THE ECLIPSING BINARY
TV CASSIOPEIAE

by

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MAJOR WORK

RESEARCH

IN ASTRONOMY

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In 1911 Astbury¹ announced the variability of TV (45.1911) Cassiopeiae, and soon afterward he derived² the following elements:

$$\text{Primary Minimum} = \text{J.D. } 2419338.33 + 1.8126 \cdot E, \text{ G.M.T.} \\ (\text{Astbury})$$

Yendell³, who observed it in 1911-12, found a range of 0^m.88 at primary minimum with no constant phase, and no trace of a secondary minimum. The observations of Lazzarino⁴ and of VanBiesbroeck⁵ confirm the period and agree in indicating no halt at minimum. Nijland's curve⁶ shows a constant phase at minimum of about 0^d.03, but here again there is no trace of a secondary. McDiarmid⁷ has published a visual curve and an orbit computed from the equations of Russell and Shapley. McDiarmid's curve shows a primary minimum of 1^m.05 with no constant phase, and a secondary of 0^m.09 coming 17 minutes before mid-period. Ellipticity in the stars and "reflection" are indicated by the manner

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1. Astbury, Astronomische Nachrichten, 189, p.357.
 2. Astbury, J.B.A.A., 22, p.279.
 3. Yendell, A.N., 190, p.430; 191, p.427.
 4. Lazzarino, A.N., 197, p.373.
 5. VanBiesbroeck, Annales de l'observatoire royal de Belgique, 13, p.181.
 6. Shapley, Princeton Contributions, No.3, p.132.
 7. Mc Diarmid, Astrophysical Journal, 42, p.412.

in which the light continues to vary outside eclipse. In the solution for the elements it was found that the smaller, fainter star, which is eclipsed at secondary minimum, contributes about 15 per cent of the light of the system.

The present discussion is based on 51 plates taken by Dr. Baker and the writer with the 5-inch camera during the period from July 1914 to October 1915. These plates were exposed about 1.5 mm. outside of focus, and were sensitomitrized and developed after the procedure described in Lows Observatory Bulletin No. 24. All are Seed 27; for plates 251 to 334 the emulsion number is 14540, for 352 to 408 number 14794, for 515 to 606 number 15194. The maximum number of exposures possible without blending of the star images was found to be 12; and, since in a sky of normal transparency 6 minutes was sufficient to give the correct density, most of the plates at and near maximum light were exposed for only 72 minutes. At primary minimum the length of a single exposure was 8 or 10 minutes. A description of the plates is found in Table I. The dates in columns (2) and (3) and the hour angle in column (8) are given for the middle exposure. The epoch, column (4), and the heliocentric phases of the first and last exposures, column (6), are based on Astbury's elements corrected by the addition of $+0.001$ to the epoch. The wedge readings on the film, column (10), are frequently high, so that especial care was necessary in order to avoid error in the use of the reduction curves.

TABLE I.

PLATES OF TV CASSIOPEAE

Plate No.	Date, G.M.T.				Julian Day	Epoch	Correction to Sun	Helio. Phase		Tel.	Hour Angle	No. Exp.	Film
(1)	(2)				(3)	(4)	(5)	(6)		(7)	(8)	(9)	(10)
	1914	d.	h.	m.			days	d.	to d.		h.		
251	July	19	21	00	2420333.8750	549	-0.0005	0.4094	0.4428	W	-1.6	9	18
259		23	20	48	337.8667	551	3	0.7796	0.8150	W	-1.4	8	13
262		24	18	04	338.7528	551	2	1.6581	1.6998	W	-4.2	7	14
266		26	17	03	340.7104	553	1	1.8033	0.0254	W	-5.1	6	12
274		28	20	25	342.8507	554	0	0.3185	0.3643	W	-1.5	9	20
275		28	21	20	342.8889	554	0	0.3692	0.3900	W	-0.6	6	15
281	Aug.	15	15	20	360.6389	564	+0.0011	1.7888	0.0575	W	-5.5	9	13
282		15	17	43	360.7382	564	11	0.0672	0.1422	W	-3.1	12	22
283		16	15	00	361.6250	564	11	0.9679	1.0137	W	-5.8	12	14
284		16	16	15	361.6771	564	11	1.0199	1.0658	W	-4.5	11	15
285		16	17	38	361.7347	564	11	1.0776	1.1234	W	-3.1	12	23
286		16	18	54	361.7875	564	11	1.1304	1.1769	W	-1.8	12	22
291		18	15	25	363.6424	565	12	1.1727	1.2186	W	-5.2	11	16
292		18	16	55	363.7049	565	13	1.2353	1.2812	W	-3.7	12	18
293		18	18	10	363.7569	565	13	1.2874	1.3332	W	-2.4	11	16
294		18	19	32	363.8139	565	13	1.3444	1.3902	W	-1.1	12	18
295		18	21	04	363.8778	565	13	1.4124	1.4499	E	+0.5	10	20
296		20	18	06	365.7542	566	14	1.4687	1.5228	W	-2.4	11	25
310	Sept.	17	21	04	393.8778	582	28	0.5997	0.6372	E	+2.4	10	18
311		17	22	05	393.9201	582	28	0.6421	0.6796	E	+3.5	10	18
320		19	18	12	395.7583	583	29	0.6803	0.6928	W	-0.3	4	clear
321		19	19	14	395.8014	583	29	0.7067	0.7532	E	+0.7	11	19
322		19	20	32	395.8556	583	29	0.7608	0.8067	E	+2.0	12	21
325		20	17	16	396.7194	583	29	1.6285	1.6664	W	-1.2	9	18
326		20	18	11	396.7576	583	29	1.6712	1.7004	W	-0.3	8	12
327		20	19	30	396.8125	583	29	1.7164	1.7664	E	+1.0	10	21
331		22	16	25	398.6840	584	30	1.7698	0.0198	W	-1.9	10	25
332		22	17	56	398.7472	585	30	0.0274	0.0691	W	-0.4	7	21
333		22	19	22	398.8070	585	30	0.0829	0.1406	E	+1.0	10	26
334		22	20	50	398.8680	585	30	0.1482	0.1941	E	+2.5	12	25
352	Oct.	17	15	55	423.6632	598	36	1.3885	1.4135	W	-0.8	7	18
358		17	19	33	423.8146	598	36	1.5316	1.5774	E	+2.9	12	31
359		17	20	55	423.8715	598	36	1.5927	1.6260	E	+4.2	9	26
366		22	13	49	428.5157	601	36	0.8479	0.9035	W	-2.6	11	27
367		22	15	20	428.6389	601	36	0.9111	0.9667	W	-1.0	11	26

TABLE I-Continued
PLATES OF TV CASSIOPEIAE

Plate No.	Date, G.M.T.			Julian Day	Epoch	Correction to Sun	Helioc. Phase		Tel.	Hour Angle	No. Exp.	Film
(1)	(2)			(3)	(4)	(5)	(6)		(7)	(8)	(9)	(10)
	1914	d.	h. m.			days	d.	to d.		h.		
383	Nov.	10	13 41	2420447.5701	611	+0.0035	1.7163	1.7718	W	-1.4	9	28
390	1915	14	13 54	451.5792	614	35	0.3069	0.3278	W	-1.0	6	24
408	Jan.	14	17 17	512.7201	647	+0.0007	1.6168	1.6626	E	+6.4	12	27
515	July	15	17 41	694.7368	748	-0.0008	0.5635	0.6010	W	-5.0	10	19
526	Sept	2	14 31	743.6049	775	+0.0021	0.4943	0.5367	W	-5.0	10	18
527		2	15 49	743.6590	775	21	0.5429	0.5873	W	-3.8	9	19
535		7	14 13	748.5924	778	23	0.0442	0.0775	W	-5.1	7	19
536		7	15 10	748.6319	778	23	0.0838	0.1213	W	-4.0	10	20
538		7	18 07	748.7549	778	23	0.2025	0.2484	W	-1.2	12	20
539		7	19 10	748.7986	778	23	0.2546	0.2796	W	-0.2	7	13
547		11	15 14	752.6347	780	25	0.4615	0.4991	W	-3.6	10	17
559		12	21 23	753.8910	780	26	1.7054	1.7624	E	+2.4	11	26
564		13	19 16	754.8028	781	26	0.8171	0.8546	E	+0.4	10	14
565		13	20 25	754.8507	781	26	0.8609	0.9025	E	+1.5	11	18
605	Oct.	14	16 50	785.7014	798	35	0.9108	0.9274	W	0.0	5	clear
606		14	17 37	785.7340	798	35	0.9351	0.9684	E	+0.7	9	18

TABLE II
COMPARISON STARS FOR TV CASSIOPEIAE

Star	B. D.		R.A. (1900)	Decl. (1900)	Adopted Photographic Magnitude	Harvard	
	No.	Mag.				Visual Mag.	Class
V	+58° 30		^h 0 13.9	+58° 35'	8.18		
a	58 28	8.0	13.8	59 10	8.18		
b	58 24	7.8	12.4	58 30	7.68		
c	58 11	7.3	5.3	59 6	6.54	6.54	A

The three comparison stars (Table II) are close to the variable and in no case farther than 18 mm. from the center of the plate. Thus the corrections for reduction to center and for atmospheric extinction are small. Star c has magnitude 6.54 in Revised Harvard Photometry, and since it is of class A this is adopted as its photographic magnitude. In measuring, four settings of the microphotometer wedge were made on each image. These wedge readings were reduced to Δ magnitudes by means of the corrected reduction curve for the emulsion concerned (see Bulletin 24, Table IX), which was derived by the writer from sensitometer images on plates of this star and RX Herculis. When the plate was fogged, the individual curve for that plate was used up to the point where it joins the clear film curve. These Δ magnitudes were then corrected for atmospheric extinction and reduced to the center of the plate by use of Tables III and IV of Bulletin No. 24. The magnitudes of the comparison stars were determined by combining the differences $c-a$, $c-b$, for an equal number of plates taken on each side of the pier. Using these adopted photographic magnitudes of a , b , and c the differences $a-v$, $b-v$, $c-v$, were converted to magnitudes v_a , v_b , v_c , and the mean of these was taken as the magnitude of the variable for each exposure. After this work was done it was discovered that the normal Laws Observatory scale requires multiplication by the factor 1.15 (see Laws Observatory Bulletin No. 24, Section VI) to bring it into agree-

ment with the Harvard Sequences. This necessitated a revision of the magnitudes for comparison stars and variable. First the Harvard magnitude was reduced to the normal Lays Observatory scale; assuming that the two scales agree at magnitude 6.00, it was found that on our scale the star ϵ would be of magnitude 6.470. Thus all the original magnitudes were too great by 0.^m070. They were converted to the revised scale by use of the formula:

$$\text{Revised magnitude} = [(m - 0.070) - 6.00] \times 1.15 + 6.00$$

where m is the unrevised magnitude.

The revised photographic magnitudes of TV Cassiopeiae, 491 in all, appear in Table III. These points were formed into 51 normal places (Table IV). Numbers 1 to 10 and 41 to 51, which are on the primary minimum, consist of about 6 observations each; for the other normal places 12 observations is the average number. When the normal magnitudes are plotted against phase we have a curve with a secondary minimum about 0.^m08 in depth, and a slightly asymmetrical primary of about 0.^m90; the curve shows the "periastron effect", being noticeably higher before than after secondary minimum; it is concave upward, which shows that the stars are elliptical; and a "reflection effect" is evidenced by a slight increase of light on both sides of the secondary.

The solution which follows is based on the theory de-

TABLE III
PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag			2420	days		mag.
251	1	333.8583	0.4094	7.35	-0.03	274	11	342.8715	0.3601	7.39	.00
	2	.8625	.4136	7.36	-0.02		12	.8757	.3643	7.35	-0.04
	3	.8667	.4178	7.38	.00						
	4	.8708	.4219	7.36	-0.02	275	1	342.8806	0.3692	7.39	.00
	5	.8750	.4261	7.37	-0.01		2	.8847	.3733	7.37	-0.02
	6	.8792	.4303	7.36	-0.02		3	.8889	.3775	7.35	-0.04
	7	.8833	.4344	7.37	-0.01		4	.8931	.3817	7.40	+0.02
	8	.8875	.4386	7.39	+0.01		5	.8972	.3858	7.35	-0.03
	9	.8917	.4428	7.39	+0.01		6	.9014	.3900	7.37	-0.01
259	1	337.8535	0.7796	7.40	-0.04	281	1	360.6125	1.7888	8.17	-0.04
	2	.8576	.7837	7.37	-0.07		2	.6180	.7943	8.29	+0.05
	3	.8618	.7879	7.47	+0.03		3	.6250	.8013	8.25	-0.01
	4	.8667	.7928	7.50	+0.06		4	.6319	.8082	8.27	.00
	5	.8722	.7983	7.45	+0.01		5	.6389	0.0026	8.25	-0.02
	6	.8778	.8039	7.41	-0.03		8	.6729	.0366	8.06	-0.07
	7	.8833	.8094	7.44	.00		9	.6799	.0436	7.99	-0.09
	8	.8889	.8150	7.45	.00		10	.6868	.0505	7.94	-0.10
							11	.6938	.0575	7.99	.00
262	1	338.7319	1.6581	7.45	-0.05						
	2	.7389	.6651	7.54	+0.02	282	1	360.7035	0.0672	7.86	-0.05
	3	.7458	.6720	7.52	-0.01		2	.7104	.0741	7.87	.00
	4	.7528	.6790	7.62	+0.06		3	.7174	.0811	7.78	-0.03
	5	.7597	.6859	7.60	+0.02		4	.7243	.0880	7.70	-0.06
	6	.7667	.6929	7.59	-0.02		5	.7312	.0949	7.67	-0.04
	7	.7736	.6998	7.61	-0.03		6	.7382	.1019	7.67	.00
							7	.7451	.1088	7.60	-0.04
266	1	340.6896	1.8033	8.33	+0.06		8	.7521	.1158	7.59	-0.02
	2	.6965	.8102	8.39	+0.12		9	.7590	.1227	7.56	-0.02
	3	.7035	0.0046	8.29	+0.02		10	.7660	.1297	7.54	-0.01
	4	.7104	.0115	8.25	.00		11	.7729	.1366	7.53	.00
	5	.7174	.0185	8.29	+0.06		12	.7785	.1422	7.48	-0.03
	6	.7243	.0254	8.23	+0.03						
						283	1	361.6042	0.9679	7.36	.00
274	1	342.8299	0.3185	7.41	+0.01		2	.6083	.9720	7.45	.00
	2	.8340	.3226	7.40	.00		3	.6125	.9762	7.40	-0.05
	3	.8382	.3268	7.44	+0.04		4	.6167	.9804	7.38	-0.07
	5	.8465	.3351	7.43	+0.03		5	.6208	.9845	7.41	-0.04
	6	.8507	.3393	7.38	-0.02		6	.6250	.9887	7.36	-0.09
	9	.8632	.3518	7.37	-0.02		7	.6292	.9929	7.44	-0.01
	10	.8674	.3560	7.37	-0.02		8	.6333	.9970	7.47	+0.02

TABLE III - Continued
PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days					2420	days		
283	9	361.6375	1.0012	7.45	.00	286	12	361.8132	1.1769	7.46	+0.4
	10	.6417	.0054	7.41	-.04						
	11	.6458	.0095	7.46	+0.01	291	1	363.6215	1.1727	7.40	-.02
	12	.6500	.0137	7.43	-.02		3	.6299	.1811	7.43	+0.01
							4	.6340	.1852	7.39	-.03
284	1	361.6562	1.0199	7.44	-.01		5	.6382	.1894	7.36	-.06
	2	.6604	.0241	7.45	.00		6	.6424	.1936	7.43	+0.01
	3	.6646	.0283	7.51	+0.06		7	.6465	.1977	7.43	+0.01
	4	.6688	.0325	7.41	-.04		8	.6507	.2019	7.44	+0.02
	5	.6729	.0366	7.41	-.04		9	.6549	.2061	7.43	+0.01
	6	.6771	.0408	7.41	-.04		10	.6590	.2102	7.41	-.01
	7	.6812	.0449	7.51	+0.06		11	.6632	.2144	7.40	-.02
	8	.6854	.0491	7.50	+0.05		12	.6674	.2186	7.39	-.02
	9	.6896	.0533	7.48	+0.03						
	11	.6979	.0616	7.50	+0.06	292	1	363.6840	1.2353	7.47	+0.06
	12	.7021	.0658	7.50	+0.06		2	.6882	.2395	7.50	+0.09
							3	.6924	.2437	7.44	+0.03
285	1	361.7139	1.0776	7.44	.00		4	.6965	.2478	7.43	+0.02
	2	.7180	.0817	7.45	+0.01		5	.7007	.2520	7.44	+0.03
	3	.7222	.0859	7.47	+0.03		6	.7049	.2562	7.45	+0.04
	4	.7264	.0901	7.45	+0.01		7	.7090	.2603	7.50	+0.09
	5	.7306	.0943	7.40	-.04		8	.7132	.2645	7.50	+0.09
	6	.7347	.0984	7.41	-.03		9	.7174	.2687	7.46	+0.05
	7	.7389	.1026	7.40	-.03		10	.7215	.2728	7.47	+0.06
	8	.7430	.1067	7.44	+0.01		11	.7257	.2770	7.39	-.02
	9	.7472	.1109	7.45	+0.02		12	.7299	.2812	7.41	-.00
	10	.7514	.1151	7.44	+0.01						
	11	.7556	.1193	7.41	-.02	293	1	363.7361	1.2874	7.43	+0.2
	12	.7597	.1234	7.39	-.04		2	.7403	.2916	7.44	+0.03
							3	.7444	.2957	7.43	+0.02
286	1	361.7667	1.1304	7.35	-.08		4	.7486	.2999	7.47	+0.06
	2	.7708	.1345	7.41	-.02		5	.7528	.3041	7.41	.00
	3	.7750	.1387	7.43	.00		6	.7569	.3082	7.40	.00
	4	.7792	.1429	7.39	-.04		7	.7611	.3124	7.40	.00
	5	.7833	.1470	7.41	-.02		8	.7653	.3166	7.40	.00
	6	.7875	.1512	7.39	-.03		9	.7694	.3207	7.41	+0.01
	7	.7920	.1557	7.46	+0.04		11	.7778	.3291	7.35	-.05
	8	.7965	.1602	7.45	+0.03		12	.7819	.3332	7.35	-.05
	9	.8007	.1644	7.45	+0.03						
	10	.8049	.1686	7.39	-.03	294	1	363.7931	1.3444	7.40	.00
	11	.8090	.1727	7.38	-.04		2	.7972	.3485	7.41	+0.01

TABLE III - Continued
PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
294	3	363.8017	1.3530	7.37	.00	310	8	393.8903	0.6289	7.38	-.02
	4	.8059	.3572	7.41	+01		9	.8944	.6330	7.40	.00
	5	.8097	.3610	7.39	-.03		10	.8986	.6372	7.40	.00
	6	.8139	.3652	7.40	+01						
	7	.8181	.3694	7.32	-.01	311	1	393.9035	0.6421	7.46	+06
	8	.8222	.3735	7.39	-.10		2	.9076	.6462	7.38	-.02
	9	.8264	.3777	7.39	-.08		3	.9118	.6504	7.47	-.03
	10	.8306	.3819	7.32	-.02		4	.9160	.6546	7.47	+07
	11	.8347	.3860	7.37	-.02		5	.9201	.6587	7.36	-.04
	12	.8389	.3902	7.39	-.02		6	.9243	.6629	7.38	-.02
							7	.9285	.6671	7.38	-.03
295	1	363.8611	1.4124	7.39	-.02		8	.9326	.6712	7.41	.00
	2	.8653	.4166	7.39	-.02		9	.9368	.6754	7.39	-.02
	3	.8694	.4207	7.41	.00		10	.9410	.6796	7.39	-.02
	4	.8736	.4249	7.39	-.02						
	5	.8781	.4294	7.39	-.02	320	1	395.7542	0.6803	7.31	-.10
	6	.8823	.4336	7.44	+02		2	.7583	.6844	7.47	+06
	7	.8861	.4374	7.38	-.04		3	.7625	.6886	7.46	+05
	8	.8903	.4416	7.35	-.07		4	.7667	.6928	7.37	-.04
	9	.8944	.4457	7.40	-.02						
	10	.8986	.4499	7.37	-.05	321	1	395.7806	0.7067	7.44	+02
							2	.7847	.7108	7.37	-.05
296	1	365.7299	1.4687	7.45	+03		3	.7889	.7150	7.51	+09
	2	.7340	.4728	7.44	+01		4	.7931	.7192	7.48	+06
	3	.7382	.4770	7.44	+01		6	.8014	.7275	7.46	+04
	4	.7430	.4818	7.43	.00		7	.8056	.7317	7.45	+03
	5	.7486	.4874	7.43	.00		8	.8101	.7362	7.45	+03
	6	.7542	.4930	7.40	-.03		9	.8146	.7407	7.50	+07
	7	.7597	.4985	7.44	+01		10	.8188	.7449	7.41	-.02
	8	.7653	.5041	7.44	+01		11	.8229	.7490	7.47	+04
	9	.7708	.5096	7.41	-.03		12	.8271	.7532	7.43	.00
	10	.7771	.5159	7.39	-.05						
	11	.7840	.5228	7.41	-.03	322	1	395.8347	0.7608	7.35	-.08
							2	.8389	.7650	7.38	-.05
310	1	393.8611	0.5997	7.31	-.08		3	.8430	.7691	7.44	+01
	2	.8653	.6039	7.36	-.03		4	.8472	.7733	7.35	-.08
	3	.8694	.6080	7.41	+02		5	.8514	.7775	7.50	+06
	4	.8736	.6122	7.43	+04		6	.8556	.7817	7.46	+02
	5	.8778	.6164	7.37	-.02		7	.8597	.7858	7.51	+07
	6	.8819	.6205	7.33	-.06		8	.8639	.7900	7.43	-.01
	7	.8861	.6247	7.43	+03		9	.8680	.7941	7.46	+02

TABLE III - Continued
PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days					2420	days		
322	10	395.8722	0.7983	7.44	.00	331	8	398.7049	0.0059	8.22	-.05
	11	.8764	.8025	7.51	+0.7		9	.7118	.0128	8.28	+0.3
	12	.8806	.8067	7.43	-.01		10	.7188	.0198	8.22	.00
325	1	396.7024	1.6285	7.39	-.07	332	1	398.7264	0.0274	8.21	+0.2
	3	.7111	.6372	7.40	-.07		2	.7333	.0343	8.16	+0.1
	4	.7153	.6414	7.40	-.07		3	.7403	.0413	8.08	-.01
	5	.7194	.6455	7.43	-.05		4	.7472	.0482	8.04	-.01
	6	.7236	.6497	7.41	-.07		5	.7542	.0552	7.96	-.04
	7	.7278	.6539	7.40	-.09		6	.7611	.0621	7.97	+0.2
	8	.7319	.6580	7.48	-.02		7	.7681	.0691	7.87	-.03
	9	.7361	.6622	7.45	-.06						
	10	.7403	.6664	7.45	-.07	333	1	398.7819	0.0829	7.76	-.03
326	1	396.7451	1.6712	7.54	.00		2	.7903	.0913	7.72	-.01
	2	.7493	.6754	7.55	.00		3	.7962	.0972	7.72	+0.2
	3	.7535	.6796	7.53	-.03		4	.8014	.1024	7.58	-.09
	4	.7576	.6837	7.59	+0.1		5	.8070	.1080	7.58	-.06
	5	.7618	.6879	7.54	-.05		6	.8167	.1177	7.60	.00
	6	.7660	.6921	7.58	-.03		7	.8222	.1232	7.60	+0.2
	7	.7701	.6962	7.64	+0.2		8	.8278	.1288	7.53	-.03
	8	.7743	.7004	7.59	-.05		9	.8333	.1343	7.58	+0.4
							10	.8396	.1406	7.46	-.05
327	1	396.7903	1.7164	7.72	.00	334	1	398.8472	0.1482	7.60	+0.10
	2	.7958	.7209	7.85	+0.9		2	.8514	.1524	7.59	+0.10
	3	.8014	.7275	7.82	+0.2		3	.8556	.1566	7.60	+0.12
	4	.8070	.7331	7.83	.00		4	.8597	.1607	7.55	+0.8
	5	.8125	.7386	7.94	+0.6		5	.8639	.1649	7.53	+0.6
	6	.8181	.7442	7.87	-.05		6	.8680	.1690	7.45	-.01
	7	.8236	.7497	7.96	.00		7	.8722	.1732	7.48	+0.2
	8	.8292	.7553	8.05	+0.4		8	.8764	.1774	7.41	-.05
	9	.8347	.7608	8.04	.00		9	.8806	.1816	7.46	+0.1
	10	.8403	.7664	8.07	-.01		10	.8847	.1857	7.48	+0.3
							11	.8889	.1899	7.41	-.04
331	1	398.6562	1.7698	8.13	+0.2		12	.8931	.1941	7.52	+0.7
	2	.6632	.7768	8.08	-.07						
	3	.6705	.7841	8.16	-.04	352	1	423.6507	1.3885	7.38	-.03
	4	.6774	.7910	8.17	-.05		2	.6549	.3927	7.45	+0.4
	5	.6840	.7976	8.21	-.04		3	.6590	.3968	7.41	.00
	6	.6910	.8046	8.21	-.06		4	.6632	.4010	7.43	+0.2
	7	.6979	.8115	8.30	+0.3		5	.6674	.4052	7.52	+0.11

TABLE III - Continued
 PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
352	6	423.6715	1.4093	7.38	+0.04	367	4	428.6278	0.9278	7.46	-0.02
	7	.6757	4135	7.46	+0.05		5	.6333	.9333	7.58	+0.02
							6	.6389	.9389	7.48	+0.02
358	1	423.7938	1.5316	7.48	+0.04		7	.6444	.9444	7.51	.00
	2	.7979	.5357	7.46	+0.02		8	.6500	.9500	7.48	+0.12
	3	.8021	.5399	7.50	+0.06		9	.6556	.9556	7.48	+0.02
	4	.8062	.5440	7.50	+0.05		10	.6611	.9611	7.41	+0.05
	5	.8104	.5482	7.52	+0.07		11	.6667	.9667	7.41	+0.02
	6	.8146	.5524	7.46	+0.01						
	7	.8188	.5566	7.46	+0.01	383	1	447.5424	1.7163	7.69	-0.03
	8	.8229	.5607	7.44	-0.01		4	.5590	.7329	7.85	+0.01
	9	.8271	.5649	7.45	.00		5	.5646	.7385	7.87	.00
	10	.8312	.5690	7.36	-0.09		6	.5701	.7440	7.92	.00
	11	.8354	.5732	7.39	-0.06		7	.5757	.7496	7.92	-0.04
	12	.8396	.5774	7.46	+0.01		8	.5812	.7551	8.08	+0.08
							9	.5868	.7607	8.07	+0.02
359	1	423.8549	1.5927	7.48	+0.02		10	.5924	.7663	8.10	+0.01
	2	.8590	.5968	7.52	+0.06		11	.5979	.7718	8.20	+0.07
	3	.8632	.6010	7.44	-0.02						
	4	.8674	.6052	7.48	+0.02	390	5	451.5708	0.3069	7.44	+0.03
	5	.8715	.6093	7.46	.00		6	.5750	.3111	7.44	+0.03
	6	.8757	.6135	7.44	-0.02		7	.5792	.3153	7.45	+0.04
	7	.8799	.6177	7.55	+0.09		8	.5833	.3194	7.45	+0.05
	8	.8840	.6218	7.39	-0.07		9	.5875	.3236	7.40	.00
	9	.8882	.6260	7.44	-0.02		10	.5917	.3278	7.43	+0.03
366	1	428.5479	2.08479	7.37	-0.08	408	1	512.6973	1.6168	7.48	+0.02
	2	.5535	.8535	7.43	-0.02		2	.7035	.6210	7.48	+0.02
	3	.5590	.8590	7.38	-0.07		3	.7076	.6251	7.46	.00
	4	.5646	.8646	7.39	-0.06		4	.7118	.6293	7.50	+0.04
	5	.5701	.8701	7.39	-0.06		5	.7160	.6335	7.47	.00
	6	.5757	.8757	7.43	-0.02		6	.7201	.6376	7.55	+0.08
	7	.5812	.8812	7.43	-0.03		7	.7243	.6418	7.51	+0.04
	8	.5868	.8868	7.47	+0.01		8	.7285	.6460	7.53	+0.05
	9	.5924	.8924	7.44	-0.02		9	.7326	.6501	7.53	+0.05
	10	.5979	.8979	7.51	+0.05		10	.7368	.6543	7.58	+0.09
	11	.6035	.9035	7.46	.00		11	.7410	.6585	7.58	+0.08
							12	.7451	.6626	7.68	+0.07
367	1	428.6111	0.9111	7.44	-0.02						
	2	.6167	.9167	7.48	+0.02	515	1	694.7201	0.5635	7.33	-0.05
	3	.6222	.9222	7.48	+0.02		2	.7243	.5677	7.28	-0.10

TABLE III - Continued
 PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	O-C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
515	3	694.7285	0.5819	7.33	-0.5	536	4	748.6278	0.0963	7.76	+0.5
	4	.7326	.5860	7.31	-0.7		5	.6319	.1004	7.72	+0.3
	5	.7368	.5812	7.36	-0.3		6	.6361	.1046	7.69	+0.3
	6	.7410	.5844	7.38	-0.1		7	.6403	.1088	7.67	+0.3
	7	.7451	.5865	7.33	-0.6		8	.6444	.1129	7.62	.00
	8	.7493	.5827	7.38	-0.1		9	.6486	.1171	7.59	-0.1
	9	.7535	.5869	7.37	-0.2		10	.6528	.1213	7.67	+0.9
	10	.7576	.6010	7.30	-0.9						
						538	1	748.7340	0.2025	7.43	-0.1
526	1	743.5882	0.4943	7.35	-0.3		2	.7382	.2067	7.41	-0.3
	2	.5924	.4985	7.36	-0.2		3	.7424	.2109	7.43	-0.1
	3	.5965	.5026	7.39	+0.1		4	.7465	.2150	7.44	.00
	4	.6007	.5068	7.36	-0.2		5	.7507	.2192	7.44	.00
	5	.6049	.5110	7.44	+0.6		6	.7549	.2234	7.45	+0.2
	6	.6090	.5151	7.44	+0.6		7	.7590	.2275	7.47	+0.4
	7	.6139	.5200	7.45	+0.7		8	.7632	.2317	7.44	+0.1
	8	.6194	.5255	7.46	+0.8		9	.7674	.2359	7.48	+0.5
	9	.6250	.5311	7.50	+1.2		10	.7715	.2400	7.41	-0.2
	10	.6306	.5367	7.50	+1.2		11	.7757	.2442	7.46	+0.3
							12	.7799	.2484	7.43	.00
527	1	743.6368	0.5429	7.33	-0.5						
	2	.6424	.5485	7.31	-0.7	539	1	748.7861	0.2546	7.39	-0.3
	3	.6479	.5540	7.38	.00		2	.7903	.2588	7.38	-0.4
	4	.6535	.5596	7.35	-0.3		3	.7944	.2629	7.39	-0.3
	5	.6590	.5651	7.35	-0.3		4	.7986	.2671	7.40	-0.2
	6	.6646	.5707	7.36	-0.2		5	.8028	.2713	7.40	-0.2
	7	.6701	.5762	7.36	-0.2		6	.8070	.2755	7.43	+0.1
	8	.6757	.5818	7.37	-0.2		7	.8111	.2796	7.40	-0.2
	9	.6812	.5873	7.38	-0.1						
						547	1	752.6180	0.4615	7.38	.00
535	1	748.5757	0.0442	8.08	.00		2	.6222	.4657	7.36	-0.2
	2	.5812	.0497	8.08	+0.4		3	.6264	.4699	7.39	+0.1
	3	.5868	.0553	8.04	+0.4		4	.6306	.4741	7.41	+0.3
	4	.5924	.0609	8.01	+0.5		5	.6347	.4782	7.40	+0.2
	5	.5979	.0664	7.98	+0.5		6	.6389	.4824	7.39	+0.1
	6	.6035	.0720	7.93	+0.5		7	.6431	.4866	7.40	+0.2
	7	.6090	.0775	7.92	+0.8		8	.6472	.4907	7.41	+0.3
							9	.6514	.4949	7.44	+0.6
536	1	748.6153	0.0838	7.81	+0.1		10	.6556	.4991	7.44	+0.6
	2	.6194	.0879	7.77	.00						
	3	.6236	.0921	7.75	+0.2	559	1	753.8618	1.7054	7.67	+0.1

TABLE III- Continued
 PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	D-C (darkened)	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	Photg. Mag.	D-C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
559	2	753.8674	1.7110	7.74	+0.5	565	4	754.8424	0.8734	7.51	+0.6
	3	.8736	.7172	7.75	+0.2		5	.8465	.8775	7.48	+0.2
	4	.8799	.7235	7.72	-0.5		6	.8507	.8817	7.47	+0.1
	5	.8854	.7290	7.76	-0.4		7	.8549	.8859	7.48	+0.2
	6	.8910	.7346	7.84	-0.1		8	.8590	.8900	7.44	-0.2
	7	.8965	.7401	7.85	-0.4		9	.8632	.8942	7.45	-0.1
	8	.9021	.7457	7.92	-0.2		10	.8674	.8984	7.47	+0.1
	9	.9076	.7512	7.90	-0.8		11	.8715	.9025	7.43	-0.3
	10	.9132	.7568	7.98	-0.4						
	11	.9188	.7624	8.00	-0.6	605	1	785.6931	0.9108	7.46	.00
							2	.6972	.9149	7.44	-0.2
564	1	754.7861	0.8171	7.44	-0.1		3	.7014	.9191	7.46	.00
	2	.7903	.8213	7.43	-0.2		4	.7056	.9233	7.45	-0.1
	3	.7944	.8254	7.48	+0.3		5	.7097	.9274	7.50	+0.4
	4	.7986	.8296	7.46	+0.1						
	5	.8028	.8338	7.44	-0.1	606	1	785.7174	0.9351	7.54	+0.8
	6	.8070	.8380	7.39	-0.6		2	.7215	.9392	7.52	+0.6
	7	.8111	.8421	7.41	-0.4		3	.7257	.9434	7.55	+0.9
	8	.8153	.8463	7.41	-0.4		4	.7299	.9476	7.53	+0.7
	9	.8194	.8504	7.40	-0.5		5	.7340	.9517	7.46	.00
	10	.8236	.8546	7.40	-0.5		6	.7382	.9559	7.44	-0.2
							7	.7424	.9601	7.45	-0.1
565	1	754.8299	0.8609	7.48	+0.3		8	.7465	.9642	7.46	.00
	2	.8340	.8650	7.54	+0.9		9	.7507	.9684	7.37	-0.9
	3	.8382	.8692	7.51	+0.6						

TABLE IV
NORMAL MAGNITUDES OF TV CASSIOPEAE

No.	Phase	θ	No. Obs.	Observed Magnitude	Observed Intensity	Rectified Intensity	Computed Magnitude (uniform)	D-C (uniform)	Computed Magnitude (darkened)	D-C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	days									
1	0.0093	1.85	4	8.263	0.448	0.505	8.263	.000	8.263	-0.00
2	.0287	5.70	5	8.176	0.485	0.542	8.179	-.003	8.184	-.008
3	.0462	9.18	6	8.036	0.552	0.611	8.063	-.027	8.072	-.036
4	.0607	12.06	7	7.971	0.586	0.646	7.964	+.007	7.971	.000
5	.0748	14.86	5	7.877	0.639	0.700	7.861	+.016	7.867	+.010
6	.0877	17.42	6	7.751	0.718	0.780	7.772	-.021	7.769	-.018
7	.0997	19.80	7	7.687	0.761	0.825	7.700	-.013	7.694	-.007
8	.1138	22.60	8	7.612	0.816	0.879	7.621	-.009	7.621	-.009
9	.1340	26.61	9	7.542	0.870	0.930	7.532	+.010	7.544	-.002
10	.1649	32.75	7	7.517	0.890	0.943	7.456	+.061	7.465	+.052
11	.1983	39.38	8	7.448	0.949	0.999	7.447	+.001		
12	.2428	48.22	12	7.428	0.966	1.001	7.429	-.001		
13	.3082	61.21	12	7.424	0.970	0.984	7.407	+.017		
14	.3653	72.55	12	7.375	1.005	1.012	7.388	-.013		
15	.4360	84.59	12	7.371	1.018	1.007	7.379	-.008		
16	.4933	97.97	12	7.400	0.992	0.979	7.377	+.023		
17	.5441	108.06	12	7.389	1.002	0.991	7.379	+.010		
18	.5824	115.67	11	7.357	1.033	1.028	7.387	-.030		
19	.6190	122.94	12	7.382	1.008	1.012	7.395	-.013		
20	.6683	132.73	12	7.406	0.986	0.999	7.405	+.001		
21	.7226	143.52	10	7.443	0.953	0.980	7.421	+.022		
22	.7693	152.79	10	7.413	0.980	1.017	7.431	-.018		
23	.7960	158.09	10	7.459	0.939	0.984	7.442	+.017		
24	.8257	163.99	9	7.438	0.958	1.008	7.447	-.009		
25	.8571	170.23	10	7.432	0.963	1.017	7.450	-.018		
26	.8850	175.77	13	7.458	0.940	0.998	7.456	+.002		
27	.9177	182.26	12	7.470	0.930	0.990	7.459	+.011		
28	.9531	189.30	16	7.466	0.933	0.992	7.457	+.009		
29	.9996	198.53	14	7.433	0.962	1.017	7.451	-.018		
30	1.0514	208.82	9	7.462	0.937	0.986	7.447	+.015		
31	1.026	218.99	11	7.429	0.966	1.006	7.435	-.006		
32	.1532	229.04	12	7.410	0.983	1.014	7.425	-.015		
33	.2038	239.09	13	7.425	0.969	0.993	7.417	+.008		
34	.2691	252.06	13	7.443	0.953	0.968	7.408	+.035		
35	.3273	263.62	12	7.401	0.991	1.005	7.406	-.005		

TABLE IV - Continued
NORMAL MAGNITUDES OF TV CASSIOPEIAE

No.	Phase	θ	No Obs.	Observed Magnitude	Observed Intensity	Rectified Intensity	Computed Magnitude (uniform)	O-C (uniform)	Computed Magnitude (darkened)	O-C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	days									
36	1.3838	274.84	13	7.389	1.002	1.015	7.405	-.016	7.470	
37	.4279	283.60	12	7.402	0.990	1.010	7.413	-.011		
38	.4999	297.90	13	7.432	0.963	1.000	7.432	.000		
39	.5646	310.75	12	7.460	0.938	0.992	7.451	+.009		
40	.6179	321.33	12	7.459	0.939	1.006	7.465	-.006		
41	.6404	325.80	7	7.471	0.929	0.996	7.467	+.004	7.470	+.001
42	.6547	328.64	7	7.492	0.911	0.981	7.484	+.008	7.493	-.001
43	.6692	331.52	8	7.531	0.879	0.949	7.516	+.015	7.529	+.002
44	.6898	335.61	7	7.580	0.840	0.912	7.590	-.010	7.593	-.013
45	.7095	339.52	7	7.680	0.766	0.838	7.691	-.011	7.686	-.006
46	.7278	343.16	6	7.806	0.682	0.751	7.801	+.005	7.800	+.006
47	.7400	345.58	6	7.884	0.635	0.701	7.884	.000	7.887	-.003
48	.7519	347.94	7	7.971	0.586	0.652	7.972	-.001	7.980	-.009
49	.7644	350.43	6	8.068	0.536	0.598	8.060	+.008	8.068	.000
50	.7845	354.42	6	8.179	0.484	0.543	8.188	-.009	8.193	-.014
51	.8052	358.53	7	8.280	0.441	0.497	8.268	+.012	8.268	+.012

veloped at Princeton by Russell and Shapley⁸. Reference to formulae and tables contained in their papers will be indicated by using red ink. First the normal magnitudes were changed into intensities by use of the formula:

$$\log l = 0.4 (m_0 - m)$$

where m is the normal magnitude and m_0 is the magnitude of the system during the intervals between eclipses. The preliminary value of m_0 was taken to be 7.357. The resulting values of l are expressed in terms of $L_1 + L_2$, the total light of the system. In Table IV, column (6), appear the observed normal intensities read from Table A; but for these the final value of m_0 was used. The preliminary values of the normal intensities on the primary minimum were plotted against the phases, computed by using Astbury's elements. The resulting curve showed the necessity for a correction to the epoch of +0.001, a correction which was not evident from a curve drawn through the normal magnitudes. The adopted elements which were used in computing the phases in column (2), are:

$$\text{Prim. Min.} = \text{J.D. } 2419338.331 + 1^d.8126 \cdot E, \text{ G.M.T.} \\ \text{(Corrected elements)}$$

It is assumed that the intensities outside of primary

8. Russell, Astrophysical Journal, 35, p.315; 36, p.54;
Russell and Shapley, Astrophysical Journal, 36, p.239;
Shapley, Princeton Contributions, No.3.

minimum can be represented by the formula:

$$I = a - b \cos \theta - c \cos^2 \theta - f \sin \theta - nd$$

where θ is the longitude from principal conjunction; a is the intensity of the system aside from the "reflection", ellipticity, and "periastron" effects and eclipse of the fainter star; b is half the amount by which that side of the fainter star turned toward the bright star exceeds the side turned away; $2c = z = \epsilon^2 \sin^2 i$, ϵ being the eccentricity of a meridian section of the prolate spheroids; f is the maximum amount by which the curve is raised midway between primary and secondary due to a "periastron" effect; d is the depth of secondary minimum, and n is the loss of light at the corresponding phase on the primary minimum, in terms of the loss at maximum obscuration. A subsequent change in the interpretation of the term $b \cos \theta$ is explained later.

The next step was to form an equation of condition for each of the normal places outside eclipse, and to solve these for the constants a , b , c , f , and d . Preliminary values of the constants were found as follows. When I^2 was plotted against $\cos^2 \theta$ the slope, $z = 2c$, of the resulting straight line was 0.115; using this preliminary value of z the intensities were rectified by adding to each I the quantity $+ c \cos^2 \theta$. When the resulting intensities were plotted the secondary entirely disappeared, showing that this drop is due to ellipticity

only. Hence we conclude that the light of the fainter star must be practically zero. The points were now examined to determine preliminary values of the other constants. To find b , comparison was made of l for large values of $\cos \theta$ equal but opposite in sign; for f , comparison was made of l for large values of $\sin \theta$; for a , terms in b , c , and f were combined with l , after changing signs. Since the curve when rectified with the preliminary z showed no secondary minimum the consideration of d was at first omitted in the least-squares solution. The equations of condition are of the form:

$$\delta a - \cos \theta \cdot \delta b - \cos^2 \theta \cdot \delta c - \sin \theta \cdot \delta f = l_o - l_c$$

When weight unity is assigned to each observation outside eclipse the following normal equations result:

$$\begin{aligned} + 30.000 \delta a + 9.348 \delta b - 13.978 \delta c - 1.161 \delta f &= + 0.009 \\ + 13.978 \delta b - 8.083 \delta c - 0.755 \delta f &= - 0.002 \\ + 10.108 \delta c + 0.562 \delta f &= + 0.033 \\ + 15.969 \delta f &= - 0.022 \end{aligned}$$

The solution gives:

	Preliminary	Correction	Final	Probable Error	Weights
$a =$	+ 0.963	+ 0.0060	+ 0.969	± 0.0031	10.212
$b =$	+ 0.0115	+ 0.0048	+ 0.016	± 0.0037	7.192
$c =$	+ 0.0575	+ 0.0156	+ 0.073	± 0.0064	2.339
$f =$	- 0.0115	- 0.0013	- 0.013	± 0.0025	15.905
$m_1 =$	7.357	+ 0.034	7.391	± 0.0034	

To get the correction to m_0 , the difference between the final a and unity was converted into magnitude. The final m_0 is the magnitude of the system between eclipses, when the effect of ellipticity and the "reflection" and "periastron" effects have been removed.

Now when the curve at primary minimum is solved for the remaining elements of the system it appears that the two stars are very little different in size, the larger star being the fainter. Here arises a difficulty concerning b , the "reflection effect" constant, and d , the depth of secondary. For if reflection from the larger star constitutes, in the maximum, 0.032 of the light of the system, then when this reflecting surface is eclipsed at secondary minimum the curve should exhibit a diminution in light from this cause even after the effect of ellipticity is removed. Then too, the visual curve and orbit by McDiarmid indicate that the rectified depth of secondary is 0.080 and that the fainter star, which is his solution is also the smaller, contributes about 0.15 of the light of the system. If the depths of the visual minima are correct, the observed decrease (0.5) in photographic intensity should be accompanied by a secondary minimum about a tenth of a magnitude in depth. For these reasons a second least-squares solution was made which included the nd terms. The function $1 + b \cos \theta$ was plotted against $\sin \theta$, and the slope (0.014) of the resulting straight line was taken as the preliminary value of d . When the normal

equations were solved the result was: $\delta d = -0.013$. This is taken as conclusive evidence that no secondary exists on the rectified curve, and the first least-squares solution is adopted as definitive. However, the first question remains: if $b = 0.016$ represents the "reflection" constant, why is there no drop in intensity when the reflecting surface is eclipsed? It appears that in this case $b \cos \theta$ cannot be interpreted as a reflection effect but must be regarded as merely a cosine oscillation in the curve. A further comparison with McDiarmid's results will be made later.

The remaining elements of the system were determined from the light curve at primary minimum. The first step was to rectify the normal intensities by means of the formula:

$$R = \frac{l + b(1 + \cos \theta) + c \cdot l \cdot \cos^2 \theta + f \sin \theta}{2 + b}$$

using the final values of the constants. The values of the rectified intensity, R , for all the points both on and outside minimum are given in column (7) of Table IV. Two solutions for the elements were made on two hypotheses: 1st, that the disks are uniformly luminous; 2nd, that they are uniformly darkened toward the limb. The uniform solution follows.

The rectified intensities, R , of the points on the primary minimum were plotted against the corrected phases; the points on the downslope were reflected onto the upslope

and through all a smooth curve was drawn. From this curve the phases, t , were read for 11 fractions, n , of the maximum loss of light, $1 - \lambda_1$. Then the value of the χ function of k and α_0 was obtained by use of formula g':

$$\frac{\sin^2 \theta(n)}{1 - z \cos^2 \theta(n)} = C \omega_1(n) + D \omega_2(n), \quad \frac{C}{D} = \chi(k, \alpha_0, t)$$

The functions $\omega_1(n)$ and $\omega_2(n)$ are taken from Table IIIa; k is the ratio of the radii of the two stars, α_0 is the maximum obscuration of the brighter star, and the χ function is defined by the equation:

$$\chi(k, \alpha_0, n) = \frac{\sin^2 \theta(n) \{1 - z \cos^2 \theta(\frac{1}{2})\}}{\sin^2 \theta(\frac{1}{2}) \{1 - z \cos^2 \theta(n)\}}$$

Now for TV Cassiopeiae most of the formulae assume a simplified form, for (assuming that the dark star is the larger) both L_1 and $1 - \lambda_1$ are zero, and $1 - \lambda_1 = \alpha_0$ can be read at once from the curve. Hence the problem reduces to the finding of a suitable value for k . The preliminary trials for $\frac{C}{D}$ proved at once that the larger star is in front at primary minimum. Consider the equation

$$\alpha_0 = 1 - \lambda_1 + \frac{1 - \lambda_2}{k^2}$$

Suppose that the dark star is the smaller. Then $1 - \lambda_1 = 0$, $1 - \lambda_2 = 0.5$, and $\alpha_0 = \frac{1 - \lambda_2}{k^2}$. Since α_0 must be equal to or less than 1, k^2 cannot be less than $\sqrt{0.5}$. Thus the limiting values for k are 1.00 and 0.71. But when trials were made assuming various values between these limits it was found that the resulting values of the χ

functions were much too great for agreement with the observed curve. This confirms the assumption that $1 - \lambda_1 = \alpha_0$, and that $L_1 = 0 = 1 - \lambda_2$. After numerous trials it was found that the value $k = 0.96$ most nearly satisfies equation g' , but a satisfactory agreement with the observed curve was not obtained by using the ω functions. Therefore a curve was computed using the formula

$$\Psi(k, \alpha_0) = \frac{\sin^2 \theta - A}{B}$$

Given $k = 0.96$ and $\alpha_0 = 0.50$, a satisfactory curve is obtained by letting $A = 0.0211$, $B = 0.0407$. Next the ordinates of this curve were converted from intensities to magnitudes and these were converted from rectified to natural magnitudes; the ordinates for values of t corresponding to those for the normal places appear in column (8), Table IV, and the residuals of the observed points from this curve appear in column (9). Using the above values of k and α_0 , the elements i and a_f were found from formula h' :

$$\begin{aligned} \cos^2 i \cos^2 \theta' + \sin^2 \theta' &= a_f^2 (1 - 2 \cos^2 \theta') (1 + k)^2 \\ \cos^2 i &= a_f^2 (1 - 2) [1 + k p(k, \alpha_0)]^2 \end{aligned}$$

where θ' corresponds to the end of eclipse, a_f is the maximum radius of the dark star, and the function $p(k, \alpha_0)$ is taken from Table I. After a_f is found the radii b_f and c_f are obtained by using the formulae:

$$b^2 = a^2 (1 - \epsilon^2), \quad \frac{c^2}{a^2} = \frac{1 - 3.4 a^2}{1 + 8.6 a^2}$$

Multiplying the resulting values by k we find a_b , b_b , c_b . If we assume equal masses, the densities of the two stars are obtained by use of the formula:

$$\rho = \frac{0.00672}{P^2 abc}$$

where P is the period in days. The absolute magnitude of the system, the hypothetical parallax, and the distance in light years are computed from formulae given in Princeton Contributions, No.3. The list of elements on the uniform hypothesis appear in Table V, column (3). The unit of length is the radius of the relative orbit, and the unit of light is the total light of the system, $L_b + L_s = L_b$.

For the darkened solution the procedure was somewhat different. The rectified curve was slightly altered, being widened and brought down at the top, while the bottom was left unchanged. The functions $\sin^2 \theta$ and $1 - z_d \cos^2 \theta$ were computed for values of θ corresponding to $n = 0, \frac{1}{4}, \frac{1}{2}$, and $\frac{3}{4}$, using $z_d = 0.091 = \frac{5}{55} z$. A series of approximations were made, to get χ functions which would give the most accordant values of k . The final solution gave $k = 0.82$. Using this value a curve was computed; the ordinates were changed from intensities to rectified magnitudes, then to natural magnitudes, and the residuals were taken as before. The ordinates of the computed darkened curve appear in column (10) of Table IV;

TABLE V.
ELEMENTS OF THE SYSTEM OF TV CASSIOPEIAE

		Uniform	Darkened
Period of revolution	T	1.8126 days	
Daily motion in longitude	μ	198°.610	
Epoch of primary minimum	T	J. D. 2419338.331 G.M.T.	
Semi-duration of eclipse		0.1662 days	0.1751 days
Inclination of orbit	i	76°.2	72°.2
Least apparent distance of centers	$\cos i$	0.238	0.306
Ratio of radii	K	0.96	0.82
Ratio of major and minor axes	$\frac{a}{b}$	1.087	1.055
Semi-major axis, bright star	a_b	0.300	0.290
Semi-minor axis, bright star	b_b	0.276	0.275
Semi-major axis, dark star	a_f	0.313	0.354
Semi-minor axis, dark star	b_f	0.287	0.335
Density of bright star	ρ_b	0.10	0.10
Density of dark star	ρ_f	0.09	0.06
Maximum obscuration during eclipse	α_0	0.50	0.50
"Radiation effect" constant	b	0.016	0.016
Light of bright star	L_b	1.000	1.000
Light of dark star	L_f	0.000	0.000
Magnitude of system		7.391	7.391
Magnitude at primary minimum		8.273	8.273
Magnitude at secondary minimum		7.459	7.459
Range of magnitude, primary		0.897	0.897
Range of magnitude, secondary		0.083	0.083
Absolute magnitude of bright star	M	-0.2	-0.1
Hypothetical parallax	π	0.0031	0.0031
Distance in light years		1052	1052

the residuals, in column (11), As before, formula h' was employed to obtain i and a_1 , the only difference being that z_1 was used in place of z , and the function $p(k, a_1)$ was this time taken from Table IX. From the new values of the radii of the two stars their densities and the parallax of the system were again computed, using the same formulae as before. The elements of the system on the hypothesis of darkened disks are found in Table V.

This table of photographic elements can be compared directly with the visual elements in *Astrophysical Journal* 42, p.432. The differences are striking. In the visual elements the fainter star is the smaller, and it contributes 14 per cent to the light of the system on the uniform and 16 per cent on the darkened hypothesis. This fundamental difference introduces many others, so that a detailed comparison is useless. However the inclination of the orbit is found to be different by only $3\frac{1}{2}$ degrees (uniform) and 2 degrees (darkened), the greater inclination being derived from the visual curve. When the investigation of this star began it was hoped that the comparison of the depths of the visual and photographic minima would give evidence on the question of a difference of spectral type for the two stars. However, since the photographic minima are in both cases shallower than the visual, no conclusions can be drawn.

Figure 3 shows the theoretical light curve and the

FIGURE 1. RECTIFIED NORMAL MAGNITUDES AND LIGHT CURVE OF TV CASSIOPEIAE DURING PRIMARY MINIMUM (DARKENED SOLUTION)

FIGURE 2. NORMAL MAGNITUDES AND THEORETICAL LIGHT CURVE OF TV CASSIOPEIAE

FIGURE 3. OBSERVED MAGNITUDES AND THEORETICAL LIGHT CURVE

individual observations on which it is based. The curve appears again in Figure 2 , with the normal magnitudes. Figure 1 gives the rectified curve and normal magnitudes during primary minimum. In all these figures the computed curve is derived on the darkened hypothesis; this solution is regarded as definitive, since the residuals of the normal places are slightly less for the darkened than for the uniform curve. For the curve during eclipse the sum of the squares of the normal magnitude residuals is 0.002675 (uniform) and 0.002595 (darkened); the resulting probable errors of a single normal place are respectively ± 0.012 and ± 0.011 . If the tenth normal place is neglected, the probable error is ± 0.008 on both hypotheses. For the portion outside eclipse the curve is the same on both hypotheses. Here the sum of the squares of the residuals is 0.006549 , and the probable error is ± 0.010 . For the entire curve on the darkened solution the probable error of a single observation is ± 0.029 .

The present photographic light curve and orbit of TV Cassiopeiae are based on 491 images of the variable on 51 plates taken with the 5-inch camera of the Laws Observatory. The system is composed of two ellipsoidal stars which revolve in an orbit inclined 72° to the plane of the sky; the diameter of the relative orbit is three times the major axis of the bright star. The fainter star,

FIGURE 4. THE SYSTEM OF TV CASSIOPEIAE AT PRIMARY MINIMUM AND AT GREATEST ELONGATION

which is slightly the larger of the two, apparently gives no light; at principal conjunction it occults one half the disk of its bright companion. Eccentricity of the orbit is shown by a "periastron effect", but data are lacking for determining its amount. A peculiarity of this system is that the "reflection" or "radiation effect" cannot be interpreted in the usual way. The probable error of a single magnitude determination is ± 0.029 .

1917 February 15

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May 9, 1917.

Dean of the Graduate School,
University of Missouri.

Dear Sir,

Miss Edith E. Cummings' thesis, entitled
The Eclipsing Binary TV Cassiopeiae and submitted
in accordance with the requirements for the Master's
degree, has my approval. It represents an investiga-
tion that has taken a great amount of time and gives
results with a degree of accuracy seldom realized
in stellar photometry. It will be published soon
as Laws Observatory Bulletin No.27.

Very truly yours,

Professor of Astronomy.

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