STABILITY OF RISK ATTITUDE ACROSS CHOICE CONTEXTS AND FRAMES

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STABILITY OF RISK ATTITUDE ACROSS CHOICE CONTEXTS AND FRAMES

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Dedication

The following people have earned co-authorship on this very important chapter of my life. I am forever grateful for their support and encouragement:

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Chapter 1: Review of Risk Attitude

Introduction

Decision Making Under Risk

Decision making under risk refers to a choice scenario that most people face in some form every day, but risk itself is sometimes difficult to define precisely. We will begin by describing a hypothetical choice situation that illustrates how important concepts in decision making under risk relate to real world decision making.

Travis is sitting at the bar he co-owns with his friends Brett and Elise listening to a radio trivia show. While the three people are friends, they are also extremely competitive. The radio host reads a trivia question worth \$500 to the first caller who answers correctly. Immediately after reading the question both Brett and Elise exclaim that they know the answer, and that Travis should give him (or her) his cellphone (the only cellphone available) to call the radio station and win the prize. However, since Brett and Elise are both competitive they refuse to say their answers out loud, instead each stubbornly insisting that they alone know the correct answer.

Travis knows that Brett is smart and quickly estimates that Brett is likely to know the correct answer, but Travis is less confident in Elise's knowledge. Elise offers to split the winnings evenly with Travis if he gives her the phone. Brett, confident in his knowledge, only offers \$100 of the winnings if Travis gives him the phone. Travis must quickly decide who to give the phone to in order to win any money.

At this point we can define several aspects of this decision environment. First, Brett and Elise each represent a risky choice alternative. Travis doesn't know whether Brett or Elise actually knows the correct answer for two reasons: 1) Travis himself does not know the correct answer and 2) neither Brett nor Elise is willing to share their information – instead withholding entirely out of competitive spite. In its current formulation, this decision problem is a decision under *uncertainty*. Travis is uncertain of the probabilities of winning with each of his friends and only estimates that Elise's chances are worse than Brett's. If we step back from reality we might assume Travis knows (or accurately estimates) objective probabilities that each friend actually knows the answer. Let's assume Travis estimates an 80% chance Brett actually knows the correct answer and a 40% chance that Elise actually knows the correct answer. If the probabilities of the outcomes are known, the problem becomes a decision under *risk*. But what are the risks? The most negative consequence is the status quo: Travis wins nothing and loses nothing, so how can there be risk without any negative consequences? Travis does not stand to lose any money per se, only the opportunity to win money.

The risk here is defined by the outcomes and probabilities of the choice alternatives. Brett offers an outcome of \$100 if he is correct and \$0 if he is incorrect. Elise offers an outcome of \$250 if she is correct and \$0 if she is incorrect. Here we see that a

negative outcome is not necessary to define a risk. We can define a risk relatively by comparing two choice alternatives:

Brett's gamble is an 80% chance of winning \$100, otherwise winning \$0 while Elise's gamble is a 40% chance of winning \$250, otherwise winning \$0. Travis has a lower chance of winning with Elise, but stands to win more if she is correct. Travis has a much higher chance of winning a lower amount of money if he chooses Brett.

Travis considers himself a "wildcard" that makes extreme decisions. He really likes the idea of winning \$250, so he starts to hand the phone to Elise. Brett sees that he is about to lose and offers the following modification: "Travis, if you give *me* the phone I'll give you \$100 even if I get the question wrong, which I obviously won't." Now Brett's gamble is made *safer* by changing the possible outcomes. Originally we could calculate the expected value (EV) of each friend's gamble: Brett's was worth \$80 in expectation ($.80 \times \$100 + .20 \times \$0$) while Elise's was worth \$100 ($.40 \times \$250 + .60 \times \$0$). The modification makes the new choice one between a sure \$100 (Brett's offer) and a probabilistic (EV of) \$100 (for Elise's offer). Travis still has a choice to make: a sure \$100 or a chance of winning \$250 (or nothing).

This represents a common choice paradigm used for eliciting risk attitudes. Brett is offering Travis the expected value of Elise's gamble *guaranteed* (Tversky & Kahneman, 1986). Thus the choice is between a lottery and a certain outcome. If Travis chooses Brett's offer, it can be taken as evidence that he prefers to avoid the risk of winning nothing if Elise is incorrect. In some decision making paradigms, the decision maker (DM) is asked to indicate the minimum guaranteed amount they would want to receive in order to not play a lottery similar to Elise's offer. This amount is called the *certainty equivalent*. Some DMs may indicate a certainty equivalent that is below the expected value of the lottery offered. In this case, the DM is willing to forego a lottery that (in expectation) is worth more than the certainty equivalent in order to avoid risk. This DM would be classified as risk averse, they're essentially willing to pay to avoid risk. On the other hand, a DM who indicates a certainty equivalent greater than the EV of the lottery would essentially be demanding additional payment to not play the gamble. This DM would be considered risk-seeking (or "risk-loving", or "risk-tolerant").

This review is concerned with decisions under risk, where the objective probabilities of risky alternatives' outcomes are known or learned through experience. Even when objective probabilities are known to the DM, there is no guarantee that those are the probabilities that DM "uses" when making a decision. Important contributions in psychology (e.g., Kahneman and Tversky, 1979) have posited that people use subjective values for probabilities and for outcomes. In other words, an individual's perception of the risks and rewards of decisions under risk are, for some theories, allowed to vary from the objectively defined values. This is easy to comprehend for Travis' scenario: Travis needed to estimate the probabilities of his friends being correct and that estimation is subject to myriad biases. Even if Travis were explicitly told the probability of each friend being correct, he may perceive the outcome and risks differently than another DM.

Theoretically, a DM will assign some utility value to all choice options when making decisions. Utility is a general term used to represent all of the benefits (or negatives) an individual receives from something. Happiness, money, enjoyment, pleasure, pain or any other feelings can constitute utility. Thus, we refer to the utility of a gamble (or prospect or choice) as some combination of all the positives and negatives a DM considers when experiencing the decision and outcomes. Utility *functions* mathematically describe how a DM will form (or express) preferences in a decision environment. For simplicity, generality and experimental expedience, decision researchers often use money as the argument of the DM's utility function. This is convenient, because money has several properties that are assumed in most utility theories. Namely, the utility of money is assumed to be monotonic: more money is always better than less money (all else equal).

We will see that variations to the basic elements of the example decision problem change the decision environment and have ramifications for choice behavior. *Descriptive* models of decision making are those that aim to accommodate various deviations from normative or *prescriptive* theories of decision making. Normative models posit that DMs are rational agents that attempt to maximize their utility in all decisions (Expected Utility Theory; von Neumann and Morgenstern, 1947). Normative theories represent how rational agents *ought* to behave. Descriptive models acknowledge the "anomalies," instances where real DMs systematically violate the predictions made by normative theories, widely observed in empirical studies of decision making. The development of risk attitude as an independent component of decision that merits its own study began with the accommodation of risk aversion, the tendency for DMs to prefer less risky (or entirely not risky) alternatives, into expected utility theory by Bernoulli (1954).

Risk attitude has become a focal point for many researchers due to its wide practical implications. Determining whether certain decision problem formulations or frames produce different risk attitudes helps decision researchers test the descriptive accuracy of decision theories. This leads to a better understanding of the decision making tendencies of DMs. Exploring whether certain groups of DMs vary in their risk attitude (such as men vs. women, older vs. younger adults, problem gamblers vs. control groups, etc.) can help researchers understand differences in decision quality and preferences and identify risk factors for groups that may be prone to engaging in unnecessarily risky activities. Marketers may want to know which groups of consumers prefer to take risks on products. Some clinical diagnoses (such as borderline personality disorder) have criteria related to risk attitude, making accurate classification of an individual's risk preferences an important concern for obtaining a diagnosis and formulating a treatment plan.

These different fields and decision contexts may have different definitions of what constitutes a "risky" choice, as well as different methods for eliciting or measuring risk attitude. Reconciling these disparate definitions and methods is a crucial endeavor for decision researchers, especially those interested in comparing choice behavior between contexts. Varying decision frames (i.e., gains or losses) and group membership can influence individual risk attitude as well, complicating the

systematic study of risk attitude across choice contexts (or domains) even further. A general method for defining risk and classifying DMs according to their *behavior* that can be applied to a wide range of contexts is necessary for a complete understanding of the effects of frame, context or other important variables on individual risk attitude. We provide a detailed review of one such approach and its recent applications in this article.

This review is organized as follows. First we review the development of risk aversion and risk attitude in three fields concerned with decision making: economics, finance and psychology. We discuss the various definitions of risk and measurements of risk attitude used in financial choice contexts. Next, we review some of the important findings and effects from empirical studies of risk attitude that inform the hypotheses of future work. Finally, we discuss a recently developed methodology for classifying DM risk attitude according to their choice behavior that is applicable to many choice contexts. We argue that methods like this are crucial to developing a complete understanding of how different aspects of the decision environment or of the individual DM affects the DM's risk attitude.

Risk Attitude

Decision making under risk involves DMs making choices between risky alternatives. These risky alternatives involve known outcomes (i.e., payoffs, returns on investments) that are associated with with different known probabilities. Risk aversion is the empirically supported tendency for DMs to prefer less risky choice alternatives when making decisions under risk.

Consider a DM deciding whether to take a 50/50 gamble to win either \$50 or \$0 versus a certain \$20. Risk aversion can be conceptualized as this DM's preference to take the guaranteed \$20 over playing the gamble. This DM prefers to eliminate uncertainty and risk by taking the sure \$20, even though the gamble's expected value is higher at \$25 - this DM would prefer to eliminate the risk in exchange for a smaller-than-expected return. The *certainty equivalent* represents the risk-free amount a DM would require in order to be indifferent between a risky alternative and a guaranteed (risk-free) amount (in this case, \$20 or less). The difference between the certainty equivalent and the expected value (EV) of the gamble is called the *risk premium*, or the compensation required in order to assume the risk of the gamble (about \$5). Risk aversion then coincides with cases wherein the DM's certainty equivalent is less than the expected value of the risky alternative, or when the DM's risk premium is negative. What motivates this "irrational" aversion to risk?

Researchers across a range of disciplines have focused on developing theories and tests to cultivate a better understanding of rational choice behavior and of actual DM choice behavior. Different fields have developed different measures and conceptualizations of risk aversion and risk attitude in general. For example, some conceptualizations of risk contend that negative outcomes must be possible for risk to be properly experienced. Here we discuss examples of the approaches to measuring and understanding risk aversion taken by economics, financial

management and psychology. We first start with a relative common ground in the form of von Neumann and Morgenstern's Expected Utility Theory (EUT; 1947), which has helped give rise to the approaches developed in each of the three fields.

Risk Attitude Across Fields : Expected Utility Theorem

Von Neumann and Morgenstern (1947) developed the expected utility theorem (EUT) to describe the axioms necessary (and together sufficient) for the expected utility hypothesis to hold in decision making under risk. This hypothesis states that a DM's subjective valuation, called "utility," of a gamble is a statistical expectation of the DM's valuations of the potential outcomes. EUT represents the expected behavior of a rational DM and is thus a normative theory of choice. Utility represents the subjective value a DM assigns to the satisfaction acquired from money, goods, services, feelings, emotions or anything else a person may want, need or experience. We denote a utility function as $u(x)$. This utility function describes the preferences of a DM over a set of x objects. A key assumption of the theory is that individuals prefer more utility (e.g., money) to less utility, or that $u(x)$ is a monotonically increasing function. Normative theories of decision making assume that DMs are rational agents that seek to maximize utility. However, vast empirical evidence demonstrates that DMs do not always make choices in line with the expected value criterion which would predict risk neutrality. EUT allows for DMs to be risk averse in the form of concave utility functions. In other words, risk aversion in EUT is simply represented by a concave utility function that illustrates a decline in the marginal utility of wealth (Bernoulli, 1954).

Risk Attitude in Economics

Arrow (1971) and Pratt (1964) independently developed a measure of risk aversion stemming from a foundation of EUT. Risk aversion in EUT is the direct result of a concave utility function. The more curved a concave utility function, the lower the DM's certainty equivalent will be, implying a higher risk premium and more risk averse behavior. The second derivative of the DM's utility function, denoted $u(x)''$, measures the curvature of the utility function. Pratt describes the second derivative of the utility function as insufficient for accurately describing risk aversion because the second derivative is not invariant to affine transformations of the utility function, a key assumption of the von Neumann and Morgenstern EUT axioms.

The Arrow-Pratt measure of risk aversion features $\frac{u(x)''}{u(x)'}$, or the ratio of the utility function's second derivative over its first derivative (denoted $u(x)'$). Any utility function must be monotonically increasing in utility (i.e., wealth) and thus the first derivative must be positive. Changing the sign of the measure produces the Arrow-Pratt measure of *absolute* risk aversion: $-\frac{u(x)''}{u(x)'}$. This measure often uses wealth (w), or an individual's total utility prior to the decision under consideration, as the argument of the utility function (but the von Neumann and Morgenstern axioms are general enough to allow for many choices for x , including wealth (static) or income (flow)). It is important to note that the reference point of $w = 0$ is intended to

indicate complete financial ruin (no wealth at all).

The Arrow-Pratt coefficient of *relative* risk aversion (RRA) is a function of the curvature of the utility function and is defined as $Ru(w) = \frac{-wu''(w)}{u'(w)}$, where w refers to the overall wealth of the DM. An Arrow-Pratt coefficient of RRA with a value greater than zero implies a concave utility function and therefore risk aversion, but the underlying utility function need not be strictly concave/convex over all w . This measure takes the wealth of the individual into account in order to describe how DMs might have different levels of risk aversion (or even be risk seeking) dependent on the diminishing marginal utility of wealth.

Conceptually, the Arrow-Pratt absolute measure of risk aversion measures the actual dollar amount a DM is willing to hold in risky assets (or the proportion of assets for relative risk aversion). Importantly, the wealth level of the individual has implications for his or her level of risk aversion. This property reflects the idea that risk aversion can be rational due to the law of the diminishing marginal utility of wealth. As an individual possesses more wealth, each additional unit of wealth is less satisfying than the last. Thus, the concept of risk aversion in economics is intimately tied to diminishing marginal utility and the curvature of the utility function. Thus, methods of eliciting risk attitude stemming from these conceptualizations rely on the estimation or specification of a utility function that directly *implies* the risk attitude of the DM(s). This consideration can be problematic since DMs may have different utility functions both between- and even within-subjects (Luce, 2000). We discuss this limitation further when we argue for the use of Luce's p -additive risk measurement model, which does not require any utility specification and is general enough to be applied to many choice contexts.

Risk Attitude in Finance

Financial management is concerned with the conflict between an investor's desire to increase returns on investments but to reduce the risk of investments. Finance has a slightly different conceptualization of risk aversion than economics and is commonly measured using the Sharpe ratio (Sharpe, 1966; 1994) or the evaluation of a ratio of risk level to expected return (e.g., risk-return ratios). Markowitz (1952) proposed a mean-variance approach to maximizing return and minimizing risk. This approach is limited in its descriptive capability by the necessity of a quadratic utility function which implies increasing absolute risk aversion (Pratt, 1964) which is not necessarily "rational" nor likely to occur in real DMs. The Sharpe ratio measures the risk premium per unit of deviation of risk for a risky choice alternative (e.g., an asset). The Sharpe ratio is defined as follows:

$$\text{Sharpe Ratio} = \frac{E(R - R_f)}{\sigma},$$

where R is the return rate of the risky asset, R_f is the risk-free return rate, σ is the standard deviation of the risk premium and $E(R - R_f)$ represents the expected return of the risk premium.

In other words, the Sharpe ratio measures which choice alternative provides the better return compared to a common, (nearly) risk-less benchmark (often chosen to be a U.S. Treasury Bill). This measure is used to evaluate the return of components of a portfolio of investments and to help DMs decide between risky choice alternatives. Within the finance literature where risk trades off with returns, an investor must expect higher returns in exchange for assuming a higher risk. No value can be obtained from the risk itself (i.e., risk inherently carries a negative utility to DMs). This implies risk-seeking behavior would only occur if the outcomes or probabilities were sufficient to entice an investor - the risk itself carries no inherent benefit. Psychological research into sensation seeking (Zuckerman, 1990) challenges the idea that DMs are averse to risk in absolute, finding that risk itself can provide positive utility. Indeed, psychology's contributions to the development of risk attitude have been primarily motivated by explaining empirically observed, systematic violations of rational choice and models thereof.

Risk Attitude in Psychology

Rabin (2000) argued that the diminishing marginal utility of wealth alone cannot explain risk aversion in decision making. Expected utility theory provides an inadequate account of risk aversion as illustrated in part by the Allais paradox (1953) and the Ellsberg paradox (1961). These paradoxes present cases where real DM behavior violates the expectations of EUT, but does so in a manner that many would argue is rational. Hansson (1988) showed that an individual with constant absolute risk aversion (CARA, from the Arrow-Pratt perspective) who is indifferent between a guaranteed \$7 and a 50-50 gamble of winning \$21 or nothing actually prefers the sure \$7 gain to *any* gamble in which the probability of gaining money is less than 45% regardless of the size of the potential gain. Thus, the shortcomings in EUT and the Arrow-Pratt measures are both empirical and theoretical in nature.

Psychologists have put forth theories to explain the behavioral tendencies of DMs when choosing between probabilistic, risky alternatives. The most influential of these theories are Prospect Theory (PT) and Cumulative Prospect Theory (CPT) developed by Daniel Kahneman and Amos Tversky (PT: 1977 and CPT: 1992). These psychological theories were developed as an alternative to the traditional expected utility theory (EUT, von Neumann & Morgenstern, 1947) as empirical results proved EUT to be inadequate for describing the choice behavior of many DMs. The most important distinction between PT and Arrow-Pratt or the Sharpe ratio is that PT is a descriptive theory of human decision making which relies on the use of a status quo reference point rather than overall wealth or income. Prospect theory attempts to provide an adequate description of real human decision making behavior, which is well documented to be marked by biases, heuristics and framing effects. In the simplest form, PT relies on the following model during an evaluation phase of the prospects:

$$V = \sum \pi(p_i) v(x_i),$$

where V is the overall expected utility of the outcomes summed over all possible outcomes x_i . π represents the probability weighting function which allows individual DMs to subjectively weight probabilities (p_i) differently than their nominal values. This accommodates the effects of overweighting small probabilities and underweighting large probabilities observed in empirical studies. v represents the subjective value function on the potential outcomes. This is critical for the concept of "loss aversion" where losses carry more subjective weight than gains of the same magnitude: it feels worse to lose \$5 than it feels *good* to find \$5.

PT and CPT view decision making under risk as a choice between prospects. In response to violations of EUT, Kahneman and Tversky theorized that DMs use subjective values (Savage, 1954) and probability weights instead of the explicit outcomes and probabilities given in choice tasks. These weights are subject to distortions from biases and heuristics and do not obey the probability axioms (for they are not probabilities). Prospect theory determines risk attitudes jointly from the subjective value and weights and not solely by the curvature of the utility (or value) function. Concavity in the value function does imply risk aversion, but empirical evidence shows that many DMs have an 'S'-shaped value function that is concave (risk averse) for gains but convex (risk seeking) for losses (Tversky and Kahneman, 1986). This framing effect is known as the "reflection effect" due to the S-shaped reflection around the origin (reference point). Kuhberger's (1998) meta-analysis of 136 empirical framing effect studies found that the reflection effect is generally a reliable finding, but that differences in research design (such as whether framing was manipulated by gain-loss versus by task wording) can produce dramatically different results. More generally, the fact that DMs can have different value functions (and thus different risk attitudes) depending on the framing of the decision as a gain or loss illustrates the importance of the status quo reference point in PT. Whereas the Arrow-Pratt and Sharpe measures operate from a reference point of overall wealth (value of all held assets), PT operates from a reference point of 0 (returns of prospects).

Prospect theory was later revised to CPT (1992) to overcome limitations in the form of violations of first-order stochastic dominance and transitivity of preferences, two essential assumptions for a theory of rational choice behavior. CPT retains the major features of PT. In particular, the asymmetry of risk attitudes for gains and losses is preserved for gains and losses of moderate or high probability (e.g., probabilities greater than .5). For small probabilities (less than .1) the prediction becomes risk seeking for gains and risk aversion for losses.

A risk need not be defined as solely volatility in the possible outcomes (as in the Sharpe ratio), but also the subjective feeling of those outcomes. Inclusion of an initial wealth variable (as in an Arrow-Pratt treatment) is one way to understand how DMs approach a decision differently from one another. Studies have shown that a variety of variables other than the variance of the outcomes affect risk attitude, such as whether the alternatives are purely expected gains or losses (Tversky and Kahneman, 1986) and the stakes of the stimuli (Holt and Laury, 2002). Depending

on how the risky outcomes are framed, the same DM can exhibit significantly different risk attitudes.

Overall, psychologists' models of risk aversion attempt to take the subjective feelings of losses and gains into account. Risk aversion is not simply uncertainty aversion, as losses produce different findings in terms of risk aversion for the same levels of probability (uncertainty) in gains. While some argue that the heuristics and biases program demonstrates a limitation of the abilities of human decision making, other researchers (i.e., Gigerenzer & Goldstein, 1996) argue that heuristics and biases are actually adaptive strategies. In any case, Kahneman and Tversky set in motion a radical change in the evaluation of risk aversion in human decision making which has permeated throughout other fields concerned with risk attitude. Fields dominated by normative models of DMs like economics (e.g., *homo rationalis*) face a challenge of reconciling the failure of normative models to adequately describe the behavior of DMs. Psychology's contribution has prompted a reevaluation of these types of models and spurred interest in developing better descriptive and normative models of choice behavior that is sensitive to risk attitude.

Stimuli-related Effects

Psychology's contributions to the study of risk attitude have focused on identifying predictable deviations from rational behavior and systematic heuristics and biases that could influence (or be influenced by) risk attitude. Many of these effects are in the traditional vein of psychological research, such as differences in risk attitude based on age or gender, but other effects are due to the subjective feelings produced by the specifics of the decision environment. Kahneman and Tversky's (1977, 1979) demonstration of the reflection effect is predicated on manipulations to the decision environment: decisions *framed* as gains tended to produce risk-averse choice behavior whereas equivalent decisions framed as *losses* produced risk-seeking choice behavior. In other words, the framing of the decision problem produced reversals in preference that were inconsistent with rational theories like EUT. We review some of the most important and relevant findings now.

Framing Effects – Gains, Losses and Mixed Gambles

The framing of a choice problem can have a significant impact on the risk attitude of a DM. Framing refers to the decision environment created to support the objective probabilities and payoffs of a choice problem: the design of the decision problem. When the choice stimuli are gambles, as is common in behavioral economics, gains, losses and mixed gambles refer to the outcomes' positions relative to some reference point. This reference point is often 0 (for money), which represents a status quo. Gains provide the DM with an outcome that is better than the reference point, losses provide the DM with an outcome that is worse than the reference point. Mixed gambles have the chance of producing either a loss or a gain from the reference point. Some studies use a variety of frames to explore whether different framings of the same objective probabilities and payoffs produce similar or different choice behavior in DMs, but some others rely solely on gains or losses. If a

researcher hopes to learn something about the general risk attitude profile of a DM, she ought to provide a sampling of frames for the DM to consider as a single frame may obscure the complexity of the DM's risk attitude.

This may be an important organizing framework for explaining the replications and failures to replicate of the framing effect that abound in the literature. This heterogeneity presents a challenge to models of decision making that predict a stable risk attitude across different frames. Not only do different DMs respond to different frames differently, but even a single DM may have a complex risk attitude profile that is sensitive to the framing of the choice problem.

Whether DMs typically have a stable risk attitude or if it varies by frame have produced results that emphasize heterogeneity. Some economic studies have found varying levels of risk *aversion*, but strict risk-seeking behavior has been difficult to observe (Holt and Laury, 2002). Recent work by Davis-Stober and Brown (2013) corroborates the substantial heterogeneity found in the risk attitude of DMs. In their study, some DMs were classified according to a consistent risk attitude across decision environments while others featured risk attitudes that varied by the decision environment. Harrison, Lau, & Rutström (2007) have found that strict risk neutrality appears rare in DMs making real choices. Thus, the search for the conditions under which DMs will maintain a consistent risk attitude remains a key endeavor for decision theorists seeking to develop general models of decision making that accomodates the heterogeneity of individual DMs.

The Reflection Effect and Loss Aversion

The reflection effect (Kahneman and Tversky, 1979) is a seminal finding and prediction of prospect theory. This effect is concerned with the tendency for people to exhibit risk aversion for prospects involving gains while exhibiting risk-seeking behavior for equal-magnitude losses. The loss side of the reflection point has been shown to be steeper than the gains side. In other words, the reflection around the origin is not a symmetric reflection. This is argued to be due to loss aversion, the tendency for losses to “loom larger” than equivalent gains and thus elicit different choices. The reflection effect and loss aversion are key psychological effects for researchers to consider. Indeed, studies are often carried out specifically to explore how the reflection effect may manifest differently for different groups of DMs (Ebner, Freund & Baltes, 2006). Many researchers have replicated Kahneman and Tversky's seminal findings (Kuhberger, 1998) in various research designs, but some recent work has encountered difficulties that spur some interesting questions moving forward. Namely, Davis-Stober and Brown (2013) and Kellen, Mata and Davis-Stober (2017) were concerned with whether individual DMs' risk attitudes satisfy the predictions of the reflection effect or if the effect is partially due to aggregating choice data across DMs. Some of their results suggest very low rates (around 3%) of individual DMs satisfying the reflection effect.

Tymula et al. (2013) report a positive correlation between risk attitude classification on gains and on losses. This finding indicates that individuals who are

more risk averse on gains are more risk averse on losses as well, contrary to the negative correlation predicted by the reflection effect. Hershey and Shoemaker (1980) show that the presence of the reflection effect at aggregate levels does not imply the presence in the individual DMs that comprise the aggregate. This finding again brings to light the importance of caution when aggregating in decision making research. Aggregating choice data across individuals is known to produce inaccurate results and has been strongly advised against (Regenwetter, Dana, & Davis-Stober, 2011). Indeed, recent results using the LPC framework (Davis-Stober & Brown, 2013; Luce, 2010a,b) described later in this article finds very little evidence in favor of the reflection effect. What evidence does exist is limited to specific framing conditions, which again highlights how slight differences in any aspect of a risk attitude task can result in different risk attitude classifications.

Stake and Incentive Effects

Holt and Laury (2002) presented DMs with a menu of ordered lottery choices to assess individual risk preferences across various payment conditions. The payment conditions were constructed to determine whether incentives (whether or not the gamble outcomes were realized or hypothetical) or stakes (the magnitude of the outcomes) significantly affected risk attitude. The menu of choices presented DMs with binary forced choices between 10 gamble pairs, ordered by the probability of obtaining the higher payoff amount in each gamble. The menus were constructed so that Option 'A' presented the risk averse (safer) option and Option 'B' presented a riskier option. The expected value of the gambles varied from favoring the less risky option in 'early' choices (e.g., when the probability of the larger payoff was smallest) to favoring the riskier option as the probability of winning the larger payoff increased for both gamble alternatives. Some menus had low overall payoffs (under \$5), while others were scaled up by factors of 20x, 50x or 90x to simulate high stakes decisions. In this paradigm, a risk-neutral agent would favor the less risky option for the first 4 choices on the menu before switching to the risky option for the remaining 6 choices. Holt and Laury measured risk aversion by counting how many choices were made in favor of option 'A' for each DM and calculated the coefficients of relative risk aversion that corresponded to each level of safe choices made.

Holt and Laury's major findings were that subjects were mostly risk averse, even for very small payoff amounts of under \$5. Risk aversion increased sharply as the scale of the cash (incentivized) payoffs increased, but there was no significant effect of the increased scale for the conditions with *hypothetical* payoffs. Risk-loving behavior (making 3 or less safe choices out of the 10) was relatively rare in Holt and Laury's experiments. Thus, Holt and Laury conclude that risk aversion tends to increase with magnitude (stakes) of the decision environment.

Findings related to the stakes of the decision environment have implications for decisions outside of the realm of money. That is, how do we translate the stakes of one's health or life, or the stakes of catastrophic climate change, into stakes that don't predict absurd levels of risk aversion? DMs routinely choose to engage in risky

activities that risk immense amounts of money (like driving intoxicated) - so how are researchers meant to generalize key findings in decision making to other choice contexts? This represents a key challenge for research in risk attitude: identifying and understanding the conditions necessary for individual DMs to maintain a stable risk attitude across contexts or, more generally, across decision environments. In order to compare risk attitude across choice contexts without relying on utility functions or estimating the value of a statistical life to properly account for stakes, we must classify DMs according to their risk attitude in relation to the alternatives they are presented with and maintain a common methodological approach for classifying risk attitude in each context.

Group-level Differences in Risk Attitude

While framing and other research design considerations clearly have implications for the risk attitude of DMs, so could demographic group membership. Age is one such variable that could impact the decision making strategies and thus risk attitude of age-group members. As a person ages, cognitive limitations and depreciation of capabilities like executive function could pose a threat to stable decision making and preferences. Research into the risk attitude of older adults could provide more clarity on whether risk attitude is impacted by various deficits in cognitive ability. For example, Rui Mata and colleagues have conducted several studies examining risk attitude over the lifespan and across cultures searching for systematic similarities and differences between groups of DMs. Of course, these group-level differences are not limited to age-related effects. We review some of the work that has been conducted in the search for group-level differences in risk attitude now.

Age-related Differences in Risk Attitude

As people age, some of their cognitive capabilities tend to decline or change. Executive function, for example, is sometimes depreciated in older adults, which has implications for decision making patterns and thus decision quality and life outcomes. Are older adults systematically more risk averse than younger adults? Do younger adults tend to take more risks and be more tolerant of risks than older adults? In which decision contexts are there age-related differences? This topic is of particular interest due to the documented decline in decision quality observed in some older adults coupled with documented declines in cognitive function as people age. This could be due to diminished cognitive ability, different risk attitudes, less stable decision processes, increases susceptibility to framing effects or many other mechanisms. It is important to disentangle the effects of individual risk attitude from other variables in order to accurately assess whether aging is a risk factor for decision quality. We review some relevant research carried out to compare older and younger adults in a variety of decision making tasks.

Roalf, Mitchell, Harbaugh and Janowsky (2012) compared the risk attitudes of a sample of 30 older adults to a sample of 29 younger adults. The authors found that older adults were less tolerant of risks and more risk averse than younger adults. These age-related differences were domain specific (e.g., some choice contexts

produce differences while others do not). The authors find that perceiving more risk was associated with poorer decision outcomes for older adults, providing some evidence that the poorer decision making outcomes for some older adults may be due to higher perceived risk and higher risk aversion.

Rui Mata and colleagues have conducted substantial work exploring age differences in risky decision making. Mata, Josef, Samanez-Larkin and Hertwig (2011) conducted a meta-analysis of age differences in risky choice. Their meta-analysis was comprised of 29 studies comparing older adults to younger adults in various experiments aimed at measuring risk taking. The authors concluded that the age-related results of these studies depend critically on the specifics of the task (i.e., the learning required, whether decisions are from experience or not). The authors found no clear age-related differences as a function of gain/loss framing. This finding is important, as age-related differences in risk attitude attributable to gain/loss framing lends support to theories of decision making that posit that cognitive deficits in older adults reduce their decision making quality.

Mata and colleagues' meta-analysis found little difference between older and younger adults in risk decisions from descriptions where risk is objectively defined, which includes paradigms like lottery versus sure outcome choices, blackjack or the Cambridge Gambling Task (CGT). Age-related differences were strongest in decisions from experience, where learning is essential and risk is learned, such as in paradigms like the Iowa Gambling Task (IGT) or the Balloon Analogue Risk Task (BART). These differences were often in the form of older adults resisting movement toward the "learned" response. For example, older adults drew more from the risky ("bad") decks in the IGT studies than younger adults. This resulted in higher risk seeking prevalence in older adults than younger adults. This highlights the importance of task structure in measuring risk attitude. The p -additive risk attitude task we describe later is general enough to be applicable to monetary and non-monetary outcomes. This helps avoid the influence of substantively different tasks being used to measure the same underlying construct of risk attitude.

Mata and colleagues' further comment that older adults may rely more heavily on simple decision rules or heuristics. Indeed, Mata, von Helversen and Rieskamp (2010) documented poorer decision performance in older adults compared to younger adults in a decision environment favoring the use of a compensatory (and more cognitively demanding) strategy. Mata, Schooler and Rieskamp (2007) conducted a study which found that older adults tended to look up less information and use simpler decision heuristics when making choices. These results point toward different decision processes at work for older and younger adults. These results further highlight the complexity of eliciting risk attitude, as task structure and many potential covariates could influence the observed risk attitudes of older and younger adults.

Harbaugh, Krause and Vesterlund (2002) examined the risk attitude of children and adults in choices between a simple gamble and the expected value of the gamble in gain and loss frames. The authors found that many participants were risk seeking

for high-probability gains and risk averse for low probability gains, while losses produced the opposite (risk aversion for high probability, risk seeking for low). The results for children are consistent with underweighting of low-probability events and over-weighting of high-probability events observed by Tversky and Kahneman (1986). The authors conclude that this tendency diminishes with age, where adults appear to use the objective probability. This finding might be at odds with other researchers' suggestion that aging increases a reliance on heuristics and affective processing – two tendencies that would likely mean older adults fail to use objective probabilities (Peters, Hess, Vastfjall and Auman, 2007). Some researchers have found that older adults show an exacerbated reflection effect due to an increased focus on loss avoidance and maintenance (Ebner, Freund & Baltes, 2006). Recent work by Kellen, Mata and Davis-Stober (2017) used the LPC approach to compare older to younger adults and found that both groups satisfied the reflection effect only about 3% of the time for pure gains and losses, indicating no clear difference between the age groups and very little evidence for the reflection effect in individual DMs in pure frame conditions.

Why do older adults produce different risk attitudes and decision making behavior than younger adults, and in which situations will age differences emerge? Peters, Hess, Vastfjall and Auman (2007) explored age differences in a wide range of decision making scenarios. They argue that three mechanisms might help explain the behavior observed in older adult decision makers in deliberative decision making tasks. The authors argue that aging is accompanied by deficits in the efficiency of systems needed for deliberative reasoning like memory and learning (Salthouse, 2006). Further, the processing and use of emotional information (making decisions based on morals or emotional feelings) appears to become more important with age and people tend to improve (or at least consider themselves improved) in emotional processing with age. One example of this emotional processing is the theorized emphasis and attention to positive information later in life, the so-called “positivity effect” documented by Mather and Carstensen (2005). A reliance on emotional information, or a disproportionate focus on only positive information, could help explain age-related differences in some tasks. These types of biases are related to the use of heuristics, simple decision rules that tend to ignore some information in the decision environment. Researchers interested in age-related differences in decision making have explored this closely related topic and have found evidence that older adults may rely on heuristics more than younger adults. Whether risk attitude differences *result* from heuristic use or if they persist on their own is an important area of exploration for decision theory.

In summary, the mixed results on age differences are complicated further by framing effects and other aspects of the decision environment. Thus, careful experimentation that systematically varies these aspects is necessary in order to sort out the sources of heterogeneity in risk attitude between age groups, or any group for that matter. Overall, risk aversion appears to be more common as people age while risk seeking behavior becomes less common. Interesting hypotheses can be gleaned from these results for researchers to validate or refute, like whether

older adults are more susceptible to framing effects like the reflection effect or if they show patterns of risk attitude that vary according to the choice context.

Gender-related Differences in Risk Attitude

Are females more risk averse than males? Do males take more risks than females? These types of questions are important to many researchers and laypeople in general. For example, studies of management make use of results on gender differences in risk taking to model the effectiveness of managers and management styles. Eckel and Grossman (2008) conducted a study which found that females were more risk averse than males. Further, when asked to predict the risk attitude of other experimental subjects, both males and females tended to forecast more risk aversion for females than for males. Decision research has produced mixed results on whether gender has a predictable effect on risk attitude (Weber, Blais and Betz, 2002), with some gender differences hinging on complicated interactions with other variables like age or the choice context. We review some of these results which highlight the need for a general method of risk attitude modeling that can tease apart the possible sources of risk attitude variability.

Harris and Jenkins (2006) asked 657 participants to assess their likelihood of engaging in different risky activities (gambling, health, recreation and social). The authors find that females reported a greater perceived likelihood of negative outcomes and a lesser expectation of enjoyment from the risky activities than their male counterparts. These findings are used to help explain why males tended to report a higher willingness to take risks than females in all contexts (domains) except for social risks. Weber, Blais and Betz (2002) explored gender differences across different risky choice domains and found similar results to Harris and Jenkins - the choice context as well as gender produced different risk attitudes in DMs. Here we see evidence that the context of the decision is an important factor to consider, but in general the evidence appears to converge toward higher risk taking intentions in males than in females. Weber et al. (2002) argue that the main differences between the genders arises out of risk *perceptions*. We note that the p -additive LPC approach described herein features gamble pair stimuli that have equal expected value and equiprobable (50/50) outcomes which limits the role that perceived/subjective probabilities could play in affecting risk attitude.

Sarin and Wieland (2016) found that females are *not* more risk averse than men when probabilities are unknown (decisions under uncertainty). However many other researchers, including Sarin and Wieland (2016), have found that women are usually more risk averse than men when probabilities are objectively defined (Byrnes, Miller and Schafer, 1999). These gender differences appear sensitive to age (diminishing with age) and decision context. Further research into the mechanisms for females' increased risk aversion has identified decision context and emotional reactions to losses as potential contributing factors. Specifically, Eriksson and Simpson (2010) attribute more pronounced loss aversion in females to explain a higher prevalence of risk aversion.

The investigation of group-level differences in risk attitude is obviously not restricted to age and gender. Many other variables like education, income, employment status and clinical diagnoses could have an impact on the risk attitudes of members of various groups. In order to explore these, methods must take into consideration the complications of framing, aggregating across DMs and choice context. With appropriate general methods, investigations into the nature and stability of risk attitudes both within and across contexts, frames and groups can be carried out.

The next chapter discusses one such methodology which we call the "LPC approach" or equivalently the " p -additive approach" developed by Luce (2010a,b) and extended by Davis-Stober and Brown (2013) as well as Kellen, Mata and Davis-Stober (2017). This methodology classifies DM risk attitude according to preferences and indifferences on gamble stimuli that can be constructed in a variety of choice contexts. The LPC provides a method for constructing gamble stimuli that distinguish between risk averse, risk seeking and risk neutral "types" of people. After classifying DMs individually according to their risk attitude in within-subjects experiments, comparisons across contexts, frames and covariate groups can be carried out to determine the generality of the results summarized in this review. We argue that this approach carries many advantages for the risk researcher. We now describe the approach and two recent experimental applications of the approach. We conclude with a discussion summarizing the review and commenting on future applications of the approach to non-financial choice contexts.

Chapter 2: p -Additive utility and Luce's proposed criterion: A general methodology for assessing risk attitude Introduction

Work on p -additive utility theory by Luce (2010a,b) led to the characterization of three types of utility representation referred to as "the three types of people." Luce (2010b) argued that individual DMs must be classified according to type prior to data analysis, due to potential confounds created by aggregating utility functions across individuals. Luce and colleagues (Luce, 2010a,b; Ng, Luce, & Marley, 2009) derived a criterion, both necessary and sufficient, for determining the appropriate p -additive representation for a DM, which is termed Luce's Proposed Criterion (LPC). This criterion is defined in terms of a DM's pairwise preferences and indifferences among a set of binary gambles. The full approach provides for the construction of general experimental choice stimuli that feature binary 50:50 gambles each with equal expected value. These stimuli can be used in conjunction with an appropriate statistical analysis framework to classify DMs according to type across decision environments and choice contexts.

Luce (2010a,b) demonstrated that these three p -additive representation types directly correspond to the risk attitude of the DM: risk-averse, risk-neutral, or risk-seeking preferences. We use the term "decision environment" to refer to the set of probabilistic outcomes under consideration, e.g., for binary gambles, all possible payouts and probabilities of winning. If a DM's risk attitude varies within a decision environment then a single p -additive representation type may not be descriptive of her choices.

Davis-Stober and Brown (2013) developed a method to extend the original LPC to evaluate DMs whose p -additive representation type is allowed to vary as a function of the decision environment. They constructed a series of p -additive representation models whose type assignments are motivated from some existing empirical results on risk attitude reviewed earlier. This extension contained the original "three types" as special cases. In a new experiment, they applied a Bayesian methodology (Karabatsos, 2005) to the framework, which allowed the direct comparison of the various p -additive models via a closed-form Bayes factor computation. This methodology *jointly* evaluated whether a DM's preferences are consistent with either single or multiple p -additive representation types within a given decision environment, and, importantly, which one(s) best account for a DM's choices. Kellen, Mata and Davis-Stober (2017) later modified this approach to classify DMs within each decision environment *independently* rather than jointly across all decision environments. Both approaches have advantages which we discuss in detail later.

This methodology can be cast as a general method to evaluate the risk attitude of a DM. By utilizing the p -additive theory, risk attitude is defined in terms of DM preferences and indifferences, as opposed to the shape of an estimated utility

function (e.g., Arrow-Pratt (1964)). The methodology is sensitive to DMs with risk attitudes that are dependent on the decision environment (i.e., on frame or context). Davis-Stober and Brown (2013) found that no single p -additive representation model emerged as the best description of the DMs' preferences. This is consistent with related analyses on choice variability (e.g., Cavagnaro and Davis-Stober (2014); Loomes, Moffatt, and Sugden (2002)). The authors found that participants were best described by stable risk attitudes (across all frames) and varying risk attitudes (which vary as a function of the decision environment) in roughly equal numbers. The empirical results suggest substantial heterogeneity among DMs with respect to risk attitude, a result that has been later replicated by Kellen, Mata and Davis-Stober (2017) using the same p -additive framework, as well as by many other studies.

We now summarize the p -additive theory and present the LPC as Davis-Stober and Brown (2013) did previously. We briefly discuss how the LPC can be extended by partitioning the space of all LPC-consistent gamble pairs into regions corresponding to models of risk attitude, or be applied granularly to classify DMs within each decision environment independently, affording greater exploratory flexibility. We show how a Bayesian statistical methodology integrates with the LPC approach and facilitates model comparison. Finally, we review the results of the experiment and compare Davis-Stober and Brown's (2013) methodology and results to those of Kellen et al. (2017).

p-Additive Utility and Luce's Proposed Criterion – Technical Description

As described in Luce (2010a,b), let X be a set of outcomes under consideration. Let $\succsim$ be a weak preference order over X , such that $x \sim y$ implies $x \succsim y$ and $y \succsim x$. Hence, we assume that $\succsim$ is a transitive binary relation. We define $\oplus$ as the binary operation of joint receipt, i.e., $x \oplus y$ defines a new consequence in which both x and y are received. Due to $\sim$ defining an equivalence relation, we assume that $\langle X, e, \oplus, \sim \rangle$ is an Abelian group with identity e . We will be working directly with the algebraic structure which we assume is a solvable, Archimedean-ordered, Abelian group with an isomorphism onto the algebraic structure $\langle \mathbf{R}, 0, +, \geq \rangle$. Hence, we assume that preference $\succsim$ is transitive and that the binary relation $\oplus$ is both associative and commutative.

Often we are interested in the special case where outcomes are monetary values and thus are represented as elements of $\mathbf{R}$. Under the standard Hölder assumptions (e.g., Luce (2000)) and by extending the representation space from the additive reals to the full reals, a p -additive utility representation is defined as follows (Luce, 2010a,b):

$$U(x \oplus y) = U(x) + U(y) + \delta U(x)U(y), \delta = -1, 0, 1,$$

where U is an order-preserving function with $U(0) = 0$. An equation on the reals of the type,

$$f(x + y) = F[f(x), f(y)],$$

is called an addition theorem. If F is a polynomial, then the most general solutions are: with $A \neq 0$, B and C arbitrary constants,

$$F(u, v) = u + v + C$$

, and

$$F(u, v) = Auv + Bu + Bv + ((B^2 - B)/A).$$

If $F(u, 0) = u$, then $C = 0$ and $B = 1$, and the general solution is,

$$F(u, v) = Auv + u + v.$$

Under the more general context where outcomes are not necessarily reals (such as in a restaurant selection or health choice context), Luce and colleagues derive that the utility function U has the form,

$$U(x \oplus y) = \delta xy + x + y.$$

In analogy with the result on the reals with standard addition, Luce (2000) terms this a *p*(olynomial) additive representation, and by a scale argument shows that no generality is lost by assuming $\delta = -1, 0, 1$. When $\delta = 0$, Eq. (1) reduces to a simple additive representation:

$$U(x \oplus y) = U(x) + U(y).$$

For the cases $\delta \neq 0$, Eq. (1) is equivalent to the following,

$$1 + \delta U(x \oplus y) = [1 + \delta U(x)][1 + \delta U(y)].$$

Let $V(x) := 1 + \delta U(x)$, now (2) becomes the multiplicative representation, $V(x \oplus y) = V(x)V(y)$. The logarithm of this representation is additive, which accounts for the name "*p*-additive" as it is the only polynomial utility representation that can be transformed into an additive representation (Luce, 2010a,b). It is important to note that U is a ratio scale representation when $\delta = 0$ and U is an absolute scale when $\delta \neq 0$. Clearly, the value of the δ parameter determines the *p*-additive representation of the DM. See Luce (2000) for a detailed overview of *p*-additive representations.

To apply the theory to uncertain alternatives, let C and D be disjoint chance events from an experiment with universal set Ω .

Let $\bar{C} := \Omega \setminus C$. Define $(x, C; y, \bar{C})$ as a simple 2-branch gamble in which x is received when C occurs and y is received otherwise. To link the *p*-additive theory to these structures, we require two important assumptions (Luce, 2010b). The first is segregation, defined as:

$(x \oplus y^{-1}, C; e, D) \oplus y \sim (x, C; y, D)$, where $y^{-1} \oplus y \sim e$. Let $S(\cdot)$ be a subjective weighting function on events. The second assumption is that of complementarity (Luce, 2010b) which requires:

$$S(C) + S(D) = 1.$$

Readers may see Luce (2010a,b) and Marley and Luce (2002) for a thorough discussion of these assumptions and other related derivations.

Luce and colleagues (Luce, 2010a,b; Ng et al., 2009) derived a series of equivalent algebraic conditions which relate the three values of δ via a series of "if and only if" statements. In other words, they provide a series of testable conditions for determining the corresponding δ value of a DM. The most applicable of these conditions is what we refer to as "Luce's Proposed Criterion." Let ">" denote strict preference and "~" denote indifference. Let $(x, C; y)$ be a simple binary gamble where an amount x is won (or lost for negative values of x) if C occurs, and y is won (or lost for negative values of y) if C' occurs. Luce's Proposed Criterion (LPC) is defined as follows.

Luce's Proposed Criterion

For all $x, x', y, y' \in \mathbb{R}$, with $x > x' > y > y'$,

$$\delta \Leftrightarrow (x \oplus x', C; y \oplus y')$$

where $S(C) = S(C') = \frac{1}{2}$. We assume that $x \oplus y = x + y$, i.e., since we are interested in monetary values, we assume that jointly receiving \$10 and \$5 is the same as receiving \$15. This assumption may or may not be valid for other, non-monetary choice contexts. Furthermore, we treat the subjective weighting of the gamble outcomes as probabilities (Luce, 2010a). The LPC, under these assumptions, is defined in terms of preferences between pairs of binary 50–50 gambles with equal expected value in which the gamble on the left has a strictly larger variance (the "risky" gamble) than the one on the right (the "safe" gamble). Ng et al. (2009) demonstrated that the images of the function δU under the three values of δ are:

In other words, under $\delta = 0$, U is unbounded from both below and above. Under $\delta = -1$, U is unbounded from below and bounded from above. Under $\delta = 1$, U is bounded from below and unbounded from above.

This work demonstrates quite clearly that the three possible values of δ directly correspond to the risk attitude of a DM. The value $\delta = -1$ corresponds to risk-averse preferences where the function V is order reversing and U is bounded from above by 1. The value $\delta = 0$ corresponds to risk-neutral preferences (via the additive ratio scale representation). The value $\delta = 1$ corresponds to risk-seeking preferences where V is order preserving and U is bounded from below by -1 .

Risk attitude has typically been operationalized as the shape of a utility function where the concavity, respectively convexity, of the function indicates risk-averse,

respectively risk-seeking, preferences (Arrow, 1971; Pratt, 1964). Measures risk attitude defined in this way typically depend upon the choice of utility function (e.g., Levy and Levy (2002)) and/or methods of estimation thereof (Luce, 2000). This can be problematic, as different choices of utility function may indicate different risk attitudes given the same set of data. In contrast, the LPC does not depend upon a specific choice of utility function (Luce, 2010a,b). In addition, the LPC contains several other risk attitude criteria as special cases (Schneider & Lopes, 1986) such as Tversky & Kahneman's (1992) paradigm. These special cases involve the comparison of a risky lottery to a degenerate case of the LPC resulting in a sure outcome of the gamble's expected value (EV). We now summarize some of the practical advantages for researchers interested in using this methodology.

Risk Attitude Classification using the LPC – Practical Advantages

- 1) *The LPC is general and not restricted to choices involving money.* Any set of outcomes (x , x' , y and y') that can be ordered in terms of preferences ($x \succ x' \succ y \succ y'$) can produce LPC-consistent stimuli. This allows risk attitude to be measured in a variety of substantive areas using the same principled approach rather than tailoring the experimental and analytical approach for each area separately. This generality is crucial for proposed work that examines risk attitude in the contexts of climate change (where stimuli are changes in temperature in degrees rather than money) and mortality (where stimuli are changes in the 10-year chance of the participant's death).
- 2) *The LPC method does not require any utility functions to be specified or estimated.* Contrary to other approaches like Arrow-Pratt (1964), the LPC is defined entirely on observed choices between gambles with equal expected value. The resulting classification can rule out entire classes of decision theories as incompatible with the observed preferences of the DM. For example, a DM who exhibits risk-seeking preferences for all decisions framed as gains would not be well described by prospect theory, which predicts risk aversion for gains, or by EUT, which would predict either risk aversion or risk neutrality. Avoiding the specification or estimation of utility functions frees researchers from potential confounds caused by different utility functions implying different risk attitudes on the same set of data.
- 3) *The LPC is defined solely on DM choice responses to simple binary gambles.* The LPC experimental stimuli are very simple to create and for DMs to understand. Each LPC-consistent gamble pair is the choice between two 50/50 gambles. These gambles might be represented as a coin flip or dice roll where the chances of any outcome are equal. This simplicity, combined with the generality of the LPC, allow for this approach to be applied broadly, even to children and individuals who struggle with understanding probability. One drawback, though, is that theories that make risk attitude predictions that depend on differences in probability (i.e., those that predict a different risk attitude for small probability events than large probability events) may not be distinguishable under this approach. Overall the LPC approach defines risk in degree using simple stimuli that can be generalized to contexts outside of monetary decisions.
- 4) *The LPC, combined with appropriate statistical methodology, permits the classification of individual DMs into models of risk attitude that may vary by*

decision environment. The original Davis-Stober and Brown (2013) approach used a Bayes factor methodology to compare models of risk attitude jointly across the different decision environments. Kellen, Mata and Davis-Stober (2017) slightly modify this approach to a more granular level by classifying DMs individually at the decision environment level. In other words, the original approach would classify an individual as risk averse if the risk aversion p -additive model produced a sufficiently large Bayes factor for the choice data across all decision environments (gains and losses, pure and mixed gambles, high and low stakes, etc.). Complex models of risk attitude make differential predictions for each decision environment, thus the models considered in Davis-Stober and Brown (2013) are jointly modeled. The granular approach would simply classify each DM within each decision environment *independently* according to one of Luce's (2010) original "three types" corresponding to risk-averse, risk-neutral or risk-seeking preferences. This allows more freedom to explore complex models of risk attitude instead of restricting the search to models already defined in the literature. Further, it will allow for new exploration into the effects of decision environment on the risk attitudes of groups of individuals.

Experiment Method and Results

Twenty-four subjects were recruited from the University of Missouri to participate in Davis-Stober and Brown's (2013) study. The study was conducted on a personal computer. Participants were presented with a series of paired comparisons among LPC consistent gambles. Figure 1 displays a sample screen of how the stimuli were presented. Participants were asked to indicate preference/indifference for a total of 26 distinct LPC gamble pairs, with each pair presented 12 times, for a total of 312 trials. All gambles used in this study are presented in Table 1. The order of presentation of the LPC gambles was fully randomized. Also, whether the risky/safe gamble was displayed on the left-hand/right-hand side of the screen was randomized and counter-balanced (six presentations each side). Participants made their choices by pressing keys on the keyboard, with the "1" key indicating the gamble displayed on the left, the "0" key indicating the gamble displayed on the right and the "Space Bar" indicating indifference between the two gambles. Participants were allowed as much as time as they needed to make each choice. Participants were each compensated \$10 for completing the hour-long session. Participants were informed at the beginning of the study that all gambles were 50–50 binary gambles. The experimental display used 'coins' (see Figure 1) to reinforce the fact that probabilities of receiving either outcome were equal for each outcome in the gamble.

Figure 1: This figure shows an example experimental trial from Davis-Stober and Brown (2013). This trial is a gain-only trial (GG) with the riskier option appearing on the left. Participants were presented with 312 trials like this and asked to indicate preference for the left gamble, the right gamble or indifference between them using a computer keyboard (pressing '1', '0' or 'Space Bar' to indicate preference, respectively).

Risk Attitude Models Considered

The six p -additive representation models Davis-Stober and Brown (2013) considered for the experiment were:

- 1) Strict Risk Aversion with $\delta = -1$ (RA)
- 2) Strict Risk Seeking with $\delta = 1$ (RS)
- 3) Strict Risk Neutrality with $\delta = 0$ (RN)
- 4) Gain-Loss Sensitivity with δ_+ (gains) = -1 and δ_- (losses) = 1 (GL)
- 5) Stake Sensitivity with $\delta' = -1$, $\delta = 1$ and $k = \$20$ (SS)
- 6) Mixed Gamble Sensitivity with $\delta_+ = -1$ and $\delta_- = 1$ with all mixed gamble pairs assigned $\delta_{mx} = -1$ (MG).

Each model makes specific predictions regarding the risk attitude (δ value) that ought to be expressed in each decision environment - determined by the stakes and whether the environment features gains, losses or both. These 26 gamble pairs were created to elicit responses to distinguish between the models of risk attitude considered. Gamble types included high- and low-stakes versions of pure gain, pure loss and mixed gambles (gambles with a chance of winning or losing). Similar to the method employed by Holt and Laury (2002), high-stakes gambles were created by multiplying the low-stakes gambles by 20 in the gain- and loss-only trials. High-stakes mixed gamble pairs were created to fall within a similar monetary range as high-stakes gain or loss pairs.

Statistical Model Comparison Framework - Accomodating Choice Variability

Choice data are comprised of DM responses to a series of gamble pair presentations, with gamble pairs constructed in a systematic way so as to differentiate the p -additive representations of interest. Once these gambles are constructed, choice data will generally be collected by repeatedly presenting gamble pairs to the DM, whereby the DM indicates his preference/indifference for each presentation trial. Repetitions of gambles are included distinguish choice variability from the underlying structural preference of the individual (Regenwetter and Davis-Stober, 2012). Because the DM is allowed to express indifference, this framework is often termed a ternary choice experimental paradigm (Regenwetter and Davis-Stober, 2012), as opposed to binary choice (see Regenwetter, Dana, and Davis-Stober (2011), for a discussion).

Since DMs are not deterministic in their choices, Davis-Stober and Brown (2013) consider the probability of a DM choosing one gamble over another. Let $\mathbf{K}$ be the set of all distinct LPC gamble pairs presented to the DM and let (G_r, G_s) be a gamble pair from $\mathbf{K}$. Define $\theta_{(G_r)}$ as the probability that G_r is chosen over G_s for the pair $(G_r, G_s) \in \mathbf{K}$, likewise $\theta_{(G_s)}$ denotes the probability that G_s is chosen over G_r . Let N_{G_r} be the number of times G_r is chosen over G_s , likewise let N_{G_s} denote the number of times G_s is chosen over G_r . Assuming a balanced design, let $\mathbf{N}$ be the number of times each individual gamble pair in $\mathbf{K}$ is presented to the DM. To simplify notation, let $\boldsymbol{\theta} = \theta_{(G_r)}, \theta_{(G_s)} \forall (G_r, G_s) \in \mathbf{K}$ and $\mathbf{N} = (N_{G_r}, N_{G_s}) \forall (G_r, G_s) \in \mathbf{K}$.

Assuming independence between trials, a DM's choice responses follow a trinomial distribution with the following likelihood,

$$L(\boldsymbol{\theta}, \mathbf{n}) = \theta_{G_r}^{N_{G_r}} \theta_{G_s}^{N_{G_s}} (1 - \theta_{G_r} - \theta_{G_s})$$

where $0 < \theta(G_r) + \theta(G_s) < 1, \forall (G_r, G_s) \in \mathbf{K}$. Importantly, this Bayesian methodology only requires the weaker assumption of exchangeability (e.g., Bernardo (1996)), not full independence, between trials. Exchangeability states that the likelihood of a DM selecting a gamble from a given LPC pair on the i_{th} presentation is the same as the likelihood of a DM selecting that same gamble, from the same LPC pair, on the j_{th} presentation of that pair.

Bayesian Methodology

Karabatsos's (2005) Bayes factor methodology is applied to evaluate choice data within the framework. This methodology applies a Dirichlet prior to a multinomial (trinomial) likelihood. For a binary choice design (where indifference is not permitted), this approach would use a conjugate Beta prior for a binomial likelihood. The approach results in a closed-form solution for the Bayes factor between any two p -additive representations. See Karabatsos (2005) for the technical details and derivations.

Similar to Karabatsos (2005), decision models are encoded via the parameters of the Dirichlet prior distribution. The conjugate prior for any multinomial distribution

with m -many categories is the Dirichlet distribution, with the following probability density function:

$$f(\theta_i; \alpha_i) = \frac{1}{B(\alpha_i)} \prod_{i=1}^m \theta_i^{\alpha_i-1}$$

where,

$$B(\alpha_i) = \frac{\prod_{i=1}^k \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^m (\alpha_i))}$$

and $\Gamma(\cdot)$ is the gamma function. For the trinomial likelihood, $m = 3$, thus, a set of three α_i values was specified for each gamble pair and model combination. α_1 is the prior value corresponding to $\theta_{(G_r)}$, α_2 is the prior value of $\theta_{(G_s)}$ and α_3 is the prior probability of choosing indifference, $(1 - \theta_{(G_r)} - \theta_{(G_s)})$. For a given LPC gamble pair, let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. Davis-Stober and Brown (2013) specified an α value of 5 for the choice predicted by the model and 1 for the remaining alternatives. This choice of prior values places heavy weight on the option predicted by the model, but also allows for the possibility of "error" on the part of the DM by placing weight on options not predicted by the model. For example, the prior values for a single gamble pair where the risky choice is predicted would be $\alpha = (5, 1, 1)$, corresponding to a roughly 20 percent error rate in observed choices of that DM for that LPC gamble pair. If K is comprised of k -many LPC gamble pairs then k -many prior specification triples, α , would need to be specified for each risk attitude model.

In summary, under a given model of p -additive representation, choice predictions for each LPC gamble pair in K are obtained. Then the appropriate prior values corresponding to each choice prediction are assigned. The marginal likelihood for a given model is the product of the likelihood with the corresponding k -many Dirichlet priors. Bayes factors (BF) are computed in order to assess the fit of the various p -additive models. The BF is simply the ratio of two marginal likelihoods and provides an estimate of the evidence for one model over another. As in Karabatsos (2005), a reference model with a Jeffreys prior was created using values of $\alpha = (0.5, 0.5, 0.5)$ for every gamble pair in K . This reference prior is diffuse and can be considered as a type of "null" model to compare the p -additive representation models against. This prior was chosen because it is invariant under one-to-one transformations of the parameter space (e.g., Karabatsos (2005)), unlike a uniform prior.

Results and Discussion

Davis-Stober and Brown (2013) found substantial heterogeneity between subjects in their classification results. Table 1 (below) shows the counts of DMs classified according to each p -additive representation model.

Model	Count

Reference	9
Risk Averse	5
Risk Neutral	2
Risk Seeking	1
Gain-Loss Sensitive	4
Stake Sensitive	1
Mixed Gamble Sensitive	2

Table 1: Davis-Stober and Brown (2013) risk attitude classification results. Nine of the 24 DMs were not fit by any p -additive representation. The remaining DM classification were heterogeneous, with some maintaining a strict risk attitude across decision environments.

Over a third of the DMs were not fit by any substantive model of risk attitude. Of those who were, roughly half expressed a strict risk attitude invariant to frame or stake manipulation while the other half showed some more complex risk attitude. The authors conclude that these results demonstrate substantial heterogeneity between subjects as well as within-subjects, providing evidence for some DMs having complex risk attitudes that depend upon the decision environment.

Kellen, Mata and Davis-Stober (2017) Comparison

Kellen, Mata and Davis-Stober (2017) recently carried out an experiment using the LPC approach as well as reanalyzed the Davis-Stober and Brown (2013) data set. The major difference between the two studies is in the classification approach. Whereas Davis-Stober and Brown (2013) classified individual DMs jointly across the decision environments, Kellen et al. (2017) carried out classification *independently* within each decision environment. This choice was made after Kellen et al. noticed the high non-classification rate observed in the original analyses. Kellen et al.'s granular approach reduced the prevalence of non-classified DMs by relaxing the strict stochastic specifications Davis-Stober and Brown (2013) used to assess the fit of empirically supported models of risk attitude. Kellen et al. argue that the granular approach provides a stronger measure of risk which facilitates exploration of precisely how individual DMs and groups of DMs respond to individual frames.

Kellen et al.'s new experiment featured an LPC-consistent financial context administered to 29 younger adults (age under 35) and 30 older adults (age 61-78). The authors use a Normalized Maximum Likelihood (NML) statistical method which has been widely used in similar contexts to carry out model comparison. Their results corroborate the substantial between-subjects heterogeneity observed by Davis-Stober and Brown (2013) and other researchers. Interestingly, Kellen, Mata and Davis-Stober (2017) find little evidence of older adults being more prone to risk aversion than their younger counterparts, and frequently found a higher prevalence of risk seeking behavior in older adults, contrary to their hypotheses and previous results. Kellen et al. find little evidence for a reflection effect (risk aversion in gains and risk seeking in losses) except for younger adults in the mixed gamble frames where about 55% of DMs satisfied the reflection effect compared to only 13% for

older adults. While most DMs varied in their risk attitude at least once across frames, older adults surprisingly showed *greater* consistency than younger adults (96% compared to 67%). These results run counter to previous results suggesting older adults are more susceptible to framing effects or have less stable preferences and decision strategies, which may lead to poor decision quality.

Kellen et al.'s reanalysis of the Davis-Stober and Brown (2013) data set demonstrated a decrease in the amount of non-classified DMs. The authors found that the reanalysis results closely resemble the younger adult results from their new experiment, in particular by a reflection effect that is restricted to mixed gains and losses and roughly similar classification distributions.

Conclusion

How people make risky decisions is a concern for many choice contexts. Whether those decisions are everyday choices like choosing between routes to work or critical ones like choosing between cancer treatment options, it is important for researchers to understand how aspects of the decision environment differentially affect DMs. Some DMs may be susceptible to "framing" effects, while others may have stable risk attitudes that guide their decisions no matter the context. Disentangling the heterogeneity between and within-subjects in risk attitude remains a key challenge for researchers. This review has summarized the development of risk attitude measurement across fields, focusing on the key empirical results psychology, economics and other fields have discovered. However, for a complete understanding of how individual DMs express risk attitude in various contexts and frames, a general methodology is required.

A recent methodology provides a general approach for classifying DMs that avoids many of the problems that alternative approaches face when exploring risk attitude within individual DMs and across choice contexts. We reviewed two recent applications of the p -additive approach which add further evidence for the heterogeneity of DM risk attitude. A lack of a substantial reflection effect in individual DMs casts suspicion on the generality of the effect. Future work with the LPC approach focuses on exploring the conditions of the decision environment that systematically influence risk attitude in DMs.

Chapter 3: Stability of Risk Attitude Across Choice Contexts and Frames

Abstract

We report the results of a large-scale study of risk attitude demonstrating substantial heterogeneity both between and within decision makers across three risky choice contexts: financial, mortality and climate change. We use a p -additive model of risk to classify decision makers according to their risk attitude in each context and frame (i.e., gains or losses). Our results indicate that while some decision makers exhibit stable risk preferences across contexts and decision frames, many others have risk preferences which vary depending on the choice environment. We find almost no evidence of the reflection effect for individual DMs or in choice data aggregated across DMs. We discuss the implications for economic and psychological research and modeling of risk attitudes, emphasizing that any one-size-fits-all approach to risk attitude such as assuming risk neutrality or risk aversion is unlikely to accurately describe the choice behavior of decision makers.

Introduction

Risk attitude has become a focal point for many researchers due to its wide practical implications. Determining whether certain decision environments produce different risk attitudes helps decision researchers test the descriptive accuracy of decision theories. Exploring whether certain groups of decision makers (DMs) vary in their risk attitude (such as men vs. women, older vs. younger adults, etc.) can help researchers understand differences in decision quality and preferences. Marketing, clinical diagnoses, public policy, finance, health decisions and myriad other contexts are concerned with the risk attitude of DMs given that oftentimes these decisions carry "risk." Many of these areas have developed their own definitions of risk and measurements thereof. These discrepancies between contexts and measurements makes comparing the risk attitudes of DMs across contexts difficult.

First, we will define some key terms for understanding risky choice environments. Risk is not defined solely as a possibility of *losing* or incurring a negative outcome. A choice between two losing alternatives may still involve both risky and safer alternatives, so we define risk relatively by comparing the outcome variance of the choice alternatives. In this way, we avoid an "absolute" assignment of riskiness to an activity and instead rely on comparisons within the choice at hand to define a risky or safe alternative. In other words, we measure risk *in degree* rather than in absolute. As we will show, the p -additive utility approach we use to define risk and classify DMs features choice alternatives with equiprobable outcomes and equal expected value, but with one alternative having a larger variance on the possible outcomes (the risky choice). We define risk in terms of outcome variance and

sidestep most concerns about subjective probability (such as over-weighting small probabilities and under-weighting large probabilities) since all outcomes are equiprobable.

A *choice context* refers to the context of the decision environment: whether the DM is faced with choices about monetary outcomes, health outcomes, food, drinks or any set of outcomes where an individual may carry preferences. The outcomes of the choice alternatives can be framed either as negative or positive, depending on the outcome's relation to a reference point or status quo (e.g., \$0 for monetary gambles). We call a decision environment that features strictly positive outcomes a "pure gain" *frame*. Previous research has shown that framing the same choice problem as a gain versus as a loss can produce preference reversals which carry implications for the DM's risk attitude and the decision theories that can accurately describe the DM's behavior (i.e., Kahneman and Tversky, 1979). These framing effects, heuristics and biases are at odds with traditional economic models of decision making which posit that DMs are rational agents with risk neutral or risk averse attitudes.

The reflection effect is a seminal finding and prediction of prospect theory (Kahneman and Tversky, 1979). This effect is concerned with the tendency for people to exhibit risk aversion for prospects involving gains while exhibiting risk seeking behavior for equal-magnitude losses. The loss side of the reflection point is also steeper than the gains side. In other words, the reflection around the origin is not a symmetric reflection. This is argued to be due to *loss aversion*, the tendency for losses to "loom larger" than equivalent gains and thus elicit different choices. The reflection effect and loss aversion are key psychological effects for researchers to consider. Indeed, studies are often carried out specifically to explore how the reflection effect may manifest differently for different groups of DMs. Many researchers have replicated Kahneman and Tversky's seminal findings, but recent work has encountered difficulties that spur some interesting questions moving forward. Namely, we are concerned with whether *individual* DMs' risk attitudes satisfy the predictions of the reflection effect or if the effect is partially due to aggregating across DMs.

Despite widespread investigation of risk attitude, relatively little work has been done that specifically aims to assess the stability of individual risk attitude across choice contexts. It is currently unknown whether individual patterns of risk attitude observed in financial choice contexts have direct implications for the risk attitude a DM exhibits for similarly framed choices in another choice context. Previous research has relied on between-subjects designs for assessing the risk attitude of DMs in different frames or contexts. This can be problematic for multiple reasons. First, a study that aggregates choice data over DMs risks producing aggregation artifacts that do not represent the preferences of the individuals within the aggregate group. Second, between-subjects studies may have difficulty teasing apart the structural preferences of the individual DMs from noise (or error) or from framing or contexts effects. A within-subjects design can assess the risk attitude of a single DM within each decision environment to better estimate the effects of frame

or context on individual risk attitudes. For example, Weber, Blais and Betz (2002) developed a series of context-sensitive measures of risk attitude and found that risk attitude was sensitive to context, with some DMs exhibiting entirely different risk attitudes between social, health, financial and other risky contexts.

We designed a large-scale experiment to explore the stability of individual DMs' risk attitudes across choice contexts and frames. We use a measurement of risk attitude based in p -additive utility theory (Luce, 2010a) that carries several important benefits to assessing risk attitude across contexts. This approach provides a criterion both necessary and sufficient for distinguishing between "types" of DMs based on their preferences and indifferences to pairs of binary lotteries. The approach is general in that outcomes need not be monetary values, which allows the construction of stimuli in a wide variety of choice contexts. This methodology was extended by Davis-Stober and Brown (2013) to accommodate DMs with complex risk attitudes that vary according to the decision environment and conducted the first experiment using the p -additive approach and criterion to explore risk attitude. We argue that this approach carries many advantages for understanding how individual risk attitude changes according to the frame and context of the decision environment, as well as due to group-level differences.

To our knowledge, this study is the first to apply this approach to non-monetary choice contexts to compare the risk attitude patterns of individual DMs across choice contexts and frames. We take an exploratory approach with relatively few firm hypotheses about the stability of risk attitude across contexts. We evaluate some hypotheses related to previously observed patterns of risk attitude in terms of the reflection effect and other framing effects.

We explore risk attitude in three choice contexts: financial choices, climate change choices and mortality choices. The financial choice context was administered to all participants to establish a baseline measure of risk attitude in our framing conditions (i.e., losses, gains and mixed gambles) and to compare to previous studies that have traditionally focused on monetary choice stimuli.

The climate change context requires DMs to choose between technologies that impact the global mean temperature and thus the environmental impacts to the earth. This context is interesting due to the general apathy encountered in empirical studies of climate decision making (i.e., Weber, 2006). The intertemporal nature of climate change choices coupled with the uncertain nature of the probabilities, location and exact outcomes of climate change add to the difficulty of disentangling DM preferences in this important context. Our approach focuses solely on the risk attitude of DMs in this context and assesses whether the financial risk attitude of DMs provide any predictive power for their attitudes in climate contexts.

The mortality context provides links to medical and health decision making, which boasts a deep literature on risky decision making as it pertains to choices with health implications. Importantly, our methodology provides a common framework for developing stimuli for each context that facilitates comparing individual risk

attitude classifications across contexts while keeping other aspects of the decision environment (like framing) constant. We now provide a brief summary of relevant research within each context which informs our hypotheses.

Financial Choice Context

The framing of a choice problem can have a significant impact on the risk attitude of a DM (Davis-Stober and Brown, 2013; Tversky and Kahneman, 1986; Luce, 2010a,b). When the choice stimuli are monetary gambles, as is common in behavioral economics, gains, losses and mixed gambles refer to the outcomes' position relative to a reference point. This reference point is often 0 (for money), which represents a status quo. Gains provide the DM with an outcome that is better than the reference point, losses provide the DM with an outcome that is worse than the reference point. Mixed gambles have the chance of producing either a loss or a gain from the reference point. Some studies use a variety of frames to explore whether different framings of the same objective probabilities and payoff magnitudes produce similar or different choice behavior in DMs, but others rely solely on gains or losses. For example, Holt and Laury (2002) used gain-only financial stimuli to demonstrate the effects of outcome magnitude and incentivization on risk attitude. That choice precluded investigating whether the same effects occur for loss-framed stimuli. If a researcher hopes to learn something about the general risk attitude profile of a DM, she ought to provide a variety of frames for the DM to consider since risk attitude is sometimes sensitive to frame (as in Kahneman and Tversky's reflection effect). Heterogeneity both within- and between-subjects presents a challenge to models of decision making that predict a stable risk attitude across different frames. Not only do different DMs respond to different frames differently, but even a single DM may have a complex risk attitude profile that is sensitive to the framing of the choice problem. We now briefly review some of the important results and effects in the study of risk attitude as it pertains to financial decisions.

Tymula et al. (2013) report a positive correlation between risk attitude classification on gains and on losses. This finding indicates that individuals who are more risk averse on gains are more risk averse on losses as well, contrary to the negative correlation predicted by the reflection effect. Hershey and Shoemaker (1980) show that the presence of the reflection effect at aggregate levels does not imply the presence in the individual DMs that comprise the aggregate. This finding again brings to light the importance of caution when aggregating in decision making research. Indeed, recent results from Kellen, Mata and Davis-Stober (2017) using the LPC framework described and used herein finds little evidence for the reflection effect in individuals.

Holt and Laury (2002) presented DMs with a menu of ordered lottery choices to assess individual risk preferences across various payment conditions. The payment conditions were constructed to determine whether incentives (whether or not the gamble outcomes were realized or hypothetical) or stakes (the magnitude of the outcomes) significantly affected risk attitude. Holt and Laury's major findings were

that subjects were mostly risk averse, even for very small payoff amounts of under \$5. Risk aversion increased sharply as the scale of the cash (incentivized) payoffs increased, but there was no significant effect of the increased scale for the conditions with *hypothetical* payoffs. Risk loving behavior (making 3 or less safe choices out of 10 possible choices) was relatively rare in Holt and Laury's experiments.

Findings related to the stakes of the decision environment have implications for decisions outside of the realm of money. Rabin (2000) argued against the marginal utility of wealth as a sufficient explanation for risk aversion, pointing out that some assumptions of traditional economic theories would imply absurd levels of risk attitude in high stakes conditions. For example, Hansson (1988) showed a DM exhibiting constant absolute risk aversion (from the Arrow-Pratt risk aversion measure) who is indifferent between a guaranteed \$7 and a 50-50 gamble of winning \$21 or nothing actually prefers the sure \$7 gain to *any* gamble in which the probability of gaining money is less than 45% regardless of the size of the potential gain. What then are the implications for risk attitude in standard economic models when a DM is considering their own mortality or environmental catastrophes? This is one key area of our exploration - are individual DMs who are risk averse for monetary decisions more likely to be risk averse for decisions that could cost their life? In order to compare risk attitude across choice contexts without relying on utility functions or estimating the value of a statistical life to properly account for stakes, we must classify decision makers according to their risk attitude in relation to the alternatives they are presented with and maintain a common methodological approach for classifying risk attitude in each context.

Climate Chance Choice Context

Decisions about climate change are difficult for people to consider due to the timing of events and uncertainty about the chances of environmental changes impacting *their* lives specifically. Research shows that Americans are aware of the potential effects of climate change, believe that it will negatively impact many millions of Americans but simultaneously believe that *they themselves* are unlikely to be affected (Howe, Mildemberger, Marlon and Leiserowitz, 2015). Research into the risk attitudes surrounding climate change has focused on risk perceptions and found a general apathy toward the probabilities and outcomes associated with climate change.

Weber (2006) reviews some of the psychological reasons the risks of climate change may not resonate with people. A central argument is that risk must be perceived, and the time-delayed, abstract and often statistically framed nature of climate change risks do not provoke strong reactions of worry. Understanding the risk attitudes that DMs have in different frames of climate change decisions could provide important evidence for designing effective policies for addressing climate change. For example, if climate change decisions framed as losses result in substantially more risk-seeking behavior, it may be the case that messages framed as losses would be more likely to spur substantial action in individuals.

Kahan et al. (2012) conducted a study to assess the role of scientific literacy on climate change attitudes. The authors concluded that members of the public with the highest scientific literacy and technical reasoning were not the most concerned about climate change. The authors eschew the attribution that public apathy over climate change is due to a deficit in comprehension, arguing instead that the conflict arises between the personal interests of the individual DM (and their close ones) and the collective interests they share in making use of science to promote common welfare. In our study, we collected education level survey data, but given these results we expect that education level is unlikely to dramatically influence the risk attitudes of DMs in the climate change context.

Mortality Choice Context

The mortality choice context features choices between alternatives that carry implications for the DM's chances of dying over the next 10 years. While this is necessarily an abstracted decision, the context relates to choices about health, risky health behaviors (like smoking or skydiving). Medical decision making research is often concerned with risk attitude of patients, specifically when choosing among treatment options that vary in their riskiness. We summarize some of the previous research that motivates our hypotheses and areas for exploration in this context.

Medical decision making researchers are often concerned with understanding how individuals might make better decisions for themselves. To this end, Felder and Mayrhofer (2014) explored whether different risk attitudes had impact on patients' decisions to pursue medical tests and treatments. The authors found that risk aversion lowers the test and treatment threshold and decreases the optimal cutoff value for diagnostic tests. In other words, a risk-averse individual is more likely to pursue treatments and tests when the risk-reducing strategy is to get treatment/tests. The authors argue this from the perspective that a risk-averse individual would want to eliminate the low-health outcome of forgoing treatment/tests when in the "sick state" (e.g., when the probabilistic positive test is accurate or the negative test is inaccurate). Criticisms of this study include that risk aversion is not simply aversion to the worst outcome and that risk attitudes were not explicitly measured in this study. Regardless, the results are corroborated with other studies that find that risk attitude has an effect on patients' decisions to seek or forgo treatment.

Within the medical decision making field, risk literacy is argued to be a potential factor in determining health behavior. Tasks like the Berlin Numeracy Task (Cokely et al., 2012) measure individual ability and comfort with dealing with numbers and probability. Reyna (2008) and others have shown that individuals with lower numeracy are less capable of understanding and appropriately weighting risks. Practically, this means that lower numeracy individuals may take more risks due to a lack of risk literacy- they are unable to accurately comprehend or weigh outcomes and probabilities. Our approach should eliminate any biases resulting from subjective probabilities used by DMs when assessing risk by using only simple 50:50 binary gambles. This simplicity provides some confidence that risk literacy is

unlikely to play a large role in a DM's choice experience in this study. While not perfectly related to numeracy, the inclusion of the education covariate could also shed some light into whether more educated, and potentially more risk literate, individuals have different risk attitudes for mortality decisions than less educated individuals. If this were the case, we may observe a higher prevalence of risk neutrality in the mortality context as education level increases.

Cher, Miyamoto and Lenert (1997) show that many models of medical decision making take a risk-neutral approach. This is potentially problematic, as most patients are not risk neutral, but risk averse. Further, research in other domains of risk attitude finds substantial heterogeneity in classifications. The authors argue that risk aversion affects the trade-offs made between short- and long-term survival, potentially leading to under- or over-estimation of the disutility of future health states. The authors show that different models of risk attitude predict different medical decisions (such as the choice to pursue surgery or expectant treatment) and encourage risk attitude to be incorporated into decision models used in medical decision making. These results highlight the need for a generalizable method for classifying DMs according to their risk attitude. Our LPC approach will allow us to explore whether a risk neutral or risk averse attitude is most common in decisions about mortality. Further, the demographic information collected and associated financial risk attitude task will allow comparisons to be made between different groups for which medical/mortality decisions may be more important (like for older adults or those in poverty).

Guiso and Paiella (2004) used household survey data and an absolute measure of risk aversion to show that risk aversion in the financial elicitation does have predictive power for other domains including exposure to chronic diseases. This study questioned whether a method of risk attitude elicitation restricted to financial decisions would predict choices in other domains and found that it could. Results like these provide some evidence for the argument that risk attitude is a relatively stable trait. We intend to test these ideas by applying the same LPC framework to both mortality and financial choice contexts. We question whether within-individual risk attitude will vary between choice contexts within the same frame. In this way, we can provide more evidence for the debate on risk attitude's status as a personality trait.

Covariate Exploration

Many researchers have investigated group-level differences in risk attitude on important variables like age and gender. Luce (2000) warned that individual DMs should be classified according to their risk attitude type prior to aggregation or group-level analyses to avoid aggregation artifacts and issues arising when aggregating over different individuals' utility profiles. The p -additive approach allows us to carry out this classification and explore whether previously observed effects are supported for individuals in a within-subjects design or across different choice contexts.

Some important results include a general consensus that females (compared to males; Weber, Blais and Betz, 2002) and older adults (compared to younger adults) tend to have more risk-averse preferences than their counterparts across a variety of choice contexts including monetary and health outcomes. However, Kellen, Mata and Davis-Stober (2017) found that older adults showed a *higher* prevalence of risk seeking than younger adults in a financial LPC task. Some of these covariate-related results show complicated interactions between the covariates and framing effects, so it is important to investigate group-level differences across all contexts and frames with the same experimental approach. We explore group-level differences in the distribution of risk attitude classifications across contexts and frames for several important covariates. We compare our results to previous work where applicable.

Previous studies of risk attitude required specifying or estimating a utility function for individual DMs and inferring the risk attitude of the DM based on the concavity (risk-averse) or convexity (risk-seeking) of the utility function (Arrow, 1971; Pratt, 1964). Further, many studies have relied on aggregated choice data to demonstrate effects such as the reflection effect preference reversal found by Kahneman and Tversky (1979). Luce (2000) and other researchers have issued warnings that aggregating choice data across DMs or across choice frames could lead to aggregation artifacts: effects that appear to represent the behavior of a group of individuals but rather only represent the behavior of the group's aggregate/average, which may not represent the behavior of any of the individual DMs who comprise the aggregate. A key strength of our approach is that it classifies individual DMs according to their risk attitude prior to any aggregation (within or between subjects) or group-level analyses. This risk measurement and classification approach has been extended by Davis-Stober and Brown (2013) to accommodate complex risk attitudes. Davis-Stober and Brown (2013) conducted an experiment on 24 undergraduate students and found substantial heterogeneity between subjects and within-subjects with some DMs showing strict, stable risk attitudes and some showing complicated risk attitudes that varied according to the frame or stakes of the decision environment.

Recently, Kellen et al. (2017) used the p -additive approach to explore age differences in risk attitude, focusing on difference between older and younger adults. One highlight of the results was a distinct lack of a reflection effect for most participants in pure frames and exacerbated for younger adults in mixed frames. The authors found only about 3% of individual DMs satisfied the predictions of the reflection effect for pure frames, calling into question whether the reflection effect accurately describes the choice behavior of *individuals*. This argument finds support when considering Kahneman and Tversky's (1977, 1979) original demonstration of the preference reversal was supported using aggregate choice data (total percent of DMs favoring the risky option for the gain frame versus the loss frame). These issues are explored in the current study which find a similarly small level of support for the reflection effect in individuals. In any case, the LPC approach used by these authors and the current study measures risk attitude individually before any aggregation occurs, allowing us to compare these results with what we would have obtained

under aggregation to determine if our choice data can support the reflection effect in any way.

***p*-Additive Utility and Luce's Proposed Criterion**

The *p*-additive utility representation assumes that there are three kinds of people, corresponding to $\delta = -1, 0$, and 1 which, in turn, correspond to risk-averse, risk-neutral and risk-seeking preferences. Luce (2010a) assumes that the joint receipt of money is additive (e.g., $\$5 \oplus \$10 = \$5 + \$10 = \$15$). Luce (2010a) demonstrated that the value of δ for each DM necessarily implies a specific preference pattern in terms of the DM's preferences and indifferences to binary 50:50 lotteries. In other words, the *p*-additive risk measurement model determines a DM's risk attitude based on their preferences and indifferences to binary "coin flip" lotteries featuring one safe and one risky alternative.

Consider a set of possible outcomes under consideration $\{x, x', y, y'\}$. So long as a preference ordering in the form of $x > x' > y > y'$ can be constructed, the approach can be used to create two binary gambles from the *joint receipt* of a combination of the original four components of the set. The resulting gamble pair features one alternative with a wider outcome variance than the other alternative. Each alternative has the same expected value and thus is centered around a common midpoint. The probabilities associated with each of the four outcomes are set equal at .5, so the resulting gamble pair is a choice between a risky option and a safe option, each with equiprobable outcomes and equal expected value. Consider the following example of the construction of a financial LPC-consistent gamble pair.

We begin with some ordered set of outcomes such as $\$20 > \$15 > \$10 > \$2$ corresponding to $x > x' > y > y'$. The LPC-consistent gamble pair results from the joint receipt of different pairs of components in the following fashion:

$$(x \oplus x', .5; y \oplus y', .5)$$

versus

$$(x \oplus y, .5; x' \oplus y', .5)$$

So for our example we have:

$$(\$20 \oplus \$15, .5; \$10 \oplus \$2, .5) \text{ versus } (\$20 \oplus \$10, .5; \$15 \oplus \$2, .5).$$

Where the risky gamble on the left has a larger outcome variance (\$35 or \$12) than the safer gamble on the right (\$30 or \$17). DM preferences or indifferences on gamble pairs such as these directly correspond to their risk attitude.

This framework allows us to develop gamble pairs that share much of the same decision environment: the probabilities of outcomes are all equal, the expected value of any two alternatives is always equal and risk attitude is measured identically regardless of the context (e.g., be it financial stimuli or otherwise). Further, we may manipulate the outcome sets used to create the LPC-consistent

pairs such that we have different framings of a similar choice problem: purely gains, purely losses or mixed gains and losses. This, the LPC provides a principled and general approach for constructing choice stimuli capable of distinguishing between risk attitudes. LPC-consistent stimuli are simple for people (including children) to understand, sidesteps subjective probability issues by maintaining constant, equal probability of outcomes and is generalizable to nearly any choice context. These advantages make the LPC approach well-suited for exploring the stability of risk attitude across choice contexts, frames and demographic groups.

The LPC framework provides for the development of stimuli, but the p -additive risk measurement model itself is deterministic - it does not allow for choice variability. We use repetitions of gambles of the same type to accommodate within-subject choice variability: the tendency for an individual DM to make different choices on identical (or nearly so) stimuli. Given this consideration, we conducted simulations to determine an appropriate number of repetitions of each type to reasonably classify DMs in our Bayesian statistical methodology. We found that as few as four repetitions were required to obtain an appropriate amount of evidence for one model over another. We chose to include more pure and mixed loss repetitions because those types tend to distinguish between competing complex models of risk attitude better than pure or mixed gains, where models tend to mimic one another.

We collected choice data consisting of DM preferences and indifferences on the LPC-consistent gamble pairs as well as responses to a demographic survey covering age, gender, income, household type, employment status, education, dwelling type and zip code. We use these demographics to explore group-level differences in the distribution of observed risk attitudes after classifying DMs individually.

Method

Experimental Design - LPC-consistent Stimuli

We constructed 24 gamble pairs that conform to the LPC for each of the three choice contexts: 4 pure gain trials where all outcomes were positive relative to the reference point, 8 pure loss trials where all outcomes were negative relative to the reference point as well as 4 "mixed gain" and 8 "mixed loss" trials. Gamble pairs with a "mixed" frame feature two alternatives each with one positive and one negative possible outcome. The "gain" or "loss" designation is determined by the expected value of the gamble pairs, with expected gains denoted as "mixed gains" and expected losses denoted "mixed losses." This allows us to assess the stability of risk attitude between pure and mixed frames of the same expected outcome (either gains or losses). Theories of decision making predict specific behaviors for each frame, often motivated by observed empirical effects like the reflection effect, loss aversion or risk aversion. Some competing models exhibit model mimicry for certain frames by predicting the same behavior which makes assessing the relative fit of the competitor models difficult. We constructed gamble frames specifically to distinguish between competing theories of decision making.

Financial Stimuli

All participants completed the financial risk attitude task. The resulting gamble pairs are equal in expected value and are essentially “coin flips” with equiprobable outcomes (50/50) with one alternative being more variable (and thus riskier) than the other. The full table of all experimental stimuli is available in the appendix.

On each trial, the participant was asked to indicate which option they would prefer if they had to make the decision for real consequences. Participants were allowed to indicate indifference between the two alternatives, meaning our paradigm is a 'ternary choice.' The nominal values used in this study would be considered 'high stakes' as possible gains and losses ranged between \$2,000 and \$20,000 (Holt and Laury, 2002). This choice was made in an attempt to make the financial stimuli about as salient as the climate change and mortality stimuli. In other words, we wanted differences in risk attitude between choice contexts to be more likely driven by the context than simply the magnitude of the possible outcomes.

We emphasize that these results are exploratory and our methodology features an intentionally lenient classification threshold (as opposed to the strict classification rules in Davis-Stober and Brown, 2013) so as to allow new or otherwise unpredicted patterns of risk attitude to emerge. As Kellen, Mata and Davis-Stober (2017) found upon reanalysis of the Davis-Stober and Brown (2013) data, strict stochastic error rates and mismatches between data and models produced substantial non-classification rates in the original study (about 33%).

Climate Change Stimuli

The climate change choice context required individuals to make decisions about which environmental policy to support/enact given that these policies have probabilistic outcomes. Unlike the mortality and financial conditions that are likely to impact only the DM and those close to them, the outcomes from different climate change mitigation policies affect many people and do so differentially (e.g., people living in US coastal regions may have more at stake than US Midwesterners due to the severity of natural disasters specific to coastal regions). Further, climate change discourse has been politicized and many people do not believe the phenomenon is reality. These important psychological considerations set this choice context apart from others and make it a very interesting context for study. In contrast to monetary stimuli, climate change stimuli could elicit different psychological reactions, and thus preferences and risk attitudes.

Similar to the other choice contexts, the climate change choice context was comprised of 24 LPC-consistent gamble pairs with 4 gain-only trials, 8 loss-only trials, 4 mixed gain trials and 8 mixed loss trials. Participants were given the following chart (Figure 1) to study before completing the task. Participants were allowed to review this chart by hovering over a link that was continuously available throughout the task.

Figure 1: The Climate Change Impact infographic links average temperature rises to corresponding environmental impacts. Participants studied this graphic for a minimum 30 seconds prior to completing the climate change LPC task. This information was continuously available to participants in each experimental trial.

Climate Infographic

Participants were told that the Intergovernmental Panel on Climate Change (IPCC, 2014) has forecasted a 2 degree Celsius (3.6 degree Fahrenheit) increase in global mean temperature by the year 2100 which would act as the reference point (status quo) for determining gains and losses. By studying this chart, participants could see the possible effects for each value of global mean temperature increase. We note that the environmental impacts estimated from each degree increase could differentially affect DMs, leading to changes in risk attitude that are not motivated by framing or context.

Gain Frame

Trials were considered 'gain-only' if all possible global mean temperature increases were below the original 2014 forecast of 3.6F. These trials represented a choice between two successful technologies for treating climate change that would reduce the realized increase in temperature from the forecasted reference point.

Loss and Mixed Frame

Trials were considered 'loss-only' trials if all possible global mean temperature increases were above the original forecast reference point of 3.6F. These trials represented a choice between two technologies that would spare the earth from even higher temperature increases (above those in the potential outcomes under

consideration), but were still guaranteed to result in an outcome that was objectively worse than the original 2 degree Celsius estimate.

Trials were considered 'mixed' if the alternatives each featured an outcome above and an outcome below the original forecast of 2 degrees Celsius, and the position of the gambles' expected value determined whether it was a mixed gain or mixed loss. In mixed frame cases, the trials represented a choice between two technologies that could either be mildly successful (spare the earth from even higher temperature increases, but still result in an increase higher than the 2C forecast) or wildly successful (reduce the global mean temperature increase to a value below the 2C forecast).

The use of a baseline forecast from the IPCC to stand as the reference point was necessary to establish a status quo. It can be argued that the chosen reference point does not accurately divide between gains and losses. If one considers the predicted effects of the status quo forecast to be losses, then even diminishing those losses may not be perceived as gains. Unfortunately, this decision was a necessary compromise for simplifying this complicated decision environment. Using a reference point of a 0 degree increase might make the loss contexts (perhaps all contexts) difficult to situate in reality.

Mortality Stimuli

The mortality risk attitude task asked participants to choose between two possible medical treatments with probabilistic (equal 50/50) health outcomes. Participants were given the 10-year average mortality rate for people fitting their self-reported gender and age range to make the stimuli as relevant as possible for each participant. Stimuli were then modified to use this average value as the individual's reference point (status quo) for determining whether outcomes would be considered gains or losses. For example, an outcome that reduced an individual's 10-year mortality rate from 8/1000 to 6/1000 would be considered a gain whereas the opposite (from 6/1000 to 8/1000) would be considered a loss. As in the other contexts, there were 4 gain-only trials, 8 loss-only trials, 4 mixed gain trials and 8 mixed loss trials.

Decision Scenario Framing

Unlike the financial choice context where decisions involving losing or gaining money are easy for DMs to relate to, the mortality choice context required a scenario in which to situate the DM. We created vignettes that briefly described the hypothetical situation each frame presents to provide a compelling reason for why a DM might realistically be in a situation where they could choose between two options that would each reduce their mortality rate (gain frames), or between two options that would both increase their mortality rate (loss frames), or between two options that may either increase or decrease their mortality rate (mixed frames).

Gains Frame

For gain-only trials, participants were told that they were being given the opportunity to undergo one of two treatments that would *reduce* their mortality rate over the next 10 years. This improvement would be a reduction from their personal base rate chance of dying within the next 10 years.

Losses and Mixed Frames

For loss-only trials, participants were told that they were to choose between two treatments to avoid a certain and immediate death. Both treatment options were guaranteed to save the individual's life immediately, but at a possible cost (or benefit) in the long run. In the loss-only trials all possible outcomes resulted in an increase in mortality rate from the individual's base rate. In essence, the treatment spares the participant's life but increases her chance of dying in the next 10 years. For mixed gambles, participants were told that the treatments had probabilistic effectiveness. Sometimes the treatment would have a negative consequence of increasing their mortality rate over the next 10 years (loss), but sometimes the treatment would work very well and reduce their mortality rate over the next 10 years (gain).

Experimental Design - Survey and Demographics

The present study administered two (of three possible) behavioral LPC risk attitude tasks along with a demographic questionnaire to a panel of respondents. The respondents were randomly selected, but efforts were made to create a representative sample by sampling across demographic variables like age, gender, education, income, geographic location and employment status.

Overall, 1,646 United States individuals provided (at least partial) responses to the survey and behavioral tasks. The entire survey, including all questions asked, is provided in an appendix. Of the 1,646 total respondents, 50 were removed due to impossible completion times (under 2 minutes) or missing data, leaving a total of 1,596 individual DM respondents. All participants completed the monetary risk attitude task and were randomly assigned to complete either the climate change or mortality risk attitude task. While a fully-crossed within-subjects design is ideal, a compromise was necessary to maintain a reasonable survey length and curb the incompleteness rate. 814 participants saw the mortality context and 782 saw the climate change context.

Participants completed an informed consent screen before continuing to the survey. Participants were paid a flat fee of \$10 (US dollars) for completing the survey and behavioral tasks and received no additional monetary incentives for performance in the behavioral tasks.

All participants completed the monetary risk attitude task and were randomly assigned to complete either the climate change or mortality risk attitude task. While a fully-crossed within-subjects design is ideal, a compromise was necessary to maintain a reasonable survey length and curb the incompleteness rate. Participants

completed the survey and behavioral tasks online through the Qualtrics survey software and were contacted via email.

Bayesian Statistical Classification

We apply Karabatsos's (2005) Bayes factor methodology to evaluate data within our framework. This methodology applies a Dirichlet prior to a multinomial (in our case, trinomial) likelihood. This approach results in a closed-form solution for the Bayes factor between any two p -additive representations. We direct the reader to Karabatsos (2005) for the technical details and derivations. The resulting Bayes factor can be considered a measure of evidence for one model over another. We classify DMs according to the model of risk attitude that best describes their choice data for each context and frame combination *independently* as in Kellen, Mata and Davis-Stober (2017). Davis-Stober and Brown (2013) used a similar methodology, but defined models jointly across all frames which resulted in more non-classifications. The granular approach used here and in Kellen et al. provides for a more exploratory approach and features a weaker threshold for classification than previous studies.

Similar to Karabatsos (2005), we encode our decision models via the parameters of the Dirichlet prior distribution. The conjugate prior for any multinomial distribution with m -many categories is the Dirichlet distribution, with the following probability density function:

$$f(\theta_i; \alpha_i) = \frac{1}{B(\alpha_i)} \prod_{i=1}^m \theta_i^{\alpha_i-1}$$

where,

$$B(\alpha_i) = \frac{\prod_{i=1}^k \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^m (\alpha_i))}$$

and $\Gamma(\cdot)$ is the gamma function. For the trinomial likelihood we are working with, $m = 3$, thus, a set of three α_i values must be specified for each gamble pair and model combination. Let α_1 be the prior value corresponding to $\theta_{(G_r)}$, α_2 for the prior value of $\theta_{(G_s)}$ and α_3 for the prior probability of choosing indifference, $(1 - \theta_{(G_r)} - \theta_{(G_s)})$. For a given LPC gamble pair, let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. We specify an α value of 5 for the choice predicted by the model and 1 for the remaining alternatives. This choice of prior values places heavy weight on the option predicted by the model, but also allows for the possibility of "error" on the part of the DM by placing weight on options not predicted by the model. Prior values for a single gamble pair where the risky choice is predicted would be $\alpha = (5, 1, 1)$, corresponding to a roughly 20 percent error rate in observed choices of that DM for that LPC gamble pair.

Since we are specifying models at this granular level only 3 models need to be specified: risk averse, risk seeking and risk neutral. We use two separate specifications for risk neutrality, as neutrality between two options might be

expressed through indifference *or* by equally distributing responses between risky and safe alternatives on identical gamble repetitions. We call the former model "risk neutral" and the latter "split-neutral." We end with the following statistical models of risk attitude:

Risk Averse corresponds to $\delta = -1$ and a prior of (5,1,1).

Risk Neutral corresponds to $\delta = 0$, and a prior of (1,1,5).

Risk Seeking corresponds to $\delta = 1$, and a prior of (1,5,1).

Split Neutral corresponds to no δ directly, rather indirectly corresponds to the equal combination of -1 and 1 (and 0) aggregating to an overall 0. The prior specification was (5,5,1) to distinguish this model from risk neutrality in the form of indifference.

Unconstrained corresponds to a null comparison model that places no prediction weight on any particular choice. The prior specification is (.5,.5,.5) as in Karabatsos (2005).

In summary, under a given model of p -additive representation, we obtain a choice prediction for the LPC gamble pair. We then assign the appropriate prior values corresponding to the choice prediction. The marginal likelihood for a given model is the product of the likelihood with the corresponding Dirichlet prior. We then compute Bayes factors (BF) to assess the fit of the various p -additive models. The BF is simply the ratio of two marginal likelihoods and provides an estimate of the evidence for one model over another. As in Karabatsos (2005), we created a reference model with a Jeffreys prior (1961), using values of $\alpha = (0.5, 0.5, 0.5)$ for every gamble pair. This reference prior is diffuse and can be considered as a type of "null" model to compare the p -additive representation models against. We choose this particular prior for our multinomial framework as it is invariant under one-to-one transformations of the parameter space (e.g., Karabatsos (2005)), unlike a uniform prior.

We classify DMs according to the model that achieves the highest Bayes factor against the unconstrained model, with the restriction that the Bayes factor be greater than 1 (which indicated equal fit between the substantive and unconstrained models). This classification threshold is purposely weak to allow for patterns of risk attitude to emerge without being cast into a "non-classified" bin. We discuss this as a potential limitation and area for future research in the Discussion section.

Results

Our large-scale experiment offers a rich environment for exploration, but we focus on only a few main results for this paper. First, we examine the overall classification rates within each context and find substantial heterogeneity in risk attitude between subjects in each frame and context. Next, we turn to exploring where stability in risk attitude occurs given the substantial heterogeneity observed both between- and within-subjects. Then we assess satisfaction of the reflection effect for individual DMs and in aggregate, finding little to no evidence in support of the effect.

Finally, we explore group-level differences in the risk attitude distributions for older adults versus younger adults.

We now list the specific hypotheses we intend to assess in addition to the general exploration of this rich set of classifications.

- 1) The reflection effect will be present for some DMs, but at low rates similar to those observed in Davis-Stober and Brown (2013) and Kellen et al. (2017).
- 2) Strict risk attitudes carried across all contexts and frames will be very rare. We do not anticipate finding strong evidence for strict risk aversion implied by traditional economic models of decision making.
- 3) Older adults will show a higher prevalence of risk aversion than younger adults.
- 4) Older adults will show an exacerbated reflection effect compared to younger adults.
- 5) Younger adults will show a higher prevalence of risk-seeking behavior.
- 6) Harrison, Lau, & Rutström (2007) argue that risk neutrality is rare in real DMs. We hypothesize that risk neutrality is not rare and likely to be influenced by choice context. We hypothesize that the climate change context will produce the highest rates of risk neutrality due to DM apathy toward this context (Weber, 2006).

Classification Graphs For Each Context

We report the classification rates for each frame within each choice context. Overall we find substantial heterogeneity between DMs.

Figure 2 (below) reports the classification rates among 1,596 DMs in each of the financial context frames. The y-axis represents the raw count of DMs classified according to a risk attitude and above each bar is the proportion of the sample. DMs whose choice data was not consistent with any substantive model (i.e., those best fit by the unconstrained model) are given the designation of 'None' in these figures.

Figure 2: Classification rates for 1,596 DMs in the financial choice context. Each DM was classified according to the risk attitude model that best described their choice data in each frame.

We find that each attitude is represented within each frame, calling into question any theories that posit that a particular risk attitude (such as risk neutrality) is absent in real DMs. Risk neutrality is significantly more prevalent in loss frames than in gain frames. Risk aversion is most prevalent in gain frames, as is risk seeking behavior.

We repeat these figures for the climate and mortality context in turn, assessing whether the patterns observed in the financial context generalize to the other contexts. Figure 3 (below) shows the classification rates of 782 DMs in the climate change context. This context produced the most uniform distribution of

classifications in the gain frames and featured the highest rates of risk (and split) neutrality, contrary to our hypothesis that climate change would produce the highest neutrality rates.

Figure 3: Classification rates for 782 DMs in the climate change choice context. Each DM was classified according to the risk attitude model that best described their choice data in each frame.

The distribution of risk attitudes in the climate change context is also heterogeneous. For pure gains, the distribution of attitudes is nearly uniform. The mixed gain frame, as in the financial context, induces a higher prevalence of risk

aversion relative to pure gains. As in the financial context, we see risk neutrality and split neutrality increase in prevalence from gain frames to loss frames. Interestingly, the climate context produced the highest overall prevalence of "neutral" attitudes, providing additional support to research demonstrating that individuals may not perceive climate change as a legitimate risk *to themselves*. Indeed, we find that risk neutrality is most common in pure loss frames in general, where the climate context finds about 78% of DMs are either risk neutral or split neutral. Risk seeking behavior was more common in gain frames than in loss frames.

Figure 4: Classification rates for 814 DMs in the mortality choice context. Each DM was classified according to the risk attitude model that best described their choice data in each frame.

The mortality context repeats most of the patterns we observe in the other contexts: risk aversion as well as risk-seeking behavior is most prevalent in gain frames. Risk neutrality is more common in loss frames than in gain frames. Surprisingly, both mixed frames produce a higher prevalence of risk aversion than their pure counterparts. The pure loss frame again produced the highest prevalence of neutrality with 70% of DMs classified as risk neutral or split neutral.

Summary

We observe the following patterns in each context:

- 1) Risk aversion is more common in gain frames than in loss frames.
- 2) Risk neutrality is more common in loss frames than in gain frames. Pure losses feature the highest prevalence of neutrality among all frames.
- 3) Risk seeking was universally more common in gain frames than in loss frames.

We previously hypothesized that the mortality context would produce the highest risk averse attitudes, but observed the opposite: risk seeking behavior was most prevalent in the mortality context. We predicted that the climate change context would produce the highest neutrality rates, but that was true for the financial context instead.

The Reflection Effect

A DM satisfies the predictions of the reflection effect if they are classified as risk averse for the gain frame and risk seeking for the loss frame. We measured the number of DMs who satisfied the reflection effect separately for each context and for pure and mixed frames. We report the total count and percentage of DMs satisfying each context by frame combination in Table 1 (below). Clearly, the reflection effect is incredibly rare in these data, with fewer than 3% of the DMs in each case satisfying the predictions of the reflection effect.

Environment	DMs Satisfying Reflection Effect (percent of sample)
Financial Pure	49 (3.07%)
Financial Mixed	33 (2.07%)
Climate Pure	18 (2.3%)
Climate Mixed	12 (1.53%)
Mortality Pure	13 (1.6%)
Mortality Mixed	9 (1.11%)

Table 1: Reflection effect satisfaction among DMs in each frame/context condition. A DM satisfied the reflection effect if they were independently classified as risk seeking for losses and risk averse for gains. We find very low support for this pattern of risk attitude.

Individual DMs in this study do not satisfy the predictions of the reflection effect, nor do the rates of satisfaction eclipse what would be predicted under random classification rates between the 5 possible classifications (including "None" non-classifications) which would be 4%. In essence, the reflection effect occurring in individual DMs is rarer than chance.

These results indicate that the reflection effect is not descriptive of individual DM's choice behavior. We explored whether aggregating choice data across DMs prior to classification (which is ill-advised) would produce an aggregation artifact that provides support for the reflection effect. To test this we aggregated DM choice responses into a single choice profile to determine if the reflection effect is satisfied for pure or mixed frames in each context. The reflection effect occurs in aggregate if the choice proportions favor the safe alternative in the gain frame and the risky alternative in the loss frame (or if the proportions shift substantially in that direction). Table 2 below reports the aggregated choice proportions for each context and frame (note: rounding may cause some rows to not sum to 1). We bold the mode choice in each environment.

Environment	Risky	Safe	Indifferent
Financial Gains	.368	.446	.185
Financial Losses	.243	.363	.393
Financial Mixed Gains	.251	.508	.241
Financial Mixed Losses	.247	.427	.326
Climate Gains	.359	.369	.273
Climate Losses	.285	.342	.374
Climate Mixed Gains	.322	.434	.244
Climate Mixed Losses	.303	.384	.312
Mortality Gains	.423	.363	.214
Mortality Losses	.305	.306	.389
Mortality Mixed Gains	.322	.448	.229
Mortality Mixed Losses	.298	.435	.266

Table 2: Reports the aggregated choice proportions for all DMs within each context and frame combination. The mode choice response is bolded for each condition. Safe choice implying risk aversion are the most common in all but 4 conditions. Indifferent choices were most common in each of the pure loss frames and risky choices were most common in the mortality pure gain condition.

First, the reflection effect is not explicitly supported via a shift in modal choice between frames - in no condition was the modal choice for gains safe and the modal choice for losses risky. Further, risky choices are less common in all loss frames in comparison to their gain frames. This indicates that we have no evidence for a shift in aggregate choice responses toward the predictions of the reflection effect. Aggregating further by combining the mixed and pure frames within a context offers no additional support for the reflection effect.

We note that our stimuli varies from those previously used to assess the reflection effect. One special case of our stimuli is a risky gamble compared to a degenerate gamble with identical payoffs – reducing to a choice between a maximally risky gamble and a sure result of the expected value of the gamble. This stimuli style was previously used by Kahneman and Tversky to demonstrate the reflection effect.

Strict Risk Attitudes

Table 3 (below) reports the number of DMs who were classified according to a single risk attitude across all frames within a context. Percentages are calculated out of the total number of DMs within a context.

Classification	Financial (N = 1,596)	Climate (N = 782)	Mortality (N = 814)
Risk Averse	81 (5.1%)	44 (5.6%)	38 (4.7%)
Risk Neutral	167 (10.4%)	125 (16%)	97 (12%)
Risk Seeking	16 (1%)	22 (2.8%)	40 (4.9%)
Split Neutral	18 (1.1%)	22 (2.8%)	14 (1.7%)
Total	282 (17.7%)	213 (27.2%)	189 (23.2%)

Table 3: Reports the total number of DMs who were consistently classified according to a given model of risk attitude (rows) across all frames within a context (columns). Overall, less than 30% of DMs maintain a strict classification across frames within a single choice context.

We see that the financial context produced the lowest percent of DMs with strict risk attitudes. Strict risk neutrality was the most common single risk attitude in each context while strict split neutrality was relatively rare.

Stability Across Contexts

We assessed whether DMs were stable in a single risk attitude classification across both the financial and either the climate or mortality context. We find that entirely stable risk attitude is relatively rare when the decision environment shifts from financial stimuli to another context. Table 4 reports the total number of DMs who were entirely consistent in risk attitude classification across all frames and contexts encountered.

Classification	Financial and Climate	Financial and Mortality
Risk Averse	10	7
Risk Neutral	52	52
Risk Seeking	2	1
Split Neutral	2	1
Total	66	61

Table 4: Reports the total number of DMs who were entirely consistent in risk attitude classification across all frames and contexts encountered. Few DMs meet this strict classification pattern.

Overall about 7.5% of the mortality context DMs and about 8.4% of the climate context DMs were entirely stable in their risk attitude across all decision environments, for a grand total of about 8% of all DMs. The distribution of types of risk attitude that are stable are nearly identical between the two conditions, suggesting no specific effect of choice context on stability.

While these results show that a strictly consistent risk attitude across contexts *and* frames is relatively rare, we can also determine how often individual DMs maintained a risk attitude across contexts within the same frame. It could be that DMs are stable across contexts, but susceptible to framing effects that makes a strictly consistent classification impossible. To assess this we calculated the percentage of DMs within each financial context frame that maintained a stable risk attitude in the other choice context's analogous frame (i.e., how often DMs classified as risk averse in pure financial gains were also risk averse for pure climate gains). These percentages are reported in Table 5 below.

Classification	Pure Gains (Climate)	Pure Losses (Climate)	Mixed Gains (Climate)	Mixed Losses (Climate)	Pure Gains (Mortality)	Pure Losses (Mortality)	Mixed Gains (Mortality)	Mixed Losses (Mortality)
Risk Averse	10.87	6.91	19.44	10.74	12.04	4.79	19.66	12.90
Risk Neutral	10.49	25.06	11.64	18.93	8.35	28.62	11.55	15.60
Risk Seeking	7.42	0.77	4.22	2.17	10.57	2.09	3.93	2.09
Split Neutral	5.37	13.43	4.22	11.25	4.91	13.76	4.05	10.20
Total	34.15	46.17	39.52	43.09	35.87	49.26	39.19	40.79

Table 5: Reports the percentage of DMs within each substantive choice context (columns) who were classified according to the same risk attitude (rows) in the corresponding financial context frame. Risk neutrality and risk aversion featured the highest rates of consistency across contexts.

Roughly 40% of individual DMs within each frame maintained a consistent risk attitude between the two choice contexts. We see that risk neutrality featured the most stability, particularly for loss frames where risk neutrality was more prevalent in all three contexts. Pure losses produced the highest rates of stable risk-neutral attitudes (as well as split neutrality) while mixed gains produced the highest rates of stable risk-averse attitudes. On the flip side, over half of individual DMs within

each frame had different risk attitudes depending on the choice context. These results show that measuring risk attitude on financial stimuli may not generalize well to other choice contexts, even when the frame and experimental paradigm are kept constant.

Older and Younger Adults

We explored whether older adults tended to be more risk averse than younger adults across all contexts and frames. We also explored whether older adults tended to exhibit *less* stable risk attitudes than their younger counterparts, as previous research has argued that older adults' "impaired" decision making is due to instability in their preferences. Finally we explored whether the reflection effect is more pronounced in older adults compared to younger adults.

Kellen, Mata and Davis-Stober (2017) used the LPC approach to classify older and younger adults in pure and mixed frames to investigate differences in the satisfaction of the reflection effect and risk aversion. They found minimal support for the reflection effect aside from in mixed gambles for younger adults (where about 55% satisfied the reflection effect).

Figure 4 (below) reports the classification results from the current study featuring 358 older adults (age > 64) and 554 younger adults (age < 35) across financial context frames. We also reproduce the classification rates from Kellen, Mata and Davis-Stober (KMDS, 2017) to compare against our results. Their study featured 30 older adults (age 61-78) and 29 younger adults (age 18-35).

Figure 4: This figure shows the aggregate classification rates for the current study (top) and Kellen, Mata and Davis-Stober's (2017) study (bottom). Each study examined the risk attitude of older and younger adults in pure- and mixed- gain and loss frames.

In general the two studies find similar classification rates. The key discrepancy appears to be in the risk neutral classification wherein Kellen et al. found much higher rates for younger adults. This is because younger adults in our sample tended to indicate neutrality via the split neutral model rather than through indifference (the risk neutral model). If we add the split neutral totals into the risk neutral totals the two sets of results would be very similar.

We assessed whether older adults exhibited less stable risk attitudes than younger adults between frames and between contexts. We find that older adults do not

appear systematically less stable than younger adults. About 36% of older adults and 33% of younger adults maintained a single risk attitude across pure frames. About 63% of older adults and 58% of younger adults did so in mixed frames.

For the mortality context we find that older adults were *more* stable than younger adults. About 44% of older adults were stable across pure frames compared to 40% for younger adults. In mixed frames, about 67% of older adults and 57% of younger adults were stable. About 49% (pure) and 60% (mixed) of older adults were stable in the climate change context compared to 48% (pure) and 51% (mixed) for younger adults. We conclude that the hypothesis that older adults are less stable, or more susceptible to framing effects, is not supported by the current data.

The reflection effect was present for 2.3% of older adults in the pure financial context and about 2% in the mixed financial context. About 4% of younger adults satisfied the reflection effect for pure financial gambles while about 1.6% did so for mixed financial gambles. These rates are similar to Kellen et al. (2017) with the exception for mixed gambles in younger adults where Kellen et al., observed 55% satisfaction.

The reflection effect was similarly scarce in the climate and mortality contexts for both age groups. In no context did the total percentage of DMs satisfying the effect eclipse 4%.

Discussion

We applied the LPC approach to multiple choice contexts in search of stability in risk attitude. Our results demonstrate substantial heterogeneity in each context, between- and within-subjects and across frames. We found little evidence of a reflection effect for individual DMs, and even aggregating across DMs and frames was unable to produce a substantial amount of evidence, calling into question whether the reflection effect is descriptive of many DM's choice behavior. If this is the case, models of decision making like (Cumulative) Prospect Theory (Tversky and Kahneman, 1992) are unable to adequately describe the choice behavior of DMs across a variety of decision environments.

Revisiting our hypotheses, we conclude the following:

The reflection effect was present for few DMs at rates similar to those observed in Davis-Stober and Brown (2013) and Kellen et al. (2017). This severe lack of a reflection effect was surprising and merits further research specifically focused on the robustness of the reflection effect in a p -additive risk framework. Strict risk attitudes carried across all contexts and frames were rare. We did not find strong evidence for strict risk aversion implied by traditional economic models of decision making. Less than half of DMs maintained a consistent risk attitude across contexts with the same frame.

Our hypotheses regarding older versus younger adults were not supported by the data. Specifically, older adults appeared no more risk averse than their younger counterparts in any choice context. These findings further support individual-level models of risk

attitude, as age group membership did not result in a clear segmentation of risk attitude profiles. Older adults did not show an exacerbated reflection effect, which could have supported a theory of aging and decision making that emphasizes instability of decision making or deprecation of executive function related to decision making. Further research could investigate the risk attitude preferences for individuals with demonstrable cognitive impairment to understand which cognitive processes may be related to risk attitude.

Harrison, Lau, & Rutström's (2007) argument that risk neutrality is rare in real DMs was not supported in our study. Risk neutrality was a very common attitude across all contexts and frames. We hypothesized that risk neutrality would be most prevalent in the climate change context due to apathy, intertemporal choice considerations as well as self-versus other (social choice) considerations. Our data did not support this hypothesis, but further study into the complicated nature of the climate change choice context is necessary.

We note several limitations of the current study. First, the number of repetitions was relatively low with as few as 4 for gain frames. This was a necessary compromise for survey length and motivated from a simulation analysis, but still resulted in non-diagnostic choice data that may have produced a classification under more repetitions. Second, the classification threshold of a Bayes factor greater than 1, while similar to Normalized Maximum Likelihood approaches (Kellen, Mata & Davis-Stober, 2017), is a weak classification threshold that could result in disagreements over whether the choice data truly represent a model well. We propose further experiments be carried out with more repetitions and a stricter classification threshold to rectify these potential issues. Finally, the climate change stimuli featured temperature change as the outcomes under consideration, but these were tied to many discrete as well as continuous environmental impacts in the form of an infographic. We acknowledge that this correspondence may not be perceived identically by all participants, thus calling into question whether participants experienced the experiment in the same way. We propose further research using the LPC method be carried out on different climate change stimuli to compare the results.

The simplicity of the LPC approach makes it an ideal choice for future work exploring risk attitude in young children or groups that experience difficulty with understanding probability. Children could be administered a version of the LPC task using non-monetary stimuli to explore the development of risk attitude over time, answering questions regarding the stability of individual risk attitude over the lifespan and across choice contexts without relying on between-subjects designs that could be influenced by cultural differences between generations. While we explored a mortality choice context, the LPC approach could be applied to symptoms or other health outcomes to more directly assess risk attitude in risky health decisions. Often, patients are required to choose between treatment options that have varying side-effects. The LPC's generality would allow for the development of symptom-based stimuli that closely resemble the actual choices DMs must make. These results could be compared with the mortality context results (in a within-subject design) to tease apart the effects of mortality and morbidity on risk attitude.

Similarly, the climate change context could be revised to use actual environmental impacts as the outcomes under consideration.

These models of risk attitude clearly have practical implications. In industry, understanding the risk attitude of consumers, and how it varies between types of products, services and offers, would be beneficial in creating personalized and mutually beneficial experiences for the company and the customer. Policy experts searching for the most effective messaging may benefit from understanding if and why individuals appear apathetic to the effects of climate change. Understanding how individuals perceive and prefer risk in mortality decisions has immediate implications for health care and health insurance decisions. Identifying individuals who appear to be entirely insensitive to health risks could help health care providers create effective messaging campaigns for preventative checkups and screening.

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