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Transmission Losses and Economy Loading by the Use of Admittance Constants

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Transmission Losses and Economy Loading by the Use of Admittance Constants

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Summary: In order to operate a power system at optimum economy, the production costs and transmission losses must be co-ordinated. A straightforward derivation of the co-ordination conditions is given. A widely adopted method of finding transmission penalty factors used so-called "B-constants." These constants are not easily modified to consider changes in the transmission system or deviation from "base-case" system conditions. By starting with the equilibrium equation of a power system written in terms of admittance constants, a concise expression for the total losses of a power transmission system which does not neglect shunt paths to neutral or the effect of off-nominal turns ratios is developed. Following this same approach, an easily modified, approximate expression for the penalty factor is formed. Any errors incurred by using this approximate penalty factor are small when compared with the inaccuracy of the original data on production costs. Several test cases on hypothetical systems are given. The generation schedules for economic dispatch, using approximate penalty factors, compare quite favorably in actual hourly costs with the costs of schedules obtained by exact penalty factors.

FOR MANY YEARS, the vast majority of electric power systems operating in the United States have dispatched generation according to the incremental production costs of the individual generating units or stations.¹ With the continued growth of transmission networks and system interconnections, however, an increasing emphasis now is being placed on the effect of transmission losses upon the over-all economy of operation of power systems. A transmission loss formula, which expressed the total transmission losses in terms of source powers, was first presented by E. E. George² in

1943. It was of the following form:

$$L = \sum_m \sum_n P_m B_{mn} P_n \quad (1)$$

where P_m and P_n are the source loadings, L is the total power loss in the transmission system, and the B_{mn} are the transmission-loss-formula coefficients or B -constants. With properly evaluated B -constants, this formula gives satisfactory answers as long as the power system is operating at conditions similar to the base case.

The B -constant form of loss formula offers a convenient method for the determination of total losses and gives a fairly simple expression for incremental losses. As will be shown later, the incremental losses have an important role in the over-all economy loading of power systems when fuel costs and transmission losses are co-ordinated. However, there is some objection to the lack of physical significance of the B -constants and to the fact that the system constants are so deeply buried beneath the transformations and mathematical manipulations required in obtaining the B -constants. Because the system constants are so absorbed into the B -constants, the effect of any change in the system on the loss formula and its accuracy cannot easily be assessed.

When new or revised loss formulas cannot be developed and put into use exactly coincident with each major system revision, or even when lines or equipment are temporarily out of service, the B -constant loss formula must be completely abandoned or misapplied. This formula is not easily revised. Although techniques for finding B -constants have improved greatly since the beginning of their use, the time and effort required for developing a new set for every system condition still limits their usefulness and effectiveness.

A method for finding the incremental losses of a power system when shifting generation from one source to another which involves functions of voltage phase angles was proposed by Brownlee³ in 1954. This method seems much more fundamental, and the system constants remain clearly visible in the formula, which was

developed for a 2-machine system and was based on the idea that changes in generator loadings were accompanied by changes in voltage phase angles as in the normal power system. The principal objections raised at the time the method was presented were, first, that it could not be extended rigorously to a system with more than two sources. By extending the formula in its original form to include multisource systems, a weakness in the formula was introduced. For the multisource case the accuracy of the determination of the incremental loss is greatly influenced by the variation in X/R ratios of the different branches of the transmission system.

The use of voltage phase angles was extended to the development of a B -constant type of loss formula for multisource systems by Cahn.⁴ However, in circumventing the need for phase-angle information in this formula, the system constants became deeply absorbed again and modification is difficult.

By writing the formulas for power flow and losses in a power system on a nodal basis, and utilizing the idea that changes in source loadings are accompanied by changes in voltage phase angles, a simple formula for use in economy loading is derived. As is shown later, this formula in most cases gives results which are as good as or better than the B -constants for economy loading. Moreover, the system constants are explicit as factors in the formula and may be readily changed for any system condition.

Conditions for Optimum Economy Considering Transmission Losses

For m plants feeding power into a transmission system, the total production cost for the entire power system will be

$$C_t = C_1(P_1) + C_2(P_2) + C_3(P_3) + \dots + C_m(P_m) \quad (2)$$

Fig. 1. General mesh network with four terminals besides neutral

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The load power of the system is

$$P_L = P_1 + P_2 + \dots + P_m - L \quad (3)$$

In this expression P_L is the load power; $P_1, P_2, \dots, P_m$ are the powers supplied to the transmission system by plants 1, 2, $\dots, m$; and L is defined as the total power loss. If it is assumed that the load powers are constant, the differential of the total load power P_L becomes:

$$0 = dP_L = dP_1 + dP_2 + \dots + dP_m -$$

$$\frac{\partial L}{\partial P_1} dP_1 - \frac{\partial L}{\partial P_2} dP_2 - \dots - \frac{\partial L}{\partial P_m} dP_m \quad (4)$$

In this expression, each dP may be assumed to be independent except for some arbitrary one, which is made dependent. By arbitrarily choosing this one dependent dP as dP_1 , equation 4 solves for dP_1 :

$$dP_1 = - \frac{dP_2 \left(1 - \frac{\partial L}{\partial P_2}\right)}{-1 \frac{\partial L}{\partial P_1}} - \frac{dP_3 \left(1 - \frac{\partial L}{\partial P_3}\right)}{1 - \frac{\partial L}{\partial P_1}} - \dots - \frac{dP_m \left(1 - \frac{\partial L}{\partial P_m}\right)}{1 - \frac{\partial L}{\partial P_1}} \quad (5)$$

The differential of the production cost relationship, equation 2, is

$$dC_t = \frac{dC_1(P_1)}{dP_1} dP_1 + \frac{dC_2(P_2)}{dP_2} dP_2 + \dots + \frac{dC_m(P_m)}{dP_m} dP_m \quad (6)$$

In this expression, the derivatives are all total derivatives because the individual cost functions are each independent of all other cost functions. Equation 6 must equal zero for minimum total cost. This is sufficient, provided the production cost curves always have positive slopes.

The value for dP_1 in equation 5 is substituted into equation 6, which, by regrouping coefficients of dP 's, becomes:

$$dC_t = dP_2 \left[\frac{dC_2(P_2)}{dP_2} - \frac{dC_1(P_1)}{dP_1} \left(1 - \frac{\partial L}{\partial P_2}\right) \right] + dP_3 \left[\frac{dC_3(P_3)}{dP_3} - \frac{dC_1(P_1)}{dP_1} \left(1 - \frac{\partial L}{\partial P_3}\right) \right] + \dots + dP_m \left[\frac{dC_m(P_m)}{dP_m} - \frac{dC_1(P_1)}{dP_1} \left(1 - \frac{\partial L}{\partial P_m}\right) \right] = 0 \quad (7)$$

Since the differential dP 's in equation 7 are all independent, each term individually must equal zero. This condition can be satisfied only by setting each coefficient equal to zero. By doing so, and separating the variables in each equation, the relationships may be written in a chain of equalities as:

$$\frac{dC_1(P_1)}{dP_1} \frac{1}{1 - \frac{\partial L}{\partial P_1}} = \frac{dC_2(P_2)}{dP_2} \frac{1}{1 - \frac{\partial L}{\partial P_2}} = \dots = \frac{dC_m(P_m)}{dP_m} \frac{1}{1 - \frac{\partial L}{\partial P_m}} \quad (8)$$

The selection of dP_1 as independent was purely arbitrary and any other choice gives the same result.

Equation 8 indicates that, for minimum total cost considering transmission losses, the incremental production cost rate times a factor should be the same for each power plant. This factor has been called a penalty factor because it is derived from the incremental losses associated with that particular plant and has its highest positive value when the incremental loss has its highest positive value. The product of the incremental cost rate and the penalty factor, then, may be thought of as being the apparent incremental cost of power delivered to a constant load from any one power plant.

Equation 8 has been presented previously (e.g., the book⁵ by Dr. Kirchmayer). The following sections are devoted to a development of penalty factors according to the terms of admittance constants.

Loss Formula in Terms of Admittance Constants and Voltage Phase Angles

Consider a mesh network with four terminals besides neutral, as shown in Fig. 1. The assumed positive direction of current is "in" to the network at all terminals. The current equations for each node are

$$\begin{aligned} I_1 &= Y_{11}E_1 - Y_{12}E_2 - Y_{13}E_3 - Y_{14}E_4 \\ I_2 &= -Y_{21}E_1 + Y_{22}E_2 - Y_{23}E_3 - Y_{24}E_4 \\ I_3 &= -Y_{31}E_1 - Y_{32}E_2 + Y_{33}E_3 - Y_{34}E_4 \\ I_4 &= -Y_{41}E_1 - Y_{42}E_2 - Y_{43}E_3 + Y_{44}E_4 \end{aligned} \quad (9)$$

The I 's are defined as the per-unit (pu) phasor current "in" at each node, the E 's are the pu voltages to neutral at each node, and the Y 's are the pu driving-point and transfer admittances. The Y 's are further defined as follows:

$$Y_{sr} = Y_{rs} \quad (10)$$

Fig. 2. Three-terminal sample power system

All voltage magnitudes = 1 pu

All admittances in pu on a common base

the pu transfer admittance, directly connecting nodes r and s .

$$Y_{ss} = \text{driving-point admittance} = \sum_{r \neq s} Y_{sr} \quad r=0 \quad (11)$$

the summation of mesh admittances connected directly to node s , zero in this case meaning neutral.

The pu phasor power "in" at each node is found by multiplying the pu voltage to neutral by the complex conjugate of the phasor current in at that node. The pu phasor power in at each node is then given by:

$$\begin{aligned} S_1 &= E_1 I_1 = E_1 \hat{Y}_{11} \hat{E}_1 - E_1 \hat{Y}_{12} \hat{E}_2 - E_1 \hat{Y}_{13} \hat{E}_3 - E_1 \hat{Y}_{14} \hat{E}_4 \\ S_2 &= E_2 I_2 = -E_2 \hat{Y}_{21} \hat{E}_1 + E_2 \hat{Y}_{22} \hat{E}_2 - E_2 \hat{Y}_{23} \hat{E}_3 - E_2 \hat{Y}_{24} \hat{E}_4 \\ S_3 &= E_3 I_3 = -E_3 \hat{Y}_{31} \hat{E}_1 - E_3 \hat{Y}_{32} \hat{E}_2 + E_3 \hat{Y}_{33} \hat{E}_3 - E_3 \hat{Y}_{34} \hat{E}_4 \\ S_4 &= E_4 I_4 = -E_4 \hat{Y}_{41} \hat{E}_1 - E_4 \hat{Y}_{42} \hat{E}_2 - E_4 \hat{Y}_{43} \hat{E}_3 + E_4 \hat{Y}_{44} \hat{E}_4 \end{aligned} \quad (12)$$

The algebraic sum of all powers into the network is equal to the losses in the network. The sum of any complex quantity and its complex conjugate equals twice the real part. By letting G be the real part of Y and δ the angle of E , adding equations 12 above to their complex conjugates, and dividing by two, the total losses in this simple transmission system may be written as:

$$\begin{aligned} L &= G_{11} |E_1|^2 + G_{22} |E_2|^2 + G_{33} |E_3|^2 + G_{44} |E_4|^2 - 2G_{12} |E_1| |E_2| \cos(\delta_1 - \delta_2) - 2G_{13} |E_1| |E_3| \cos(\delta_1 - \delta_3) - 2G_{14} |E_1| |E_4| \cos(\delta_1 - \delta_4) - 2G_{23} |E_2| |E_3| \cos(\delta_2 - \delta_3) - 2G_{24} |E_2| |E_4| \cos(\delta_2 - \delta_4) - 2G_{34} |E_3| |E_4| \cos(\delta_3 - \delta_4) \end{aligned} \quad (13)$$

Generalizing for an n -terminal mesh network, the total losses are

$$L = 2 \sum_{s=1}^n G_{ss} |E_s|^2 - \sum_{r=1}^n \sum_{s=1}^n G_{rs} |E_r| |E_s| \cos(\delta_r - \delta_s) \quad (14)$$

Equation 14 has positive and negative terms of the same order of magnitude; therefore, in combining the terms as shown the accuracy of the total loss will not be great unless great care is exercised in computing the individual terms. The formula may be rearranged by substituting for G_{ss} by analogy with equation 11 and regrouping the multipliers of the G 's as follows:

$$L = \sum_{s=1}^n G_{s0} |E_s|^2 + 1/2 \sum_{r=1}^n \sum_{s=1}^n G_{rs} [|E_r|^2 + |E_s|^2 - |E_r| |E_s| \cos(\delta_r - \delta_s)] \quad (15)$$

All terms in these summations are positive since, by the law of cosines from trigonometry, the multiplier of G_{rs} is simply $|E_r - E_s|^2$. The choice of which of the above expressions to use for losses and for the partial derivatives of losses, depends on the form of the voltage information and the computing facilities available.

An Approximate Incremental Loss Factor

For an a-c power system with relatively constant voltage magnitudes and having primarily reactive impedances, changes

in power are accompanied by changes in angles. If the system is in stable equilibrium, an increase in power input to the system at any terminal is accompanied by an advance in angle of the voltage at that terminal. Since the change in power is accompanied by a change in angle at the same terminal, holding the powers at all except one generating station will have a strong tendency to hold the phase angles constant at the terminals where the power does not change. By using this consideration, the incremental loss associated with plant n for a constant-voltage system is approximately given by:

$$\frac{\partial L}{\partial P_n} \approx \frac{\frac{\partial L}{\partial \delta_n}}{\frac{\partial P_n}{\partial \delta_n}} \quad (16)$$

In this expression the angles of all generators are held fixed instead of holding all power constant except P_n as in an exact expression.

To determine just how approximate the equation 16 might be, a simple example is selected and examined in detail. The example chosen is a 3-terminal system with two generators and one load. Since there are three angles, the load angle is assumed to remain fixed and is used as a reference. Considering terminal no. 1 of this simple system as being one of the generator terminals, the derivative of P_1 with respect to itself will be given by

$$\frac{\partial P_1}{\partial P_1} = 1 = \frac{\partial P_1}{\partial \delta_1} \frac{\partial \delta_1}{\partial P_1} + \frac{\partial P_1}{\partial \delta_2} \frac{\partial \delta_2}{\partial P_1} \quad (17)$$

The derivative of P_2 with respect to P_1 ,

assuming that P_2 is not changed, will be

$$\frac{\partial P_2}{\partial P_1} = 0 = \frac{\partial P_2}{\partial \delta_1} \frac{\partial \delta_1}{\partial P_1} + \frac{\partial P_2}{\partial \delta_2} \frac{\partial \delta_2}{\partial P_1} \quad (18)$$

The derivative of the loss with respect to P_1 is given by:

$$\frac{\partial L}{\partial P_1} = \frac{\partial L}{\partial \delta_1} \frac{\partial \delta_1}{\partial P_1} + \frac{\partial L}{\partial \delta_2} \frac{\partial \delta_2}{\partial P_1} \quad (19)$$

By assuming that the losses and powers are expressed as functions of the admittance constants, voltages, and phase angles of the simple system, the foregoing three equations may be solved simultaneously to determine the exact incremental losses of generator no. 1. The desired solution is given by:

$$\frac{\partial L}{\partial P_1} = \frac{\frac{\partial P_2}{\partial \delta_1} \frac{\partial L}{\partial \delta_2} - \frac{\partial L}{\partial \delta_1} \frac{\partial P_2}{\partial \delta_2}}{\frac{\partial P_2}{\partial \delta_1} \frac{\partial P_1}{\partial \delta_2} - \frac{\partial P_1}{\partial \delta_1} \frac{\partial P_2}{\partial \delta_2}} \quad (20)$$

By forming equations similar to equations 17, 18, and 19, but holding P_1 fixed instead of P_2 , the exact incremental loss associated with generator no. 2 may be written as:

$$\frac{\partial L}{\partial P_2} = \frac{\frac{\partial P_1}{\partial \delta_2} \frac{\partial L}{\partial \delta_1} - \frac{\partial L}{\partial \delta_2} \frac{\partial P_1}{\partial \delta_1}}{\frac{\partial P_2}{\partial \delta_2} \frac{\partial P_1}{\partial \delta_1} - \frac{\partial P_1}{\partial \delta_2} \frac{\partial P_2}{\partial \delta_1}} \quad (21)$$

For this simple system, the generator powers expressed in terms of admittance constants, voltages, and phase angles are given by the following equations:

$$P_1 = |E_1|^2 G_{11} - |E_1| |E_2| G_{12} \cos(\delta_1 - \delta_2) + |E_1| |E_2| B_{12} \sin(\delta_1 - \delta_2) - |E_1| |E_L| G_{1L} \cos(\delta_1 - \delta_L) + |E_1| |E_2| B_{1L} \sin(\delta_1 - \delta_L)$$

$$P_2 = |E_2|^2 G_{22} - |E_2| |E_1| G_{21} \cos(\delta_2 - \delta_1) + |E_2| |E_1| B_{21} \sin(\delta_2 - \delta_1) - |E_2| |E_L| G_{2L} \cos(\delta_2 - \delta_L) + |E_2| |E_L| B_{2L} \sin(\delta_2 - \delta_L) \quad (22)$$

By writing an equation similar to those for the generator input powers, expressing the input power at the third terminal in terms of admittance constants, voltages, and phase angles, and then adding all three equations, the total power loss in this simple system may be written as:

$$\text{Loss} = |E_1|^2 G_{11} + |E_2|^2 G_{22} + |E_L|^2 G_{LL} - |E_1| |E_2| (G_{12} + G_{21}) \cos(\delta_1 - \delta_2) + |E_1| |E_2| (B_{12} - B_{21}) \sin(\delta_1 - \delta_2) - |E_1| |E_L| (G_{1L} + G_{L1}) \cos(\delta_1 - \delta_L) + |E_1| |E_L| (B_{1L} - B_{L1}) \sin(\delta_1 - \delta_L) - |E_2| |E_L| (G_{2L} + G_{L2}) \cos(\delta_2 - \delta_L) + |E_2| |E_L| (B_{2L} - B_{L2}) \sin(\delta_2 - \delta_L) \quad (23)$$

By assuming that this is a constant-voltage system, the various partial derivatives

Fig. 3. Incremental loss of generator no. 1 versus its own angle

Fig. 4. Incremental loss of generator no. 2 versus its own angle

Fig. 5. Power system used to check and co-ordinate losses; admittance values in pu on 100-megavolt-ampere base

with respect to angles δ_1 and δ_2 all of which are factors in the exact incremental loss expressions, equations 20 and 21, and some of which factor in the approximate incremental loss expression, equation 16, may be written as follows:

$$\frac{\partial P_1}{\partial \delta_1} = |E_1| |E_2| G_{12} \sin(\delta_1 - \delta_2) +$$

$$|E_1| |E_2| B_{12} \cos(\delta_1 - \delta_2) +$$

$$|E_1| |E_L| G_{1L} \sin(\delta_1 - \delta_L) +$$

$$|E_1| |E_L| B_{1L} \cos(\delta_1 - \delta_L)$$

$$\frac{\partial P_1}{\partial \delta_2} = -|E_1| |E_2| G_{12} \sin(\delta_1 - \delta_2) -$$

$$|E_1| |E_2| B_{12} \cos(\delta_1 - \delta_2)$$

$$\frac{\partial P_2}{\partial \delta_1} = -|E_2| |E_1| G_{21} \sin(\delta_2 - \delta_1) -$$

$$|E_2| |E_1| B_{21} \cos(\delta_2 - \delta_1)$$

$$\frac{\partial P_2}{\partial \delta_2} = |E_2| |E_1| G_{21} \sin(\delta_2 - \delta_1) +$$

$$|E_2| |E_1| B_{21} \cos(\delta_2 - \delta_1) +$$

$$|E_2| |E_L| G_{2L} \sin(\delta_2 - \delta_L) +$$

$$|E_2| |E_L| B_{2L} \cos(\delta_2 - \delta_L)$$

$$\frac{\partial L}{\partial \delta_1} = |E_1| |E_2| (G_{12} + G_{21}) \sin(\delta_1 - \delta_2) +$$

$$|E_1| |E_2| (B_{12} - B_{21}) \cos(\delta_1 - \delta_2) +$$

$$|E_1| |E_L| (G_{1L} + G_{L1}) \sin(\delta_1 - \delta_L) +$$

$$|E_1| |E_L| (B_{1L} - B_{L1}) \cos(\delta_1 - \delta_L)$$

$$\frac{\partial L}{\partial \delta_2} = |E_2| |E_1| (G_{21} + G_{12}) \sin(\delta_2 - \delta_1) +$$

$$|E_2| |E_1| (B_{21} - B_{12}) \cos(\delta_2 - \delta_1) +$$

$$|E_2| |E_L| (G_{2L} + G_{L2}) \sin(\delta_2 - \delta_L) +$$

$$|E_2| |E_L| (B_{2L} - B_{L2}) \cos(\delta_2 - \delta_L) \quad (24)$$

By examining the above factors and their relative positions in the exact expressions for incremental losses, equations 20 and 21, it is seen that if Y_{12} and Y_{21} are both

Fig. 6. Power system used to check and co-ordinate losses; admittances in pu on 200-megavolt-ampere base

made smaller and smaller, the exact incremental loss expression will approach the value of the approximate incremental loss factors, equation 16. In the limit, as Y_{12} and Y_{21} both equal zero, the two expressions for incremental losses will give the same value.

If the admittance constants used in the equations of the system were those of an actual network, the mutual terms would be reciprocal. In this case the cosine terms in the last two of equations 24 would have zero coefficients. Then for the case having all angles equal, both the exact and approximate incremental loss equations would equal zero for both generators. As the angles diverge, however,

the two expressions do not yield the same result for the incremental loss factor.

To evaluate further the correlation of the exact and the approximate incremental loss factors as the angles between the generators were varied, a numerical example was selected and the incremental loss factors calculated by both equations. A diagram of the numerical example chosen and checked is shown in Fig. 2. The coupling between the two generators was purposely chosen to be greater than the coupling between either generator and the load because it is this factor which would cause the two forms of incremental loss factor to diverge.

The effect of varying the angle of the generator upon both forms of incremental loss factor was calculated over a range of angle variation from -20 to $+20$ degrees for each generator terminal. The results of these calculations showing the effect of this angular shift on both forms of incremental loss factor for generator no. 1 are shown in Fig. 3. Similar results for generator no. 2 are shown in Fig. 4. As may be noted from these figures, the exact and approximate incremental loss factors always increase or decrease together as the angle of the station concerned is increased or decreased. As was mentioned when the analytical expressions were formed, the two forms give the same value when all angles are zero. The values separate somewhat at the larger angles but this is not considered as too serious since both forms show the same general increase or the same general decrease as the angle itself is varied.

Table I. Comparison of Incremental Losses in Sample Power System of Fig. 5

	Generator Powers, Mw*	Per-Unit Incremental Loss		
		B-Constant Method	Exact Method	Approximate Method
Case A-2: peak load.....	G_1600.00.....	0.0475.....	0.0229.....	0.0769
	G_2200.00.....	0.0004.....	0.0137.....	0.0008
	G_3411.33.....	-0.0142.....	0.0072.....	0.0439
Case B-1: median load (base case for B-constants).....	G_1200.00.....	-0.1397.....	-0.1386.....	-0.1485
	G_2400.00.....	0.1767.....	0.1755.....	0.1483
	G_3348.44.....	0.1089.....	0.1082.....	0.0479
Case C-5: valley load.....	G_1300.00.....	0.0024.....	0.0054.....	0.0287
	G_2200.00.....	0.0630.....	0.0590.....	0.0637
	G_3107.68.....	-0.0086.....	-0.0109.....	-0.0104

* Megawatts.

Table II. Comparison of Incremental Losses in Sample Power System of Fig. 6

	Generator Powers, Mw	Per-Unit Incremental Loss		
		B-Constant Method	Exact Method	Approximate Method
Case 3: median load (base case for B-constants).....	G_1144.000.....	0.0260.....	0.0146.....	0.0312
	G_210.668.....	-0.0037.....	-0.0151.....	-0.0108
	G_334.600.....	0.0582.....	0.0609.....	0.0830
Case 3-III-3: peak load.....	G_1161.173.....	0.0256.....	0.0213.....	0.0242
	G_245.369.....	-0.0058.....	-0.0030.....	-0.0031
	G_345.396.....	0.0848.....	0.0811.....	0.1072

Table III. Comparison of Economic Dispatches of Generation for the Power System of Fig. 5

		B-Constant Dispatch	Exact Penalty Factor Dispatch	Approximate Penalty Factor Dispatch
Peak load case...	P_{G1} (mw)	298.18	299.96	273.56
	P_{G2} (mw)	489.65	496.82	489.88
	P_{G3} (mw)	478.36	469.50	511.41
	λ_1	4.2348	4.2522	4.0446
	λ_2	4.2300	4.2471	4.0049
	λ_3	4.2652	4.2344	4.0454
	Total losses (mw)	66.19	66.28	75.16
	Cost in \$/hour	\$4,511.50	\$4,510.49	\$4,515.34
	P_{G1} (mw)	200.00	200.00	180.08
	P_{G2} (mw)	400.00	400.00	390.66
Median load case	P_{G3} (mw)	348.44	348.44	383.21
	λ_1	4.0024	4.0000	3.8226
	λ_2	4.0061	4.0000	3.8453
	λ_3	4.0029	4.0000	3.8226
	Total losses (mw)	48.44	48.44	53.75
	Cost in \$/hour	\$3,274.03	\$3,274.03	\$3,277.26
	P_{G1} (mw)	106.34	105.77	94.16
	P_{G2} (mw)	305.44	300.73	288.14
	P_{G3} (mw)	222.42	225.96	252.34
	λ_1	3.7657	3.7683	3.6459
Valley load case..	λ_2	3.7684	3.7653	3.6446
	λ_3	3.7643	3.7683	3.6408
	Total losses (mw)	33.20	32.47	34.63
	Cost in \$/hour	\$2,109.75	\$2,108.91	\$2,110.89

Calculations of exact and approximate incremental loss were also made for two other simple power systems. These systems were selected because each had been used previously to illustrate the validity of the B -constant type of loss formula. Fig. 5 shows a loop system with four loads and three generators. This system has many of the properties of an actual power system and represents an extreme case of voltage phase angles not being independent. This feature is the one which will place the most severe handicap on the use of the approximate versus the use of the exact incremental loss factor. Comparisons of exact incremental losses, B -constant incremental losses, and approximate incremental losses as calculated using equation 16 for three different load levels, are shown in Table I. As may be noted from the table, the approximate incremental loss has the same sign but differs in magnitude from the exact incremental loss.

Fig. 6 shows part of an actual power system with both radial and loop features.⁷ Comparisons of exact incremental losses, B -constant incremental losses, and approximate incremental losses calculated for two different load levels on this system are shown in Table II. As may be noted from the table, the approximate and the exact incremental loss have the same sign but differ in magnitude for this system also.

Comparison of Penalty Factors

A good test of the validity of the assumption that the approximate penalty factors could be used to arrive at an

optimum generation schedule would be actually to use these factors to obtain an economic generation schedule and compare the cost of operation under this schedule with that obtained using exact incremental losses. Economic dispatch generation schedules have been determined for three different load levels for the power system of Fig. 5. Since this system had been used previously to check the accuracy of the B -constant loss formula, the B -constant incremental losses were also used to obtain an economic generation schedule for this system. The results of using the three methods (approximate incremental losses, exact incremental losses, and B -constant incremental losses) to obtain an economic generation schedule are shown in Table III. As the table shows, the variation in total cost per hour between the three methods of obtaining an economic dispatch is only of the order of 1/10 of 1%. This variation is much less than the variation due to the practical limitations on accuracy of original data, such as system constants and production cost data.

The same three methods of obtaining an economic dispatch of generation were used for two different load levels for the power system of Fig. 6. All of these results are shown in Table IV. As indicated in the table, the total costs per hour are nearly equal for each method of determining penalty factors.

Economy Loading When B -Constant Assumptions Are Violated

The assumptions made in using the B -constant form of loss formula offer some

apparently severe restrictions on the operation of a practical power system. These assumptions are as follows:

1. All loads are fixed complex fractions of the total system load.
2. The voltage magnitudes and angles at all generator and tie points are the same as for the base case.
3. All shunt paths to neutral, line charging, and synchronous condenser reactive powers are lumped with the system loads.
4. Transformation ratios are unity around each closed loop of the network.
5. The generator and tie Q/P ratios are the same for the base case.

The first two assumptions tend to compensate each other. Violating the fourth assumption will cause the third and fifth assumptions to be violated. This violation of the fourth assumption might easily occur in obtaining a more desirable distribution of reactive power or in maintaining scheduled voltage levels as the load and generation change.

With the economic dispatch for the peak load case of the system of Fig. 5, the reactive power generation of generator 1 seemed quite high, while that of generator 2 is quite low. In order to give a more equitable distribution of reactive power between generators 1 and 2, a transformer was assumed to be inserted in the 1-2 circuit near terminal 2. The transformer was assumed to add no extra impedance to the 1-2 circuit, but only to boost the voltage toward terminal 1. Two settings of the transformer were considered. In the first case, a setting of 112% was assumed. In the second case, since the 112% setting of the transformer still leaves a relatively unsatisfactory distribution of reactive power generation, a setting of 120% for the transformer was assumed. Economic dispatches were found using B -constant penalty factors, exact penalty factors, and the approximate penalty factors. In comparing the results of these modifications, the costs of operation did not change very much between the three methods of obtaining an economic dispatch.

As a further test of the validity of using B -constants when their restrictions are violated, the circuit and voltages of the system of Fig. 6 were altered from the base case as follows:

1. The voltage level of station 2 was raised from 98.0% to 101.0%.
2. In order to maintain the reactive power generation at station 1, a transformer was assumed to be inserted near station 2 in the 1-2 circuit and set to boost 3.0% toward station 2.
3. The voltage level of station 3 was raised from 95.3% to 100.0%.

4. The voltage level of bus 5 was lowered from 98.1% to 97.2% to absorb the additional reactive power of stations 2 and 3.

As in the previous case, an economic schedule was obtained for each of the three methods of determining the penalty factor. Also as in the previous case, each of the three methods of obtaining an economic dispatch arrives at an hourly cost which is nearly the same.

Conclusions

The methods developed in this paper and the results of the test cases lead to

1. The B -constants give reasonable economy loading indications when their assumptions are violated, but with no major system changes such as lines or equipment out of service.
2. The approximate penalty factor, which is considerably simpler, gives results equally as good.

Making a major system change such as taking a circuit out of service for maintenance or adding a new facility would, however, put any loss formula which did not take this system change into account to a severe test. Because of its greater simplicity and ease of modification, the admittance constant form of loss formula is at a distinct advantage.

Appendix. Incremental Fuel Cost Expressions

For the system of Fig. 5, the incremental fuel cost expressions are Watson's:⁸

Table IV. Comparison of Economic Dispatches of Generation for the Power System of Fig. 6

		B-Constant Dispatch	Exact Penalty Factor Dispatch	Approximate Penalty Factor Dispatch
Base case loads....	P_{G1} (mw)	144.000	144.000	143.325
	P_{G2} (mw)	10.668	10.668	15.109
	P_{G3} (mw)	34.600	34.600	30.397
	λ_1	2.5266	2.4972	2.5332
	λ_2	2.5266	2.4982	2.5318
	λ_3	2.5266	2.4939	2.5343
	Total losses (mw)	2.868	2.868	2.592
	Cost in \$/hour	\$373.95	\$373.95	\$374.05
All loads increased one-third.....	P_{G1} (mw)	161.173	161.173	162.254
	P_{G2} (mw)	45.369	45.369	48.958
	P_{G3} (mw)	45.396	45.396	40.957
	λ_1	2.6700	2.6583	2.6764
	λ_2	2.6953	2.6717	2.6902
	λ_3	2.6894	2.6789	2.6895
	Total losses (mw)	3.778	3.778	3.481
	Cost in \$/hour	\$534.02	\$534.02	\$535.63

$$\Delta CP_1 = 3.80 + 0.3772000P_{G1}$$

$$\Delta CP_2 = 3.00 + 0.0745000P_{G2}$$

$$\Delta CP_3 = 3.20 + 0.1053836P_{G3}$$

In these expressions, the incremental fuel costs are in dollars per megawatt-hour and P 's are pu on 100-megavolt-ampere base.

For the system of Fig. 6, the incremental fuel cost expressions are given or derived from data in the paper by Glimm et al.⁷

$$\Delta CP_1 = 1.2800000 + 0.00820000P_{G1}$$

$$\Delta CP_2 = 2.49218339 + 0.00413460P_{G2}$$

$$\Delta CP_3 = 2.11699698 + 0.00759007P_{G3}$$

In these expressions, the incremental fuel costs are in dollars per megawatt-hour and the P 's are in megawatts.

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