THE UNIVERSITY OF MISSOURI
Bulletin
ENGINEERING REPRINT SERIES
Reprint Number 50
Engineering Experiment Station
Columbia, Missouri

Static Modulus of Elasticity of Concrete as Affected by Density

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Reprinted from

American Concrete Institute Journal, Title No. 57-32 ; V.32 ; No. 6
December 1960 (Proceedings V. 57), P.O. Box 4754, Redford Station,
Detroit 19, Michigan.

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THE UNIVERSITY OF MISSOURI BULLETIN

VOL. 61, NO. 60 ENGINEERING REPRINT SERIES NO. 50

Published by the University of Missouri at the Office of Publications, Columbia, Missouri. Entered as second-class matter, January 2, 1914, at post office at Columbia, Missouri, under Act of Congress of August 24, 1912. Issued five times monthly.

600
December 17, 1960

Title No. 57-32

Static Modulus of Elasticity of Concrete as Affected by Density

By ADRIAN PAUW

The elastic modulus of concrete is an important parameter in reinforced concrete design and analysis. With the increased use of lightweight aggregates for structural concrete a better understanding of the relationship between weight, strength, and the elastic modulus is needed. In this study the static modulus for a large variety of aggregates and concrete strengths was analyzed and an empirical formula was derived which is applicable to both lightweight and normal weight structural concretes. The formula is in excellent agreement with recognized empirical formulas for normal weight concrete.

■ IN REINFORCED CONCRETE and especially in prestressed concrete a knowledge of strains as well as of stresses is important. Strains may be classified into four types: elastic strains, lateral strains, creep strains, and shrinkage. This paper is limited to a consideration of elastic strains.

The term elastic strain when applied to concrete is somewhat ambiguous since the stress-strain curve for concrete is seldom a straight line, even at normal working stress levels; and, furthermore, the strains for the first loading cycle are seldom entirely recoverable. If creep strains are eliminated from consideration, the lower portion of the instantaneous stress-strain curve is usually found to be relatively straight after the specimen is subjected to an initial load cycle resulting in a compressive load equivalent to about one-half the compressive strength. The slope of this curve may be considered as the static modulus of elasticity. This modulus varies with such factors as strength of concrete, age of concrete, properties of aggregates and cement, rate of loading, and type and size of specimen, as well as of the definition of the term modulus of elasticity itself, whether initial, tangent, or secant modulus.

ELASTIC MODULUS OF NORMAL WEIGHT CONCRETE

To date no standard test has been adopted for the determination of the static modulus of elasticity for concrete, thus further adding to the ambiguity of the term. Several empirical formulas have previously been proposed to serve as a guide for use when the modulus cannot be determined by test. Examples of these follow.

1. ACI Committee 318 ACI Building Code formula:

$$E_c = 1000\ f_c' \qquad (1)$$

where E_c = modulus of elasticity, psi
f_c' = 28-day compressive strength

2. ACI-ASCE Committee 323 formula:

$$E_c = 1{,}800{,}000 + 500\ f_c' \qquad (2)$$

3. Empirical formula proposed by Jensen:[1]

$$E_c = \frac{6 \times 10^6}{1 + 2000/f_c'} \qquad (3)$$

4. Empirical formula developed by Inge Lyse:[2]

$$E_c = 1{,}800{,}000 + 460\ f_c' \qquad (4)$$

All of the above formulas are applicable only to normal weight concretes. Eq. (1) is a simple approximation which is reasonably accurate for concretes having a compressive strength of about 3000 psi, but for

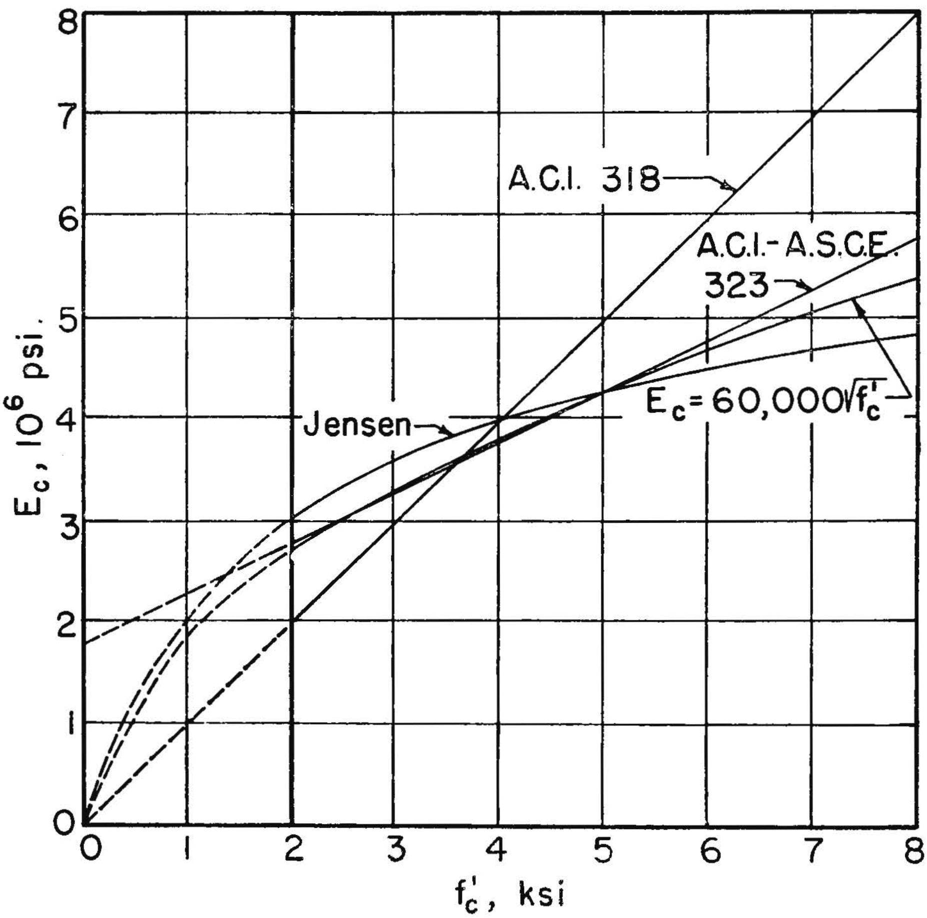

Fig. 1—Empirical formulas for E_c

ACI member **Adrian Pauw,** professor of civil engineering, University of Missouri, Columbia, is now on leave from the university as a Fulbright lecturer at Delft Technological Institute, Delft, Netherlands. He is chairman of ACI Committee 213, Properties of Lightweight Aggregates and Lightweight Aggregate Concrete, and a member of Committee 337, Strength Evaluation of Existing Concrete Structures. Professor Pauw's areas of special interest, in addition to lightweight aggregate concrete, include structural theory and machinery foundations.

higher strength concretes this formula yields values for E_c which may be as much as 50 percent too high. Eq. (2), (3), and (4) yield essentially similar values for concrete strengths ranging from 2000 to 7000 psi but their applicability is limited to normal weight concrete. The basic form of Eq. (2) and (4) limits their applicability to a specific range of compressive strength values since for very low values of f_c' these formulas yield values of E_c greater than 1,800,000 psi. Similarly the formula proposed by Jensen limits the maximum value of the elastic modulus to 6,000,000 psi.

With the rapid expansion of the use of lightweight aggregate concrete in recent years, the need for a suitable empirical relationship for the elastic modulus of these concretes has become increasingly apparent. An earlier study showed that the empirical relationship

$$E_c = 60{,}000\sqrt{f_c'} \qquad (5)$$

yields values for E_c within the limits of the values given by the formulas proposed by Lyse and Jensen for normal weight concretes having compressive strengths greater than 2000 psi as demonstrated by the plot in Fig. 1. This relatively simple formula has the further advantage of yielding reasonable results for concretes with both very low and very high compressive strengths.

PROPOSED EMPIRICAL FORMULA

It has been observed by many investigators that the elastic modulus of lightweight aggregate concretes is considerably lower than the values of normal weight concretes of comparable compressive strength and that the modulus appears to be a function of the weight. It is known that all mineral aggregates have about the same absolute specific gravity. The difference in weight of various types of concrete is therefore primarily the result of voids in the concrete, whether they be due to purposely entrained air, or due to the vessicules in lightweight aggregate. From these considerations it was suspected that it might be possible to obtain a satisfactory approximation by expressing the value of the elastic modulus by an empirical relationship of the form

$$E_c = aw^{3/2}\sqrt{f_c'} \qquad (6)$$

where E_c = static modulus of elasticity of the concrete, psi
w = air-dry weight of the concrete at time of test, pcf
f_c' = compressive strength of the concrete at time of test, psi
a = a suitable constant

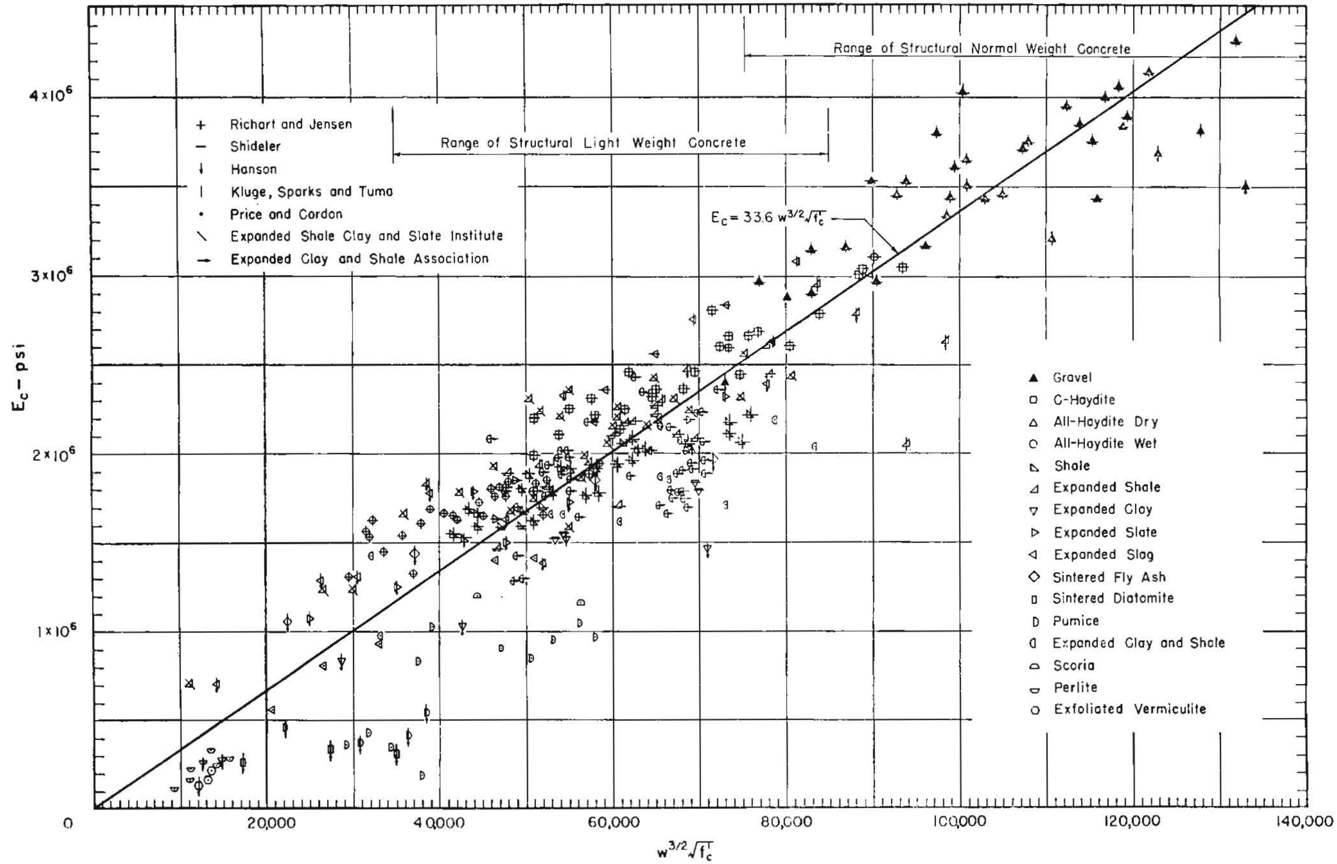

Fig. 2—Correlation of test data

TEST DATA

In searching the literature for data which would be useful in testing Eq. (6), several difficulties were encountered. The first difficulty was the result of the lack of a standard test for measuring elastic modulus—some investigators reporting the initial tangent modulus whereas others, a secant modulus at some given stress level. In some instances the method used to determine the static modulus was not even defined. Then it was found that in many cases the unit weight of the concrete at time of test was not reported and therefore such data could not be further considered. In a few instances only the wet weight of the concrete was reported; these data were used by adjusting the weight by applying reduction factors based on experience with similar aggregates. The reports in which suitable data were found are listed as references.

The most important source of information was the report of the work of Richart and Jensen.[3] Materials tested included gravel, C-Haydite, and all-Haydite, both dry and moist. Nine mixes were tested ranging from 1:1.5:2.5 to 1:3.5:2.5. The static modulus values reported are initial-tangent modulus values. Reported values of E_c ranged from 55 to 75 percent of the values obtained for gravel concrete of comparable strength.

The results of 32 tests were obtained from the work reported by Shideler.[4] The static modulus values reported, based on the secant modulus at about 0.3 f_c' range from 53 to 82 percent of the values for sand and gravel concrete of comparable strength. The aggregates tested included shales, slates, and clays expanded in rotary kilns as well as aggregates produced by the sintering process. Expanded blast-furnace slags were also tested. Materials tested were obtained from a wide range of geographical locations.

The results of 18 tests reported by Hanson[5] are also included. Only the weight of the concrete in the plastic condition was reported and these weights were therefore adjusted on the basis of information given in Reference 4.

The paper by Kluge, Sparks, and Tuma[6] furnished data on a variety of aggregates including exfoliated vermiculite, sintered diatomite, perlite, expanded blast-furnace slags, sintered fly ash, pumice, as well as expanded shale, slate and clay. The modulus of elasticity was determined by both static and dynamic test methods. The observed modulus of elasticity for some of the concretes produced with the weaker aggregates was found to be very low.

Data for aggregates produced in the western part of the United States were obtained from a paper by Price and Cordon.[7] Represented are two expanded shales and clays, an expanded slag, scoria, four samples of pumice, five samples of perlite, and two exfoliated vermiculites. Most of the weaker aggregates produced concretes having very low moduli of elasticity. All specimens were fog-cured for 7 days fol-

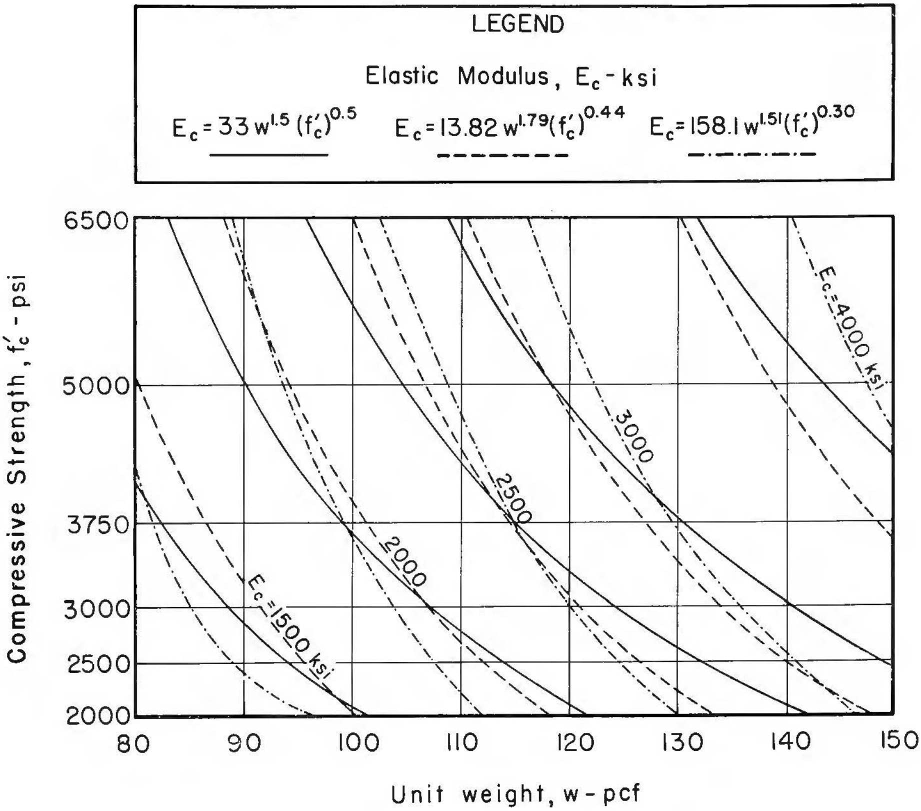

Fig. 3—Comparison of empirical formulas for E_c

lowed by 21 days of curing at 50 percent humidity. Tests results were for age 28 days and the static modulus values were reported. The basis used for determining the static modulus was not reported.

Additional data for concretes produced from expanded shales, slates, and clays were obtained from unpublished reports furnished by the Expanded Shale, Clay and Slate Institute.[8,9] Both sonic and secant modulus values were reported but only the latter were considered in this study.

ANALYSIS OF DATA

The value of the constant a in Eq. (6) was determined by considering the ratio $E_c/(w^{3/2}\sqrt{f_c'})$. Fig. 2 is a plot of E_c as a function of $w^{3/2}\sqrt{f_c'}$. Using the method of least squares the value of the constant a in Eq. (6) was found to be 32.43. Since most of the data reported by Price and Cordon were for concretes produced with relatively weak aggregates such as the pumices and perlites, the value of a was recomputed, omitting these data. The value of a on this basis was found to be 33.6.

This value is believed to be a reasonable compromise since most of the concretes tested by Price and Cordon cannot be considered to be of structural quality.* Thus for concretes of structural quality the static modulus of elasticity may be determined by the approximate empirical equation

$$E_c = 33\ w^{3/2} \sqrt{f_c'} \qquad (7)$$

After the computations for the constant a were completed some additional data were obtained from the Expanded Clay and Shale Association.[10] These data, for 16 different lightweight concrete mixes, have also been plotted in Fig. 2 but they were not included in the least-square analysis. It is believed however that inclusion of these data would not have produced any significant change in the final results.

Considering the limitations of the data available, the correlation between the reported elastic modulus and the values predicted by Eq. (7) appears rather remarkable. In view of the many other variables involved in concrete design it is believed that this equation is adequate for most design purposes. As a further check on the validity of the proposed formula, a generalized form of Eq. (6) was analyzed:

$$E_c = A\ w^B\ (f_c')^C \qquad (8)$$

By writing Eq. (8) in logarithmic form

$$\log E_c = \log A + B \log w + C \log f_c' \qquad (9)$$

a linearized equation was obtained which was susceptible to a non-orthogonal regression analysis. This analysis yielded the formula

$$E_c = 13.82\ w^{1.79}\ f_c'^{0.44} \qquad (10)$$

with all the data included, and the formula

$$E_c = 158.1\ w^{1.51}\ f_c'^{0.30} \qquad (11)$$

when the data for concretes having a compressive strength of less than 2000 psi were omitted. The exclusion of these data is believed to be justified since elastic modulus is generally of interest only in the case of structural quality concretes. A number of the weaker concretes were made with friable aggregates and such concretes are not suitable for structural work. It may be noted from Eq. (11) that the value of the coefficient B seems to confirm the predicted value used in Eq. (6), while the predicted value of the coefficient C appears to be somewhat high. However, considering the limitations of the data used, a further refinement of Eq. (7) does not appear to be warranted until new data determined on a standardized test basis becomes available. Fig. 3 shows the value of elastic modulus obtained by Eq. (10) and (11) as compared with the value obtained by the proposed equation. It may be noted that all these equations predict reasonably similar results and

*Structural quality concrete is here defined as a concrete having compressive strength of 2500 psi or higher.

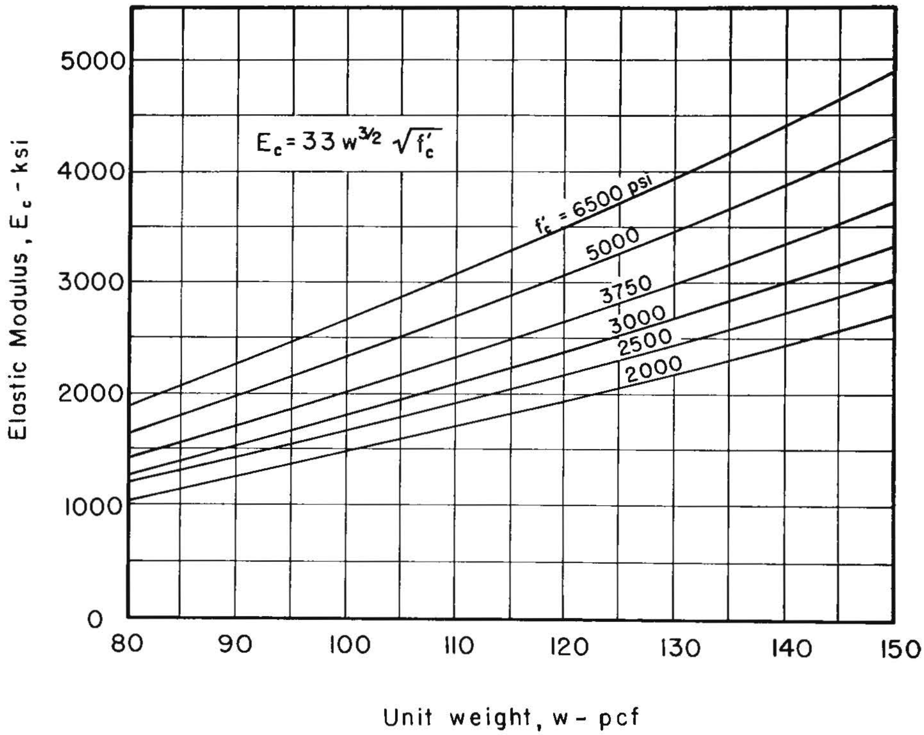

Fig. 4—Elastic modulus as a function of strength and weight of concrete

that the value of modulus is much more sensitive to small changes in the weight of the concrete than it is to the compressive strength.

SUMMARY AND CONCLUSIONS

The static modulus of elasticity of both normal and lightweight structural concrete may be approximately determined by the empirical formula

$$E_c = 33\ w^{3/2} \sqrt{f_c'}$$

The value of the elastic modulus is more dependent on the weight of the concrete and the method of test used to determine it than on the compressive strength of the concrete. The reliability of the proposed formula appears to be as good as recognized and acceptable empirical formulas for normal weight concretes. The proposed formula should therefore prove useful to the designer by providing a single uniform method for estimating the modular ratio for any structural quality concrete. A convenient graphical form of the relationship is given in Fig. 4.

ACKNOWLEDGMENTS

The assistance of Isaac Gordon, graduate research assistant, Department of Civil Engineering, and Roy F. Keller, acting director, University of Missouri Computer Center, in processing and analyzing the data is gratefully acknowledged. The author is indebted to Frank G. Erskine of the Expanded Shale, Clay and Slate Institute for his suggestion that the elastic modulus might be related to the weight and for furnishing unpublished test data. The author also wishes to express his appreciation for the data furnished by Lucas E. Pfeiffenberger of the Expanded Clay and Shale Association.

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Received by the Institute Mar. 17, 1960. Title No. 57-32 is a part of copyrighted Journal of the American Concrete Institute, V. 32, No. 6, Dec. 1960 (Proceedings V. 57). Separate prints are available at 50 cents each.

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Discussion of this paper should reach ACI headquarters in triplicate by Mar. 1, 1961, for publication in the June 1961 JOURNAL.

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