May 17, 1916.

My dear Professor Hedrick:-

It is customary for the Graduate Committee to refer dissertations, submitted by candidates for the degree of Master of Arts, to some member of the Group who is not connected with the Department in which the candidate's work has been done. I am sending you herewith a dissertation which has been submitted by J. L. Weatherwax.

I shall be greatly obliged if you will kindly examine the same at your earliest convenience and report to us for the Graduate Committee whether in your opinion the dissertation meets the general standard which has been established in this University for the Master's dissertation.

Very truly yours,

A handwritten signature in cursive script, reading "Walter Miller". The signature is written in dark ink and is positioned above the printed name of the signatory.

Chairman, Graduate Committee.

INVESTIGATION OF THE STRETCH MODULI

by

James Lloyd Weatherwax, A. B.

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INVESTIGATION OF THE STRETCH MODULI

In 1678 Robert Hooke published a paper from which the following is quoted: "About two years since I printed this Theory in an Anagram at the end of my Book of the Descriptions of Helioscopes, viz. Ut Tensio sic vis: That is the power of any spring is in the same proportion with the tension thereof. From all, which it is very evident, the Rule or Law of Nature in every springing body is that the force or power thereof to restore itself to its natural position is always proportionate to the distance or space it is removed therefrom, whether it be by rarefaction, or separation of its parts, the one from the other, or by condensation, or crowding the parts nearer together".

This general statement has since been known as Hooke's Law, and in more modern language would be: Any component of stress at any point in a body is a linear function of the component in the same direction of the strain at that point.

Numerous tests have been made and it has been found that for most materials, including all metals except cast metals, the measurable extension is proportional to the applied tension, provided that the tension is not too great.¹ In interpreting this conclusion it is assumed that the tension

1--The Encyclopedia Britannica, Vol. 9, p. 141.

is uniform over the cross section of the material and the extension of longitudinal filaments is uniform throughout, the results then take the form of a proportionality connecting the stress and the strain.

Following out the theory that the stress is proportional to the strain, enunciated by Robert Hooke, Thomas young in 1807 introduced physical constants called "Moduli of Elasticity" of a body, which have since been known as Young's Moduli, different for different materials. It is very easily seen that if a rubber band is stretched the cross section becomes smaller; a similar change in cross section occurs when any elastic material is stretched but the strain is so small that it is not so easily observed as in the case of a rubber band.

The infallibility of Hooke's Law in expressing the relation of the stress to the strain as a linear function and the constants of modulus of elasticity have for a considerable time past been questioned. Numerous investigations have been made on the determination of Young's Modulus under different conditions. A summary of the various re-determinations of the moduli of elasticity under different conditions are given as follows:

About 1890 Bach gives an empirical formula,, in his book on Strength of Materials, expressing the relation between the stress and the strain which he believes fits the facts more closely than Hooke's Law. This formula is $T = AF^m$, where T is the relative extension or contraction, F is the force per unit area and A and m are constants of the material. The formula was the result of various tests

made on steel, concrete, and other materials, both for elongation and compression.

With Bach's formula in mind, F. Kohlrausch and E. Gruneissen¹ made a series of determinations on rods of cast iron, wrought iron, brass and slate for small elastic displacements by an optical method. It is agreed that Bach's formula, or a similar one with a series of powers, expresses the connection between the stress and strain more accurately than Hooke's Law. The importance of such a law for technical purposes is also admitted but the theoretical claim by Bach is doubted. The validity of extending the supposition at all beyond the experimental range is denied and the need for investigation, especially in the low values is insisted upon. For soft iron it was found that Hooke's Law holds to within experimental error; however, brass did not follow Hooke's Law, but there was a much less deviation than found by Bach. For grey cast iron there was an agreement with Bach's results. With the belief that Bach assumes too much in saying that his law holds true for small values, Kohlrausch and Gruneissen propose a formula:

$$\frac{Y}{X} = A + B\sqrt{X}$$
 where X equals the stress, Y equals the strain, and A and B are elastic constants.

In view of determining the values of Young's Modulus for small initial loads and comparing the results obtained with Young's Modulus found by using larger loads up to the elastic limit, Charles P. Weston² experimented on the metals copper, brass and steel in the form of bars 66 cm. x 1 cm.

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1-F.Kohlrausch and E.Gruneissen Pruess.Akad.Wiss. Berlin
 Sitz.Ber. 46 pp 1086-1091 N ov. 1901.
 2-Phys. R^{ev}. volume 8, p297, 1899.

x 0.5 cm. in dimensions. These bars were supported at each end and the weights applied at the center. The deflections shown by the addition of weights as small as 0.5 grams were measured by an interferometer in terms of wave lengths of sodium light, using .00005893 cm. as the value of the wave length. The following table will show results obtained for a steel bar.

Mass in grams	Defl. in Sodium Waves	Defl. in cm.	Defl. in cm. per 0.5 grams
0.5	3	.00008839	.00008839
1.0	6	.00017678	
1.5	9	.00026517	
2.0	12	.00035356	

Young's Modulus equals 19.19×10^{11}

It was concluded that for brass, copper and steel, the ratio between the deflection and load is a constant quantity ranging from 0.5 gram to the load that would give the bar a permanent set.

W. C. Popplewell¹ investigated Bach's formula, $T = AF^m$ and finds that it holds for various metals on compression.

H. W. Bearce² determined the modulus of elasticity for small loads at the elastic limit by the use of an interferometer. The results were obtained by bending the specimens of iron, steel, copper and brass until permanent set could be detected. It was found upon removal of the load and upon continuing loading by very small amounts the following values for Young's Modulus:

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1-Manchester Lit. & Phil. Soc. Mem. 49 pp 1-18, 1905.

2--Phys. Rev. 24, pp 191-194, Feb. 1907.

Metal	At Elastic Limit Young's Modulus	Within the Elastic Limit, Large Loads.
Iron	20.96 x 10 ¹¹	-----
Steel	21.04 "	19.56 x 10 ¹¹
Copper	12.26 "	12.00 "
Brass	11.59 "	10.62 "

These experiments seem to show a slight increase in the elastic moduli as bars reach their elastic limit. This is only another way of saying that Hooke's Law fails..

A comparison of the moduli of torsional and longitudinal elasticities of metals as determined by the statical and kinetical methods was made by H. Tomlinson.¹ It was found that the value of the modulus of longitudinal elasticity for hard drawn metals as determined by the statical method of loading accords with the value obtained by the method of longitudinal vibrations provided the deformations are sufficiently small. These values of the moduli of torsional elasticity, as determined by the statical method, accord with the value obtained by the method of torsional vibrations for most metals in the hard drawn state, provided the deformations are small. For annealed iron, the modulus obtained by the second method slightly exceeds that obtained by the first method, by an amount which is greater than can be attributed either to the heating and cooling effects of contraction or expansion, or to the errors of observations.

The effect of drawing on the elasticity of a copper wire was investigated by J. R. Benton², and it was observed that if a copper wire is first annealed by heating electrically to redness and is drawn, its modulus of rigidity decreases,

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1-Proceedings of the Royal Soc. Vol. 42--1887.

2-Phys. Rev. 13, pp 234-45, Oct. 1901.

and Young's Modulus increases. Each successive drawing causes a further diminution of the modulus of rigidity and a further increase of Young's Modulus, the amount of change for each modulus becoming less with each drawing. If the wire is annealed after having been drawn several times, the modulus of rigidity returns to a value considerably greater than its original value and Young's modulus to one considerably less than its original value.

The dependence of elastic properties of steel upon composition and treatment was observed by Waurzinik¹. Steels containing from 0.53 to 1.13% carbon were tested and the modulus of elasticity was found to decrease with increasing carbon composition from 2,121,000 kg./cm² with .063 % carbon to 2,098,000 kg./cm.² with 1.13 % carbon.

The variation of the density and elasticity of metals was investigated by Angenheister², and it was found that an increase in density of a metal leads to an increase in Young's Modulus.

The elastic coefficient of elongation of Platinum was determined with varying temperatures by A. Winkelmann.³ At 20 degrees Centigrade the coefficient of elasticity had a value of 16,926; after many heatings to 400 degrees C. its value was 18,380; at the end of about 10 months' rest, the value was 17,508. It is not known whether the coefficient would have attained its original value if left undisturbed for a longer time.

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1-Metallurgie 4, pp 810-815, Dec. 1907.

2-Ann. d. Physk. 11 pp 188-201, April 1903.

3-Annal. Phys. Chem. 63, 1, pp 117-123, 1897.

Miss Noyes¹ made determinations of the influence of heat, of the electric current, and of magnetization upon Young's Modulus. The results obtained seemed to show that neither magnetization nor an electric current through a wire produced any effect upon the size of Young's Modulus aside from that which can be attributed to accompanying heat. All wires show a permanent increase in elasticity produced by heating them, the increase being the greater for silver and least for copper. The modulus of stretch of a piano wire, 0.40 mm in diameter changes 5% in a change of 100 degrees. An annealed piano wire 0.45 mm in diameter changes 3.4% in a change of 100 degrees, while there was a permanent change of 1.27%. The modulus of a silver wire, 0.48 mm in diameter, changed 8.2%, while there was a permanent change of 1.8%, the elastic limit increasing 69.2% when heated.

F. C. Lea and O. H. Crowther² also investigated Young's Modulus of elasticity and other properties of metals with a variation of temperatures. Specimens of mild steel, copper and high tension brass were tested by means of a specially designed extensometer while being heated by an electrical furnace. In the case of mild steel, the modulus decreases slowly from about 29.5×10^6 lbs. per sq. in. at 0 degrees Centigrade to about 27.5×10^6 at 300 degrees. Above this temperature the decrease becomes more rapid; the value at 600 degrees being only about 12.5×10^6 . Copper
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1-Phys. Rev. Vol. 3, p 432, 1895-96.

2-Engineering 98 pp 487-489, Dec. 1913.

shows a continual and fairly constant decrease in the modulus as the temperature rises.

Recently H. L. Dodge¹ has investigated the change in elasticity of stretch of a copper wire with current and external heating. This investigation was undertaken to determine whether heating by an electric current had any other effect on the elasticity of a copper wire than that caused by accompanying increase in temperature. The following results were obtained.

Amperes	Variation in Length mm.	Variation in Stretch mm.
12.8 Alt. Current	.0000	.0000
12.8 Direct Current	.0065	.0009
External Heating D.C.	.0059	.0009
External Heating A.C.	.0032	.0014
12.2 Direct Current	.0018	.0009

This variation corresponds to about a temperature variation of one degree. In every test made it was found that Young's Modulus decreased with an increase in temperature.

The effect of sudden cooling on the elastic properties of metals was observed by J. Muir.² Specimens of mild steel, iron, copper, brass and aluminum were tested in tension after being quenched from a known temperature of about 700 degrees Centigrade, the strain being measured by an extensometer. In each case the material had no range of elasticity after being quenched, Hooke's Law being departed

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1-Phys. Rev. 2, Ser . 2, pp 431-49, Dec. 1913.
2-Roy. Soc . Proc. 71, pp 80-91, Oct. 1902.

from at the beginning of the load and to a greater extent towards the end.

A most striking deviation from the laws of elasticity was observed by K. E. Guthe¹, and investigated along with Seig,² using wires of platinum-iridium. They found the period of torsional vibration for these wires was by no means a constant, but decreased rapidly with the amplitude of vibration. Platinum-iridium wires containing over 30% iridium show striking peculiarities in the elastic behavior which become more marked as the percentage of iridium is increased. For small amplitudes, the period of torsional vibration is nearly proportional to the amplitude; while for larger amplitudes the rate of increase of the period becomes smaller and for larger amplitudes the period tends towards a constant value. The relation of the logarithmic decrement to the amplitude reaches a maximum, after which it decreases and probably reaches a maximum for very large amplitudes, the maximum being more pronounced the larger the original distortion. However, if the wire has been allowed to rest some time before the torsional vibrations are produced, the total number of vibrations necessary to bring the system to rest is the smaller the larger the initial amplitude. The wires when tested by the static method closely obey Hooke's Law. The torsional moment calculated from the static experiment is larger than the largest torsional moment found by the dynamic method.

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1--Proc. Iowa Ac. Sci. 15, p 147, 1908.

2--Phys. Rev. 31, pp 421-462, Oct. 1910.

At the suggestion of K. E. Guthe, an investigation of the elastic properties of Bismuth was taken up by J. E. Harris¹ who found some of the same peculiarities. When the wires were vibrated torsionally, the period of vibration and the logarithmic decrement were observed to decrease considerably with a decrease of amplitude. A few characteristics of platinum-iridium wires observed were not present in the Bismuth wires due to the low elastic limit of Bismuth. It was found that, in two experiments, in which the mass of the vibrating system was kept constant but the moment of inertia varied, the period-amplitude curves in the two cases could be made to coincide by applying a reduction factor to the periods in one set of observations. This reduction factor was found to be equal to the ratio of the square roots of the moments of inertia used in the two experiments. This indicates that the mathematical relation between the period and amplitude does not depend upon the rapidity of motion of the vibrating system. The logarithmic decrement-amplitude curves in the two cases, where the moment of inertia was varied, were found to coincide. However, when the load as well as the moment of inertia was raised it was found that when the proper reduction factor was applied to the periods, the two curves, representing the variation of the moment of inertia, did not coincide, the curve corresponding to the vibrating system having the smaller mass being flatter than the other. If the length of the wire was raised, it was found upon applying a reduction factor to the periods and

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amplitudes, the period-amplitude curves for different experiments could be made to coincide. Also wires of different diameters were used with suspended weights of such a mass that the stresses to which the wires were subjected were the same in each case. Again the period-amplitude curves can be made to coincide by proper reduction factors. The logarithmic decrement curve for the smaller wire was found to lie above that for the wire of larger diameter, indicating a greater damping effect in the former.

Probably the earliest quantitative measurements that are obtainable on the elastic constants of India Rubber are found in Thomson and Tait's Natural Philosophy. It is here stated that the resilience of vulcanised india rubber, i.e. the amount of work restored by the substance when allowed to return to its equilibrium form, after having been stretched to a maximum short of rupture, is stated to be equivalent to its own weight (12.3 grams) raised through 1200 meters.

The variation of the rigidity of vulcanised india rubber with tension was investigated by T. C. Hebb¹ and it was found upon using a fresh, or partially fatigued, vulcanised rubber cord when subjected to an increasing tension that the rigidity at first diminished, then reached a maximum and finally increased; while in the case of a sufficiently fatigued cord, the minimum point seemed to disappear.

An investigation of a slow stretch in india rubber

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1--Nova Scotian Inst. Trans. 10 pp 273-286, 1900.

was made by P. Phillips¹. The relation between the stretch X of a specimen of india rubber and the time t for which the given load is applied is found to be: $X = a + b \log t$, where a and b are constants for the particular pull. After the first few seconds, this equation represents the facts with considerable accuracy. When the loads are removed, the equation: $X = b \log t/t_0$, where t is the time of application of load and t_0 is the time of removal of load, represents the relation of the stretch to the load.

The extensibility of vulcanised rubber was determined by Cheuveau and Heim.² It was found that the equation $Y = kX + a \sin^2 bX$ represented the relation between the elongation and the load, where k and a are constants characteristic of the initial and mean elongation, Y is the extension in centimeters, X is the load in kilograms, b is characteristic of the ultimate elongation. It was observed that K and a are higher for better quality of rubber, while bx increases as the quality of rubber deteriorates. K , a , and b only assume steady values after third application of load.

The thermal phenomena in india rubber was investigated by A. Schwartz and P. Kemp,³ and it was found that when rubber is stretched in the cold at first it shows a slight cooling and then as the stress is increased there is a heating. This phenomenon was also observed by Joule and by Cheuveau, the latter ascribing the phenomenon to two effects:

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1--Phil. Mag. 9, pp 513-531, April 1905.

2--Comptes Rendus 152, pp 320-22, Feb. 1911.

3--Mech. Eng. 30 p 257, Aug. 30, 1912.

a displacement of the position of the molecular groups relatively to their positions of equilibrium and a change in the dimensions of the inter-molecular spaces. This view does not explain why the two effects are only observed when the experiments are made in the cold, and it is suggested that two or more types of molecular aggregation exist.

The elongation of india rubber was determined by S. Ono¹, and the following formula is given as best representing the results obtained: $S = A (1 - 2/(R-B)) e^{h(p-R)^2}$, where S is the stress per unit of original area, R is the strained length divided by the original length, B is the strained breadth divided by the original breadth, and A, h, and p are constants. The results for several specimens of rubber gave $A/\sqrt{h}$ as approximately 400 grams per square millimeter for all specimens, and p is roughly 3.3 for all specimens. Tests were made at a constant temperature.

Young's Modulus and the modulus of simple rigidity were each measured in two ways, statically and dynamically by A. Mallock.² The statical measurements were made by observing a known extension and angle of torsion produced by a known force and moment, while for the dynamical measurement the frequencies of the vibrations were noted which the respective elasticities produced when acting on a known mass and moment of inertia. The values obtained in these two ways do not agree, those given by the dynamical method being in all cases greater than the statical values. One property possessed by india-rubber, and to which part of the

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- 1--Mathematics-Physical Soc. Tokyo, Proc. 7 pp 36-43, Moh.1913.
2--Proc. of the Royal Society Vol. 46, 1889.

difference between the dynamic and static values of elasticities is due, is that when strained by a given force, the extension due to the force increases rapidly at first and then gradually for days. When the force is removed, the contraction takes place gradually in the same way, but not at the same rate. The material appears to take a subpermanent set, which ultimately becomes a definite fraction of the extension to which it is subjected. Another property of india rubber, especially conspicuous in the soft grey kind, is that when stretched to a definite point, the resistance to further extension increases very rapidly, so rapidly indeed as to suggest that a structure of the material brings some sort of a mechanical stop into action. This action is similar to a mechanical mixture such as putty. If a lump of putty be rolled or beaten it will be found to be slightly elastic, but beyond the elastic limit to be easily stretched for a certain distance and then to become almost hard, while at the same time the surface changes from a smooth oily character to a dull granular one. The explanation of this is that the hard particles of the mixture are separated each from its neighbor by a wall of fluid of finite thickness. When the material is distorted the particles separate from one another in one direction and approach one another in a direction at right angles to this. As long as this approach merely involves the flow of the intervening fluid, the distortion takes place with comparative ease; but

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when the approach of the particles brings them into contact, the conditions change. There is no longer a store of fluid between the vertical layers, perpendicular to the action of the force, of particles which can be drawn on to supply the increased distance between the horizontal layers, and if the strain is increased it must mean either a distortion of the hard particles themselves or an increase of volume of the whole mass. The latter is what happens in the case of the putty, the dull surface being the result of the fluid being sucked, or rather pushed, inwards by atmospheric pressure to supply the extra volume required in the interior, thus leaving the surface comparatively dry.

It is very evident from the various investigations that have been made on the elasticity of various materials under such widely different conditions that there is a conflict of opinion as to the relation of stress to strain. Professor E. R. Hedrick, after plotting numerous curves from data obtained by the Watertown Arsenal, on testing strength of materials, found that in a great many determinations of the relation of the stress to the strain Hooke's Law was not always followed. This deviation from a direct proportion led Professor Hedrick to suggest an exponential function as best representing the relation between the stress and the strain. Curves were plotted on logarithmic paper, from which an equation similar to the one proposed by Bach was suggested, i.e. $T = AF^m$, where T is the extension or contraction, F is the force per unit area, and A and m are constants of the

material. In arriving at this equation, Professor Hedrick was unaware that Bach had proposed the same equation in his book on Strength of Materials.

The present investigation of the stretch moduli was started upon the suggestion of Professor Hedrick to determine if possible the relation between the stress and strain testing the proposed equation.

An investigation of the elasticity of stretch of grey vulcanised india rubber, obtained from the B. F. Goodrich Co., was made by means of a cathetometer. The specimen of india rubber, rectangular in shape, dimensions being 2.5 cm. x 2.5 cm. x 50 cm., was hung vertically and a scale pan was attached at the lower end. Identification marks were made on the rubber about 10 cm. from each end for setting the cathetometer. To obtain a series of readings of the elongation as the load was increased, it was necessary to set and read the cathetometer first on the lower identification mark, then on the upper mark returning to reread the lower mark. The double reading of the lower mark was made necessary because rubber elongates with time. However, if the same time has occurred between readings and if an average of the two lower readings is taken, the average will be equivalent to taking the lower reading at the same time as the upper reading.

A series of readings was taken as stated above and the loads were increased and decreased as fast as the readings could be made. In the first column is found the change in load, in the second and fourth columns the lengths, and in

the third and fifth columns the elongations.

Table 1.

Load in Grams	Length in cm. Increasing Load	Elongations cm.	Length in cm. Decrease Load	Elongations cm.
1850	29.595		30.048	
2000	30.078	0.517	30.690	0.542
4000	30.660	1.065	31.365	1.317
6000	31.365	1.770	32.194	2.146
8000	32.087	2.492	32.913	2.965
10000	32.850	3.260	33.796	3.748
12000	33.710	4.115	34.672	4.624
14000	34.618	5.013	35.621	5.573
16000	35.557	5.962	36.534	6.486
18000	36.567	6.972	37.660	7.612
20000	37.712	8.118	38.815	8.767
22000	38.848	9.253	39.937	9.889
24000	40.159	10.564	41.025	10.977
26000	41.418	11.823	42.066	12.018
28000	43.705	12.705	43.930	13.882

The loads are plotted against the corresponding elongations, and figure 1, curves 1 and 2 show the relation between elongations with increasing and decreasing loads. It is seen that upon decreasing the loads, the elongations lag behind those for increasing loads.

Since india rubber elongates with time, a series of readings of elongations of the same specimen was taken after 16,000 grams were placed in the scale pan, total weight applied at once, and the corresponding time in minutes observed. The elongations are computed from a reading taken immediately after loading.

Load

30000

16000

22000

18000

14000

10000

6000

2000

0

FIG. 1.

Table 2

Time in Minutes	Elongation cm.
5	.282
10	.442
16	.562
20	.638
25	.694
30	.752
35	.812
40	.860
45	.892
50	.932
56	.972
60	1.002
65	1.022
70	1.058
75	1.072
90	1.140
106	1.192
137	1.292
155	1.328
217	1.466
280	1.552
323	1.702
1273	2.130
1393	2.148

In figure 2, curve 1, the results of table 2 are plotted. Abscissae are times in minutes after the load was put on and ordinates the corresponding extensions beyond the initial length immediately after the addition of the load.

With the same specimen of rubber, and investigation was made to show the effect of a load increasing by increments of 2000 grams up to 32,000 grams, a considerable interval of time being left between changes in load. The observations were taken as follows: First, 2000 grams were placed in the scale pan, and a setting taken on the lower mark as quickly as possible. Then settings were made on this mark

FIG. 11

at frequent intervals of time until the rate of movement did not exceed .2 mm. in 5 minutes. Settings were then taken in the following order,- lower mark, upper mark, and lower mark. The difference between the reading on the upper mark and the average of those on the lower mark, was taken as the length for 2000-gram load. This procedure was repeated after the addition of 2000 grams more to the scale pan, and so on, till the maximum load of 32000 grams was reached. Table 4 gives in column 1 the load, in column 2 the corresponding length, as defined above, and in column 3 the elongations. In figure 3 the elongations are plotted as ordinates against the loads as abscissae.

For the purpose of comparison with the results indicated in table 2 and in figure 2, curve 1, there is given in table 3 the readings taken, in the course of obtaining the data for table 4, when the load on the rubber was 16000 grams, the load for which table 2 is given. In table 3, the first column is the time in minutes, second column the corresponding readings of the lower mark and in the third column is the elongation in centimeters.

Table 3.

Time in Minutes	Readings of Lower Mark	Elongation cm.
0	23.304	
1	23.264	.040
2	23.244	.060
3	23.224	.080
4	23.210	.090
5	23.200	.104
10	23.164	.140
15	23.138	.166
20	23.110	.194
25	23.095	.209
30	23.074	.230
35	23.056	.248
40	23.046	.258

Table 4

Load in Scale Pan (1850 g)	Length in cm.	Elongation in cm.
0	29.690	
2000	30.368	.678
4000	30.923	1.233
6000	31.645	1.955
8000	32.327	2.637
10000	33.263	3.573
12000	34.244	4.554
14000	35.264	5.574
16000	36.403	6.713
18000	37.459	7.769
20000	38.821	9.131
22000	40.147	10.557
24000	41.692	12.002
26000	43.118	13.428
28000	44.680	14.990
30000	46.428	16.738
32000	47.782	18.092

Figure 3, shows a curve plotted from data taken from table 4.

Summing up the elasticity of stretch of india rubber, it is evident in giving any formula representing the relation of the stress to the strain, it is necessary that the previous history of the specimen be known and also the time that the stress is acting must be considered. This is shown by observing the length, 29.595 cm. for the scale pan empty and 43.705 cm. for 28,000 grams, Table 1; the length 29.690 cm. for the scale pan empty and the length 44,680 cm. for 28,000 grams, Table 4.

FIG. III

The apparatus used in determining the elongation of wires for increasing and decreasing loads is shown by figure 4. A concrete base was built with an upright at each end. A movable iron rod passed through upright M. A hole is bored through one end of the rod for fastening the wire, and a nut is placed on the other end to allow for a change in the length of the iron protruding from the concrete. On top of upright N is placed a pulley, and a dividing engine is set on top of the concrete base between uprights M and N. The wire to be tested is fastened to the iron rod at R, passes over the dividing engine, over the pulley at N to a scale pan W. Fine scratches were made on the wire at d_2 and d_1 . A micrometer-microscope, rigidly fastened to the dividing engine at d_2 was focused on one edge of the scratch; and was used to measure the elongation of the wire from R to d_2 , i.e. to measure the change of the point d_2 . The smallest reading that could be made on the micrometer-microscope at d_2 was .0005 mm. Another microscope was attached to the movable part of the dividing engine and focused on the edge of scratch d_1 . The screw of the dividing engine that regulated the movable microscope had a pitch of 1 mm. and could be read to .001 mm. The differences between successive readings of d_1 minus the difference between successive readings of d_2 gives the elongation for successive increments of loads.

One of the most important features of the apparatus is the construction of the pulley, since it is very important

that friction be eliminated as far as possible. As shown in figure 4, the pulley instead of revolving on an axle, rests on points p and p' constructed from hardened tool steel. These points rest in grooves so that the pulley and wire can be moved horizontally. It was necessary to lower the center of gravity to the points p and p' in order to have the pulley in equilibrium at different positions. This was accomplished by counter weights as shown in figure 4. Friction was eliminated as far as could be detected, for it was possible to duplicate readings to within .003 mm. However, this error may be due to setting microscope d_1 on the scratch, for a difference of .003 mm. is made between two settings of microscope d_1 . An error of .002 mm was possible in setting the micrometer-microscope at d_2 .

The weights used in changing the load were 500-gram brass weights taken from 1000-gram sets used in the laboratory. The weights were applied carefully so as not to produce any sudden strain.

Metallic connections t_1 and t_2 were made to the electrical power circuit for the purpose of annealing the wires. The connections at t_1 and t_2 were made by twisting fine wire around the wire to be tested.

In the following experiments, the wires were always loaded with a zero load, which included the weight of the scale pan, the value of the zero load being shown in the tables. The amount of zero load was determined by the weight necessary to straighten the wire free from kinks.

FIG. IV.

The distance between points d_1 and d_2 was measured. The microscopes d_1 and d_2 were set on the identification marks and read; an increment of load was added to the scale pan and the microscopes d_1 and d_2 again set and read. The difference between the two readings at d_1 minus the difference between the two readings at d_2 gives the elongation for the given increment of load. This same procedure was followed for decreasing loads. If the elongation is desired between the reading with the zero load and after say ten increments of load have been placed in the scale pan, it is only necessary to take the difference between the readings of d_1 at zero load and load required minus the difference of similar readings of d_2 . Upon diminishing the load, the recovery is computed by taking the difference between readings of d_1 for the maximum load and after the increment of load has been removed minus similar readings of d_2 . The difference obtained subtracted from the maximum elongation gives the elongation remaining after the increment of load has been removed.

The tables show the data arranged as follows: The first column contains the load in grams, the second column the corresponding elongation for increasing loads, and the third column the corresponding elongation for decreasing loads. Curves are drawn for each set of readings showing the elongations as ordinates and the loads as abscissae. The points on the curve representing the relation of the elongation to the loads as the loads were increased are marked "x", and as the loads are decreased they are marked "O".

The elasticity of stretch for a specimen of hard drawn brass wire, length 66.73 cm. and diameter 0.50 mm., was first investigated. Tables 5 and 6 give the data taken for two sets of readings from which curves, figure 5 and 6 are plotted.

A current of 15.5 amperes was then passed through the wire, while loaded with 1000 grams, heating it to brightness, after which it was cooled suddenly by plunging it into water. A length of 66.03 cm. was observed and loads were added until at 3500 grams the elastic limit was passed. The loads were increased to 6000 grams drawing the brass wire considerably past its elastic limit and at the same time hardening the wire to some extent. The readings of this test is not given, owing to the movement of the identification marks outside of the range of observation. After allowing the wire to rest for a day, a series of readings was made. The data taken for the different runs is shown in tables 7 to 13 inclusive, from which curves are plotted, figures 7 to 13, inclusive.

A brass wire, length 64.32 cm., diameter 0.570 mm., was next investigated to the breaking point. Loads were added in increments of 1000 grams and the elongation observed up to the point where the wire broke, as shown in table 14. A curve, figure 14, is plotted from data taken from table 14.

Table 5.

Load in grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
2000		.019
2500	.141	.156
3000	.268	.285
3500	.390	.424
4000	.519	.556
4500	.657	.683
5000	.779	.816
5500	.909	.845
6000	1.047	1.076
6500	1.175	1.211
7000	1.314	1.337
7500	1.443	1.458
8000	1.570	1.596
8500	1.695	1.721
9000	1.837	1.846
9500	1.957	1.975
10000	2.098	2.098

Table 6

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
1500		.018
2000	.138	.164
2500	.270	.305
3000	.389	.444
3500	.520	.565
4000	.643	.695
4500	.771	.883
5000	.900	.964
5500	1.029	1.089
6000	1.161	1.211
6500	1.287	1.340
7000	1.412	1.471
7500	1.542	1.600
8000	1.668	1.732
8500	1.800	1.856
9000	1.932	1.984
9500	2.074	2.104
10000	2.203	2.333
10500	2.336	2.351
11000	2.471	2.479
11500	2.602	2.602

FIG. V

E1x5

DEPARTMENT OF ENGINEERING, UNIVERSITY OF MO.
13
14
15
16

12
11
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3
2
1

PLATE A - FOR SALE AT THE UNIVERSITY CO-OPERATIVE STORE, COLUMBIA, MO.

FIG. VI

Table 7

Load in grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
500		
1000	.101	.172
1500	.175	.310
2000	.292	.428
2500	.291	.524
3000	.501	.637
3500	.609	.728
4000	.725	.837
4500	.849	.927
5000	.985	1.019
5500	1.108	1.108

Table 8

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
500		
1000	.104	.081
1500	.182	.166
2000	.273	.271
2500	.368	.394
3000	.475	.502
3500	.587	.610
4000	.687	.707
4500	.810	.886
5000	.945	.986
5500	1.088	1.088

Table 9

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
500		-.022
1000	.096	.076
1500	.171	.177
2000	.262	.278
2500	.360	.382
3000	.447	.496
3500	.541	.601
4000	.634	.696
4500	.725	.802
5000	.843	.892
5500	.980	.980

FIG. VII

FIG. IX

Table 10

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
500		.000
1000	.075	.082
1500	.161	.179
2000	.255	.285
2500	.346	.389
3000	.447	.486
3500	.537	.594
4000	.628	.690
4500	.720	.784
5000	.837	.874
5500	.960	.960

Table 11

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
500		.016
1000	.090	.098
1500	.174	.195
2000	.262	.300
2500	.352	.405
3000	.441	.507
3500	.539	.599
4000	.630	.703
4500	.735	.787
5000	.865	.877
5500	.973	.973

Table 12

Load in Grams	Elongation in mm. Increasing Load.	Elongation in mm. Decreasing Load.
500		.011
1000	.095	.112
1500	.196	.207
2000	.285	.327
2500	.376	.428
3000	.460	.516
3500	.560	.612
4000	.655	.705
4500	.750	.797
5000	.856	.893
5500	.980	.980

FIG. X

FIG. XI

FIG XII

Table 13

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
500		.016
1000	.088	.082
1500	.173	.181
2000	.235	.279
2500	.348	.379
3000	.434	.481
3500	.530	.576
4000	.622	.667
4500	.724	.748
5000	.835	.845
5500	.941	.941

Table 14

Load in Grams	Elongation in mm. Increasing Load.
2000	
3000	.245
4000	.483
5000	.738
6000	.989
7000	1.237
8000	1.497
9000	1.758
10000	2.010
11000	2.292
12000	2.589
13000	2.930
14000	3.334
15000	3.702
16000	4.080
17000	4.462
18000	Wire broke.

FIG. XIII

E1.X.2.5

11
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3
2
1

0 1000 3000 5000 7000 9000 11000 13000 15000 17000

FIG. XIV

A piano wire was tested and the change in elongation with the load is shown in tables 15, 16, and 17, from which curves of figures 15, 16, and 17 are plotted.

Table 18 gives the results obtained after annealing the wire by sending a current through it. The wire was heated to brightness and was allowed to cool down slowly by reducing the current gradually. Figure 18 shows the curves plotted from the results given in table 18.

A German make of steel wire, No. 8, length 65.6 cm., diameter .492 mm. was tested. Tables 19, 20, and 21 give the results obtained and figures 19, 20 and 21 represent the curves plotted. The same wire was then annealed, first by sending a current of 8 amperes through the wire, and cooled slowly by gradually reducing the current. The elastic limit was passed when the load was increased to 11,000 grams, as is observed from curve, figure 22, showing the results of the test. A current of 11 amperes was again sent through the wire and kept at a bright red heat for 15 minutes and cooled slowly by reducing the current gradually. There was one portion of the wire that seemed to heat much more slowly, for which the cause is unknown. Table 23 and figure 23 give the results obtained and the curve.

Another steel wire, No. 8, taken from the same coil, length 65.6 cm. and diameter .492 mm. was substituted for the annealed wire. A test was made to determine the behavior of the elasticity as the load was increased beyond the elastic limit. Tables 24 and 25 give data taken, and curves shown in

figures 24 and 25 represent the behavior. It will be seen from the curve in the figure 25 that the elastic limit is reached at about 24,000 grams.

Table 15

Load in grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
1000		.059
1500	.108	.176
2000	.212	.280
2500	.321	.378
3000	.423	.483
3500	.535	.588
4000	.638	.692
4500	.741	.796
5000	.850	.893
5500	.955	.983
6000	1.068	1.090
6500	1.175	1.189
7000	1.285	1.292
7500	1.391	1.391

Table 16

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
1000		.018
1500	.122	.131
2000	.225	.232
2500	.328	.322
3000	.429	.438
3500	.537	.550
4000	.638	.652
4500	.734	.752
5000	.832	.851
5500	.938	.952
6000	1.038	1.055
6500	1.140	1.156
7000	1.230	1.259
7500	1.351	1.357
8000	1.455	1.455

FIG. XV

Load

FIG. XVI

Table 17

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
1000		.024
1500	.120	.128
2000	.228	.248
2500	.328	.350
3000	.432	.455
3500	.543	.549
4000	.646	.653
4500	.746	.751
5000	.834	.859
5500	.947	.961
6000	1.051	1.068
6500	1.146	1.167
7000	1.251	1.267
7500	1.357	1.368
8000	1.454	1.444
8500	1.551	1.570
9000	1.674	1.674

Table 19

Load in Grams	Elongation in mm. Increasing Load.	Elongation in mm. Decreasing Load.
1000		.004
2000	.213	.206
3000	.414	.430
4000	.606	.621
5000	.817	.804
6000	1.001	1.021
7000	1.196	1.210
8000	1.380	1.402
9000	1.578	1.582
10000	1.801	1.801

E1. x5

FIG. XVII

FIG. XVIII

Table 19

Load in Grams	Elongation in mm. Increasing Load.	Elongation in mm. Decreasing Load.
1000		.019
1500	.085	.100
2000	.163	.180
2500	.249	.266
3000	.327	.343
3500	.418	.429
4000	.503	.509
4500	.588	.585
5000	.680	.673
5500	.753	.757
6000	.834	.842
6500	.916	.931
7000	.996	1.013
7500	1.042	1.098
8000	1.168	1.175
8500	1.243	1.254
9000	1.332	1.346
9500	1.411	1.425
10000	1.492	1.498
10500	1.570	1.579
11000	1.552	1.658
11500	1.732	1.732
12000	1.815	1.815

Table 20

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
1000		.063
1500	.104	.140
2000	.187	.225
2500	.319	.305
3000	.403	.388
3500	.485	.461
4000	.561	.556
4500	.641	.655
5000	.716	.737
5500	.799	.819
6000	.886	.896
6500	.966	.978
7000	1.049	1.054
7500	1.126	1.137

FIG. XIX

Table 20 (Continued)

Load in grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
8000	1.209	1.256
8500	1.280	1.296
9000	1.364	1.377
9500	1.442	1.450
10000	1.525	1.540
10500	1.608	1.618
11000	1.690	1.694
11500	1.770	1.770

Table 21

Load in grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
1000		.017
1500	.093	.099
2000	.177	.182
2500	.255	.259
3000	.349	.336
3500	.413	.418
4000	.307	.505
4500	.589	.587
5000	.671	.669
5500	.754	.760
6000	.840	.849
6500	.919	.920
7000	1.001	.996
7500	1.074	1.078
8000	1.154	1.155
8500	1.235	1.237
9000	1.319	1.321
9500	1.399	1.401
10000	1.479	1.481
10500	1.563	1.562
11000	1.644	1.637
11500	1.724	1.714
12000	1.800	1.800

FIG. XX

FIG. XXI

Table 22

Load in Grams	Elongation in mm. Increasing Load.
1000	
15000	.084
2000	.167
2500	.254
3000	.336
3500	.421
4000	.508
4500	.588
5000	.673
5500	.755
6000	.825
6500	.912
7000	1.001
7500	1.093
8000	1.180
8500	1.260
9000	1.344
9500	1.428
10000	1.510
10500	1.593-----Past elastic limit.

Table 23

Load in Grams	Elongaion in mm. Increasing Load	Elongation in mm. Decreasing Load.
1000		.012
1500	.089	.103
2000	.166	.199
2500	.258	.293
3000	.347	.395
3500	.437	.489
4000	.519	.578
4500	.616	.668
5000	.703	.770
5500	.799	.860
6000	.897	.957
6500	.989	1.046
7000	1.085	1.132
7500	1.181	1.226
8000	1.272	1.318
8500	1.374	1.413
9000	1.474	1.507
9500	1.572	1.588
10000	1.683	1.683

FIG. XXII

FIG. XXIII

Table 24

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
2000		.035
3000	.161	.190
4000	.320	.352
5000	.473	.512
6000	.640	.673
7000	.799	.850
8000	.964	.993
9000	1.125	1.156
10000	1.290	1.316
11000	1.450	1.476
12000	1.614	1.640
13000	1.769	1.804
14000	1.939	1.967
15000	2.107	2.128
16000	2.268	2.296
17000	2.423	2.447
18000	2.594	2.609
19000	2.760	2.770
20000	2.935	2.935

Table 25

Load in Grams	Elongation in mm. Increasing Load.
2000	
3000	.161
4000	.323
5000	.478
6000	.640
7000	.800
8000	.963
9000	1.128
10000	1.291
11000	1.454
12000	1.617
13000	1.780
14000	1.941
15000	2.098
16000	2.262
17000	2.425
18000	2.588
19000	2.753
20000	2.920
21000	3.083
22000	3.251
23000	3.413
24000	3.588
25000	3.772
26000	3.949

Table 25 (Continued).

Load in Grams	Elongation in mm. Increasing Load.
27000	4.129
28000	4.301
29000	4.486
30000	4.673
31000	4.863
32000	5.040

FIG. XXIV

Load

FIG. XXV

El. x 2

Tests were made of the elasticity of stretch of a copper wire 65.9 cm. in length and .77 mm in diameter. Tables 27 and 28 give the data on the change in elongation as the loads are increased and decreased, and from this data curves are plotted as shown in figures 27 and 28.

Table 27

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load
1500		.119
2000	.067	.182
2500	.124	.267
3000	.194	.327
3500	.262	.392
4000	.328	.452
4500	.397	.514
5000	.459	.578
5500	.530	.640
6000	.599	.704
6500	.673	.749
7000	.757	.818
7500	.832	.879
8000	.910	.941
8500	.997	.997

Table 28

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
1500		.029
2000	.060	.102
2500	.118	.169
3000	.172	.231
3500	.240	.295
4000	.313	.360
4500	.387	.417
5000	.449	.482
5500	.517	.545
6000	.582	.603
6500	.648	.662
7000	.711	.722
7500	.778	.783
8000	.844	.857
8500	.913	.913

Load

FIG. XXVI

E/x/E

FIG. XXVII

A series of readings was made on constantan wire, length 66.65 cm. and diameter .810 mm. The data is shown in table 29, and from the curve it is observed that the elastic limit was reached at 6500 grams. Table 30, figure 30, gives the results obtained when the wire is loaded to, or near, the elastic limit. Table 31, figure 31, gives the variation of the elongation with the length, well within the elastic limit.

Table 29

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load
1500		
2000	.054	Not taken
2500	.096	
3000	.144	
3500	.190	
4000	.232	
4500	.273	
5000	.319	
5500	.358	
6000	.407	
6500	.456	
7000	.505	
7500	.582	
8000	.662	

01 x 10

PLATE A - FOR SALE BY THE UNIVERSITY CO-OPERATIVE STORE, COLUMBIA, MO.

FIG. XXVIII

Table 30

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
1500		.021
2000	.041	.054
2500	.080	.078
3000	.123	.121
3500	.165	.166
4000	.207	.207
4500	.247	.250
5000	.287	.290
5500	.327	.330
6000	.368	.369
6500	.411	.411

Table 31

Load in Grams	Elongation in mm. Increasing Load	Elongation in mm. Decreasing Load.
1500		.008
2000	.043	.031
2500	.124	.121
3000	.203	.203
3500	.279	.274
4000	.359	.358
4500	.434	.434
5000	.528	.528

FIG. XXIX

E1.2.10

Load

FIG. XXX

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PLATE A - FOR SALE BY THE UNIVERSITY CO-OPERATIVE STORE, COLUMBIA, MO.

E/x/10

In order to test the formula as enunciated by Bach in his book on Strength of Materials, and independently by Professor E. R. Hedrick, that $T = AF^m$, curves are plotted on logarithmic paper. The ruling of logarithmic paper represents the logarithms of the numbers on the paper to the base ten. The advantage of logarithmic paper in plotting a logarithmic curve lies in the fact that the values as observed can be plotted directly. Cross section paper could be used, but it would necessitate plotting the logarithms of the elongations against the logarithms of the loads. This would be a tedious process. In plotting the curves on logarithmic paper, the elongations are taken as ordinates and the loads as abscissae. The results obtained from tests made on a brass wire, given in tables 5, 6 and 10 are shown by curves in figures 32, 33, 34 and 35. Tests made on a steel wire, the results of which are given in tables 21, 22, and 25, are shown by curves in figures 36, 37 and 38. The

In order to be able to plot these results on logarithmic paper, it is necessary to know the initial load. It will be observed from figure 34 that the data as taken from table 6 gives a curve which would never pass through zero, that is it would never give zero elongation for zero load. However, if the proper constant be added to each of the elongations and these results plotted against the load, the curve will be a straight line, if the exponential law holds true. This constant is determined by assuming a value for the elongation produced by the initial load necessary to take the kinks out

of the wire. The constant is added to each elongation, as shown in the tables, and the result plotted against the corresponding load. If the constant happens to be the correct one, the curve results in a straight line, but if it is too large the resulting curve is bent towards the positive direction of the elongations, or to the left; and if it is too small, the curve is bent towards the positive direction of the loads, or to the right. This fact is shown in figure 34.

If Hooke's Law expresses the relation between the elongations and the loads, the curve as plotted on logarithmic paper, will also result in a straight line. According to Hooke's Law, $T = AF$, where T is the tension, F is the force and A is a constant different for different materials, the exponential law, $T = AF^m$, will reduce to Hooke's Law when m is unity.

Assuming the exponential law, and taking the logarithm of each side of the equation,

$$T = AF^m$$

$$\log T = \log A + m \log F$$

At two different times in the test, let the stresses and strain be represented by, F_1 , T_1 , and F_2 , T_2 . Then

$$\log T_1 = \log A + m \log F_1$$

$$\log T_2 = \log A + m \log F_2$$

Subtracting and solving for m , we have,

$$m = \frac{\log T_2 - \log T_1}{\log F_2 - \log F_1}$$

This is the expression for the tangent of the angle that the curve in logarithmic coordinates makes with the axis of stress.

If the elongation be taken for 10,000 grams and the curve, as shown in figure 36, extended to the axis of elongation, the exponent m can be read very easily by taking the difference between the reading where the curve crosses the ordinate axis and the elongation for 10,000 grams plus the additive constant. From the curve shown in figure 36,

$$m = 1.089$$

$$\log T = \log A - m \log F$$

$$F = 10,000 \text{ grams}$$

$$T = .1479 \text{ cm. for a load of 10,000 grams.}$$

$$\log .1479 = \log A + 1.089 \log 10,000.$$

$$= \log A + 1.089 \times 4$$

$$\log A = \log .1479 - 4.356$$

$$\log A = 5.8136 - 10$$

$$A = .0000651$$

$$\text{True } E = .0000651 F^{1.089}$$

$$\text{Observed } E = .0000651 F^{1.089} - .0177$$

Where .0177 cm. is the additive constant to make the curve in figure 36 become a straight line.

74

See additional file for unfolded version.

Increasing Loads Table 5

FIG XXXII

75

See additional file for unfolded version.

Decreasing Loads Table 5

FIG XXXIII

76

See additional file for unfolded version.

$T = AF^{0.995}$

Increasing Loads Table 6

FIG. XXXIV

Table 10

FIG XXV

78

See additional file for unfolded version.

Increasing Loads Table 21

FIG XXXVI

79
See additional file for unfolded version.

80

Se additional file for unfolded version.

Increasing Loads Table 25

FIG XXXVIII

81.

See additional file for unfolded version.

$T = AF^{1.000}$

The conclusions to be drawn from the various tests made on different materials are as follows:

Tests on the elasticity of stretch of india rubber show that in order to express the relation of the elongation to the load it is necessary to know the previous history of the rubber, the manner of loading and the time the load is applied.

The elongations for decreasing loads for hard drawn brass wire lag behind the elongations for increasing loads. The percentage of lag is more than can be accounted for by the errors made in measuring the elongations.

The tests made on annealed brass wire show a decided hysteresis effect. The area between the curves will be seen to decrease somewhat after traversing a cycle. This reduction of the hysteresis effect seems to indicate that the brass wire is being hardened by repeated loadings.

From the observations made on the different specimens of steel wire, a slight hysteresis effect was observed. This effect was most prominent in the initial test, and may be due to the removal of kinks that were still present in the wire after the zero load had been applied.

The tests made on copper wire show a decided hysteresis effect.

The values of the elongations obtained for constantan wire for increasing and decreasing loads follow the same

path during a cycle within the limits of measurements.

The curves, as plotted on logarithmic paper after the initial elongation is added to the observed elongations, gives for hard drawn brass wire and steel a variation of the exponent m of 1% to 4% from unity. To arrive at the possibility of error in straightening the elongation-load curve on logarithmic paper, i.e. adding the initial elongation to the observed elongations, the data from table 22 was plotted in figures 37 and 39. It is seen that the exponents vary by about .2%. In plotting the curve, shown in figure 39, different values were added to the observed elongation to determine, if possible, the variation of the added constant and still get a straight line with the accuracy of lining up the points. It was found that a variation of less than 1% would produce a bending of the curve from a straight line.

The logarithmic curves plotted from the observations made on annealed brass show that upon increasing the load, Hooke's Law is not obeyed.

The softer metals tested do not obey Hooke's Law absolutely. It is shown from the curves plotted on coordinate paper that the relation of the stress to the strain for annealed brass, copper and constantan wires deviate from a straight line. If the exponential law is assumed as expressing the relation of the stress to the strain, the curves will become straight lines as plotted on logarithmic paper. As far as the experiments have been carried out, the exponential law seems to represent the relation between the stress and strain better than Hooke's Law for soft metals.

The tests made on the elasticity of stretch for hard drawn brass wire and steel wire are not sufficient to justify any conclusion as to the relative accuracy of Hooke's Law as against the exponential law in expressing the relation between the stress and strain.

In conclusion I wish to thank Professor Hedrick for suggesting the problem and the valuable advice given while the experiments were being carried on. I wish also to thank Professor Rentschler for the valuable assistance and advice given, and to thank Professors Stewart, Reese and Dike for the interest shown in my work.

UNIVERSITY OF MISSOURI
COLUMBIA

COMMITTEE ON ENTRANCE EXAMINATIONS

May 18/19

Dean Walter Miller,
The Graduate School
University of Missouri.

Dear Sir - I have examined the thesis
of Mr. J. L. Weatherway, submitted to
me with your letter of May 17th,
and I find it very satisfactory and
I recommend that it be accepted
by the Graduate Committee in
fulfillment of the thesis requirement
for the degree of Master of Arts.

Yours I. R. Hedrick.

UNIVERSITY OF MISSOURI
COLUMBIA

DEPARTMENT OF PHYSICS

May 16 - 1915

Dear Walter Miller,
Academic Hall

Dear Sir, -

I recommend the acceptance of Mr. J. L. Weatherwax's thesis for the Master's Degree. I have looked over it in detail, & the other members of the department have left the matter in my hands.

Yours resp

H. M. Reese

University of Missouri - Columbia

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