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THE IMPEDANCE OF A COIL PLACED ON A CONDUCTING PLANE

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The Impedance of a Coil Placed on a Conducting Plane

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FELLOW AIEE

Summary: Probe coils are useful in determining the properties of metals on which the coil is placed. An analysis and an equivalent circuit of the coil on a metal surface are needed and are presented here. The coil was assumed to have a ferromagnetic core, and the constants of the equivalent circuit were computed from the analysis. Experimental measurements of a probe coil on a metal surface, and with varying frequency and conductivity, were made to verify the theory.

PROBE COILS ARE USED for measuring the conductivity of metal

Fig. 1. Cross section of probe core

samples,¹ for testing metal products non-destructively,² and for determining the temperature of metal surfaces.³ If the coil is placed near a piece of metal, a varying current in the coil sets up eddy currents in the metal. The eddy currents react on the coil to produce an apparent change of the coil impedance. From this apparent change, various properties of the metal may be deduced. An analysis of a coil near a conductor has been made,⁴ although, under the assumptions made, it was found that the solution was inaccurate for small spacings between coil and metal. The present study was made to present theoretical and experimental results for a coil on or very close to a conducting surface.

Theory

In this study, it was decided to use a coil with a circular cylindrical core of ferromagnetic material whose cross section is shown in Fig. 1. The coil is

wound on the central leg of the core, and the magnetic flux is assumed to pass through this leg, enter the metal, and return by way of the outer shell of the core. The core is assumed to have such a high permeability that none of the flux passes through the air and all of it passes through the core and then into the conductor. This is probably the most serious assumption made, and its validity should be checked by the experimental results. Other assumptions made are that the magnetomotive force is zero in the core and that the metal is infinite in extent and depth. The theory is presented in some detail in the Appendix.

The results in the Appendix indicate that the equivalent circuit of the idealized probe coil might consist of a parallel combination of two admittances, $Y_m = (1/Z_m)$ —the apparent admittance of the conductor as viewed from the probe coil, and

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Part of the theoretical work was done at the Argonne National Laboratory, and the remainder was completed under a National Science Foundation grant to the University of Missouri, Columbia Mo.

Fig. 2 (left). L equivalent circuit of probe coil

$Y_p = (1/Z_p)$ —the admittance of the probe coil except for the apparent admittance of the conductor. An actual probe coil will, in addition, have a series resistance and inductive reactance representing the ohmic resistance of the wire in the coil and the leakage reactance of the coil. The leakage reactance in the actual probe coil would be caused by the magnetic flux lines flowing in the air rather than in the conductor.

Thus, the probe coil equivalent circuit might be taken as that of Fig. 2, which consists of Y_m at the output terminals of the 2-terminal pair network shown here as an L network. The admittance Y_B would approach Y_p as the magnetic permeability of the ferrite core became much greater than that of the conductor and as the leakage flux in the air approached zero. The input impedance is

$$Z = Z_A + \frac{1}{Y_B + Y_m} \quad (1)$$

The admittance Y_m may be obtained from

$$Y_m = \frac{G(\alpha, \beta, \gamma)}{j\omega\mu b\pi N^2} \quad (2)$$

where ω is the angular frequency, μ is the permeability of the metal in mks (meter-kilogram-second) units, b is the dimension of Fig. 1 in meters, and N is the number of turns on the probe coil. The quantity of $G(\alpha, \beta, \gamma)$ is a function of the geometrical shape of the probe, the frequency, and the metal constants, and is given by equation 23 of the Appendix where

Table I. Values of $G(\alpha, \beta, \gamma)$ for $\alpha=0.47$, $\beta=0.25$

$ \gamma $	$G(\alpha, \beta, \gamma)$
0.0.....	1.047+j0.0
0.1.....	1.047+j0.007708
0.3.....	1.048+j0.02305
1.0.....	1.052+j0.07552
3.0.....	1.079+j0.2139
10.0.....	1.231+j0.5942
30.0.....	1.678+j1.2781
100.0.....	2.782+j2.5618
300.0.....	4.688+j4.5608
1,000.0.....	8.477+j8.4074
3,000.0.....	14.644+j14.602
10,000.0.....	26.710+j26.685

Fig. 3 (right). Real and imaginary parts of $G(\alpha, \beta, \gamma)$ for $\alpha=0.47$, $\beta=0.25$

$$\alpha = (a/b)$$

$$\beta = (c/b)$$

$$\gamma = j\omega\sigma\mu b^2 \quad (3)$$

An electronic computer has been employed to calculate $G(\alpha, \beta, \gamma)$ over a large range of values of α , β , and γ . These values are given by V. E. Schuster and T. J. Russell in two research reports entitled, "The Impedance of a Coil Placed Against a Plane Metal Conductor." Although unpublished, these reports are available at the Engineering Library, University of Missouri, Columbia, Mo. This library will supply a microfilm or photostatic copy at a nominal cost. Table I gives the values of $G(\alpha, \beta, \gamma)$ for $\alpha=0.47$ and $\beta=0.25$ over a large range of the magnitude of γ , while a plot of these data is shown in Fig. 3. As γ approaches zero, the imaginary part of $G(\alpha, \beta, \gamma)$ approaches zero while the real part approaches a value given by equation 25. Physically, this indicates that, with the probe coil resting on a nonconductor, the losses are zero and Z_m becomes a

pure inductive reactance. As γ becomes large, both the real and imaginary parts of $G(\alpha, \beta, \gamma)$ approach each other and become large as indicated in Table I.

Anyone wishing to use values $G(\alpha, \beta, \gamma)$ other than those given in this paper would save much time and effort by using the computed values in the two reports. As shown in the Appendix, the admittance Y_m from equation 2 becomes approximately

$$Y_m = \frac{(1-j) \ln 1/\alpha}{2\sqrt{2}\pi N^2} \sqrt{\frac{\sigma}{\mu\omega}} \quad (4)$$

This shows that, for the probe coil resting on a good conductor, or a material with high permeability, or for very high frequencies, the admittance has equal real and imaginary parts, similar to the surface admittance of a very thick homogeneous conductor with a plane electromagnetic wave incident on the conductor.

An example of the variation of $G(\alpha, \beta, \gamma)$ with α and β is shown in Figs. 4 and 5 where $G(\alpha, \beta, \gamma) = G_r + jG_i$. Fig. 4 pre-

Fig. 4. Real part of $G(\alpha, \beta, \gamma)$ for $|\gamma| = 1.0$

Fig. 5. Imaginary part of $G(\alpha, \beta, \gamma)$ for $|\gamma| = 1.0$

sents the real part G_r , while Fig. 5 shows the imaginary part G_i for the magnitude of γ equal to unity. There is practically no change in $G(\alpha, \beta, \gamma)$ when β is greater than 10, and this corresponds to the case of a very thick outer shell on the probe core.

Experimental Work

The feeling prevailed that experimental work on an actual probe coil placed on various metal plates would help to verify the theory and also point out possible applications. Accordingly a number of ferrite cores were constructed out of commercially available ferrite rods, tubes, and sheets. In addition, several complete ferrite cores were obtained commercially, and the tests on one of these cores will be described. The particular core chosen is shown in Fig. 6 and had

$\alpha = 0.47$ and $\beta = 0.25$, values of α and β , the same as those chosen for Table I. The coil consisted of 125 turns of no. 38 copper wire, wound on the center leg of the core.

Metals of various conductivities were obtained in plates about 1 foot square. They were at least 1/4 inch thick, which was sufficient so that, at frequencies of 4 kc and above, the return from the second metal-to-air boundary could be neglected. The conductivities of the metal samples were measured by passing direct current through a piece of known cross-sectional area and measuring the voltage drop across the sample. The conductivities measured are given in Table II.

The measured impedance of the probe coil when placed against a conductor was obtained by use of the bridge circuit of Fig. 7. The quantities to be measured (R_x , L_x) were rather low in impedance, so R_2 was made a 2-ohm precision resistor. Decade resistors (R_3) and capacitors (C_3) were used along with a precision 0.1- μ f (microfarad) capacitor at C_1 . A wave analyzer was found to make a good detector. One problem that arises in measurements of this type is that the probe coil overheats because too much power is supplied to the coil. The voltage across the coil was kept below 5 volts for this reason although, if it were made

Fig. 6 (left). Experimental probe core

Fig. 7 (right). Circuit to measure impedance of probe coil

Table II. Measured Conductivities

Metal	Conductivity, 10^8 Mhos (Units of Conductance) Per Meter
B, Stainless Steel.....	1.4
C, Zirconium.....	2.0
D, Lead.....	4.8
E, Tin.....	8.6
F, Brass.....	14
G, Aluminum Alloy.....	23
H, Pure Aluminum.....	35
J, Copper.....	57

Table III. Calculated Values of Z_A and Y_B

Frequency, Kc	Z_A , Ohms	Y_B , Mhos
4.....	$5.59 + j13.05$	$0.009 - j0.0321$
8.....	$5.74 + j26.79$	$0.0035 - j0.0195$
16.....	$6.48 + j53.9$	$0.00143 - j0.0073$

too small, not enough voltage would be available for best operation of the detector. Measurements of the impedance were made at 1-kc intervals from 1 to 16 kc for each of the metal samples. Fig. 8 shows measured values of R and ωL where $Z = R + j\omega L$ for three frequencies. The upper curve is for a frequency of 16 kc; the middle, 10 kc; and the lower, 4 kc. The letters represent the conductors involved: A for air, B for stainless steel, and so on as given in Table II.

Comparison of Theoretical and Experimental Results

The first step was to determine the impedance Z_A and admittance Y_B of Fig. 2 at one frequency by using the measured values of the input impedance Z and the theoretically determined values of Y_m as taken from equation 2 and Table I. Two measured values of Z and two theoretical values of Y_m were needed, and these were taken as those for air and for copper. Air and copper were used because, in the measurement of any other probe coil, these might be the easiest measurements to obtain since they would require only a thick piece of fairly pure copper. Of course, to make the imped-

Fig. 8 (left). Measured impedance of probe coil

Fig. 9 (right). Comparison of calculated and measured values of probe-coil impedance, using L equivalent circuit

ance measurements, a bridge circuit similar to that of Fig. 7 would be needed. Values of Y_m could be obtained from graphs such as Figs. 4 and 5, or the more complete series of values are to be found in the research reports previously mentioned as being available in microfilm or photostatic copies. The length of the tables in these reports has discouraged their publication.

Values of Z_A and Y_B —calculated for 4, 8, and 16 kc—are given in Table III. The real part of Z_A is somewhat greater than the 5.52 ohms resistance of the coil as measured using direct current. The imaginary part of Z_A is almost directly proportional to frequency, indicating that it acts much like a constant inductance. The admittance Y_B , on the other hand, does not appear to have such a simple dependence on frequency and probably depends, for the most part, upon the properties of the ferrite core.

As a further check upon the calculated values of Y_m , the impedance Z was calculated, using the Y_m for each metal as given in equation 2 and Table I, along with the values of Z_A and Y_B as given in Table III. The results of these calculations for a frequency of 4 kc are shown in Fig. 9, where the scale has been expanded appreciably to bring out the differences between calculated and measured points. The end points for air (A) and copper (J) have the same calculated and measured

values, but all of the other metals will have calculated and measured values that differ from each other. The metals shown in Fig. 9 are zirconium (C), tin (E), and aluminum alloy (G). The calculated points are the upper ones of each set in Fig. 9, and probably indicate that the variation in inductive reactance for the calculated points is slower than that for the measured points as conductivity is increased. This seemed to indicate that possibly additional impedance between Y_B and Y_m of Fig. 2 might make the agreement even better.

The equivalent circuit of Fig. 10 was tried as a consequence of this idea. If $Z = (V_1/I_1)$,

$$Z = \frac{C + AZ_m}{B + Z_m} \quad (5)$$

where

$$A = Z_{11}$$

$$B = Z_{22}$$

$$C = Z_{11}Z_{22} - Z_{12}^2$$

Fig. 10 (left). T equivalent circuit of probe coil

Fig. 11 (right). Same as Fig. 9, except resistance is compared, using T equivalent circuit

For this equivalent circuit at a given frequency, the three constants A , B , and C may be determined by using three different conductivities and the corresponding three pairs of values of Z as measured and Z_m as obtained from equation 2. This was done, using stainless steel, aluminum alloy, and copper. At 4 kc, for example, the resulting values of A , B , and C are

$$A = 24.31 + j13.53 \text{ ohms}$$

$$B = 23.22 - j8.61 \text{ ohms}$$

$$C = 263.2 + j254.9 \text{ (ohms)}^2 \quad (6)$$

When A , B , and C from equation 6 are put in equation 5, the calculated value of Z may be obtained for any conductivity if the corresponding Z_m from 2 is also put in 5. The results of these calculations for four different frequencies are shown in Figs. 11 and 12. Again the scales have been expanded substantially so that

Fig. 12. Same as Fig. 9, except inductance is compared, using T equivalent circuit

measured and calculated differences are clearly distinguishable.

Conclusions

A method of analysis for a probe coil with a ferrite core has been presented, along with some of the calculated results. Some experimental measurements have also been presented and compared with the calculated results by means of two equivalent circuits for the probe. This information should be useful in designing probe coils and in planning electronic circuitry associated with the coils. Extensions of this analysis and experimental work would be helpful in connection with:

1. Probe coils separated by air or non-metallic material from a conducting surface.
2. Probe coils with air as the core.
3. Pulsed probe coils.
4. Probe coils over a conductor with layers of different conductivities.

Appendix. Derivation of the Coil Equations

From Fig. 1, the magnetic field intensity H_z in the Z direction in the conductor may be obtained from the following wave equation:

$$\nabla^2 H_z = \sigma \mu p H_z \quad (7)$$

where ∇^2 is the Laplacian; σ and μ are the conductivity and permeability, respectively, of the conductor; $p = j\omega = j2\pi f$; and f is the frequency of the sinusoidal wave of current flowing in the coil. The Z axis is the same as the axis of the core with the positive direction of the axis pointing from the conductor toward the core. From equation 7, using cylindrical co-ordinates

$$H_z = g e^{k_1 z} J_0(k_2 r) \quad (8)$$

where g and k_1 are arbitrary constants, J_0 is the Bessel function of the first kind of zero order, and

$$k_2^2 = k_1^2 - \sigma \mu p \quad (9)$$

The other field components may be obtained from Maxwell's equations

$$H_r = -\frac{g k_1}{k_2} e^{k_1 z} J_1(k_2 r) \quad (10)$$

$$E = -\frac{g \mu p}{k_2} e^{k_1 z} J_1(k_2 r)$$

where J_1 is the Bessel function of the first order.

To satisfy the boundary conditions, using equation 8, put H_z in the form of the following equation:

$$H_z = \int_0^\infty g(\lambda) e^{kz} J_0(\lambda r) \lambda d\lambda \quad (11)$$

where $k = \sqrt{\lambda^2 + \sigma \mu p}$, λ is a variable of integration, and $g(\lambda)$ is an arbitrary function. Similarly, for the other field components

$$H_r = -\int_0^\infty g(\lambda) k e^{kz} J_1(\lambda r) d\lambda$$

$$E = -p \mu \int_0^\infty g(\lambda) e^{kz} J_1(\lambda r) d\lambda \quad (12)$$

The field components indicate that eddy currents in the conductor are circular, are in planes perpendicular to the Z axis, and have their centers on this axis. Next it was assumed that the total magnetic flux in the center leg of the core was Φ and that the flux density was the same everywhere over the cross section of this leg. The flux density in the center leg is then

$$\Phi / \pi a^2 \quad (13)$$

and in the outer shell

$$\Phi / \pi [(b+c)^2 - b^2] = \Phi / \pi c(2b+c) \quad (14)$$

where a , b , and c are dimensions as shown in Fig. 1. Using equations 11, 13, and 14, the magnetic field intensity in the Z direction at the surface of the conductor is

$$H_z|_{z=0} = \int_0^\infty g(\lambda) J_0(\lambda r) \lambda d\lambda = \begin{cases} -\Phi / \mu \pi a^2, & 0 < r < a \\ 0, & a < r < b \\ \Phi / \mu \pi c(2b+c), & b < r < (b+c) \\ 0, & (b+c) < r \end{cases} \quad (15)$$

where μ is the permeability of the conductor. Equation 15 is in the form of a Hankel transform.⁵ The inverse transform of 15 is as follows

$$g(\lambda) = \int_0^\infty [H_z|_{z=0}] J_0(\lambda r) r dr = \frac{\Phi}{\mu \pi} \left\{ -\frac{J_1(\lambda a)}{(\lambda a)} + \frac{(b+c)^2}{c(2b+c)} \frac{J_1[\lambda(b+c)]}{\lambda(b+c)} - \frac{b^2}{c(2b+c)} \frac{J_1(\lambda b)}{(\lambda b)} \right\} \quad (16)$$

The magnetomotive force NI is found by integrating the magnetic field intensity around a closed path, or

$$NI = \int_a^b (H_r|_{z=0}) dr + (NI)_{\text{probe}} \quad (17)$$

The first integral is the magnetomotive force for the conductor, while the second term is that for the ferrite core of the probe coil. Assuming the resistance of the coil is zero for the moment, the admittance Y of the probe coil is

$$Y = \frac{\frac{1}{N} \int_a^b (H_r|_{z=0}) dr + \frac{(NI)_{\text{probe}}}{N}}{N j \omega \Phi} = Y_m + Y_p \quad (18)$$

where $N j \omega \Phi$ is the voltage across the coil, Y_m is the portion of the admittance ascribable to the conductor, and Y_p that for the coil itself. Then

$$Y_m = \frac{1}{j \omega \Phi N^2} \int_a^b (H_r|_{z=0}) dr \quad (19)$$

From equations 12 and 16

$$H_r|_{z=0} = \frac{\Phi}{\mu \pi} \left\{ \int_0^\infty k \frac{J_1(\lambda a)}{\lambda a} J_1(\lambda r) d\lambda - \frac{(b+c)^2}{c(2b+c)} \int_0^\infty k \frac{J_1[\lambda(b+c)]}{\lambda(b+c)} J_1(\lambda r) d\lambda + \frac{b^2}{c(2b+c)} \int_0^\infty k \frac{J_1(\lambda b)}{\lambda b} J_1(\lambda r) d\lambda \right\} \quad (20)$$

Now let

$$F(\alpha, \delta, \gamma) = \int_0^\infty \sqrt{\lambda^2 + \gamma} J_1(\lambda \delta) \times [J_0(\lambda \alpha) - J_0(\lambda)] \frac{d\lambda}{\lambda^2} \quad (21)$$

$$\alpha = (a/b)$$

$$\beta = (c/b)$$

$$\gamma = j \omega \sigma \mu b^2$$

Then, by using equations 19 and 20 and interchanging the order of integration

$$Y_m = \frac{1}{j \omega \mu b \pi N^2} \left[\frac{1}{\alpha} F(\alpha, \alpha, \gamma) - \frac{(1+\beta)}{\beta(2+\beta)} F(\alpha, 1+\beta, \gamma) + \frac{1}{\beta(2+\beta)} F(\alpha, 1, \gamma) \right] \quad (22)$$

where $F(\alpha, \delta, \gamma)$ may be obtained from equation 21. If

$$G(\alpha, \beta, \gamma) = \frac{1}{\alpha} F(\alpha, \alpha, \gamma) - \frac{1+\beta}{\beta(2+\beta)} F(\alpha, 1+\beta, \gamma) + \frac{1}{\beta(2+\beta)} F(\alpha, 1, \gamma) \quad (23)$$

then the apparent impedance introduced by the conductor is

$$Z_m = \frac{1}{Y_m} = \frac{j\omega\mu b\pi N^2}{G(\alpha, \beta, \gamma)} \quad (24)$$

For the special case of a perfect non-conductor ($\sigma=0$, $\gamma=0$) and from equation 23

$$G(\alpha, \beta, 0) = \frac{2}{\pi\alpha} - \frac{1}{2} \left[{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 2; \alpha^2\right) \right] + \frac{2(1+\beta)}{\pi\beta(2+\beta)} \left[E\left(\frac{1}{1+\beta}\right) - E\left(\frac{\alpha}{1+\beta}\right) \right] + \frac{2}{\pi\beta(2+\beta)} [E(\alpha) - 1] \quad (25)$$

where ${}_2F_1(1/2, 1/2; 2; \alpha^2)$ is a hypergeometric function and is given by the series^{6,7}

$${}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 2; \alpha^2\right) = 1 + \frac{\left(\frac{1}{2}\right)^2}{2} \left[\frac{\alpha^2}{1!} \right] +$$

$$\frac{\left(\frac{1}{2}\right)^2 \left(\frac{3}{2}\right)^2}{2 \cdot 3} \left[\frac{\alpha^4}{2!} \right] + \frac{\left(\frac{1}{2}\right)^2 \left(\frac{3}{2}\right)^2 \left(\frac{5}{2}\right)^2}{2 \cdot 3 \cdot 4} \times \left[\frac{\alpha^6}{3!} \right] + \dots$$

and $E(\alpha)$ is the complete elliptic integral of the second kind.⁸

The other important special case is that of a perfect conductor ($\sigma \rightarrow \infty$, $\gamma \rightarrow \infty$), and from equation 22, it is found^{6,7} that when σ is very large but finite, the admittance Y_m from the same equation is approximately

$$Y_m \approx \frac{(1-j)\ln(1/\alpha)}{2\sqrt{2\pi N^2}} \sqrt{\frac{\sigma}{\mu\omega}} \quad (26)$$

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