THE GROWTH OF SCHOOL MATHEMATICS: KOREAN SECONDARY
GIFTED STUDENTS' COLLABORATIVE PROBLEM SOLVING USING THE
WIKI

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DOCTOR OF PHILOSOPHY

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by
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THE GROWTH OF SCHOOL MATHEMATICS: KOREAN SECONDARY
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WIKI

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DEDICATION

To Sohyun and Yurim

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LIST OF ABBREVIATIONS

ADM	Administrator	96
AGMT	Agreement	90
AM	Acquisition Metaphor	19
CC	Collective Cognition	8
CE	Closed Ended	51
CFM	Confirmation	93
CFP	Collective Focal Point	8
CMS	Collective Mental Space	8
CoI	Community of Inquiry	1
CPS	Collaborative Problem Solving	12
CSCL	Computer-Supported Collaborative Learning	8
ECS	Explicitly Contradictory Situation	78
EM	Error-Maker	90
ESMCE	Educational Setting for Mathematical Creativity and Errors	7
ESMPR	Educational Setting for Mathematical Practice and Readiness	6
FIX	Fixing errors detected by REF*	92
FIX*	Fixing errors without pinpointing errors	92
ICS	Implicitly Contradictory Situation	78
JPS	Joint Problem Space	8
KBE	Knowledge Building Environment	7
KF	Knowledge Forum	52
MF	Math Forum	46
MKB	Mathematical Knowledge Building	5

NEW	New solution or idea	90
OB	Old Babylonian	3
PM	Participation Metaphor	19
PoW	Problems of the Week	46
RFT	Refuting by checking a result	90
RFT*	Refuting by pinpointing errors	91
RPT	Repetition	90
RR	Repairing	90
SM	School Mathematics	5
STU	Student participant	90
SUPPL	Supplementation	90
VMT	Virtual Math Teams	46

THE GROWTH OF SCHOOL MATHEMATICS: KOREAN SECONDARY
GIFTED STUDENTS' COLLABORATIVE PROBLEM SOLVING USING THE
WIKI

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Dr. Barbara Reys, Dissertation Supervisor

ABSTRACT

Electronic networks have provided new intellectual environments where people can accumulate their cognitive power and have increased the necessity to provide students opportunity to *learn together* in online CoI (Community of Inquiry) beyond classroom walls. In this respect, this study, as a design experiment, investigated the applicability of the Wiki as a collaborative problem solving tool in Korean secondary gifted educational environments.

Fourteen Korean secondary gifted students enrolled in a calculus II course, including two females, in a gifted school in Korea collaboratively solved ten calculus problems for 70 days. In this study, the Wiki functioned as a collective cognitive tool by coalescing individual thinking processes into collective thinking processes on the one hand and by distributing cognitive loads of problem solving over individual cognitions along the logic of *solutions and refutations* on the other hand. This study provides evidence that the Wiki provided Korean secondary gifted students opportunity to experience collaborative learning and growth of school mathematics and thus realized *connected knowing via mathematical contents* in online CoI in spite of the inconvenience of writing mathematical expressions using the Wiki.

CHAPTER 1. INTRODUCTION

Hence the greatest and most difficult problem to which man can devote himself is the problem of education. For insight depends on education, and education in its turn depends on insight. It follows therefore that education can only advance by slow degrees, and a true conception of the method of education can only arise when one generation transmits to the next its stores of experience and knowledge, each generation adding something of its own before transmitting them to the following. (Kant, 1900, p.15)

1.1 Statement of the Problem

The process of teaching and learning mathematics is not a simple transmission of mathematical facts and techniques. Instead, it involves teachers and students creating products through dynamic, creative, and error-reducing activities such as making new problems with scenarios, making conjectures, finding fallacies, removing errors, devising more efficient ideas and solutions, and discovering new solutions. This implies the necessity to consider teachers and students as a Community of Inquiry (CoI) and build CoI in mathematics classrooms. Building a CoI depends on sharing products emergent from the educational setting to facilitate MKB (Mathematical Knowledge Building) process.

Recent development of informational technologies provide various tools that make it possible to build a CoI on the Internet; products of teachers and students are stored and shared with tools in Computer-Supported Collaborative Learning (CSCL) or Knowledge Building Environment (KBE) where an online CoI is expected. Since educational experience based on a CoI can provide students opportunities to construct their own mathematical knowledge, “the challenge educators face today is creating a CoI in a virtual environment” (Garrison, Anderson, & Archer, 2000, p.92). However, it is difficult to build a CoI in CSCL or KBE (Litz, 2007). In addition, whereas a CoI has been considered important to develop students’ higher-order thinking (Garrison

et al., 2000) and thus essential for mathematics gifted education, there has been no attempt to build a CoI in the field. In this respect, as a design experiment, this study addresses the following general problem: How can we build and support an online CoI of secondary gifted students for collaborative mathematical problem solving? In what ways and to what extent can secondary gifted students collaboratively build mathematical knowledge while they are engaging in mathematics problem solving via a Wiki environment?

1.2 Purpose of the Study

Mathematics educational environment in Korea is characterized as strongly competitive and individualistic. This situation is more acute in gifted educational settings than in regular education settings in Korea. In this respect, most Korean students, especially gifted, are deprived of opportunities to experience learning mathematics within a supportive social environment. By reviewing literature, I suggest that building online CoI can be a practical way to provide Korean secondary gifted students in math and science opportunities of collaborative learning in mathematics. This study is intended to test the use of a virtual CoI environment for Korean gifted students. More specifically, the design of the experimental study is to develop a model for introducing Wikis as a collaborative mathematical problem solving tool and to investigate the applicability of Wikis as a tool for local and global growth of SM.

I seek to identify the factors that influence building a CoI of secondary gifted students on the Internet, and to investigate the features of collaborative mathematical problem solving process in the Wiki system. The primary focus of the study is on Korean secondary gifted students in mathematics and science as they study calculus. As a design experiment, this study will provide insights to inform creation of an online CoI in Korean secondary gifted educational settings and provide researchers,

teachers, and curriculum developers information about online collaborative problem solving and the usability of Wikis for educational purposes.

1.3 Assumption of This Study

The development of mathematics is related not only to cultural aspects such as art and literacy but also to social systems such as schooling. That is, the growth of mathematics may partly depend on some functions of school mathematics and some aspects of school mathematics likely contributed to the development of mathematics (e.g., Kleiner, 1995, p.226). Specifically, some part of pure and abstract mathematics that developed in the OB (Old Babylonian) period (2000 – 1600 BC) may have been developed from schooling purposes rather than from purely mathematical research or due to mathematical amusement. Recent context-based research on Mesopotamian mathematics shows that OB mathematics was from the schooling process and requests us to consider OB mathematical tablets as school products rather than as research products or as products from specific professional tradition (Høyrup, 2007; Robinson, 2000a; 2000b; 2001; 2002a). For example, Plimpton 322, a tablet of Pythagorean triples, needs to be interpreted as a pedagogical tool rather than as a product of research mathematics (Robson, 2001; 2002b; 2002c; 2007, Katz, 1993). Since Plimpton 322 contains the intrinsic mathematical intellectual landscape, it still can be used to create educational settings for modern school mathematics (e.g., Nelson, 1993, p.65) even though they may not be the same as the educational setting for OB school mathematics (e.g., Katz, 1993, p.1; Robson, 2002a). From Popper’s perspective, this is possible because Plimpton 322 as a tenet of ‘world 3’, “the world of the logical contents” (Popper, 1989, p.74), provides us the mathematical landscape to be situated and share with.

Schools’ inherent function is traditionally considered the transmission of culture and subject matter to students for the needs of society and students’ self-

fulfillment (NCTM, 1989). However, this does not explain how school education of mathematics also contributes to the development of human knowledge in a systemic way. For example, this cannot answer why challenging non-practical problems such as finding “the length of a rectangle with area 2500 acres if the width is $7\frac{1}{5}$ inch” in modern terms were created and used in OB schools (Høystrup, 2007, p.262). Those mathematics problems were fabricated from known mathematical knowledge to provide an “educational setting for mathematical creativity” for OB students to develop their mathematical potentials to the fullest (Høystrup, 2007; Robson, 2001, p.200). Similarly, modern mathematics educators fabricate mathematics problems from known mathematical knowledge to create educational settings. In this respect, I assume that there exists an inherent function of school education of mathematics in a culture, which is operating beyond the difference of cultures and time. In other words, I assume that school education of mathematics has some mechanism, which is “functional invariant” for the development of school mathematical knowledge (Piaget, 1971). If such a mechanism exists and is transplanted in the Internet, then it will equally operate in virtual space for education of mathematics as in the real world.

Based on this assumption, even though there is no historically comprehensive description of development of school mathematics across both OB period to today, I seek to find a common mechanism of today’s and the OB school education of mathematics and suggest a chain of ESMPPR (Educational Setting for Mathematical Practice and Readiness) and ESMCE (Educational Setting for Mathematical Creativity and Errors) as a mechanism working for growth of SM (School Mathematics). From this, I design an experiment to investigate usability of the Wiki as a CPS tool by structuring and embedding ESMCE in the shared world.

By adopting Garrison and Arbaugh’s (2007) Elements of an Educational Experience and Practical Inquiry Model as a conceptual framework, I approach to the problem. In the first phase, mathematical content analysis will be applied to students’

solutions stored in the Wiki system. To understand individual students' mathematical thinking process more precisely, some of students will be interviewed. Based on this analysis, in the second phase, a cognitive map of students' solutions for each problem will be made to examine the structure of students' collaborative solutions on a cognitive presence level. It is expected that this mapping will reveal CoI's MKB pattern in the virtual environment. Other types of notes will be analyzed on social and teaching presence level. These analyses will reveal the effect of the design factors. In the third phase, students' primary sources of ideas and solutions will be analyzed from interviews. In the fourth phase, students' opinions and their mathematics teacher's opinion about the Wiki as a CPS tool will be surveyed. In the last phase, students and the teacher will be asked to nominate three students who contribute to CPS for each problem. This will reveal the difference between the teacher's perspective and students' perspectives on growth of SM in the virtual space.

1.4 Definition of Terms

In the same way that archeologically excavated mathematical tablets represent OB school mathematics, the work of today's teachers and students' represent modern school mathematics. From this perspective, I define several terms that will be used throughout this report.

- Mathematical Knowledge Building (MKB): Contribution to growth of school mathematics through creating or coherently organizing mathematical contents, discovering new math problems, solutions, or proofs, and improving solutions by eliminating redundancies, finding short cuts, and developing ideas through conjectures, refutations, and reducing errors in artifacts such as spoken language, texts, hypertexts, diagrams, graphs, multi-media files etc.
- School Mathematics (SM): SM is divided by two levels: macro and micro. SM on a macro level is defined as a cultural product of a community of people re-

lated to the schooling process such as mathematics teachers, mathematicians, parents, policy makers, and researchers. Thus, SM on the macro level includes products *for* the schooling process such as mathematics curriculum, mathematics textbooks, educational policies, research in mathematics education and so on. SM on a micro level is a set of products of a community of teachers and students while they are engaging in the MKB process. Thus, SM on the micro level is a product *from* the schooling process such as a teacher's way of explanation, models for conceptualization, discussions in a math class, a teacher's writing on the black/white board, students' solutions in their notebooks, or on exam papers, homework, and so on.

- Growth of SM: Growth of SM on the macro level is mainly caused by societal change, growth of mathematics, efforts to reflect results from research of mathematics education, and so forth. Growth of SM on the micro level consists of global growth and local growth. Local growth of SM results from actions by an evolving math class community. In terms of quantity, local growth of SM means whole outputs of local community from the beginning of a course of a school year to the end of the course of the year. In terms of quality, local growth of SM can be evaluated by considering mathematical products from the MKB process of a local community of teachers and students as *oeuvres*. Similarly, global growth of SM is possible when classroom communities are connected and share their products and engage MKB together by appropriate tools such as publication or social software. For instance, if *error-reduced*, more coherently organized, and creative products are publicized and shared with other teachers and students beyond a local community through the internet or other media, then local growth of SM can spur global growth of SM.
- Educational Setting for Mathematical Practice and Readiness (ESMPR): This

setting is built to provide “interpretive support necessary for making sense of” the didactically transposed mathematical knowledge from scholarly mathematics (Lave & Wenger, 1991, p.98), which is from SM on the macro level. By creating ESMPR, math teachers introduce new mathematical concepts and tools with illustrative examples and simple problems, which are designed to help students understand and practice as teaching aids or didactical tools to carefully guide their students to acquire new concepts, methods of solutions.

- Educational Setting for Mathematical Creativity and Errors (ESMCE): This setting is built by providing challenging problems with respect to cognitive tools which students have acquired in ESMPR. In this setting, students are expected to improve their mathematical knowledge by applying, inspecting, and sharpening acquired cognitive tools, and thus move to a higher level of mathematics. It may be the case that mathematics teachers allot more class time for ESMPR than ESMCE. However, ESMPR and ESMCE can operate relative to the state of the learners’ knowledge. For instance, the example of Gauss (see section 1.5.1) is not challenging to high school students who are taking algebra courses but very challenging to elementary students. As another example, a problem of finding the area of triangle ABC when $a = 5, b = 4, A = 90^\circ$ creates ESMPR for students who have learned Pythagorean theorem. Although a problem of finding the area of triangle ABC when $b = 6, c = 4, A = 30^\circ$ looks similar to the above problem, it may be challenging to students who have just learned the Pythagorean theorem and creates ESMCE. However, it is not challenging to students who have learned a proof of $\triangle ABC = \frac{1}{2}bc \sin A$ and thus create ESMPR. In this respect, ESMPR is providing stepping stones, which learners are expected to use as a cognitive tools and ESMCE is helping learners jump from one stepping stone to the next one.

- Knowledge Building Environment (KBE): “Any environment (virtual or otherwise) that enhances collaborative efforts to” build knowledge (Scardamalia & Bereiter, 2003, p.269).
- Computer-Supported Collaborative Learning (CSCL): “CSCL is a field of study centrally concerned with meaning and the practices of meaning-making in the context of joint activity, and the ways in which these practices are mediated through designed artifacts” (Koschmann, 2002, p.20).
- (Mathematical) Error: A statement or action leads to absurd and/or an incorrect mathematical conclusion (result). In this study, the term ‘error’ broadly subsumes simple mistakes in calculation, fallacies resulting for “an apparently logical explanation of why the result is correct” (Bunch, 1982, pp.1-2), and cases results from lack of rigor (Borasi, 1996).
- Collective Mental Space (CMS): This space consists of the intellectual landscape from the shared world and individual cognitions in the private world (see figure 2.3). To participate in the MKB process, individual cognitions in the private world bring into CMS their resources such as time, effort, ways of understanding, their background mathematical knowledge etc. CMS can be open or closed according to the presence of individual cognitions. In addition, CMS keeps changing through the learning of individual cognitions. When CMS is formed for CPS, I call it “Joint Problem Space” (JPS).
- Collective Focal Point (CFP): The act of individuals concentrating their efforts and resources on a specific part of the intellectual landscape in the shared world while engaging in the MKB process.
- Collective Cognition (CC): Mathematical problems and contents in the shared world provide the intellectual landscape in CMS. I call the interaction process

between the intellectual landscape and individual cognitions in CMS as Collective Cognition (CC). CC consists of teaching and learning. One's cognitive process is coalesced into the current intellectual landscape through CC. This characterizes *collaborative learning*, *mutual learning*, or *connected knowing via mathematical contents*. CC is different from what Stahl (2006a; 2006b; 2009h) calls group cognition as discourse of a small group because CC is not confined to small groups and is not direct discourse but is characterized as *indirect mathematical discourse via the shared world* (see "VMT" in section 3.2.1 for group cognition).

- Teaching: When the intellectual landscape environment influences one's cognitive process, it is said that the landscape environment *teaches* or *situates* the student in CMS.
- Learning: When an individual changes the current intellectual landscape, it is said that s/he *learns* or *participates* in the MKB process. In this study, learning, participating, and posting are synonymously used.

1.5 Examples of School Mathematics

The following examples are SM on the micro level as products of teachers and students in ESMCE. These examples show that teachers and students' products can develop SM but do not contribute to the growth of mathematics or that of SM on the macro level such as a curriculum, which consists of a fixed body of contents that students should know. Most teachers and students' products such as creative solutions or erroneous ideas are temporarily left within dimensions of an individual or a few people's experience and destined to disappear if not being shared among teachers and students. This explains why SM does not seem to grow in a continuous and systemic way.

1.5.1 An Historical Example

A famous anecdote about the school experience of Carl Friedrich Gauss (1777-1855), illustrates an important point. Boyer (1991, p.497) describes the situation as follows:

One day, in order to keep the class occupied, the teacher had the students add up all the numbers from one to a hundred, with instructions that each should place his slate on a table as soon as he had completed the task. Almost immediately Carl placed his slate on the table, saying, “There it is.” The teacher looked at the results, the slate of Gauss was the only one to have the correct answer, 5050, with no further calculation. The ten year boy had computed mentally the sum of the arithmetic progression: $1 + 2 + 3 + \cdots + 100$, presumably through the formula:

$$\sum_{k=1}^n k = \frac{n(n+1)}{2} \quad (1.1)$$

This story shows not only Gauss’ creative mathematical thinking but also the teacher’s role. The impractical and challenging problem was suggested after students learned adding numbers in ESMPR. The teacher provided ESMCE; If students had already learned the formula (1.1) then the problem is an illustrative example or a simple problem given in ESMPR. But clearly, elementary students in the classroom did not learn the formula and therefore the problem placed students in ESMCE. As a result, a brilliant student’s creative product emerged from ESMCE. However, it is not clear whether Gauss’ idea was shared in the class.

1.5.2 An Example from Mathematics Competition

An example of the growth of SM can be found in one mathematical competition in the USA. At a competition for students in grades 6-8, the following problem was given:

What is the greatest number of bags that can be used to hold 190 marbles if each bag must contain at least one marble, but no two bags may contain the same number of marbles? (MATHCOUNTS Foundation, 1999, p.8)

---

¹I added the left side of this equation

Figure 1.1: Bags in Bags

If we think: put one marble into the first bag, put two marbles into the second bag ... and put n marbles into the n th bag, then the solution is the n satisfying an equation: $1 + 2 + 3 + \cdots + n = 190$. From the equation (1.1),

$$\frac{n(n+1)}{2} = 190 \quad (1.2)$$

Solving equation (1.2) when $n > 0, n = 19$. This was the standard solution that the authors of the problem expected of students. It is likely the authors made the problem “backward from a known solution” of $\sum_{k=1}^n k$ in scholarly mathematics by putting a scenario from practical mathematics (Høyrup, 2007, p.270). However, this was not the only “right way” to solve it because a bag could contain other bags (Figure 1.1). It is interesting because there is a democratic process for correcting the prepared solution:

Historical note: When this problem was originally written for the 1992 MATHCOUNTS National Competition, the authors believed the answer to be 19 bags: 1 marble in the first bag, 2 marble in the second bag, 3 marbles in the third bag, and so on, with 19 marbles in the nineteenth bag. ($1 + 2 + 3 + \cdots + 19 = 190$.) At the competition, one student protested the answer of 190, using the explanation given above. As a result, the correct answer of 190 was accepted. This is especially noteworthy, since prior to that student’s protest the problem had been reviewed by nearly 40 people, many of whom were college and university professors, junior and senior high school math teachers, and technical professionals, and none of them had considered this alternative solution. (p.45)

Gauss' teacher and authors of this problem used the same formula (1.1) to provide ESMCE but in different ways. This example shows that teachers and students can build mathematical knowledge together by finding and removing errors. Suppose that a teacher made the problem for a test in an algebra course and all students in the course including the teacher could not find the real solution, then there would be no growth in SM except the new problem as the teacher's product. On the contrary, if the solution came out, the teacher and students could explore the meaning of "contain" and the condition of the problem to decide whether or not to accept "bags in bags" and thus *learn together* (figure 1.1).

This example is important for two points. Above all, it shows that the social system of refutation and approval is working for MKB and thus growth of SM is possible. Thus, it supports fallibilists' view on the nature of mathematical knowledge: mathematics as man-made knowledge is tentatively true as long as not being refuted. In addition, publication of the solution from growth of SM on the local level contributes to growth of SM on the global level by preventing other math teachers, who can similarly fabricate problems backwards from formula (1.1), from making the same error. In this study, I stand on fallibilists' view and focus on growth of SM on the micro and local level.

1.6 Research Questions

The specific questions addressed in this study are:

1. What design elements facilitate students' collaborative problem solving using the Wiki? What are the students' opinions about the Wiki as a CPS (Collaborative Problem Solving) tool or as a learning tool?
2. In what ways and to what extent do secondary gifted students collaboratively build mathematical knowledge while they are engaging in mathematics problem

solving via a Wiki environment? What patterns emerge from secondary gifted students' MKB process?; Linear pattern or thread pattern, parallel pattern, or other patterns?

3. What are main sources that gifted students use for their MKB? Class, face-to-face discussion with other students, teacher, textbooks, Internet, other students' solutions stored in Wiki, themselves, etc.
4. What is the teacher's opinion about the use of Wiki as a teaching tool? How does the teacher utilize the Wiki in his/her teaching mathematics?
5. What is the difference between the teachers' perspective and students' perspective when they are evaluating others' contributions to the growth of SM?

1.7 Significance of the Study

This study is significant in four areas of inquiry: gifted education, learning theory, mathematical problem solving, and technology. Although research in the field of mathematics education has been increasing, "scant attention has been paid to gifted education at the secondary mathematics level" (Sriraman & Steinthorsdottir, 2008, p.395). Especially, in terms of problem solving, "at secondary level, there are very few studies that document and describe how gifted students approach problem solving, abstract, and generalize mathematical concepts" (Sriraman, 2003, p.151). Moreover, there are few studies that document and describe how secondary gifted students in mathematics and science build mathematical knowledge while engaging in CPS. This study may contribute to the field of gifted mathematics education by discovering secondary gifted students' MKB patterns and their characteristics.

Second, a growing awareness of the social nature of mathematical knowledge with a paradigm shift characterized as "social turn" (Lerman, 2000) in the field of mathematics education has raised concerns about the appropriateness of the tra-

ditional model of didactic instruction and individual learning (Boaler, 1998; 1999; Smith, 1996). In response, one of the shifts in the teaching of mathematics necessary to modernize mathematics classrooms is to view classrooms as mathematical communities, instead of as a collection of individuals (NCTM, 1991). However, a tradition of assessing individual learners' cognitions and a rarity of tools for assessing learning in mathematical communities has obstructed the ability to introduce communities of learners into the schooling process. In this respect, the concept of *growth of SM* and comparing teacher's perspective and students' on contribution to it can support development of a tool for assessing learning in a community of learners. In this respect, MKB and growth of SM using the Wiki can be a new perspective to unify two opposite learning paradigms: Acquisition and participation.

According to Kang and Kilpatrick (1992), at the 58th annual meeting of the National Council of Teachers of Mathematics in Seattle, Kilpatrick raised the question: "Is problem solving bookable?" This is to ask if anyone can "put into the linear, static form of a textbook the multidimensional dynamic process of solving a problem"; "How do you "book" blind alleys, incorrect solutions, reformulations of the problem, hints, and so forth?" (Kang & Kilpatrick, 1992, p.3). A contemporary version of this question is, "Is problem solving bookable electronically?" This study is an attempt to explore the question within a Wiki environment.

Lastly, there have been repeated calls for introducing new technologies to the field of mathematics education (Kaput & Thompson, 1994; NCTM, 2000; Hollenbeck, Wray, & Fey, 2010). Educators have been increasingly paying attention to the social software of Web 2.0 such as blogs, Wikis, podcasts, and RSS feeds, which can create an online community by connecting people (Parker & Chao, 2007; Richardson, 2009). Especially, "the needs met by Wikis – easy authoring of Web content, open access, unrestricted collaboration – are not being satisfied by other available ICT tools" (Ben-Zvi, 2007, p.15). In this respect, Wikis as emblems of Web 2.0 are oppor-

tunities “for students to develop 21st-century skills such as expert thinking, complex communication, and new media literacy” (Reich, Murnane, & Willett, 2012, p.7). Even though “wikis appear to offer opportunities for publishing, communication, and collaboration that are rarely available in the U.S. classrooms” (Reich et al., 2012, p.13), “most U.S. K-12 wikis provide few opportunities for 21st-century skill development. The majority of wikis are abandoned immediately or are teacher-centered, content delivery devices” (Reich et al., 2012, p.12). This situation is not so different in Korea. In this respect, this study is a response to calls in terms of responsibility to take “a closer look at the role of the technology in learning or cognition—how it can be used to support problem solving” (Kaput& Thompson, 1994, p.677).

It is extremely rare to find a model of collaborative mathematics problem solving using Wikis, even though Wikis are known as powerful tools to facilitate collaboration (Notari, 2006). This study as a design experiment aims to identify the applicability of Wikis as a collaborative mathematical problem solving tool and provide a design framework for further studies to help researchers understand the characteristics of asynchronous written communication in the Wiki environment.

CHAPTER 2. CONCEPTUAL FRAMEWORK

The constant search for control, liberation, and expansion characterized man from his first appearance, and part of his success in devising effective mechanisms rested in his ability to secure the perpetuation and refinement of such skills through the procedures of education. Since man's environment from the end of the Neolithic period was as much social and intellectual as physical, the process of education took on social and intellectual cast and increasingly through the course of history has been mediated symbolically. The operation of the process changed the character of man's environment: with the extension of symbolic control the physical environment diminished in significance and social aspects predominated, until, with increased dependence on conceptual achievements, the social environment in turn became less fortuitous, and the intellectual or noetic environment became paramount. (Bowen, 1972, p.2)

2.1 Practical Inquiry Model

Garrison, Anderson, and Archer (2001) and Garrison (2007) suggest a conceptual framework to identify educational experiences in a Community of Inquiry (CoI). The framework is based on the philosophy of John Dewey and consistent with constructivist approaches to learning (Garrison & Arbaugh, 2007). This framework consists of three elements: social presence, teaching presence, and cognitive presence (figure 2.1).

Social presence "is defined as the ability of participants in the Community of Inquiry to project their personal characteristics into the community" (Garrison, Anderson, & Archer, 2000, p.89). The purpose of social presence in an educational context is to create conditions for inquiry and quality interaction to achieve worthwhile educational goals collaboratively. Social presence for educational purposes cannot be separated from the other two factors: teaching and cognitive presence.

Teaching presence consists of two general functions: design of the educational experience and facilitation. In this study, a teacher creates ESMCE by fabricating challenging problems and organizing problems aligned to the his/her aim. In addition, s/he can facilitate students' participation in CPS activities by suggesting questions,

Figure 2.1: Elements of an Educational Experience^a

^aNote. From "Researching the community of inquiry framework: Review, issues, and future directions" by Garrison, D. R. and Arbaugh, J. B. (2007), *The Internet and Higher Education*, 10(3), p.158. Copyright 2007 by Copyright Holder. Reprinted with permission.

hints, comments, feedbacks etc. Although these functions are the primary responsibility of the teacher, students can take over the responsibility in the CoI framework because students can select, make, and post problems, organize and revise others' solutions, and assess other members' ideas, solutions, and contribute to micro growth of SM in the shared world.

Cognitive presence is defined as "the extent to which learners are able to construct and confirm meaning through sustained reflection and discourse" (Garrison & Arbaugh, 2007, p.161). Cognitive presence is described "in terms of a cycle of practical inquiry, where participants move deliberately from understanding the problem of issue through to exploration, integration, and application" (Garrison & Arbaugh, 2007, p.162). Garrison, Anderson, and Archer (2001) operationalized cognitive presence in terms of a practical inquiry model resulting in a four-phase process (Figure 2.2), which are interpreted for online CoI in the present study as follows: (1) a triggering event, where the uploaded problems or projects and some errors of conjectures or

Figure 2.2: Practical Inquiry Model ^a

^aNote. From “Researching the community of inquiry framework: Review, issues, and future directions” by Garrison, D. R. and Arbaugh, J. B. (2007), *The Internet and Higher Education*, 10(3), p.161. Copyright 2007 by Copyright Holder. Reprinted with permission.

solutions are identified for further inquiry; (2) exploration, where students explore the issues and try to solve them through critical reflection and discourse; (3) integration, where learners find new ideas, construct conjectures or solutions from the exploration; (4) resolution, where learners suggest and apply their ideas and constructed conjectures and solutions or fix errors in a shared world. In this model, teacher and students in the private world can acquire their knowledge from the shared world and CoI can build mathematical knowledge in the shared world. While engaging in MKB in the shared world, teachers and students experience “a network of inter-realizations” of concepts (Stahl, 2009b). Therefore, Garrison and Arbaugh’s (2007) practical inquiry model explains the relationship between acquisition metaphor (AM) working for the private world and the participation metaphor (PM) working for the shared world and how two different worlds can be complimentary. The present study focuses on the process of MKB in the shared world with iterated circulation of cognitive presence in the practical inquiry model.

2.2 Acquisition Metaphor, Participation Metaphor, and School Mathematics

According to ontological stances, learning is mainly conceptualized through two prevalent metaphors: the acquisition metaphor (AM) and the participation metaphor (PM) (Sfard, 1998). AM assumes that knowledge is out there to be discovered and internalized, and that learning is the act of acquiring concepts that “are to be understood as basic units of knowledge that can be accumulated, gradually refined, and combined to form ever rich cognitive structures” (Sfard, 1998, p.5). Constructivists attack the dominating conception of teaching in AM, namely that “knowledge can be transferred from a teacher to student” because shared meaning is an illusion (von Glasersfeld, 1983, p.41); “sharing meaning, ideas, and knowledge is like sharing an apple pie or a bottle of wine; None of the participants can taste the share another is having” (von Glasersfeld, 1990, p.311). However, this argument raises a problem of incommensurability of the concept. Thus, although constructivists attempted to overcome AM by establishing individuals or societies as epistemic subjects who are “the builders of their own conceptual structures” (Sfard, 1998, p.7), whatever version of constructivism inevitably confronts a new dilemma of the “incommensurability of concept” between individuals or between societies (Sfard, 1998, p.7).

In contrast, within the paradigm of PM, all human intellectual activities are construed as “inherently social in nature whether performed individually or in a team” (Sfard, 2000, p.161) and “a community of practice is an intrinsic condition for the existence of knowledge” (Lave & Wenger, 1991, p.98). That is “all of our thoughts and actions are social construct” (Restivo, 1992, p.3); From Durkheim’s perspective, concepts and ideas are collective representations and collective elaborations (Restivo, 1992, p.xii), and “knowledge statements and systems are always parts of *networks* of ideas, concepts, and facts” (Restivo, 1992, p.xiii). “Thus, participation in the cultural

practice in which any knowledge exists is an epistemological principle of learning” (Lave & Wenger, 1991, p.98). Since thinking is conceptualized as “an instance of discursive activity” in PM (Sfard, 1998, p.6) and knowledge as a permanent entity is substituted with knowing or discourse that “has a very broad meaning and refers to the totality of communicative activities, as practiced by a given community” (Sfard, 2000, p.160), learning mathematics means becoming a participant in social practice, especially in a mathematical discourse (Sfard, 2000). “On this view, knowing and communicating are in their nature highly interdependent, indeed virtually inseparable” (Bruner, 1996, p.3). In this respect, “studying mathematical communication has been central for many researchers” (Sfard, 2000, p.161). From the perspective of PM, meaning is culturally situated and “meaning making involves situating encounters with the world in their appropriate cultural contexts in order to know “what they are about”” (Bruner, 1996, p.3). Thus, “whether “private meanings” exist is not the point; what is important is that meaning provide a basis for cultural exchange” (Bruner, 1996, p.3).

To circumvent quandaries of AM, PM “provides an alternative talk about learning as making an acquisition” (Sfard, 1998, p.9) by simply getting rid of “objectifying” knowledge (Sfard, 1998, p.8). That is, the knowledge is deleted from the glossary list in PM. However, disobjectification of knowledge in turn entraps PM in failing to explain continuous growth of learners’ experience. Within the paradigm of PM, explaining learning with changing learners’ roles in part-whole relationship between learners and a community or learners’ contribution “to the existence and functioning of a community of practitioners” (Sfard, 1998, p.6). However, what does it mean by a student’s learning mathematics, if it “does not take into account the actor’s previous learning” of mathematics (Sfard, 1998, p.10). Similarly, PM does not explain how knowledge grows in a society.

In spite of the incommensurability between AM and PM, they are not incom-

patible (Sfard, 1998). Thus, finding a new theory that balances AM and PM has been primary for learning theorists. For example, Cobb considers PM as “the conditions for possibility of learning” because “participation enables and constrains learning but does not determine it” (Cobb & Yackel, 1996, p.185) on the one hand and AM as the window through which “what students learn and the processes by which they do” can be looked into on the other hand (Cobb, 1994, p.13). More recently, Sfard (2008) coined the term “Comognition” to erase the compartment between individual’s mind and social practice.

For practitioners, this means moving the pendulum from AM toward PM in response to calls for reform of mathematics education from didactic teaching toward discussion-based teaching (NCTM, 1989; 1991; 2000; Becker & Jacob, 2000). While AM combined with drill-and-practice has been criticized as it forces students to learn “Parrot math” (Becker & Jacob, 2000, p.532), PM combined with discussion-based teaching also often leads “to fears that students are not learning enough, that they are left to wander in different, unproductive directions, and they learn only ‘fuzzy’ mathematics” (Boaler, 2002, p.45). On the one hand, “a clearly delineated content” of SM (Sfard, 1998, p.10), which is inherited and didactically transposed from the culture of scholarly formalized mathematics, “only makes sense in an AM framework!” (Sfard, 1998, p.12). On the other hand, sociocultural aspects of mathematical knowledge, which are related to development of identity and agency based on the culture of practical mathematics, need to be realized in school education; Invoking Kant’s words, AM without PM is blind while PM without AM is empty. How can these two sides of learning be harmonized in school education? Since “the main problem, it seems, is that of a gradual disappearance of a well-defined subject matter” in applying PM to the math classroom (Sfard, 1998, p.10), a math teacher’s concern is how to provide math class practices which afford “growth of connected knowing” in terms of PM (Boaler & Greeno, 2000, p.177) while “the knower’s being connected with” the

mathematical contents in terms of AM (Boaler & Greeno, 2000, p.191).

One reason for the tendency to emphasize AM in the school environment is that “the ultimate value of schooling is judged by what students individually carry away from it” and thus the idea of knowledge having primarily social existence does not fit school educational thought (Scardamalia, Bereiter, & Lamon, 1994, p.206). AM combined with individual assessment is helpful in working in the private world, but it is essentially mute with respect to the shared world (Scardamalia et al., 1994). In this respect,

..... one cannot measure the learning-even the individual learning-that takes place in collaborative situations with the use of pre- and post- test that measure capabilities of the individuals when they are working alone. To get at what takes place during collaborative learning, it does not help to theorize about mental models in the heads of individuals, because that does not capture the shared meaning making that is going on during collaborative interactions. (Stahl, Koschmann, & Suthers, 2006, p.416)

In order to encourage students, especially gifted, toward collaborative learning, we need assessment that reflects the social aspect of learning mathematics. However, “nothing of this sort is operational yet. Consequently, we focus on individual assessments” (Scardamalia et al., 1994, p.211). The problem is, then, how to assess students’ progress as growth of connected knowing while they are working collaboratively.

If the assessment of learning in “the learner’s personal world (reflective and meaning-focused)” is based on evaluating the amount of knowledge acquired by the individual student, the assessment of collaborative learning in “the shared world (collaborative and knowledge-focused) associated with a purposeful and structured educational environment” is to evaluate the quantity and the quality of mathematical discourse that a classroom community produces (Garrison, Anderson, & Archer, 2000, p.92). Although growth of verbal discourse is difficult to assess on a large scale, written discourse can be assessed when it is stored in the shared world. In this respect,

Figure 2.3: Collective Mental Space and Growth of SM

Wikis shed a light on assessment in the collaborative learning environment.

To approach this problem, I modify the practical inquiry model (Garrison & Arbaugh, 2007) in terms of growth of SM as figure 2.3. While participating in MKB process, an individual can acquire knowledge from the shared world through internalization. Individually accumulated knowledge in the private world in turn provides fuel for continuing MKB and growing knowledge in the shared world through participation. In this respect, AM and PM are complementarily functioning. In figure 2.3, an individual students' cognitive achievement can be evaluated by tracing their trajectory of participation in the shared world as well as traditional assessment tools in terms of AM. On the other hand, in order to evaluate collaborative learning, that is growth of SM, in the shared world, two perspectives need to be introduced: Cultural psychological approach and Popper's world 3.

2.3 SM as Oeuvres of Mutual Learners

Schoenfeld (1996) reports that in his class, members of CoI, including himself, learned from others' learning. Members of online CoI are the mutual learners and the power to sustain CoI comes from mutual learning. According to "the cultural-psychological approach to learning" (Bruner, 1996), learning per se is construed as mutual learning:

..... where human beings are concerned, learning (whatever else it may be) is an interactive process in which people learn from each other, and not just by showing and telling. It is surely in the nature of human culture to form such communities of mutual learners. Even if we are the only species that "teaches deliberately" and "out of the context of use," this does not mean that we should convert this evolutionary step into a fetish. (Bruner, 1996, p.22)

From this perspective, mutual learning as a cultural phenomenon happens in any culture. Admittedly, teachers can learn from students and students can learn from teachers (e.g., section 1.5.2). From a cultural psychological perspective, mutual learning is possible through interaction between culture and mind, and the interaction between culture and mind is possible via stored knowledge:

In some way, this stored knowledge, replete not only with information but with prescriptions for how to think about it, comes to shape mind. So, in the end, while mind creates culture, culture also creates mind. (Bruner, 1996, pp.165-166)

Since the stored knowledge is collaborative work, mutual learning is possible because "works and works-in-progress create *shared* and *negotiable* ways of thinking in a group" and thus create meaning (Bruner, 1996, p.23). Referring to Bruner's (1996, p.22) explanation about French psychologist, I. Meyerson's view: "the main function of all collective cultural activity is to produce "works" – *oeuvres*," SM needs to be considered as oeuvre. This leads us to consider how SM as a cultural product *teaches* or *situates* us in an intellectual landscape because "learning and thinking are always

situated in a cultural setting and always dependent upon the utilization of cultural resources” (Bruner, 1996, p.4). Therefore, it is important to design educational settings in which students can be situated and learn from each other, by considering what they bring into educational settings from outside.

Asynchronous interactions through the Wiki system give prominence to mutual learning. In figure 2.3, I refer to CMS (Collective Mental Space) as a space where mathematical contents in the shared world and individual cognitions in the private world interact with each other. In CMS, educational settings provide the initial intellectual landscape that triggers individual cognitions on the MKB process. Students *learn* by adding their products which emerge from their individual cognitive process, especially in a form of mathematical texts in this study, to the shared world, based on their understanding the intellectual landscape in the shared world. Students’ learning changes the current mathematical landscape which *teaches* other students to *learn*. In this way, members of CoI can learn from each other. Thus, every member of online CoI including a teacher can learn from each other via mathematical landscape developed by individual’s learning and contribute to growth of SM.

One’s learning, while being situated in the current mathematical landscape, immediately changes the current mathematical landscape, which in turn teaches others in CMS. I refer to such interaction between the intellectual landscape and individual cognitions as CC (Collective Cognition). In figure 2.3, arrows from the private world to the shared world denote *learning* and dotted arrows *teaching* (see definitions in section 1.2). Although verbal discourse is difficult to assess because of its instant disappearance, written discourse can be assessed because it is realized as oeuvre in the shared world. In this respect, in order to assess collaborative learning in the shared world, it needs to consider SM as oeuvre. I suggest students’ collaborative mathematical solutions from the present experiment as oeuvres for readers’ assessment (see Appendix 2). This will show that growth of one’s knowledge can be assessed in terms

of AM and growth of SM also can be assessed in terms of PM.

Mutual learning entails individual students' cognitive developments in terms of AM but the converse is not true because social parts in mutual learning are irreducible to individual student learning in terms of AM. That is, although growth of SM in the shared world entails individuals' acquisition of knowledge, some aspects of connected knowing are irreducible to growth of individual students' knowledge in the private world. In this respect, Korean students may easily lose mutual learning opportunities in such individual and competitive educational environments. Korean secondary gifted students need opportunities to experience social aspects of mathematics in the balanced educational situation between AM and PM. Gifted students in math and science need to "learn to *do* mathematics"¹ (Schoenfeld, 1996, p.11). Thus, this study aims to find a way to realize mutual learning in the Korean educational system regardless of teaching methods. In such a way, this study attempts to test the applicability of the Wiki as a tool for renewing Korean school culture by "creating community of mutual learners" (Bruner, 1996, p.84).

2.4 Growth of SM as Mutual Learning

From Popper's (1989) perspective, mutual learning is anchored in the shared world. Mathematical thoughts and solutions in the shared world independently exist from individual learners as math books do in world 3. Thus, mathematical landscapes in the shared world autonomously grow as objective mathematical contents in world 3. While cultural psychology guides us to focus on educational settings that can change the individual knower in the private world into "collaborative thinker" in the shared world (Bruner, 1996, p.50), Popper (2002) provides us a tool for predicting and analyzing mutual learning in the shared world.

The interaction between culture and mind is very similar to what Popper (1978,

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¹The author's emphasis.

p.160) calls *feedback relationship* between world 2 (“*thought process*”) and world 3 (“*thought contents*”). Although culture and world 3 are commonly “the products of the human mind” (Popper, 1978, p.144), Popper (1978) reduces culture to smaller world 3 by differentiating world 3 abstract objects from world 1 embodiments of them. From this, Popper highlights the objectivity and the autonomy of world 3. From Popper’s viewpoint, one’s learning as acquisition can be understood in terms of world 3 just like one’s subject act of understanding can be understood “only through its connections with third-world objects” (Popper, 1989, p.164). Thus, Popper (1989, p.164) explains that we *learn* by using “*the method of conjecture and refutation*”: $P_1 \rightarrow TT \rightarrow EE \rightarrow P_2$.² From Popper’s perspective, learning means growth of knowledge in world 3.

Combining these two views, the example of MATHCOUNTS can be better understood. Cultural situatedness explains why authors of the MATHCOUNTS problem and other investigators could not think about the possibility of putting bags in bags (figure 1.1) since it rarely happens in ordinary life. Culture situates them in ordinary context where we put marbles in bags. In this respect, they brought cultural resources in ordinary life into CMS. On the other hand, Popper’s world 3 explains why the logic of “bags in bags” is accepted in the community; Bags in the problem, as abstract world 3 objects with the function of containing other stuff rather than as physical world 1 objects, create the logical possibility of putting bags in bags. From this, we *learned* a new *better* solution, which characterizes growth of SM. Our learning as discovering is possible because world 3 has “*its own domain of autonomy*” (Popper, 1989, p.118) and relation between a problem and a solution belongs to world 3:

It is important to note that the relationship between a solution and a problem is a logical relationship and thus an objective third-world relationship. (Popper, 1989, p.165)

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² P_1 : a problem, TT : Tentative Theory or can be written as TS (tentative solution), EE : error-elimination, P_2 : new problem.

In figure 2.3, CC as interaction between the private world and the shared world is similar to Popper's *feedback relationship* between world 2 and world 3. In this study, Popper's world 3 is reduced to the shared world in virtual space. Mathematical problems and contents of the shared world create intellectual landscapes for individual cognitions to view, interpret, and understand. That is, it stimulates the individual cognitive process in the private world. On the other hand, the intellectual landscape in the shared world is changed as soon as one learns, which is not his/her subjective state but is detached from him/her as texts and thus acquire a citizenship of the shared world as world 3 immigrates its citizens from world 2. Since individual cognition's learning constitutes CC with the intellectual landscape in CMS, the intellectual landscape bridges individual cognitions. This bridging process is very slow in world 3, which is represented by books with objective knowledge, but become faster in the shared world with Wikis; Wikis can quickly connect individual cognitions to create an converging intersubjective mental space. In this respect, the Wiki as a collective cognitive tool can realize *connected knowing via mathematical contents*. Thus, the Wiki can be a tool for educators to understand the nature of learning by creating a laboratory environment for the experiment with respect to the growth of SM in virtual space. That is, the Wiki makes, as *a tool for epistemic experiment*, it possible to conduct an experiment with the MATHCOUNTS problem in virtual space.

Figure 2.3 shows how CC connects individual cognitions in the private world via the intellectual landscape in the shared world; individual cognitions interacts via the intellectual landscape that mathematical contents form in the shared world through CC. In CMS, on one hand members of CoI change the intellectual landscape (learn or participate in MKB process) and on the other hand the changed one develops (teaches or situates) their individual cognitions. In this respect, ongoing process of CC impacts both the intellectual landscape in the shared world and individual cognitions in the private world. In this respect, CC characterizes *connected knowing via mathematical*

contents.

Since SM is stored in the shared world, qualitative and quantitative differences between the final state of the shared world and the initial state of it defines the growth of SM. Growth of SM and growth of an individual's knowledge support each other through CC. It is noteworthy that a math teacher's cognition is connected to students' cognitions via mathematical contents in figure 2.3. Via mathematical writing as products in the shared world, individual cognitions including their teacher's are connected to each other in CMS. Thus, *growth of SM* implies *growth of connected knowing*. Although growth of verbal discourse is difficult to assess on a large scale, written discourse can be assessed when it is stored in the shared world. Wikis, as tools for accumulating individual cognitions on the shared world, visualizes MKB process. Thus, the Wiki can make it possible to assess growth of SM and growth of connected knowing.

CHAPTER 3. LITERATURE REVIEW

I think we need to reach culture with respect to science, so that one finally recognizes that what is important in science is not the distinction between true and false. This might be strange to mathematicians, but I will say that if I had the choice between an error which has an organizing power of reality (this could exist) and a truth which is isolated and meaningless in itself, I would choose the error and not the truth. (Thom, 1992, p.12)

3.1 Collaborative Learning

In this section, I compare collaborative learning and cooperative learning in general and examine issues of using cooperative learning in gifted education. From this, I suggest that building an online Community of Inquiry (CoI) in Computer Supported Collaborative Learning (CSCL) can be a pedagogical model for improving gifted educational environments in Korea, which can be characterized as highly competitive and individualized from the perspective of Acquisition Metaphor (AM).

3.1.1 Cooperative Learning and Collaborative Learning

Although different theorists characterize *cooperative* learning and *collaborative* learning differently, the term *cooperative learning* has historically been associated with “particular instructional strategies and particular theorists” such as P.W. Johnson, R.T. Johnson, and R.E. Slavin (Ormrod, 2009, p.437). Small-scale laboratory research on cooperation started in the 1920’s and “research on specific applications of cooperative learning to the classroom began in the early 1970’s” (Slavin, 1991, p.72). Cooperative learning refers to “classroom techniques in which students work on learning activities in small groups and receive rewards or recognition based on their group’s performance” (Slavin, 1980, p.315). In cooperative learning, group activities are organized “so that learning is dependent on the socially structured exchange of information between learners in groups and in which each learner is held accountable

for his or her own learning and is motivated to increase the learning of others” (Olsen & Kagan, 1992, p.8). Thus, cooperative learning has a dual structure; to cooperate in a group task and to help one another achieve individual cognitive objectives. “The challenge, then, is to find an appropriate balance between affirmation of the group process and individual accountability” (Reynolds et al., 1995, p.6).

Whereas cooperative learning refers to a particular set of techniques that attempt to foster individual students’ learning in terms of AM through highly structured group activities, collaborative learning “views learning as construction of knowledge within a social context and which therefore encourages acculturation of individuals into a learning community” (Oxford, 1997, p.443). More specifically, collaborative learning is a “reacculturative process that helps students become members of the knowledge communities whose common property is different from the common property of knowledge communities they already belong to” (Oxford, 1997, p.444). Thus, learning occurs socially as the construction of knowledge in collaboration within the community of learners rather than individually; “students are not engaging in individual-learning activities but group actions like negotiation and sharing, while learning in cooperative groups is viewed as something that takes place individually” (Stahl, 2006a, p.3).

In the paradigm of collaborative learning, students learn through actively participating in knowledge building as members of a group where knowledge is created through social interaction; students learn by doing, that is, by “problem-solving activities shared over distributed learning environments” (Litz, 2007, p.31). In this respect, Garfinkel (1967) suggests, as cited in Jordan and Henderson (1995, p.42), that collaborative learning needs to be understood “as a distributed, ongoing social process, where evidence that learning is occurring or has occurred must be found in understanding the ways in which people collaboratively do learning and do recognizing learning as having occurred.” The focus is no longer on what may be taking place

“in the heads” of individual learners, but what is taking place in the process of their interactions within collaborative learning environments (Stahl, 2006a, p.7).

In collaborative learning, students work on each aspect of a project by contributing and building on each other’s ideas, along with sharing the workload. Although cooperative learning (i.e., distributing work among team members) is part of collaborative learning, it is not the essential characteristic. Instead, the key characteristic of collaborative learning is knowledge building through interactions with others. Collaborative learning is more flexible and student-oriented whereas cooperative learning is more directive, task-oriented, and teacher oriented (Panitz, 1999). In summary, while cooperative learning is based on the motto: “two heads are better than one” (Bruffee, 1995, p.12), collaborative learning “involves individual learning, but is not reducible to it” (Stahl. 2006a, p.3).

3.1.2 Cooperative Learning and Gifted Students

In spite of the fact that there is evidence supporting cooperative learning in regular mathematics classrooms, the effectiveness of this method has been called into question for gifted education because of its structure. “Cooperative learning began with the observation that competition among students sometimes impedes learning” (Bruffee, 1995, p.16). In cooperative learning, students work together in small groups to achieve a common group goal and contribute “by improving over their own past performances” (Slavin, 1991, p.73). Effectiveness of cooperative learning depends on two key elements: group goals and individual accountability (Slavin, 1991). Individual accountability means “the success of the group must depend on the individual learning of every group member” (Slavin, 1991, p.76). These two elements motivate all group members to help each other learn for group rewards (Slavin, 1990a; 1991). Therefore, “an important goal of cooperative learning is to hold students accountable for learning collectively rather than in competition with one another” (Bruffee, 1995, p.16). This

structure of cooperative learning provoked debate as it relates to gifted education in the 1990's (Slavin, 1990a; 1990b; Robinson, 1990a; 1990b). Although researchers have investigated heterogeneity in students' ability with relation to tracking and grouping in gifted education, it is still controversy; "Some say that grouping and tracking can be beneficial, since they help schools achieve excellence in education. Others say that grouping and tracking are harmful, since they keep schools from achieving educational equality" (Kulik, 2003, p.268).

In cooperative learning, accountability makes problems of heterogeneity more acute. Advocates of cooperative learning methods argue that the structure of cooperative learning generally works equally for all types of students, including gifted students (Slavin, 1990a; 1990b). However, some gifted educators worry that cooperative learning deprives gifted students of opportunities for suitable acceleration and enrichment because they frequently work in their groups as a "junior teacher" (Davis, 2006, p.20). It is, also claimed that when high ability, low ability, and medium ability students are grouped together, the high ability student tutors the low ability students, and the medium ability student is left out of the group interaction (Webb & Palincsar, 1996).

Critics suggest there is evidence that cooperative learning is not effective in gifted education: many gifted students do not view cooperative learning positively (Clinkenbeard, 1991; Matthews, 1992); cooperative learning is not significantly related to higher achievement for gifted secondary students (Li & Adamson, 1992). On the other hand, proponents of cooperative learning methods do not consider these negative results to be "compelling answers about outcomes of cooperative learning for gifted students" (Patrick, Bangel, Jeon, & Townsend, 2005, p.92). They insist that gifted students can profit academically from cooperative learning. As a group, gifted students score equally high on achievement tests after either cooperative learning or regular individualized classroom instruction (Neber, Finsterwald, & Urban, 2001).

Furthermore, in terms of the achievement of gifted students, there is no significant difference between working in a cooperative learning group with gifted and working grouped with nongifted group members (Kennedy, Archambault, & Hallmark, 1995). The debate “between proponents of cooperative learning and advocates of gifted education is ongoing” (Patrick et al., 2005, p.91).

The present study concerns the implications of the debate rather than the problem of individual accountability and heterogeneity as such. While the critics implicitly assume that gifted students are a uniform, homogenous group, in reality, gifted and talented students are not a homogeneous population (Patrick et al., 2005). They form a considerable heterogeneous group (Patrick et al., 2005; Moon & Dixon, 2006). Therefore, applying cooperative learning methods to classes of gifted students may cause the same conflict as grouping mixed ability students. Although additional study regarding use of cooperative learning methods with classes of gifted students is needed, it is not the primary focus of this study.

Considering that “even with perfect teaching methods, individual differences, in the sense of different levels of ability, will not be obliterated” (Krutetskii, 1976, p.5), “the optimal learning environment would be one in which the level and pace of instruction is individually matched to each student” (Clark, 1997, recited in Reed, 2005, p.19). “In reality, however, individual instruction is rarely possible in public school classrooms” (Renzulli & Prucell, 1996, recited in Reed, 2005, p.19). Furthermore, most educators agree with Dewey’s doctrine that “school is primarily a social institution and that experience is education” (Bruffee, 1995, p.14). Individual instruction may not catch the vivid social interaction of learning mathematics, while “mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engage in the science of patterns” according to Schoenfeld (1994) (recited in Schoenfeld, 1992, p.335). Therefore, there exists a dilemma between two goals of mathematics education for gifted students: knowing

mathematical content knowledge and knowing about mathematics itself; to pull out an individual student's mathematical intellectual power to the fullest extent and to give students opportunities to experience collaborative mathematical problem solving in social surroundings. Although Johnson, Johnson, and Smith (1991, p.1:11) argue that "competitive and individualistic learning situations discourage active construction of knowledge and the development of talent by isolating students and creating negative relationships among classmates and with instructors," gifted students tend to prefer individualistic or competitive environments to cooperative environments (Li & Adamson 1992); and individual and competitive learning goals may motivate high achieving students to learn.

Since researchers in the field of gifted education have a tendency to focus on the individual gifted student's ability and dispositions rather than the collaborative learning environment, it is hard to find research related to gifted students' collaborative learning. Thus, "we need to understand better the dynamics collaborative interactions, particularly for gifted students" (Diezmann & Watters, 2001, p.7).

3.1.3 Cooperative Learning and Community of Inquiry (CoI) in Mathematics Problem Solving

Educators underscore importance of students' sharing "their thinking and approaches with other students and teachers" while solving mathematics problems (NCTM, 1989, p.23). Emerging social theories of learning mathematics have called for re-conceptualizing mathematical problem solving as "doing mathematics" in a societal dimension rather than in an individual dimension (e.g., Ernest, 1994; 1998; Cobb, 1994; Cobb & Yackel, 1996). To make mathematical problem solving "an interpersonal enterprise" rather than individual practice (Johnson & Johnson, 1990, p.108), mathematics educators have been paying considerable attention to cooperative learning methods over the past 30 years (Patrick et al., 2005; Davidson & Kroll,

1991).

There is a body of research that indicates the use of cooperative learning methods significantly increases students' achievement in mathematics and motivation to learn mathematics (e.g., Slavin, 1991; Qin, Johnson, & Johnson, 1995). Many mathematics researchers recommend this method as a supplement to the traditional instruction methods of mathematics and to promote successful problem solving (Gokhale, 1995; Slavin, 1991; Reynolds et al., 1995). Cooperative learning has been applied to mathematics classrooms in various settings and for various purposes. Curriculum developers have also encouraged forms of cooperative learning by designing activities which are based on cooperative tasks (Kramarski & Mevarech, 2003).

Setting aside whether or not cooperative learning enhances individual gifted students' mathematical ability, it is still problematic because it "assumed that knowledge resided in the individuals, and that group interaction was most useful as a way of transferring knowledge from one individual to another or as a way of motivating individuals to perform better" (stahl, 2009e, p.58). That is, the paradigm of cooperative learning does not attend to some components of the social aspect of learning mathematics in terms of PM. From the frame of PM, "participation in social practice is what learning mathematics is" (Boaler & Greeno, 2000, p.172).

Doing mathematics constitutes a social practice rather than an individual pursuit and thus collaboration as a social nature of mathematics matters. From this perspective, students learn not only specific topics of mathematics but also "a particular set of practices and associated beliefs" (Boaler, 2000a, p.3). Gifted students need to learn not only mathematical topics but also experience mathematics itself as a growing product from human endeavor, cultivating a culture of mathematical inquiry. Therefore, it is necessary that gifted students in mathematics and science should know about what mathematicians do in practice (Howson, 2005) and how mathematical knowledge evolves in the community. Thus, providing gifted students

opportunities to participate in CPS (Collaborative Problem Solving) activities is essential for deep and meaningful educational experiences.

One of the current reform efforts in mathematics education is to foster student discussion (NCTM, 1989; 1991; 2000). In traditional instruction, learning practices are characterized as “listening and watching the teacher demonstrate mathematical procedures” and memorizing and reproducing the procedures by practicing textbook exercises (Goos, 2004, p.259). Learning practices, on the other hand, consist of discussion and collaboration in reform-oriented instruction and are characterized as “*inquiry mathematics*” (Goos, 2004, p.259). In that CoI is “essential to support collaborative learning and discourse” in the math classroom (Garrison & Arbaugh, 2007, p.158), there has been an effort to build CoI in the math classroom environment (e.g., Schoenfeld, 1996; Lampert, 1990; Boaler, 1999; Goos, 2004). Although this research indicates that classes structured as CoI have positive effects, “researchers have not yet systematically compared the academic achievement of community of learners with the achievement of more traditional classes” in terms of AM (Ormrod, 2009, p.447). Moreover, since “most math education research is still focused on individual learning ..... the collaborative nature of knowledge building at either the professional or student level is not well studied or documented” (Stahl, 2009b, p.13). Particularly in gifted education, studies on the CoI model are rare. This situation becomes worse with respect to the research area of online models of CoI.

Despite the fact that a classroom community is different from a research community in that “it is “manufactured,” managed, and will disband at the end of the semester” (Schoenfeld, 1996, p.13), Schoenfeld (1996) reported that CoI was a key component of the classroom environment built in his undergraduate mathematical problem solving course. His students’ experience was not so much different from that of researchers; the students could “learn to *do* mathematics”¹ in CoI (Schoenfeld,

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¹The author’s emphasis.

1996, p.11) and even reached “mathematical results new to” him (Schoenfeld, 1996, p.16).

Schoenfeld’s course was specifically designed for mathematical problem solving rather than usual mathematics content such as number theory, algebra, geometry, calculus etc. In this respect, he did not have to provide ESMPR and he could focus on ESMCE. To create ESMCE, he started a lesson by distributing a set of non-standard problems, which provoked “immediate and widely divergent intuitive reaction from students” (Schoenfeld, 1996, p.15). Secondly, he gave students sufficient time to solve problems in groups and the opportunity to share ideas and solutions through presenting at the board and in whole class discussion. His role as a teacher was to circulate to check students’ questions and progress, to ask questions (“can you solve it another way? are there extensions? generalizations?”), to decide whether or not students were making sufficient progress on the problem, and to do a wrap-up (Schoenfeld, 1996, p.13). In the case that “the students believed they were “on the edge” of knowledge for the field,” he played a role as “*consultant* rather than a *certifier*”² (Schoenfeld, 1996, p.16).

While Schoenfeld (1996) built CoI within a college classroom, Lampert (1990) built it in the elementary classroom. Lampert (1990, p.44) set two teaching agendas: students’ acquiring “knowledge *of* mathematics, or content of mathematics” and “knowledge *about* mathematics, or mathematical practice.” Lampert (1990, p.46) created ESMCE for fifth graders by writing a problem on the blackboard (“What is the last digit in $5^4?6^4?7^4?$ ”). Since the expression of power of exponents is not typical in elementary school students’ culture of ordinary mathematics, she introduced it to her students through ESMPR. After writing the question, she walked around the classroom to watch and listen to what her students did. When most students had time to think about the problem, she started class discussion. Since the problem had “the

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²The author’s emphasis.

capacity to engage all of the students in the class in making and testing mathematical hypotheses” (Lampert, 1990, p.39), all students could create and participate in JPS (Joint Problem Space). In discussion, she played a role as a manager by saying “Can anyone explain what they thought so-and-so was thinking?” and “Why would it make sense to think that?” (Lampert, 1990, p.40) and “sometimes participated in the argument, refuting a student’s assertion” (Lampert, 1990, p.50). She reported that her students discovered “a new pattern in square numbers” (Lampert, 1990, p.45).

Goos (2004) reported the case of CoI built in a secondary math classroom using the school curricula. In one of the reported lessons, after students learned the formula on the previous day $a \cdot b = |a||b| \cos \theta$, the simple problem of finding the angle between two vectors $(3, 2)$ and $(5, 1)$ was given in ESMPR. Most students initially worked in groups and pairs to discuss the problem without the teacher’s help. The teacher then nominated a student to present his solution on the white board. When the student started to calculate the value of $(3, 2) \cdot (5, 1)$, the teacher asked him to show the general solution. Other students participated in solving the equation by offering suggestions and hints. When the student introduced the general scheme for the solution, the teacher “pulled the students toward sense-making” of the equation by raising questions about the cases when two vectors were parallel or perpendicular (Goos, 2004, p.277). These questions stimulated the students’ thinking and led them to participate in a discussion exploring the meaning of the formula. It is noteworthy that these students “were remarkably well tuned to their teacher’s goals and were aware that their classroom operated differently from others they had experienced” (Goos, 2004, p.272). This case “highlights the teacher’s central position in assisting students to appropriate mathematics as cultural knowledge” (Goos, 2004, p.282). In addition, this example is significant because “accountability pressures associated with syllabus coverage and high-stakes assessment might seem to favor more traditional approaches to teaching at this level of schooling” (Goos, 2004, p.281).

These example cases of CoI in math class provide several implications to this study. First, the teachers had clear instructional goals and effectively steered the process of discussion toward their goals. Without a clearly delineated goal or well-structured guidance, “the whole process of learning and teaching is in danger of becoming amorphous and losing direction” (Sfard, 1998, p.10). Second, students had enough time to think about the problems individually or as a small group and then had opportunities to put their thoughts and results into JPS (Joint Problem Space) and participated in collaborative activities (questioning and responding, suggesting conjectures, justifying, making refutations). Third, students learned how to participate or talk within the classroom environment. Learning about discourse and how to use new cognitive tools such as vocabulary and symbols leads to “learning about the nature of mathematical knowledge and about their responsibilities as participants in the activity of learning mathematics” (Lampert, 1990, p.54).

3.1.4 Korean Secondary Gifted Students and Online Community of Inquiry

According to Borasi (1996; 1994), there have been many critiques of traditional direct teaching and transmission pedagogy and these critiques are based on the recognition that such instructional environments do not prepare students for the various future needs of jobs. However, most mathematics classes are still places where teachers are the active demonstrator and students are passive imitators (Swan, 2002; Boaler & Greeno, 2000). Whole class lecture does not necessarily mean that there is no opportunity for students’ collaboration; it is possible to give a lecture and conduct a whole-class discussion in a collaborative manner. However, as long as traditional whole-class instruction is based on an individual task structure and competitive goals structure (Robinson, 2003), students have little time to collaborate via discussion of ideas, explaining methods for solving problems or proofs, finding

fallacies and correcting errors in others solutions, discussions of conjectures, developing ideas together, and refuting by suggesting counter-examples. Thus, there is a conflict between individual task & competitive goal structure and the social aspect of learning mathematics.

In Korea, as a whole, the environment of mathematics education emphasizes mathematical problem solving from the perspective of AM and is characterized as strongly competitive and individualistic. Each student within a class completes his/her seatwork and is evaluated for grades through course tests. Norm-referenced school testing and the college-bound secondary school curriculum reinforce a competitive, individualistic environment. Teachers are asked to rank all students linearly³; students must be treated equally in terms of content, teaching method, test problems, test time, and so forth. Secondary institutions for gifted students in Korea also provide their students with an individualistic and competitive mathematics learning environment based on a strict school evaluation system. The individual and competitive atmosphere is more severe at institutions for secondary gifted students of mathematics and science in Korea because students are continuously being asked to prove their superior ability through entrance examinations on national or international competitions of mathematics and science. Thus, Korean secondary gifted students are accustomed to studying mathematics as competitive individuals rather than as collaborative groups and are accustomed to experiencing learning in terms of AM rather than PM.

In this learning environment, Korean secondary gifted students may lose important experiences in learning mathematics, as Litz (2007) notes:

If the culture of a traditional discipline is particularly individualistic, as in the case of mathematics, then it may be that students do not even conceive of the possibility of collaborating, where they have always seen mathematics

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³In some respects, this has a tendency to amplify ESMCE in a school because math teachers should carefully make test problems not to have multiple perfect scorers from the same grade in a school.

problem-solving as a competitive activity undertaken between two or more individuals racing to achieve an answer first. (p.24)

Furthermore, there are students who are motivated to do well in school and who apply themselves seriously to school tasks, but who are not in fact pursuing cognitive goals (Ng & Bereiter, 1991). Individual and competitive learning environments combined with those students' goals of getting better scores in comparison to others further exacerbate the situation. Many Korean mathematics educators worry about this situation and have been urged to take responsibility for seeking a remedy for students suffering from extremely individualistic and competitive school environment in Korea (Woo, 2006).

To lead students to the social aspect of mathematical learning, new evaluation methods need to be introduced. However, the system of Korean secondary education is not flexible with regard to evaluation based on the individual and competitive environment. It is difficult to affect educational change in circumstances dominated by examination pressures (Swan, 2002). In addition, secondary gifted students tend to dislike the use of the cooperative learning method due to the individualistic and competitive evaluation environment (Matthews, 1993). To eliminate a conflict between an individual task & competitive goal structure and cooperative learning model, Johnson et al. (1991) emphasized that success of cooperative learning depends on paradigmatic change entailing evaluating systems in education; "a cultivate and develop" philosophy must replace a "select and weed out" philosophy. That is, "emphasis is placed on learning rather than upon evaluation" (Davidson, 1990, p.339). Therefore, there are constraints that hinder the cooperative learning model from being operated under an individualistic and competitive environment of evaluation in institutions for secondary gifted students in Korea. What is needed is a way to harmonize the reality of gifted students' learning environments with the educational needs of gifted students.

In this environment, Acquisition Metaphor (AM) draws people "apart rather

than bring them together” (Sfard, 1998, p.8). In addition, AM counts “the means of gaining” knowledge as “a highly priced possession” and “the gifts and potentials, like other private possessions, are believed to be measurable and may therefore be used for sorting people into categories. In this climate, the need to prove one’s “potential” sometimes overgrows his or her desire to be useful” (Sfard, 1998, p.8). As long as AM is the dominant practice in the Korean educational system, “its normative implications are usually taken for granted” (Sfard, 1998, p.8). Since changing this focus is neither desirable nor possible (Sfard, 1998), PM has to play a role of leading “to a new, more democratic learning and teaching” in harmony with AM’s hegemony in the Korean educational environment (Sfard, 1998, p.9). Therefore, the task is to insert a culture of students’ CPS into a math class based on individual and competitive settings. Litz (2007) suggests that online technology can be a possible approach.

In this study, I explore the roles of Computer Supported Collaborative Learning (CSCL) environments for Korean secondary gifted education in mathematics and science. Although collaborative learning can be designed for face-to-face interaction and take place in the classroom, collaborative learning is conducive for online CoI (Ashcraft & Treadwell, 2007). That is, collaborative learning makes it possible to build online CoI regardless of the teacher’s preference for instruction in a class or constraints in an individualistic and competitive educational environment.

By adopting the Wiki as an collaborative learning tool, this study is part of an effort to introduce student-centered online CoI into individualized and competitive Korean mathematics educational environment in parallel to the formal school system. A goal of the present study is to make the irreducible part of collaborative learning visible in the shared world and to investigate CPS process and growth of SM in online CoI.

3.2 Collaborative Mathematical Problem Solving In Virtual Environment

Information Technology has progressed rapidly over the last 20 years. With the shift from Web 1.0 to Web 2.0, the role of users has been changed from consumers of given content based on read-only technology to producers of information with technological tools of reading, writing, and creating visual and audio content. Communication technology supports creating new virtual environment where people live their virtual lives via blogs, twitters, virtual worlds etc. In contrast to students' quick adjustment to new types of the technology, the school environment has been slow to incorporate these new technologies to support student learning. Thus, there exists “*digital disconnect* between students' use of technology in and out of school” (Engstrom & Jewett, 2005, p.6). To address this disconnect, educators are examining “new models of education that incorporate the use of information and communication technologies as part of the basics of a 21st century education” (Engstrom & Jewett, 2005, p.14).

Keats and Schmidt (2007, p.3) warn that “education 2.0 happens when the technologies of Web 2.0 are used to enhance traditional approaches to education” and anticipates that future education will operate with social networking outside traditional boundaries of courses or schools focusing on flexible learning activities by creating room for student creativity. They call this *education 3.0* and argues that there is a growing emergence of *education 3.0* that demands educators to prepare for the rapid change of educational environment, which has almost arrived at the *tipping point*. Although it is difficult to predict what kinds of problems or opportunities students will face in the future, it is clear that they will be more connected by electronic networks and more frequently asked to solve challenging problems *together*. In this respect, many educators believe that “internet-based computer-mediated communication technologies” will shape the future of education (Brandon & Hollingshead, 1999,

p.109). The responsibility to explore how to appropriately use this new technology and revise mathematics curriculum to reflect the changes of technological environments belongs to the responsibility of mathematics educators (Fey, Hollenbeck, & Wray, 2010).

New communication technology has caused a shift – making educators across disciplines move from computer based individualistic learning models such as Computer Assisted Instruction (CAI), Intelligent Tutoring System (ITS), and LOGO, to network-based learning models such as CSCL and KBE, which can be synchronously or asynchronously implemented online. This represents “a shift between using technology to support the individual to using technology to support interactions and relationships between individuals” (Ben-Zvi, 2007, p.1). Whereas research regarding an individualistic learning paradigm is concerned with instructional efficacy, learning transfer, and students’ understanding, research in network-based learning environments is concerned with the language of learners, meaning making, and social factors in collaborative settings of CSCL and KBE. CSCL research commonly focuses on participants’ online talk, seeking ways to create learning communities, or virtual online communities (Koschmann, 1996; Stahl, 2006b). A stream of educational technology has been developing to “replace classroom discourse patterns with those having more immediate and natural extensions to knowledge-building communities outside school walls” (Scardamalia & Bereiter, 1993, p.265). Although CSCL and KBE commonly support collaborative learning, KBE focuses on process of creation and development of knowledge (Scardamalia & Bereiter, 2003) while CSCL does on meaning making as such (Koschmann, 2002).

In recent years, “CSCL has grown exponentially” based on positive results of research in various domains of education (Litz, 2007, p.26). Technology is used in CSCL as a mediational tool to support collaboration among participants (Koschmann, 1996) and the development of CSCL technology has increased applicability of new

pedagogical models based on collaboration, which can be seen as mutual participation in “a coordinated effort to solve problems together” (Roschelle & Teasley, 1995, p.70). In this respect, “CSCL has been shown to alter the nature of student learning” (Litz, 2007, p.36). More promisingly, “the use of technology has markedly improved the learning outcomes” (Lehtinen et al., 1999, in summary). Thus, many educators expect that CSCL entails not only a technical change but also a dramatic pedagogical one (Brandon & Hollingshead, 1999). Even though many research in CSCL has shown promising results, “what is still lacking is the evidence that the same results could be achieved widely in normal classrooms” (Lehtinen et al., 1999, in summary) and “there is not much experience of CSCL in mathematics” (Hurme & Järvelä, 2005, p.49).

In the next section, three prominent software systems (PoW, VMT, and KF) used in CSCL or KBE are described in terms of mathematical problem solving.

3.2.1 Math Forum (MF)

Math Forum (<http://mathforum.org>) was founded in 1992 to support improving math learning and teaching using the Web at Drexel University. It provides a variety of programs for mathematics education. One of its famous services is Ask Dr. Math, which responds to math questions for K-12 students. Among MF’s services, I focus on two types of online courses: Problems of the Week (PoW) and Virtual Math Teams (VMT). PoW is based on collecting and distributing individual students’ solutions and mentors’ feedback while VMT supports synchronous online collaborative problem solving for groups of students.

Problems of the Week (PoW)

In 1993, MF started Problems of the Week (PoW) to provide students an opportunity to solve mathematical problems through the Internet or e-mail. A problem

in each of seven strands (Primary, Math Fundamentals, Pre-Algebra, Algebra, Geometry, Discrete Math, Trig & Calculus) is posted on the site every two weeks and students are invited to send their solutions within the due period. Participants are asked to write their solutions including both their process and their reasoning.

PoW problems vary from typical problems found in most textbooks to newly developed ones with scenarios. Problems are structured with notes and a related bonus question. For example, the following problem with a scenario of cooling colas is titled as “Cooling Colas [Problem #1506]”: (retrieved in <http://mathforum.org/pows/submission-flow.do?execution=e4s1> Jul 30, 2012)

A few weekends ago we invited some relatives over and served them lunch. The day before we had bought some cans of soda, but we didn't have enough room in our refrigerator to cool the colas. Since it was February and we live in Pennsylvania, my husband suggested that we leave the cans outside overnight to cool. The overnight temperature was to be in the twenties, so I was afraid they might freeze; however, I figured out a way to approximate the temperature of the cola at any given time.

First, I read the thermometer in our apartment. It said 72°F . Next I read our outdoor thermometer, and it read 25°F . I put the cola outside for 30 minutes and then brought one of the cans inside to have a drink. Before I drank the soda, I measured the temperature. The temperature of the soda was 60°F .

Based on this information, how long would it take the cola to cool to 35°F ? Assume that the outdoor temperature remains constant during the cooling process.

Note: The temperature of the cola is not decreasing linearly. Bonus: Water freezes at 32°F . Will cola also freeze at this temperature? Explain.

Students are invited to send their solutions with their name, age, school name, and district. Participants' solutions come from all over the world. Nine participant provided solutions are posted at the end of the time period. Since participants have no unified tool for writing mathematical expressions, they use various ways to express mathematical formulas and graph. For example, one student submitted a solution with a form of JPG file and other students typed mathematical expressions or graphs (figure 3.1).

$$\begin{array}{l} \text{integrate:} \\ \int \frac{1}{(T - 25)} dT = \int k dt \\ \ln (T - 25) = kt + C \\ T - 25 = e^{(kt + C)} = e^{kt} * e^C \end{array}$$

Figure 3.1: A Student's Math Expressions From Sample Solutions of "Cooling Colas"

Mentors provide individualized feedback based on a rubric to participants. Since 1994, PoW has sent individualized feedbacks for more than one hundred thousand students to revise and improve their solutions (Alejandre, 2006). For nearly 20 years, problems and solutions from all over the world have been archived in the PoW library. Teachers may use the stored materials for problem solving in their class by registering with some cost and opening an account. They then can create and manage online classes. The PoW program also supports a math teachers' community for discussion and sharing of information.

PoW suggests an online model for the global growth of SM by supporting individual learning by constructing ESMCE with challenging problems with scenarios, providing feedback, accumulating students' various solutions, and disseminating its products. However, students cannot share their solutions or errors with each other in PoW or engage in MKB. Therefore, it is hard for students to experience growth of SM. PoW is designed for individual learning and does not provide shared space for MKB. In the PoW system, students do not experience the social aspect of mathematics.

Virtual Math Teams (VMT)

The Virtual Math Teams (VMT) project is an NSF-funded research program, which has been directed by Gerry Stahl since 2002 at Drexel University. The VMT project aims "to develop an online environment that will effectively foster student mathematical discourse and collaborative knowledge building ... to understand the nature of team interaction during mathematical discourse within this new environ-

ment” (Stahl, 2009a, p.3).

The VMT project began with an experiment called PoW-wow, which extended PoW (Çakir, Xhafa, & Zhou, 2009). While PoW started at the time of Web 1.0 and cannot support synchronous mathematical online communication, PoW-wow supported synchronous chat communication with AOL Instant Messenger (AIM) software, which was popular at the time. The online chat revealed chat confusion; since participants often typed simultaneously, it was not clear who was the recipient of the posting message (Strijbos, 2009). This caused difficulty to analyze chat data (Strijbos, 2009) and deterred “the enactment of some group dynamics that require precise understanding of the chat conversation” (Fuks & Pimentel, 2009, p.396). To avoid the confusion, the VMT project adopted ConcertChat, which “allowed chat participants to connect their postings to previous chat postings or other items by means of an arrow representing an explicit reference” (Fuks & Pimentel, 2009, p.374).

The VMT system provides an online synchronous collaborative environment and multimodal affordances. The VMT chat consists of two spaces for interaction: a shared whiteboard (left) and a chatwindow (right) (figure 3.2). Participants can chat while seeing math expressions and figures on the whiteboard. A shared whiteboard supports drawing geometric figures with different brush thickness and color. Participants can chat with each other using the chat box. A chat window allows users to visually connect their chat postings to previous postings (*message-to-message* reference) or to objects on the whiteboard via arrows (*message-to-whiteboard* reference) (Çakir et al., 2009). Chat windows show typed words and “social awareness messages indicating who is currently typing and drawing or entering and leaving the chat room” (Stahl, 2009d, p.44). A history of participants activity both in the whiteboard and the chat can be scrolled by users. The VMT chat is equipped with multiple functions such as summary of work, task description, Geogebra, view online resources such as VMT wiki pages and Math Forum’s digital library (Çakir et al., 2009).

Figure 3.2: The VMT-Chat

Stahl’s team found that characteristics of small group discourse in the chat room is the same as that of individual’s cognitive process for problem solving. That is, “the group has solved the problem by constructing an argument much like what an individual might construct” in one’s mind (Stahl, 2009e, p.69). For example, “cognitive conflict” in individual cognition is superseded with “inter-animation of perspectives” in group cognition (Toledo, 2009, p.161). As cognitive conflict impels an individual’s learning, the interactional process of resolving differences between perspectives brings a richer shared understanding of group participants (Stahl, 2009f) and “drives the learning activity of the virtual math team by structuring the continuity of the discourse” (Toledo, 2009, p.161).

Stahl (2009a, p.4) defines group cognition as “the interactive processes by means of which small groups of people can solve problems, build knowledge and achieve other cognitive accomplishments through joint efforts”. Hence, group cognition is apparent when a group of people produce a sequence of utterances that perform a cognitive act, and in principle cannot be reduced to mental representations of an individual or of a sum of individuals (Stahl, 2007). A body of research in the VMT

project has been conducted to understand the mechanism of group cognition for the last ten years and the overall result is documented in the book: *Studying Virtual Math Teams* (Stahl, G. (Ed.), 2009).

VMT chats were largely sustained and driven by “math proposal adjacency pairs” (stahl, 2009f, p.80). Math proposal adjacency pairs are sets of an individual’s proposal and another member’s acceptance or rejection. They “provide a social order for discussions of mathematical problems in small groups” (Stahl, 2009g, p.524). A social order consists of a temporal dimension, JPS, and an interaction space. The temporal dimension is related to “bridging activity” which connects the past, the current, and the possible future events (Sarmiento-Kalpper, 2009a, p.90). JPS is the place where objectification of mathematical objects proceeds. Since a mathematical object is represented in multiple ways such as figures, mathematical symbols, and narrative forms in the VMT chat, those representation of it need to be objectified and “united by the group discourse” (Stahl, 2009h, p.576). The *interaction space* is conceptualized as a space for intersubjective relations. It has been analyzed in terms of *positioning*. In the VMT chat, participants position themselves as explorer, questioner, explainer, instructor, reporter, listener, etc. while engaging in collaborative problem solving. The agency depends on the relationship between an actor and his/her mathematical knowledge.

In the VMT project, building JPS is the major focus. The early VMT system used traditional closed ended (CE) math problems from standardized multiple-choice problems of SAT and problems of PoW. Since these problems had a correct answer, “they were not well suited for collaborative knowledge building” (Stahl, 2009h, p.562) because “discourse around them was often confined to seeing who thought they knew the answer and then checking for correctness” (Stahl, 2009h, p.562). Wee and Looi (2009, p.481) report that traditional CE problems from textbooks did not promote “quality mathematical reasoning between participants” in the VMT chat environ-

ment. In contrast, Wee (2007) designed a new type of problem which is called a guided collaborative critique (GCC). A GCC problem is a CE problem with an erroneous solution; the problem is proposed with an erroneous solution and students are asked to correct errors in the solution. When GCC problems were used for junior college students (ages 16-17 years) in Singapore who were among the top 20% of their cohorts in terms of academic ability (Wee, 2007), they “not only collaboratively explored mathematical concepts learned in class but also reasoned out the feasibility of their application in various GCC problems” (Wee & Looi, 2009, p.482).

3.2.2 Knowledge Forum (KF)

Computer Supported Intentional Learning Environments (CSILE) is designed “to make advanced knowledge processes accessible to all participants, including children”, and “to foster the creation and continual improvement of community knowledge” (Scardamalia, 2004, p.183). By redesigning CSILE in 1995, Scardamalia and Bereiter and their team at the Center for Applied Cognitive Science at the University of Toronto, have developed a Knowledge Forum (KF) (<http://www.knowledgetforum.com>). KF is a network-based educational software system designed to create and support KB communities by providing a collaborative space in which to share ideas and materials, cite reference materials, organize and develop ideas, analyze data, and discuss. In the form of notes, participants contribute theories, working models, plans, evidence, reference material, and so forth to this shared space (Scardamalia, 2002).

Initially, students start by writing their ideas in an empty *New Note*. Students can use graphics to communicate in addition to text. Notes can contain graphics, video clips, applets, and so on. Students can link their notes to others’ and make references and quotes from existing notes. Notes and views can be revised at any time. Students can make “Rise-above notes” or “Rise-above view” which subsumes sets of related notes or graphics (Scardamalia & Bereiter, 2006). The teacher and

Figure 3.3: Window for New Note

Figure 3.4: Window for View

his/her students can form a KB community and advance their knowledge through discussion, comments, and collecting materials.

Scardamalia and Bereiter (2006) argue that the note-taking, searching, and organizational features of this tool allow any type of community to build knowledge and KF has been successfully used for building the KB community in classrooms in the domain of geology, literacy, and science. However, establishing and maintaining a KB community in the domain of mathematics, “has been found to be a rather intractable problem” (Nason & Woodruff, 2004, p.104). Nason and Woodruff (2004) have suggested two major reasons why collaborative mathematical problem solving

activity is difficult in KF: an issue of representing mathematical expressions and the nature of school math textbook problems. First, the KF system is limited in providing mathematics representational tools for mathematical discourse (Nason & Woodruff, 2004). Since this is not problematic on the elementary mathematics level where sophisticated mathematical expressions are not needed, Moss and Beatty (2006) could construct online CoI of grade 4 students (n=68) from three classrooms in a diverse urban setting using KF. On the secondary level, Hurme and Järvelä (2005) adopted a way of inserting formulas of mathematical expressions into the notes as graphics to overcome the drawback of KF in mathematical communication. In this way, they constructed online CoI of Finish secondary school students (n=16).

On the other hand, most mathematics textbook problems are CE and thus inappropriate to elicit MKB activities. The math text problems' lack of open-endedness, relevance, and authenticity hinder or restrict engagement in mathematical discourse and MKB (Nason & Woodruff, 2004). In this respect, Nason and Woodruff (2003) attempted to build the MKB community by suggesting context-based problems such as finding the best city to live in. Twenty one grade 6 female students built models in groups and posted their models in KF. Then they revised and improved their models by comparing with other groups. Nason and Woodruff (2003, p.359) reported that students understood the nature of mathematics as “the outcome of social processes” and “fallible and eternally open to revision” from the MKB.

Moss and Beatty (2006) and Hurme and Järvelä (2005) refute Nason and Woodruff's (2004) view that math textbook problems do not elicit meaningful MKB. Moss and Beatty (2006) investigated how grade 4 students solved mathematical generalizing problems such as finding the functional relationship of $y = mx + b$ using KF. To be sure, the generalizing problems created ESMCE for grade 4 students. They reported that the multiple cycles of testing and refining occurred as part of MKB and KF supported students to revise their conjectures of rules and to provide evidence

and justification for them. Clearly, ESMCE of finding the functional relationship created CMS. Similarly, while geometry and probability courses were taught following the Finish curricula, Hurme and Järvelä (2005) provided ESMCE by asking Finish secondary school students to find a definition of polygon, formulas for calculating the area and perimeter, and to create examples and problems. This ESMCE successfully created CMS in which participants collaboratively solved mathematical problem using KF.

Above cases show that even the text-based nature of KF may benefit students by providing opportunities to communicate with each other and enhance their understanding by making their thinking visible with school mathematical problems in spite of shortcomings of KF when ESMCE sparks off CMS (Nason & Woodruff, 2003; Hurme & Järvelä, 2005; Moss & Beatty, 2006). Thus, Moss and Beatty (2006) conclude that KF can be a pedagogical tool for the establishment of classroom culture in which students “analyze and evaluate the mathematical thinking and strategies of others; communicating mathematical thinking coherently and clearly to peers; and make and investigate mathematical conjectures” (NCTM, 2000, p.460).

3.2.3 Wikis

Wikis have been considered as an ideal online learning platform for the 21st Century classroom. Reich, Richard, and Willett (2012, p.8) suggest that Wikis “support emerging models of innovative, online pedagogies that can foster the development of essential competencies for a networked age.” They examined a random sample of 1% of about 180,000 Wikis used in U.S. K-12 environment. They reported that 99% of the sample was used for simple storage of information such as teachers’ resources, content to deliver, individual student assignments and portfolios, while only 1% for collaborative works, and that 50 % of the sample did not survive after 13 days of establishment. They also noted that, “the percentage of wikis created in Title I school

that fail on the first day is twice as high as the percentage of early failures among wikis created in non-Title I schools” (Reich et al., 2012, p.14).

Asynchronous Writing and Collective Mental Space

Unlike VMT chat, Wikis provide an asynchronous interaction environment. Asynchronous postings and interactions in the Wiki are different from synchronous chat conversations. Messages posted in a chat session cannot be edited or altered like spoken words but every posting in the Wiki is open to editing and revising. Thus, knowledge co-constructively formed in the Wiki is “always ‘in formation’” as long as people can approach it (Ruth & Houghton, 2009, p.148). However, most simple information stored in the Wiki cannot survive for long time from its birth (Reich et al., 2012).

VMT chat provides an intersubjective space for group cognition of 3-5 participants. In contrast, Wikis provide CMS for a community’s MKB where its members share equal authority and responsibility for MKB (Ruth & Houghton, 2009). Thus, Wikis may function as educational platforms where an online CoI can support the local growth of SM. Since it is possible to connect online CoIs and share oeuvres from an online CoI of their class, global growth of SM is possible. Theoretically, there is no size limit for the participating group in the Wiki environment.

While interaction through VMT chat occur in rapid and frequent ways, interaction process in the Wiki is delayed, very slow, static, but highly organized and focused on mathematical contents. Thus, Wikis “may be appropriate for long-term knowledge building in classroom communities” (Stahl, 2009b, p.13). Since chatting is typically informal and reflects “the linguistic subculture of teenagers” such as abbreviation, emoticon, etc (Stahl, 2009h, p.561), “it provides a space for emotions and decreasing the feeling of impersonality” (Fuks & Pimentel, 2009, p.375). VMT chat sessions typically proceed in the order of greetings and socializing, tool training for

	PoW	VMT	KF	WIKI
Group Size	Flexible	Small	Small	Flexible
Use of Editor	Easy	Very Easy	Very Easy	Easy
Mathematical Expression	Not Efficient	Efficient	Not Efficient	Efficient
Interaction	Teacher	Moderator	Teacher	Teacher
	Individual	Group	Group	Group
	Student	of students	of students	of students
Community of Math Teachers	Exist	Exist	Not Found	Exist

Table 3.1: Comparison of Tools of CSCL

new users, starting with one of the participants’ suggestions for working on the given problem (Stahl, 2009c, p.26). However, participation in the Wiki starts with reading precedent postings individually and socializing elements are not necessarily needed for collaboration through the Wiki. In this respect, VMT chat provides a better environment than Wikis “for a learner to better perceive herself as part of the group, minimizing the feeling of isolation that is notoriously identified as one of the main causes of disappointment in distance courses” (Fuks & Pimentel, 2009, p.375).

Wiki writing needs more elaborate processes than chatting. Since this writing “produces a *record* of our mental efforts, one that is “outside us” rather than vaguely “in memory”” (Bruner, 1996, p.23), it naturally leads writers to deeper and reflective thinking on their writing. Once completed, results of a VMT need to be written in an organized way for readers because the whole body of informal chat dialogue is cumbersome. In contrast, in the Wiki environment, students are expected to take turns of writing and stretch their sequences of writing in a coherent way by reflecting others’ writing and thus different thoughts and ideas are interwoven into oeuvres that are worthwhile to read. See a comparison of features of tools in CSCL environment in table 3.1.

When a VMT team posted its result on the Wiki, other teams working on the same problem “used, validated and advanced” it and sometimes added new solutions

to it (Sarmiento-Klapper, 2009b, p.234). Interestingly, in one instance, a team found an error in another team’s posted solution “and proceeded to re-work the original solution and post the corrected result back to the wiki” (Sarmiento-Klapper, 2009b, p.234). This example demonstrates the potential of Wikis as a platform for MKB in ESMCE. Wikis provide not only repository space but also create CMS where students’ mathematical knowledge can evolve, which satisfies the primary condition of the collaborative environment; “must enable students to create online intersubjective space that adequately supports the students’ cooperation” (Larsson & Alterman, 2009, p.372). That is, Wikis do not belong to typical asynchronous media that “encourage mere exchange of individual opinions more than co-construction of progressive trains of joint thought” (Çakir, 2009, p.101). Thus, Wikis provide learners CMS for a community’s knowledge building, shared authority, responsibility for their own knowledge, and students centered environment (Ruth & Houghton, 2009).

Several online MKB patterns have been found in CSCL environment. For instance, math conversation in VMT chat “proceeds through sequences of math proposals, which themselves tend to have a typical structure of group interaction” (Stahl, 2009c, p.27). However, it is not known how CPS using the Wiki proceeds. As a design experiment, thus, the purpose of this study is to create JPS using the Wiki and investigate what CPS patterns emerge from JPS.

Wikis and Mathematics Education

Compared with face-to-face learning, the asynchronous nature of the Wiki allows students to participate in a manner in accordance to their learning style and needs by providing them flexibilities in time and space constraints. However, Wikis are primarily used as repository tools (Sheehy, 2008) and the majority of research on Wikis has focused on technical aspects (e.g., Tazzoli, et al., 2004). In this respect, “research into ‘wikis for learning’ is in its infancy” (Ruth & Houghton, 2009, p.137).

Furthermore, most successful models of Wiki usage in educational settings are set in other domains of subjects such as history, geography, and science. Information technology as “a meditational tool to support collaboration” is rarely used in teaching and learning mathematics (Hurme & Järvelä, 2005, p.49). In recent years, some math teachers on the college level have begun to look for possibilities for changing mathematics educational environments that incorporate Wikis (e.g., Ben-Zvi, 2007; Narasimhan, 2009; Carter, 2009; Peterson, 2009). Their informal reports indicate the usefulness of Wikis in the mathematics educational environment.

There exist online Wiki communities for mathematics education such as PlanetMath (planetmath.org) and Art of Problem Solving (<http://www.artofproblemsolving.com>). PlanetMath provides virtual space of articles, problems, and questions for mathematics educators on college level. In contrast, Art of Problem Solving provides resources for mathematics competition such as AMC 8, AMC 10, AMC 12, AIME, USAMO. These sites form online CoIs and contribute global growth of SM. However, these sites primarily apply Wikis to accumulate, store, and convey mathematical contents rather than to support CPS. A case that applies Wikis as CPS tools on secondary mathematics course level has not been reported.

CHAPTER 4. DESIGN AND METHOD

A primary goal for a design experiment is to improve the initial design by testing and revising conjectures as informed by ongoing analysis of both the students' reasoning and learning environment. (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p.11)

4.1 A Design Experiment

The purpose of this study is to investigate the application of Wikis to the practice of school systems, especially to a calculus course for Korean secondary gifted students in mathematics and science. The study can be characterized as a *design experiment*, in which the researcher “attempts to engineer innovative educational environments and simultaneously conduct experimental studies of those innovations” (Brown, 1992, p.141). The goal of conducting a design experiment is “to develop theories, not merely to empirically tune ‘what works.’ These theories are relatively humble in that they target domain-specific learning processes” (Cobb et al., 2003, p.9).

As a design experiment this study has three features. First, it has “two faces: prospective and reflective” (Cobb et al., 2003, p.10). The assumption is that Wikis, as collaborative writing tools, support CPS (Collaborative Problem Solving) and MKB (Mathematical Knowledge Building). Thus, the theoretical intent of the study as a design experiment is to investigate the applicability of Wikis to gifted educational practice as a CPS tool. Accordingly, pedagogical design matters rather than technological design.

In designing the experiment, I considered six factors: motivation, mathematical writing, norms of participation, epistemic agency, selection of problems and error-makers, and the design of the pages where problems and solutions are posted. On the

reflective side, the design as a conjecture is to be tested. Combining the prospective and reflective aspects of design experiments provokes “iterative design” with new conjectures (Cobb et al., 2003, p.10). However, this study focuses only on the first experiment in an effort to document implications for iterative design. The next step will be left for later study.

Second, the feature of the design research is “the highly interventionist nature of the methodology” (Cobb et al., 2003, p.10) and this study needs active intervention “to investigate the possibilities for educational improvement by bringing about” online CPS. However, intervention is confined to the assignment of online tasks as homework and will have no direct impact on the face-to-face class. Thus, this study does not address the relationship between online homework and face-to-face class participation.

Lastly, this study aims to develop a local theory about online CPS of gifted students in the Wiki environment, it does so by attempting to identify factors which contribute to the emergence of online JPS (Joint Problem Space) and patterns that emerge from secondary gifted students’ CPS process in JPS.

4.1.1 Anticipated Results From The Design Experiment

In a design experiment, it is necessary to “specify the significant disciplinary ideas and forms of reasoning that constitute the prospective goals or endpoints for student learning” (Cobb et al., 2003, p.11). Although Wikis can serve for online CoI as “a knowledge platform” (Parker & Chao, 2007, p.58), the design of this study focuses on CPS and MKB using the Wiki. An anticipated outcome from gifted student CPS using the Wiki is clarified through a thought experiment, which attempts to draw upon Lakatos’ (1976) spirit.

Suppose that a teacher wants to nurture students problem solving development. However the teacher worries that there are lots topics to cover in class and furthermore since the more difficult problems require more time for students to think

about and work on, the allotted class time is not likely sufficient for deep and thoughtful discussion. If CMS is open for MKB in the internet, s/he does not have to say “Time is up. We have to stop. We’ll continue on Monday” (Lampert, 1990, p.51). Instead, the teacher decides to provide ESMCE (Educational Setting for Mathematical Creativity and Errors) to stimulate CC (collective cognition) by uploading challenging problems and topics on the Wiki site created for his/her math class only; students can collaboratively work on the problems at any time that is convenient and discuss their thoughts with other students beyond the class time limit. The teacher informs students of this plan, and asks them to post their solutions and ideas and freely revise and develop them in the shared world.

Further, suppose the teacher posts the “Gauss problem” in the shared world using the Wiki: Evaluate

$$1 + 2 + 3 + \cdots + n \tag{4.1}$$

At first, a student α suggests his solution as follows:

$$\begin{aligned} 1 + 2 + \cdots + 99 + 100 &= (1 + 100) + (2 + 99) + \cdots + (50 + 51) \\ &= \overbrace{101 + 101 + \cdots + 101}^{50} \\ &= 101 \times 50 \\ &= 5050 \end{aligned}$$

If we take 100 with n and apply the same method to equation (3.1) then,

$$\begin{aligned} 1 + 2 + \cdots + (n-1) + n &= \overbrace{(n+1) + (n+1) + \cdots + (n+1)}^{\frac{n}{2}} \\ &= \frac{n}{2}(n+1) \end{aligned}$$

Another student δ reads this solution and refutes: “ n is not necessarily even”, and revises the solution as follows: (i) If n is an even number then,

$$\begin{aligned} 1 + 2 + \cdots + (n-1) + n &= \overbrace{(n+1) + (n+1) + \cdots + (n+1)}^{\frac{n}{2}} \\ &= \frac{n}{2}(n+1) \end{aligned}$$

(ii) If n is an odd number then,

$$\begin{aligned} 1 + 2 + \cdots + (n-1) + n &= \overbrace{(n+1) + (n+1) + \cdots + (n+1)}^{\frac{n-1}{2}} + \frac{n+1}{2} \\ &= \frac{n-1}{2}(n+1) + \frac{n+1}{2} \\ &= \frac{n^2 + n}{2} \\ &= \frac{n}{2}(n+1) \end{aligned}$$

From (i) and (ii), therefore $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$. Still another student β solves the problem as follows: Let

$$S = 1 + 2 + \cdots + (n-1) + n \tag{4.2}$$

Figure 4.1: An Inductive Conjecture

By rearranging

$$S = n + (n - 1) + \cdots + 2 + 1 \quad (4.3)$$

By adding equation (4.2) and (4.3), $2S = (n + 1) + (n + 1) + \cdots + (n + 1) = n(n + 1)$

Therefore,

$$1 + 2 + \cdots + (n - 1) + n = \frac{n(n + 1)}{2} \quad (4.4)$$

Another student γ makes an inductive conjecture using diagrams in figure 4.1. Student δ leaves γ a note that says: “Wonderful, however, how can this method be always applied to the equation (4.4) for every natural number n ? Do you think that your method guarantees that the equation (4.4) holds for arbitrary natural numbers?” Student γ responds “Why not? I think you are always doubtful.” Student ϵ reads the discussion between δ and γ , and writes: “I think δ wants you to prove this conjecture.” γ says: “Why should I? My method is not a conjecture. If I have to prove this, then I think α and β should do the same thing.” Student discussions continue. The teacher participates in the discussion by posting a new, related, homework problem: Prove $\sum_{k=1}^n k = \frac{n(n+1)}{2}$.

Student λ suggests a proof:

Since $(k+1)^2 - k^2 - 1 = 2k$, for $k = 1, 2, \dots, n$:

$$2^2 - 1^2 - 1 = 2 \times 1$$

$$3^2 - 2^2 - 1 = 2 \times 2$$

$$4^2 - 3^2 - 1 = 2 \times 3$$

$\dots$

$$n^2 - (n-1)^2 - 1 = 2(n-1)$$

$$(n+1)^2 - n^2 - 1 = 2n$$

By adding up above equations,

$$(n+1)^2 - 1^2 - n \times 1 = 2(1 + 2 + \dots + n)$$

$$\text{Therefore, } \sum_{k=1}^n k = \frac{n(n+1)}{2}$$

Student ζ leaves a link connected to a site having the proof of their solution.

As a result, students learn a method of proof: the mathematical induction.

This example illustrates a way to build mathematical knowledge utilizing a Wiki environment. In terms of growth of School Mathematics (SM), α , β , and γ contribute different solutions to the problem (parallel pattern), and γ 's conjecture and ζ 's proof are structurally connected to each other (thread or linear pattern). It illustrates learning mathematics through experiencing local growth of SM in CMS using the Wiki. The problem that will be addressed in this study is how to design the Wiki environment to support and encourage this type of collaborative and mutual learning.

4.1.2 A Pilot Study

The purpose of the pilot study was to check whether or not ESMCE can create JPS using the Wiki and to investigate what is going on in JPS. The pilot experiment was conducted over 6 days and involved three participants.

S Science High School (SSHS) and Its Wiki system

The pilot study was conducted from June 30 to July 5, 2011 at S Science High School (SSHS) in Seoul, Korea, which is a boarding school for gifted students in math and science. The school had been running a server devoted exclusively to the Wiki system since September, 2008. The server was installed in the school for the purpose of research from January, 2008 to December, 2009, supported by Seoul Metropolitan City Department of Education. The focus of the research was to study to activate teaching and learning through the school homepage in particular using the Wiki. Since all teachers and all freshmen in 2008 and 2009 in SSHS participated in the research and some teachers were using the Wiki for their classes after the research period, some students had experience of using the Wiki in their classes when the pilot study started. This provided a good environment to conduct the pilot study.

The server in SSHS adopted LYNIX as the operating system and MediaWiki. A system administrator of the school cared for the server. Although MediaWiki supports LaTeX for mathematical writing, to support the function of writing Korean & special symbols and provide a pallet for mathematical expression, additional software was installed in the Wiki system: Kotex, Tex Box, CharInsert, Embedded Document, PaserFunctions, SyntxHighlight, Wiki2LaTex; The Wiki system at SSHS also supported 2D and 3D graphs. On the Wiki site, all the subjects taught in SSHS had their own pages. In the case of mathematics, the site was mainly used for repository space of mathematical contents, which mathematics teachers posted for their students.

Figure 4.2: The Problem Posted in The Pilot Study

Procedure for Pilot Study

A calculus problem (see Appendix 1), which had been suggested several years ago as a sample problem for the entrance exam of a university in Seoul, Korea, was used for this pilot study. The problem is about modeling a ship oil leak accident on the sea by using integral and binomial expansion. This problem consists of three subproblems; one asks students to state the suggested model and give their opinion about whether the model works for anticipating the situation. The other two problems are to calculate the anticipated time when the leaked oil will arrive at the sea shore based on the suggested model. One of math teachers at SSHS recruited participants and posted the problem on the Wiki system on June 30th, 2011 (figure 4.2) with the following sentences:

Read the following problem and write your solutions. You may revise others' solutions and raise questions about their solutions. When you revise others' solution, use "editing" (button on the menu bar)¹. And when you raise questions about solutions, use "discussion" (button on the menu bar).

- Period: Jul. 1 ~ Jul. 5
- Norms

¹I added the parenthesis for readers' understanding because it is written on the top line in Korean in the figure 4.2.

- Ms. Yangsoo will reward the first solver.
- Participants need to edit solutions more than 7 times except correcting typos.

Results and Implications

Two students out of three posted their solutions to the problem on the Wiki. A third student submitted solutions to the three subproblems written on paper to the teacher. Later, the teacher scanned these solutions into PDF files and posted them on the Wiki. Although the data for this pilot is limited to participants' solutions stored in the shared world, I elicit some implications for the present study. Above all, students had no experience in writing mathematical expression using LaTeX even though they might have used the Wiki in other classes such as History, Sociology, English. There were two ways to type mathematical expressions in the Wiki environment of the school; using "help" on the left of the given problem and using a palette for math expression in the editor's window. Students might have learned basic LaTeX command words for their solutions with "help," which connected to information pages for writing mathematical expressions. Using those words, they could have typed mathematical expressions needed in their solution. However, since students needed to put effort and time to learn LaTeX words, they were not expected to do it. The second way was easier. When students were editing, they could use a "mathematical expression" button, which showed a palette for math expressions. Students could type mathematical expressions just by clicking each expression button needed in their problem solving process. However, neither of the students posting in the Wiki used this tool. As a result, the first problem solver's poor mathematical writing made it difficult for others to understand his solution in the shared world. Thereby, the second student just added his solution below the first one rather than attempting to edit or revise it, even though the teacher asked them to edit or revise the others'

solutions. There was no interaction between the users in shared world and thus CC was not activated in CMS.

Second, students needed to create JPS for mathematical interaction. While the potential existed for students to notice and examine differences among their understanding of the conditions of the problems, they did not investigate others' ideas. It has been shown that "what determines the success of the collaborative learning experience is the interactional manner in which this intersubjective problem space is created and used" (Sarmiento-Klapper, 2009a, p.90). Third, they were confronted with the difficulty of communicating to each other via LaTeX. This difficulty of expressing mathematical thought as math formula and graphs in the Wiki hindered students' ability to communicate their thinking. Furthermore, the two students who posted their solution in the shared world, did not suggest graphs for $f_1(x)$, $f_2(x)$, $f_3(x)$. Indeed, they may have drawn the graphs on paper. However, there is no data for ascertaining whether or not they did so. Fourth, the third student misunderstood the condition of the problem. He did not use the given condition of $f_1(x)$ and did not draw graphs of $f_1(x)$, $f_2(x)$, $f_3(x)$.

In the pilot study, ESMCE was able to be embedded in the shared world since the given problem was sufficiently challenging for participants to collaborate; Their solutions were erroneous and revealed authors' misunderstanding of the problem. However, CPS in JPS did not occur. That is, students did not edit or revise peers' solutions.

According to Litz (2007), factors of students' resistance to online learning are: lack of motivation, aversion to technology, reluctance to gain proficiency in courseware, and sense of isolation in the absence of a traditional classroom. In regards to two of the factors: motivation and proficiency in courseware, the pilot study suggests several implications for the present study. It is not surprising that Korean high school students are reluctant to participate in the experiment because they are so

busy with their studies. In this respect, a critical focus in designing a Wiki learning environment is in motivating students to participate in the experiment. In addition, participants did not voluntarily learn the technical skills needed to use the Wiki system such as editing and using LaTeX. Students' lack of experience using LaTeX and the Wiki's editor was a huge stumbling block in enhancing mathematical communication in the Wiki. Thus, in designing the present study, it is necessary to support students' learning LaTeX on a basic level and the necessary functions of Wikis. In addition, participants need to be accustomed to reading and editing others' solutions. It is known that in the beginning, students in Wiki-based classes are reluctant to edit others work (Ben-Zvi, 2007).

The asynchronous nature of Wiki writing takes a long time for participants to do. Probably, the five day period of the pilot experiment was too short for participants to interact. Therefore, the period of the experiment needs to be sufficiently long for participants to learn basic skills for mathematical interactions and grow accustomed to reading and editing others' hypertexts in the Wiki. Therefore, the present study is designed on a course level for a semester period and starts with providing opportunities for participants to learn how to communicate in the Wiki. In addition, problems used to embed ESMCE in the shared world are succinct for better concentration and initial participation.

4.2 A Design of The Experiment

The primary task of the present study is to design ESMCE to support gifted students' collaboration in mathematical problem solving in the Wiki environment. In this respect, it needs to consider "the logical and practical *preconditions* for students to" engage in CPS using the Wiki (Stahl, 2009e, p.525). Based on the results of the pilot study and Garrison's Practical Inquiry Model (figure 2.2), to activate students' editing and revising their solutions emergent from ESMCE embedded in the shared

world, I considered six factors: motivation, writing practice of LaTeX, norms of participation in setting climate, epistemic agency for supporting discourse, problems and error-makers in selecting content, and the design of problem and solution pages.

4.2.1 Motivation

“In naturalistic settings, it is known that it is hard to sustain collaborative learning in online communities of learners because of discontinuities or gaps in group interaction over time” (Sarmiento-Klapper, 2009a, p.90). Motivation is a primary factor to bridge these gaps. Litz (2007, p.3) studied motivation that can move students “from positions as individually oriented problem solvers to online problem solving collaborators.” She identified four factors: teachers’ encouragement for students’ participation, incentive such as extra credits in math courses, difficulty in using software, and uncommonness of the software. Peterson (2009) reported that her students were highly motivated to contribute two or three entries per block to the glossary in the Wiki because it benefited students in two ways: a small amount of points were assigned to the task; and glossary activities were beneficial to students in relation to their upcoming exam in the course. Thus, to secure the teacher’s continuing encouragement for students’ participation and motivating students with some incentives, the design experiment needs to be embedded within a required course with assigned problems for homework (figure 4.3). Since the course is face-to-face based and the Wiki is used for doing homework, students are not isolated from other students.

4.2.2 Writing Practice For Mathematical Expression in the Wiki

In spite of students’ experience using Wikis, the pilot study revealed that students might not be accustomed to writing mathematical formula using LaTeX. In the pilot study, students failed to create JPS and the Wiki was functioning only as a repository space for separated individuals’ solutions because of incommunicability

Figure 4.3: A Design Level of The Study

Figure 4.4: Editor's Window of The Wiki

between students. The first stage of the experiment is for students to learn how to use buttons on the editor's window (figure 4.4) and how to type mathematical expressions using LaTeX in a computer room. At least, one to two class periods (50-100 minutes) are allotted for this training.

For students' easy acquisition and use of writing skills of LaTeX in this experiment, five mathematical expressions, which are most frequently used in a calculus course, were selected (figure 4.5) and taught in the class. In a computer room, students practice LaTeX by editing erroneous solutions. The exercise problem is given

간단한 수식 목록(LIST FOR WRITING MATHEMATICAL EXPRESSIONS)

입력	출력
<code>\frac{3}{2}</code>	$\frac{3}{2}$
<code>{ }_n C_k</code>	$n C_k$
<code>\sum_{k=1}^N k^2</code>	$\sum_{k=1}^N k^2$
<code>\lim_{n \rightarrow \infty} x_n</code>	$\lim_{n \rightarrow \infty} x_n$
<code>\int_1^3 x^2 dx</code>	$\int_1^3 x^2 dx$

Figure 4.5: A Short List For Mathematical Expressions Posted in The Wiki

as follows:

The following is a problem and a student's solution from a quiz of a calculus course in a foreign country. Reading the solution, you can find that it has the correct answer but errors in the process of calculation. You might have experienced similar things when you solve problems. In such cases, we can help each other fix the erroneous solutions. In addition, since different solutions can be discovered while solving a problem, collaborative problem solving is beneficial to anyone. Lets practice to revise the erroneous solution.

When students click a link below the paragraph, students can see a problem. Twenty independent pages are prepared for individual practice space. Through this practice, participants can envisage what to do in the Wiki and reduce reluctance for editing others' solutions. This practice is also expected to increase students' familiarity of the Wiki site (e.g., the address of the site, experience of editing, using LaTeX, structure of Wiki pages).

4.2.3 Online Community of Inquiry (CoI)

The goal of "wiki learning" is for participants to contribute to each other's learning through CPS and thus to become members of a CoI (Ruth & Houghton, 2009). However, "creating a sense of community is a complex challenge for the teacher and students" (Ben-Zvi, 2007, p.10). To build online CoI in the Wiki based model, I consider three factors: norms of participation for setting climate, epistemic agency

for supporting discourse, problems and structuring ESMCE with two types of Explicit Contradictory Situations (ECS) and Implicit Contradictory Situations (ICS) for selecting mathematical contents.

Norms of Participation

In the beginning of Wiki-based classes, students contributed their solutions and showed a reluctance to edit or revise other students' writing by considering "it rude" and the teacher as the only one who has power of authority to do it (Ben-Zvi, 2007, p.11). To change this atmosphere, Ben-Zvi (2007) suggested whole-class discussions for establishing class social norms, which helped his "students to gradually get over this hesitation and start exploring a review – discuss – suggest – revise process in their writing to synthesize individual contributions and improve groups' products" (p.11). In this respect, class time is needed for whole class discussion and for the teacher to set up ground rules of participation through a negotiation before beginning the experiment. For this study, Norms of Participation were prepared before the whole class discussion as follows:

- You should not read others' solutions before you read and solve problems by yourself.
- The Wiki is an open editing system. Thus, anyone who thinks the posted solution is not right can freely edit or revise others' solutions.
- In some cases, problems may contain errors. Thus, you may edit or revise problems as well as others' solutions. In addition, you may post problems that you could not solve by yourself or that you created.
- If you are the first solver, then just post your solution.
- When you post your solution, let others know it is yours by using the button of "Your signature with timestamp," which is located at the second from your right on the editor's window.
- Since the signature shows your IDs, you do not have to feel shame or fear even though your solutions are only immature ideas or may have errors.
- If there is others' solutions, then look into others' solution before posting yours on the solution pages.
- If the ideas used in others' solutions are similar to yours, then develop the solutions by editing and revising them.

- If others' solutions are different from yours, then press the button of "Horizontal line," which is placed at the first from your right, and add your solutions below the horizontal line.
- If you have any question while reading others' solution, then post your questions.
- Whenever you use outer sources such as books and contents on the Internet, leave references or links.

This list of proposed norms was provided to the teacher of the target class and s/he was encouraged to work with her/his students to set up shared social norms before CPS begins in the Wiki. In addition, at the end of experiment, students were asked to evaluate other students' contributions to the local growth of SM and the teacher also evaluated students' ideas and contributions.

Epistemic Agency

In designing online CoI, "release of agency" is one of guiding principles (Scardamalia, 2002). "Epistemic agency refers to the amount of individual or collective control people have over the whole range of components of knowledge building – goals, strategies, resources, evaluation of results, and so on" (Scardamalia & Bereiter, 2006, p.109). Thus, the development of agency is "an intrinsic part of any learning environment" (Boaler & Greeno, 2000, p.171) and critical to support mathematical discourse via the Wiki.

Epistemic agency describes "acts of initiatives taken by students" to post their ideas and to negotiate with others' solutions by editing and revising them in the Wiki (Charles & Shumar, 2009, p.209). According to Lave and Wenger (1991, p.98), power relation is one of factors defining "possibilities for learning (i.e., for legitimate peripheral participation)." In this respect, an equal distribution of power and knowledge is critical to support CPS (Goos & Galbraith, 1996). Wikis afford 'Open Editing' functions that equally distribute powers over members of online CoI. However, the power relation in the classroom may be transferred to CMS because this experiment

is designed on a course or class level. In face-to-face math class, interactions for collaborative problem solving are strongly influenced by students' bias such as "David regarded himself as more competent than Rick" (Goos & Galbraith, 1996, p.257) or "because that's what Tommy said, and he's usually right" (Lampert, 1990, p.56). To prevent this influence, participants will anonymously register for the Wiki site using only IDs. The anonymity in the Wiki is a very important element for this study because it makes it possible for error-makers to play a role in collaborative problem solving processes (see the next section).

Second, students' "willingness to see themselves as members of a community" in turn support their community identity (Charles & Shumar, 2009, p.209). In this respect, Brett, Nason, and Woodruff (2002) see that community identity and epistemic agency mutually constitute the students' engagement in community discourse. The community identity increases "collective cognitive responsibility" for activating the MKB process (Scardamalia, 2002). Since Wikis provide a "signature" function in writing, every change in writing is accompanied by identified authorship. This will help students establish their identities as members of online CoI. In this respect, participants are asked to use the "signature" function whenever they post new solutions or revise texts.

A participant as a member of online CoI in this study is expected to have responsibility for MKB by developing competency, posting new tasks and related resources for further mathematical inquiry, evaluating own and others contribution to local growth of SM, and setting up goals and strategies together etc. To enter into CMS in the Wiki, participants need to be responsible for their understanding of problems. In this respect, students are asked not to read others' solutions before completing their attempts to solve the problem for themselves. This is necessary to avoid students' "impulsiveness" in face-to-face CPS (Goos & Galbraith, 1996).

ESMCE and Two Types of Contradictory Situations

Since students' collaborative problem solving may initially depend on the nature of the problems posted, the selection/development of appropriate problems for collaboration is important. Nason and Woodruff (2003; 2004) argue that most school mathematics problems do not provide students adequate opportunity to collaborate because they focus on correct answers. In addition, it is reported that students in VMT chat "tend to jump to finding one solution that works rather than taking the opportunity to search for alternative solutions" (Cui, Kumar, Chaudhuri, Gweon, & Rosé, 2009, p.336). In this respect, open problems are better than closed ones for collaborative work (Cui et al., 2009). However, according to Diezmann and Waters (2001), mathematically gifted students collaborate when the task is sufficiently challenging. Thus, I choose typical but challenging calculus problems for this study.

In terms of the private world (figure 2.3), the individual students' goal is to successfully solve assigned problems. Creativity and errors in problem solving have been considered as signs of individual students' mathematical ability. For example, the Gauss' creative solution exemplifies his mathematical genius; whereas students' errors are treated as symptoms of misunderstanding or not knowing something and analysis of errors has been helpful for diagnosis, remediation, and successful problem solving in the end (Borasi, 1994; 1996).

In contrast, in terms of the shared world, students' mathematical products emergent from ESMCE are objects of questioning about their assumptions, methods, strategies, and generalization etc. Those questions are results of students' critical and reflective activities in scrutinizing their products in the shared world. This is fuel for SM as products to evolve in the shared world. However, the pilot study implies that even though embedding ESMCE in the online space is possible, students may not raise such questions and thus students' products emerging from CC does not evolve automatically in the shared world. Therefore, in designing the study, I

focused on how to stimulate students' critical and reflective activities in CMS, given that participants can mathematically communicate with each other. To structure ESMCE to lead students' activity gradually into MKB, I adopt fallibilists' view:

The difference between the amoeba and Einstein is that, although both make use of the method of trial and error elimination, the amoeba dislikes erring while Einstein is intrigued by it: he consciously searches for his errors in the hope of learning by their discovery and elimination. The method of science is the critical method. (Popper, 1989. p.70)

To use errors as a tool for stimulating CC, I specify two types of contradictory situations for this study: Implicitly Contradictory Situation (ICS) and Explicitly Contradictory Situation (ECS). ICS implies, in a broad sense, fallibilists' view; Mathematical knowledge is potentially fallible and thus can be developed further through reflective and critical activities. That is, all products emergent from ESMCE create ICS. In a narrow sense, ICS includes erroneous proofs or solutions in the history of mathematics, which were refuted and whose errors were found later. For example, Cauchy's proof of a conjecture, which says that "the limit of any convergent series of continuous function is itself continuous" (Lakatos, 1976, p.128), was accepted as right before it was found not to be true. Such historical examples can be used for educational purpose to create ICS in math class. Also, recall the example of MATHCOUNTS. The prepared solution of the MATHCOUNTS problem was accepted as right within CoI before a student's new solution came up. This characterizes ICS. As soon as the old solution is challenged with a new creative solution, ICS turns into ECS and other activities of CoI follow to resolve ECS. In this respect, it is important to design the experiment by structuring ESMCE around errors to generate energy for students to continuously engage in MKB.

To structure ESMCE, I purposefully insert errors in the shared world, which are expected to cause cognitive conflict to CC and thus break cognitive equilibrium in CMS. I differentiate ECS into two types. The 1st type of ECS is a proof or a solu-

tion leading to a conclusion that directly contradicts common background knowledge within CoI. In the example of Cauchy's proof, ICS was turned into the 1st type of ECS when a global counterexample was found. The 1st type of ECS triggered proof analysis within mathematicians' CoI.

On the other hand, as error-makers, I post erroneous solutions with different results. Since different results for a mathematical problem cannot be simultaneously right in most cases, they cause tension in CMS and thus create the 2nd type of ECS. This situation is expected to trigger students' finding-error activities by comparing, analyzing, and revising solutions. In the example of MATHCOUNTS, ICS was turned into the 2nd type of ECS where two solutions with different answers of 19 and 190 were posed and competed with each other. Tension between two solutions generated the driving force for CoI to settle the contradictory situation by investigating solutions and thus finding and removing errors. Both examples, however, show that the transition from ICS to ECS did not happen easily within CoI.

In terms of deploying problems, I consider three steps to induce students' gradual participation in MKB: (1) The 1st type of ECS: Problems include erroneous solutions leading to contradictory conclusions such as " $0 = 1$ "(problem 1-2). Since such a result contradicts common knowledge shared within the CoI and thus is obviously wrong, it provokes cognitive conflicts in CMS. Thus, the 1st type of ECS elicited from such conclusions stimulates individual cognitions to focus on investigating the given solution and finding errors in it. (2) The 2nd type of ECS: To cause tension among solutions, I play a role of error-maker, posting erroneous solutions before or after students post their solutions in the Wiki. Since students register for the Wiki site only with ID's, error-makers' solutions are expected to be equally treated as other students' solutions within CoI. Error-makers' activity is expected to trigger students' activities such as comparing and analyzing solutions, and finding and removing errors. (3) ICS: Some problems are posted without any erroneous solution or error-makers'

activity. Students are expected to change ICS into ECS by testing, exploring, and investigating others' solutions without any interference such as error-makers' activity.

4.3 Participants

4.3.1 G Science High School (GSHS)

GSHS was established in 1983 as a boarding school for gifted students in mathematics and science in Suwon, Korea. Grade 10 to 12 students attend GSHS. In spring, 2010, the legal status of GSHS was changed from a local school to a national school. For the present study, this is relevant for two reasons: First, GSHS recruits talented students from across Korea. Therefore, it is not appropriate to compare students' mathematical ability in GSHS with other Korean high school students. However, it can be said that GSHS ranks in the top 10% of Korean high schools in mathematical ability because only 120 students are selected among around 1500 applicants every year and a primary criteria for admittance is evidence of giftedness in science and mathematics. Second, GSHS does not have to follow the Korean national curriculum. In this respect, calculus courses in GSHS are different from those based on the Korean national curriculum. Teachers can freely adopt any mathematics books as textbooks.

4.3.2 Teacher

Mr.O is pursuing a doctoral degree in mathematics education from one of the universities in Seoul, Korea, from which he acquired his master and bachelor degrees. He has been teaching mathematics for 14 years. He started to work at GSHS in spring 2008 and had no experience using Wikis during his previous teaching career. In spring 2012, Mr.O opened two sections of Calculus II. The course prerequisite was the completion of Calculus I, in which students learned the first half of "Thomas' Calculus: Early Transcendentals" (Thomas et al., 2010). Students took Calculus I in spring, 2011. Ten students enrolled in one section and six students enrolled in the

other section.

Mr.O's philosophy of these classes is based on students' discussion and communication in math class rather than by telling. While being interviewed at the end of the experiment, he described his philosophy on learning mathematics as participation. When asked which is more important, individual learning or collaborative learning, he replied:

"I think that collaboration is essential (in learning mathematics). While engaging in problem solving, students may not notice it, but they are using things in problem solving such as symbols, which someone created and were agreed to being used, and they are appropriating even thinking processes or methods, which are similarly made, thus, a community (of learners) can be said to be a mathematical community. Entering into a frame or process of thinking that such a community has built is to learn mathematics and to study mathematics and so (entering into) the process means discussing while collaborating and listening to others rather than entering into the pre-established world, and so it (discussing while collaborating) is a part of active participation rather than passive one. I think that it is real learning."

4.3.3 Students

The Calculus II students at GSHS were informed of the experiment when they enrolled. Mr.O distributed consent forms at the beginning of the course and guided students to decide whether or not participate. All of the sixteen students enrolled in Calculus II courses agreed to participate but two of them did not participate even though they had registered with the Wiki site. Thus, the actual number of participants was 14. Participants ranged in age from 17 to 18. Two of the 14 participants were female.

4.4 Procedure

I created the basic frame pages in the Wiki site such as main pages, norms of participation, and practicing mathematical writing using LaTeX. While constructing the site, I conferred with Mr.O via text, e-mail, and telephone to decide on the

2012	Feb			Mar			Apr			May			Jun		
Spring SEM				›	›	›	›	›	›	›	›	›	›	›	›
Const. of Site	›	›	›	›	›	›									
Problem Set 1							›	›	›	›	›	›	›		
Problem Set 2										›	›	›	›	›	
A Project									›	›	›	›	›	›	
INTV & SRVY														›	›

Table 4.1: A Time Table for The Procedure

selection of problems (e.g., how many problems to be posted, when to post them, and so on). Participants worked collaboratively for about 70 days (table 4.1). At the end of the spring semester, I interviewed Mr.O and 9 students. Mr.O and the students were asked to evaluate students' contribution to the local growth of SM. In addition, students were surveyed about their experience of using the Wiki as a problem solving tool (see Appendix 3).

4.4.1 Constructing A Site of Wiki

I created the site "gshs.educatewiki.com" for the experiment at educatewiki.com whose server was located in the U.S. It was possible for students to type Korean alphabet but everything was run in English at educatewiki.com. Furthermore, palette for mathematical expressions was not supported. In these respect, constraints of the Wiki were increased when compared to the system run in SSHS. Since anyone could visit the site differently from the site used in the pilot study, outside people could come to the site. However, I thought this a rare possibility.

In constructing the site, I adhered to the principle that the site should be simple and sufficiently informative for any new visitor to be able to know what to do and how to do it. On the main page, information about how to edit in the Wiki was uploaded as a PDF (Appendix 4). Each problem was separately assigned to a page. Students' solutions were to be posted on a solution page independently from its problem page because if the solution is placed below the problem statement, it

may distract students' focus while they are reading the problem. Thus, I made a link to a separate solution page with respect to problems under the problem statement.

4.4.2 Conference With Mr.O and Change of Plans

I corresponded with Mr.O before and during the experiment via texting and e-mail. Based on this communication, I kept informed about the course schedule and the extent to which the computer room was available for his class. While communicating, we agreed on several changes as follows:

- **Numbers of Problems:** Mr.O thought that his students were too busy to visit the site every week. So he asked me to use only five problems and to post them at once rather than post 1-2 problems every one or two weeks. Thereby, I changed the format of the problem posting from “Problems of The Week” to “Problem Set 1” that consisted of 5 problems. At first, it was planned that there would be only problem set 1 to be posted because of students' busy situation. About one month later, Mr.O asked me to post the other problems on the list of problems that I had sent to him before because students wanted more problems. So I posted Problem Set 2.
- **A Format of Posting Solutions:** Mr.O wanted to count the number of individual students' postings on a solution page. Although this can be seen in one's contribution page, which Wikis automatically provide, he wanted to see who edited solutions on solution pages. In this respect, students were asked to copy others' solutions and paste them below the solution and to edit the copied one. This method were expected to show roles of participants such as a refuter, a checker, consenter, suggester, etc. rather than results of evolving solutions as oeuvres. This is the most critical change from the initial design because the function of ‘open editing’ was used in a restrictive way.
- **A project of Proving The Isoperimetric Theorem:** Mr.O explained particular interest in the problem of proving the Isoperimetric Theorem and thought that the project would provide meaningful opportunities for students to learn. However, he told me that his class was based on discussion and the courses might end without students' learning Green's Theorem, which is placed in the last part of the textbook. Later, we decided that I would build pages for the project in advance even though students might not learn Green's Theorem in classes (see Table 4.1).
- **Discussion for Norms of Participations:** Mr.O could not spare class time on discussions for setting up norms of participations because it was estimated that the class time was not enough to cover the entire contents in the textbook. As a result, students did not discuss norms of participation in their class.

- **Practicing Editing and Mathematical Writing:** In the initial plan, one or two periods of class time were supposed to be allotted for students' practicing editing and mathematical writing with LaTeX. However, my school was far from GSHS and it was impossible for students to take extra classes for practicing after school. In this respect, Mr.O said that he could teach students in classes after he learned how to edit and write mathematical writing using LaTeX. So he practiced editing and writing in the Wiki through online chatting with me for about an hour. Later, he could not find a computer room available in his class time. Thus, he asked me to upload a PDF file explaining how to edit and write mathematical expressions using the LaTeX on the main page (Appendix 4). As a result, there was no class for students to practice LaTeX.
- **Evaluation Paper:** In the initial plan, students were supposed to evaluate others' contributions to the growth of SM on evaluation paper in a class because all students needed to participate in the evaluating process. However, their course schedule did not allow it. Therefore, students were asked to evaluate contributions by responding to online survey questions after school.
- **Mr.O's Role:** In the initial plan, Mr.O was asked to participate in MKB. That is, he was supposed to lead the MKB process by suggesting hints and feedback to make progress on the problems and raise questions about students' solutions for further development. However, he was too busy to come by the site regularly and to post his comments on students' work. This caused a big conflict for me because I had to choose my role as researcher or teacher.

4.4.3 Process of Embedding ESMCE in The Shared World

Mr.O explained that his teaching method emphasized student presentation and discussion in class. If he assigned problems to students as homework in advance of class, they were expected to be prepared to present their solutions and discuss them in the next class rather than submitting them in written form to him. In terms of evaluation, Mr.O assigned students scores according to their participation. To give students more chances to participate in discussion, Mr.O encouraged students to participate in CPS using the shared world after school. That is, a posting in the shared world would be equivalent to one in-class discussion. In addition, he made it clear to students that problems posted in the shared world would not be discussed in class.

Mr.O reviewed 12 problems drawn from Barbeau (2000) and Williams and

Hardy (1988). Mr.O investigated them in terms of whether or not they were challenging for his students. The problems prepared for the experiment needed to have an appropriate level of difficulty to embed ESMCE in the shared world; if too simple, then students' motivation to work on solutions will not happen; if too hard, building knowledge and identifying solutions may be discouraged. On the other hand, the problem statement needed to be short to enhance student concentration as suggested from the result of the pilot study. Although erroneous solutions from Barbeau (2000) were intriguing, most problems were typical and not so challenging. In this respect, I was worried that problems might not create ESMCE. Mr.O replied to me that problems would be somewhat easy for students because they had studied these topics but the erroneous solutions might stimulate students' mathematical thinking and create ESMCE.

Two Problem Sets and A Project

For this study, I used two sets of problems and one project. Each problem set consisted of five problems and a project to prove Isoperimetric Theorem using Green's Theorem. I deployed problems following three steps to structure ESMCE: 1st type of ECS, 2nd type of ECS, and ICS. According to the conference between Mr.O and I before the experiment began, I posted a set of five problems for ECS on March, 24. Problem 1-1 and 1-2 include solutions with erroneous results ("a function x^n is a constant for every natural number n " and " $0=1$ ", respectively) and thus were expected to create the 1st type of ECS. In problem 1-3, 1-4, and 1-5, error-makers worked to create the 2nd type of ECS.

Responding to the request for additional problems from Mr.O, I posted Problem Set 2 that also consisted of five problems on April, 27. In posting problems, I followed the three steps of structuring ESMCE. Problem 2-1 including an erroneous solution with a result of " $\cosh x = \sinh x$ " for the 1st type of ECS. It is similar to

		Intended Type	Error- Maker	Constructed Type	Posted Date	Students' 1st posting
PROB Set 1	1-1	1st ECS	N	1st ECS	Mar, 24	Mar, 26
	1-2	1st ECS	N	1st ECS	Mar, 24	Mar, 26
	1-3	2nd ECS	Y	<i>2nd ECS</i>	Mar, 24	Mar, 26
	1-4	2nd ECS	Y	2nd ECS	Mar, 24	Mar, 26
	1-5	2nd ECS	Y	2nd ECS	Mar, 24	Mar, 28
PROB Set 2	2-1	1st ECS	N	1st ECS	Apr, 27	May, 2
	2-2	2nd ECS	Y	1st ECS	Apr, 27	Apr, 30
	2-3	2nd ECS	Y	2nd ECS	Apr, 27	Apr, 28
	2-4	ICS	N	1st ECS	Apr, 27	Apr, 28
	2-5	ICS	N	1st ECS	Apr, 27	May, 8
PROJ	3.3.1	Reading	N	–	Apr, 2	Mar, 26
	3.3.2	ICS	N	1st ECS	Apr, 2	Apr, 15
	3.3.3	1st ECS	N	1st ECS	Apr, 2	Apr, 27
	3.3.4	ICS	N	–	Apr, 3	May, 29

Table 4.2: Types of Problem Set 1 & 2 and Project

problem 1-2, which is about the integral constant in applying the integral by part. Error-makers were working for problem 2-2 and 2-3. Since problem 2-2 was beyond the students' knowledge boundary, students could not post their solution. As a result, in spite of an error-maker's solution, the 1st type was created rather than the 2nd type of ECS. Since students were not likely to learn Green's Theorem in their class, problems for ICS were needed. In this respect, problems 2-4 and 2-5 were given for ICS as typical close ended problems, which can be found in many Calculus textbooks. Problem 2-4 was an exercise to apply Green's theorem and the final destination of intellectual enterprise was still to prove the Isoperimetric Theorem by applying Green's theorem.

4.4.4 Interviews and Surveys

I interviewed Mr.O on June, 13 in a teacher's tea room. I interviewed 9 students in a mathematics classroom of GSHS from June, 20 to June, 26. The focus of the interviews was on students' solutions (see Appendix 3). Thus, I printed out all solution pages stored in the Wiki. In the interview, students could see what they

[View logs for this page](#)

Compare selected revisions

posted on the printed pages. Before the interviews, I compared every two adjacent edits (figure 4.6) and printed out pages showing changes of every two adjacent edits (figure 4.7). Based on the printed pages as shown in figure 4.7, I made tables for every edit of the pages (table 4.3). To interview students more effectively, I repeatedly read tables and checked the printed solution pages while interviewing students. After the interview, these tables were used for coding to draw cognitive maps for each set of solution pages.

Some students had difficulty remembering their postings, which had been written one or two months before interviews. In such cases, I showed them printed solution pages of their writings. In problem 2-5, some of students were asked to correct some errors in their postings on the printed pages of solutions with pencil-and-paper.

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문제2-3의 풀이

From CollaborativeProblemSolving

(Difference between revisions)

Revision as of 09:02, 28 April 2012 (edit)

Jihoonc2 (Talk | contribs | block)

(Created page with "~~~~~$u=\cos x$라 놓자. <math>\int_0^{\frac{3}{4}\pi} \frac{\sin x}{1+\cos^2 x} dx = \int_1^{-\frac{\sqrt{2}}{2}} \frac{-\frac{1}{1+u^2}}{\frac{1}{1+u^2}} du = \int_{\frac{\sqrt{2}}{2}}^1 \frac{1}{1+u^2} du = \frac{1}{2} \left[\tan^{-1} u \right]_{\frac{\sqrt{2}}{2}}^1 = \frac{1}{2} \left(\tan^{-1} 1 - \tan^{-1} \frac{\sqrt{2}}{2} \right) = \frac{1}{2} \left(\frac{\pi}{4} - \frac{\pi}{4} \right) = 0") (show/hide)
← Older edit

Line 1:

```
--[[User:Jihoonc2|Jihoonc2]] 19:02, 28 April 2012 (EST)
<math>u=\cos x</math>라 놓자.
<math>\int_0^{\frac{3}{4}\pi} \frac{\sin x}{1+\cos^2 x} dx = \int_1^{-\frac{\sqrt{2}}{2}} \frac{-\frac{1}{1+u^2}}{\frac{1}{1+u^2}} du = \int_{\frac{\sqrt{2}}{2}}^1 \frac{1}{1+u^2} du = \frac{1}{2} \left[ \tan^{-1} u \right]_{\frac{\sqrt{2}}{2}}^1 = \frac{1}{2} \left( \tan^{-1} 1 - \tan^{-1} \frac{\sqrt{2}}{2} \right) = \frac{1}{2} \left( \frac{\pi}{4} - \frac{\pi}{4} \right) = 0</math>
```

Revision as of 09:02, 28 April 2012 (edit) (undo)

Jihoonc2 (Talk | contribs | block)

(show/hide)

Newer edit → [Mark as patrolled]

Line 1:

```
+ <math>u=\cos x</math>라 놓자.
+ <math>\int_0^{\frac{3}{4}\pi} \frac{\sin x}{1+\cos^2 x} dx = \int_1^{-\frac{\sqrt{2}}{2}} \frac{-\frac{1}{1+u^2}}{\frac{1}{1+u^2}} du = \int_{\frac{\sqrt{2}}{2}}^1 \frac{1}{1+u^2} du = \frac{1}{2} \left[ \tan^{-1} u \right]_{\frac{\sqrt{2}}{2}}^1 = \frac{1}{2} \left( \tan^{-1} 1 - \tan^{-1} \frac{\sqrt{2}}{2} \right) = \frac{1}{2} \left( \frac{\pi}{4} - \frac{\pi}{4} \right) = 0</math>
+
+ --[[User:Jihoonc2|Jihoonc2]] 19:02, 28 April 2012 (EST)
```

Revision as of 09:02, 28 April 2012

$u = \cos x$ 라 놓자.

$$\int_0^{\frac{3}{4}\pi} \frac{\sin x}{1+\cos^2 x} dx = \int_1^{-\frac{\sqrt{2}}{2}} \frac{-\frac{1}{1+u^2}}{\frac{1}{1+u^2}} du = \int_{\frac{\sqrt{2}}{2}}^1 \frac{1}{1+u^2} du = \frac{1}{2} \left[\tan^{-1} u \right]_{\frac{\sqrt{2}}{2}}^1 = \frac{1}{2} \left(\tan^{-1} 1 - \tan^{-1} \left(\frac{\sqrt{2}}{2} \right) \right) = \frac{1}{2} \left(\frac{\pi}{4} - \frac{\pi}{4} \right) = 0$$

--Jihoonc2 19:02, 28 April 2012 (EST)

[Mark as patrolled]

Retrieved from "http://gshs.educatewiki.com/wiki/%EB%AC%B8%EC%A0%9C2-3%EC%9D%98_%ED%92%80%EC%9D%B4"

Figure 4.7: Changes Between Two Adjacent Edits

Seven participants responded to both survey questions and evaluation questions (see Appendix 3). Since the rate of response was low 50%, Mr.O suggested that I send the survey questions and evaluation questions to students one more time in August. When I sent them again, there was one more respondent to both questions. However, he replied to all evaluation questions as "I cannot remember." So I left his response out of the data. Two other students did not offer their reasons for evaluating contributors with respect to problem 1-1 to 1-5, and did not responded with respect to problem 2-1 to 2-5, either. One other student responded with respect to problem 2-1 to 2-5 as "I can't evaluate because I did not solve them." As a result, I collected 8 responses to survey questions and 4 responses to questions of evaluating contributors to the growth of SM.

4.5 Data Collection

In this study, students were asked to actively edit, revise, and evaluate others' solutions and to discuss suggested changes. Entire texts that students and I created and revised were stored with the information and time log in the Wiki system. As a result, I collected five types of data: students' solutions stored in the Wiki system; interview logs of the teacher and students; students' responses to the survey of their experience using the Wiki; students' responses to the survey for evaluating contributors to the growth of SM and Mr.O's paper for evaluating contributors. In addition, there are subsidiary data such as chatting files with Mr.O, e-mail files with Mr.O, GSHS's homepage, and the Wiki system data such as the number of each student's log in, the number of page views, the time of log in and out, etc.

4.6 Methods of Analysis

With respect to research question 1 and 5, I analyzed interview transcripts of individual students and Mr.O. To find patterns of gifted students' CPS using the Wiki, I drew cognitive maps with respect to each solution page to visualize patterns of secondary gifted students' MKB in CMS. One math teacher at SSHS (Mr.C), Mr.O, and I analyzed students' solutions and ideas to understand what Koran secondary gifted students did in terms of mathematical contents. I analyzed the difference between Mr.O's and the students' perspectives on the growth of SM. In addition, to improve the initial design of the Wiki environment, survey papers of participants were analyzed.

4.6.1 A Process of Coding

I made tables for coding work. New solutions are given 1, 2, 3, ... according to the time order of postings as their thread numbers and the first editing in each postings following new solutions are assigned as 1-1, 1-2, ..., 2-1, 2-2, ..., 3-1, 3-2,

.... In addition, an author's editing following his/her first editing in each posting was coded 0. Table 4.3 shows that there are two new solutions in the solution page of problem 2.3 (STU8² is the first solver and EM2³ is the second one) and that EM2s' erroneous solution is threaded by three other students whereas STU8's is followed by one.

By comparing two adjacent edits, every change on each solution page was coded. For example, the Wiki system allows to see what changed between two edits as shown in figure 4.7 when two adjacent edits in figure 4.6. In figure 4.7, STU8 added space-line (Sline) and moved his signature (Sign.) below what he posted on the first edit. These changes were written in the column of changes of table 4.3 as "Sline, Sign." and coded as RR (Repair). Based on these changes, I induced an initial code scheme from my categorization as New(NEW), (RFT), Supplementation(SUPPL), Fixing errors (FIX), Agreement(AGMT), Repetition(RPT), Repair (RR), etc. In table 4.3, STU8 posted his new solution in his first edit (NEW), and then he added an empty space-line (Sline) and moved his signature from the front to the end of his writing in his second edit (RR).

At first, I adopted an edit as an unit of coding. However, since a sentence in an edit might have multiple meanings, one code for an edit was not sufficient. Thus, I adopted a meaning of a clause in a student's sentence as unit of coding. From this new unit of coding arose the necessity to elaborate the initial coding scheme while I was working on analyzing back and forth between students' solutions and the interview transcripts. In this way, I referred to interview logs to interpret students' postings correctly and specify codes. As a result of elaborating the coding scheme, the initial coding scheme has been changed; RFT, AGMT, and FIX have been differentiated into RFT/RFT*, AGMT/CFM, and FIX/FIX* respectively.

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²I denote participants as STU1 to STU14. I assigned each number according to the order in the list of users' IDs, which the Wiki system provided.

³I denote two error-makers as EM1 and EM2.

No	ID	Date & Time	Changes	Thread	Code
1	STU8	4/28/12 09:02	New Solution	1	NEW
2	STU8	4/28/12 09:02	Sline, Sign.	0	RR
3	EM2	4/30/12 06:49	Suggest an erroneous sol	2	NEW
4	STU14	5/12/12 14:38	Refute EM2 by testing	2-1	RFT
5	STU14	5/12/12 14:40	Pinpoint an error in EM2's Suggest an idea to fix	0	RFT* FIX
6	STU14	5/12/12 14:40	Sline	0	RR
7	STU1	6/03/12 06:54	Adding an idea to STU14's	2-2	SUPPL
8	STU1	6/03/12 06:55	Agree with STU14's idea Confirm STU8's solution	0	AGMT CFM
9	STU10	6/05/12 06:47	Sline, Hline Sugg. similar idea with STU14's	2-3	RR RPT
10	STU12	6/06/12 10:04	Sline, Hline Agree with STU8's sol Mentioning about similarity between prob.1-3 & 2-3	1-1	RR AGMT SIM
11	STU12	6/06/12 10:05	Sign.	0	RR

Table 4.3: A Coded Table With Respect To Solution 2-3

In table 4.3, STU14's first edit contains two sentences ("Since EM2's result is negative, it is not right. $\frac{\sin x}{1+\cos^2 x}$ is positive on $(0, \frac{3}{4}\pi)$." and the second one shows an additional sentence ("Since $\sec x$ is not defined at $\frac{1}{2}\pi$, the interval should be split for integrating."). These two edits had been coded as RFT and FIX respectively according to the initial coding scheme. The sentence in the second edit, however, needed to be specified as RFT* and FIX rather than FIX because STU14 refuted again EM2's solution by pinpointing an error in it by saying that " $\sec x$ is not defined at $\frac{1}{2}\pi$ " (RFT*). Comparing the two sentences, STU14's first one refutes EM2 by testing an erroneous solution with a well-known theorem within CoI: $f(x) \geq 0$ on (a, b) then $\int_a^b f(x)dx \geq 0$ whereas the first clause of the sentence in the second edit refutes it again by pinpointing an error in it. That is, the first clause of the second sentence narrows down errors rather than merely repeats the first refutation. In this respect, I differentiate refutation into RFT and RFT*. These two refutations were coded as RFT and RFT* respectively; STU14's RFT shows that EM2's solution was erroneous because it did not pass the test whereas her RFT* pinpointed a spot where

an error was found in the solution and provided a reason for getting rid of it. Lastly, STU14 suggested how to revise the solution (FIX).

RFT* is not the only code related to finding-error activity. Sometimes students fixed a solution as soon as they found errors in it. This is denoted as FIX*. Although RFT* and FIX* commonly contain students' finding-error activity, the process of finding-error of FIX* is not open to readers while that of RFT* works as a kind of conjecture clearly expressed as a written form of sentences; readers need to compare an erroneous solution with a revised solution to find which part of the erroneous solution has been changed. SUPPL refers to improving posted ideas by expressing them with mathematical schemes or adding missing parts to them. In figure 4.8, for example, when STU14 fixed EM2's erroneous solution (FIX), STU1 supplemented STU14's idea with a more specific idea of using limits to integrate (see branch 2 of solution 2-3 in Appendix 2).

AGMT signifies acceptance of the validity of others' ideas. In table 4.3, STU1 agreed with STU14's idea of splitting the given interval to integrate by saying, "If we integrate it by splitting the interval (into two small intervals) as mentioned above (STU14's solution), then we can find the right answer (as STU8's)." According to the initial coding scheme, I coded this STU1's sentence as AGMT; It was clear that he accepted STU14's idea whereas it was not clear what "the right answer" meant because he did not suggest any mathematical expression for it. From the interview transcript of STU1, however, it turned out that the second clause of the sentence had a different meaning from AGMT. While interviewing STU1, he said that he solved problem 2-3 following STU14's idea and got the same results as STU8's. This was clearly different from STU12's sentence of AGMT for STU8's idea in the same table: "Like the first solution (STU8's), it seems to be desirable to solve by substituting $\cos x$ (with u)." From this difference, I could understand that STU1 confirmed STU8's solution as "the right answer" by getting the same answer as STU8's. Thus, I made

a new code of CFM that is to decide whether or not a result of a solution is correct, and coded the second part of STU1's sentence as CFM.

CFM plays a strong role in ECS. In the 2nd type of ECS of figure 4.8 where STU8' solution and EM2's solution were competing with each other, STU1's posting confirmed STU8's solution. This reminds us of J. L. Austin's theory of speech act; STU1's CFM declared that STU8's solution was right as a judge who decides a winner in a match. This looked similar when a new or revised solution brings in the same answer with one of competing solutions. In this respect, I coded a posting as an implicit CFM when a posting showed the same result with a previously posted one even though a student did not express confirmation in his/her writing (see a dotted arrow of STU2 in figure 5.9). Similarly, I interpreted there is an implicit AGMT by considering logical aspects of relation between two postings, even though a writer does not explicitly express her/his AGMT; Because if s/he did not agree with other's RFT or RFT*, s/he would not attempt to find errors in the solution failed to pass a test or remove the errors detected with RFT* (see figure 5.2, 5.3, and 5.4).

4.6.2 Cognitive Map

Students' writing in the shared world can be drawn as cognitive maps. I constructed cognitive maps for each solution page using coded tables. I identified a posting with its author in a solution page. Thus, I assign an author's ID on a node in a cognitive map. Also, I assign codes of a posting on links, which represent relations with other postings. For instance, students' postings on the page of solution 2-3 were drawn as figure 4.8. In a cognitive map, a node and a link are drawn as a box and a directed arrow respectively. In a case of drawing a posting that is not shown on a solution page, I use a dotted line to draw it. In figure 5.9, STU6's posting is drawn with a dotted line because STU6 deleted his posting when he found that EM1's solution was not for the "problem 1-4 with the condition of $x \rightarrow \text{infinity}$ ". In

Figure 4.8: A Cognitive Map of Problem 2-3

addition, in a case that I interpret that there is an implicit AGMT/ CFM, I draw a link with a dotted line because it is not shown in a posting.

In drawing cognitive maps, I consider time flow, direction of arrow, and logical aspects of the relationship between postings. I place nodes from left to right according to the time order of their postings. An arrow with codes connects two postings. When a new posting refutes (RFT/RFT*) or agrees with an existing one (AGMT/CFM), an arrow is drawn from the new one to the old one because one targets the other by analyzing and testing ideas or solutions existing in the shared world. On the contrary, I draw arrows for FIX and SUPPL from old ones to new ones because FIX and SUPPL produce revised solutions that are open to further activity such as RFT and AGMT in the shared world. In this way, an arrow shows how a posting is influenced by old postings and how it impacts new ones.

For example, figure 4.8 is a cognitive map drawn from table 4.3 and the page of solution 2-3. It shows that the time sequence of postings is an order of problem 1-3, problem 2-3, STU8, EM2, STU14, STU1, STU10, and STU12. Although an array of postings on a solution page 2-3 corresponds to the time order of nodes in figure 4.8,

an array of postings sometimes did not correspond to time order of postings because a participant inserted his/her writing between others' writing rather than placed it on the last.

A node of EM2, an error-maker, is shaded differently from other nodes in figure 4.8. Thus, it can be seen that STU8's solution and EM2's solution is competing with each other and thus creates the 2nd type of ECS. Following nodes, STU14 fixed EM2's solution (FIX) after refuted EM2's solution in two different ways (RFT/RFT*) and then, STU1 showed his agreement with STU14's RFT* (AGMT), supplemented STU14's revised solution (SUPPL), and confirmed STU8's solution (CFM). STU10 repeated STU1's idea (RPT). Lastly, came STU12. He showed his agreement on STU8's solution (AGMT) and mentioned the similarity between problem 1-3 and 2-3 (SIM). Since problem 1-3 does not exist on the page of solution 2-3, it was drawn with a dotted line in figure 4.8.

Since the thread of EM2's erroneous solution is longer than that of STU8's in figure 4.8, we can see that students actively participated in improving the erroneous solution by refuting and revising it. In this way, cognitive maps can visualize relationship and dynamics among postings and thus show how postings weave contents in the shared world. It is expected that cognitive maps will reveal students' MKB patterns in CMS.

In a case that a student keeps developing his/her writing in the same compartmented area on the solution page with several visiting, I draw an old one as a dotted line on a cognitive map. In figure 5.10, for example, STU11 made RFT* and FIX on STU8's solution at first. When I raised a question (Q) for justification as ADM (administrator) the next day, he responded to my question by elaborating his posting on solution page 1-4. As a result, he developed his writing for two days. I drew his earlier posting with a dotted line and his later one with a solid line. Since he also deleted my question included in my posting as ADM, the relationship between ADM

and STU11 did not show up on the solution page. Thus, I drew it with a dotted line too.

4.6.3 Mathematical Content Analysis

Reducing students' mathematical discourse to several codes can eliminate mathematical contents and details (Stahl, 2006b). "This is a paradox of reliable and valid coding efforts in exploratory CSCL research" (Strijbos, 2009, p.419). In this respect, mathematical content analysis is needed to understand the MKB process in the shared world in a complimentary way to cognitive maps drawn with codes. Comparing cognitive maps and solutions with my codes attached in Appendix 2 can show vivid sights of acts of CC in CMS by combining two different level of analysis.

The mathematical content analysis in this study is based on connoisseur's perspective (Eisner, 2001). One math teacher at SSHS (Mr.C) and Mr.O participated in analyzing mathematical content. I requested that they analyze students' postings stored in the Wiki in terms of the following questions:

- What errors can you find in students' postings in the Wiki?
- What are solutions or ideas that students could not include in their postings?
- If you are a member of online CoI as a math teacher, then what will you suggest in the Wiki as a hint, comment, or feedback in the case that students are stuck on something and do not develop their solution or discussion?
- Freely suggest your opinions about students' postings from your mathematical perspective.

By synthesizing two math teachers' analysis and mine on mathematical content of students' postings, I address research question 2.

4.6.4 Comparative Analysis

The evaluation questions ask participants to nominate the first, second, and third contributor in collaborative problem solving for each problem and to state their rationale for the rankings (see Appendix 3). By comparing 4 students' responses and

Mr.O' response to the evaluation question, I analyze the differences in Mr.O's and students' perspective on evaluating contributors. I expect this analysis to reveal the difference between two perspectives on the development of SM.

4.7 Limitations of The Experiment

While conducting the experiment, I had a conflict between a role of researcher and that of math teacher because Mr.O could not participate in the MKB process. I perceived that students' postings were influenced by my intervention both in quantitative and qualitative ways during the experiment. In this respect, I think that an ideal form of this research is action research where the classroom teacher rather than an outsider manages the Wiki.

There were several changes in the process of the experiment compared to the initial plan due to restraints imposed at GSHS (G Science High School). For instance, there was no class time for practicing LaTeX and no class discussion for establishing norms of participation in the Wiki environment. These changes might have impacted the results of the experiment. For example, using a strategy of 'copy, paste, and revise' as a norm of participation instead of 'open editing' in the Wiki may have impacted the MKB process. In addition, although a project was a main focus of this experiment, students could not conduct it because Green's Theorem was placed in Mr.O's class at the end of the semester.

I made mistakes in managing the space in the Wiki, for example, I did not separate solution pages, thereby STU9 posted her solution of problem 1-4 on the solution page of problem 1-3 and this caused confusion (see figure 5.10); I posted different conditions for the problem 1-4 and changed the condition of the problem, which caused STU6 to delete his refutation to an error-maker's solution.

Since the experiment lasted for a full semester, some students could not remember the reasons for what they posted earlier in the semester. In this respect,

they could not respond completely to interview questions and survey questions. Furthermore, students were too busy to respond to survey questions. Thus, I collected a poor data set from the survey. In addition, two students were interviewed together because they wanted to be interviewed at the same time to save their time. This also might influenced interview process.

Since I developed coding scheme and coded students' postings by myself, coding process was not triangulated. In this respect, this may considerably limit reliability of the coding scheme.

CHAPTER 5. DESCRIPTION OF DATA AND ANALYSIS

Mathematical activity is human activity. . . . . But mathematical activity produces mathematics. Mathematics, this products of human activity, ‘alienates itself’ from the human activity which has been producing it. It becomes a living, growing organism, that *acquires a certain autonomy*¹ from the activity which has produced it; it develops its own autonomous laws of growth, its own dialectic. The genuine creative mathematician is just a personification, an incarnation of these laws which can only realise themselves in human action. Their incarnation, however, is rarely perfect. The activity of human mathematicians, as it appears in history, is only a fumbling realisation of the wonderful dialectic of mathematical ideas. (Lakatos, 1976, p.146)

In this chapter, I suggest patterns of students’ postings based on the Wiki environment established for this study. Based on Garrison et al’s (2007) elements of an educational experience (figure 2.1), I suggest design elements for building on-line CoI (Community of Inquiry) for MKB (Mathematical Knowledge Building). In terms of social presence, which relates to the quality of CC (Collective Cognition) in CMS (Collective Mental Space), I describe the relation between design and motivation, emergence of implicit norms, relation between collective responsibility of CoI and open editing, epistemic agency, source of knowledge imported to CMS, and the possibility of blended learning. In terms of cognitive presence, I describe patterns of MKB, the nature of problems and solutions, students’ use of pencil-and-paper while participating in CPS (Collaborative Problem Solving), and relation between errors and JPS (Joint Problem Space). In terms of teaching presence, I describe my role as a designer and a facilitator. Lastly, I suggest the teacher’s and students’ opinions about the use of the Wiki learning environment.

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¹The author’s emphasis.

No of Content Pages	Avg. edits per page	View total	Views per edit
56	5.53	6,166	13.95

Table 5.1: Basic Data from The Wiki System

Ranks	1	2	3	4	5	6	7	8	9	10
Page	Cont	Main	P1-1	P1-4	P1-2	P1-3	S1-4	P1-5	S1-3	Steiner's Pf
Views	1,258	671	262	241	228	226	193	188	177	173

Table 5.2: Most View Pages in The Wiki

5.1 Summary of Wiki Postings

The data in tables 5.1 and 5.2 were retrieved on Feb, 23, 2013 from the Wiki system. Fifty six content pages were created including 21 pages that were created for students to practice writing mathematical expressions using LaTeX (table 5.1).

In table 5.2, the number of views for content pages outnumbers those of other pages because participants used the page for contents as a hub to move to problem pages. The main page is a portal of the site and everyone has to pass through it whenever s/he comes to the site. Since problem set 1 was exposed to students a month longer than problem set 2, it is not surprising that problem set 1 has more page views than problem set 2. Since the condition of problem 1-4 was changed about 10 days after it was posted on Mar 24, both problem 1-4 and solution 1-4 have many views. It is interesting that Steiner's proof page, which was available from April 2, has more views than solution pages for 1-1, 1-2, and 1-5, problem set 2, and historical background page. This suggests that students were more interested in Steiner's proof than other problems.

Regarding patterns in postings, seventy nine percent of edits after the first edit took less than 2 minutes. This suggests that most second edits were likely used to repair the first edits for readers in terms of mathematical expressions and easy reading. As noted in table 5.3, 48.6 % of edits were for repairing (RR) and 45.6%

Code	RR	NEW	RFT (RFT*)	AGMT	FIX (FIX*)	SUPPL	CFM	Q (RE)	Others
Freq	84	26	7 (16)	2	7 (5)	5	3	4 (4)	10
Pct	48.6	15.0	12.3	1.2	6.9	2.9	1.7	4.6	5.8

Table 5.3: Frequencies of Codes For Problem Set 1& 2

were for NEW, RFT/RFT*, AGMT, FIX/FIX*, SUPPL, CFM, Q/RE. Other edits included Repetition (RPT), Changing Problem (CP), Related Material (RM) etc. Although students rarely raised questions of each other, as Administrator (ADM) I raised three questions to encourage students to explain their ideas in a clear manner.

As indicated in table 5.4, the average number postings per solution page is about 7. The average number of branches per problem is 3.2 and the average number of the longest length of thread per problem is 4.2. On a cognitive map, that is, three postings are placed unrelated each other along horizontal axis underneath a nod of a problem and one of them is developed to the longest thread with 4 postings. Students posted in response to problem 1-4 more than other problems. This problem consisted of two questions with different conditions. In contrast, they posted least in response to problem 2-4. Problems 1-1 to 2-3 consisted of erroneous solutions or ones that error-makers posted, so that students could ‘copy, paste, and revise’ to write their own solutions or edits. For problem 2-4 and 2-5, students needed to type mathematical expressions using LaTeX. The averages of edits per posting in solution 2-4 and 2-5 are 3.0 and 4.2 respectively – higher than those in problem 1-1 to 2-3.

The average number of edits per posting across all problems is 2.2. Since the data include numbers from my posting and editing as an ADM and two error-makers (EM1, EM2), the average does not exclusively show students’ activity. To find a more precise number, I used the history function of the Wiki to count the number of students’ postings and editings (table 5.5). Table 5.5 shows students’ average edits per posting is still around 2. This means that the typical structure of a posting consists

Sol. No	No of Post.	No of Edits	No of Branches	Average Edits Per Posting	The Longest Length of Thread
1-1	6	19	5	3.2	2
1-2	7	9	3	1.3	5
1-3	9	11	4	1.2	6
1-4	13	30	6	2.3	6
1-5	5	11	2	2.2	4
SUM	40	80	20		23
AVG	8.0	16.0	4.0	2.0	4.6
2-1	4	7	3	1.8	1
2-2	7	11	3	1.6	4
2-3	6	11	2	1.8	5
2-4	3	6	2	3.0	2
2-5	9	38	2	4.2	7
SUM	29	73	12		19
AVG	5.8	14.6	2.4	2.6	3.8
SUM	69	153	32		
AVG	6.9	15.3	3.2	2.2	4.2

Table 5.4: Basic Data of Problem Set 1 & 2

of a main idea and repairing it; the main idea comes first as a new idea or solution (NEW), Refutation (RFT/RFT*), Fix (FIX/FIX*), or Supplement (SUPPL) and then follows the second edits for repairing (RR). Thus, a posting with two edits has a typical structure of: NEW + RR, RFT/RFT* + RR, FIX/FIX*/SUPPL + RR.

The number of a participant's visits cannot be exactly counted because although a student may visit the site, s/he might not have edited. In student interviews, I asked participants how many times they had visited the site. Students answered 2 to 4 times and these numbers were similar with the count available in the Wiki system. Therefore, the overall average number of students' visits is about 2.5 for 70 days. Students who worked as constructors, which are coded as NEW, FIX/FIX*, and SUPPL, edited their postings more than others who worked as refuters, Which are codes as RFT/RFT*. In this respect, STU2, STU7, STU11, and STU14's average numbers of edits per posting are relatively higher in table 5.5.

No	No of Visits	No of Postings	No of Edits	Avg. No of Edits Per Posting
STU1	4	6	8	1.3
STU2	4	5	13	2.6
STU3	1	2	3	1.5
STU4	2	2	4	2
STU5	2	3	9	3
STU6	2	5	7	1.4
STU7	2	3	14	4.7
STU8	3	5	8	1.6
STU9	1	2	3	1.5
STU10	3	8	10	1.3
STU11	4	4	13	3.3
STU12	1	2	3	1.5
STU13	2	4	4	1
STU14	4	5	22	4.4
SUM	35	56	121	
AVG	2.5	4.0	8.6	2.2

Table 5.5: Individual Participant’s Data For Problem Set 1 & 2

5.2 Social Presence

5.2.1 Motivation

The Wiki problems were assigned as “extra” material rather than homework because Mr.O did not want that participation in the experiment causes any trouble to students in terms of time management of school life. In this respect, he expected that the Wiki activity provided students additional opportunity to participate in discussion and attempted to encourage them to participate by adding up the number of students’ postings to the number of participation in class discussion. These aspects influenced students’ motivation and participation; Students tended to look around and pick problems that they were of interest or that they thought they could solve at first glance. For example,

STU14: “At first, I read whole problems. Once, I attempted to solve some problems with a pen that no one touched. Then, I started with ones that I could sketch their solution in my mind.”

Lee: “Did you skip problems that seemed to take you too much time?”

STU14: “Yes. And I attempted to solve the one with double integrals because it looked interesting.”
 Lee: “Aren’t there similar problems in Thomas’ (text)book?”
 STU14: “I saw many with one integral but there was no double one.”
 Lee: “Ah, there was not. It looked interesting (to you)?”
 STU14: “Yes, it did. I wanted to solve it because it looked to be solved (with my idea).”

Sometimes students did not attempt to solve problems when they looked too challenging and might take an extended period of time. This was their strategy to harmonize their busy school life by participating in the experiment. For example, when I asked STU14 why she did not come to the pages of (proving) Isoperimetric Theorem, she replied:

STU14: “I knew that it was difficult (to prove) and its proof was very long. So I did not enter its pages because it seemed to take too much time (to prove it).”
 Lee: “Then didn’t you read others’ postings (with respect to it), either?”
 STU14: “No.”

As previously mentioned, Mr.O informed students that the number of individual student’s postings would be counted as part of their participation in class discussion. Whereas Mr.O’s way of evaluation motivated students to participate, it likely also led a few students to post their writing only to increase their number of postings. For example, STU10 visited the site three times for three days right before the semester ended, posting 8 times, which is the highest number of postings. This is shown in table 5.6, where he posted for problem 1-1 on June, 3, which was the first day of his visit and three days before the experiment ended. Nine minutes later, he posted for problem 1-2 and he posted for problem 1-3 four minutes later. In addition, eleven minutes later, he posted for problem 2-1.

Table 5.6 shows that STU10 posted many writings during a short period of time. This was possible because he did not write mathematical expressions while

No	Problem	Time	Date	Code
1	1-1	18:40 (EST)	June, 3	RFT*
2	1-2	18:49 (EST)	June, 3	RFT*, RPT
3	1-3	18:53 (EST)	June, 3	SUPPL, AGMT
4	2-1	19:04 (EST)	June, 3	RFT*
5	1-4	19:26 (EST)	June, 4	RPT
6	2-2	19:31 (EST)	June, 4	RFT*
7	2-3	16:47 (EST)	June, 5	RPT
8	2-2	16:53 (EST)	June, 5	RFT*

Table 5.6: A List of STU10's Postings

posting. His strategy of participation seemed to positioning himself as a refuter rather than a constructor. In this respect, his average number of edits per posting is 1.3, which is relatively lower than others in table 5.5. Furthermore, his postings for problem 1-4 and 2-3 were very similar to others' postings. Comparing STU10's posting with STU2's solution, it turns out that STU10 explained STU2's mathematical expressions with words (see branch 4 of solution 1-4 in Appendix 2). STU10's posting for problem 2-3 was also coded as RPT because it is very similar to STU1's posting (see branch 2 of solution 2-3 in Appendix 2). In his posting, he used the word "seems" implying that he did not solve the problem. The pressure of evaluation might have led STU10 to repeat others' ideas and this depreciates the quality of the CPS process. In an interview, STU14 pointed out the quality of postings as follows:

Lee: "Please tell me about good points and bad points of the Wiki at this point after you have experienced problem solving using the Wiki."

STU14: "Since I have my own style (method) in solving problems, I always stick to it. From others' postings, I came to know new methods. It was good to get better, shorter, and more perfect solutions because others fixed errors (in solutions) even though they were trifle. The bad point was that there were some who posted meaningless ideas because (the number of) postings were related to assessment."

5.2.2 An Implicit Norm and Design of The Experiment

In this study, students were asked to revise an erroneous solution by applying two norms: ‘copy, paste, and revise’ and ‘put horizontal line and signature’. The reasons for introducing these two norms were to relieve difficulty in writing mathematical expressions using LaTeX and for Mr.O to easily count each student’s number of postings for assessment. These two norms allowed students to compartmentalize their writing with a horizontal line and a signature while revising others’ solutions. Thus, they created an individual subspace for their thoughts and solutions under identified authorship on solution pages (see pictures of solution pages in Appendix 2). In this respect, most postings on solution pages were placed according to time flow and were not structured around similarity of ideas and methods of solutions.

The most significant implicit norm that students created and shared was a *norm of application of ‘copy, paste, and revise’*, which says: ‘You don’t have to revise a solution as long as it contains a right answer.’ Based on this norm, students decided what was worthwhile to do for CPS. Thus, it determined the intellectual landscape of the shared world. This aspect of the MKB process showed up in an interview with STU5:

Lee: “Were they (others’ comments on solutions) helpful for you to solve problems?”

STU5 : “But when we comment (on solutions), emotionally.....in other words, we pointed out errors in solutions rather than elaborated them. Errors...When we read a solution, we frequently neglected superfluous parts of it as long as it was not wrong. Because it yielded a right answer anyway.”

Participants did not pinpoint trivial errors in a solution as long as “it yielded a right answer”. They attempted to concentrate on critical errors that prevented a solution from yielding a right answer rather than on trivial errors that did not influence a result. Based on this norm for working on a critical line, they did not

correct trivial errors such as wrong usage of terminology and typos. For example, despite the fact that STU8's new solution included a simple error in the calculation process in figure 4.8 (see STU8' solution for problem 2-3 in Appendix 2), no one fixed it. Misuse of terminology (see STU8' solution for problem 2-4 in Appendix 2) and a typo in mathematical expressions were also not corrected either.

In addition, students did not elaborate others' solutions if it included a correct result. They left a supplement of it for its author on a constructive line as long as it was correct. For example, although students might have developed STU8's solution for problem 2-4 by supplementing with mathematical expressions, they did not because it included a correct result of " πab ". In contrast, they revised or supplemented others' solutions when they did not contain a correct answer. For example, STU10 supplemented STU14's idea by adding his idea and STU7 supplemented STU1's idea by translating it into mathematical expressions (see their solution for problem 1-3 in Appendix 2). Thus, whether or not a correct answer was included in a solution was an important criterion for students to react to another student's solution.

Some students did not investigate solutions from the first line till their results were checked as wrong because it was "boring" to investigate them (see an interview transcript of STU8 in "students ability and interest" in section 5.3.1). In this respect, students might not have noticed small errors in solutions with right answers. However, it is hard to imagine that many students could not find an error in STU8's solution for problem 2-3, which was competing with EM2's erroneous solution in the 2nd ECS, because students looked into solutions when their results turned out to be contradictory. Furthermore, it cannot be explained why students did not supplement STU8's solution whereas STU14's idea was supplemented on the same solution page (see Appendix 2). Thus, the possibility of missing trivial errors cannot explain students' action.

Identified authorship influenced students' judgement of worthwhileness to do.

Students adhered to identified authorship while participating in the MKB process. For example, STU1 refuted an erroneous solution given in problem 2-1 but forgot to post his signature. When he came by the solution page three weeks later and found that his signature was missing in his posting, he added his signature (see STU1's RR in figure 5.16). Students' adherence to identified authorship increased students' responsibility for their own postings and might develop a notion that elaborating a solution by supplementing with mathematical expressions and removing trivial errors is the author's responsibility. In this respect, it was STU6 himself who developed his solution for problem 1-5 by removing a simple error in his previously posted solution. Similarly, STU11 elaborated on his own solution by adding a reason to his posting for problem 1-4 when the ADM raised a question(Q). Although students could remove trivial errors by using 'open editing' of the Wiki, they did not because it was not related to their identified authorship. Thus, small errors might be neglected and thus eliminating them being attributed to their authors' responsibility.

5.2.3 Collective Responsibility and Open Editing

As seen in the previous section, identified authorship with signing increased students' individual responsibility for their own writing. Moss and Beatty (2006) reported that individuals' responsibility for their own writing develops an epistemic agency for progressive advancement and increases mathematical knowledge in the online CoI. However, individual responsibility does not necessarily increase collective responsibility of CoI for growth of SM. For example, for problem 1-4, I mistakenly posted a different problem instead of the original problem and also posted an erroneous solution with respect to the original problem as EM1. STU5 came to the solution page 2 hours later and posted a new idea for the original problem because he did not find the mismatch between the posted problem and EM1's erroneous solution. Two days later, STU6 refuted EM1's solution (RFT*). Right after he posted

his RFT*, he found that EM1's solution did not match with the posted problem and his RFT* on EM1's solution became pointless. Thus, he deleted his posting but he did not inform other students of this mismatch. Seventeen days later, STU8 found what was wrong on the page of solution 1-4 and made RFT*.

Undoubtedly, STU6 was responsible for his own writing and avoided making the mismatched mathematical landscape worse by deleting his RFT*. However, it might be said that he did not embrace collective responsibility of CoI for growth of SM because he did not inform others of the mismatch. If STU6 did, the mathematical landscape in the shared world would've changed and CC could've developed solutions more efficiently. As a result, no progress was made until STU8 found the mismatch. In this respect, STU6 was uninvolved in the collective responsibility of CoI and he lost his opportunity to contribute to the growth of SM. In this example, individual responsibility with respect to identified authorship does not necessarily carry collective responsibility of CoI.

As mentioned earlier, the two given norms allowed students to create individual subspace on solution pages for their thoughts and solutions with compartment and signature. This subspace tagged with signature might also hinder students from using 'open editing', whereas it increased individual responsibility for their own writing. Although subspace with identified authorship was better in showing relationship between students' writing than 'open editing', it was not economical to collaboratively develop a solution in terms of time and space. Actually, when students applied this strategy to fix STU14's solution with long mathematical expressions on a page of solution 2-5, it was annoying to repeatedly read almost the same long mathematical expressions. For example, in an interview, STU14 said, "They looked the same as mine but their answers were all different. So I did not feel like to read (solve) them." If students applied this norm to every error in solutions, the intellectual landscape would have become less attractive to readers. In this respect, the compartment of a

solution page needs to be applied to different solutions and students need to develop a solution within a block on a page by removing errors in it with ‘open editing’ according to my initial design. In this case, ‘open editing’ can function as a economic tool of CC for MKB in the shared world.

The ‘open editing’ feature of the Wiki may enhance collective responsibility of CoI for growth of SM. However, in terms of authorship, most students interviewed did not like ‘open editing’ even though the Wiki provided a history of contributors. For example, STU9 responded to a question about this feature as follows:

Lee: “What do you think about others correcting your solution in the Wiki without your permission?”

STU9: “I feel like I hate it”

Lee: “Hate it?”

STU9: “Yes.”

Lee: “Do you feel bad even if someone removes errors in your solution?”

STU9: “Yes. I don’t like anyone freely changing my solution. Rather, s/he may say (to me) that this needs to be changed.”

When I asked the same question to STU6, he had a similar opinion as STU9 and preferred ‘copy, paste, and revise’ to ‘open editing’:

“Uh....I don’t feel good about it (if others revise my solution) because I wrote it. If others want to revise it, then they can do it by using ‘copy and paste’ after they suggest their opinions. I prefer this.”

Lessening students’ dislike of ‘open editing’ is important to develop SM as oeuvres. An interview suggests that students may begin to like it when they get used to it. For example, STU5, who had experienced open editing in his class, was the only one who had a positive opinion about it. He replied:

“Uh...We did it several times although it was in class rather than online. So, not too much similar...If a student wrote his/her solution (on the blackboard) in Physics class, then others came to revise it or suggested a new solutions (on the blackboard). As for me, I really like it.”

Thus, it appears that in order to facilitate involvement and collective responsibility of CoI, educational settings should be designed to allow for ‘open editing’ in the shared world.

5.2.4 Epistemic Agency

Every member of online CoI is responsible for others’ learning. In this respect, students partly share the teacher’s role. A math teacher’s role is to encourage students to learn by designing and embedding educational setting and by directing the MKB process. Students voluntarily participated in building an educational setting in the shared world and directing the MKB process. When students extend the range of their role to include the teacher’s role, students’ epistemic agency is revealed. In terms of epistemic agency, students serve as an organizer, a director, an editor, and a questioner in CMS.

In this study, students acted as organizers by collecting and managing material. After posting problem set 1, I planned to develop and post a summary of historical background of the Isoperimetric Theorem. I began by writing the title, “Historical Background of the Isoperimetric Theorem” and left the rest of page vacant for a while until I collected and summarized reading materials. Interestingly, STU5 wrote a statement about the Isoperimetric Theorem before I posted it (see table 4.2). STU9 also posted a modern mathematician’s work with respect to a generalized form of the theorem. Similarly, before posting detailed material about the Isoperimetric Theorem using Green’s Theorem, I titled a page as “To Restate Isoperimetric Theorem With Green’s Theorem”. STU4 voluntarily posted Green’s Theorem from Wikipedia at 23:51, 29 May 2012. Five days later, STU1 made a link to another geometrical proof of the Isoperimetric Theorem below STU4’s posting. These examples show that students were willing to voluntarily search for information related to topics and to post results even though there was only a title to guide them.

In some instances, students also acted as a director in CMS. For example, in problem 1-4, STU13 suggested a new objective for CoI: “As a result (of STU14’s solution), the derivative of the numerator and that of denominator need to converge to 0 as well as numerator and denominator. To prove this, we need to discuss it more.” Even though others did not react to his suggestion, his suggestion could’ve directed CFP (Collective Focal Point) toward a new topic if someone had replied.

As an editor, some students removed superfluous writing for the readers’ sake. In problem 1-4, STU11 pinpointed an error in STU8’s new solution (RFT*): “@STU8 the condition of $\lim_{x \rightarrow \infty} \frac{(\sin 3x)}{x^3} \geq 0$ is needed (in your solution)”. When I as ADM asked STU11 to explain why, he elaborated his writing instead of directly answering my question. He added the following sentence to his posting of RFT*:

Since STU8’s solution showed that an upper bound of the problem is 0 but did not seek its lower bound, it cannot be guaranteed that the limit is 0.

Since my question induced STU11’s elaboration of his RFT*, other members of CoI had no reason to read it. In fact, STU11 deleted my question for the readers’ sake. As a result, students could not read my questions as long as they did not attempt to see the history of the solution page.

As a questioner, STU12 asked STU1 to clarify his meaning (see branch 2 of solution 2-5 in Appendix 2). Even though no one responded to STU12’s posting, his questioning of justification for STU1’s solution might’ve led CC to a new path of MKB.

5.2.5 Source For MKB

In this study, students’ primary source for MKB in the shared world was from searches on the Internet. Although there was one student, STU14, who used her textbook as a resource, most students searched for related information on the Internet when they came across new things such as Green’s Theorem, and the Isoperimetric

Theorem. For example, STU11 searched for a definition of a function on the Internet while solving problem 2-1:

Lee: “Have you asked (mathematics) teachers, or referred to books or the Internet?”

STU11: “I used the Internet.”

Lee: “What did you search for?”

STU11: “Something like $\sinh$ ”.

Using a definition of $\sinh$ that he found on the Internet, he suggested a new solution for problem 2-1. Since students could use the Internet immediately while being connected to the Wiki, it was a convenient tool for them. Students also searched related materials, new proofs, answers etc. STU4 indicated that he searched for Green’s Theorem on the Internet and posted search results in the shared world even though the same information was included in his textbook. In addition, STU1 searched another geometric proof for the Isoperimetric Theorem and made a link to it. STU9 searched three papers related to Isometric Theorem and made references of them in the shared world.

Whereas the Internet is a convenient and powerful tool for students to use while participating in the MKB process, dependence on the Internet may cause some problems. Above all, it may deprive students of opportunities for deep thinking in the private world. For example, on a page which contained an error in Steiner’s Proof for the Isoperimetric Theorem, STU9 simply but correctly pinpointed an error in the proof. However, she did not write any reason for the error. In an interview, she told me that she read some papers about Steiner’s Proof and was informed of what was wrong with the proof.

Lee: “You came to a page of Steiner’s proof.”

STU9: “I seemed to write short sentence after I searched for some papers (on the Internet). They were all in English. So I could not....”

.....

STU9: "I found three papers."
 Lee: "Did you?"
 STU9: "Yes. I could just partly understand them. I posted them on the page for references."
 Lee: "Your posting says that it (Steiner's proof) did not show the existence (of the figure). Did you think about it for yourself or read about it (in some papers)?"
 STU9: "I read about it."
 Lee: "Read it?"
 STU9: "Yes."

When I asked her about the necessity of showing the existence of the figure with the maximum area in the proof, she could not remember why this was important. It did not seem that she thought deeply about it although she suggested a correct answer from her search on the Internet.

Lee: "What do you think about the necessity to show the existence? How can you show the existence?...How can you show the existence of the figure?"
 STU9: "Its area is the same as...."
 Lee: "The area of the figure is maximum. (What do you think) about showing the existence of (the figure with) maximum area?"
 STU9: "I almost forgot it now."

Suggesting answers found outside of CMS in the shared world may entangle other members of CoI in silence rather than encourage them to mathematically communicate in CMS. This situation may discourage members of CoI to attempt to solve problems and thus may impede development of other student's idea.

5.2.6 A Possibility of Blended Learning

Since a primary focus of this study was on CPS using the Wiki, connection between face-to-face interaction and the MKB process using the Wiki was not considered from the outset. That is, students engaged in online homework, which was not related to face-to-face class work. However, CC in CMS could not be insulated from face-to-face interaction as long as students shared a common interest in a course.

For example, a few students discussed some Wiki problems and erroneous solutions face-to-face before their regular class.

Lee: "Have you discussed problems posted (in the Wiki) with other students in your class?"

STU5: "I directly discussed with others 2 or 3 times. Since most problems posted in the Wiki were concerning calculation, I don't have to discuss with others much."

Lee: "Then, you discussed (face-to-face) with others 2 or 3 times about problems?"

STU5: "Although I cannot remember what they were, I asked others about a problem because I could not understand its solutions."

Lee: "Other students' solutions?"

STU5: "When I read a solution, others said that it was wrong but I could not understand why...So..."

STU5's reply indicates that he wanted to understand how errors occurred in the solution. His desire to understand led him to ask others about solutions in a face-to-face environment. An interview with STU11 and STU3 shows that problems and solutions in the shared world caused them to interact in a face-to-face environment even though such interaction was brief.

Lee: "Have you ever discussed the problems posted in the Wiki with others?"

STU3: "Hyperbolic sine..."

STU11: "Yes."

STU3: "You asked me about integration by parts."

Lee: "Did you discuss about it?"

STU3 & STU11: "Yes."

STU11: "A little."

Lee: "When you started discussion, how did it go?...Was it started by anyone's question? or...?"

STU11: "It started by my question and he answered to me..."

Lee: "How long did your discussion last?"

STU3: "In our case, I could immediately answered to it because I had thought about it..."

STU11: "Our discussion did not last long..."

STU3: "It never lasted long."

Lee: "Was this the only case? Or anything else?"

STU11: "I am not sure."

STU3: "About others' solutions..."

STU11: “Yes.”
STU3: “We talked (about it).”
STU11: “Or we talked about what problems in the Wiki were like.”
Lee: “About your whole impression (about problems)?”
STU11: “Like...they looked very difficult...”
STU3: “It was online space where problems were posted.
So we did not actively discuss them outside of the space.”

Conversely, knowledge from face-to-face class interaction can influence CPS in JPS. For example, in his RFT* with respect to the erroneous solution included in problem 1-1, STU4 recalled class discussions by saying: “This error in applying mathematical induction is similar to what we learned in the mathematics seminar class” (see branch 5 of solution 1-1 in Appendix 2).

These examples indicate the possibility of “blended learning”, where social practices change as students move back and forth between face-to-face and online interaction (Stahl, 2006a, p.410). Although interaction through hypertext in the shared world was not active as much as face-to-face interaction, posting solutions and participating in discussion in the shared world could provide new opportunities for inactive or passive students in class. Thus, the Wiki environment can be helpful to develop and equalize power relations in building CoI in a face-to-face class. In that MKB process in the shared world led students to face-to-face interaction, this study indicates that the MKB process in the shared world can coincide with face-to-face class discussion. Thus, students learning while engaging in MKB can be developed with face-to-face learning in complimentary ways.

5.3 Cognitive Presence

5.3.1 Patterns of MKB

Cognitive maps drawn with respect to each solution page are used to examine the structure of CC in JPS. Cognitive maps visualize MKB patterns emerging from

JPS. Analysis of the cognitive maps revealed how CC was organized around errors in JPS. In particular, two CPS patterns were revealed: Distributed form and inverse directional double helix. In addition, two patterns of confirmation triangles are used to increase members' sense of truthfulness about results within CoI. Confirmation triangles indicate how CC organizes branches While an inverse directional double helix shows how CC develops a solution or an idea. These patterns are discussed in the following section.

Distributed Form

Problem 1-1 includes an erroneous proof in applying mathematical induction (see Appendix 2). This proof contradicts well-known knowledge about differentiation of x^n for integer n within CoI. Thus, it creates the 1sttype of ECS (Explicit Contradictory Situation). In figure 5.1, STU5, STU14, STU13, STU10, and STU4 attempted to refute it by pinpointing errors hidden in the proof (RFT*). Even though some of RFT*s looked similar in terms of mathematical ideas, students did not mention others' postings. Instead, they suggested their opinions rather than refute or develop others' ideas. As a result, students' postings were scattered as branches along the time sequence rather than threaded and thus constituted a distributed form.

This form also appeared in a cognitive map of solution 2-2 (figure 5.2). When EM1 suggested an erroneous solution, STU8 tested it by applying the relation between antiderivative and derivative and refuted it (RFT): "If $\int \ln(\sin x) dx = (\ln 2)x + C$ then $\frac{d}{dx}[(\ln 2)x + c] = \ln(\sin x)$ but it is not. -STU8 02:22, 12 May 2012 (EST)". This successful RFT created the 1sttype of ECS, which triggered members of CoI to attempt to spot errors in EM1's solution (RFT*): STU2's RFT* and STU10's RFT*s. However, none of these RFT*s was accepted within CoI or led to others' activity related to CPS in JPS. As a result, students' postings were configured into a distributed form around EM1's erroneous solution in figure 5.2.

Figure 5.1: A Cognitive Map of Solution 1-1

Figure 5.2: A Cognitive Map of Solution 2-2

When a member of CoI suggests a detected error in a solution (RFT*) as a result of finding-error activity, RFT* works as a kind of conjecture. That is, it may be accepted or not within CoI. In the later case, RFT*s are not developed but rather are left untouched. Thus, when RFT*s are placed along a time sequence without any development, a cognitive map of them shows a distributed form.

Inverse Directional Double Helix

Members of CoI may make one of two types of refutation (RFT or RFT*) about a solution posted in the shared world. In the case that RFT* is raised and accepted, members of CoI revise solutions by eliminating detected errors (FIX). In figure 5.3, STU6 attempted to refute by spotting an error in EM2's solution (RFT*) and then STU14 removed the error revealed in STU6's RFT* (FIX). Later, STU10 implicitly agreed with STU6's RFT* (AGMT) and then supplemented STU14's revision (SUPPL). This pattern of collaborative problem solving is diagrammed as $NEW \rightarrow RFT^* (\rightarrow AGMT) \rightarrow FIX (\rightarrow SUPPL)$

Figure 5.4 shows that STU11 refuted STU14's solution by showing that the result of her solution is negative whereas it should be positive (RFT). STU11's successful RFT triggered other students' finding-error activity; STU3 implicitly agreed with his refutation (AGMT) and attempted to find and remove an error in STU14's solution (FIX*). STU5 refuted STU3's solution again by applying the same test that STU11 had used (RFT). STU7 also implicitly agreed with his refutation (AGMT) and fixed the solution again (FIX*). Thus, this pattern of CPS is summarized as $NEW \rightarrow RFT (\rightarrow AGMT) \rightarrow FIX^*$.

The CPS process as illustrated in figure 5.3 and 5.4 can be diagrammed as in figure 5.5—revealing how members of CoI collaborate to solve problems from S1 to S3 in terms of their action (RFT/RFT*, AGMT, FIX*/FIX/SUPPL). However, it neglects some aspect of the dynamics of members' postings in JPS; postings themselves

Figure 5.3: A Cognitive Map of Solution 1-3

and relationships between postings are not clearly visualized. For example, STU11's posting for RFT cannot be placed on figure 5.5. In this respect, postings need to be drawn to reflect a relation between postings in terms of directed arrow. Using this idea, figure 5.5 is redrawn as figure 5.6.

Figure 5.6 illustrates that a new solution leads to revised solutions following two lines: a line of RFT/RFT*/AGMT (a critical line) and a line of NEW/FIX/-FIX*/SUPPL (a constructive line). A critical line puts force on a previous posting in the opposite direction of a time sequence because action along a critical line looks backwards to test or analyze existing ideas and solutions in JPS whereas action along a constructive line goes in the same direction of time flow and creates solutions or revises a solution for the next critical action. Therefore, I characterize the model as *inverse directional double helix*. Figure 5.6 shows that a constructive line and a critical line alternatively interweave mathematical knowledge in JPS and intersect at a point where an error occurs in a solution. Thus, CPS proceeds along an inverse

Figure 5.6: Inverse Directional Double Helix

Figure 5.7: A Logic of Collaborative Problem Solving

directional double helix rather than a single thread.

The pattern of CPS depicted in figure 5.6 is similar to the process of *proofs and refutations* as “a simple pattern of mathematical discovery – or of the growth of informal mathematical theories” (Lakatos, 1976, p.127). That is, when a global counterexample is found to refute a proof (RFT), which is suggested with respect to a primitive conjecture, proof-analysis activity is launched within CoI. While re-examining the proof, mathematicians treat a global counterexample as a local one – to find errors in the proof (RFT*). When RFT* is accepted, new conjecture is suggested by refining related concepts causing errors in the proof (FIX). Similarly, when a solution fails to pass a test (RFT), members of CoI begin investigating solutions and attempting to spot errors in it (RFT*). If a RFT* is accepted, then a detected error is removed (FIX) and a revised solution is supplemented (SUPPL). RFT* is used for detecting errors in a solution as local counterexample is a tool for finding error-part in a proof. In this respect, the roles of RFT and RFT* are similar to those of a global counterexample and a local one respectively in the process of proofs and refutations.

Now, a critical line is split into two branches of RFT and RFT*. This characterizes a process of *solutions and refutations* as the logic of CPS, which can be shown as in figure 5.7.

Every posting as a result of an individual cognitive process changes a current mathematical landscape in the shared world, which in turn stimulate individual cognitions. This relationship between learning and teaching in CMS, what Bruner (1996) calls as the interaction between culture and mind (see section 2.3) or what Popper (1978) calls as the feedback relationship (see chapter 6), provides fuel for CC to advance. Relationships between postings, which are structured in the shared world, position an individual's mathematical thinking in the current mathematical landscape. In this process, CC follows a logic of *solutions and refutations* in JPS and consists of two factors: relations in the shared world as social aspects, and individual cognitions in the private world. This can be illustrated by combine figure 5.7 with figure 2.2 into figure 5.8. A constructor's implicit agreement is denoted with a dashed arrow from the new posting of a revised solution to old posting of a refutation, which connects parts of a critical line as in figure 5.6 and 5.7. In figure 5.8, the shared world and the private world are joined together with an inverse directional double helix. In this respect, figure 5.8 shows how individual members of CoI are connected via the shared world and how students' experience is interwoven and shared in the CPS process.

Positioning Following Double Helix

While students are engaging in CPS, work load of mathematics problem solving in the shared world is distributed over individual cognitions in the private world along the inverse directional double helix. In other words, students take their positions as refuters or constructors following the double helix in CPS process. This is depicted with a critical line and a constructional line, which separately move via an individual's

Figure 5.8: Double Helix and Collective Cognition

mind in the private world (figure 5.8). In this study, students' positioning varies according to factors including: the current mathematical landscape in the shared world, the nature of errors, and their interest and ability. Each of these features are discussed in the sections that follow.

Mathematical Landscape in the Shared World. Logical aspects of the current mathematical landscape in the shared world influence an individual's mathematical thinking process. The following interview excerpt with STU6 shows how the intellectual landscape determined his action for RFT*. When STU6 was confronted with the 2nd type of ECS of figure 5.3 where EM2's solution and STU13's were competing with each other, he found that his solution and answer were similar to STU13's. Thus, he took up his role as a refuter by posting RFT* on EM2's solution rather than RFT or SUPPL.²

Lee: "You pointed out an error in this solution (EM2's). Didn't you attempt to check the other solution (STU13's)?"

STU6: "No...It looked to be right."

Lee: "Then, did you think it would yield this because the

²Still, STU6 can refute EM2's solution by using the fact that $\int_{-1}^1 \frac{1}{x^2+1} \geq 0$ because $\frac{1}{x^2+1} \geq 0$ (RFT). Or he can supplement STU13's solution. See branch 1 and 2 of solution 1-3 in Appendix 2.

problem was not complicated despite that this person (STU13) did not write his solution in a specific way?”

STU6: “Hum... I thought in the same way as this person did.”

Lee: “Did you solve it on a paper?”

STU6: “Yes, I did.”

Lee: “Then, was your solution similar to this?”

STU6: “Yes. I solved it by substitution as this person did and found the same answer. So I just wanted to point out what was wrong with this solution.”

Lee: “Did you think that there was no necessity to post your solution because an idea of your solution was already posted (by others)?”

STU6: “Yes.”

When STU6 was asked why he did not fix EM2’s solution with his RFT*, he replied as follows:

Lee: “Here, you pointed out that it needed + or – according to the range (of x in EM2’s).”

STU6: “Yes.”

Lee: “Why didn’t you fix (EM2’s solution) with this?”

STU6: “Uh...Uh...I thought about it but others solved it (in this way)...their answers were different like these... So I just wanted to point out what was wrong with this solution.”

This example implies that STU6’s role was as a refuter to spot an error in EM2’s solution because he got the same result with STU13’s and he chose not to supplement it. STU6’s action of RFT* changed the intellectual landscape in the shared world. This intellectual landscape in turn determined STU14’s position as a constructor in figure 5.3.

In general, when students were confronted with ECS and refutations, they reflected others’ postings and attempted to elaborate their thoughts. In figure 5.9, after STU6 posted his new solution, EM1 posted an erroneous solution with a different result from STU6’s. This 2nd type of ECS led STU6 to investigate his own solution and fix a simple error ³ in his solution. Then he made RFT* on EM1’s solution (see

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³he changed $\int_0^1 x^3 - \sqrt{x}dx$ with $\int_0^1 \sqrt{x} - x^3dx$

Figure 5.9: A Cognitive Map of Solution 1-5

branch 2 of solution 1-5 in Appendix 2). In an interview, STU6 explained how the erroneous solution impacted his thought and why he made RFT*:

“At first, I thought that average meant that (length of each chord) was evenly cut along y axis so that this was the concept of definite integral. Thus, I solved like this but EM1 did along x axis. I thought that the average would be changed if we chose a different axis. Something was wrong. I started to think about what the average was. It meant that summing whole values and dividing it (with the number of values). To evenly count (length of chords), I thought, it would be better to do it along the y axis than along the x axis. Thus, I posted like this (RFT* on EM1).”

In this case, the mathematical landscape in the shared world seems to have influenced individual thinking processes and positioning. Since the design of this experiment created ECS in most solution pages with erroneous solutions and error-makers’ activity, students were expected to position themselves as refuters. Actually, RFT/RFT*s were most frequently shown in two problem sets except RR ⁴ (see table 5.3).

Nature of Errors In figure 5.3, an error in EM2’s solution is related to the conceptual aspect of the relationship between an integrand and a primitive function. STU6 pointed this out for others (RFT*) and thus was positioned as a refuter. In

⁴Since I posted 6 erroneous solutions as error-makers (EM1 and EM2), which were coded as NEW, the number of students’ RFT/RFT* is bigger than the number of students’ NEW.

Figure 5.10: A Cognitive Map of Solution 1-4

contrast, when an error occurred in the calculating process, students would get rid of it as soon as they found it. In figure 5.4, an error in STU14's solution occurred in the calculating process and STU3 fixed part of it, because he concluded it an error (FIX*). Similarly, STU7 fixed STU3's solution as soon as he found an error in it (FIX*). They did not suggest results from finding-error activity as a form of texts but fixed calculating errors in the solution (FIX*). In this respect, they are positioned as constructors rather than as refuters. This example illustrates that the nature of errors can make a difference in positioning students in the MKB process.

Student Ability and Interest In figure 5.10, STU14 pointed out an error in

EM2's solution (RFT*) and then removed it (FIX). Thus, she worked as a refuter and as a constructor at the same time. In figure 5.4, when STU5 found that STU3's solution did not pass a test (RFT), he could have attempted to find errors and to fix the solution (FIX*) but he did not work as a constructor. When I asked STU5 whether or not he checked the solution to find errors, he replied:

“After roughly reading that (STU14's new solution)...I saw this (STU3's revised solution). I thought that $\ln 2$ would be small as soon as I saw it. So I calculated and found the answer was less than 0. In fact, I could not understand it thoroughly... What should I say... I tried to write a solution like this (STU3's revised solution) but could not solve (revise) it.”

This illustrates why STU5's role was confined to that of a refuter. Thus, positioning along an inverse directional helix can vary according to the individual student's ability.

In addition, students' interest influences their positioning and action. In figure 5.2, for example, STU8 took the position as a refuter for his first posting because he was not interested in investigating EM1's erroneous solution. When asked why he did not attempt to find errors in EM1's solution, STU8 replied that it was “boring” to check it:

Lee: “You saw that (EM1's solution) was wrong. Didn't you?”

STU8 : “Yes.”

Lee: “When you saw this (solution), did you investigate it from the start or from the end?”

STU8: “Once I saw its result, it did not make sense at all. So...”

Lee: So...?

STU8: “I started to look into it from the beginning... I did not specifically think about what was wrong in it.”

Lee: “Then why didn't you find the error in it?”

STU8: “Um...It was just boring (laugh).”

Lee: “Boring? About a month later, you visited the site and posted again. Had you been thinking about the problem during that period?”

STU8: “No.”

Lee: “Then you just came by...”

STU8: “I visited to check my posting and look around for others. Since there was no solution, I tried to think about a

solution and found imaginary numbers in it. I felt that something was strange. So I started to search (on the internet)”

Lee: “After your search, did you look into the first solution?”

STU8: “I saw it again. It was similar to this proof for real number range (pointing out the link “http://www.math.ucsd.edu/20B/7_DefInt.pdf”, which he made in his posting).”

Lee: “How did you find it?”

STU8: “I just typed “integrate $\ln \sin x$ ” then it came out.”

Lee: “Where?”

STU8: “Google.”

STU8 was not initially interested in investigating EM1’s erroneous solution. However, the fact that “there was no solution” even a month later led him to be interested in solving the problem and to investigate it. When he realized that it could not be solved using his current knowledge, he started to search for an answer on the internet. Thus, his interest led him to be positioned as a constructor.

Internalized Inverse Directional Double Helix

In many cases of this study, a member of CoI was doubly positioned as a refuter (RFT*) and a constructor (FIX). This pattern of double positioning is found in solution 1-2 (figure 5.17), 1-4 (figure 5.10), 1-5 (figure 5.9), 2-1 (figure 5.16), and 2-3 (figure 4.8). In a similar way to figure 5.8, double positioning in CPS is drawn as figure 5.11. In this figure, two lines of the double helix move together via individual’s mind where mathematical thinking process with double action of RFT* and FIX occurs. In this case, agreement is not shown because s/he does not have to agree with herself/himself, which makes this pattern a shortened form of a double helix.

In contrast to figure 5.8 where two roles are separately taken up in the shared world and thus a critical line and a constructive line alternatively interweave the CPS process, two lines move together between the shared world and the private world in figure 5.11. Two different movements of the double helix in figure 5.8 and 5.11 show that there exist relationships between positioning and the individual mathematical

Figure 5.11: Internalized Inverse Directional Double Helix

thinking process. The MKB process in the shared world and the individuals' mathematical thinking process in the private world are commonly mediated by language.

From Vygotsky's perspective, inner speech is an internalized form of outer speech (Vygotsky, 1962). Individual mathematical thinking as inner speech reflects positioning as relation between outer speeches. That is, students' double positioning can be interpreted as an internalized form of separate positioning in outer speech. In this respect, an inverse directional double helix in the shared world seems to be internalized into the individual private world in one's cognitive developmental process. Thus, I characterize the shortened form of the double helix with RFT* and FIX in figure 5.11 as *internalized inverse directional double helix*.

If an individual's mathematical thinking process is internalized movement of an inverse directional double helix, it will follow the logic of the CPS process depicted in figure 5.7; $NEW \rightarrow RFT \rightarrow RFT^* \rightarrow FIX$. In this study, STU14's posting for solution 2-3 shows that her thinking process followed the logic of the CPS process (see branch 2 of problem 2-3 in Appendix 2 and figure 4.8). In 2nd type of ECS of

figure 4.8 where STU8' solution and EM2's were competing, STU14 refuted EM2's by checking (RFT), pinpointed an error in it (RFT*), and then fixed it (FIX). Thus, an individual student's mathematical thinking process proceeds following the inverse directional double helix in the private world like the CPS process does in the shared world. If the MKB process in the shared world and one's mathematical thinking process developed together along two lines of a double helix, then growth of SM in the shared world and growth of an individuals' knowledge in the private world would resemble each other.

It is noteworthy that two lines of inverse directional double helix come across at places where errors occur in solutions. Students decide their positions when they are situated at these places in ECS. When a student is positioned as a refuter, s/he attempts to find an error in a solution. However, students may or may not clearly reveal their thinking process by writing about it as seen in the previous section. In contrast to STU14's clear expression of her thinking process (figure 4.8), members of CoI may participate in the MKB process without a clear expression of their thinking process in the shared world. When a member of CoI, who is doubly positioned, does not reveal her/his act of eliminating errors in the shared world, two lines of the inverse directional double helix do not intersect in shared world and thus error-spots that s/he has detected cannot be shared in CMS. This is more likely to happen in an automatized cognitive process such as calculation.

In figure 5.4, STU7 and STU3's eliminating-error action without RFT* produced revised solutions (FIX*). Even though they did not express RFT* and AGMT in their writing in the shared world, they certainly worked on a critical line while scrutinizing the solution to find errors and worked on a constructive line while removing them. To be sure, those actions had occurred as combined actions of RFT* and FIX in their private worlds before results of them were posted in the shared world. In this respect, FIX* is interpreted as combined action of RFT* and FIX. Thus, a con-

Figure 5.12: Inseparable Two Strands of Double Helix in The Private World

structive line in figure 5.4 is actually a form of combined inverse directional double helix, which has internalized into one's cognitive process in the private world and two lines of double helix are so tightly intertwined that the distance between them is zero and they look like a single thread in figure 5.12. A RFT* branch of a critical line in figure 5.7 is merged with a constructive line. This implies that two different roles of a refuter or a constructor are sometimes inseparable in one's automatized cognitive process of the private world. Since individual cognitions take up more portions in CMS in figure 5.12, two different positioning as a constructor and as a refuter does not occur in a separate way.

Since results from finding-error activity imbedded in FIX* are not suggested in a form of text on a critical line, errors detected from finding-error activity may not be opened up in a clear way to other members of CoI as RFT*. In this respect, it is more difficult for members of CoI to follow any changes in revised solutions through FIX*. This implies that CoI needs more specified tools to test revised solutions that resulted from FIX* or needs more refined lenses to investigate it thoroughly. In figure

5.4, students kept checking revised solutions using the test (RFT) that was applied to the first solution, but did not scrutinize changed parts from FIX* in the calculation process. As a result, students could not find errors in the solution. Furthermore, since students could not find new errors made from FIX* in the private world, they transmitted not only old ones but also new ones to other revised solutions.

Two Types of Confirmation Triangles

Developed knowledge or solutions within CoI need to be verified as true. In this respect, members of CoI need to develop a tool for increasing reliability with respect to developed knowledge or for confirming correct solutions in the MKB process. In this study, CC used two patterns to confirm a solution: Backward confirmation triangle and forward conformation triangle. Confirmation triangles relate different branches of solutions while inverse directional dual helix relates postings in one branch of a solution.

In a backward confirmation triangle, members of CoI revised an erroneous solution to get a right answer or used a new solution as a tool of confirmation. In figure 4.8, for example, STU8's solution and EM2's erroneous one were competing, STU1 supplemented STU14's revisal of EM2's erroneous one (SUPPL). Then he confirmed STU8's solution (CFM) by ascertaining that he got the same result with STU8's by revising EM2's solution; "If we integrate it by splitting the interval (into two small intervals) as mentioned in above (STU14's posting), then we can find the right answer (as STU8's). -STU1 16:54, 3 June 2012 (EST)". In this way, STU14's FIX and STU1's SUPPL revised EM2's erroneous solution to confirm STU8's answer as right following an inverse directional double helix.

In another illustration (see figure 5.3), EM2's answer '0' (NEW) and STU13's ' $\pi/2$ ' are conflicting, STU1 confirmed STU13's result by showing that his new solution had the same result with STU13's; he wrote " In this case, we ..... can find the same

Figure 5.13: A Backward Confirmation Triangle

answer with STU13's. –STU1 12:48, 3 June 2012 (EST)” (see branch 3 of solution 1-3 in Appendix 2). In this case, STU1's new solution raised STU13's hand over EM2's as a judge does in a match by declaring STU13's answer right. The above two examples can be described as a pattern of a backward conformation triangle (figure 5.13). This pattern is descriptive of other examples (see figure 4.8, 5.3, 5.4, and 5.10). For example, figure 5.10 illustrates a double confirmation triangle: STU14 revised EM2's erroneous solution to get the same result with STU2 and thus confirmed STU2's solution; STU13 confirms STU14 by suggesting a new solution with the same result. Thus, a small backward confirmation triangle is nested in a bigger one.

In contrast to a backward confirmation triangle where a new alternative solution or revised solution is used to confirm an existing solution, established knowledge within CoI can be used to confirm newly introduced methods as reliable. In figure 5.14, STU8 suggested his solution in an intuitive way rather than from a rigid calculation. Other students did not seem to be interested in developing STU8's posting further. When I asked STU8 why he had not used integral calculus to show how to get a result, he said, “I just wrote πab because it is well known and I thought that I did not have to show it using integral calculus,” confirming that it is natural to consider what is commonly shared with in the CoI.

Figure 5.14: A Cognitive Map of Solution 2-4

Figure 5.15: A Forward Confirmation Triangle

After the problem was changed by adding the condition of “using Green’s Theorem” (CP), STU7 posted his solution showing the same result as STU8’s. Since Green’s Theorem had been newly introduced to students (they just started to learn Green’s Theorem in Mr.O’s class in the first week of June when STU7 posted his solution), STU7’s solution needed to be tested. To test it, members of CoI needed to use the well-known result of πab to confirm a new method introduced to CoI. Although this experiment ended a few days after STU7’s solution was posted, STU8’s solution seemed to be accepted because no one refuted it. In this respect, STU8’s result is interpreted to implicitly confirm STU7’s new solution. Thus, an arrow of CFM is drawn from STU8 to STU7, which characterizes a forward confirmation triangle of figure 5.15. In a case where common background knowledge in CoI is used to confirm new knowledge, the forward confirmation triangle of figure 5.15 represents the situation. Since this problem is frequently given as an example for Green’s Theorem in textbooks, this problem may have provided ESMPR rather than ECMSE.

Although a confirmation triangle was an useful mechanism for CoI, it might deceive members of CoI to accept an incorrect solution. For solution 2-5 (figure 5.4), STU11 refuted STU14’s result (RFT) and STU3 fixed STU14’s solution (FIX*). This pattern was repeated with STU5’s RFT and STU7’s FIX* (see branch 1 of solution 2-5 in Appendix 2). At this time STU1’s new solution, which changed the order of integrations with respect to x and y , falsely confirmed STU7’s wrong answer (see branch 2 of solution 2-5 in Appendix 2). This backward confirmation triangle did not operate properly but deceived members of CoI to believe that STU7’s solution was correct. This backward confirmation prevented even Mr.O and Mr.C from finding errors in STU7’s solution because they did not have to investigate STU7’s solution in the situation that STU7’s solution was confirmed. This shows that the confirmation triangle may malfunction when the every detailed process of CPS is not clearly opened up in JPS.

5.3.2 Nature of Problems and Solutions

Although open-ended problems may more easily facilitate MKB than problems with fixed answers (Nason & Woodruff, 2003; 2004), I did not use open-ended problems for this study. Instead, I structured ESMCE using erroneous solutions. Since students did not show interest in problems that they deemed as easy, structuring ESMCE is important. For example, in problem 2-4, which is the easiest one and posted without any ECS (see Appendix 2), I expected students to post various solutions but they did not because they were not interested in solving a simple problem. This caused me to intervene CC by changing the problem (CP).

Students attempted to build mathematical knowledge beyond their course of Calculus II. Problem 2-2 was the most challenging problem in set 1 and 2 because it belongs to the domain of Complex Analysis, even though it looks like a typical problem in a Calculus course (see Appendix 2). When STU8 created the 1st type of ECS in figure 5.2 by testing EM1's erroneous solution (RFT), this ECS caused students to spot errors in it; STU2 and STU10 attempted RFT*s. As seen in figure 5.2, they were not accepted and thus scattered to a distributed form. STU8 noticed that problem 2-2 was beyond their knowledge as soon as he attempted to solve it because $\ln(\sin x)$ is imaginary when $\sin x$ is negative. Then, he sought out an answer on the Internet and got it from "Wolframalpha" and made link to it (see branch 3 of solution 2-3 in Appendix 2).

Patterns of CPS differed according to two types of erroneous solutions: fixable and non-fixable. If an erroneous solution can be fixed to get a right answer, it is fixable. If not, then it is non-fixable. Erroneous solutions embedded in problems 1-1, 1-2, and 2-1 and some of the error-makers' solutions (see Appendix 2) are non-fixable and the others are fixable.

In non-fixable cases, students' constructive activities are restricted to critical action of RFT*. For example, although an erroneous solution suggested in problem

1-1 and STU14's solution for problem 2-5 commonly created the 1st type of ECS, their patterns of CPS emergent from ECS were different. Since a solution embedded in problem 1-1 is non-fixable, students' activity occurred along a critical line only, that is, they attempted to pinpoint hidden errors in the process of applying the mathematical induction to the proof (RFT*) and they did not revise the erroneous solution. Thus, a constructional line does not appear in figure 5.1. In contrast, figure 5.4 shows that members of CoI are alternatively working as a refuter and a constructor and CPS occurs along an inverse directional double helix. This happened because STU14's solution was fixable.

In the 2nd type of ECS of figure 5.10, EM1's erroneous solution is non-fixable whereas EM2's is fixable (see Appendix 2). EM1's solution starts with a wrong assumption of the existence of limit; $L = \lim_{x \rightarrow 0} \frac{(\sin 3x)}{x^3}$. Without this assumption, EM1's solution is not sustainable and thus cannot be revised. As a result, when STU6 and STU2 found errors in it (RFT*), they did not attempt to fix them. Thus, in a case of non-fixable erroneous solutions, members of CoI focus on revealing every possible error. In this respect, despite that the CFP moved to EM2's solution from EM1's three weeks later after EM1 posted an erroneous solution in figure 5.10, STU2 added his RFT* on EM1's solution as follows (see branch 1 of solution 1-4 in Appendix 2).

On the other hand, in the same figure, since EM2's erroneous solution is fixable, it is developed following an inverse directional double helix; it is refuted (RFT*) and fixed (FIX) by STU14. In this way, CPS depends on the nature of the given solutions in ECS.

In a non-fixable case of figure 5.16 of solution 2-1, STU1 showed that the given solution's result was wrong by pointing out errors. However, since the given solution was non-fixable, his revisal did not lead it to the right answer. In this respect, STU11 suggested a new alternative solution (NEW) after he revised STU1's solution (FIX*) (see branch 2 of solution 2-1 in Appendix 2).

Figure 5.16: A Cognitive Map of Solution 2-1

5.3.3 Pencil-and-paper vs.The Wiki

In figure 5.8, to connect the private world and the shared world with an inverse directional double helix, products from individual cognitions in the private world need to be encoded into the shared world and hypertexts in the shared world need to be decoded for the individual cognitive process in the private world. Although the Wiki as a collective cognitive tool provided media such as LaTeX, Editor, Images, Graphs, etc.for encoding and decoding, students used pencil-and-paper to record or use mathematical expressions while participating in CPS. When students did not use a recording tool, they frequently made errors. This implies that pencil-and-paper works as an encoding and decoding tool connecting individual cognitive process with CPS in JPS.

As seen earlier, students in this study had difficulty in writing mathematical expressions using LaTeX. This difficulty caused students to avoid writing mathematical expressions in the shared world. In this respect, students frequently stopped working at the place where they could contribute to CPS further. While interviewing STU3 and STU11, they said:

Lee: "Here (on the page of solution 1-4)...You said that the condition was needed. Why didn't you show it (with mathematical expressions)."
 STU11: "This was when I participated for the first time.
 It was difficult to type mathematical expressions."
 STU3: "It was really difficult."
 Lee: "Was it that difficult?"
 STU3: "Yes, we've never used it before."

Typing mathematical expressions of their written solution on papers in the Wiki took them additional time and effort. This caused students to be reluctant to detail their solutions in terms of mathematical expressions in the shared world. For example, STU1 recorded his ideas but not his complete solution even though he had solved problem 2-1 with pencil-and-paper. An interview with him revealed that he did not want to put additional time and effort into typing mathematical expressions in the shared world:

Lee: "It is 2-1. (You wrote that s/he) did not consider (the first) integral constant, and the second one, either. Did you calculate it, too?"
 STU1: "I calculated it with pencil-and-paper. It would take too much time to write the entire (solution in the shared world).
 So I posted my idea only."

As a result, the shared world in the Wiki may not reflect the entire individual students' thinking process in the private world.

When individual students' solutions were not placed under investigation in JPS, errors could be transferred and even new errors were added to solutions. In the process of investigation, students needed pencil-and-paper as part of their workspace. In an interview, when I asked STU3 and STU11 why they failed to revise STU14's solution for problem 2-5, they pointed out the importance of pencil-and-paper while investigating solutions as an individual cognitive tool:

STU3: "I think it happened because I roughly fixed it. I could've found the right answer if I used pencil-and-paper."
 STU11: "Typing (mathematical expression) on the site..."
 Lee: "Did you directly type (your solution) without writing it on a paper?"

STU3: “Yes, so I could not process it in my mind.”
STU11: “Typing on the site was not good for my thinking process. We used to solve the problem on paper. We could’ve make it right...(if we used pencil-and-paper)”.

In summary, to follow mathematical expressions in a solution in the shared world, they needed individual workspace where they could process mathematical expressions. However, the Wiki did not provide such space so that pencil-and-paper was used instead to process the cognitive load emergent from the encoding and decoding process of mathematical expressions.

5.3.4 Errors and Joint Problem Space

Since it is analyzed to be important for students to have enough time to think as an individual or as small group before participating CPS in section 3.1.3, students were asked to individually work in the private world before joining in JPS. Since a failure to create an individual workspace or a problem space using pencil-and-paper or other tool may cause errors in the shared world, passing through such individual workspaces was considered in designing this experiment. One of the norms of participation in this study was to solve a problem before reading others’ solutions on a solution page. This was considered as a precondition for students to attach themselves to JPS based on a common understanding of problems. When students did not abide by this norm, that is, when they did not pass through their individual problem space, they made errors in the shared world.

When I mistakenly posted a different problem in figure 5.10, STU5 and STU6 did not notice my error because they did not solve the problem before they read EM1’s solution. Although they glanced through the problem, they could not remember the problem while investigating EM1’s solution. Thus, they posted a new idea (NEW) and refutation (RFT*) with respect to EM1’s solution, respectively.

Lee: “Here, you wrote ‘We can also solve this by using

l'Hôpital's Rule.”’
 STU5: “Yes, I did.”
 Lee: “Then, you did not notice that the condition of the problem was
 (for x) to go to infinity.”
 STU5: “No, I did not seem to. I just read this (EM1's) solution.
 I thought that it could have been solved like this but it also could be
 solved by applying l'Hôpital's Rule. So, I posted mine.”

The following transcript of an interview illustrates that STU6 did not solve the problem before he read EM1's solution:

STU6: “When I saw the problem again after I posted mine, (I found that
 the condition of) the problem was to go to infinity.”
 Lee: “Then, did you enter into the solution page without solving the
 problem?”
 STU6: “Yes, it seems that I did.”
 Lee: “(You said) To solve like this (EM1's solution), this (existence of
 solution) needs to be guaranteed. Did you just delete (your posting)
 because the problem was not matched with yours?”
 STU6: “Yes.”

Unlike STU5 and STU6, STU8 solved the problem before he read EM1's erroneous solution. Thus, he instantly found the mismatch between the problem and EM1's solution. Understanding a problem is critical for students to enter into JPS. Reading a problem was not sufficient for really understanding the problem. Only when students attempted to solve a problem, could they understand and remember it.

5.4 Teaching Presence

Despite that I, as a researcher, was cautious not to directly intervene in the students' CPS process, it turned out that I was the primary participant in CPS process as well as the designer and organizer. Comparing table 5.4 with 5.5, I posted 13 times as error-makers (6 times) and as administrator (7 times including providing related materials (RM) twice, helping students writing mathematical expressions (RR) twice,

raising a question (Q) three times, and changing conditions of problems (CP) twice⁵). Since I posted more than students did, I was the primary contributor to growth of SM. In this respect, my role in this experiment needs to be taken into account.

5.4.1 Designing and Organizing the Educational Experience

I created ESMCE to provide the initial intellectual landscape in the shared world by selecting and adjusting problems. I designed the mathematical landscape to facilitate individual and collaborative attention around finding and eliminating errors in the given solutions in problem 1-1, 1-2, and 2-1, and sometimes moved it toward a new direction. I also worked as error-makers to spark CC by causing cognitive conflicts in CMS. In addition, I considered some educational aspects in structuring ESMCE while designing this experiment such as students' interest and ability, and relatedness between mathematical content.

In designing the project, I selected the Isoperimetric Theorem as a topic because it was related to Green's Theorem that students in Calculus II were expected to learn. I expected the historical background of the Isoperimetric Theorem to induce students' interest. As expected, it evoked students' interest and voluntary participation. In fact, most interviewed students replied that the project was the most interesting problem in the Wiki material. For example, STU9 voluntarily posted related reading material. In her interview, she indicated interest in the Isoperimetric Theorem which led her to a voluntary search for related material.

Lee: "Were you more interested in the Isoperimetric Theorem than other problems?"

STU9: "Yes, I was."

Lee: "I was interested in your posting. Where did you find it?"

STU9: "On the internet."

Lee: "Internet?"

STU9: "Yes."

Lee: "What did you use for searching?"

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⁵Two postings of ADM include two codings of RM & Q (for problem 1-1) and CP & Q (for problem 2-4), respectively.

STU9: “Google.”

.....

STU9: “I cannot remember well now. Probably a problem (posted in the Wiki) was about a flat plane but this was related to a curved plane. I thought that it was wonderful. So I posted it (below yours).”

As a designer and an organizer, I expected that students could relate common errors across solutions. For example, Steiner’s proof and EM1’s erroneous solution for problem 1-4 contain a common error: starting from the assumption that a solution exists. As I expected, STU6 exactly pointed out this error in his RFT* on EM1’s solution for problem 1-4 (see branch 1 of solution 1-4 in Appendix 2). Far from my intention, however, this REF* did not impact the process of students’ discussion about Steiner’s proof. There are two reasons for this. Firstly, since Green’s Theorem was placed in the last part of Calculus II, a project of proving the Isoperimetric Theorem was not a main concern to students. In this respect, students’ participation in the project was not activated. In addition, I mistakenly posted problem 1-4 with different conditions and STU6 deleted his posting when he found the mismatch between problem 1-4 and EM1’s erroneous solution. Thus, most students could not read his posting on the page of solution 1-4. This example shows that a teacher’s intention as a designer may not be realized in the MKB process. In CMS, students’ learning does not necessarily follow paths that a facilitator intends or anticipates.

5.4.2 Facilitating the MKB Process

Directing

As a director of the MKB process, a facilitator can form and move CFP in the MKB process. In this study, I changed problems (CP) twice. Whenever I changed problems, students’ CFP moved from the old problem to a new one in CMS (see figure 5.10 and 5.14). In figure 5.10, I mistakenly posted a different problem for problem

1-4 and EM1 posted an erroneous solution for the original problem as planned. This caused a mismatch between problem 1-4 and the erroneous solution. STU8 pointed out this (RFT). As soon as I found STU8's posting, I changed the condition of problem 1-4 from $\lim_{x \rightarrow \infty}$ to $\lim_{x \rightarrow 0}$ (CP). This moved CFP to the changed condition. The CPS process with respect to the old problem with the condition of $\lim_{x \rightarrow \infty}$ stopped even though it could be developed further. For example, someone might have supplemented STU11's revised solution as I suggested in branch 2 of solution 1-4 in Appendix 2. In contrast, the CPS process was activated around the new problem with the changed condition. Thus, a new CFP was formed around the changed problem.

In addition, I changed problem 2-4 by adding a condition of "using Greens' Theorem" to its statement (see figure 5.14). Students were not interested in solving problem 2-4 because it was too easy for them. Even though there are at least three different methods to solve this problem such as substitution, Jacobian Determinant, and Green's Theorem, students did not post their ideas during the 5 weeks after STU8 posted his solution. Thereby, CC was not activated and thus the CPS process was poor. I decided to intervene to change the intellectual landscape in the shared world by changing the problem (CP). As a result of my intervention, CFP was moved from an answer itself to a method; STU7 posted a new solution "using the Greens' Theorem".

Since resources for the MKB process are finite, a facilitator needs to consider which path is the most effective for the MKB process in the shared world. A teacher can move CFP to a place that s/he wants students to concentrate on while they are engaging in CPS. S/he directs CC toward a place worthwhile for the MKB process from her/his perspective. Sometimes students' voluntary activity may go beyond the range of knowledge that is expected. For example, when STU1 found another geometric proof for the Isoperimetric Theorem on the Internet and made a link to it, I needed to decide whether to encourage pursuit of this approach. I asked students

to discuss it for research sake because I had the desire to see how students build mathematical knowledge. As mentioned in 3.1.3, however, if I had a clear intentional goal as a teacher rather than as a researcher, I might have made a different decision to direct CFP toward the educational goal. As a result, no one reacted to my suggestion.

Clearly, when directing the MKB process, a facilitator needs to consider factors such as time, mathematical relatedness, and students' interest and ability etc. The decision will reflect a facilitator's perspective. For example, in figure 5.2, when STU8's RFT created the 1st type of ECS, individual students had two choices: revising solution (RFT*/FIX) or developing alternative solution (NEW). That is, they might take their positions as refuters or as constructors. At first, STU2 attempted to point out an error in error-maker's solution (RFT*) (see branch 1 of solution 2-2 in Appendix 2). About an week later, STU1 suggested a new idea (see branch 2 of solution 2-2 in Appendix 2). An interview with STU1 shows that he had an idea for solving problem 2-2 but did not develop a solution:

Lee: "Next here, you just suggested an idea. Didn't you calculate it?"

STU1: "Yes, I did. I found it in a form of series expansion."

Lee: "Since it came in a form of series expansion, (you calculated it) approximately..."

STU1: "Yes."

Lee: "Did you think that it could be sought only (in a approximate way)?"

STU1: "Yes."

At this point, STU10 attempted to point out an error in an error-maker's solution. On the next day, STU10 again attempted to point out an error in the error-maker's solution (see branch 1 of solution 2-2 in Appendix 2). Lastly, STU8's posting showed that problem 2-2 was beyond their course and there was a necessity to learn advanced mathematical knowledge of complex analysis. Since students' postings did not form CFP, they were scattered to form a distributed form of figure 5.2. Confronting STU8's suggestion, a teacher might confine the extent of students' MKB to a Calculus II course rather than encourage students to study more advanced complex

analysis.

A facilitator's own interest can influence his/her directing of the MKB process. For example, Mr.O was interested in the fact that two different erroneous solutions of EM1's and EM2's for problem 2-3 yielded the same result. In this respect, he was curious that there exists a relationship between characteristic of limit and l'Hôpital's Rule's condition. Thus, he used the instructional strategy of "*capitalizing on errors as springboards for inquiry*" to lead CC to concentrate on digging into why this happened (Borasi, 1994; 1996). In this way, the direction of students' MKB process will change according to a teacher's decision.

Providing Related Materials

In forming CFP, the facilitator can extend the students' MKB process by providing related materials. To stimulate CC, I provided related material (RM) twice (see figure 5.1 and 5.17). As seen in figure 5.1, RM did not influence students' learning and thus failed to form CFP because STU14, STU10, and STU4's postings were scattered rather than threaded.

In contrast, RM influenced activity as shown in figure 5.17. In the figure, STU4 posted his RFT* with conclusion: "Here, C is -1 " (see branch 1 of solution 1-2 in Appendix 1-2). When I raised a question: "Why $C = -1$? -ADM 10:10, 17 April 2012 (EST)", STU2 answered me saying: "It is trivial. -STU2, 14:50, 19 April 2012 (EST)". I felt something needed to stimulate CC. Thus, I posted related materials (RM) (see branch 2 of solution 1-2 in Appendix 1-2). My suggestion regarding related material (RM) elicited STU2's posting (RE). In addition, STU13 applied STU2's idea to fix the given solution (FIX*). What I posted as RM directly led STU2 to think about the meaning of the integral constant. Then STU2's thought influenced STU13 to fix STU4's error of taking $C = -1$. Thus, my action as a provider of related materials influenced the action of these students.

Figure 5.17: A Cognitive Map of Solution 1-2

Questioning

A facilitator can stimulate CC by asking her/his students to clarify meaning, justify, show specific calculation process, give an example, think in another way, and so on. Since students' writing in this experiment was frequently ambiguous, I needed to ask them to articulate their explanation to stimulate CC and activate the MKB process.

When I raised a question of "Why $C = -1$?" with respect to STU4's posting in figure 5.17, STU2's reply was trivial. My posting of RM implicitly refuted STU2's idea. This was possible because CFP was clearly targeted toward the meaning of the integral constant C by my question. In contrast, my question in figure 5.1 was "how about explaining your thoughts by comparing the following proof with Youngbin's solution?" This question was too broad to form CFP around finding-error activity in STU5's posting. As a result, there was a confusion and students' postings were scattered.

Editing and Helping

Until students get accustomed to using new skills in CMS, their collaborative MKB can be distracted because of difficulty in writing mathematical expressions and editing their Wiki pages (Carter, 2009). Because of this, I edited students' writing of mathematical expressions and edited their postings in the shared world. As expected, students had trouble writing mathematical expressions with LaTeX. Many students mentioned it in the interviews. Most students spent more time on writing mathematical expressions in the shared world than on solving problems with pencil and paper. In an interview, STU14 stated:

Lee: "How long did it take you to solve it (problem 2-5)?"

STU14: "It took me less than 5 minutes. But it took me much more to write my solution (using the Wiki)."

Lee: "How long did it take you to write it?"

STU14: "About 10 to 20 minutes. Whenever (I write a solution using LaTeX)."

In figure 5.4, I repaired STU14's mathematical expression (RR) in response to her asking for help; STU14, who was the first solver of problem 2-5, left a help-message after she had attempted to fix her mathematical expressions 7 times: "The enter key does not work for making a new line ¶ Someone, who writes after me, please makes lines in my expression by aligning equal signs _/-STU14 22:35, 8 May 2012 (EST)." This was the sole expression written in a casual way with emoticons. Since STU14's mathematical expression was connected with many equal signs and very long rather than wrap around, her mathematical expression was off the page. The shaded part of the figure 5.18 shows the part of her mathematical expression that was off the page. On the next day, I repaired her expression as ADM (RR) in figure 5.4 and deleted the above sentence. This helped CC advance the MKB process by relieving the cognitive load of writing mathematical expressions.

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \sum_{k=1}^n \frac{j}{j^2+k^2} = \lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{j=1}^n \sum_{k=1}^n \frac{j}{\frac{j^2}{n^2} + \frac{k^2}{n^2}} = \int_0^1 \int_0^1 \frac{x}{x^2+y^2} dx dy = \int_0^1 \left[\frac{1}{2} \ln(x^2+y^2) \right]_0^1 dy = \int_0^1 \left(\frac{1}{2} \ln(y^2+1) - \ln y \right) dy = \frac{1}{2} [y \ln(y^2+1)]_0^1 -$$

enter처도 줄이 안바뀌네ㅠ 다음줄이 쓰는사람 내꺼 등호에 맞춰서 줄줄 바꿔주어\^_/\

--통가통가 22:35, 8 May 2012 (EST)

Figure 5.18: STU14's Mathematical Expression Out of the Page and Her Asking For Help

위의 링크에 접속하면 다음과 같은 증명이 나옵니다. 다음은 보기 편하도록 수식을 편집한 것입니다. 다음의 증명에 대한 여러분의 의견을 알려주세요...

증명)

위의 그림에서 AB....Z는 볼록 다각형의 꼭지점이다.

다각형의 둘레를 L, 넓이를 A 라 하고,

다각형에서 선분 AQ 를 다음을 만족하도록 긋는다.

(1) $AB + BM + \dots + PQ = L/2$

(2) 다각형 ABM....PQA 의 넓이를 A_1 이라 할 때 $A/2 \leq A_1$

점 O 는 선분 AQ의 중점이고, 점 M은 다각형 ABM....PQA 의 꼭지점 중에서 점O 에서 가장 멀리 있는 점 이다.

선분 OM=R이라하고 중심이 O 이고, 반지름이 R 인 원을 그린다.

Figure 5.19: Part of Researcher's Editing A Proof From Outer Source

In addition, I edited students' mathematical writing as ADM to be readable (RR). For example, STU1 posted a link for a geometric proof of the Isoperimetric Theorem. When I visited the site to which STU1 made a link, it was hard to read the proof because it was written like the second students' solution in the pilot study (Appendix 1). Thus I edited it and made a discussion page for the proof on the next day (figure 5.19).

5.5 Mr.O's Opinion and Students' Opinion About The Use of Wiki

Mr.O identified some positives from using the Wiki: first, a teacher can suggest mathematical topics to students that s/he cannot cover in her/his class due to the

limitation of class time. Students can post their ideas and keep elaborating them in the Wiki whenever they have available time.

Second, the Wiki environment provides an alternative way to participate for students who do not want to participate in class discussion. Although Mr.O emphasized students' mathematical communication in his class through discussion and presentation, he had a few students who were too shy to present their solutions in front of others. Mr.O said that they participated in CPS using the Wiki even though they did not present their solutions or ideas in his class. In this respect, the Wiki provided them an opportunity to participate in CPS. This is consistent with Carter's findings that the Wiki provides a new learning opportunity to a "shy student who never spoke in class and never came to office hours" (Carter, 2009, p.9).

Third, Mr.O pointed out that accumulated material in the shared world could be used in his course. This supports my viewpoint of considering products from CPS as oeuvres. Finally, he mentioned that there was a student who asked him whether or not s/he could suggest her/his own problems in the shared world, he said to her/him "yes". Although s/he did not post her/his own problems in the shared world, he thought that the Wiki could be a powerful tool for students, who used to passively solve given problems, to discover their own mathematics problems, and to participate in MKB. He valued this aspect of the Wiki environment because he felt it helped students experience doing mathematics as mathematicians.

Mr.O made one suggestion to improve the Wiki environment: he claimed it would be better if the Wiki site instantly showed on the front page and sent messages to students' cell phones what had changed on the Wiki pages.

Regarding student opinions, 8 out of 14 participants responded to the survey (table 5.8). Seven of these 8 students said that others' comments and solutions were helpful to develop their thoughts (Q1) and the Wiki was useful to improve their solutions (Q2). Seven out of 8 students replied that the ideas and solutions of others

No	Questions	Yes	No	NR
Q1	Did other students' comments or solutions were engaging in stimulate your new thought while you solving the posted problems? If "Yes", in what points? If "No", Why?	87.5%(7)	12.5%(1)	
Q2	Is the Wiki useful for you to improve the solutions by removing errors? 1. Yes 2. No	87.5%(7)	12.5%(1)	
Q3	Did you actively participate in solving problems using the Wiki? If "Yes", Why? If "No", Why?	50%(4)	37.5%(3)	12.5%(1)
Q4	How many times did you stop by the Wiki site?			
Q5	How many times did you write in the Wiki system?			
Q6	Did you have any trouble in writing mathematical expression using the Wiki while you were engaging in solving problems? If "Yes", then what is your suggestion for making the writing system easier to write the math expression?	75%(6)	25%(2)	
Q7	Do you think that it is necessary to set up norms of participation for collaborative problem solving using the Wiki? If "Yes", why? If "No", why?	12.5%(1)	87.5%(7)	
Q8	Do you think that collaborative mathematics problem solving is more efficient than individual problem solving? If "Yes", why? If "No", why?	100%(8)	0%(0)	
Q9	Which problem was the best one for collaborative problems solving using the Wiki? And which one was the worst?			
Q10	What are your suggestions to improve the design of the Wiki pages?			

Table 5.7: Survey Questions and Students' Response

helped them think in other ways and improve their own solutions. Most interviewees also showed similar opinions:

STU3: “A good point is that it helped me save time for solving and think starting from a certain level where others already led the process (of solution) while it took me more time to solve by myself because I had to start from the bottom. On the other hand, when I read problems, I ...”

STU11: “Mostly, we individually have solved problems by ourselves. In addition, I cannot find various solutions in problem books. While experiencing the Wiki, even though I could not approach a solution (by myself), I was able to use others’ ideas to easily approach it. And it was good that I could find more solutions than I solved by myself.”

Lee: “What were bad points (of the Wiki)?”

STU11: “Uh...bad point was...I could not find anything bad about the system itself. It was difficult for me to type or draw pictures for explanation.”

Four of the 8 students replied to Q3 that they actively participated in collaborative problem solving using the Wiki because problems were different from those in their class and were interesting to them and errors in others’ solutions invoked them to revise solutions. Three students indicated that it was annoying to turn on the computer and connect to the Wiki and to type mathematical expressions using the Wiki, and one of them said that s/he was not in the mood to solve problems when other students’ solutions were already posted in the shared world.

Excluding the weeks during the mid-term exam period, four students stopped by the site once or twice a week and three students did so once per two weeks (Q4). One student came to the site once a month. Six of the 8 respondents indicated that LaTeX was too complicated and inconvenient to type mathematical expressions because they had no time to practice LaTeX and had to search for how to type a new sign whenever they needed to, and it was hard to check whether or not they were typing correctly and to revise what they wrote. In contrast, two of the 8 students did not have trouble using LaTeX. One student said that it was OK because s/he could

get used to LaTeX fast. The other student said that it was not bad because it was similar to some Korean Word Software.

Seven of the 8 students indicated that norms for CPS were not necessary because ‘no rule’ would give them freedom to express. Only one student replied that setting up norms of participation was needed to promote the purpose of the Wiki by preventing small groups of students from dominating the whole CPS process.

All respondents agreed that CPS is more beneficial than individual problem solving (Q8). They said that others’ ideas and solutions made it possible for them to think in various ways, to know what they did not know, and to reduce errors when problems were sufficiently challenging.

With respect to Q9, four students, who suggested reasons for their choice, agreed that problems providing things to discuss and think about were better for CPS than problems of simple calculation. Students suggested several things to improve the design of the Wiki in this experiment (Q10). Four students pointed out that easy typing of mathematical expressions was needed. One of them suggested that it would be better to make it possible to ‘copy and paste’ mathematical expressions typed in Korea Word Software. Two students mentioned the specific problems used in this study. One said that more problems needed to be posted and they should be better for discussion and have potential to be developed further as a project problem. The other said that the difficulty level of problems need to be systematic. One of students suggested that it would be better to be able to connect to the Wiki with a Smart Phone.

5.6 Difference between Teacher’s Perspective and Students’ Perspectives on Contribution to Growth of SM

Mr.O used words such as “meaningful”, “showed”, and “explained” to explain on how contributors facilitated learning opportunities by letting others compare,

Evaluator	The 1st Contributor Reasons	The 2nd Contributor Reasons	The 3rd Contributor Reasons
Mr.O	STU14 She pointed out that there existed an interval on which the substituted function could not be defined and thus, she contributed in the most meaningful way. Thereby, she suggested the necessity to integrate it on split intervals.	STU1 He explained that it could be solved by applying improper integral.	STU8 He helped others students compare two solutions by suggesting an alternative solution.
Student 1	STU8 S/he suggested a right solution first.	EM2 S/he provided a room for discussion by suggesting a different solution.	STU14 S/he suggested a new method of splitting the interval.
Student 2	STU8 A right solution.	EM2 S/he suggested a plausible derivative but it had an error on the interval. Thus, discussion was activated.	
Student 3	STU8 S/he suggested a right and clean solution.	STU14 S/he pinpointed an error in the erroneous solution.	
Student 4	STU14 S/he solved the problem well in an easy way.	EM2 S/he provided something to discuss.	STU12 S/he suggested an idea.

Table 5.8: Mr.O and Students' Evaluation of Contributors For Problem 2-3

investigate, reflect, and know. In contrast, students used words such as “right”, “wrong”, and “perfect” to describe contributors. In this respect, Mr.O evaluated contributors based on what students did but students’ judgements were based on whether or not a contributor’s solution right. For example, table 5.9 shows that Mr.O’s and students’ selections of contributors do not match. Three out of four students nominated STU8 as the 1stcontributor for problem 2-3 because his solution was “right” whereas Mr.O selected STU14 because she suggested RFT* “in the most meaningful way.” The students’ norm of evaluation is consistent with the implicit norm in section 5.2.2; ‘You don’t have to revise a solution as long as it contains a right answer’. However, it is not clear which one comes first; Did students bring the norm of judgment into MKB process? or did the implicit norm forge the norm of evaluation?

Interestingly, three out of 4 students nominated EM2 as the 2nd contributor for problem 2-3 in table 5.9. They commonly suggested their reason for this as “discussion was activated.” This indicates that students recognized that the CPS process was structured around erroneous solutions and error-makers’ solutions facilitated their learning. In contrast, there was no student who nominated STU10, who posted most frequently (table 5.5), as a contributor with respect to 10 problems.

5.7 Summary

Fourteen students posted 2 to 8 times for 70 days of the experiment. The average number of visiting per student was 2.5 and the average number of posting per student was 4 (table 5.5). The average of edits per posting was 2. The average number of students’ postings per problem was about 5.6 during the experiment (table 5.4). When 13 postings of mine as error-makers and administrator is counted, the average number of postings per problem increases to around 7. This indicates that my role as a facilitator was critical in CPS process. Since my cognition was part of

CC in JPS, my learning facilitated MKB process and students' learning. In terms of teaching presence, my role was analyzed as designer and organizer of educational experience, director of MKB process, provider of related materials, questioner, editor and helper. Making e-book of students' solutions through CPS as oeuvre belongs to a mathematics teacher's responsibility.

In terms of social presence, the teacher's evaluating strategy of counting the number of postings motivated students to participate in CPS but also impacted the quality of their postings. In an interview, one student mentioned that some postings were meaninglessly and depreciated quality of CPS process due to the teacher's strategy. Two norms of participation ('copy, pasted, and revise' and 'put horizontal line and signature') were beneficial for readers to construe relations between postings and follow changes of CFP in CMS whereas they hindered students from developing solutions in an economic way in terms of time and space. In addition, they enforced students' identified authorship and caused an implicit norm saying 'you don't have to revise a solution as long as it contains a right answer.' Based on this implicit norm, students neglected small errors that did not influence the results. Although identified authorship increased students' individual responsibility in CPS process, individual responsibility did not necessarily support collective responsibility of CoI. In this respect, students need to experience the benefit of 'open editing' function of the Wiki. When students assumed a teacher's roles in MKB process, they revealed epistemic agency. Students' primary source for MKB process was from the Internet. They searched for information needed in MKB process. Although this experiment was designed to be irrelevant to face-to-face interaction, students' interest and curiosity led them to discuss a few solutions face-to-face before the class.

In terms of cognitive presence, cognitive maps revealed some patterns of CPS process. When students' ideas and solutions were scattered into different branches, distributed form appeared. When students' postings were threaded in a branch, they

were intertwined along the *inverse directional double helix* consisting of a constructive line and a critical line. Work load of problem solving was distributed over individual students who were positioned as constructors or as refuters along inverse directional dual helix. Students' positioning varied according to the current mathematical landscape in the shared world, the nature of errors in solutions, and their interest and ability. When students were doubly positioned, two lines of the inverse directional double helix moved together. In addition, when students did not reveal their thinking process in a clear manner, two lines looked as one line and social interaction decreased and thus errors increased in JPS. Students used backward confirmation triangle to confirm correct solutions while they used forward confirmation triangles to verify newly introduced knowledge. The nature of problems and solutions influenced patterns of CPS. Students needed pencil-and-paper while participating CPS. Without it, they frequently made errors because the Wiki did not provide individual workspace. When students did not solve problems before entering their solution pages, they could not create and enter JPS.

CHAPTER 6. CONCLUSION

Our relationship to our work is a feedback relationship: our work grows through us, and we grow through our work. (Popper, 1978, p.167)

6.1 Summary of Problem

Invention of online writing systems has created a new social environment and made it possible for human beings to collaborate in virtual space. Many educators expect that this will stimulate dramatic pedagogical changes (Brandon & Hollingshead, 1999). However, Oliver and Kellogg's (2010) survey indicates that an issue of digital disconnect is more severe in mathematics courses than any other courses (re-cited in Hodges & Hunger, 2011). On the other hand, since educational environments for Korean secondary gifted students in mathematics and science lack opportunities for students to engage in collaborative learning of mathematics, Korean mathematics educators have been asked to find a way to improve educational environment for the gifted students. Among newly invented 21st century communication tools, Wikis are conceived as strongly reliable for collaboration in CSCL (Computer-Supported Collaborative Learning) and have been attracting educators' attention for providing collaborative learning opportunities in CSCL environment (Augar, Raitman, & Zhou, 2004; Notari, 2006). This study investigated the applicability of the Wiki environment to Korean secondary gifted educational environment in terms of collaborative learning of mathematics.

This design experiment addressed the general question: Can a Wiki problem-based environment support collaborative learning among gifted secondary Korean mathematics students? In this chapter, I summarize the design of the study and organize findings by the specific research questions posed in Chapter 1.

6.2 Design of Study

Fourteen 12th grade students from GSHS, a science high schools for gifted students in mathematics and science in Suwon, Korea, participated in the study. They were all enrolled in Mr.O's Calculus II course. The Wiki was used as a collaborative mathematical problem solving tool for students to do extra assignments after school. I posted two sets of 5 calculus problems and one project for reconstructing proof of the Isoperimetric Theorem. In order to stimulate students to focus on eliminating errors emergent from the CPS (Collaborative Problem Solving) process, I posted erroneous solutions both in explicit and implicit ways. To structure the mathematical landscape from ESMCE (Educational Setting for Mathematical Creativity and Errors), I created two types of contradictory situations: The 1st type of ECS (Explicitly Contradictory Situation) consisted of solutions in problems which contradicted commonly shared knowledge; the 2nd type of ECS included solutions with different, competing results. Lastly, two problems were posted without any erroneous solutions or error makers' postings.

In this study, the Wiki functioned as an epistemological experiment tool by providing a laboratory environment for investigating the growth of SM (School Mathematics). The Wiki site provided virtual space for participants to post, share, collaborate, and thus I could investigate their CPS process without any nosy data. To design the experiment, I adopted Garrison and Arbaugh's (2007) Practical Inquiry Model as the conceptual framework. The experiment produced main data including students' solution in the shared world, interview logs of Mr.O and nine students, responses of eight students to a survey, and Mr.O and four students' evaluation of contributors to growth of SM.

While analyzing data, I drew on two perspectives: Cultural psychology and Fallibilists' view. Cultural psychology helped in extracting design factors of ESMCE

to provide the mathematical landscape to induce mutual learning in CMS. Fallibilists' perspective provided insight while analyzing data. I conducted mathematical content analysis for coding with respect to student collaborative solutions based on connoisseur's perspective (Eisner, 2001). From my coding work, a cognitive map for each solution was drawn. Cognitive maps visualized the CPS process for growth of SM so that I could find patterns of CPS process in JPS (Joint Problem Space).

6.3 Findings of the Study

6.3.1 Design Elements

- What design elements facilitate CPS using the Wiki in terms of social, teaching, and cognitive presence?

Social Presence

Motivation The Wiki problems were assigned as extra material rather than homework because the teacher did not want participation in this design experiment to interfere with students in terms of their time management. In addition, he encouraged students to participate in the Wiki problem solving by adding up the number of individual student's postings to the number of their participations in class discussion. Fourteen students posted 2 to 8 times during the period of the experiment. The teacher's approach to evaluation influenced the quality of SM (School Mathematics). More specifically, students tended to pick interesting or easy problems that might not take much time to solve. This resulted in a few students' postings repeated or meaningless ideas. For example, STU10 most frequently posted for three days but three of his eight postings repeated others' solutions (table 5.6). This kind of act may depreciate the quality of SM and suppress others' willingness to contribute to collaborative learning in CoI.

Norms of Participations Participants had no class time for establishing norms

of participation at the beginning of the experiment. Instead, students were given two norms: ‘copy, paste, and revise’ and ‘put horizontal line and signature’. The norm of ‘copy, paste, and revise’ was beneficial for students to view and follow the whole collective thinking process, and to construe flows of meanings and the relationships between postings including signature. In this respect, participants did not have to use the ‘history’ function to check out changes of solutions. However, since the norm of ‘copy, paste, and revise’ increased the amount of similar text in the shared world, it did not support the development and organization of solutions in an economic way. Some participants were reluctant to read similar solutions and had difficulty pinpointing errors. As a result, new errors were created, transmitted, and retransmitted to other solutions. Students showed their tendency to adhere to identified authorship. For example, they typed their ID’s when they did not login and they added their signature later when they found their postings without their signatures. The norm of ‘put horizontal line and signature’ enforced this tendency.

Participants created implicit norms by following the first solver’s writing pattern and these types of implicit norms were easily created and disappeared. Differently from these norms, two given norms created an implicit norm, which impacted the MKB process by saying: ‘You don’t have to revise a solution as long as it contains a correct answer.’ This is evidenced by the fact that students did not eliminate simple errors in others’ solutions as long as they did not influence their results because it was not worthwhile to revise under their identified authorship. They attributed the elimination of trivial errors such as typos, misuse of terminology, and simple errors in calculation to their authors’ responsibility because eliminating them was not worth doing so under the identified authorship. In this respect, correcting trivial errors in a solution with a right answer belonged to their authors’ responsibility whereas correcting critical errors or supplementing an idea without a right answer was considered as worthwhile work.

Collective Responsibility and Open Editing Identified authorship increased individual students' responsibility for their own writing. However, individual responsibility did not necessarily foster collective responsibility for the growth of SM. Lack of collective responsibility diminished the quality of collaboration and interaction. In interviews, most students said that they did not like others' free revisal of their own solutions. However, students might appreciate others' open editing of their solutions if it was a part of the culture of their regular class. Since the implicit norm and identified authorship weakened students' active use of the 'open editing' function, educational settings must be designed to coincide with normal practices so that students get used to and actively use 'open editing' as a keystone to develop SM as oeuvres by enhancing collective responsibility in CoI.

Epistemic Agency When students took over a facilitator's role in CMS, their epistemic agency was revealed. Students voluntarily searched and organized mathematical materials and attempted to direct CFP (Collective Focal Point) by suggesting, for instance, "To prove this, we need to discuss it more". In addition, they edited their writing for the readers' sake. In addition, one student raised questions for clarification, justification, and legitimacy.

Teaching Presence

Designing and Organizing This study indicates that a challenging problem can create ESMCE (Educational Setting for Mathematical Creativity and Errors) but ESMCE is not sufficient to trigger students' interaction in the shared world. In this respect, a teacher's role as a designer and organizer is important. When the mathematical landscape in the shared world was structured with two types of ECS, it induced more students' participation and interaction. For example, three students evaluated error-maker's solution as the second contributor in table 5.9 because it activated discussion. Erroneous solutions contained in problems and error-makers'

solutions provided “something to discuss”. Although errors can be used to organize students’ collective thinking process, it is not easy to find interesting erroneous solutions to stimulate CC (Collective Cognition) in CMS by clarifying the spot to be focused.

Facilitating Although a pdf file, which provides information of how to use the Wiki editor, was posted to help students get accustomed to new system, most of interviewed students did not read it. This indicates that students may prefer a trial-error method to learning a new system by reading guiding materials. In this respect, students had difficulty in expressing their ideas and solutions using the Wiki. A teacher’s role includes being an editor of e-book of students’ collaborative solutions and a helper to guide students to get accustomed to the Wiki system.

The number of my postings as administrator or error-makers was 13 among 69 postings on solution pages. I posted 6 times as error-makers and 7 times as administrator. Since this is up to 18.8%, I was the primary contributor to CPS process. To thread students’ scattered thoughts, I attempted to stimulate CC by posting related materials twice. When the material did not clearly reveal its relation with the erroneous solution, the students felt confused. To form CFP, an additional act such as questioning was needed to disclose the spot on which CFP should be put. Even though an asynchronous characteristic of the Wiki delayed question-response process, questioning was important to stimulate CC.

Students did not suggest various solutions with respect to the easy problem. In this case, I directed the MKB process by changing the conditions of problems. Changing conditions of problems instantly prompted students’ learning. Since students may bring new material from the outside and suggest various ideas and thoughts, student learning might follow unexpected path in CMS even though ESMCE was designed by considering students’ interest and ability, and relatedness between mathematical contents. In this respect, a teacher needs to direct CFP toward the educational pur-

pose and select an efficient way for the MKB process by considering resources of CoI. This process may reflect a teacher's own interest and mathematical perspective.

Cognitive Presence

Nature of Problems and Solutions The Korean gifted students in this study did not show an interest in simple or easy problems. Once the answer for easy problems was posted, no one participated in further action such as supplementing the posted solution or suggesting alternative ones. When the condition of the easy problem was changed, students responded to this change to post alternative solutions. In a case of a challenging problem that was beyond the course range, students actively participated in CPS. They pointed out the reason why they could not solve it with knowledge that they learned. In this respect, these students showed their mathematical ability and insight.

In this study, two types of erroneous solutions were used to structure CMS: fixable solution and non-fixable one. In a case of a fixable solution, participants revised erroneous solution to get the right answer by spotting, removing errors, and making necessary corrections. In a case of a non-fixable solution, students pointed out errors in it but could not fix it to get the right solution. In this case, they suggested new alternative solutions.

Although solution pages were made for each problem and JPSs were separately created with respect to each solution page, participants showed the possibility of connecting JPSs across solution pages. For example, some students mentioned the similarity between problems and between solutions.

Pencil-and-Paper and Individual Workspace Encoding the individual thinking process in the private world and decoding mathematical solution in the shared world is critical for students to participate in CPS process. While posting or investigating solutions, students needed individual workspace to encode their solutions in the private

world and decode mathematical expressions in the shared world. That is, individual workspace is needed to connect the private world and the shared world. However, the Wiki did not provide students individual workspace. In this respect, students used pencil-and-paper to create individual workspace to process encoding and decoding. When students did not use pencil-and-paper while encoding and decoding, they made errors in CPS process.

On the one hand, pencil-and-paper provided students workspace for solving problems and temporary repository space before typing their solutions using LaTeX. When students attempted to directly solve problems in the shared world without creating workspace using pencil-and-paper, they frequently made errors in their solutions. Thus, students needed to translate their solutions from individual workspace to the shared world with additional time and effort. Furthermore, although it is essential that the thinking process in the private world is encoded into the shared world in an open form for easy decoding, difficulty in using LaTeX increased students' reluctance to post a full account of solutions in the shared world. These deterred many students from committing themselves to others' learning by revealing their thoughts in shared world. On the other hand, students had difficulty in following mathematical expressions while investigating solutions in the shared world without creating individual workspace with pencil-and-paper. This caused students to transmit old errors and create new errors in revised solutions.

Errors and JPS One of intended norms of participation in this Wiki environment was: 'You should not read others' solutions before you read and solve problems by yourself.' However, there was not an opportunity to discuss this norm prior to the start of the design experiment and some students did not abide by this norm. As a result, they failed to enter into JPS and did not contribute to MKB.

6.3.2 Patterns of MKB

- What patterns emerge from JPS while students are engaging in CPS?

Patterns of Student Postings The average number of participants' postings per problem was 5.6 over the 70 days period of the experiment. Students posted 2 to 8 times with respect to 10 problems. Since students posted 0.8 posting per day, suggesting that the CPS proceeded very slowly. The average number of postings per student with respect to 10 problems was 4.0 and the average number of students' edits was 8.6. Thus, students edited twice per posting. When students posted their ideas, they often called for additional editing. Thus, a pattern of students' posting was: main idea + repairing (NEW/RFT/FIX+RR). The frequency order of codes was RR-RFT(RFT*)-NEW-FIX(FIX*)-Q(RE)-SUPPL-CFM-AGMT (table 5.3). For each problem, 3.2 branches of new ideas or new solutions including two error-makers' solutions were made. The average number of postings in the longest thread in a solution page was 4.2 (table 5.4).

Distributed Form and Inverse Directional Double Helix When students' postings were not threaded, a cognitive map formed a distributed form. In this study, two cognitive maps showed distributed forms. ECS caused individual cognitions to question postings and led to additional investigation and identification of errors (RFT*). If a suggested RFT* was not accepted, it was abandoned and isolated in JPS. In a case that most RFT*s were not developed by others, they were scattered to a distributed form. That is, individual cognitions were not connected via the mathematical landscape. This pattern supports the idea that the CPS process depends on the nature of solutions and difficulty of problems. In problem 1-1, since the erroneous solution was non-fixable, RFT*s were not used to develop better solutions. In problem 2-2, students' RFT*s were not successful and scattered because it was beyond the course level.

On the other hand, cognitive maps of most solution pages exhibited *inverse directional double helix*. Inverse directional double helix consists of a constructive line and a critical line that are respectively powered by what Popper (2002) says: imagination and rational criticism. The Wiki operated as a collective cognitive tool to assemble the imaginative and critical power of individual cognitions. CPS proceeded alternatively along two lines. This process followed the logic of *solutions and refutations* similar to what Popper (1989) calls “the method of conjectures and refutations”. Students’ CPS process was diagramed as $S_1 \rightarrow \text{RFT/RFT}^* \rightarrow \text{AGMT} \rightarrow \text{FIX}^* / \text{FIX} \rightarrow S_2$. This study shows that two lines of an inverse directional double helix intersect at error-parts in the shared world. This is a critical point where imagination and a rational criticism as two axes of growth of knowledge generate power to develop knowledge. Acquisition Metaphor as well as Participation Metaphor is possible along progress of alternative interweaving of an inverse directional double helix. In other words, “knowledge in the subjective sense” in the private world and “knowledge in an objective sense” in the shared world can develop together through CC (Popper, 1989, p.108).

Participants tested answers of solutions to check the validity of them by applying their background knowledge. When a solution failed to go through a test (RFT), ECS emerged from the current mathematical landscape. Multiple solutions with different results (RFT) also caused cognitive conflict in CMS (ECS). ECS triggered participants’ solution-analysis within CoI; Members of CoI attempted to find all the possible RFT*s through investigating a solution. In a case of fixable solutions, two ways of revising solutions by removing errors in the CPS process were shown: $\text{RFT}^* + \text{FIX}$ and FIX^* . A refuter marks off detected error-places for CoI (RFT*) and then a constructor comes to eliminate them (FIX). This process is similar to what Lakatos (1976) describes in the history of mathematics; when a global counter example against a theorem or a conjecture is suggested (RFT), mathematicians begin

to look into proofs and find error-spots to revise it (RFT*/FIX). In a case of a solution where the individual cognitive process is dominant in CPS such as the calculating process, students fixed the solution without pointing out errors (FIX*).

Positioning Each participant positioned himself/herself in JPS as a constructor or a refuter following *inverse directional double helix* (figure 5.8). Refuters tested and analyzed solutions, and found errors in them while constructors created new solutions or revised erroneous solutions by removing detected errors. Since refuters and constructors worked on a critical line and a constructive line respectively in CPS process, positioning separately occurs along two lines of an inverse directional dual helix. When positioning clearly occurs from a contradictory situation within CoI, two lines of dual helix move in a separate way between the shared world and the private world.

The current intellectual landscape, the nature of errors, and students' interest and ability influenced their positioning. The current intellectual landscape and the nature of errors suggested to students what to do. For example, when the 1st type of ECS was caused by RFT, students started to work on finding errors in the solution. If the detected error was related to a mathematical concept, then students were positioned as refuters, whereas if it was a simple error emergent from a calculating process, then they were positioned as constructors by removing and fixing the error simultaneously. Students occasionally took both roles of a constructor and a refuter according to their interest and ability. When double positioning happened, both a constructive line and a critical line moved together between the private world and the shared world (figure 5.11). I characterized this as an *internalized inverse directional double helix* from Vygotsky's perspective on the relationship between outer speech and inner speech.

The more portion of CPS appropriates individual cognitive process, the less separate positions occur. Since double positioning occurs along an internalized inverse

directional double helix, a critical line and a constructive line are closer in this case than those in an inverse directional double helix. In an automatized cognitive process such as calculating, students' individual cognitive process for pinpointing errors was not clearly revealed in JPS and thus two lines of inverse directional double helix looked to be inseparable. When attempts to pinpoint errors were opened up as RFT* in the shared world and there followed a social process of deciding whether this RFT* was accepted or not, members of CoI could more clearly recognize and remove errors at spotted-error-places (FIX). In this respect, RFT*+FIX was more efficient in eliminating errors than FIX*. For example, on the page of solution 2-5 (figure 5.4), FIX* created new errors and transferred them to other solutions rather than removing original errors. Thus, the growth of SM depends on how to transfer the individual cognitive process into the shared world in a clear form. That is, to commit himself/herself to the growth of SM, every member of CoI shares a responsibility of encouraging others' learning by clearly revealing her/his own ideas and solutions in the shared world. In this respect, online CoI needs to be an open society to foster students' critical and creative mind because "a free, open society is one that best encourages a free flow of ideas" (Bailey, 2000, p.129).

Two Types of Confirmation Triangles The students in this study confirmed their solutions or validity of new methods by applying two types of strategies: Backward confirmation triangle and forward confirmation triangle. In the 2nd type of ECS where solutions showing different results were competing, students confirmed the right solution by suggesting a new solution with the same result or by correcting fixable erroneous solutions to get the right answer. On the other hand, when it was necessary to check the validity of the newly introduced mathematical method, students used the forward confirmation triangle. The forward confirmation triangle seemed to be more closely related to ESMPR rather than ESMCE.

Although conformation triangles as a social mechanism increased truthfulness

in CoI, they might malfunction to confirm erroneous solutions. In a case of calculating problem 2-5, a new solution with an error, which coincidentally produced the same wrong result with the existing solution, falsely confirmed it. This shows that confirmation triangles may not operate properly when members of CoI put overrated beliefs on others' work and fail to diversify their tools of investigation. Thus, to reveal hidden assumptions of knowledge causing errors as targets of refutation, CoI should never stop to investigate their knowledge.

6.3.3 Source For MKB

- What do students bring into CMS (Collaborative Mental Space) as their source of mathematical knowledge?

Since students were connected to the Internet while participating in CPS, they typically relied on the Internet search feature. For example, they searched the definition of a function, command words of LaTeX for new mathematical expressions, statement of a theorem, alternative proofs or solutions, information about modern mathematicians' works, related articles in mathematics, etc. Some of searched information on the Internet was imported into the shared world as texts, pictures, and links. Other information such as definitions of functions was consumed in the private world for understanding problems while engaging in CPS. In this respect, the primary source for MKB was the Internet.

Findings from this study indicate that material from outside searches might not be food for individual thinking in the private world and not stimulate CC in JPS, either. Although a student got information about an error in the posted proof from Googling on the Internet, she did not sufficiently reflect the information in the private world. In this respect, she could not justify her writing. In addition, although a student searched an answer for a problem, which is beyond their course level, from "Wolframalpha", his search did not necessarily stimulate CC because the exact answer

from the outer world might hide potential of other student's viable idea in the shared world. In these respects, the outer source on the Internet may deprive individual students of an opportunity for deep mathematical thinking and cause a rapid change of authority structure.

6.3.4 Mr.O's and Students' Opinions about the Wiki

- What are students' and teacher's opinions about the Wiki as a CPS tool?

Mr.O suggested that the Wiki can be beneficial for a mathematics teacher to extend the range of mathematical content beyond the limitation of class time, to provide students alternative ways of participation, serve as a tool for doing mathematics, and produce material as oeuvre to be used in other courses.

All eight students who responded to survey questions agreed that CPS is more efficient than individual problem solving. Seven out of 8 students replied that the Wiki is useful to develop solutions by removing errors and others' comment and that others ideas and solutions are helpful to improving their thoughts and solutions. Four of these students were active participants of the Wiki. The others did not participate frequently because they reported it was annoying to turn on computer to participate. Most students indicated that they had trouble in using LaTeX. Four students suggested that problems evoking discussion and thinking are better than simple calculation problems. Seven out of 8 students indicated that setting up norms of participation is not necessary because it may restrict their freedom to express.

6.3.5 Difference between Mr.O's and Students' Perspectives on Contribution to Growth of SM

- What is the difference between the teacher's perspective and students' perspectives in evaluating contributors of growth of SM (School Mathematics)?

Students evaluated contributors based on whether or not their solutions were

correct with words of “right” and “wrong”. In contrast, Mr.O evaluated based on what they contributed to mutual learning by using “meaningful”, “showed”, and “explained”. In this respect, Mr.O evaluated contributors by focusing on the process of collaborative learning while students’ evaluation was based on judging whose solution is right. This tendency of students is consistent with the implicit norm that allows students to decide what to do under identified authorship.

6.4 Implications

Although some aspects of CPS in the experiment were different from my thought experiment due to characteristics of the Wiki and changes of the initial plan, students collaboratively solved mathematical problems using the Wiki and thus learned mathematics together as was anticipated.

Above all, this study provides some evidence that school problems can create ESMCE in the virtual world and that ESMCE operates in the shared world as well as in classroom. The initial mathematical landscape triggered students’ collaborative learning and potential errors in solutions provided fuel for the MKB process. In this process, the Wiki operated as a collective cognitive tool by accumulating individuals’ thoughts and solutions in the current mathematical landscape on the one hand and delivering it to the private world on the other hand. When individual cognitions were situated in the current mathematical landscape, individual students related their thinking process with the landscape in the shared world. Since the envisioned current mathematical landscape promotes a collective thinking process, students had opportunities to *think about collective thinking*. In that “thinking about thinking has to be a principal ingredient of any empowering practice of education” (Bruner, 1996, p.19), the Wiki can be an important mathematics educational tool for *thinking about collective thinking*.

Students’ asynchronous interaction via the Wiki was different from face-to-face

interaction or synchronous VMT chat interaction. Students did not have to greet or establish peer relationship and CPS proceeded slowly. Students used more formal language and spent considerable time transferring their thoughts and solutions to the shared world. Since students could self pace, they potentially had sufficient time to think deeply about mathematical content while engaging in the CPS process. Students' ideas or solutions were combined with the current mathematical landscape in JPS. This characteristic of JPS was demonstrated by student use of different strategies in the CPS process. While participants in the VMT chat mainly used question-response to sustain their mathematical discourse and to create JPS (Zhou, 2009), the frequency of question-response Q(RE) in this study was a low 4.6 % in the CPS process using the Wiki. Instead, high frequency of students' action were new solution (NEW), refutation (RFT/RFT*), and eliminating errors (FIX/FIX*) (table 5.3). There is evidence that the Wiki connected individual cognitions via mathematical contents in the shared world and students used the method of *solutions and refutations* to sustain asynchronous mathematical written discourse using the Wiki. This study provides support for the notion that the method of *solutions and refutations* operates as a textile machine interweaving multiple voices of deep thinkers in the shared world.

In terms of social presence, collective responsibility for collaborative learning is seen to be a critical factor in terms of social presence. Individual assessment can motivate students to participate but not necessarily to learn together. In addition, students' individual responsibility may not be aligned to the purpose of online CoI, growth of SM. Thus, this implies that it needs to be designed to enforce collective responsibility for growth of SM by providing students opportunities to experience the benefit of the 'open editing' function of the Wiki. Since students focus on right answers while mathematics teachers on the MKB process itself, designing to support collective responsibility will be critical to change students into mutual learners.

In terms of teaching presence, it is critical for a mathematics teacher to play his/her role in CMS. In figure 2.3, a math teacher's cognition constitutes CC, which characterizes her/him as a mutual learner like other students. However, a mathematics teacher's role is different from other learners' because s/he makes others initiate new inquiries and direct them to an educational goal. In this respect, Bruner (1996, p.xv) characterized a teacher as "the enabler, *Primus Inter Pares*"¹ in the community of mutual learners. To stimulate CC and direct the MKB process toward her/his goal by constantly reshaping the intellectual landscape in the shared world, s/he needs to consider finite resources for growth of SM and technological constraints of the Wiki system. S/he observes individuals' imaginative products following a constructive line and controls their rational critic activity following a critical line. S/he is responsible to make students conscious of their fallibility including hers/his and contribute to the growth of SM. In this respect, s/he plays a metacognitive role in CC. Thus, s/he is an *epistemic agent* who leads the MKB process toward the growth of SM. As an epistemic agent, a mathematics teacher needs to design and change the intellectual landscape to stimulate CC, and consider resources such as time, students' mathematical ability and interest, their background knowledge, affordances and constraints of virtual environment, outer resources on the Internet, etc. to direct CFP toward the goal.

In terms of cognitive presence, although the Wiki operated as a collective cognitive tool by collecting and delivering students' ideas and thoughts, it was not sufficient for all students' active participation. This study provides evidence that students needed workspaces to objectify their thinking in the private world before encoding and to decode to assimilate mathematical contents in the shared world. Most students had to spend much more time writing mathematical expressions using the Wiki due to the difficulty in writing mathematical expressions using LaTeX than solv-

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¹The author's emphasis.

ing problems individually. Furthermore, students needed individual workspace while decoding mathematical expressions in the shared world. However, since the Wiki did not provide individual workspace, students used pencil-and-paper to create individual workspace. Since participants were reluctant to write mathematical expressions with LaTeX, this raises the most critical issue of the Wiki as a mathematical CPS tool. Since this technological constraint of the Wiki system strongly influenced elements of the educational experience in virtual space, easier mathematical writing tools are needed to support and sustain students' willingness to participate in the MKB process. This difficulty of representing mathematical symbols is common in CSCL tools, despite the development of various online mathematical writing tools. In this respect, mathematics educators "still asserts that communicating math on the Internet is "a fairly difficult task""(Hodges & Hunger, 2011, p.40). Even if easier mathematical writing tools for the Wiki are developed, students still need to learn how to write using them. That is, a mathematics teacher and her/his students have to put time and effort to learn new writing tools and the Wiki system. This raises a question about introducing the Wiki into a mathematics class: should a mathematics teacher and students learn how to write mathematical expressions using the Wiki as a future skill? Although there have been educators' calling for teaching social software including the Wiki as the skill of the future and preparing students to make innovative uses of them as a common requirement of education (Parker & Chao, 2007; Hodges & Hunger, 2011) and a few mathematics educators agreed that Wikis are powerful tools for the collaborative learning of mathematics (Ben-Zvi, 2007; Carter, 2009; Narasimhan, 2009; Peterson, 2009), there has been no wide agreement among mathematics educators on whether or not to introduce the Wiki as a CPS tool into mathematics class for gifted students in mathematics and science.

This study indicates that students situated their cognitive process in the current mathematical landscape following the method of *solutions and refutations* and

the Wiki provided a way “in which jointly negotiated thought can be communally externalized as oeuvres” (Bruner, 1996, p.25). Since oeuvres as joint products “help *make* communities of mutual learners”² (Bruner, 1996, p.22), the Wiki was an efficient educational tool to realize mutual learning in virtual space. The current landscape in the shared world at the end of this experiment showed growth of SM. As a result, various mathematical solutions as oeuvres were developed (see Appendix 2). In this respect, the Wiki, by its very nature, created “socially constituted practice” of a community of mutual learners (Rogoff, 1996) where they could participate and contribute to others’ learning. Thus, the Wiki was an efficient tool for building a CoI or a community of mutual learners in virtual space. Since SM in the shared world, in principle, can endlessly grow as long as individual cognitions attend CMS just like mathematical objects in World 3 do, the Wiki is “an *epistemological device*”(Ruth et al., 2009, p.148). An *e-book* produced from the CPS process is an oeuvre worthy to be reused, reexamined, and reappreciated in other mathematics classes. In this respect, the Wiki can be a tool for the global growth of SM, including a potential rich tool for gifted students.

6.5 Limitations

Since this study aimed to improve Korean secondary gifted students’ learning environment by investigating the applicability of the Wiki to CPS process in virtual space, this study was conducted at GSHS, which is an institution for Korean secondary gifted students in mathematics and science who are placed above top 10% of Korean high school students in terms of mathematical ability. In this respect, applying results of this study to normal mathematics courses will be the different problem.

In terms of the size of participants, fourteen is small. However, this is a design experiment, since there is no data to compare the data of this study with at the present

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²The author’s emphasis.

time, it is hard to discuss the sample size. Theoretically, asynchronous interaction using the Wiki has no size limit unlike synchronous VMT-chat, which can be used only for three to four participants.

The primary focus is on growth of SM as oeuvre rather than on whether or not a specific individual student actively participates. In this respect, numbers of individual participation, which varied from 2 to 8, were not analyzed in terms of quantity. However, quality of participants' learning and how social aspects of mathematical learning were realized in the shared world were analyzed. Certainly, heterogeneity of gifted students or individual students' learning in the private world were not concerned in this study.

This study has several limitations in conducting the initial plan, collecting data, and analyzing data. According to the situation of GSHS and the teacher, some parts of the initial plan were changed. Above all, the teacher did not participate in MKB process. That is, he did not post any writing except for writing practice with LaTeX. In this respect, I played the role of both developer and facilitator. Second, the students did not jointly discuss or develop norms of participation. Instead, the teacher informed students of two norms of posting solution: 'put a horizontal line and signature' and 'copy, paste, and revise.' In addition, there was no class time devoted to practicing LaTeX. Lastly, although many participants were attracted to the prepared project of proving the Isoperimetric Theorem, they conducted only the first part of the task due to time limit. Although there were some data from students' postings for part of project, the data were not analyzed in terms of mathematical content.

While interviewing students, two students came together to be interviewed because they wanted to save their time. Even though this informed me of new facts such as they solved some problems together in the experiment, this might influence the interview process. In addition, only four students replied to survey for evaluating contributors, data were not sufficient to be analyzed to answer to question 5.

In analysis, I developed a coding scheme and worked in coding process by myself; The coding process was not triangulated. Since reliability of the coding schemes was not attained, there is a reliability issue with the codings. To allow others to view the coding, I provided students' solutions with my codings in Appendix 2.

6.6 Future Research

According to Sthal (2009a), studies in CSCL have reported that the teacher's role is important in the the MKB process. As seen in the pilot study, just telling participants to post their solutions in the shared world is not sufficient because there may not be social acts for the MKB process (Sthal, 2009a). Their thoughts need to be connected via mathematical contents. To stimulate CC in CMS, a mathematics teacher needs to keep encouraging students to share their thoughts and probing beyond what they find (Kumar, Chaudhuri, Gweon, & Rosé, 2009). In this experiment, since the teacher did not participate in the MKB process, I played the role of a facilitator. As a researcher, however, I did not actively perform the mathematics teacher's roles in this study. In this respect, the interventionist nature of the design experiment was restricted. Thus, the present experiment needs to be conducted again as action research to apply blended learning to build online CoI including a mathematics teacher and students.

This study indicates that an attempt to build online CoI or online community of mutual learners using the Wiki evokes the need to change not only in "task structure" but also in "reward structure" and "authority structure" (Slavin, 1980, p.315). In terms of task structure, follow-up design experiments need to focus on projects, which could not be conducted due to time constraint in this experiment, because the project type work is known as more suitable to collaborative learning. In terms of the reward structure, it needs to enforce collective responsibility to change students from passive problem solvers to active collaborators. The survey result implies that

difference between the teacher's and students' perspectives may hinder mathematics teachers from introducing a new way of evaluation of contributors to growth of SM to their class. To embed online collaborative educational environment to individualized and competitive Korean educational environment, further study concerning the difference between teacher's and students' perspectives on the growth of SM and reward structure in online CoI is needed.

This study indicates that the Wiki may shift the authority structure for knowledge. Although the Wiki can create a more egalitarian environment than face-to-face class because students can anonymously participate, the online environment may introduce a new authority issue in mathematics class. Since CMS created in virtual space is always connected to the Internet world, online CoI is part of the Internet ecology. In this respect, students imported not only what they learned in the class but also what they searched on the Internet into CMS. When students import outer knowledge on the Internet, they may be more dependent on it. For instance, when a student imports a right answer from "Wolframalpha" into CMS and simply stigmatize others' solution as wrong, students may put excessive authority on imported knowledge from outer Internet world and lose an opportunity to learn together by investigating their solutions, spotting and removing errors, and developing new alternative solutions. In this respect, the next study needs to consider authority structure in blended learning.

6.7 Summary

In this study, the Wiki functioned as a CPS tool. Some students collaboratively solved mathematical problems using the Wiki. This study also provides some evidence that individual learning is interwoven into collaborative learning or mutual learning and thus into growth of SM. As a design experiment, this study induced as design elements for facilitating CPS using the Wiki: motivation, norms of partici-

pations, collective responsibility, epistemic agency, source of knowledge in terms of social presence, designing and organizing, and facilitating in terms of teaching presence, and nature of problems and solutions, individual workspace, and errors and JPS in terms of cognitive presence.

While engaging in CPS, students' thoughts were alternatively interwoven with constructive activity and critical activity. When students are not clearly positioned as constructors or as refuters, two lines are not separated and individual portions take up more space in JPS. This increases the possibility of making errors and decreases the possibility of removing errors in solutions.

Some students attempted to relate what they learn in their class and extend range of MKB by importing new knowledge on the Internet into CMS. Considering Korean secondary gifted students' mathematical ability and their ability to use the Internet as a tool for MKB, this study demonstrates the potential for developing the MKB process. However, importing outer knowledge on the Internet into CMS may cause authority issues when students are blindly dependent on it.

The Wiki provided students opportunities to put their ideas and solutions in front of others and work collaboratively in virtual space. Most students agreed that the Wiki is a useful CPS tool to improve their thought and solutions. However, most students had difficulty in writing mathematical expressions using LaTeX and did not like to spend additional time and effort to post their solutions in the shared world. The teacher indicated that the Wiki provided an alternative way of participation for students who are reluctant to present their ideas face-to-face and experience of doing mathematics.

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APPENDIX 1. DATA DESCRIPTION OF THE PILOT STUDY

A Problem in The Pilot Study

This problem is about an oil leaking accident in a sea. Read the following description of (1) to (3) about the accident and answer the questions from [1-1] to [1-3]

(1) Suppose that there is 10,000 ton of oil leaked from an oil ship that is staying on the sea 10 km away from the seashore. The leaked oil is moving to the seashore following the flow of seawater and the direction of the wind. The leaked oil accumulated in the seashore will cause a disaster in ecological environment and economy. If we anticipate the movement of the leaked oil, we can effectively do control task to reduce loss.

(2) Although it is hard to describe the movement of the leaked oil because the flow of seawater and the direction of the wind change every moment, we can make a mathematical model by simplifying several factors influencing the movement of the leaked oil. When a hours elapse from the accident, let the amount of oil within x km from the seashore $F(x)$. (a is a real number). Suppose 10,000 tons of the leaked oil are between 9 and 10 km away from the seashore a hours later from the initial time of the accident. That is, if $0 \leq x < 9$ then $F(x) = 0$ and if $10 \leq x$ then $F(x) = 10000\text{ton}$. $F(x)$ is differentiable when $0 < x$. Define $f_1(x) = \frac{d}{dx}F(x)$ when $0 < x$. Define $f_2, f_3, \dots$ as follows:

$$f_{n+1}(x) = \frac{4}{5}f_n(x) + \frac{1}{5}f_n(x+1) \quad (0 \leq x) \quad (n = 1, 2, 3, \dots)$$

$\int_0^x f_n(s)ds$ means the amount of the leaked oil within x km from the seashore when $a \times n$ hours elapsed from the initial moment of the accident. In addition, S_n is the amount of the leaked oil accumulated on the seashore when $a \times n$ hours elapse from

Oil Leaking

the initial moment of accident and can be expressed as follows:

$$S_1 = 0, S_{n+1} = S_n + \frac{1}{5} \int_0^1 f_n(x) dx \quad (n = 1, 2, 3, \dots)$$

According to this model, it is then anticipated that the leaked oil will reach the seashore 5 hours later from the initial moment of accident.

(3) The task force for the accident has 1,000 volunteers. Every volunteer can continuously work for 4 hours at a time and is supposed to go back home. The leaked oil will be removed as soon as it reaches the seashore.

Answer the following questions:

[1-1] (a) State your thought on whether or not the above model can correctly reflect the situation.

(b) To understand the above model, suppose $f_1(x) = \begin{cases} 1 & (9 \leq x < 10) \\ 0 & (0 \leq x < 9, x \geq 10) \end{cases}$

Draw graphs of f_1, f_2, f_3 . Express f_3 with respect to f_1 and figure out $f_n(x)$.

[1-2] Seek the first time when the leaked oil reach the seashore and explain the change of amount of the leaked oil accumulated for two hours from the first time.

[1-3] When $a = 2$, the task force decides to put some volunteers into workplace every 4 hours after 16 hours form the accident. Explain the optimizing way to arrange the number of volunteers for cleaning work based on the model.

(b) 3708 정회원
 $f_1(x) = 1 \ (9 \leq x < 10) \ 0 \ (x \geq 10 \text{ 또는 } x < 9)$
 정의된 수식을 이용해서 f_2 를 구하면
 $f_2(x) = 4/5 \ (9 \leq x < 10) \ 1/5 \ (8 \leq x < 9) \ 0 \ (x < 8)$
 다시 f_3 를 구하면
 $f_3(x) = 4/5 * 4/5 = 16/25 \ (9 \leq x < 10) \ 1/5 * 4/5 + 4/5 * 1/5 = 8/25 \ (8 \leq x < 10) \ 1/5 * 1/5 = 1/25 \ (7 \leq x < 9) \ 0 \ (x < 7)$
 다시 f_4 를 구하면
 $f_4(x) = 16/25 * 4/5 = 64/125 \ (9 \leq x < 10) \ 8/25 * 4/5 + 16/25 * 1/5 = 48/125 \ (8 \leq x < 10) \ 1/25 * 4/5 + 8/25 * 1/5 = 12/125 \ (7 \leq x < 9) \ 1/125 \ (6 \leq x < 7) \ 0 \ (x < 6)$
 $f_3(x) = 4/5 \ f_2(x) + 1/5 \ f_2(x+1) = 4/5(4/5 \ f_1(x) + 1/5 \ f_1(x+1)) + 1/5(4/5 \ f_1(x+1) + 1/5 \ f_1(x+2)) = (4/5)^2 \ f_1(x) + 2(4/5)(1/5) \ f_1(x+1) + (1/5)^2 \ f_1(x+2)$
 이 식을 일반화시키면
 $f_{n+1}(x) = (4/5)^n \ f_1(x) + nC1 \ (4/5)^{(n-1)} \ (1/5) \ f_1(x+1) + \dots + nCn \ (1/5)^n \ f_1(x+n)$

Part of Student A's and B's Solutions Posted in The Pilot Study

Solutions of Three Students

Student A: The first student solved subproblem [1-1] (a) and [1-1] (b). In his ³ solution of problem [1-1] (a), he simply agreed that the model appropriately reflected the situation. His solution for [1-1] (b) was very hard to understand and mathematically incorrect. In figure 4.3, he typed “ $f_1(x) = 1 \ (9 \leq x < 10) \ 0 \ (x \geq 10 \text{ 또는 } x < 9)$ ” for the expression:

$$f_1(x) = \begin{cases} 1 & (9 \leq x < 10) \\ 0 & (x < 9, x \geq 10) \end{cases}$$

Then, he wrote as “ $f_2(x) = 4/5 \ (9 \leq x < 10) \ 1/5 \ (8 \leq x < 9) \ 0 \ (x < 8)$ ”, which can be written as:

$$f_2(x) = \begin{cases} \frac{4}{5} & (9 \leq x < 10) \\ \frac{1}{5} & (8 \leq x < 9) \\ 0 & (x < 8) \end{cases}$$

Since $f_n(x)$ is a step function, $(9 \leq x < 10)$ and $(8 \leq x < 9)$ makes sense

³I did not meet the students or have any information about them. However all participants wrote their student numbers and names on the top of their solutions as if they did when they took exams. All boys names were recorded.

for

$$f(x) = \begin{cases} 1 & (9 \leq x < 10) \\ 0 & \text{otherwise} \end{cases}$$

and

$$f(x) = \begin{cases} 1 & (8 \leq x < 9) \\ 0 & \text{otherwise} \end{cases}$$

respectively.

In calculating $f_3(x)$, the expression is hard to understand because he incorrectly used many '=' in one expression. The expression " $f_3(x)=4/5*4/5=16/25$ ($9 \leq x < 10$) $1/5*4/5 + 4/5*1/5 = 8/25$ ($8 \leq x < 10$) $1/5*1/5=1/25$ ($7 \leq x < 9$) $0(x < 7)$ " meant:

$$f_3(x) = \begin{cases} \frac{16}{25} & (9 \leq x < 10) \\ \frac{8}{25} & (8 \leq x < 9) \\ \frac{1}{25} & (7 \leq x < 9) \\ 0 & (x < 7) \end{cases}$$

Actually this is correct except " $(7 \leq x < 9)$ " in which 9 is guessed as a typo of 8 because " $(7 \leq x < 8)$ " is shown in calculating $f_4(x)$. The first student stopped in calculating $f_4(x)$ even though problem [1-1] (b) asks to seek $f_n(x)$.

Student B: The second student solved [1-1](b) and [1-2]. He might have read the first solution but added his solution below the first student's solution without changing it. Probably, he did not spend enough time on reading the first solution to understand it. In his solution, " $f_3(x)=4/5 f_2(x) + 1/5 f_2(x+1) = 4/5(4/5 f_1(x) + 1/5(4/5 f_1(x+1))) + 1/5 (4/5 f_1(x+1) + 1/5 f_1(x+2)) = (4/5)^2 f_1(x) + 2(4/5)(1/5) f_1(x+1) + (1/5)^2 f_1(x+2)$ " is translated as $f_3(x) = \frac{4}{5}f_2(x) + \frac{1}{5}f_2(x+1) = \frac{4}{5}(\frac{4}{5}f_1(x) +$

$\frac{\partial f}{\partial n} = \frac{1}{5} \frac{\partial f}{\partial x}$
 $\therefore f_{n+1}(x) - f_n(x) = \frac{1}{5} f_n(x+1) - \frac{1}{5} f_n(x)$
 $\Rightarrow f_{n+1}(x) = \frac{4}{5} f_n(x) + \frac{1}{5} f_n(x+1)$
 $\Rightarrow f_3 = \frac{4}{5} f_2(x) + \frac{1}{5} f_2(x+1)$
 $= \frac{16}{5} f_1(x) + \frac{8}{5} f_1(x+1) + \frac{1}{5} f_1(x+2)$

Handwritten notes and graphs:

- Graphs of functions f_1, f_2, f_3 showing increasing complexity.
- Equation: $f_{n+1}(x) = \frac{4}{5} f_n(x) + \frac{1}{5} f_n(x+1)$
- Equation: $f_n(x) = \frac{4^n}{5^n} f_1(x) + \frac{n}{5^{n-1}} \frac{4^{n-1}}{5} f_1(x+1) + \dots + \frac{n}{5} \frac{4^{n-1}}{5} f_1(x+n)$

Student C's Written Solution Posted as a PDF file in the Wiki

$\frac{1}{5}(\frac{4}{5}f_1(x+1)) + \frac{1}{5}(\frac{4}{5}f_1(x+1) + \frac{1}{5}f_1(x+2)) = (\frac{4}{5})^2 f_1(x) + 2(\frac{4}{5})(\frac{1}{5})f_1(x+1) + (\frac{1}{5})^2 f_1(x+2).$

Simply saying “generalizing this expression,” he listed the general formula: “ $f_{n+1}(x) = (\frac{4}{5})^n f_1(x) + nC_1 (\frac{4}{5})^{n-1} (\frac{1}{5}) f_1(x+1) + \dots + nC_n (\frac{1}{5})^n f_1(x+n)$ ” which means:

$$f_{n+1}(x) = \left(\frac{4}{5}\right)^n f_1(x) + {}_nC_1 \left(\frac{4}{5}\right)^{n-1} \left(\frac{1}{5}\right) f_1(x+1) + \dots + {}_nC_n \left(\frac{1}{5}\right)^n f_1(x+n)$$

Student C: The last student solved the entire problem. He wrote his solutions on paper instead of posting them in the shared world and freely explained his ideas with mathematical expressions and graphs. His calculation of $f_3(x)$ was incorrect and he did not suggest any expression for $f_n(x)$. Moreover, he defined a new function $g_n(x) = \sum (\int_{9-i}^{10-i} f_n(x) dx) x^i$ and then sought $g_n(x) = (\frac{x+4}{5})^{n-1}$ and answered to [1-3] without using the given condition of $f_1(x)$.

APPENDIX 2. PROBLEMS AND CPS IN THE DESIGN EXPERIMENT

A Problem For Practicing Editing and Using LaTeX

Integrate $\int \frac{1}{x+1} dx$ (Barbeau, 2000, p.103). Revise the following solution:

$$\begin{aligned}\int \frac{1}{x+1} dx &= \int \left(\frac{1}{x} + \frac{1}{1} \right) dx \\ &= \int \frac{1}{x} dx + \int \frac{1}{1} dx \\ &= \ln x + \ln 1 \\ &= \ln(x+1) + C\end{aligned}$$

Problem Set 1

Problem 1-1

Youngbin proved the following proposition: Let n be a nonnegative integer. The function x^n is constant (Barbeau, 2000, p.91).

Youngbin's proof

Use the mathematical induction. (i) If $n = 0$, $(x^0)' = 0$. (ii) Assume that the derivative of x^n is zero for $n = 0, 1, 2, \dots, k$, that is $(x^0)' = x' = (x^2)' = \dots = (x^k)' = 0$. Then

$$\begin{aligned}(x^{k+1})' &= (x \cdot x^k)' \\ &= x' \cdot x^k + x \cdot (x^k)' \\ &= 0 \cdot x^k + x \cdot 0 \\ &= 0\end{aligned}$$

Post your thought on Youngbin's proof and Discuss with others.

Problem 1-2

Gyoungbin found out an odd result while he was solving “Integrate $\int \frac{1}{x \ln x} dx$ ” (Barbeau, 2000, p.103).

Gyoungbin’ result

Let $J = \int \frac{1}{x \ln x} dx$. Integrating J by parts with $u = \ln x, v = \frac{1}{\ln x}$,

$$\begin{aligned} J &= \ln x \cdot \frac{1}{\ln x} - \int \ln x \cdot \left[-\frac{1}{(\ln x)^2} \cdot \frac{1}{x} \right] dx \\ &= 1 + J \end{aligned}$$

$$\therefore 0 = 1$$

Post your thought on Gyoungbin’s result and Discuss with others.

Problem 1-3

Evaluate $\int_{-1}^1 (1+x^2)^{-1} dx$ (Barbeau, 2000, p.110).

As an error-maker, EM2, I uploaded the following solution.

Solution Since $\left(\frac{1}{2} \arccos \frac{1-x^2}{1+x^2} \right)' = \frac{1}{1+x^2}$,

$$\begin{aligned} \int_{-1}^1 (1+x^2)^{-1} dx &= \left[\frac{1}{2} \arccos \frac{1-x^2}{1+x^2} \right]_{-1}^1 \\ &= 0 \end{aligned}$$

Problem 1-4

Find the limit (Barbeau, 2000, p.98).

$$\lim_{x \rightarrow 0} \frac{(\sin 3x)}{x^3}$$

As an error-maker, EM1, I uploaded the following solution:

Suppose that $L = \lim_{x \rightarrow 0} \frac{(\sin 3x)}{x^3}$. Then

$$\begin{aligned}
 L &= \lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x^3} \\
 &= 3 \lim_{x \rightarrow 0} \frac{\sin x}{x^3} - 4 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^3 \\
 &= 3 \lim_{t \rightarrow 0} \frac{\sin(3t)}{(3t)^3} - 4 \\
 &= \frac{1}{9} \lim_{t \rightarrow 0} \frac{\sin 3t}{t^3} - 4 \\
 &= \frac{1}{9} L - 4 \\
 \therefore L &= -\frac{9}{2}
 \end{aligned}$$

As EM2, I uploaded the following solution.

Using l'Hôpital's Rule:

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{(\sin 3x)}{x^3} &= \lim_{x \rightarrow 0} \frac{\cos 3x}{x^2} \\
 &= -\frac{3}{2} \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \\
 &= -\frac{3}{2}(3) \\
 &= -\frac{9}{2}
 \end{aligned}$$

Problem 1-5

Let Ω be the closed region in R^2 bounded by the curves of equations $y = x^3$ and $y = \sqrt{x}$. What is the average length of all horizontal chords AB of the region Ω ? (Barbeau, 2000, p.111).

As EM1, I posted the following solution:

Solution The chords in the question have endpoint $(x, \sqrt{x})$ and $(x^{\frac{1}{3}}, \sqrt{x})$

for $0 \leq x \leq 1$, so the lengths are $x^{\frac{1}{6}} - x$. The average chord length is

$$\int_0^1 \left(x^{\frac{1}{6}} - x \right) dx = \frac{5}{14}$$

Problem Set 2

Problem 2-1

Hyunbin found out an odd result while he was working on “Integrate $\int e^x \sinh x dx$ ” (Barbeau, 2000, p.104).

Hyunbin’s result

Solution Integrating by parts twice yields:

$$\begin{aligned} \int e^x \sinh x dx &= e^x \cosh x - \int e^x \cosh x dx \\ &= e^x \cosh x - \left\{ e^x \sinh x - \int e^x \sinh x dx \right\} \\ \therefore e^x (\cosh x - \sinh x) &= 0 \\ \therefore \cosh x &= \sinh x \end{aligned}$$

Post your thought on Hyunbin’s result and Discuss with others.

Problem 2-2

Integrate $\int \ln(\sin x) dx$ (Barbeau, 2000, p.104). As EM1, I posted the following erroneous solution.

Solution Let $I = \int \ln(\sin x) dx$ Then

$$\begin{aligned} I &= \int \ln \left(2 \sin \frac{x}{2} \cos \frac{x}{2} \right) dx \\ &= \int (\ln 2) dx + \int \ln \left(\sin \frac{x}{2} \right) dx + \int \ln \left(\cos \frac{x}{2} \right) dx \\ &= (\ln 2)x + \int \ln \left(\sin \frac{x}{2} \right) dx + \int \ln \left(\cos \frac{x}{2} \right) dx \end{aligned}$$

Let $u = \frac{x}{2}$. Then

$$\begin{aligned}
 I &= (\ln 2)x + 2 \int \ln(\sin u) du + 2 \int \ln(\cos u) du \\
 &= (\ln 2)x + 2I + 2 \int \ln(\cos u) du \\
 I &= -(\ln 2)x - 2 \int \ln(\cos u) du \\
 &= -(\ln 2)x - 2 \int \ln \left\{ \sin \left(\frac{\pi}{2} - u \right) \right\} du
 \end{aligned}$$

Letting $t = \frac{\pi}{2} - u$ yields

$$\begin{aligned}
 I &= -(\ln 2)x + 2 \int \ln(\sin t) dt \\
 &= -(\ln 2)x + 2I \\
 \therefore I &= (\ln 2)x + C
 \end{aligned}$$

Problem 2-3

Evaluate $\int_0^{\frac{3\pi}{4}} \frac{\sin x}{1+\cos^2 x} dx$ (Barbeau, 2000, p.109).

As EM2, I posted the following solution:

Solution Since $\{\arctan(\sec x)\}' = \frac{\sin x}{1+\cos^2 x}$,

$$\begin{aligned}
 \int_0^{\frac{3\pi}{4}} \frac{\sin x}{1+\cos^2 x} dx &= [\arctan(\sec x)]_0^{\frac{3\pi}{4}} \\
 &= \arctan(-\sqrt{2}) - \arctan 1 \\
 &= -\arctan \sqrt{2} - \frac{\pi}{4}
 \end{aligned}$$

Problem 2-4

Determine the area enclosed by the ellipse with equations (Barbeau, 2000, p.112).

$$x = 4 \cos \theta \quad y = 3 \sin \theta \quad (0 \leq \theta \leq \pi).$$

As ADM, I added the condition of “using Green’s theorem” to the problem later.

Problem 2-5

Evaluate the limit

$$L = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \sum_{k=1}^n \frac{j}{j^2 + k^2}$$

(Williams & Hardy, 1988, p.23).

Students’ Solutions with Codes As Oeuvres

I provide students’ solutions posted in the Wiki to show how e-bookable the CPS process is. I attached my codes to the students’ writing to provide the whole view of CPS in a cognitive map. Students’ postings are literally translated into English and parenthesis are given to facilitate reading. I supplement students’ solutions with mathematical expressions when it is needed. My supplement is given on a footnote. In some cases, I omit undeveloped solutions or refutations and some of the students’ postings in cognitive maps may not be seen in the following solutions. Comparing the following solutions as oeuvres with cognitive maps will show how CC solves problems along the inverse directional double helix in JPS. To follow threads on cognitive maps, all postings were placed along the cognitive maps.

Solution 1-1 (figure 5.3)

Branch 1

RFT*: “For Youngbin’s solution to hold, when $k = 0$, $(x^0)' = 0$ $(x^{k+1})' = (x^{0+1})' =$

$(x^1)' = x' \cdot x^0 + x \cdot (x^0)' = x'$. Substantially, since it is not guaranteed that x' is 0 in the solution, the mathematical induction cannot hold.” (STU5)

Q: ““It is not guaranteed that x' is 0 in the solution” and “the mathematical induction cannot hold” are not logically connected. Please suggest more clear reasons and explanations. For example, how about explaining your thoughts by comparing the following proof with the Youngbin’s solution?

RM:

$$(x^n)' = nx^{n-1} \text{ for natural number } n$$

(Proof)

- i) If $n = 1$ then $x' = 1$ (the proof of this is left out). Thus it holds.
- ii) Suppose that if $n = k(k = 1, 2, 3, \dots)$ then it holds.

$$(x^{k+1})' = (x^k)' \cdot x + x^k \cdot x' = kx^{k-1} \cdot x + x^k \cdot 1 = (k+1)x^k$$

From i) and ii), therefore, it holds for natural number n .” (ADM)

Branch 2

RFT*: “A reason that the mathematical induction does not hold in Youngbin’s solution is: (he) looked into $k+1$ by assuming it holds from 0 to k in the step ii) but the assumptions used in his proof were the cases of k and $k-1$. Accordingly, it should be guaranteed to check the case of $n = 1$ by assuming the case of 0 that the cases of 0 and -1 hold. But this is meaningless because -1 is not a natural number. Therefore, if the case of $n = 1$ is contained in the initial condition, the proof can be correct. But it is easy to know that it does not hold when $n = 1$.” (STU14)

Branch 3

RFT*: “The fact that the mathematical induction holds and satisfying the given conditions are two different problems. For the proposition proven with the mathematical induction to holds, (we should) check whether or not it holds by keeping substituting

n (with natural numbers). In fact, it cannot be guaranteed that inductive propositions keep holding. However, we can know whether or not proposition proven with the mathematical induction is true because mathematical proposition can be clearly judged as true or false.” (STU13)

Branch 4

RFT*: “Youngbin proved the initial case of $n=0$ while applying the mathematical induction and assumed that it holds in the case of n is 0 to k . He made a mistake while proving by using the value of derivative when n is 1. Since he proved only the initial case that n is 0, he should not recklessly use the case of 1.” (STU10)

Branch 5

RFT*: This error in applying mathematical induction is similar to what we learned in the mathematics seminar class. To apply mathematical induction, it needs to show that it holds for the first and the second term but Youngbin only showed that it holds when $n = 0$. We come to know that it does not hold when $n = 1$.” (STU4)

Solution 1-2 (figure 5.17)

Branch 1

RFT*: “Indefinite integral always needs integral constant C . In fact, 1 with respect to uv in Gyunhchul’s solution is not 1 but needs to be written as $1 + C$. Hence, the given solution is wrong.

FIX: Here, C is -1 .” (STU4)

Q: “Why $C = -1$?” (ADM)

RE: “It is trivial.” (STU2)

Branch 2

RFT: “Let’s talk about the meaning of the integral constant by comparing the given solution with the following.

RM:

$$\begin{aligned}
 \int \frac{\sqrt{2} \cos x}{\sin(x + \frac{\pi}{4})} dx &= \int \frac{\sqrt{2} \cos(t - \frac{\pi}{4})}{\sin t} dt = \int \frac{\cos t + \sin t}{\sin t} dt^4 \\
 &= \int \frac{\cos t}{\sin t} + 1 dt = \int \frac{\cos t}{\sin t} dt + \int dt = \ln |\sin t| + t + C \\
 &= \ln \left| \sin \left(x + \frac{\pi}{4} \right) \right| + x + \frac{\pi}{4} + C = \ln \left| \sin \left(x + \frac{\pi}{4} \right) \right| + x + C
 \end{aligned}$$

” (ADM)

RE: “An integral constant is needed to seek a function according to an initial condition after integration. Since an initial is not given above (ADM’s posting), we can consider C as a kind of variable and thus can write $\frac{\pi}{4} + C = C$. In a more rigid way, C s in both sides are different and thus it is better to write $\frac{\pi}{4} + C = C'$.” (STU2)

FIX*, AGMT: “Indefinite integral means a function that becomes an integrand when it is differentiated. Thus, the first J may be different from the J which is yielded from integration. The reason why we always put the integral constant on a function yielded from integration is that indefinite integrals of an integrand are infinitely many. Thus, it is right to express $J = J + C' + C$. From this, it is wrong that $0 = 1$.” (STU13) ⁵

Branch 3

RPT: “Gyungchul did not construe the meaning of the indefinite integral. J is a function of x . Since the definition of indefinite integral means that a derivative of an indefinite integral is to change the indefinite integral into the original function (integrand), it needs to add a constant to the expression of the function to satisfy the value of the indefinite integral.

RFT*: It is not correct without a constant. Hence, (he) should add a constant to J . Since J is not a fixed function or value, J in the right side cannot be compared with J in the left side and denoted as such.” (STU10)

⁴I mistakenly omitted ‘dt’ at the end of this expression

⁵STU13 must have typed his ID as “BY STU13” because his IP address was shown as “-122.203.53.66, 9 May 2012 (EST) BY STU12”, which was different from others signature forms shown as, for instance, -STU2 08:47, 24 April, 2012 (EST).

Solution 1-3 (figure 5.3)

Branch 1

NEW : “Substitute x with $\tan t$ ” (STU9).

SUPPL: “Let $x = \tan t$ $dx = \sec^2 t dt$. $t = \frac{\pi}{4}$ when $x = 1$ and $t = -\frac{\pi}{4}$ when $x = -1$.

Thus, answer is $\frac{\pi}{2}$.” (STU13)

Branch 2

NEW: EM2’s erroneous solution.

RFT*: “If x is greater than 0, then $\left(\frac{1}{2} \arccos' \frac{1-x^2}{1+x^2}\right) = \frac{1}{x^2+1}$. If x is less than 0, then $\left(\frac{1}{2} \arccos' \frac{1-x^2}{1+x^2}\right) = -\frac{1}{x^2+1}$. Thus, the first solution is wrong” (STU6).

FIX, AGMT: “Since ‘ $-$ ’ is problematic, use the absolute value” (STU14)

SUPPL, AGMT: “Recall that the integrand is an even function with respect to x .

Although the above solution holds only if the primitive function is positive, multiplying 2 to integration from 0 to 1 will do because of a characteristic of even function⁷” (STU10)

Branch 3

NEW: “From the above, the inverse function of $\cos$ has been found (in EM2’s posting) but my calculating leads to the inverse function of $\tan$. In this case, we do not have to consider the absolute value and

CFM: can find the same answer with STU13’s.” (STU1).

SUPPL: “Since $(\arctan x)' = \frac{1}{1+x^2}$, $\int_{-1}^1 \frac{1}{1+x^2} dx = \arctan(1) - \arctan(-1) = \frac{\pi}{2}$.” (STU7)

Branch 4

Sol1-4: “Since the range of $\sin 3x$ is $[-1, 1]$, applying the squeeze theorem to find an answer 0.” (STU9)

Solution 1-4 (figure 5.10)

$${}^6 \int_{-1}^1 \frac{1}{1+x^2} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\tan^2 t} \sec^2 t dt = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} dt = [t]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{2}$$

$${}^7 \int_{-1}^1 \frac{1}{1+x^2} dx = \int_{-1}^1 \left| \frac{1}{2} \arccos' \frac{1-x^2}{1+x^2} \right| dx = 2 \int_0^1 \left| \frac{1}{2} \arccos' \frac{1-x^2}{1+x^2} \right| dx = \arccos 0 - \arccos 1 = \frac{\pi}{2}$$

이곳을 클릭하고 여러분의 풀이를 올려 주세요.

set 1-3 여기서 x를 tan t로 치환하면 된다.

$$\left(\frac{1}{2}\arccos\frac{1-x^2}{1+x^2}\right)' = \frac{1}{x^2+1} \text{이므로}$$

$$\int_1^{\frac{1}{x^2+1}} dx = \left[\frac{1}{2}\arccos\frac{1-x^2}{1+x^2}\right]_{-1}^1 = \frac{1}{2}(\arccos 0 - \arccos 0) = 0$$

따라서 구하는 값은 0이다.

--Xlot 19:09, 26 March 2012 (EST)

set 1-4 $\sin 3x$ 의 치역은 $[-1, 1]$ 이므로 squeeze theorem을 사용하면 답은 0.

set 1-5 $x = \tan t$ 라 치환하자. $dx = \sec^2 t dt$ 이고 $x=1$ 일때 $t = \pi/4$ $x=-1$ 일때 $t = -\pi/4$ 이다. 따라서 정리하면 답은 $\pi/2$ --Thinkone 00:23, 27 March 2012 (EST)

$$\left(\frac{1}{2}\arccos\frac{1-x^2}{1+x^2}\right)' = \frac{1}{x^2+1}$$

는 x 가 0보다 클 때 성립한다. x 가 0보다 작다면 $\left(\frac{1}{2}\arccos\frac{1-x^2}{1+x^2}\right)' = -\frac{1}{x^2+1}$ 이 된다. 따라서 첫번째 풀이는 틀렸다. --Haha 19:03, 28 March 2012 (EST)

+가 문제이므로 위의 풀이에 절댓값을 붙이면 해결될 것 같다. --통가통가 22:38, 20 April 2012 (EST)

위에서 적분을 할 때 $\cos$ 의 역함수가 나왔는데, 정리하면 $\tan$ 의 역함수가 나온다. 이때는 절댓값을 고려해주지 않아도 답이 thinkone의 답과 같이 나온다. --219jy 12:48, 3 June 2012 (EST)

적분을 할 식이 x 에 대한 우함수임을 생각하자. 위의 풀이는 원시함수가 양수 조건일 때만 성립하지만 우함수의 특징으로 적분값이 음수 구간에서도 똑같은 것이기 때문에 0에서 1까지 적분값을 2배해주면 될 것이다. --Ladgladg 18:53, 3 June 2012 (EST)

$$(\arctan x)' = \frac{1}{1+x^2} \text{이므로} \int_{-1}^1 \frac{1}{1+x^2} dx = \arctan(1) - \arctan(-1) = \frac{\pi}{2} \text{이다. --Hr2307 22:27, 4 June 2012 (EST)}$$

Whole Solution Page of Problem 1-3

문제 1-4의 풀이

--풀이) $L = \lim_{x \rightarrow 0} \frac{\sin 3x}{x^3}$ 이라고 하자.

$$L = \lim_{x \rightarrow 0} \frac{3\sin x - 4\sin^3 x}{x^3} = 3\lim_{x \rightarrow 0} \frac{\sin x}{x^3} - 4\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^3 = 3\lim_{t \rightarrow 0} \frac{\sin(3t)}{(3t)^3} - 4 = \frac{1}{9}\lim_{t \rightarrow 0} \frac{\sin 3t}{t^3} - 4 = \frac{1}{9}L - 4$$

$$\therefore L = -\frac{9}{2} \text{ --Ss2003 19:50, 26 March 2012 (EST)}$$

로피탈의 정리를 활용하는것도 좋은 풀이가 될 수 있다. --Gjalfhdi 21:43, 26 March 2012 (EST)

@Ss2003 문제는 $\lim_{x \rightarrow 0} \frac{\sin 3x}{x^3}$ 이 아니라 $\lim_{x \rightarrow \infty} \frac{\sin 3x}{x^3}$ 이다.

$$\lim_{x \rightarrow \infty} \frac{\sin 3x}{x^3} \leq \lim_{x \rightarrow \infty} \frac{1}{x^3} = 0$$

이므로 답은 0이다. --Jihoonc2 18:02, 15 April 2012 (EST)

@Jihoonc2 $\lim_{x \rightarrow \infty} \frac{\sin 3x}{x^3} \geq 0$ 라는 조건이 필요하다.

Jihoonc2의 풀이는 준식의 상계가 0이라는 것만 밝혔을 뿐, 준식의 하한을 구하지 않았으므로, 준식의 극한값이 0이라는 것을 보장할 수 없다.

처음부터 샌드위치 정리를 이용하는 것이 바람직할 것 같다. --WOWfantasticbaby 11:04, 17 April 2012 (EST)

문제의 조건을 $\lim_{x \rightarrow \infty}$ 에서 $\lim_{x \rightarrow 0}$ 로 바꾸었습니다. --SiB96 10:01, 17 April 2012 (EST)

$$\text{로피탈의 정리를 이용하면 } \lim_{x \rightarrow 0} \frac{\sin 3x}{x^3} = \lim_{x \rightarrow 0} \frac{\cos 3x}{x^2} = -\frac{3}{2} \lim_{x \rightarrow 0} \frac{\sin 3x}{x} = -\frac{3}{2}(3) = -\frac{9}{2} \text{ --Xlot 09:00, 17 April 2012 (EST)}$$

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{x^3} = \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \cdot \frac{3}{x^2} = \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \cdot \frac{3}{x^2} = \lim_{x \rightarrow 0} \frac{3}{x^2} = \text{발산(무한대)이다. --Bryan D. Lee 22:36, 19 April 2012 (EST)}$$

Xlot의 로피탈정리에서 $\frac{\cos 3x}{x^2}$ 는 $\frac{1}{0}$ 꼴이므로 더이상 로피탈을 쓸 수 없고, 따라서 무한대로 발산한다. --통가통가 22:41, 19 April 2012 (EST)

Part of Solution Page of Problem 1-4

I posted a problem: $\lim_{x \rightarrow \infty} \frac{\sin x}{x^3}$. This problem is different from what I intended to post.

Branch 1

NEW: EM1's erroneous solution

RFT*: "To use such a method, the convergence of $\lim_{x \rightarrow 0} \frac{\sin 3x}{x^3}$ should be guaranteed. If it does not converge, then L cannot exist. Hence the equation cannot be established either." (STU6)

RFT: "@EM1 The problem is not $\lim_{x \rightarrow 0} \frac{\sin x}{x^3}$ but $\lim_{x \rightarrow \infty} \frac{\sin x}{x^3}$.⁸(STU8)

RE, CP: "The above problem of $\lim_{x \rightarrow \infty} \frac{3}{x^2}$ was changed with $\lim_{x \rightarrow 0} \frac{\sin x}{x^3}$ " (ADM)

RFT: "@EM1, In EM1's solution, to be $\lim_{x \rightarrow 0} \frac{3 \sin x - 4 \sin^3 x}{x^3} = 3 \lim_{x \rightarrow 0} \frac{\sin x}{x^3} - 4 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^3$, each term in (the right of) the expression needs to converge. However, it cannot be because $3 \lim_{x \rightarrow 0} \frac{\sin x}{x^3} = 3 \lim_{x \rightarrow 0} \frac{\sin x}{x} \frac{1}{x^2} = 3 \lim_{x \rightarrow 0} \frac{1}{x^2} = \text{divergence(infinity)}$." (STU2)

Branch 2

NEW: "Since $\lim_{x \rightarrow \infty} \frac{\sin x}{x^3} \leq \lim_{x \rightarrow \infty} \frac{1}{x^3} = 0$, the answer is 0."⁹ (STU8)

RFT*: "@STU8 A condition of $\lim_{x \rightarrow \infty} \frac{1}{x^3} \geq 0$ is needed.

FIX: It will be better to start with Sandwich Theorem." (STU11)

Q: "Please, explain in detail why the reason is needed.

RE: "@STU8 A condition of $\lim_{x \rightarrow \infty} \frac{1}{x^3} \geq 0$ is needed.

SUPPL: Since STU8's solution showed only its upper bound is 0 but did not seek its lower bound, it is not guaranteed that its limit is 0. It will be better to start with Sandwich Theorem.¹⁰" (STU11)

Branch 3

NEW: EM2's erroneous solution

RFT*: "In EM2's solution, since $\frac{\cos 3x}{x^2}$ is a type of $\frac{1}{0}$, l'Hôpital's Rule cannot be applied any more."¹¹

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⁸This RFT is the first half of STU8's posting

⁹This NEW is the second half of STU8's posting

¹⁰ $0 \leq \lim_{x \rightarrow \infty} \left| \frac{\sin x}{x^3} \right| \leq \lim_{x \rightarrow \infty} \left| \frac{1}{x^3} \right| = 0$

¹¹ $\lim_{x \rightarrow 0} \frac{\sin x}{x^3} = \lim_{x \rightarrow 0} \frac{\cos 3x}{x^2} = \infty$

FIX & CFM: Thus, it diverges.” (STU14)

Branch 4

NEW: “ $\lim_{x \rightarrow 0} \frac{\sin 3x}{x^3} = \lim_{x \rightarrow 0} \frac{\sin 3x}{3x \cdot \frac{x^2}{3}} = \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \frac{3}{x^2} = \lim_{x \rightarrow 0} \frac{3}{x^2} = \text{divergence(infinity)}$ ” (STU2)

RPT: “We can know that the value of $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$ is 3 in $\lim_{x \rightarrow 0} \frac{\sin 3x}{x^3}$. The other part, x^{-2} , diverges to infinity when x converge to 0. Since we know that a product of a diverging term and converging term, which is not 0, diverges to infinity, we can easily decide that it diverges.” (STU10)

Branch 5

CFM: “As a result (of STU14’s solution), the derivative of the numerator and that of denominator need to converge to 0 as well as numerator and denominator. NEW: To prove this, we need to discuss it more.” (STU13)

Solution 1-5 (figure 5.9)

Branch 1

NEW: “ $x = 0$ for $y = \sqrt{x}$ to exist. When x is nonnegative, solutions of $x^3 = \sqrt{x}$ are 0 and 1. Thus, the average of lengths of horizontal lines equals the area of the range divided by length of y axis. Thus, $\int_0^1 \sqrt{x} - x^3 dx$ is the area and the length of y axis is 1. Since $\int_0^1 \sqrt{x} - x^3 dx = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$, the average of lengths of horizontal lines is $\frac{5}{12}$.” (STU6)

FIX*: STU6 changed $\int_0^1 x^3 - \sqrt{x} dx$ with $\int_0^1 \sqrt{x} - x^3 dx$ in the above solution.

Branch 2

NEW: EM1’s erroneous solution.

RFT*: “The average length of all horizontal chords is the limit of average of horizontal chords in subregions evenly split with lines parallel to x axis. That is, this means integral of length of horizontal chords with respect to y axis. So we need to consider

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¹²A typo of x^{-2}

it as integral of length of horizontal chords with respect to y axis by using concept of definite integral. However, distances of subregions are not constant in EM1's solution. Thus, we need to consider this." (STU6)

CFM: "In EM1's solution, to get the right answer,

FIX: s/he needs to integrate using (x^2, y) and $(x^{\frac{1}{3}}, y)$ ¹³

RFT*: rather than $(x, \sqrt{x})$ and $(x^{\frac{1}{6}}, \sqrt{x})$." (STU2)

Solution 2-1 (figure 5.16)

Branch 1

RFT*: "Integral constant is missing in the first equation. It is similar in the second equation.

FIX: Integral constants for both sides are $1/2$ are $-1/2$ (respectively) and thus the right side of the second equation has 1 rather than 0." (STU1)

FIX*: "We need to be cautious about integral constant while applying indefinite integral. Solving the problem by considering integral constant, it yields:

$$\begin{aligned}\int e^x \sinh(x) dx &= e^x(\cosh x + c) - \int (\cosh x + C) dx \\ &= e^x(\cosh x + C) - \int e^x \cosh x dx - C \int e^x dx \\ &= e^x(\cosh x + C - \left\{ e^x(\sinh x + D - \int e^x(\sinh x + D) dx \right\} - C \int e^x dx\end{aligned}$$

Subtracting $\int e^x \sinh x dx$ from both sides,

$$e^x(\cosh x + C) - C \int e^x dx = e^x(\sinh x + D) - D \int e^x dx$$

$$e^x \cosh x + C' = e^x \sinh x + D'$$

$$\text{Since } \cosh x = \frac{e^x + e^{-x}}{2}, \sinh x = \frac{e^x - e^{-x}}{2}, e^x \cdot \frac{e^x + e^{-x}}{2} + C' = e^x \cdot \frac{e^x - e^{-x}}{2} + D'$$

$$\frac{e^{2x} + 1}{2} + C' = \frac{e^{2x} - 1}{2} + D' \quad D' - C' = 1 \quad \text{Hence, the error does not occur if we consider}$$

$$^{13} \int_0^1 x^{\frac{1}{3}} - x^2 dx = \left[\frac{3}{4} x^{\frac{4}{3}} - \frac{1}{3} x^3 \right]_0^1 = \frac{5}{12}$$

integral constants C' and D' . But, we cannot find indefinite integral that we want.”¹⁴
(STU11)

Branch 2

NEW: “(Instead,) We can integrate $\int e^x \sinh x dx$ as follows:

$$\begin{aligned}\int e^x \sinh x dx &= \int e^x \cdot \frac{e^x - e^{-x}}{2} dx \\ &= \int \frac{e^{2x} - 1}{2} dx \\ &= \frac{1}{2} \left[\frac{1}{2} e^{2x} - x \right] + C \\ &= \frac{1}{4} e^{2x} - \frac{1}{2} x + C \quad ^{15}\end{aligned}$$

” (STU11)

Branch 3

RFT*: “The fact that he did not consider (integral) constant after finding definite integration while finding indefinite integration in the definite integration caused an error. If an integral constant is not considered while finding integration, it is the same with the case that a special case of (indefinite) integration is considered. Thus, a case that does not hold can emerge. When an integral constant is not considered, it results solving a problem by assuming that nonequivalent propositions are equivalent”
(STU10)

Solution 2-2 (figure 5.2)

Branch 1

NEW: EM1’s erroneous solution.

RFT: “If $\int \ln(\sin x) dx = (\ln 2)x + C$ then $\frac{d}{dx}[(\ln 2)x + c] = \ln(\sin x)$ but it is not.”
(STU8)

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¹⁴This is the first part of STU11’s posting.

¹⁵This is the second part of STU11’s posting.

RFT*, AGMT: “In the solution by substituting $u = \frac{x}{2}$, it is not legitimate to consider $\int (\sin u) du$ same as $I = \ln(\sin x) dx$ ¹⁶ because the two have different integral ranges.” (STU2)

RFT*, AGMT: “In EM1’ solution, an error is related to the integral constant. It is not right not to include an integral constant in finding an indefinite integral. For example, when a function is denoted as a result of evaluating the indefinite integral, if the function does not contain an integral constant then it means that the constant is 0, but this causes errors in calculating an equation by substituting the indefinite integral with I .” (STU10)

RFT*, AGMT: “(In EM1’ solution) s/he split a form of addition into two (parts). I don’t think it possible to consider them as same after those two (parts’) range of integration are made different.” (STU10)

RFT, AGMT: “The function $\ln(\sin x)$ is a real number when $\sin x > 0$ but a complex number when $\sin x < 0$. Hence, we cannot evaluate it within the range of what we have learned”¹⁷ (STU8)

Branch 2

NEW, AGMT: “Differentiate $x \ln(\sin x)$ then it gives $\ln(\sin x) + x \cot(x)$ and thus it can be solved if we integrate $x \cot(x)$ ¹⁸. This indefinite integral can be denoted as a series by using Taylor expansion and term by term integration.” (STU1)

Branch 3

NEW: “and should learn Theory of Complex Function. Referring to

<http://www.wolframalpha.com/input/?i=int+ln%28sinx%29dx>, the result is $\frac{1}{2}i(x^2 + \text{Li}_2(e^{2ix})) - x \ln(1 - e^{2ix}) + x \ln(\sin x)$ where $\text{Li}_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n}$. If the integral is given such as $0 \sim \frac{\pi}{2}$ ¹⁹, where the function is real, then we can evaluate it by using substitu-

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¹⁶STU2 omitted an integral sign here.

¹⁷This is the first part of STU8’s posting.

¹⁸cor is STU1’s typo of cot

¹⁹His informal expression of $[0, \frac{\pi}{2}]$

tion (Refer to the following link) http://www.math.ucsd.edu//20B/7_DefInt.pdf”²⁰ (STU8)

Solution 2-3 (figure 4.8)

Branch 1

NEW: “Let $u = \cos x$.

$$\begin{aligned} \int_0^{\frac{3}{4}\pi} \frac{\sin x}{1+\cos^2 x} dx &= \int_1^{-\frac{\sqrt{2}}{2}} -\frac{1}{1+u^2} du = {}^{21}\int_{-\frac{\sqrt{2}}{2}}^1 \tan^{-1} u du = \tan^{-1} 1 - \tan^{-1}\left(-\frac{\sqrt{2}}{2}\right) \\ &= \frac{\pi}{4} + \tan^{-1}\left(\frac{\sqrt{2}}{2}\right)” \text{ (STU8)} \end{aligned}$$

AGMT: “It seems desirable to use substitution of $\cos x$ like the first solution (STU8’s).

SIM: This substitution makes the same formula seen in problem 1-3. Thus, we can solve this problem by applying a method of solution 1-3.” (STU12)

Branch 2

NEW: EM2’s erroneous solution.

RFT: “EM2’s answer is not right because it is negative. $\frac{\sin x}{1+\cos^2 x}$ is positive on $(0, \frac{3}{4}\pi)$.

RFT*: Since $\sec x$ is not defined at $\frac{1}{2}\pi$,

FIX: the interval should be split for integrating.” (STU14)

SUPPL: “The problem of defining $\sec(x)$ seems to be solved by using limit. Although the function can be defined as a limit at $\frac{\pi}{2}$, it is not differentiable because it is a discontinuous function and thus not an indefinite integral.

AGMT: If we integrate it by splitting the interval (into two small intervals) as mentioned in the above (STU14’s posting),

CFM: then we can find the right answer²² (as STU8’s)” (STU1)

RPT: “A function that EM2 has provided has an interval on which it cannot be defined. Its domain needs to be changed according to the integral scheme. Or it seems to be solved by applying a method of taking limit.” (STU10)

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²⁰This is the second part of STU8’s posting

²¹Here, a simple error occurred. $\int_{-\frac{\sqrt{2}}{2}}^1 \tan^{-1} u du$ needs to be changed as “ $\int_{-\frac{\sqrt{2}}{2}}^1 \frac{1}{1+u^2} du = [\tan^{-1} u]_{-\frac{\sqrt{2}}{2}}^1$ ”

²²In an interview, I asked STU1 to write down and explain his solution. His solution can be

Solution 2-4 (figure 5.14)

Branch 1

NEW: “Since $9x^2 + 16y^2 = 144(\sin^2 \theta + \cos^2 \theta) = 144$, the given curve is an ellipse. Its long axis is 4, short axis ²³ is 3, ellipse’s area is denoted as πab (a is long axis and b is short axis), and thus answer is 12π .” (STU8)

CP: A condition of “using Green’s Theorem” was added to the problem. (ADM)

Branch 2

NEW: “ $\int_{\partial D} F dr = \iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dx dy$ $F = (f(x, y), g(x, y))$ Since $\iint_D dx dy$ is an area (of D), $\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}$ should be a constant. Let $f = x + y, g = 2x$. Then, $\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}$ is 1, the right side (of above equation) become an area (of D). Let’s calculate the right side.

$$\begin{aligned} \int_{\partial D} F dr &= \int ((x + y)dx + (2x)dy) \\ &= \int_0^{2\pi} ((4 \cos \theta + 3 \sin \theta)(-4 \sin \theta) d\theta + 2 \times 4 \cos \theta 3 \cos \theta d\theta) \\ &= \int_0^{2\pi} (-16 \sin \theta \cos \theta - 12 \sin^2 \theta + 24 \cos^2 \theta) d\theta \\ &= 12\pi \end{aligned}$$

Hence, $S = \iint_D dx dy = 12\pi$ ” (STU7)

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re-written as follows:

$$\begin{aligned} \int_0^{\frac{3}{4}\pi} \frac{\sin x}{1 + \cos^2 x} dx &= \lim_{t \rightarrow \frac{\pi}{2} + 0} \int_t^{\frac{3}{4}\pi} \frac{\sin x}{1 + \cos^2 x} dx + \lim_{t \rightarrow \frac{\pi}{2} - 0} \int_0^t \frac{\sin x}{1 + \cos^2 x} dx \\ &= \lim_{t \rightarrow \frac{\pi}{2} + 0} [\arctan(\sec x)]_t^{\frac{3}{4}\pi} + \lim_{t \rightarrow \frac{\pi}{2} - 0} [\arctan(\sec x)]_0^t \\ &= \lim_{t \rightarrow \frac{\pi}{2} + 0} \left(\arctan(-\sqrt{2}) - \arctan(\sec t) \right) + \lim_{t \rightarrow \frac{\pi}{2} - 0} \left(\arctan(\sec t) - \arctan\left(\frac{\sqrt{2}}{2}\right) \right) \\ &= \frac{\pi}{2} - \arctan(\sqrt{2}) + \frac{\pi}{2} - \frac{\pi}{4} \\ &= \tan^{-1}\left(\frac{\sqrt{2}}{2}\right) + \frac{\pi}{4} \end{aligned}$$

²³I literally translated all postings. STU8 used axis rather than the length of the short axis.

Solution 2-5 (figure 5.4)

Branch 1

NEW:

$$\begin{aligned}
 \text{“} \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \sum_{k=1}^n \frac{j}{j^2 + k^2} &= \lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{j=1}^n \sum_{k=1}^n \frac{\frac{j}{n}}{\frac{j^2}{n^2} + \frac{k^2}{n^2}} \\
 &= \int_0^1 \int_0^1 \frac{x}{x^2 + y^2} dx dy \\
 &= \int_0^1 \left[\frac{1}{2} \ln(x^2 + y^2) \right]_0^1 dy \\
 &= \int_0^1 \left(\frac{1}{2} \ln(y^2 + 1) - \ln y \right) dy \\
 &= \frac{1}{2} [y \ln(y^2 + 1)]_0^1 - \int_0^1 \left(\frac{y^2}{y^2 + 1} \right) dy - [y \ln y - y]_0^1 \\
 &= \frac{1}{2} \ln 2 - \int_0^1 \left(1 - \frac{1}{y^2 + 1} \right) dy - 1 \\
 &= \frac{1}{2} \ln 2 - 1 - [y - \tan^{-1} y]_0^1 \\
 &= \frac{1}{2} \ln 2 - 2 + \frac{\pi}{4} \text{” (STU14)}
 \end{aligned}$$

RFT: “Since your answer is negative, it is not correct.” (STU11)

FIX*, AGMT:

$$\begin{aligned}
 {}^{24}\text{“} &= \frac{1}{2} [y \ln(y^2 + 1)]_0^1 - \int_0^1 \left(\frac{y^2}{y^2 + 1} \right) dy - \frac{[y \ln y - y]_0^1}{2} \\
 &= \frac{1}{2} \ln 2 - \int_0^1 \left(1 - \frac{1}{y^2 + 1} \right) dy - \frac{1}{2} \\
 &= \frac{1}{2} \ln 2 - \frac{1}{2} - [y - \tan^{-1} y]_0^1 \\
 &= \frac{1}{2} \ln 2 - \frac{3}{2} + \frac{\pi}{4} \text{” (STU3)}
 \end{aligned}$$

RFT: “This answer is also less than 0. The answer is $\frac{1}{2} \ln 2 - \frac{3}{2} + \frac{\pi}{4}$, but it is less

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²⁴Since the first part of the solution is same with STU14’s due to a strategy of ‘copy, paste, and revise’, it was not written

than 0 while considering $\ln 2$ is around 0.7.” (STU5)

FIX*:

$$\begin{aligned}
^{25} &= \frac{1}{2} [y \ln(y^2 + 1)]_0^1 - \int_0^1 \left(\frac{y^2}{y^2 + 1} \right) dy - \frac{1}{2} [y \ln y - y]_0^1 \\
&= \frac{1}{2} \ln 2 - \int_0^1 \left(1 - \frac{1}{y^2 + 1} \right) dy + \frac{1}{2} \\
&= \frac{1}{2} \ln 2 + \frac{1}{2} - [y - \tan^{-1} y]_0^1 \\
&= \frac{1}{2} \ln 2 + \frac{\pi}{4} - \frac{1}{2} \text{ ” (STU7)}
\end{aligned}$$

Branch 2

NEW: “How about changing the order of integration of x and y ? If you integrate with respect to y first, then you will find $\arctan(1/x)$ by substituting $t = y/x$.

CFM: You can easily seek (the answer) ²⁶ by integrating $1 \times \arctan(1/x)$ by parts.

We can see the importance of the order of integration (from this).” (STU1)

Q: “I have a question concerning 219ljy’s opinion above.

RFT*: Since there is no difference between (role of) x and (that of) y in the mathematical expression (of $\int_0^1 \int_0^1 \frac{x}{x^2 + y^2} dx dy$), it doesn’t matter whether or not they are interchanged. So I don’t understand what you mean by “integrate with respect to y first”. Since we can still use the same substitution even though we integrate with

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²⁵Since the first part of the solution is same with STU3’s due to a strategy of ‘copy, paste, and revise’, it was not written

²⁶In an interview with STU1, he replied that he found the same answer with STU7’s by changing the order of integration. Since STU1’s new solution leads to the correct answer as follows: Let $t = y/x$ ($0 \leq t \leq 1/x$). Then

$$\begin{aligned}
\int_0^1 \int_0^1 \frac{x}{x^2 + y^2} dx dy &= \int_0^1 \int_0^1 \frac{x}{x^2 + y^2} dy dx = \int_0^1 \int_0^{\frac{1}{x}} \frac{1}{t^2 + 1} dt dy = \int_0^1 [\arctan t]_0^{\frac{1}{x}} dy = \int_0^1 \arctan \frac{1}{x} dx \\
&= \left[x \arctan \frac{1}{x} \right]_0^1 + \int_0^1 \frac{x}{x^2 + 1} dx = \frac{\pi}{4} + \frac{1}{2} [\ln(x^2 + 1)]_0^1 = \frac{\pi}{4} + \frac{1}{2} \ln 2
\end{aligned}$$

I guess that STU1 made an error by omitting multiplication of derivative of inner function while calculating: $\frac{1}{x}$ while integrating $\arctan 1/x$ by parts as $\int_0^1 \arctan \frac{1}{x} dx = \left[x \arctan \frac{1}{x} \right]_0^1 - \int_0^1 \frac{x^3}{x^2 + 1} dx = \frac{\pi}{4} + \frac{1}{2} \ln 2 - \frac{1}{2}$. This result is same as STU7’s. As a result, STU1’s new erroneous solution coincidentally confirmed STU7’s old erroneous solution.

스타이너 (Jakob Steiner)의 증명

19세기에 들어 스타이너 (Jacob Steiner; 1796-1863)는 천재적인 다양한 방식으로 등주 정리의 증명을 시도하였다. 그 중 가장 명쾌한 것은 다음과 같이 기하를 이용한 것이다. 스타이너의 증명은 4 단계로 이루어져 있다.

*스타이너의 증명

1단계) 길이가 L 인 폐곡선 중 면적이 최대인 곡선을 C 라 하자.

2단계) 곡선 C 는 볼록이다.

증명) 곡선 C 는 볼록이 아니라고 하면 다음 그림 1과 같이 곡선 C 위의 적당한 두 점 O 와 P 에 대하여 $\overline{OP}$ 가 곡선 C 의 외부에 있다. 직선 OP 에 대하여 호 OQP 의 대칭인 호를 $OQ'P$ 라 하고 새로 만들어진 곡선을 C' 이라 하면 두 곡선 C 와 곡선 C' 의 둘레 길이는 L 로 같지만 곡선 C' 은 곡선 C 보다 면적 I 와 II 만큼 더 가지게 되므로 1단계의 가정에 모순된다. 따라서 곡선 C 는 볼록이다.

Part of the Page for Steiner's Proof

respect to x first, I don't think that changing the order of integrations causes any difference.” (STU12)

Project (Reconstructing Hurwitz's Proof for The Isoperimetric Theorem)

The Isoperimetric Problem is known as: What is the largest area among those of all closed curves with a given length L ? The answer has been known as $\frac{L^2}{4\pi}$, that is the circle has the largest area. Although this problem was already given in ancient greek times and looks so obvious, successful proof for the problem was not given and “only modern methods have yielded a rigorous proof” (Courant and Robbins, 1996, p.373). Jakob Steiner(1796-1863) devised various ingenious ways of proving this theorem, of which the following is geometric one.

The proof consists of four steps:

1. Suppose that the curve C is the closed curve with the length L and maximum

area.

2. The curve C is convex.
3. Let A, B be two points that divide the curve C into two arcs C_1, C_2 with length $\frac{L}{2}$ then areas of C_1 and C_2 are equal.
4. C_1 is a semicircle.

Although his proof is simple and elegant, it is conditional: "If there is a curve of length L within maximal area then it must be a circle." (Courant and Robbins, 1996, p.375). As we can see problem 7, assuming the existence of a solution can lead us to an wrong conclusion. Thus a proof for the existence of a solution is needed. Dirichlet pointed out that there was no demonstration of the existence of a figure that maximize the area in Steiner's proof. Historically, "because of the apparent obviousness of the premise that a solution exists, mathematicians until late in the nineteenth century paid no attention to the logical point involved, and assumed the existence of a solution to extremum problems as a matter of course" (Courant and Robbins, 1996, p.367).

APPENDIX 3. INSTRUMENTS

Interview Protocol (Students)

Name _____ Date _____

Grade _____ Gender _____ Phone _____

Interview Location _____ Interviewed by _____

Greeting

Thank you for allowing me to interview you. For last three months, you participated in the experiment. I would like to ask you several questions. The information you provide in this interview will be used only for my study. My interest is in how you get to solve the problems. The interview takes about forty minutes. The interview will tend to focus on your problem solving process.

Experience of The WIKI

- Have you ever been solved collaboratively mathematical problem using Wiki or any other similar tools?
- Please tell me about your Wiki experience. What, if anything, did you like about it? What, if anything, did you dislike about it?
- How long does it take you to write your solution in the Wiki system?
- Is the Wiki useful for your learning? In what sense? If not, why?
- What is your opinion about a mathematics course like this calculus course using the Wiki? In what way did it change the way you study for the course?
- Did you discuss about homework problems with other students face-to-face? Or only in the Wiki?

Problem Solving

- How long did you work on this problem?
- How many times did you read problems?
- Did you read other students' solutions or comments thoroughly?
- Did you find the problem easy, medium or difficult? Did you know how to solve the problem when you first read it? How did you find the solution?
- How did you make this conjecture?
- Do you think that the Wiki stimulate your creativity in problem solving?
- Do you think that the Wiki is an efficient tool to improve your or others' solutions / to get rid of any errors in your or others' solutions?
- Why did you change your writing at this part?
- Why didn't you revise the solution rather than just suggest which part of it is wrong?
- Why didn't you write the solution more precisely/clearly?
- How did you make sure that this is correct or incorrect?
- What do you think the most important idea in this problem?
- Why did you connect these ideas?
- When other students change this process, did you understand it or not?

Idea Source

- Where did you get this idea? That is, what is the source of your idea? Books, website, the teacher, your friends?

- How long did it take you to find an idea in this source?

Collaboration and Mathematics

- Did the comments posted on the Wiki by other students assist you in solving the problem? If so, in what way?
- How did you feel when you change other's solutions?
- Do you think it is efficient to solve problems together?
- How do you think mathematicians do their work?
- Whether or not is collaboration important in mathematical problem solving? Individual work (collaborative work) is more important? Why?

Evaluation

- Did you feel any pressure regarding evaluation?
- What do you think about peer evaluation?
- If you are not evaluated, are you still going to participate in the Wiki problem solving?

Thanks for your time.

Interview Protocol (Teacher)

Name _____ Date _____

Grade _____ Gender _____ Phone _____

Interview Location _____ Interviewed by _____

Greeting

Thank you for allowing me to interview you. I appreciate your help during the experiment. The information you provide in this interview will be used only for my study. My interest is in what you think about the Wiki. The interview takes about forty minutes.

General Questions

- How long did you teach math?
- When did you start to work at this school?
- What do you think about your students?

Experience of The WIKI

- Have you ever been used the Wiki in your class?
- How do you think about the Wiki? Good points and Bad points
- Will you use the Wiki in other class too?
- Is the Wiki useful for your teaching? In what sense? If not, Why?
- What is your opinion about mathematics course like this calculus course using the Wiki?
- What course will most need the Wiki?
- What do you think your students learned from the work with wiki?

- Did you find the problem easy, medium or difficult?
- Do you think that the Wiki stimulate your students' creativity in problem solving?
- Do you think that the Wiki is an efficient tool for students to improve their solutions / to get rid of any errors in their solutions?
- Why do you think some students did not revise their solutions?

Collaboration and Mathematics

- Do you think collaboration is essential in students' learning mathematics?
- What do you think students learned in this experiment?
- How do you think mathematicians do their work?
- Whether or not is collaboration important in mathematical problem solving?
Individual work (collaborative work) is more important? Why?

Evaluation

- Do you think that evaluation is sufficient motivation factor for using the Wiki?
- What do you think students' peer evaluation?
- Which solution do you think best? Why?

Design

- What is your opinion for developing the Wiki environment better? Thanks for your time

Survey Questions

The information you provide in this survey will be used only for my study. Questions are about your experience in solving problems with the Wiki. The survey takes about ten minutes.

Experience of The WIKI

Q1. Did other students' comments or solutions stimulate your new thought while you were engaging in solving the posted problems? If "Yes", in what points? If "No", Why?

Q2. Is the Wiki useful for you to improve the solutions by removing errors? 1. Yes
2. No

Participation

Q3. Did you actively participate in solving problems using the Wiki? If "Yes", Why? If "No", Why?

Q4. How many times did you stop by the Wiki site?

Q5. How many times did you write in the Wiki system?

Writing Practice

Q6. Did you have any trouble in writing mathematical expression using the Wiki while you were engaging in solving problems? If "Yes", then what is your suggestion for making the writing system easier to write the math expression?

Community of Inquiry

Q7. Do you think that it is necessary to set up norms of participation for collaborative problem solving using the Wiki? If "Yes", why? If "No", why?

Q8. Do you think that collaborative mathematics problem solving is more efficient than individual problem solving? If "Yes", why? If "No", why?

Q9. Which problem was the best one for collaborative problems solving using the Wiki? And Which one was the worst? If "No", then of which was the worst one?

The best: The Worst:

Design of Pages

Q10. What are your suggestions to improve the design of the Wiki pages?

Evaluation Form (Teacher and Students)

The following problems were posted in the Wiki and you solved them for last three months. Evaluate other students' solutions and state reasons.

Problem 1-1	Name	Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Problem 1-2	Name	Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Problem 1-3	Name	Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Problem 1-4	Name	Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Problem 1-5	Name	Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Problem 2-1

Name

Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Problem 2-2

Name

Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Problem 2-3

Name

Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Problem 2-4

Name

Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Problem 2-5

Name

Why

The 1st contributor

The 2nd Contributor

The 3rd Contributor

Others

Project

Proof 1	Name	Why

The 1st contributor		

The 2nd Contributor		

The 3rd Contributor		

Others		

Discussion 2	Name	Why

The 1st contributor		

The 2nd Contributor		

The 3rd Contributor		

Others		

Statement 3	Name	Why

The 1st contributor		

The 2nd Contributor		

The 3rd Contributor		

Others		

Proof 4	Name	Why

The 1st contributor		

The 2nd Contributor		

The 3rd Contributor		

Others		

APPENDIX 4. A GIDE TO USE THE WIKI

위키 사용 안내문

1. 로그인 및 등록 방법

- ① <http://gchs.educatewiki.com>에 접속하면 다음과 같은 메인페이지가 뜬다.
메인페이지 오른쪽 상단에 있는 Login/create account를 클릭한다.

- ② 등록을 하기 위해 create an account를 클릭한다(로그인 할 때는 Username과 Password를 입력하여 바로 들어간다.)

Log in / create account

Log in

Don't have an account? **create an account** ←클릭

You must have cookies enabled to log in to EducateWiki.

Username:

Password:

☐ Remember my login on this computer

- ③ 다음과 같은 계정만들기 창에 자신이 정한 Username, Password 등을 입력한다. 이때 처음에 제시된 질문에 답변하는 것을 잊으면 안 된다. 이러한 종류의 질문은 위키의 특별한 특징인데 나중에 그림과일을 업로드하거나 외부링크를 걸 때에도 나온다. Username을 정할 때는 Username의 사용자가 누구인지 알아볼 수 있도록 정하는 것이 좋다.

Log in / create account

Create account

Already have an account? Log in

Do not create account without answering the question below

What is the secret name in the English alphabet? ←질문에 답

Username:

Password:

Repeat password:

Email:

☐ Remember my login on this computer

④ 계정을 만들고 나면 자신이 정한 Username과 Password를 이용하여 로그인한다. 로그인 후 상단의 메뉴가 바뀌며 접속 IP Address가 Username으로 바뀐다. 이제 화면에서 “• 미적분학(CALCULUS)” 항목을 클릭한다.

2. 콘텐츠와 구조

① 콘텐츠 목록

미적분학 항목 아래에 콘텐츠 목록이 다음과 같이 나오며 현재 1. 협력 문제해결을 위한 참여규칙(NORMS OF PARTICIPATION), 2.수식 기본 사용법, 3. 이 주의 문제(PROBLEMS OF THE WEEK) 등으로 구성되어 있다. 콘텐츠 목록에서 해당 항목을 누르면 화면이 해당 항목에 대한 설명이 있는 부분으로 이동한다.

② 협력 문제해결을 위한 참여규칙(NORMS OF PARTICIPATION)과 간단한 수식 목록(LIST FOR WRITING MATHEMATICAL EXPRESSION)을 클릭하고 각각의 내용을 한 번 읽어 본다.

3. 편집

① 콘텐츠 목록에서 2.2 수식입력을 위한 연습문제를 클릭한다. 다음 화면에서 “연습을 위한 수식문제 1” 항목을 클릭한다.

② 다음 화면에서 선생님께서 자신에게 지정해준 번호를 클릭한다.

③ 지정한 번호를 통해 들어오면 다음과 같이 어떤 외국 학생의 잘못된 풀이가 나온다. 이 풀이를 고치기 위해서 오른쪽 상단에 있는 “Edit” 항목을 클릭한다.

④ 다음의 에디터 창에서 위의 잘못된 풀이를 수정할 수 있다.

4. 에디터창의 버튼 사용법

① 다음에서 편집창의 각 버튼의 용도를 살펴본다. 우선 왼쪽의 두 버튼은 텍스트를 굵게 하거나 기울일 때 사용한다. 버튼을 각각 누르면 "Bold text" "Italic text"와 같이 편집창에 나타나고 음영진 곳에 필요한 텍스트(그림에서는 "굵은 글자체", "기울여진 글자체")를 입력하고 아래의 "Show Preview" 버튼을 눌러보자.

② 다음은 "Show Preview" 버튼을 눌렀을 때 에디터 창 위에 그 결과를 미리 보여주고 있다. 미리보기의 결과가 자신의 원하는 바와 일치하면 "Save" 버튼을 누르면 편집한 내용이 최종 저장된다.

③ 왼쪽에서 3번째 “Internal Link” 버튼은 새로운 페이지를 만들 때 사용한다. 예를 들어 자신의 설명 중에 “함수”라는 용어가 들어 있고 이를 설명하거나 정의하려 한다면 이 버튼을 이용할 수 있다. 이 버튼을 누르면 “[[Link title]]”이 나타나고 여기에 “[[함수]]”와 같이 입력하여 저장하면 다음과 같이 나타난다.

위의 함수를 누르면 함수에 관한 새로운 페이지로 들어가게 되며 이곳에 함수에 관한 내용을 입력하면 된다.

④ 왼쪽에서 네 번째 버튼 “External Link”는 외부의 사이트에 링크를 걸 때 사용한다. 자신의 풀이와 관련하여 참고한 사이트가 있다면 이 버튼을 이용하여 링크를 만들면 유용한 참고 자료를 제공할 수 있다. 예를 들어 경기과학고의 홈페이지에 링크를 걸고 싶다면 버튼을 누르고, 나타나는 [http://www.example.com link title]에 “[http://www.gs.hs.kr 경기과학고]”라고 입력, 저장하면 등록할 때와 같이 질문이 화면이 나타난다.

위의 질문에 답변을 하고 저장을 누르면 링크가 다음과 같이 생성된다.

⑤ 왼쪽에서 다섯 번째 위치한 “Level 2 Headline” 버튼은 글의 제목을 만들 때 사용한다. 버튼을 누르면 “-- Headline text --”가 나타나고 이곳에 원하는 제목을 입력하면 된다. 예를 들어 “제1장”이라고 입력하면 다음과 같이 “제1장”이라는 제목이 나타나고 그 제목 아래 페이지는 나누는 가로줄이 들어가 있는 것이 보인다. 제목의 레벨을 더 낮추려면 “-”를 하나 더 추가하여 “--- 제1절 ---”이라고 입력하면 된다. 다음에서 “제1절”이라고 입력한 것이 “제1장” 아래 보이고 있다.

⑥ 왼쪽에서 6번째 위치한 버튼인 "Embedded file"은 그림이나 pdf 파일을 업로드 할 때 사용한다. 지원되는 확장자는 png, gif, jpg, jpeg, pdf 등이고 다른 확장자는 지원되지 않는다. 따라서 hwp 파일을 업로드할 때는 pdf로 변환 저장하여 업로드할 수 있다. 버튼을 누르면 "[[File:Example.jpg]]"이 나타나고 여기에 업로드하려는 파일명을 넣는다. 예를 들어 "[[File:바닷가.jpg]]"라고 입력하고 저장하면 아래 그림과 같이 "File:바닷가.jpg"가 나타난다.

위의 페이지에서 "File:바닷가.jpg"를 클릭하면 다음과 같은 화면이 나타나고 소스파일을 "찾아보기"를 클릭하여 입력한 후 맨 아래 "Upload file" 버튼을 누른다.

파일이 성공적으로 업로드되면 다음과 같이 페이지에 그림이 나타난다.

1반1번

$$\int \frac{1}{x+1} dx = \int \left(\frac{1}{x} + \frac{1}{1} \right) dx = \int \frac{1}{x} dx + \int \frac{1}{1} dx = \ln x + \ln 1 = \ln(x+1) + C$$

⑦ 왼쪽에서 7번째 “File link” 버튼은 미디어 파일을 업로드할 때 사용하며 올리는 방법은 그림파일 올릴 때와 같다.

⑧ 가장 중요한 버튼은 “Mathematical formula(LeTex)”이다. 수식을 입력하기 위해서는 이 버튼을 누르고 “$Insert formula here$”이 나타나면 수식명령어를 입력하며 수학적상수 용어 앞에 “\”를 붙인다. “\”는 보통 한글자판에서 “W”를 입력하면 된다. 예를 들어, 분수 $\frac{2}{5}$ 을 표현하기 위해서는 “$\frac{2}{5}$”와 같이 입력한다. 그 결과 다음과 같이 수식이 나타난다. 기본적인 수식명령어는 “간단한 수식목록”을 참조한다.

1반1번

Preview

Remember that this is only a preview. Your changes have not yet been saved!

$$\int \frac{1}{x+1} dx = \int \left(\frac{1}{x} + \frac{1}{1} \right) dx = \int \frac{1}{x} dx + \int \frac{1}{1} dx = \ln x + \ln 1 = \ln(x+1) + C$$

Mathematical formula(LeTex)

$\int \frac{1}{x+1} dx = \int \left(\frac{1}{x} + \frac{1}{1} \right) dx = \int \frac{1}{x} dx + \int \frac{1}{1} dx = \ln x + \ln 1 = \ln(x+1) + C$

$\int \frac{1}{x+1} dx = \int \left(\frac{1}{x} + \frac{1}{1} \right) dx = \int \frac{1}{x} dx + \int \frac{1}{1} dx = \ln x + \ln 1 = \ln(x+1) + C$

⑨ 오른쪽에서 8번째에 있는 "Ignore wiki formatting"은 위키에서 지원하지 않는 특수부호를 입력가능하게 한다. 예를 들어 이 버튼을 누르면 나타나는 "<nowiki>Insert non-formatted text here</nowiki>"에 한글에서 한자를 복사하여 "<nowiki>數學</nowiki>"와 같이 붙여넣기를 하면 이를 입력할 수 있다.

⑩ 오른쪽에서 두 번째 "Your signature with time stamp" 버튼은 입력을 마친후 누르면 자신의 Username과 수정 시간이 입력된다. 다음과 같이 시각은 미국동부시간(EST)로 나타난다.

⑪ 오른쪽 첫 번째 버튼인 "Horizontal line"은 페이지를 구분할 때 쓴다. 예를 들어 나의 줄이를 다른 사람과 구분하여 나타나게 하고 싶다면 이 버튼을 누르면 다음과 같이 줄로 페이지를 구분할 수 있다.

VITA

Seoung Woo Lee was born in Seoul, South Korea, on January 11, 1967. After graduating Seoul National University in February, 1993, he started his career as a mathematics teacher at one of middle schools in Korea, in March, 1994. He received an M.S. in mathematics education from Seoul National University in February, 2001. He completed a doctoral program in the Department of Mathematics Education at the same graduate school in February, 2003. He published three articles in Korean in the field of mathematics education and coauthored two Korean high school mathematics textbooks. He entered the Mathematics Education Program at The College of Education in the Department of Learning, Teaching, and Curriculum at the University of Missouri at Columbia, in August, 2009. He is currently teaching mathematics at a secondary gifted school in Seoul, Korea.