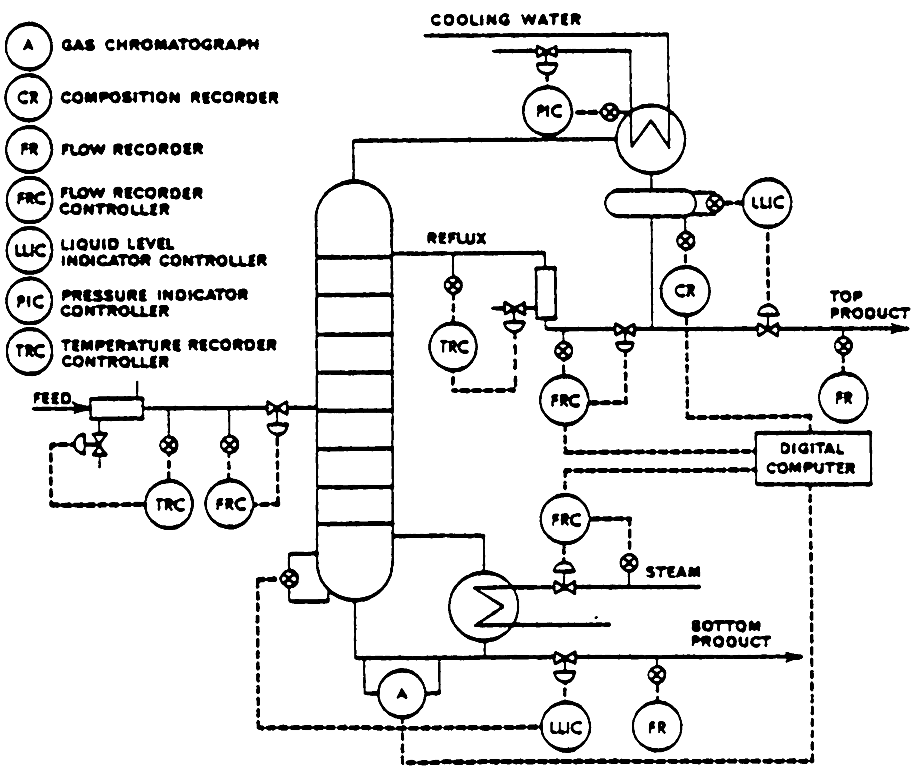
y_2 = bottoms mole fraction methanol,

u_1 = overhead reflux flow rate,

u_2 = bottoms steam flow rate, and

d = column feed flow rate.

The transfer functions were established by pulse testing time. The process reaction curves were assumed to have first-order-lag-plus-dead-time model:



Figure(4-1). Schematic.diagram of the distillation column (Wood and Berry, 1973).

$$G_m(s) = \frac{Ke^{-\theta s}}{\tau s+1}$$

where K is the process gain, θ the dead time, and τ the time constant. The parameters of the assumed model were determined from the transient data. The steady state values for the overhead and bottoms compositions were 96.25% and 0.5% methanol respectively. The transfer function matrices in equation (4-1) were:

$$G(s) = \begin{bmatrix} \frac{12.8\ e^{-s}}{16.7s + 1} & \frac{-18.9\ e^{-3s}}{21.0s + 1} \\ \\ \frac{6.6\ e^{-7s}}{10.9s + 1} & \frac{-19.4\ e^{-3s}}{14.4s + 1} \end{bmatrix} \tag{4-2}$$

and

$$G_d(s) = \begin{bmatrix} \frac{3.8\ e^{-8.1s}}{14.9s + 1} \\ \\ \frac{4.9\ e^{-3.4s}}{13.2s + 2} \end{bmatrix} \tag{4-3}$$

where the time constants and time delays are given in minutes. Figure (4-2) shows the transfer function model fit to the data.

A state variable representation of the model was presented as (Ogunnaike and Ray, 1979):

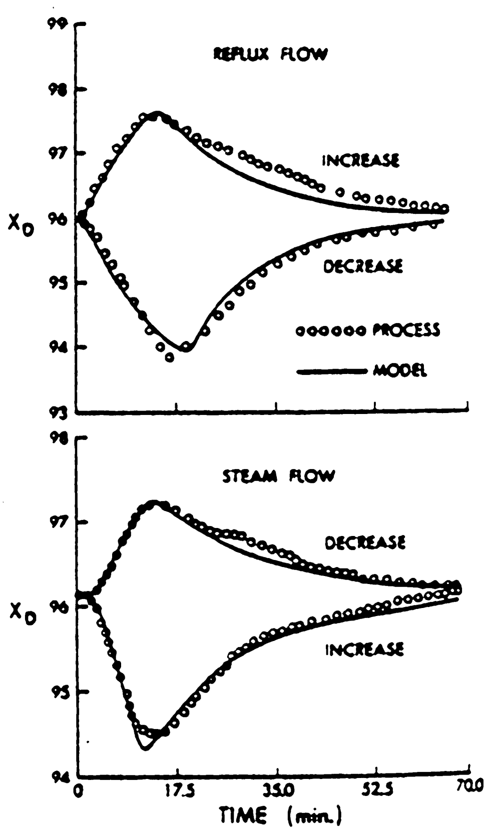


Figure (4-2) Transfer function model fit to the experimental data (Wood and Berry, 1973).

The state vector,

$$\left.\begin{aligned}
\dot{x}_1(t) &= -0.06\ x_1(t) + 0.768\ u_1(t-1)\\
\dot{x}_2(t) &= -0.04762\ x_2(t) + 0.9\ u_2(t-3)\\
\dot{x}_3(t) &= -0.09174\ x_3(t) + 0.6055\ u_1(t-7)\\
\dot{x}_4(t) &= -0.06944\ x_4(t) + 1.3472\ u_2(t-3)
\end{aligned}\right\} \tag{4-4}$$

along with the load vector,

$$\left.\begin{aligned}
\dot{x}_{L_1}(t) &= -0.0671\ x_{L_1}(t) + 0.255\ d(t-8.1)\\
\dot{x}_{L_2}(t) &= -0.0576\ x_{L_2}(t) + 0.3712\ d(t-3.4)
\end{aligned}\right\} \tag{4-5}$$

and the output vector,

$$\left.\begin{aligned}
y_1(t) &= x_1(t) - x_2(t) + x_{L_1}(t)\\
y_2(t) &= x_3(t) - x_4(t) + x_{L_2}(t)
\end{aligned}\right\}. \tag{4-6}$$

The derivation of the state-variable representation [equations (4-4) to (4-6)] from the transfer function model [equations (4-1) to (4-3)] is given in Appendix A.

To put the state-variable representation in a more concise vector-matrix form; make the following change in variables:

$$\left.\begin{array}{ll} x_1 = x_1(t) & u_1 = u_1(t-1) \\ x_2 = x_2(t) & u_2 = u_2(t-3) \\ x_3 = x_3(t) & u_3 = u_1(t-7) \\ x_4 = x_4(t) & u_4 = d(t-8.1) \\ x_5 = x_{L_1}(t) & u_5 = d(t-3.4) \\ x_6 = x_{L_2}(t) & \end{array}\right\} \quad (4\text{-}7)$$

with this change in variables equation (4-4)-(4-5) can be written as:

$$\underset{(6\times 1)}{\dot{\underline{x}}} = \underset{(6\times 6)}{A} \cdot \underset{(6\times 1)}{\underline{x}} + \underset{(6\times 5)}{B} \cdot \underset{(5\times 1)}{\underline{u}} \quad (4\text{-}8)$$

where A is the diagonal matrix whose diagonal elements are

Diag(A) = (0.06, - 0.04762, - 0.09174, - 0.06944, - 0.06971, -0.07567)

and

$$B = \begin{bmatrix} .768 & 0. & 0. & 0. & 0. \\ 0. & 0.9 & 0. & 0. & 0. \\ 0.6055 & 0. & 0. & 0. & 0. \\ 0. & 0. & 1.3472 & 0. & 0. \\ 0. & 0. & 0. & 0.255 & 0. \\ 0. & 0. & 0. & 0. & 0.3712 \end{bmatrix}.$$

The discrete analogous to equation (4-8) is:

$$\left.\begin{aligned} \underline{x}_{K+1} &= \phi(T)\underline{x}_K + \Delta(T)\underline{I}_K \\ \phi(T) &= e^{AT} \text{ and } \Delta(T) = \int_0^T e^{A\bar{\tau}} B \, d\tau \end{aligned}\right\} \qquad (4\text{-}9)$$

where T is the sampling period. Derivation of equation (4-9) from (4-8) is given in Appendix B. For T = 2.5 minutes, the diagonal matrix $\phi(2.5)$ is given by:

$$\text{Diag}(\phi(2.5)) = (0.8607, .8878, .7951, .8406, .8456, .8275)$$

and

$$\Delta(2.5) = \begin{bmatrix} 1.7830 & 0. & 0. & 0. & 0. \\ 0. & 2.1205 & 0. & 0. & 0. \\ 0. & 0. & 3.0925 & 0. & 0. \\ 0. & 0. & 0. & 0.5648 & 0. \\ 0. & 0. & 0. & 0. & 0.8452 \end{bmatrix}.$$

Since the system is initially at steady state then the outputs $y_1(0)$ and $y_2(0)$ are zero. The DMC was implemented to keep the outputs at zero despite changes in disturbance inputs. The variables of equation (4-7) are incorporated into the synthesis of the design of the DMC to maintain awareness of time delays. The disturbance inputs u_4 and u_5 are measured and used to predict the necessary moves on the manipulated inputs u_1, u_2, and u_3 to keep the outputs y_1 and y_2 as close to zero as possible. The responses of y_1 and y_2 were considered for 40 intervals of time and the movement on each manipulated variable was considered for 20 intervals. The problem has the following dimensions:

p = 2; two interacting outputs (y_1 and y_2)

q = 3; three manipulated variables (u_1, u_2 and u_3)

n = 40; forty elements in the error projection for each output

m = 20; twenty control moves on each manipulated input.

Hence, for this example:

$$\underline{E} = (2*40)\times 1 = 80\times 1$$

$$\underline{I} = (3*20)\times 1 = 60\times 1$$

$$A = 80\times 60.$$

The coefficient vectors, a_i and b_i, needed to construct matrix A and vector $\underline{E}$ are obtained from equation (4-9) as discussed in Appendix B.

Knowing $\underline{E}$ and A, equation (2-10) can be solved for I. The results of the application of DMC Algorithm to the distillation column are presented and discussed in Chapter V.

CHAPTER V

DISCUSSION OF RESULTS

A. Gas Absorber

To show the performance of DMC Algorithm on the gas absorber in Figure (3-1), the uncontrolled response due to a given set of initial conditions is plotted in Figure (5-1). The results of using the DMC Algorithm are tabulated in Table (5-1) and shown graphically in Figure (5-2). It is noticed that the output variable x_6 and x_3 approach the final desired zero values much faster than the uncontrolled response in Figure (5-1).

To show the effect of the suppression factors, suggested by Culter and Ramaker (1979) and discussed in Chapter 2 in this work, an $\varepsilon = .005$ was chosen. The results of using a suppression factor of 1.005 is tabulated in Table (5-2) and graphed in Figure (5-3).

The absolute sum of the manipulated moves (ASUM) is shown in Tables (5-1) and (5-2) to indicate smoothness of the manipulated inputs curve. The sum of squares of the error (SERROR) is also included in the above tables to show how good the output response is. From the values of ASUM, it is noticed that a smoother manipulated input curve can be obtained by the use of a suppression factor. From the

Table 5-1.

DIAGONAL OF SQUARE MATRIX AT*A MULTIPLIED BY 1.000

TIME	UNCONT X6	UNCONT X3	CONT X6	CONT X3	U1	U2
1.0	-0.1273	-0.0788	0.0006	-0.0440	2.9233	0.3006
2.0	-0.0733	-0.0785	0.0013	0.0135	-4.4883	-0.2765
3.0	-0.0508	-0.0760	0.0088	0.0043	1.5848	-0.0599
4.0	-0.0387	-0.0713	-0.0242	-0.0039	0.1349	-0.0867
5.0	-0.0311	-0.0656	0.0078	-0.0042	0.0	0.0
6.0	-0.0258	-0.0555	0.0017	-0.0033	0.0	0.0
7.0	-0.0218	-0.0536	0.0013	-0.0016	0.0	0.0
8.0	-0.0187	-0.0480	0.0010	-0.0009	0.0	0.0
9.0	-0.0162	-0.0429	0.0013	-0.0006	0.0	0.0
10.0	-0.0141	-0.0382	0.0028	-0.0003	0.0	0.0

ABSOLUTE SUM OF INPUT MOVES 9.85498447

THE SQUARE ERROR IS 0.293243D-02

Table 5-2.

DIAGONAL CF SQUARE MATRIX AT*A MUTIPLIED BY 1.0C5

TIME	UNCONT X6	UNCONT X3	CCNT X6	CONT X3	U1	U2
1.0	-0.1273	-0.0788	-0.0252	-0.0709	0.6273	0.2403
2.0	-0.0733	-0.0785	0.0101	-0.0450	-0.0725	-0.1496
3.0	-C.0508	-0.0760	0.0111	-0.0178	-0.4149	-0.0657
4.0	-0.0387	-0.0713	-0.0192	-0.0030	-0.0967	-0.1156
5.0	-0.0311	-0.0656	-0.0169	0.0014	0.0	0.0
6.0	-0.0258	-0.0555	-0.0047	0.0007	0.0	0.0
7.0	-0.0218	-0.0536	0.0046	-0.0019	0.0	0.0
8.0	-0.0187	-0.0480	0.0056	-0.0049	0.0	0.0
9.0	-C.0162	-0.0429	0.0109	-0.0080	0.0	0.0
10.0	-0.0141	-0.0382	0.0056	-0.0108	0.0	0.0

ABSOLUTE SUM OF INPUT MCVES 1.78271331

THE SQUARE ERRCF IS 0.944616D-02

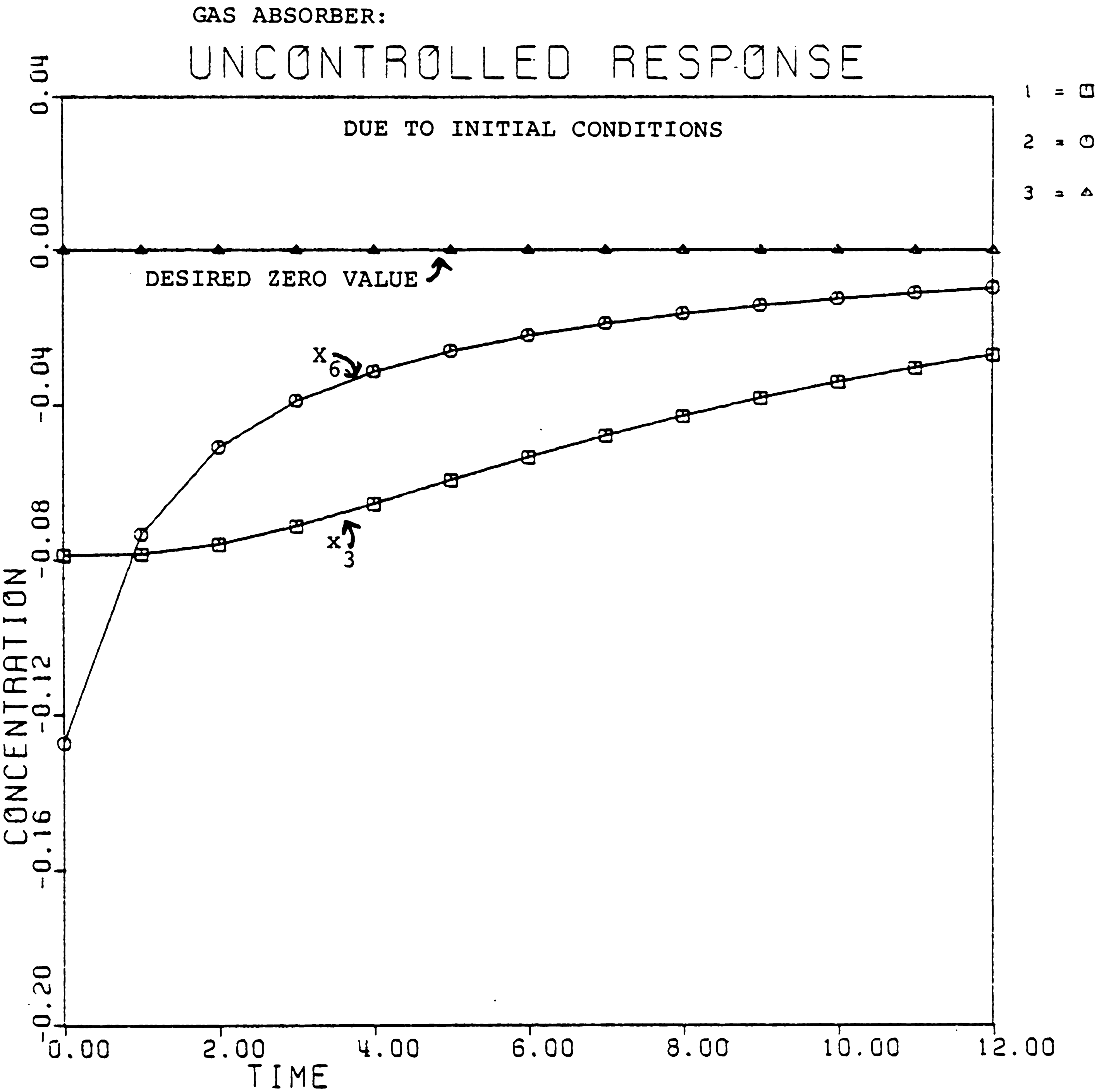

GAS ABSORBER:
UNCONTROLLED RESPONSE
DUE TO INITIAL CONDITIONS
1 = □
2 = ○
3 = △
DESIRED ZERO VALUE
x_6
x_3
0.04
0.00
-0.04
-0.08
-0.12
-0.16
-0.20
CONCENTRATION
0.00
2.00
4.00
6.00
8.00
10.00
12.00
TIME

Figure (5-1).

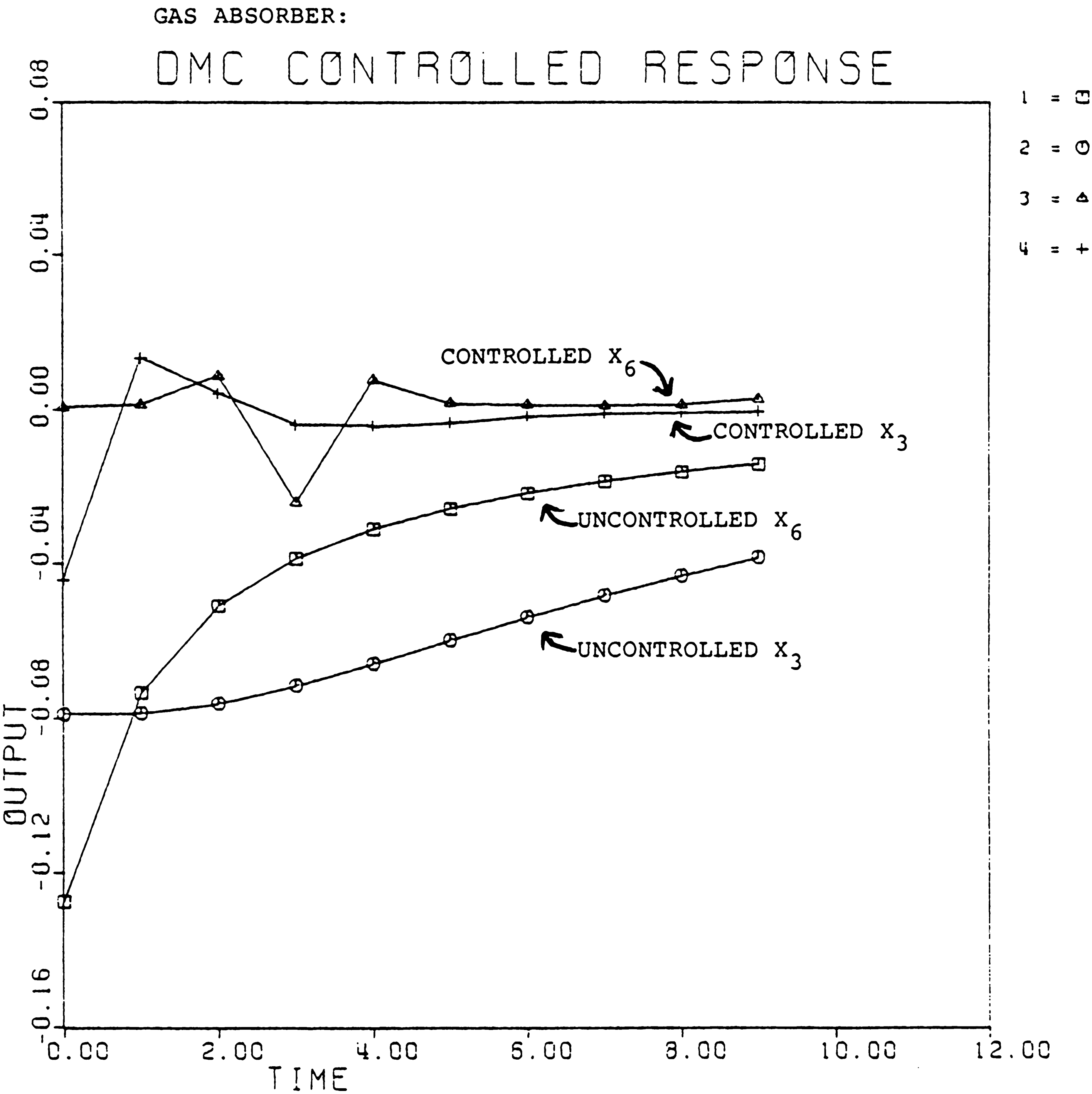

GAS ABSORBER:
DMC CONTROLLED RESPONSE
OUTPUT
TIME
0.08
0.04
0.00
-0.04
-0.08
-0.12
-0.16
0.00
2.00
4.00
6.00
8.00
10.00
12.00
1 = □
2 = ○
3 = △
4 = +
CONTROLLED X_6
CONTROLLED X_3
UNCONTROLLED X_6
UNCONTROLLED X_3

Figure (5-2).

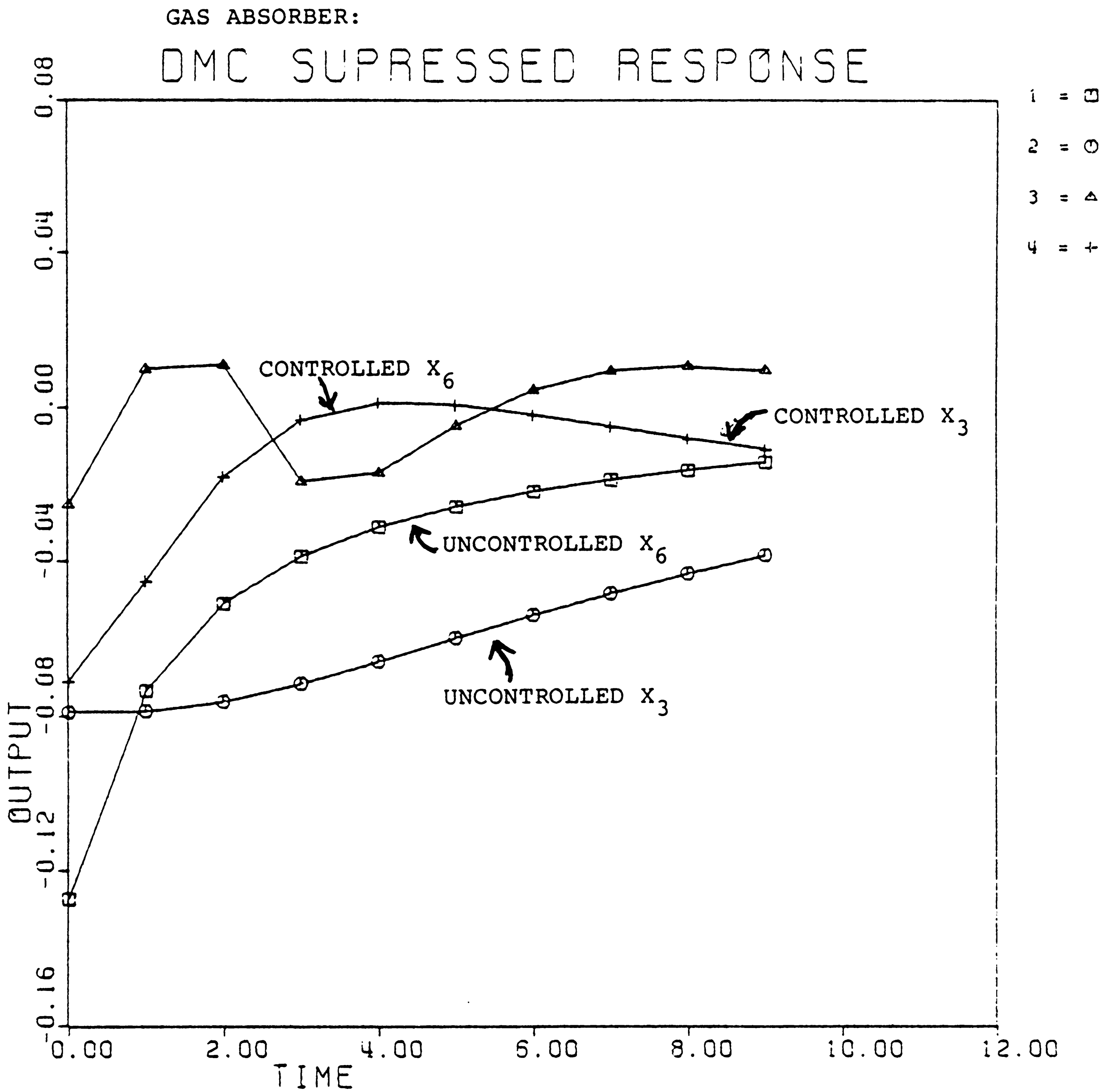

GAS ABSORBER:
DMC SUPRESSED RESPONSE
CONTROLLED X_6
CONTROLLED X_3
UNCONTROLLED X_6
UNCONTROLLED X_3
1 = □
2 = ○
3 = △
4 = +
OUTPUT
TIME
0.08
0.04
0.00
-0.04
-0.08
-0.12
-0.16
0.00
2.00
4.00
6.00
8.00
10.00
12.00

Figure (5-3).

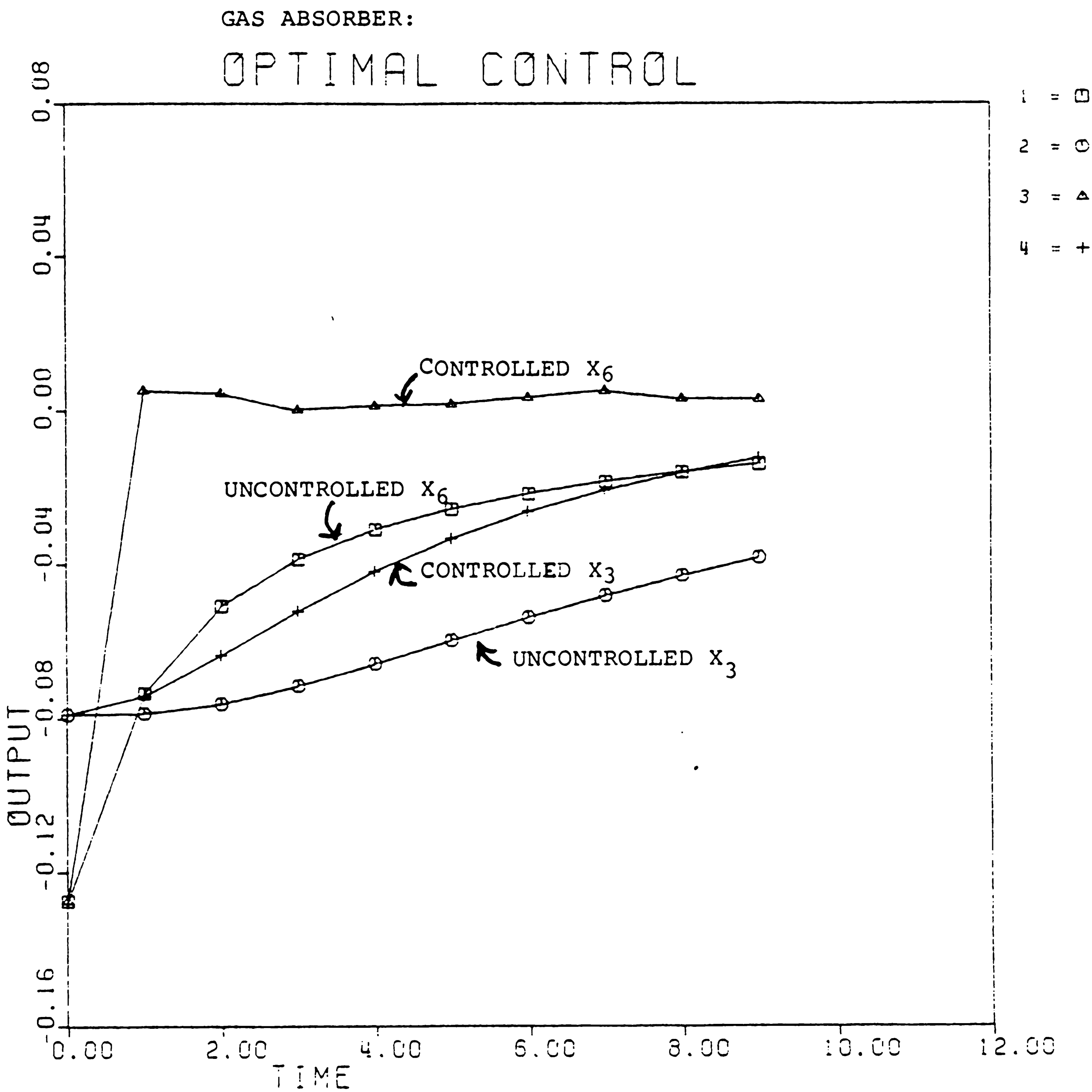

GAS ABSORBER:
OPTIMAL CONTROL
0.08
0.04
0.00
-0.04
-0.08
-0.12
-0.16
OUTPUT
0.00
2.00
4.00
6.00
8.00
10.00
12.00
TIME
1 = □
2 = ○
3 = △
4 = +
CONTROLLED X6
UNCONTROLLED X6
CONTROLLED X3
UNCONTROLLED X3

Figure (5-4).

SERROR values, it can be concluded that the use of suppression factors doesn't change the output response by much. Figure (5-4) shows the simultaneous control of x_3 and x_6 of the same example system (gas absorber) using the least squares optimal control (Lapidus and Luus, 1979). The comparison of Figures (5-3) and (5-4) shows that the DMC control gives a better result than the least squares optimal control.

B. Distillation Column

The results of using DMC Algorithm to control the distillation column in Figure (4-1) are tabulated in Table (5-3) and plotted in Figures (5-5) and (5-6). Curve 1 in Figure (5-5) represents the uncontrolled response of the overhead composition to a disturbance of (.34 lb/min). Curve 2 of Figure (5-5) represents the response of the overhead composition of DMC is used. Curve 1 and Curve 2 in Figure (5-6) represent the uncontrolled and the DMC controlled response of the bottoms mole fraction to the same disturbance (.34 lb/min). The controlled responses of the outputs in Figures (5-5) and (5-6) are almost identical to the desired final values.

The results of using a suppression factor of 1.005 is shown in Table (5-4) and Figures (5-7). From the values of the absolute sum of the manipulated variables, it is noticed that the use of the suppression factor in DMC Algorithm results in a smoother manipulated curve.

Table 5-3. DMC Controlled Distillation Column.

TIME	UNCONT Y1	UNCONT Y2	CONT Y1	CONT Y2	U1(T-1)	U2(T-3)	U1(T-7)	U1	U2
2.5	0.9625	0.0050	0.9625	0.0050	-0.1078	0.1338	0.1384	0.0	0.0
5.0	0.9625	0.0636	0.9625	0.0050	-0.4068	0.5015	0.1480	-0.1078	0.1338
7.5	0.9625	0.1122	0.9625	0.0050	0.0578	-0.0885	-0.0981	-0.4068	0.5015
10.0	1.0969	0.1523	0.9625	0.0050	-0.7085	-0.3304	0.8944	0.1962	-0.0885
12.5	1.1286	0.1856	0.9625	0.0050	0.5062	0.1172	-0.7043	-0.5605	-0.3304
15.0	1.1554	0.2131	0.9625	0.0050	0.0043	-0.0063	-0.0177	0.4081	0.1172
17.5	1.1781	0.2358	0.9625	0.0050	-0.0085	0.0099	0.0004	0.8987	-0.0063
20.0	1.1973	0.2547	0.9625	0.0050	0.0047	-0.0069	-0.0141	-0.7128	0.0099
22.5	1.2135	0.2702	0.9625	0.0050	-0.0092	0.0111	0.0053	-0.0130	-0.0069
25.0	1.2272	0.2831	0.9625	0.0050	-0.0023	0.0020	-0.0020	-0.0088	0.0111
27.5	1.2388	0.2938	0.9625	0.0050	0.0305	-0.0409	-0.0443	-0.0164	0.0020
30.0	1.2486	0.3026	0.9625	0.0050	-0.0046	0.0072	0.0050	0.0359	-0.0409
32.5	1.2569	0.3099	0.9625	0.0050	0.1338	-0.1673	-0.1742	-0.0066	0.0072
35.0	1.2639	0.3160	0.9625	0.0050	-0.0875	0.1131	0.1200	0.0895	-0.1673
37.5	1.2698	0.3210	0.9625	0.0050	0.0481	-0.0595	-0.0635	-0.0825	0.1131
40.0	1.2749	0.3251	0.9625	0.0050	-0.0105	0.0157	0.0240	-0.1261	-0.0595
42.5	1.2791	0.3285	0.9625	0.0050	-0.0180	0.0238	0.0140	0.1095	0.0157
45.0	1.2827	0.3314	0.9625	0.0050	-0.0091	0.0120	0.0156	-0.0815	0.0238
47.5	1.2857	0.3337	0.9625	0.0050	0.0003	-0.0015	-0.0003	0.0150	0.0120
50.0	1.2883	0.3357	0.9625	0.0050	0.0007	0.0008	-0.0012	0.0143	-0.0015
52.5	1.2904	0.3373	0.9624	0.0050	0.0	0.0	0.0	0.0163	0.0008
55.0	1.2923	0.3386	0.9625	0.0050	0.0	0.0	0.0	-0.0003	0.0
57.5	1.2938	0.3397	0.9625	0.0050	0.0	0.0	0.0	-0.0012	0.0
60.0	1.2951	0.3406	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
62.5	1.2962	0.3414	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
65.0	1.2972	0.3420	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
67.5	1.2980	0.3425	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
70.0	1.2987	0.3429	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
72.5	1.2992	0.3433	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
75.0	1.2997	0.3436	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
77.5	1.3001	0.3438	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
80.0	1.3004	0.3440	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
82.5	1.3007	0.3442	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
85.0	1.3010	0.3443	0.9626	0.0050	0.0	0.0	0.0	0.0	0.0
87.5	1.3012	0.3443	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
90.0	1.3013	0.3444	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
92.5	1.3015	0.3445	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
95.0	1.3016	0.3446	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
97.5	1.3018	0.3447	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
100.0	1.3018	0.3447	0.9625	0.0051	0.0	0.0	0.0	0.0	0.0

Table 5-4. Suppressed DMC Controlled Distillation Column.

TIME	UNCONT Y1	UNCONT Y2	CONT Y1	CONT Y2	U1(T-1)	U2(T-3)	U1(T-7)	U1	U2
2.5	0.9625	0.0050	0.9553	-0.0119	-0.0394	-0.0156	-0.0556	0.0	0.0
5.0	0.9625	0.0636	0.9363	0.0132	-0.0549	-0.0446	-0.0529	-0.0394	-0.0156
7.5	0.9625	0.1122	0.9040	0.0186	-0.0553	-0.0726	-0.0277	-0.0549	-0.0446
10.0	1.0969	0.1523	0.9547	0.0132	-0.0466	-0.0790	-0.0061	-0.1109	-0.0726
12.5	1.1286	0.1856	0.9635	0.0073	-0.0129	-0.0267	0.0029	-0.0995	-0.0798
15.0	1.1554	0.2131	0.9735	0.0041	0.0028	0.0030	0.0028	-0.0406	-0.0267
17.5	1.1781	0.2358	0.9667	0.0033	0.0073	0.0154	-0.0006	-0.0033	0.0030
20.0	1.1973	0.2547	0.9630	0.0039	0.0066	0.0175	-0.0036	0.0102	0.0154
22.5	1.2135	0.2702	0.9614	0.0046	0.0042	0.0147	-0.0048	0.0094	0.0175
25.0	1.2272	0.2831	0.9611	0.0051	0.0023	0.0105	-0.0045	0.0036	0.0147
27.5	1.2388	0.2938	0.9613	0.0054	0.0011	0.0067	-0.0033	-0.0013	0.0105
30.0	1.2486	0.3026	0.9617	0.0054	0.0004	0.0039	-0.0020	-0.0038	0.0067
32.5	1.2569	0.3099	0.9621	0.0052	0.0001	0.0022	-0.0010	-0.0041	0.0039
35.0	1.2639	0.3160	0.9623	0.0051	-0.0000	0.0013	-0.0003	-0.0032	0.0022
37.5	1.2698	0.3210	0.9624	0.0050	-0.0002	0.0009	0.0000	-0.0020	0.0013
40.0	1.2749	0.3251	0.9624	0.0053	-0.0002	0.0007	0.0003	-0.0011	0.0009
42.5	1.2791	0.3295	0.9623	0.0051	-0.0001	0.0007	0.0007	-0.0005	0.0007
45.0	1.2827	0.3314	0.9621	0.0049	-0.0002	0.0009	0.0006	-0.0001	0.0007
47.5	1.2857	0.3337	0.9621	0.0049	-0.0003	0.0018	0.0002	0.0000	0.0009
50.0	1.2881	0.3357	0.9624	0.0050	0.0001	0.0037	-0.0001	0.0004	0.0018
52.5	1.2904	0.3373	0.9626	0.0050	0.0	0.0	0.0	0.0007	0.0037
55.0	1.2923	0.3386	0.9628	0.0050	0.0	0.0	0.0	0.0002	0.0
57.5	1.2938	0.3397	0.9629	0.0050	0.0	0.0	0.0	-0.0001	0.0
60.0	1.2951	0.3406	0.9629	0.0050	0.0	0.0	0.0	0.0	0.0
62.5	1.2962	0.3414	0.9629	0.0050	0.0	0.0	0.0	0.0	0.0
65.0	1.2972	0.3420	0.9629	0.0050	0.0	0.0	0.0	0.0	0.0
67.5	1.2980	0.3425	0.9628	0.0051	0.0	0.0	0.0	0.0	0.0
70.0	1.2987	0.3429	0.9628	0.0050	0.0	0.0	0.0	0.0	0.0
72.5	1.2992	0.3433	0.9627	0.0051	0.0	0.0	0.0	0.0	0.0
75.0	1.2997	0.3436	0.9627	0.0050	0.0	0.0	0.0	0.0	0.0
77.5	1.3001	0.3438	0.9626	0.0050	0.0	0.0	0.0	0.0	0.0
80.0	1.3004	0.3440	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
82.5	1.3007	0.3442	0.9625	0.0050	0.0	0.0	0.0	0.0	0.0
85.0	1.3010	0.3443	0.9624	0.0050	0.0	0.0	0.0	0.0	0.0
87.5	1.3012	0.3443	0.9624	0.0049	0.0	0.0	0.0	0.0	0.0
90.0	1.3013	0.3444	0.9623	0.0049	0.0	0.0	0.0	0.0	0.0
92.5	1.3015	0.3445	0.9622	0.0050	0.0	0.0	0.0	0.0	0.0
95.0	1.3016	0.3446	0.9621	0.0049	0.0	0.0	0.0	0.0	0.0
97.5	1.3018	0.3447	0.9621	0.0050	0.0	0.0	0.0	0.0	0.0
100.0	1.3018	0.3447	0.9620	0.0050	0.0	0.0	0.0	0.0	0.0

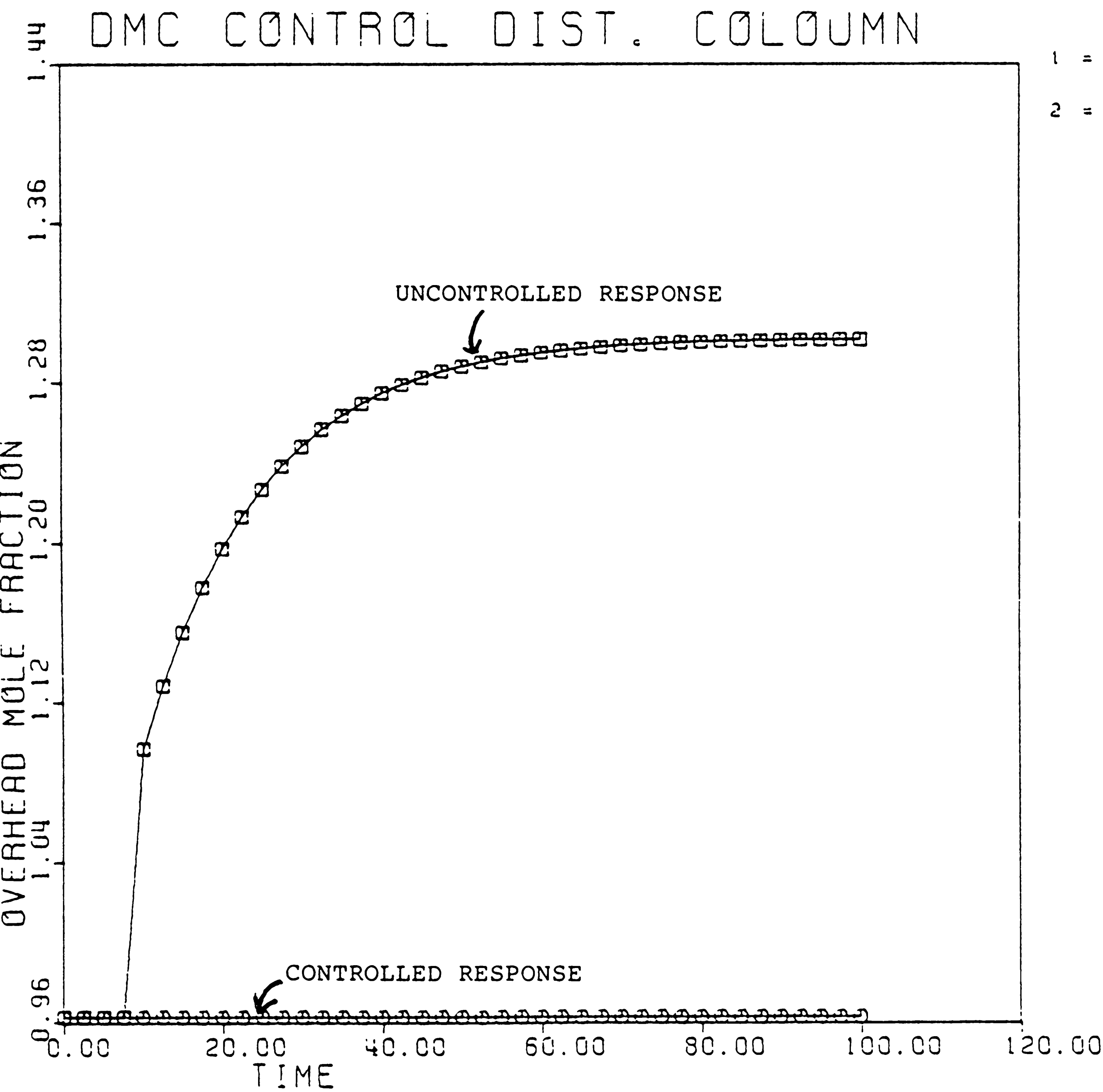

DMC CONTROL DIST. COLOUMN
1 =
2 =
UNCONTROLLED RESPONSE
CONTROLLED RESPONSE
OVERHEAD MOLE FRACTION
0.96
1.04
1.12
1.20
1.28
1.36
1.44
0.00
20.00
40.00
60.00
80.00
100.00
120.00
TIME

Figure (5-5).

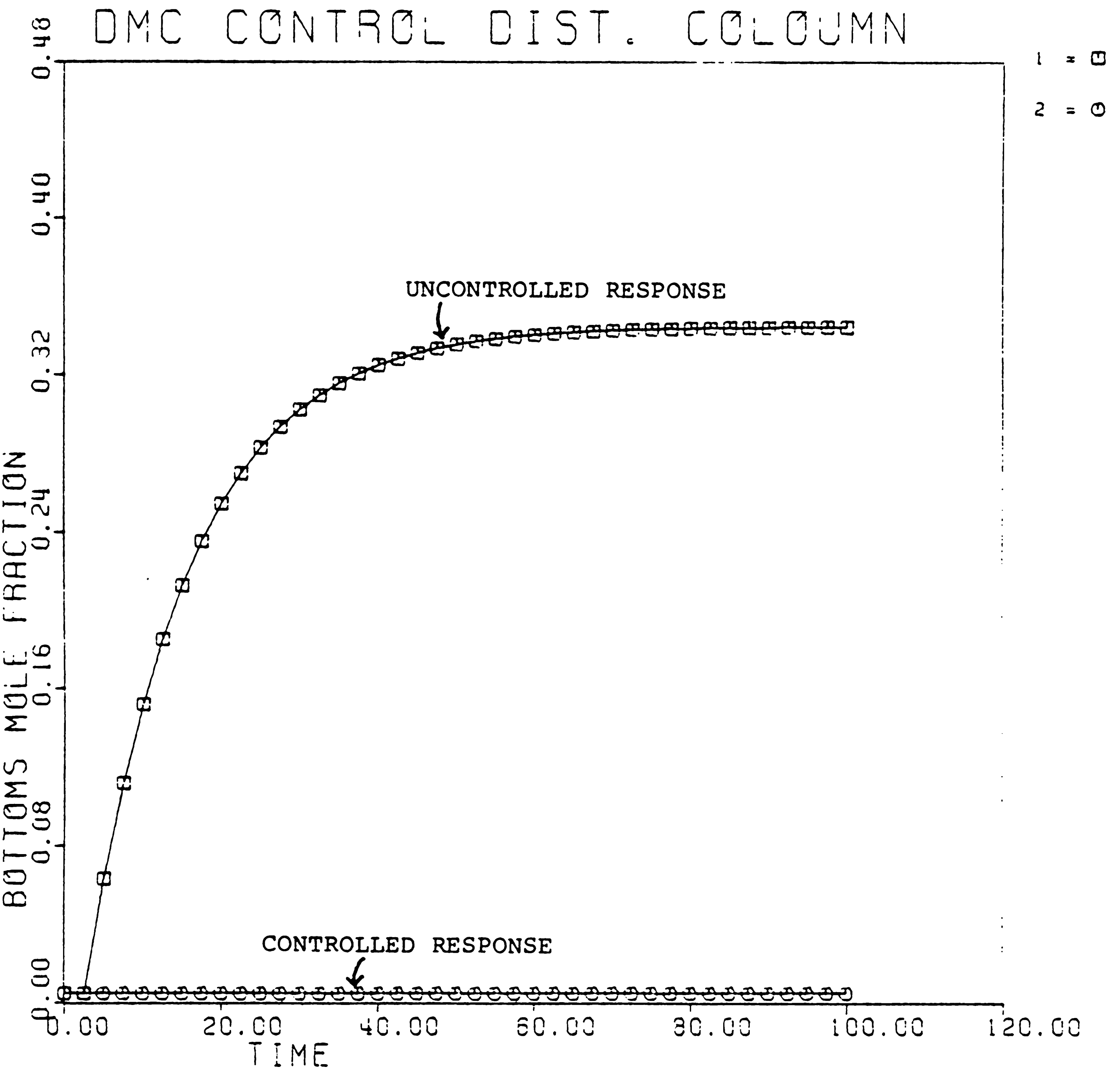

DMC CONTROL DIST. COLOUMN
1 = ⊟
2 = ⊖
UNCONTROLLED RESPONSE
CONTROLLED RESPONSE
BOTTOMS MOLE FRACTION
0.00
0.08
0.16
0.24
0.32
0.40
0.48
0.00
20.00
40.00
60.00
80.00
100.00
120.00
TIME

Figure (5-6).

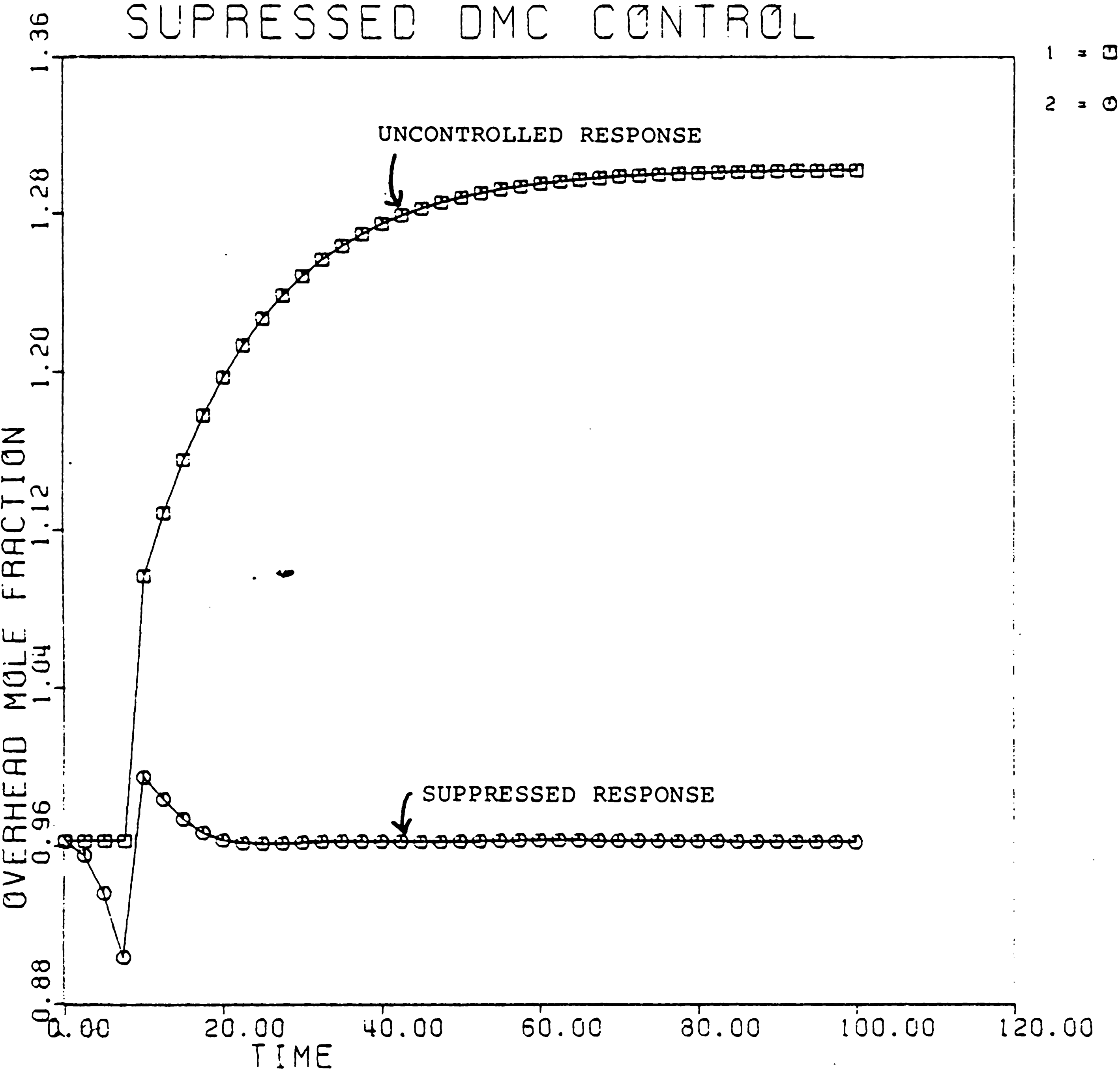

SUPRESSED DMC CONTROL
1 = □
2 = ○
UNCONTROLLED RESPONSE
SUPPRESSED RESPONSE
OVERHEAD MOLE FRACTION
0.88
0.96
1.04
1.12
1.20
1.28
1.36
0.00
20.00
40.00
60.00
80.00
100.00
120.00
TIME

Figure (5-7).

Figures (5-8) and (5-9) show the responses of the overhead and bottoms mole fraction to a disturbance of (.34 lb/min) using conventional control techniques. Comparison of Figure (5-5) to Figure (5-10) shows the DMC Algorithm give better results.

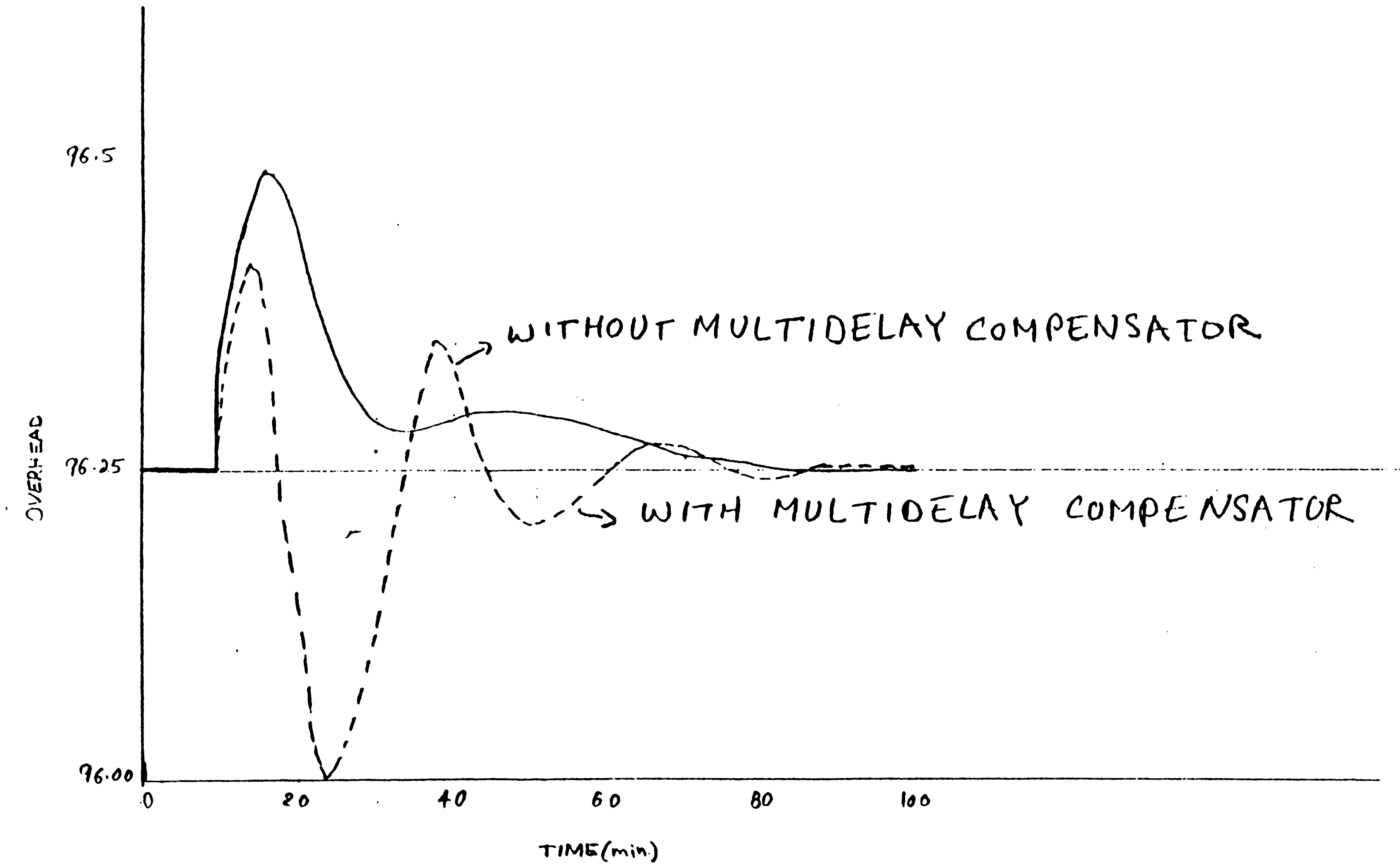


Figure (5-8). Response of overhead mole fraction to (.34 lb/min) disturbance using PI control.

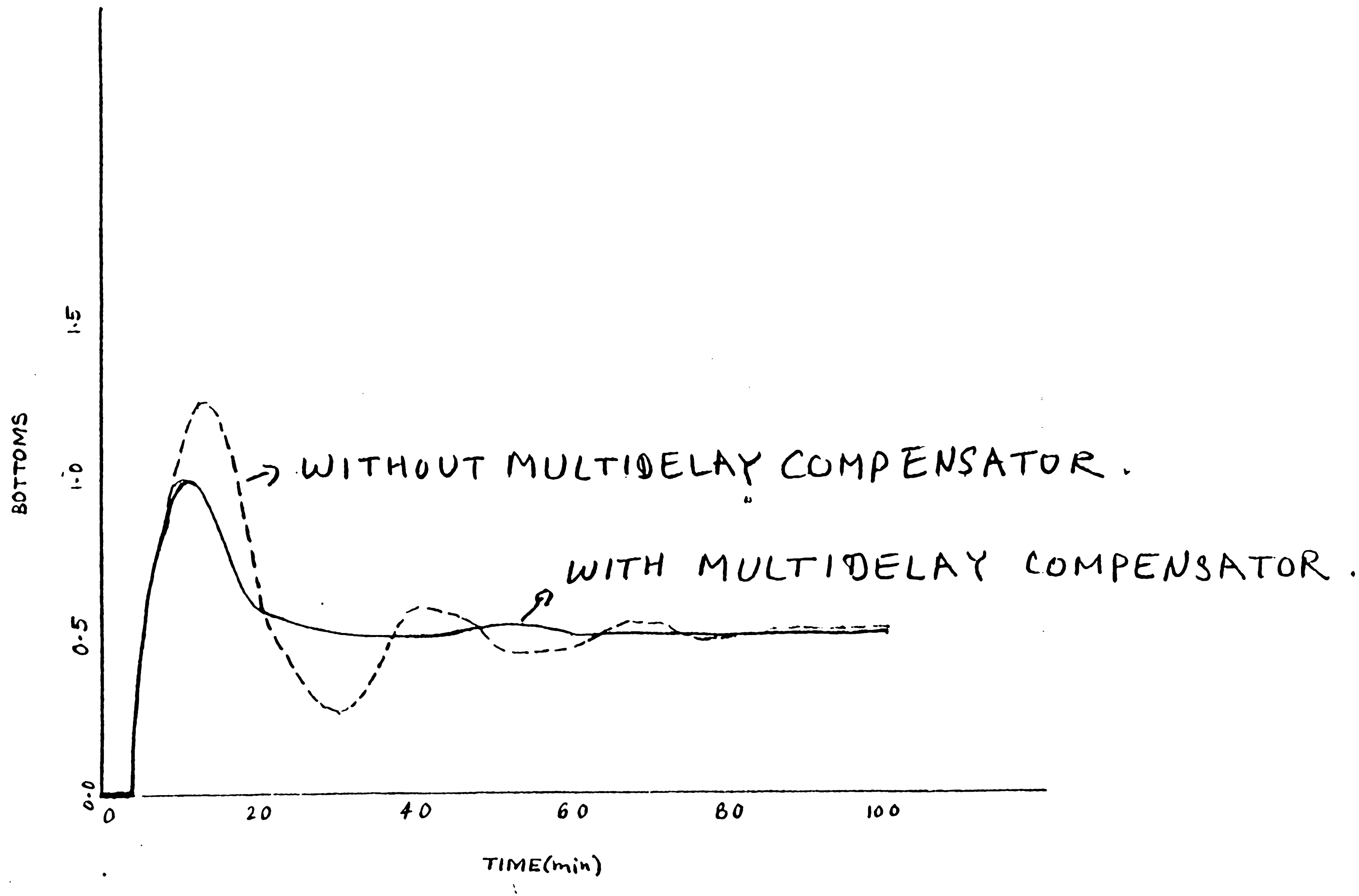


Figure (5-9). Response of bottoms mole fraction to (.34 lb/min) disturbance using PI control.

CHAPTER VI

PLACE IN THEORY

Feedback and feedforward control methods have been employed in a wide range of industrial applications. Feedback (FB) control may be described as the use of deviation of the controlled variable from a certain desired value to cause a corrective manipulated input to be applied to the system. The correction by the manipulative moves are not felt by the system until the new conditions pass through the whole loop, i.e., through the lag of each element.

On the other hand, in feedforward (FF) technique the manipulated variable move is based on the measurement of the disturbance input. The magnitude of the measured disturbance is used to predict the manipulative action that will tend to keep the output as close as possible to the desired value before any error is detected in the loop. Once the manipulated input is applied to the system, there is no chance for correcting the input.

While each of the techniques has some advantages, a combination of them (FF/FB) seems much more desirable than just a single one (Smith, 1972). In practice the inclusion of (FF) is encountered by some difficulties. Each disturbance and manipulated variable must be predictable qualitatively and quantitatively. Also some control equipment must be associated with each of the measured disturbances.

The DMC Algorithm uses a combined (FF/FB) control for multivariable systems. The numerical coefficients in A and $\underline{b}$ (of sec. 2.2-A) give an accurate representation of the dynamics of the system without a deep knowledge of the process itself. In the DMC Algorithm, the design of the control equipment depends only on the measures of the disturbance and not its type. So, in contrast to the conventional FF control, only one piece of control equipment is needed for all the measured disturbances. Moreover, the numerical representation of the dynamics of the system makes the deadtime and other unusual dynamic behavior more manageable. This fact was seen in the performance of the DMC Algorithm on the distillation column.

CHAPTER VII

RECOMMENDATIONS

(1) In this work, the DMC Algorithm was considered for multivariable systems which were essentially unconstrained. Further investigation should consider the effects of constraints on the system variables as suggested by Prett and Gillette (1979).

(2) The number of time intervals for which the output and input variables were chosen arbitrarily in this work. More investigation for the optimal dimensions of the dynamic matrix A should be considered.

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APPENDIX A

TRANSFER FUNCTION MODEL FOR THE BINARY DISTILLATION COLUMN

A transfer function model for the binary distillation column was given by equations (4-1) to (4-3). Substituting (4-2) and (4-3) into (4-1):

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} \begin{bmatrix} \frac{12.8e^{-s}}{16.7s + 1} & \frac{-18.9\ e^{-3s}}{21.0s + 1} \\ \frac{6.6\ e^{-7s}}{10.9s + 1} & \frac{-19.4\ e^{-3s}}{14.4s + 1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} + \begin{bmatrix} \frac{3.8\ e^{-8.1s}}{14.9s + 1} \\ \frac{4.9\ e^{-3.4s}}{13.2s + 1} \end{bmatrix} \cdot d(s) \quad (A-1)$$

(A-1) can be resolved into:

$$\left.\begin{aligned} y_1(s) &= \frac{12.8\ e^{-s}}{16.7s + 1} u_1(s) - \frac{18.9\ e^{-3s}}{21.0s + 1} u_2(s) + \frac{3.8\ e^{-8.1s}}{14.9s + 1} d(s) \\ y_2(s) &= \frac{6.6\ e^{-7s}}{10.9s + 1} u_1(s) - \frac{19.4\ e^{-3s}}{14.4s + 1} u_2(s) + \frac{4.9\ e^{-3.4s}}{13.2s + 1} d(s) \end{aligned}\right\} (A-2)$$

Choose the state variables x_1, x_2, x_3, x_4, x_{L_1} and x_{L_2}; such that:

a. $x_1(s) = \frac{12.8\ e^{-s}}{16.7s + 1} u_1(s)$

$\Leftrightarrow 16.7s\ x_1(s) + x_1(s) = 12.8\ e^{-s}\ u_1(s)$

$\Leftrightarrow sx_1(s) = -0.06\ x_1(s) + 0.077\ e^{-s}\ u_1(s)$.

Take inverse Laplace transform:

$$\dot{x}_1(t) = -0.06\ x_1(t) + 0.77\ u_1(t-1). \qquad \text{(i)}$$

Similarly the other state variables are:

b. $$x_2(s) = \frac{18.9\ e^{-3s}}{21.0s + 1}\ u_2(s)$$

$$\leftrightarrows\ \dot{x}_2(t) = -0.05\ x_2(t) + 0.90\ u_2(t-3). \qquad \text{(ii)}$$

c. $$x_3(s) = \frac{6.6\ e^{-7s}}{1.09s + 1}\ u_1(s)$$

$$\leftrightarrows\ \dot{x}_3(t) = 0.09\ x_3(t) + 0.61\ u_1(t-7). \qquad \text{(iii)}$$

d. $$x_4(5) = \frac{19.4\ e^{-3s}}{14.4s + 1}\ u_2(s)$$

$$\leftrightarrows\ \dot{x}_4(t) = 0.07\ x_4(t) + 1.35\ u_2(t-3). \qquad \text{(iv)}$$

e. $$x_{L_1}(s) = \frac{3.8\ e^{-8.1s}}{14.9s + 1}\ d(s)$$

$$\leftrightarrows\ \dot{x}_{L_1} = -0.07\ x_{L_1}(t) + 0.26\ d(t-8.1). \qquad \text{(v)}$$

f. $$x_{L_2}(s) = \frac{4.9\ e^{-3.4s}}{13.2s + 1}\ d(s)$$

$$\leftrightarrows\ x_{L_2}(t) = -0.08\ x_{L_2}(t) + 0.37\ d(t-3.4). \qquad \text{(vi)}$$

Notice that equations (i) to (iv) constitute equation (4-4) and equations (v) and (vi) constitute equation (4-5).

Equation (**):

$$y_1(s) = x_1(s) - x_2(s) + x_{L_1}(s)$$

$$y_2(s) = x_3(s) - x_4(s) + x_{L_2}(s).$$

In time domain (A-2) becomes:

$$y_1(t) = X_1(t) - X_2(t) + X_{L_1}(t). \qquad \text{(vii)}$$

$$y_2(t) = X_3(t) = x_4(t) + X_{L_2}(t). \qquad \text{(viii)}$$

Equations (vii) and (viii) constitute (4-6).

APPENDIX B

DYNAMIC REPRESENTATION

The linear, stationary system behavior is described by (Koppel, 1967):

$$\frac{d\underline{x}}{dt} = A\ \underline{x} + B\ \underline{I} \tag{B-1}$$

where $\underline{x}$ is a n-dimensional state vector, $\underline{I}$ is an r-dimensional control vector, A is a n×n system matrix and B is a r× control matrix.

For sampled-data systems equation (B-1) can be written as:

$$\underline{x}(t) = \phi(t-t_K)\underline{x}(t_K) + \Delta(t-t_K)I(t_K);\ t_K < t \leq t_K \tag{B-2}$$

where the transition matrix $\phi(t) = e^{At}$ and matrix $\Delta(t)$ is defined by

$$\Delta(t) = \int_0^t \phi(\tau)Bd\tau \tag{B-3}$$

The behavior of the system at the sampling instants is described by

$$\underline{x}_{K+1} = \phi(T)\underline{x}_K + \Delta(\tau)I_{\underline{K}} \tag{B-4}$$

where T is the sampling period and $\underline{x}_K$ is identical to $\underline{x}(t_K)$. The solution of equation (B-4) is given by

$$\underline{x}_K = \phi^K \underline{x}_0 + \sum_{i=0}^{K-1} \phi^{K-i-1} \Delta \cdot \underline{I} \qquad \text{(B-5)}$$

where $\underline{x}_0$ denotes the initial conditions of the state vector $\underline{x}_K$.

To get the coefficient representation employed by DMC Algorithm, suppose that

$$\underline{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix},$$

where I_1 is a manipulated input and I_2 is a disturbance. Introduce the identity matrix equation (B-5)

$$\underline{x}_K = \phi^K \underline{x}_0 + \sum_{i=1}^{K-1} \phi^{K-i-1} \cdot \Delta \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\underline{x}_K - \phi^K \underline{x}_0 = \sum_{i=1}^{K-1} \phi^{K-i-1} \cdot \Delta \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} I_2 \right\} \qquad \text{(B-6)}$$

Assume that the final desired value $\underline{x}_K = 0$, and let

$$a_K = \sum_{i=0}^{K-1} \phi^{K-i-1} \cdot \Delta \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } b_K = \sum_{i=0}^{K-1} \phi^{K-i-1} \cdot \Delta \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

then equation (B-6) becomes:

$$\phi^K \cdot \underline{x}_0 + b_K I_2 = -a_K \cdot I_1 \qquad \text{(B-7)}$$

Equation (B-7) has the same form as that of equation (2-1).

In general if $\underline{I}$ is an $(I+n)^{th}$ vector with I manipulated inputs and n disturbances, i.e.,

$$\underline{I} = [I_1 \; I_2 \cdots I_I \; d_{I+1} \; d_{I+2} \cdots d_{I+n}]^T$$

The coefficient vectors $\underline{a}_K$ and $\underline{b}_K$ are given by

$$\underline{a}_{1K} = \sum_{i=0}^{K-1} \phi^{K-i-1} \cdot \Delta \cdot [\underbrace{1 \; 0 \cdots 0}_{I} \quad \underbrace{0 \cdots 0}_{n}]^T$$

$$\underline{a}_{2K} = \sum_{i=0}^{K-1} \phi^{K-i-1} \cdot \Delta \cdot [\underbrace{0 \; 1 \cdots 0}_{I} \quad \underbrace{0 \cdots 0}_{n}]^T$$

$$\vdots$$

$$\underline{a}_{IK} = \sum_{i=0}^{K-1} \phi^{K-i-1} \cdot \Delta \cdot [\underbrace{0 \cdots 1}_{I} \quad \underbrace{0 \cdots 0}_{n}]^T$$

$$\underline{b}_{1K} = \sum_{i=0}^{K-1} \phi^{K-i-1} \cdot \Delta \cdot [\underbrace{0 \cdots 0}_{I} \quad \underbrace{1 \cdots 0}_{n}]^T$$

$$\vdots$$

$$\underline{b}_{nK} = \sum_{i=0}^{K-1} \phi^{K-i-1} \cdot \Delta \cdot [\underbrace{0 \cdots 0}_{I} \quad \underbrace{0 \cdots 1}_{n}]$$

and

$$\phi^K \underline{x}_0 + \underline{b}_{1K} \, d_{I+1} + \cdots + b_{nK} \, d_{I+n} =$$

$$a_{1K} \, I_1 + \cdots + a_{nK} \, I_I \, .$$

APPENDIX C

SAMPLE COMPUTER PROGRAM

```
LEVEL 2.3.0 ( JUNE 78)                OS/360  FORTRAN H EXTENDED                DATE 80.318/10.16.03        PAGE   1
REQUESTED OPTIONS: AUTODBL(DBLPAD)
OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(MAX) AUTODBL(DBLPAD)
                   SOURCE EBCDIC NOLIST NODECK OBJECT NOMAP NOFORMAT GOSTMT NOXREF ALC NOANSF NOTERM IBM FLAG(I)

              C---------------------------------------------------------------
              C                    PROGRAM NO. 3                               |
              C     THIS PROGRAM ILLUSTRATES THE APPLICATION OF DMC ALGORITHIM |
              C     ON A MULTIVARIABLE MULTIDELAY SYSTEM (BINARY DISTILLATION  |
              C     COLUMN).THE SYSTEM HAS:                                    |
              C       TWO INTER ACTING OUTPUTS TO BE CONTROLLED:               |
              C          Y1=OVERHEAD MOLE FRACTION METHANOL,                   |
              C          Y2=BOTTOMS MOLE FRACTION METHANOL.                    |
              C       TWO MANIPULATED INPUTS TO CONTROL THE OUTPUTS:           |
              C          U1=OVERHEAD REFLUX FLOW RATE .                        |
              C          U2=BOTTOMS STEAM FLOW RATE.                           |
              C       AND ONE DISTURBANCE INPUT:                               |
              C          D=COLUMN FEED FLOW RATE.                              |
              C---------------------------------------------------------------
              C---------------------------------------------------------------
              C     SUBROUTINES USED:                                          |
              C     NAME         PURPOSE                                       |
              C     LEQT2P       SOLUTION OF LINEAR EQUATIONSWITH POSITIVE     |
              C                  DEFINITE MATRICES-SYMMETRIC STORAGE MODE.     |
              C     MULT         MATRIX MULTIPLICATION.                        |
              C     VCVSTF       MATRIX STORAGE MODE MODE CONVERSION.          |
              C                  (FULL TO SYMMETRIC).                          |
              C---------------------------------------------------------------
              C
ISN 0002            DIMENSION B(60,1),AA(4020),WKAREA(4000),
                   $A(80,60),AT(60,80),AP(60,60),AI(60,60),
                   $DD(80,1),DI(60,1),DE(80,1),
                   $DD1(40),DD2(40),DI1(40),DI2(40),DI3(40),
                   $U11(40),U13(40),U1(40),U2(40),T(40),
                   $Y1(40,10),Y2(40,10),
                   $B11(80,1),B21(80,1),B12(80,1),B22(80,1),DR(80,1)
              C
              C *** INITIALIZE THE VARIABLES
              C
ISN 0003            DATA DD1/40*0./,DD2/40*0./
ISN 0004            DATA DIST1,DIST2/2*0.0/
              C
              C *** READ IN COEFFICIENT MATRIX A AND VECTORS B
              C
ISN 0005            READ(5,100)((A(I,J),I=1,80),J=1,60)
ISN 0006            READ(5,110)(B11(I,1),B21(I,1),I=1,40)
ISN 0007            READ(5,110)(B12(I,1),B22(I,1),I=1,40)
              C
              C *** PROJECTED OUTPUT DUE TO A POSITIVE DISTURBANCE
              C
ISN 0008            DO 1 I=1,40
ISN 0009            IF(I.GT.1)DIST2=.34
ISN 0011            IF(I.GT.3)DIST1=.34
ISN 0013            DD1(I)=DD1(I)+B11(I,1)*DIST1+B21(I,1)*DIST2
ISN 0014            DD2(I)=DD2(I)+B12(I,1)*DIST1+B22(I,1)*DIST2
ISN 0015            DD(I,1)=DD1(I)
ISN 0016          1 DD(I+40,1)=DD2(I)
              C
```

```
LEVEL 2.3.0 ( JUNE 78)          MAIN          GS/360   FORTRAN H EXTENDED          DATE 80.318/10.16.03          PAGE   2
                C *** SOLVE EQUATION (AT*A*DO=-AT*DI)                                     00000580
                C                                                                         00000590
ISN 0017              DO 2 I=1,80                                                         00000600
ISN 0018              DO 2 J=1,60                                                         00000610
ISN 0019            2 AT(J,I)=A(I,J)                                                      00000620
ISN 0020              CALL MULT (AT,QQ,B,60,80,1)                                         00000630
ISN 0021              CALL MULT (AT,A,AP,60,80,60)                                        00000640
ISN 0022              DO 30 I=1,60                                                        00000650
ISN 0023              DO 30 J=1,60                                                        00000660
ISN 0024              IF(I.NE.J)GO TO 29                                                  00000670
ISN 0026              AI(I,J)=1.005*AP(I,J)                                               00000680
ISN 0027              GO TO 30                                                            00000690
ISN 0028           29 AI(I,J)=AP(I,J)                                                     00000700
ISN 0029           30 CONTINUE                                                            00000710
ISN 0030              CALL VCVTFS (AI,CC,60,AA)                                           00000720
ISN 0031              CALL LEQT2P (AA,1,60,60,B,IDGT,DI,D2,WKAREA,IER)                    00000730
ISN 0032              DO 3 I=1,60                                                         00000740
ISN 0033            3 DI(I,1)=-B(I,1)                                                     00000750
ISN 0034              CALL MULT (A,DI,CE,80,60,1)                                         00000760
ISN 0035              DO 4 I=1,80                                                         00000770
ISN 0036            4 DR(I,1)=DC(I,1)+DE(I,1)                                             00000780
ISN 0037              DO 5 I=1,40                                                         00000790
ISN 0038              DO1(I)=CO(I,1)                                                      00000800
ISN 0039              Y1(I,2)=.9625 +DO1(I)                                               00000810
ISN 0040              Y1(I,1)=.9625 +DR(I,1)                                              00000820
ISN 0041              DO2(I)=QQ(I+40,1)                                                   00000830
ISN 0042              Y2(I,2)=DO2(I)+.0050                                                00000840
ISN 0043            5 Y2(I,1)=DR(I+40,1)+0.0050                                           00000850
ISN 0044              DO 6 I=1,20                                                         00000860
ISN 0045              DI1(I)=DI(I,1)                                                      00000870
ISN 0046              DI2(I)=DI(I+20,1)                                                   00000880
ISN 0047            6 DI3(I)=DI(I+40,1)                                                   00000890
ISN 0048              DO 7 I=21,40                                                        00000900
ISN 0049              DI1(I)=0.                                                           00000910
ISN 0050              DI2(I)=0.                                                           00000920
ISN 0051            7 DI3(I)=0.                                                           00000930
ISN 0052              U11(1)=0.                                                           00000940
ISN 0053              U2(1)=0.                                                            00000950
ISN 0054              DO 9 N=1,3                                                          00000960
ISN 0055            9 U13(N)=0.                                                           00000970
ISN 0056              DO 10 N=2,40                                                        00000980
ISN 0057           10 U11(N)=DI1(N-1)                                                     00000990
ISN 0058              DO 11 N=2,40                                                        00001000
ISN 0059           11 U2(N)=DI2(N-1)                                                      00001010
ISN 0060              DO 12 N=4,40                                                        00001020
ISN 0061           12 U13(N)=DI3(N-3)                                                     00001030
ISN 0062              DO 13 N=1,40                                                        00001040
ISN 0063           13 U1(N)=U11(N)+U13(N)                                                 00001050
ISN 0064              DO 14 I=1,40                                                        00001060
ISN 0065           14 T(I)=FLOAT(I)*2.5                                                   00001070
ISN 0066              WRITE(6,220)                                                        00001080
ISN 0067              DO 15 I=1,40                                                        00001090
ISN 0068           15 WRITE(6,210)T(I),Y1(I,2),Y2(I,2),Y1(I,1),Y2(I,1),DI1(I),DI2(I),DI300001100
                     $(I),U1(I),U2(I)                                                     00001110
                C                                                                         00001120
                C *** FORMAT OF THE OUTPUT                                                00001130
ISN 0069          100 FORMAT(10F5.4)                                                      00001140
ISN 0070          110 FORMAT(2F5.4)                                                       00001150
```

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LEVEL 2.3.0 (JUNE 78)            MAIN            OS/360  FORTRAN H EXTENDED          DATE 80.318/10.16.03        PAGE   3
                    C                                                                                00001160
   ISN 0071           200 FORMAT(///)                                                                00001170
   ISN 0072           210 FORMAT(' ',4X,F5.1,9F12.4)                                                 00001190
   ISN 0073           220 FORMAT('0',5X,'TIME',4X,'UNCONT Y1',3X,'UNCONT Y2',5X,'CONT Y1'           00001200
                          $,5X,'CONT Y2',5X,'U1(T-1)',5X,'U2(T-3)',5X,'U1(T-7)',5X'U1',8X,'U200001210
                          $ ')                                                                       00001220
   ISN 0074           245 FORMAT(1H0,2F12.8)                                                         00001230
   ISN 0075           300 FORMAT('1',///)                                                            00001240
   ISN 0076               STOP                                                                       00001260
   ISN 0077               END                                                                        00001270
*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(MAX) AUTODBL(DBLPAD)
*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK OBJECT NOMAP NOFORMAT GOSTMT NOXREF ALC NOANSF NOTERM IBM FLAG(I)
*STATISTICS*     SOURCE STATEMENTS =      76, PROGRAM SIZE =    216736, SUBPROGRAM NAME =  MAIN
*STATISTICS*  NO  DIAGNOSTICS GENERATED
****** END OF COMPILATION ******                                          164K BYTES OF CORE NOT USED
```

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LEVEL 2.3.0 (JUNE 78)                    OS/360  FORTRAN H EXTENDED          DATE 80.318/10.16.05       PAGE   1

REQUESTED OPTIONS: AUTODBL(DBLPAD)

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(MAX) AUTODBL(DBLPAD)
                   SOURCE EBCDIC NOLIST NODECK OBJECT NOMAP NOFORMAT GOSTMT NOXREF ALC NOANSF NOTERM IBM FLAG(I)

   ISN 0002          SUBROUTINE MULT (A,B,C,I,J,K)                                       00001280
   ISN 0003          REAL A(I,J),B(J,K),C(I,K)                                           00001290
   ISN 0004          DO 10 NI=1,I                                                        00001300
   ISN 0005          DO 10 NK=1,K                                                        00001310
   ISN 0006          C(NI,NK)=0.                                                         00001320
   ISN 0007          DO 10 NJ=1,J                                                        00001330
   ISN 0008       10 C(NI,NK)=C(NI,NK)+A(NI,NJ)*B(NJ,NK)                                 00001340
   ISN 0009          RETURN                                                              00001350
   ISN 0010          END                                                                 00001360

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(MAX) AUTODBL(DBLPAD)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK OBJECT NOMAP NOFORMAT GOSTMT NOXREF ALC NOANSF NOTERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =        9, PROGRAM SIZE =       720, SUBPROGRAM NAME =  MULT

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                 188K BYTES OF CORE NOT USED

*STATISTICS*  NO  DIAGNOSTICS THIS STEP
```

VITA

Zaki Amin Shanaa was born at Sidon, Lebanon, on [redacted].

Zaki received his elementary and preparatory education at (Ein-El-Heloueh Camp) Sidon, Lebanon. He attended Sidon schools until graduation from Sidon Official High School in June 1975. In August 1975 he left Lebanon for the United States for higher studies. He attended Winona State University (Minnesota) for one year of pre-engineering. He transferred to Iowa State University from which he received a B.S. in Chemical Engineering. The degree was received in May, 1979.

In June, 1979, he entered the Department of Chemical Engineering at the University of Missouri-Columbia, to pursue advanced degrees, and worked as a Teaching Assistant in the Department of Chemical Engineering and the Mathematics Department.

The undersigned, appointed by the Dean of the Graduate Faculty, have examined a thesis entitled

INVESTIGATION OF DYNAMIC MATRIX CONTROL

presented by Zaki A. Shanaa

a candidate for the degree of Master of Science in Chemical Engineering

and hereby certify that in their opinion it is worthy of acceptance.

[redacted]