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A STUDY
OF THE SPECTROSCOPIC BINARY α ANDROMEDAE

by

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SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS

in the

GRADUATE DIVISION
(COLLEGE OF ARTS AND SCIENCE)

of the

UNIVERSITY OF MISSOURI

1912

Spectrum analysis may perhaps be said to date from the year 1666 when Sir Isaac Newton discovered that rays of light of different colors are of different refrangibility. Later he discovered that a beam of white light coming from a small round opening and passed through a prism would be spread out into a band of color. Fraunhofer in 1823 substituted for the round hole a narrow slit, and found that in the case of the sun the band of color was crossed by many fine dark lines. He also turned the instrument on some of the brighter stars, and found their spectra in general different from that of the sun. For example some showed bright lines. But the true explanation of the bright and dark lines remained a mystery until the announcement in 1859 of Kirchhoff's famous law.

The law of Kirchhoff, briefly stated, is that a substance absorbs exactly those wave lengths of light which under the same conditions it emits. The sodium flame gives as its spectrum two bright yellow lines. A beam of white light passed through

the flame will give a spectrum crossed by two dark lines at exactly the position of the yellow sodium lines. Since each element has its own characteristic spectrum we can analyze a substance by studying the light it emits. This is one of the most delicate tests known to the chemist. And since the distance of the source of light appears to make no difference we can study not only the laboratory spark, but also the sun. Or we can attach our spectroscope to a powerful telescope and photograph the spectrum of a distant star.

Another important discovery was that of the Doppler principle, in 1842. This principle as developed by ^[Fizeau] states that if a source of light is receding from the spectroscope, the lines will be displaced toward the red end of the spectrum. If the source is approaching, the displacement will be toward the violet. As the amount of displacement is proportional to the velocity of the source, the instrument can be used for the determination of velocity of approach or recession.

The pioneers in spectroscopic analysis and classification were Huggins, Secchi, Donati, and Rutherford. The work of Secchi and Huggins has been especially valuable. In 1868 Huggins first applied the Doppler principle in his work, and began the determination of radial - line of sight - velocities of the stars. As instruments improved the determination of these velocities opened up a new and unexpected field for investigation - that of spectroscopic binaries.

A binary star is a system of two stars revolving about one another, or more accurately about their common center of mass. If we are able to see the two components separately in the telescope we term it a visual double. If we can detect the duplicity with the spectroscope only it is called a spectroscopic double. If the plane of the orbit is not exactly perpendicular to the line of sight the two stars should alternately approach and recede from us with respect to their center of mass. Further, when the one star is approaching us most rapidly the other will be receding, and in consequence one spectrum will be

displaced toward the blue, the other toward the red. The lines will at this time appear double provided both stars give measurable spectra. When the two stars are moving across the line of sight neither will have its lines displaced and the spectrum of the double will appear normal. In case only one star is bright enough to give a noticeable spectrum the lines will not appear double at intervals, but will appear to oscillate back and forth. The measured velocity will vary.

The first spectroscopic binary discovered was announced by Pickering of Harvard in 1889. It was Mizar, the well known star at the bend of the handle of the Dipper, and was detected by the regular doubling of the lines of its spectrum. Mizar illustrates the first method of detecting as doubles stars which are telescopically single. Another binary of this character, Algol, illustrates the other method. In 1889 Vogel of the Potsdam observatory found a regular variation from rapid approach to rapid recession in the velocity of this famous variable. His observations confirmed the theory that Algol

has a dark satellite and that the falling off at regular intervals of the light is an eclipsing of the bright component by this body. He could detect no trace of the spectrum of the companion. After these first two*, two more were very soon announced one at Harvard and one at Potsdam. As time went on other observatories took up the work, slowly at first and later more rapidly until now there are several hundred such systems known.

The honor of devising a method and computing the first stellar orbit from spectroscopic measures belongs to Arthur A. Rambaut of the Dunsink observatory. Professor Pickering in the Harvard Bulletin published a graph, the time as abscissas and the measured velocities as ordinates, to show how the observations on β Aurigae all lay very nearly on a smooth sine curve drawn through them. The observations plotted were all taken in the month of December 1889. Taking the velocity determinations from this figure Mr. Rambaut computed the orbit the one component of β Aurigae describes with respect to the other.

Note *Algol was not one of the first two binaries announced. It was probably third or fourth.

Having only a few measures, and these not the original determinations but scaled from Professor Pickering's figure, his orbit was not exceedingly accurate. However it illustrated his method.

The method of Rambaut has naturally been improved upon considerably by later investigators. Among the more important papers on the subject are articles by Wilsing, Lehmann-Filhés, Schwarzschild, Zuhellen and Russell. A valuable discussion of the different methods by Dr. H. D. Curtis of the Lick Observatory will be found in the Publications of the Astronomical Society of the Pacific, Volume XX page 133 and following. The method of Wilsing is analytical. Russell's method is merely an extension of that of Wilsing to make it more general. The other methods are largely graphical. The analytical methods should give better results in certain special cases. They may obtain a solution where the graphical methods would fail utterly (See Russell's paper). But ordinarily the graphical processes are as accurate and easier. Of these the one which appears to be

most generally satisfactory and is now used by most computers is that of Lehmann-Filhes'. This method will be used in the derivations made in what follows.

That the radial velocity of the bright star Alpheratz, α Andromedae, is variable was discovered by V. M. Slipher of the Lowell Observatory in 1903, and announced in the Lowell Observatory Bulletin No. 11 July 1904. The variable radial velocity was independently discovered by H. D. Curtis at the Lick Observatory about the same time, but the Lowell announcement preceded the Lick. In November 1907 Dr. H. Ludendorff of the Potsdam Observatory, using the Lowell and Lick measures and a series of his own, published a set of provisional elements for the orbit of the star. In April 1908 using the same measures and in addition a further series of his own observations Dr. Ludendorff published his final definitive orbit. The elements were derived by the method of Lehmann-Filhes'. Just before his final orbit appeared, Dr. R. H. Baker then at the Allegheny Observatory of Pittsburgh published his elements for

the orbit of the star. Dr. Baker used the early Lowell and Lick measures to determine the period, but in the final determination of the other elements used only the Allegheny observations from August 1907 to February 1908 inclusive. The orbit was derived by the method of Lehmann-Filhés and adjusted by the method of least squares.

Dr. Ludendorff used in the determination the Lowell, Lick, and Potsdam measures. Rejecting those earlier than 1903 there remain sixty-seven extending from 1903 to 1908. The early observations are rejected because an error in the assumed period would have too great an effect on them. These will be discussed for purpose of comparison with the Allegheny observations. The period was first determined by a systematic comparison of early observations with late of about the same value. Observations at a time when the velocity of the star was changing rapidly were of most value here. A period of 96.7 days appeared quite satisfactory. The next step was to ascertain whether any systematic difference ~~is~~

in the measures of the different observers existed. To do this the observations were all reduced to the period of one revolution and plotted on the velocity curve from Dr. Baker's elements. A comparison of the residuals showed that to combine the Lowell observations with the Potsdam a constant correction of $+1.5$ km. should be added to the Lowell. No correction was necessary in the case of the Lick measures. The Lowell observations are corrected by this amount in the following

TABLE OF OBSERVATIONS

Date G.M.T	Phase	Velocity	Observatory
1904 Feb. 17.7	0.1ds.	-3.5 Km.	Lowell
1908 Feb. 9.2	1.9	-17	Potsdam
1904 Dec. 6.2	2.3	-31	"
1908 Feb. 10.2	2.9	-22	"
1904 Dec. 9.3	5.4	-30	"
1904 Dec. 12.2	8.3	-41	"
1904 Dec. 13.2	9.3	-18	"
1908 Feb. 19.3	11.9	-34	"
1904 Feb. 29.7	12.1	-35.5	Lowell

TABLE OF OBSERVATIONS (Cont.)

Date G.M.T	Phase	Velocity	Observatory
1904 Dec. 16.2	12.3 <i>ds.</i>	-28 Km.	Potsdam
1903 Nov. 25.7	12.8	-40.5	Lowell
1903 Nov. 26.7	13.8	-38.5	"
1904 Mar. 4.7	16.1	-43.5	"
1903 Nov. 30.7	17.8	-36	Lick
1903 Nov. 30.7	17.8	-35	"
1904 Mar. 6.7	18.1	-43.5	Lowell
1903 Dec. 1.7	18.8	-32.5	"
1908 Feb. 28.2	20.9	-43	Potsdam
1904 Sept. 19.4	21.2	-28	"
1904 Sept. 19.4	21.2	-26	"
1906 Nov. 2.3	21.5	-38	"
1904 Sept. 20.3	22.2	-23	"
1906 Nov. 6.3	25.5	-33	"
1904 Sept. 24.3	26.2	-28	"
1904 Sept. 25.3	27.2	-21	"
1906 Nov. 8.4	27.6	-33	"
1906 Nov. 9.3	28.5	-34	"

TABLE OF OBSERVATIONS (Cont.)

Date G.M.T	Phase	Velocity	Observatory
1907 Dec. 1.3	28.7ds.	-29 Km.	Potsdam
1904 Sept. 28.3	30.2	-30	"
1907 Dec. 4.3	31.7	-41	"
1903 Dec. 14.7	31.8	-25.5	Lowell
1904 Sept. 30.3	32.2	-17	Potsdam
1907 Dec. 6.4	33.8	-16	"
1906 Nov. 15.3	34.5	-22	"
1904 Oct. 4.3	36.2	-26	"
1903 Dec. 19.7	36.8	-22.5	Lowell
1903 Dec. 21.7	38.8	-28	Lick
1903 Dec. 21.7	38.8	-26	"
1904 Oct. 8.3	40.2	-21	Potsdam
1906 Nov. 23.3	42.5	-25	"
1905 Jan. 16.2	43.3	-15	"
1906 Nov. 24.2	43.4	-26	"
1905 Jan. 17.2	44.3	-26	"
1904 Oct. 13.3	45.2	-18	"
1903 Sept. 22.7	45.5	-26	Lick
1905 Jan. 20.2	47.3	-23	Potsdam

TABLE OF OBSERVATIONS (Cont.)

Date G.M.T.	Phase	Velocity	Observatory
1904 Oct. 16.3	48.2 d_s	-18 Km.	Potsdam
1905 Jan. 21.2	48.3	- 8	"
1906 Dec. 1.2	50.4	-23	"
1903 Sept.27.7	50.5	- 8	Lick
1903 Sept.27.7	50.5	-10	"
1904 Oct. 19.2	51.1	-18	Potsdam
1906 Dec. 7.2	56.4	-13	"
1906 Dec. 8.2	57.4	-10	"
1903 Oct. 5.7	58.5	- 2	Lick
1907 Dec. 31.2	58.6	- 6	Potsdam
1908 Jan. 1.3	59.7	-14	"
1908 Jan. 2.2	60.6	-10	"
1903 Oct. 12.7	65.5	- 6	Lick
1903 Oct. 12.7	65.5	- 4	"
1906 Dec. 22.2	71.4	- 6	Potsdam
1904 Nov. 10.4	73.2	- 6	"
1908 Jan. 21.2	79.6	0	"
1903 Jan. 18.3	88.0	+19	"
1904 Feb. 10.7	89.8	+17.5	Lowell

TABLE OF OBSERVATIONS (Cont.)

Date G.M.T.	Phase	Velocity	Observatory
1904 Feb. 11.7	90.8 ds.	+21.5 Km.	Lowell
1907 May 22.7	95.1	+11.5	"

These observations were combined into seventeen normal places, bringing together those of nearly the same phase. The places were weighted according to the number of observations represented. The plates were considered of equal weight since not enough was known about them to justify attempting any other system. The resulting normal places are tabulated below. The phase from periastron is denoted by Φ .

NORMAL PLACES

Φ	Limits of Φ	Observed Velocity	Computed (Ludendorff)
1.00 ds.	0.1-1.9 ds.	-10.25 Km.	- 5.93 Km.
3.53	2.3-5.4	-27.67	-19.87
11.12	8.3-12.8	-32.83	-36.29
17.07	13.8-18.8	-38.17	-36.33
22.08	20.9-25.5	-31.83	-34.33

NORMAL PLACES (Cont.)

Φ	Limits of Φ	Observed Velocity	Computed (Ludendorff)
28.07 ds.	26.2-30.2 ds.	-29.17 Km.	-31.23 Km
33.37	31.7-36.2	-24.58	-28.23
40.07	36.8-43.3	-22.92	-24.30
45.65	43.4-48.2	-22.83	-20.90
50.16	48.3-51.1	-13.40	-18.03
58.53	56.4-60.6	- 9.17	-12.30
65.50	65.5-65.5	- 5.00	- 6.93
72.30	71.4-73.2	- 6.00	- 0.86
79.60	79.6	0.00	+ 6.95
88.00	88.0	+19.00	+16.57
90.30	89.8-90.8	+19.50	+17.64
95.10	95.1	+11.50	+ 8.23

NORMAL PLACES (Cont.)

Φ	O-C Ludendorff	O-C Least Squares	Number Obs.	Weight
1.00 ds.	-4.32 Km.	+1.32 Km.	2	0.33
3.53	-7.80	-1.74	3	0.50
11.12	+3.46	+2.97	6	1.00
17.07	-1.84	-4.07	6	1.00

NORMAL PLACES (Cont.)

Φ	O-C Ludendorff	O-C Least Squares	Number Obs.	Weight
22.08 ds.	+2.50 Km	-0.12 Km.	6	1.00
28.07	+2.06	-0.50	6	1.00
33.37	+3.65	+1.32	6	1.00
40.07	+1.38	-0.54	6	1.00
45.65	-1.93	-3.45	6	1.00
50.16	+4.63	+3.46	5	0.83
58.53	+3.13	+2.67	6	1.00
65.50	+1.93	+2.05	2	0.33
72.30	-5.14	-4.49	2	0.33
79.60	-6.95	-6.05	1	0.17
88.00	+2.43	+1.77	1	0.17
90.30	+1.86	-0.01	2	0.33
95.10	+3.27	+3.60	1	0.17

Assuming for simplicity the A, B, and γ of the Potsdam elements, which seemed to satisfy these points quite well, an orbit was derived by the method of Lehmann-Filhes. The following elements resulted:

$$A = 32.4 \text{ Km.} \quad \gamma = -14.4 \text{ Km} \quad \omega = 67^\circ 42'$$

$$B = 22.1 \text{ Km.} \quad e = 0.50$$

The "e" and ω are nearly those derived by Ludendorff; so his elements are used for the preliminary orbit in the least squares solution which follows. The following elements result:

$$A = 32.4 \text{ Km} \quad P = 96.7 \text{ ds.} \quad e = 0.50$$

$$B = 22.1 \text{ Km} \quad \mu = 0.06498 = 3^\circ.723$$

$$\gamma = -14.4 \text{ Km} \quad \omega = 69^\circ 24' \quad u_1 = 100^\circ 54'$$

$$T = 1904 \text{ Dec. 2.} \quad a \sin i = 31,400,000 \text{ Km}$$

From these elements an ephemeris was computed and the residuals for the normal places formed. Lehmann-Filhes has shown that the equations of condition are of the form

$$\delta \frac{dz}{dt} = (\cos u + e \cos \omega) \delta K + \left[\cos \omega - \frac{\sin u \sin v}{1-e^2} (2 + e \cos v) \right] K \delta e$$

$$- (\sin u + e \sin \omega) K \delta \omega + \sin v (1 + e \cos v)^2 \frac{K \mu \delta T}{(1-e^2)^{3/2}}$$

The period is probably determined better from the early observations than we can get it from this solution. So it has been assumed and the term in $\delta \mu$ omitted. Adding a term in $\delta \gamma$ and putting for simplification $K \delta e = \delta e'$; $K \delta \omega = \delta \omega'$; $\frac{K \mu \delta T}{(1-e^2)^{3/2}} = \delta T'$

we have

$$\delta \frac{dz}{dt} = \delta \gamma + (\cos u + e \cos \omega) \delta K + \left[\cos \omega - \frac{\sin u \sin v}{1 - e^2} (2 + e \cos v) \right] \delta e' - (\sin u + e \sin \omega) \delta \omega' + \sin u (1 + e \cos v)^2 \delta T'$$

Computing the equations of condition for the normal places we have the following

EQUATIONS OF CONDITION

	$\delta \gamma$	$+0.311 \delta K$	$-0.378 \delta e'$	$-1.459 \delta \omega'$	$+2.192 \delta T'$	$=$	Weight
						-4.32	0.33
1	-0.201	-1.632	-1.394	+1.731	$=$	-7.80	0.50
1	-0.803	-0.164	-0.672	+0.174	$=$	+3.46	1.00
1	-0.805	+0.736	-0.273	-0.106	$=$	-1.84	1.00
1	-0.732	+0.998	-0.048	-0.174	$=$	+2.50	1.00
1	-0.618	+1.028	+0.140	-0.201	$=$	+2.06	1.00
1	-0.508	+0.926	+0.262	-0.212	$=$	+3.65	1.00
1	-0.363	+0.705	+0.374	-0.216	$=$	+1.38	1.00
1	-0.238	+0.475	+0.442	-0.229	$=$	-1.93	1.00
1	-0.133	+0.266	+0.483	-0.238	$=$	+4.63	0.83
1	+0.077	-0.167	+0.527	-0.266	$=$	+3.13	1.00

EQUATIONS OF CONDITION (Cont.)

						Weight
$1\delta\gamma + 0.274\delta K - 0.559\delta e' + 0.527\delta\omega' - 0.303\delta T' = +1.93$						0.33
1	+0.497	-0.939	+0.479	-0.356	= -5.14	0.33
1	+0.784	-1.207	+0.326	-0.429	= -6.95	0.17
1	+1.137	-0.400	-0.191	-0.300	= +2.43	0.17
1	+1.176	+0.354	-0.469	+0.001	= +1.86	0.33
1	+0.831	+1.214	-1.224	+1.631	= +3.27	0.17

These give the following Normal Equations.

NORMAL EQUATIONS

$$\begin{aligned}
 11.160\delta\gamma - 2.986\delta K + 3.372\delta e' - 0.037\delta\omega + 0.096\delta T' &= +10.258 \\
 +3.735 &-2.715 &+0.090 &+0.522 &= -6.170 \\
 +6.626 &+1.453 &-2.027 &= +16.912 \\
 +3.622 &-3.307 &= +8.087 \\
 +4.030 &= -10.605
 \end{aligned}$$

The normal equations solved by the method of elimination gave the following corrections to the elements, with their probable errors.

$$\delta\gamma = +0.46 \pm 0.60$$

$$\delta T = -0.78 \pm 0.68$$

$$\delta K = +0.46 \pm 1.16$$

$$\delta\omega = -1^\circ 4' \pm 4^\circ 4'$$

$$\delta e = +0.072 \pm 0.033$$

with their probable errors
The final elements then are:

$$\gamma = -1.394 \text{ Km} \pm 0.60 \text{ Km}$$

$$\mu = 0.06498$$

$$K = 27.71 \text{ Km} \pm 1.16 \text{ Km}$$

$$T = 1904 \text{ Dec. } 2.92 \pm 0.68$$

$$\omega = 68^\circ 20' \pm 4^\circ 4'$$

$$e = 0.572 \pm 0.033$$

From these elements the velocities at the observed points were computed and the residuals O - C formed. These residuals compared with those obtained by direct substitution in the equations of condition show the error caused by neglecting terms of the second degree. The result is tabulated below.

RESIDUALS.

ϕ	O	C	O-C	By Dir. Subs.	Res.
1.00ds,	-10.25Km.	-11.57Km.	+1.32Km.	-0.26Km	+1.58Km
3.53	-27.67	-25.93	-1.74	-1.99	+0.25
11.12	-32.83	-35.80	+2.97	+3.71	-0.74

RESIDUALS (Cont.)

Φ	O	C	O-C	By Dir. Subst.	Resid.
17.07d5.	-38.17Km.	-34.10Km.	-4.07Km.	-3.75Km.	-0.32Km.
22.08	-31.83	-31.71	-0.12	+0.02	-0.10
28.07	-29.17	-28.67	-0.50	-0.51	+0.01
33.37	-24.58	-25.90	+1.32	+1.28	+0.04
40.07	-22.92	-22.38	-0.54	-0.57	+0.03
45.65	-22.83	-19.38	-3.45	-3.48	+0.03
50.16	-13.40	-16.86	+3.46	+3.44	+0.02
58.53	- 9.17	-11.84	+2.67	+2.65	+0.02
65.50	- 5.00	- 7.05	+2.05	+2.05	0.00
72.30	- 6.00	- 1.51	-4.49	-4.50	+0.01
79.60	0.00	+ 6.05	-6.05	-6.15	+0.10
88.00	+19.00	+17.23	+1.77	+1.50	+0.27
90.30	+19.50	+19.51	-0.01	-0.07	+0.06
95.10	+11.50	+7.90	+3.60	+2.90	+0.70

The residuals near periastron are larger than one would ordinarily want to permit. But, as will be seen later, the accuracy of the observations hardly justifies the labor of a second solution. In case the observations were better we would take the

elements and the "O-C"'s just obtained, compute equations of condition and solve as before. The corrections would certainly be small enough in another solution to make the terms of the second degree negligible.

For purpose of comparison the three sets of elements, by Ludendorff, by Wylie from Ludendorff's observations, and by Baker are given in parallel columns.

		Ludendorff	Wylie	Baker
P	=	96.7ds,	96.7ds,	96.67 d S,
e	=	0.50	0.572±0.03	0.525±0.011
T	=	1904	1904	1907
		Dec. 2	Dec.2.92±0.68	Nov.2.40±0.28ds,
ω	=	69° 24'	68° 20'±4° 4'	76° 21±1° 71
K	=	27.25Km.	27.71±1.16km,	30.75±0.48 Km,
γ	=	-14.4Km.	-13.94±0.60Km	-11.55±0.24km.
A	=	32.4 Km.	33.56 Km.	34.60 Km,
B	=	22.1 Km.	21.86 Km	26.90 Km.
a sin i	=	31,400,000 km	30,220,000 km	34,790,000 km,

In Figure I the seventeen normal places are plotted, also the velocity curve computed from Dr. Ludendorff's elements and that from the elements given by my least squares solution. The curve from Ludendorff is in black, the other in red. The change in form of the velocity curve is quite evident. At $\Phi = 3.5$ the red curve bends outward to approach a lone observation. From $\Phi = 10$ to $\Phi = 65$ the red curve strikes through the line of observations better than the black. Between 70 and 80 the red curve drops below the black to come nearer two evidently ~~awkward~~ observations. And at the maximum the red reaches the observations the better. That the solution has improved the orbit is forcibly shown by the fact that $[p.v.v]$ has been reduced from 131.09, in the case of Dr. Ludendorff's orbit to 75.73.

The scarcity of observations in the upper part of the curve is quite noticeable. In the table of observations we find only four with

a velocity greater than zero recorded. As a consequence there are few normal places on the upper part and these are of light weight. A single bad observation here would then change the form of the curve considerably and consequently change the resulting elements. The two normal places between $\Phi = 70$ and 80 together represent one half the number of observations in a normal place of weight unity, that is they represent three of our original sixty-seven velocity determinations. Yet to approach them the curve dips below three normal places between $\Phi = 50$ and $\Phi = 70$ and below one at $\Phi = 88$. The pulling down of the curve decreases the area of the upper part and consequently decreases the value of γ . The steepening of the curve has a tendency to increase the eccentricity.

As a matter of curiosity simply the orbit was computed neglecting these two points.

The following elements resulted:

$A = 32.4 \text{ km}$	$\omega = 70^\circ 51'$
$B = 23.2 \text{ km}$	$e = 0.504$
$K = 27.8 \text{ km}$	$\gamma = -12.6 \text{ km}$

The threemeasures here discarded have had considerable effect on the final result. These new elements are closer to Baker's than the others in every point but the "A". The greatest change is in the γ . The velocity curve computed from these elements is plotted in red in Figure 2. Dr. Ludendorff's curve is plotted in black. A least squares solution would improve the residuals some but would probably not alter the elements much.

The velocity curve from my least squares solution and that from Dr. Baker's elements are plotted in Figure 3. The curves run fairly close together part of the way, but between $\Phi=40$ and $\Phi=90$ Dr. Baker's curve runs higher. This is the region in which I had few observations. Dr. Baker's curve and velocity of center of mass stand above the other in somewhat the same manner as my curve of Figure 2 does.

In comparing the parallel columns of elements we notice that the probable errors after Dr. Baker's elements are less than half those after my elements. This would indicate, either that the

Allegheny measures are more accurate than those of the other observatories, or that they are better distributed over the orbit and so determine the velocity curve better. Probably both these things are true. Dr. Baker's observations are fairly uniformly distributed over the curve while we have seen that the upper portion of Dr. Ludendorff's curve is uncertain ^{because of the scarcity of observations}. On looking over the list of observations we have used we see considerable variation in measured velocities at about the same phase. The variation is much greater than in the Allegheny observations.

There is some slight evidence of a secondary oscillation in the velocity curve. To investigate it the residuals were plotted and an attempt made to put a sine curve through which would lessen the [p.v.v]. The best result appeared to be obtained assuming that the secondary had a period one third that of the primary. But the improvement was too slight to give any evidence of the reality of the disturbance. There is a slight indication of a secondary oscillation in

the Allegheny observations. But Dr. Baker was unable to get any marked improvement by assuming the existence of the disturbance. We get further evidence of the nonreality of the oscillation by comparing the curves which better the two sets of residuals. We find the curve for the Allegheny residuals is radically different from the best we can obtain. The period, phase and time of passage of node are all different. A curve which betters one set would make the other worse. Hence we believe the variations are accidental and do not exist in the star orbit.

The two orbits, that from Dr. Baker's elements and that from my elements, are plotted to scale in Figures 4 and 5, a $\sin i$ is plotted as the semimajor axis. We do not know "a" exactly. We only know "a" multiplied by the sine of the inclination which can not be determined from spectroscopic measures alone. The difference in the two values for ω and e can be detected in the figure, but the greatest difference is in a $\sin i$. The smaller

value of K , and the larger value of e both tend to make the $a \sin i$ in my orbit smaller.

Fig. 1

Figure 1

Ludendorff's Elements compared with Wylie's from the same observations.

W. L. LUDENDORFF

Fig. 2.

Figure 2

Showing the change in Lydenorff's
Elements when three questionable observations
are ignored.

Fig. 3.

Figure 3
Wyllie's Elements Compared
with Baker's Elements

Fig. 4.

Figure 4

The Orbit from Baker's
elements

$$a \sin i = 34890000$$

F = Center of mass

of system

S_0 = ascending node.

S_1 and S_2 = points of zero

velocity in the line of

sight.

Fig. 5

The orbit of a Andromeda from
Wyllie elements
 $a \sin i = 36,200,000$
 F = center of mass of syst
 ω = ascending node
 Ω = descending node
 S_1 and S_2 = points of zero
 velocity in line of sight.

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