**BIFURCATION ANALYSIS AND DAMAGE DETECTION IN
MECHANICAL STRUCTURES**

A Dissertation presented to
the Faculty of the Graduate School
University of Missouri-Columbia

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

By
IQBAL ALSHALAL
Dr. Frank Z. Feng, Dissertation Supervisor

December 2018

The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled:

**BIFURCATION ANALYSIS AND DAMAGE
DETECTION IN MECHANICAL STRUCTURES**

presented by **Iqbal Alshalal**,
a candidate for the degree of **doctor of philosophy**,
and hereby certify that, in their opinion, it is worthy of acceptance.

Professor Zaichun Frank Feng

Professor Guoliang Huang

Professor Craig Kluever

Professor Ahmed S. El-Gizawy

Professor William D. Banks

ACKNOWLEDGEMENT

I would like to thank my Lord Allah for his blessings and benefactions. I have been achieved my PHD degree successfully under his custody and blessings.

I would like to thank my dissertation advisor, Professor Zaichun “Frank” Feng for his time, his patience, his advice, his kindness and his help at every step of my doctoral program in university of Missouri-Columbia. He has always been available to sit with me and guide me to achieve my goal.

I would like to sincerely thank Dr. Pai for all his support, his help and guidance to accomplish my study. Without his help, this dissertation would not be possible.

I would like to thank Prof. El-Gizawy, Prof. Huang, Prof. Kluever, and Prof. Banks for serving in my PhD committee. Special thanks go to the Mechanical & Aerospace Engineering department and the nice faculty and staff there who make my PhD studying at MU profitable and joyful.

I would like to thank my sponsor, the Ministry of Higher Education and Scientific Research (MOHESR) - University of Technology in Iraq. MOHESR has granted me five years of scholarship to conduct my PhD study in USA and provided me continuous funds despite of the hard circumstance issues in Iraq.

During my stay at MU, I have come across a lot of colleagues and real friends who have supported and helped me. Special thanks to my friends Faten Al Zubaidi and Hui Chen.

Finally, the words cannot ever express my feelings toward my big family: father, mother, brothers and sisters, for their love, care, wishes, and unlimited support, they have made my study much easier and successful.

TABLE OF CONTENTS

Acknowledgment.....	ii
List of figures.....	vii
List of tables.....	x
Nomenclature.....	xii
Abstract.....	xiv
Chapter 1: Introduction.....	1
1.1 Background.....	1
1.2. Gesa and Ansys.....	3
1.3. Geometric Nonlinearity.....	4
1.5. One and Two-Dimensional Structures.....	4
1.4. Candidate Structure.....	5
1.6. Finite Element Method.....	5
1.7. Natural Frequency and Mode shape.....	6
1.8. Objectives.....	7
Chapter 2: Literature review.....	8
2.1 Previous works for bifurcation analysis.....	8
2.2 Previous works about damage detection methods.....	15
2.3. Summary.....	20
2.3 Organization of the dissertation	21
Chapter 3: Symmetry breaking bifurcation.....	22

3.1 Introduction	22
3.2 Symmetry breaking bifurcation of a Von Mises arch.....	22
3.2.1 Snap-through bifurcation and the ultimate load limit of the symmetric structure	23
3.2.2 Finite element calculations (Gesa).....	26
3.2.3 Finite element calculations (Ansys).....	27
3.2.4 Bifurcation and stability phenomena in Von Mises arch.....	29
3.2.5 Symmetry breaking for arch with Gesa.....	34
3.3 Symmetry breaking bifurcation in shallow arches with seven bars.....	39
3.3.1. Theoretical analysis.....	39
3.3.2 Bifurcation study by Gesa.....	44
3.3.3 Bifurcation study by Ansys.....	49
3.4 Conclusion.....	51
Chapter 4: Damage detection using residual error method.....	53
4.1 Introduction.....	53
4.2 Residual error method.....	53
4.2.1 Application on a bar (one dimensional structure)	55
4.2.2 Application on a beam (one dimensional structure).....	57
4.2.3 Application on a plate (two dimensional structures).....	59
4.3 Results of residual error method.....	61
4.3.1 Result for the bar.....	61
4.3.2 Results for the Beam.....	66
4.3.3 Results for the Plate.....	71

4.4 Comparative study by using mode shape curvature.....	82
4.4.1 Mode shape curvature for one dimension.....	82
4.4.2 Mode shape curvature for two dimensions.....	83
4.5 Effect of noise.....	84
4.6. Applying the residual error method on plate with hole.....	91
4.7 Conclusion.....	93
Chapter 5: Conclusion and Recommendations.....	95
5.1 Conclusions.....	95
5.2 Recommendation for Future Work.....	97
REFERENCES.....	98
Appendix A	
Calculation the difference between Cauchy strain and Green.....	104
strain.....	
Appendix B	
Procedure to use Ansys for bifurcation analysis.....	106
Appendix C	
Derivation of mass and stiffness matrices for plate.....	111
Appendix D	
Matlab code for beam with residual error and curvature methods.....	119
VITA.....	124

LIST OF FIGURES

Figure 1.1. Critical points on the equilibrium path.....	2
Figure 3.1. Symmetric truss supporting load.....	21
Figure 3.2. The analytical load-deflection relationship (a) $\alpha_0 = 20^\circ$ and (b) $\alpha_0 = 15^\circ$...	23
Figure 3.3. Load deflection curves for truss with two elements and initial angle $\alpha_0 = 15^\circ$ obtained with different methods.....	26
Figure 3.4. The values of $\frac{k_{11}}{k}$ and $\frac{k_{22}}{k}$ for (a) $\alpha_0 = 11\pi/30$ and (b) $\alpha_0 = 2\pi/5$	29
Figure 3.5. (a) The relation between the load and displacement (y/L) in bifurcation point, (b) The bifurcation point with $\alpha_0 = 72^\circ$	31
Figure 3.6. Load deflection curve for truss initial angle $\alpha_0 = 72^\circ$ (a) With horizontal force (black line for symmetric bifurcation and red line for asymmetric bifurcation (b) Small deviation on geometry (black line for symmetric bifurcation and red line for asymmetric bifurcation (c) Some of asymmetric steps loading (d) The asymmetric loading steps (5, 7, 11, 12 and 13) on the geometry of truss.....	35
Figure 3.7 The load deflection curve for truss angle $\alpha_0 = 80^\circ$ with horizontal force using Gesa.....	36
Figure 3.8. Symmetric pinned truss structure with a roller at the node 3.....	37
Figure 3.9. Change of stability as function of Δ/L for $\alpha_0 = \frac{\pi}{12}$, $k_2 = 0$ (a) $k_3 = (\cos \pi/12)/2 k_1$ (b) For $k_3 = \frac{1}{10} (\cos \pi/12)/2 k_1$, (c) k_{11} , $\alpha_0 = \frac{\pi}{12}$	41
Figure 3.10. (a) Symmetric load deflection curve for truss loading in node 4 and 5 with $\alpha_0 = 15^\circ$ (b) A truss with symmetric loading with $\alpha_0 = 15^\circ$	43
Figure 3. 11. Asymmetric load deflection curve (a) Small axial force perturbation (b) Small horizontal deviation perturbation.....	44

Figure 3.12. Symmetric and asymmetric truss angle $\alpha_0 = 15^\circ$ loading with R, at node 4 node 5 and small axial displacement in node 2 when changing cross section area to $A/10$ for bar and 2.....46

Figure 3.13. The symmetric and asymmetric curves in nodes 4 & 5 for arch angle $\alpha_0 = 15^\circ$ with 7 elements using Ansys data (a) All bars with same cross section area (b) Changing cross section area to $A/10$ in the bottom bars.....48

Figure 4.1. Fixed-free ends bar geometry with its element.....54

Figure 4.2 (a) Rectangular plate (b) Plate element.....57

Figure 4.3 Residual error detection for fixed-free bar: (a) 1st residual error mode with 10% damage at element 15 (b) 2nd residual error mode with 20% damage at element 99 in fixed bar (c) 1st residual error mode with 10% damage at elements 15 and 99 (d) 2nd residual error mode with 20% at elements 15 and 99.....63

Figure 4.4 Residual error detection for cantilever beam: (a) 1st residual error with 10% damage at element 10 (b) 1st residual error mode with 10% damage at elements 10 & 40 (c) 2nd residual error mode with 10% damage at elements 10 and element 40 (d) 3rd residual error with 20% at elements 10 and 40.....68

Figure 4.5. The geometry for undamaged rectangular meshed plate and damaged elements 24, 98 and 324.....71

Figure 4.6 Damaged and undamaged mode shape for hinged plate with 30% damage at elements 24, 98 and 324 (a) first mode shape (b) second mode shape.....75

Fig 4.7 Residual error detection on plate with damage in elements 24, 98 and 324 (a) 1st residual mode with 5% damage (b) 2nd residual with 5% damage (c) 1st residual mode with 10% (d) 2nd residual mode with 10% damage (e) 1st residual mode with 20% damage (f) 2nd residual mode with 20% damage.....79

Figure 4.8 10% damage at elements 24, 98 and 324 (a) first residual error with noise level 1 (b) first (ADMSC) with noise level 1 (c) first residual error with noise level 1 (d) first (ADMSC) with noise level 2.....85

Figure 4.9 10% damage at elements 24,98and 324(a) second residual error with noise level 1(b) second (ADMSC) with noise level 1(c) second residual error with noise level 1(d) second (ADMSC) with noise level 2.....88

Figure 4.10 Geometry for rectangular plate with hole in the center.....89

Figure 4.11 Residual error with 10% damage at element (65) and four elements in center with zero mass and stiffness.....90

Figure 4.12 Residual error with 20% damage at element (65) and four elements in center with zero mass and stiffness.....90

LIST OF TABLES

Table 3.1 The geometrical and material properties.....	24
Table 3.2 Nodes coordinate of Von Mises arch.....	25
Table 3.3 The values of the load and downward displacement at bifurcation point	32
Table 3.4 The node coordinates for truss with seven elements.....	47
Table 4.1 Boundary conditions for bar	54
Table 4.2 Boundary conditions for beam.....	56
Table 4.3 Boundary conditions for plate.....	58
Table 4.4 Damage cases for bar.....	59
Table 4.5 Material and dimensions for bar.....	60
Table 4.6 Six natural frequencies of fixed bar with different damage scenarios...	60
Table 4.7 Damage cases for beam.....	64
Table 4.8 Material and dimensions for beam.....	65
Table 4.9 Six natural frequencies for cantilever beam with different damage scenarios.....	66
Table 4.10 Damage cases for plate.....	70

Table 4.11 Material and dimensions for plate.....	70
Table 4.12 Six natural frequencies for hinged plate with different damage scenarios.....	72
Table 4.13 Natural frequencies change ratio for six modes.....	74

NOMENCLATURE

α_0	Initial angle
α	Angle after deformation
k	Spring with spring constant
E	Young's modulus
A	Cross sectional area
L	Unloaded length of each compressive member
Δ	Downward displacement of the top node
Δ/L	Dimensionless displacement
ρ	Mass density
μ	Poisson's ratio
M	Mass matrix
K	Stiffness matrix
$\ddot{x}$	Acceleration
x	Displacement vector
K_u	Stiffness matrix of undamaged structure
M_u	Mass matrix of undamaged structure
Λ^*	Diagonal matrix of natural frequencies

X^*	Mode shapes matrix of damaged structure
E_r	Matrix represents the error, caused by damage, in the equation of motion
γ	Stiffness reduction factor
$w(x)$	Displacement mode shape
I	Moment of inertia
M_b	Bending moment
D	Bending stiffness of the plate
w_{lk}	Noisy eigenvector where
w_{lk}^0	Free noise eigenvector
ψ_l^k	Random number to generate noise
e_i	Element length to subscript i
k^i	Elemental stiffness
m^i	Elemental mass matrix
ω_n	Natural frequency

BIFURCATION ANALYSIS AND DAMAGE DETECTION IN MECHANICAL STRUCTURES

Iqbal Alshalal

Dr. Frank Z. Feng, Dissertation Supervisor

Department Mechanical and Aerospace Engineering

ABSTRACT

Bifurcation and damage are two phenomena prompting any structure to unpredicted failure. Their early detection is crucial to maintaining structural health and integrity. In this work, we investigate two topics in bifurcation and damage detection. Trusses with geometric and loading symmetries have been used in many structures to reduce complexity design. Here, bifurcation analysis has been conducted for a two-bar truss and a shallow arch structure with seven bars. Two program packages Gesa and Ansys based on the finite elements method (FEM) have been used to detect the symmetry breaking bifurcation points. The theoretical examination uncovers that the bifurcation prompts a much lower critical load in the presence of little asymmetry in comparison to the symmetric case.

The outcomes of bifurcation detection by using Gesa program in Matlab for fully nonlinear analysis and Ansys commercial program show the two programs give results close to the results acquired from the theoretical analysis. Our study opens the door for

the researcher to use the two programs for more complicated structures for bifurcation detection analysis since analytically will be hard to use.

As for damage detection, the residual error method has been used. This is a technique that relies on observing the residual error in the equation of motion specified for free vibration analysis so as to reveal any changes in the structural dynamic characteristics. The method has been applied on bar, beam and plate to demonstrate its validity. Several numerical simulations with different damage scenarios are presented to assess the robustness and limitations of the method. The sensitivity of the method to noise has been tested with different noise levels as well. Results obtained with the residual error method are compared with those obtained from the absolute difference mode shape curvature (ADMSC) method. The comparison demonstrates that the residual error method can detect and locate damage in the simulated structures with low level of noise.

CHAPTER ONE

INTRODUCTION

1.1 Background

Any structural design is affected by the environmental and service conditions, so the likelihood of being damaged is prospective. Damage can be defined as any weakening in the structure that influences its performance negatively. Damage might make the structure no longer functional as it may ideally be, even if it is still practically functional (Worden, 2007). Damage is caused by the changes in the properties of material or in original geometry dimensions and shapes. It results in undesirable stresses, displacements and vibrations. In addition, damage may occur due to fatigue, cracks, manufacturing errors, casting defects or mechanical treatment of the material and earthquakes in civil structure (Mohan, 2013), earliest detection to damage from the first stage is of the upmost importance in the mechanical, civil and aerospace engineering communities in order to avoid unpredicted failure.

On the other hand, many failures in building structures are caused by stability loss. Instability resulting from bifurcations is an important failure mode in many thin walled or skeletal structures. The bifurcation point is the point on a load deflection curve which has two or more directions to go when the load varies (Pai, 2007). The bifurcation point is one of the critical points in the equilibrium path when the system changes from being stable to unstable (Kala, 2016), where the equilibrium path is the load deflection curve of

an elastic structure in static state. Each point on this curve corresponds to an equilibrium state in the equilibrium path as in figure (1.1). Theoretically, the structure is considered snapped when the equilibrium path emerges from unloaded state to a loaded state and loses stability to another more stable point. Then, the structure reaches a new stable configuration (Thompson, 1963).

To maintain structural health and integrity, this study is concerned about damage detection in structures with a new nondestructive method based on vibration analysis. In addition, it is concerned about bifurcation detection in symmetric truss structure with a new technique based on using two package programs (Gesa) and (Ansys) to prevent instability mode and eventually catastrophic events in mechanical structures.

Figure.1.1. Critical points on the equilibrium path

1.2. Gesa and Ansys.

GESA (Geometrically Exact Solution Structural Analysis) has been used in this study. It was created by Pai (2007). The main reason to use GESA is because only geometric nonlinearities are considered in it and it is convenient for our study since it is just including non- linear geometry. GESA is a code program in Matlab based on the fully nonlinear structural theories for structures undergoing of large rotation, finite strain and large displacement. In addition, both anisotropic and isotropic materials are used. Ansys, a commercial software package based on FEM is adopted in this study to verify the results from Gesa.

1.3. Geometric Nonlinearity

The geometric nonlinearity must be considered in the structure when formulating the equations of equilibrium if the displacements, rotations, or strains become large enough compared to the dimension to the structure.

1.5. One and Two-Dimensional Structures

One -dimensional structures need one coordinate to describe the unreformed reference line whereas two-dimensional structures need two coordinates to describe the unreformed reference line. Bars can be defined as one- dimensional structures that can sustain compressional, extensional, and torsional loads. The bar is called a rod if it is only subjected to longitudinal tensile loads; however, it is called a column if it is only subjected to a longitudinal compressive load. Both columns and rods are only subjected to two aligned forces (one on each end) and thus are called two-force members. Trusses are composed of members with two-force in line. Beams are also one- dimensional structures having one dimension much larger than the others resulting in bending of the other reference axes. In general, the beam is a structure that can tolerate bending, compression, extension, transverse shear (flexure), and twisting loads. Plates are considered two-dimensional structures that have two dimensions much larger than the third one in their geometry. Plates sustain compression, extension, shear, bending, twisting and transverse shear loads (Pai, 2007).

1.4. Candidate Structure

Truss has been used in the first part of our study because it is slender geometry. It follows nonlinear geometry behavior. It can show snap-through and bifurcation Analysis. Two kinds of truss structure have been used; Von-Mises arch and shallow arch with seven bars. Both structures are symmetric. Bar and beam have been used in the second part of the study as one-dimensional structure and plate as two-dimensional structure to identify damage in them.

1.6. Finite Element Method

The finite element method is a numerical technique for structural analysis assuming the deformation of the entire structure can be represented by some discrete points called nodes and the deformation of an arbitrary point and the surrounding nodes in an element can be interpolated by using low order polynomial so partial differential equations are discretized into ordinary differential equation. In addition, the accuracy of the FEM solution can be improved by increasing the number of total degrees of freedom (nodal variables) adopted in the modal analysis.

1.7. Natural Frequency and Mode shape

Natural frequency: When the system oscillates under initial condition excitation and it is independent of the initial position or the velocity of the system. Each system has its own natural frequency and it is dependent on stiffening and the mass distribution of the system.

Mode shapes: is the configuration of a system when it vibrates with respect to initial condition excitation. A spatial initial displacement causes it to vibrate harmonically. It describes the relative displacement of the system. Each mode shape is associated with its natural frequency. A system with N degrees of freedom there will be N natural frequencies and N of mode shapes. A continuous system will have infinite Natural frequencies and mode shapes. The natural frequency and mode shape can be obtained mathematically by using the Eigen value problem. Both natural frequency and modes have been used in vibration analysis algorithm to identify damage in structures.

1.8. Objectives

1. Detect bifurcation points in two-bar truss (Von-Mises arch) and with shallow arch has seven bars by using two programs Gesa and Ansys in addition to the analytical analysis.
2. Present parametric study for a symmetric truss with different initial angles under compressive loading with nonlinear geometry.
3. Use a new method based on vibration technique to identify and detect and quantify damage in bars, beams, and plates.

CHAPTER 2

LITERATURE REVIEW

This chapter provides the reader with a summary of some of the significant research to date on the following topics; bifurcation analysis, and damage detection methods.

2.1 PREVIOUS WORKS FOR BIFURCATION ANALYSIS

A system may lose stability at a limit point bifurcation. It can be said that the system will snap towards a far stable equilibrium position and the structure buckles once it reaches the bifurcation point at the maximum loading.

Kim (2014) explains that the fundamental reason for system instability is system nonlinearity. The nonlinearity sources in solid mechanics are: force nonlinearity, kinematic nonlinearity, material nonlinearity, and geometrical nonlinearity. In general, the geometric nonminority is considered when the relations among displacement, rotation, and strain are nonlinear. It occurs when the deformation is large. However, Pintea (2012) concluded from his study of pre-stressed cable suspended structures under static loading that linear analysis is sufficient for the design because of the difference between the linear and nonlinear response of the cable is small. The material is assumed to behave linearly and only the geometric nonlinearity is being accounted for the study. He used NELSAS software in which the cables were modeled using straight elements connected in nodes.

Different methods to predict and solve the problem of bifurcation analysis of a structure have been used and presented in the literature. The analytical solution is very

important to clarify the fundamentals of nonlinear behavior. Such a solution is used in the current study to the name Von Mises (1923) who first use this kind of model truss which is a simple truss with two pinned bars.

Many researchers have used simple structures in their studies to explain the basic concepts of stability behavior. Pecknold et al. (1985) studied and examined exact closed solutions for the load deflection response of a simple two-bar planar truss. The truss has been analyzed to illuminate the snap-through and the bifurcation behavior of the structure. Three kinds of loading are considered in the study such as vertical, horizontal, and combination of horizontal and vertical. The exact solutions from this study can be used to construct more complex problems for evaluation of numerical solutions for nonlinear structural analysis.

Gerco et al. (2009) presented an analytical method to analyze the behavior of Von Mises truss which is consisted of two bars made of different material and cross section areas. The normal forces and the nodal positions of the members can be found analytically after the nonlinear equilibrium equations have been written. The method depends on root finding procedure to solve nonlinear equilibrium equations using mathematical software MATHCAD to calculate the equation response.

The geometrical nonlinearity behavior is often seen in slender structures such as arches, trusses and membranes. Based on nonlinearity analysis, Jansen et al. (2015) displayed a powerful approach to deal with topology advancement to gain well-performing designs that are insensitive to imperfections. Geometric nonlinearities are accounted by limiting the end-consistence while utilizing a Total Lagrangian formulation to model large displacements.

Bazzucchi et al. (2017 a) investigated the interaction between the Eulerian instability and the snap-through in shallow structures. Due to the interaction of the two different instabilities modes, the authors proposed a theoretical evaluation of critical load. 2D Von Mises arches were examined with an incremental displacement nonlinear analysis. Different parameters were tested like shallowness and slenderness. The shallowness ratio is the height to length ratio which may also be quantified by the initial angle of the arch. The slenderness ratio is the ratio between the length of a structural element and the thickness of the element. The results showed the amount of imperfection does not influence the type of the interaction. The study can predict the imperfection magnitude of the structural and the stability behavior.

Wang et al. (2018) studied the snap-through nonlinear deformation of an inflated balloon. The post bifurcation evolution of spherical and ellipsoidal balloons subjected to pressure has been investigated. The growth of involving both primary and bifurcated branches has been explored and studied numerically using finite element method.

Wu, et al. (2015) used multiple-short truss elements with nonlinear analysis procedure to model the bent effects of long cables. A methodology, along with its software development is presented to simulate the different construction stages associated with cable stayed bridges to obtain satisfied geometric configuration for each stage to ensure correct final design.

Liu et al (2012) proposed a method that can trace the load-deflection equilibrium path. Total global stiffness matrix equation of star dome truss has been established using nodal coordinates as unknowns. The method can solve the nonlinear problems in geometry and material, and pass limit point easily.

Nistor et al. (2017) presented a study for a simple pinned-pinned beam initially flat under transverse loading formulated based on classical Euler-Bernoulli theory. The study has reveals the connection between Euler buckling and the load deflection curve. Introducing simple calculation and this connection relation can provide information about the number of unstable equilibrium without utilizing any simulation. The study has been done without any imperfection in the geometry of the structure.

Kala (2012) studied the effect of initial random imperfections on the load carrying capacity. Finite element methods with the assumption of the geometrical nonlinearity have been used to solve stability problem of frame column whilst assuming constant slenderness of the columns.

Hrinda (2007) presented and analyzed the responses of highly geometrically nonlinear structures. The response of such structure exhibits complex snap through and snapback behavior. These responses can be modeled using the finite element approach with employment of the arc-length method. The arc-length method can follow the nonlinear equilibrium path and pass the critical points while the Newton-Raphson method fails when snap-back behavior occurs along the loading path and cannot follow the response immediately after snap-through.

Tsiatas et al. (2017) investigated the behavior response of non-uniform shallow arch with linear and nonlinear geometrical response. The arch is examined under central concentrated force and with cross section varies along its axis. The study included a numerical method called (AFE) Analog Equation Method based on an integral equation approach. The method depends on replacing the two coupled nonlinear differential equations by two uncoupled linear equations. The nonlinear equations are solved by

using arc length method which can follow the nonlinear equilibrium path to overcome bifurcation and limit points.

Some structures can exhibit snap-through buckling behavior under lateral loading which is appeared in curved structures, such as panels, beams, and arches. Virgin et al. (2017) studied such behavior with a number of clamped arches. The study showed the relation between the geometry and the corresponding configuration of the equilibrium and the condition when only stable shape is possible for the initial condition. The study is extended to obtain results under thermal condition.

Pai (2007) explained that the snap-through and snap-back phenomena are very common and dominant in highly flexible truss structures. They eventually lead to large deformation in the truss. Trusses are more likely to have local buckling and bifurcated solution than continuous structures. Therefore, different examples of truss structures have been studied to demonstrate different phenomena of highly flexible trusses.

The field of deployable structures has many potential applications in space for their advantages of small volume that can be occupied. Gantes et al. (1989) investigated the concept of deployable structures featuring stable and stress-free states in both deployed and collapsed configurations. The study particularly is focused on the nonlinear behavior of these structures during their deployment procedure. This behavior is associated with geometric compatibility requirements.

Degertekin et al. (2008) proposed an algorithm for optimum design of nonlinear steel space frames geometry. A structure can lose the stability before reaching the theoretical ultimate load. The judgment of stability can be evaluated by finding the

determinant of the stiffness matrix for the system extracted from first order derivative of potential energy. The instability takes place when the determinant of the matrix is negative.

Danso and Eduard (2017) introduced an analytical method using a third order derivative of the potential energy function to analyze multi stable truss structure and unit cells of engineered materials. The concept of catastrophe theory has been utilized to find the cusp point singularity. The locus of all cusp points is the basis to predict any bistable hysteric behavior of the truss system. Such a point represents the onset of bistability in which the system has two stable equilibrium states.

Gioncu (1994) presented the general theory of coupled instabilities in many practical cases. Instability can be detected by checking two or more eigenmodes at coincident or nearly coincident critical loads. The coupled instability might give rise to severe imperfection sensitivity.

Fu (2013) presented a new geometric nonlinear formulation for large-deflection in static analysis. It is based on the finite element method to solve problems for complex space truss structure. The formulation depends on using nodal position instead of nodal displacement space. The good results showed that the proposed formulation is valid.

Hu et al. (2015) have presented a review about how to induce buckling in smart application. In which an instability phenomenon can be avoided through special design modification. However, in recent studies, researchers have been attempting to transform the negative effect to the positive one, as a beneficial behavior to be used in the design of smart devices.

Based on the changes in the shape of the structure concept, buckling is induced in micro-electromechanical systems (MEMS) devices. The advantage of buckling motivated recent research in MEMS application. A wide range of MEMS devices have been manufactured with low production cost and with advanced and integrated circuit technologies relying on the benefits of the buckling (Batra et al. 2007).

Bazzucchi et al. (2017 b) has proposed the tracing of interaction domains and interaction curves. The structure has been analyzed regards to its shallowness ratio, slenderness, and imperfection patterns. The imperfection was introduced in Von Mises arch as a deviation in mid-span of each beam from the line axis in the two patterns. The first used symmetric imperfection where the deviation applied downward in each beam. The second reverses direction for the adjacent beams as an asymmetric imperfection. However, in our study, small load and displacement perturbation in the initial conditions are used as asymmetric perturbation.

In this study, asymmetric perturbation is used to detect the bifurcation. Bifurcation analysis of two structures are carried out and due to the simplicity of these structures, stability criteria can be obtained analytically. Under asymmetry perturbations, a much lower critical load was found for each structure. Using the same two structures, we then demonstrate the use of two packages based on finite element methods for the bifurcation analysis. Although the same critical loads are obtained, the analysis using FEM is more suitable for complex structures. However, care must be taken when using these packages; the computer programs may fail to detect some bifurcations unless an asymmetric perturbation, such as the loading or in the geometry, is inserted. The newly detected bifurcations of the two truss structures are important since the critical loads at

these bifurcations are much lower than the ultimate load corresponding to the limit point of an arch. A parametric study for a truss with different initial angles has presented. The detected bifurcations may also provide designers novel options when utilizing nonlinear structural responses in devices.

2.2 PREVIOUS WORKS ABOUT DAMAGE DETECTION

METHODS

Most current methods to detect damage are either visual or experimental such as radiography, acoustic magnetic field methods, and ultrasonic and eddy current (Doherty, 1987; (Zoidis, 2013). The experimental techniques are localized and need to know the vicinity of the damage before inspection. Because of this limitation of the experimental methods, it's opened the door to the development of many studies and the continued research of global methods. These methods rely on the examination of changes in the vibration characteristics of the structure. The global methods can be applied on complex and large structures (Doebling, 1998).

Structural identification refers to the identification of physical structural parameters; mass, damping and stiffness to update and calibrate the structural model. Tracking the changes in theses parameters are the base for nondestructive method techniques to detect damage in structures (Koh, 2009).

Natural frequencies and mode shapes are the dynamic characteristics to a system. They are functions of the changes in physical structural parameters. These changes provide a means to assess the structural integrity and safety (Doebling, 1998; Heckman,

2014). Based on vibration analysis, many different methods have been developed to detect damage in structures. Some research used frequency domain and mode shape for identification. The eigenfrequencies are affected by the severity of crack depth in the beam thickness. The values of eigenfrequencies decreased with the accumulation of cracks in the beams (Ndambi 2002).

Rytter (1993) classified the identification of a damage in a structure can be under four levels: (i) identify that damage is present in the structure or not, (ii) detect the location of the damage, (iii) estimate the severity of the damage, (iv) and predict the remaining service life of the structure.

Kim (2003) clarified that a few natural frequencies and mode shapes can be used to identify damage. Two damaged frequency and mode shapes are used to locate damage and detect the size of the crack in a beam.

Mousavi and Amir (2016) proposed a method that just used one mode shape and its corresponding eigenvalue to carry on the damage detection from structure using modal data. It is one of the hybrid modal reduction techniques. Several different types of structures have been examined to detect the damage using complete and incomplete measured data. The results showed the ability of the method to detect damage for 2D and 3D frame structures.

Yang et al. (2017) presented experimental and theoretical study depended on eigenvalue problem perturbation to get the defect modal response from the damaged structure. To demonstrate the performance of the study, finite element method was used as a numerical method to obtain the modal parameters from an aluminum beam. The

internal defects were located and evaluated by using damage indicator and damage estimator based on multiple mode biorthogonal wavelet coefficients. A greater enhancement of the damage detection was observed with damage indicator than by using individual modes in wavelet coefficients.

Abdo (2012) proposed a method examined the capability of using higher-order mode shape derivatives on simply and cantilever steel plate model structures to detect and locate the damage. The method used numerical analysis with six scenarios for investigation. The method showed good results to identify the damage location. However, the drawback of using changes in fourth derivative of mode shapes is sensitive to measurement noise.

Abdo (2014) presented ideas on using mode shape curvature as a concept to identify damage in a structure. A numerical study was used for a comparison investigation between two methods to show the robustness of the methods. One of them depends on the curvature mode shape simulated using the finite difference method and the second one depend on using the flexibility of the structure to detect damage using a few modes. The results revealed that both methods can find single damage in a beam with different damage ratios.

Rucevskis et al. (2015) developed a method to be applied on plate structure which was originally applied on beam structures. This method can detect and locate damage in a structure like-plate using mode shape curvature. The significance of the proposed method is that it only needs information for damaged mode shape from the damage structure. This method had showed good accurate results under medium damage

ratio. A comparison had been done between experimental results from test and the simulated results to demonstrate the validity of the method.

Kumar (2016) studied a method depends on the curvature of the frequency response functions (FRF) to identify the damage in structures, such as a plate or a beam. The damage index was extracted from the difference between damaged and undamaged modal data. The damage index is a dependent factor on a frequency range. This method requires numerical differentiation and integration of the input spatial data. The method showed successful results to detect and quantify the damage

Rucevskis et al. (2016) developed another method that can detect and locate damage in a structure based on using data from damaged structure without knowing the history data for the undamaged structure. To be used, this method requires only the damaged mode shape curvature in the proposed algorithm. An aluminum plate with mill cut to represent damage was investigated as an example to demonstrate the method's validity. The modal data was extracted from finite element method. The measured data comes from scanning data vibrometer. The damage index is the difference between the damaged measured mode shape curvature and the undamaged smoothed polynomial mode shape curvature. Smoothing it with regression analysis by approximate polynomial to estimate undamaged mode shape curvature. The results gave good indicator of the method to be valid.

Bayissa (2007) used a parameter to identify the damage in the structure known as spectral strain energy (SSE) depends on vibration response, curvature response, natural frequency, modal damping and mode shape. These parameters were used to drive SSE. Mean-square value MSV, a statistical parameter of the response need to be known, to

compute SSE with the assumption that the excitation force is a static. Eventually, various damage detection and localization in simply supported plate and beam elements were presented.

Radzieński (2013) presented a study was applied to detect damage and locate it based on the changes in propagating waves. Experimental investigation was applied on an aluminum plate with riveted two L-shape stiffeners. The wave excited by piezoelectric and then Laser Scanning Doppler Vibrometer was used to measure and analyzed the waves. The results showed that damage was detected with this fast and precise approach.

Ghodrati (2011) identified a method called discrete wavelet transform to analyze damaged mode shapes. Any sharp changes in wavelet coefficient displayed the rejoin of the damage. The ratio of wavelet coefficients of the damaged structure to the undamaged structure has been used for damage detection in plate structures.

In this work, a method called residual error method used to detect damage for beam developed to detect damage for structure with two dimensions like plate. The method is applied on structure with one dimension, bar and beam for validation. The residual error method was proposed by (Genovese, 2000) and then adopted by (Brasiliano, 2004) to apply on continuous beam with validation of experimental data laboratory. This method is used to detect damage in a structure by observing the error present in the equation of motion when the stiffness and mass matrices of the undamaged structure and the modal parameters (natural frequencies and mode shapes) of damaged structure are used in the equation. A program coded with MATLAB using finite element method to illustrate the method. From literature with many papers, the damage in this work is characterized as a stiffness reduction, but the mass is stationary. To verify the

efficiency of the proposed method, absolute difference curvature mode shape method has been used to compare the results. Moreover, the sensitivity to the noise has been test for the two methods.

2.3. SUMMARY:

Previous works on bifurcation analysis and stability concept have been mentioned and presented. Some researches presented an analytical method to analyze the behavior of Von Mises truss. Other researchers investigated the behavior response of non-uniform shallow arch with linear and nonlinear geometrical response explained and presented. From the previous work that we presented we found that no one has investigated the area of arch with step angle and its stability analysis analytically. In addition, we adapted the software Ansys to capture the bifurcation point and we did the same with Gesa program coded in Matlab. Several kinds of damage detection method have been presented in the literature. We nicely chose residual error method to use in damage detection. We adapted the method to detect damage in two dimensions in addition to one dimension, since no one use this method precisely with plate structure or with bar structures. This proposed method is significant because of its accuracy in result and simplicity of the performance of the computation.

2.3 ORGANIZATION OF THE DISSERTATION

Chapter I: a general introduction and background with some conception is presented in the dissertation. The justification and problem description is discussed. The dissertation objectives are defined.

Chapter II: a literature review of the previous works on the subject of bifurcation analysis and structural damage identification methods are presented. Organization of the dissertation with its chapter are includes in this chapter.

Chapter III: contains a brief introduction to symmetry breaking bifurcation with theory analysis for two examples of truss structures. Finite element calculations using (Gesa) and Finite element calculations using (Ansys) are presented and explained. Results, discussion and conclusions for each subject are included in this chapter.

Chapter IV introduces damage detection using residual error method with numerical examples for one and two dimensional structures are prepared to illustrate the effectiveness of the procedure to locate damage. Results, discussion and conclusions for each damage detection methods on structure are included.

Chapter V: the last chapter of the dissertation that contains overall conclusions and recommendations for future work.

In addition to four appendices and references.

CHAPTER 3

SYMMETRY BREAKING BIFURCATION

3.1 INTRODUCTION

Trusses with geometric and loading symmetries have been used in many structures to reduce the complexity of the design. Slight asymmetries in geometry and in loading could lead to bifurcation in the structural response. Failure of such structures still occur occasionally causing major damage to the property and to the human lives. A fully nonlinear structural analysis is expected to detect such symmetry breaking bifurcations. Two kinds of trusses have been studied in our study. The two structural examples used serve to illustrate the robustness and the limitations of widely used finite element analysis packages for bifurcation analysis.

3.2 Symmetry breaking bifurcation of a Von Mises arch

Two assumptions are considered throughout our analysis. First, the Von Mises arch is linearly elastic; geometric nonlinearity refers to large nodal displacement and moderate axial strains (<0.05). Second, high Euler buckling is assumed to neglect the possibility of buckling instability (Danso and Eduard, 2017; Ligaro and Valvo, 2006). On the assumption of moderate strain, we found that the difference between Cauchy strain and Green strain is (0.027%), which is insignificant as in appendix A. Figure 3.1

shows an arch, traditionally named Von Mises arch, with two elastic hinged members under a compressive load. We include the elastic deformation in the equilibrium equation and treat the two compressive members as spring with spring constant $k = \frac{EA}{L}$, where E , A and L are the Young's modulus, the cross sectional area, and the unloaded length of each compressive member, respectively.

Figure 3.1. Symmetric truss supporting load

3.2.1 Snap-through bifurcation and the ultimate load limit of the symmetric structure

Let Δ denote the downward displacement of the top node, α_0 the initial angle, and α the angle after deformation. Figure 3.1 with trigonometric relationships leads to the followings:

$$\sin \alpha = (L \sin \alpha_0 - \Delta)/l \quad (3.1)$$

$$L \cos \alpha_0 = l \cos \alpha \quad (3.2)$$

The force P is calculated using the force in the truss based on:

$$P/\sin\alpha = 2k(L - l) \quad (3.3)$$

Substituting (3.2) into (3.3), we have

$$\frac{P}{kL} = 2 \sin \alpha \left(1 - \frac{\cos \alpha_0}{\cos \alpha} \right) \quad (3.4)$$

$$\sin \alpha = \frac{L \sin \alpha_0 - \Delta}{\sqrt{(L \cos \alpha_0)^2 + (L \sin \alpha_0 - \Delta)^2}}, \quad \cos \alpha = \frac{L \cos \alpha_0}{\sqrt{(L \cos \alpha_0)^2 + (L \sin \alpha_0 - \Delta)^2}} \quad (3.5)$$

Substituting (3.5) into (3.4), we get

$$\frac{P}{kL} = 2[(L \sin \alpha_0 - \Delta)(L^2 - 2\Delta L \sin \alpha_0 + \Delta^2)^{-\frac{1}{2}} - \frac{L \sin \alpha_0 - \Delta}{L}] \quad (3.6)$$

This can be written in the following dimensionless form:

$$\frac{P}{kL} = 2(\sin \alpha_0 - \frac{\Delta}{L}) \left[\left(1 - 2 \frac{\Delta}{L} \sin \alpha_0 + \left(\frac{\Delta}{L} \right)^2 \right)^{-1/2} - 1 \right] \quad (3.7)$$

Figure 3.2. The analytical load-deflection relationship (a) $\alpha_0 = 20^\circ$ and (b) $\alpha_0 = 15^\circ$

The relation between dimensionless load (P/kL) and the dimensionless displacement (Δ/L) is nonlinear as seen from Equation 3.7 and it is demonstrated by Figure 3.2 for $\alpha_0 = 20^\circ$ and $\alpha_0 = 15^\circ$. Both curves snap-through at the critical points (limit points) to another stable path. Larger initial angle corresponds to larger ultimate load. The curve for force and deflection will follow the same symmetric pattern for differential initial arch angles. The coordinates for the ultimate point on the curve of load displacement is (0.11, 6.98e-3) for $\alpha_0 = 15^\circ$, and the coordinates with angle 20° is (0.14, 1.61 e-2).

3.2.2 Finite element calculations (Gesa).

Since a truss consists of bar elements and the external loads are applied on the nodes, the derived governing equations are discrete and exact if fully nonlinear strains are employed. Gesa (Geometrically exact structural analysis) which is coded in MATLAB has been used for this purpose. Only geometric nonlinearities are considered which account for nonlinear structures undergoing large deformation, rotation and finite strain. Gesa is a total Lagrangian displacement-based finite element code for analyzing highly flexible structures (Pai, 2007). Riks method has been used in nonlinear static analysis to overcome limit-point problems. Material properties for the arch truss with two elements and angle $\alpha_0 = 15^\circ$ are listed in Table 3.1 for Aluminum Alloy 2024 T3 [49]. We must choose appropriate initial load to cover the whole range of the equilibrium path. We have applied (-35585.77 N) at the top of the arch specifically. The nodal coordinates are listed in Table 3.2.

Table 3.1

The geometrical and material properties

Length for each bar	L	0.3048	m
Young's modulus	E	73.08443 e6	KPa
Area of each bar	A	0.00059	m^2
Mass density	ρ	2783.0456	$\frac{kg}{m^3}$
Poisson's ratio	μ	0.33	—

Table 3.2

Nodes coordinate of Von Mises arch

Node	X	Y	Z
1	0	0	0
2	0.588	0	0
3	0.294	0	0.078

3.2.3 Finite element calculations (Ansys)

The arch truss in Figure 3.1 has been analyzed in Ansys version 16.0 (Moaveni, 2011), to verify the results from the theory part and as well as the results coded by Gesa in MATLAB using finite element method. The analysis is done considering the nonlinear analysis. The truss with angle $\alpha_0 = 15^\circ$ treated each bar as one element. The element type defined as (link180) is 3-D spar valid for truss structure and variety of engineering applications. The element is a uniaxial tension-compression with two nodes in each element. There are three translation degrees of freedom in x, y, and z directions at each node; no bending is considered, and the loads are applied at the nodes. By default, (link180) allows change in cross sectional area as a function of axial elongation. For nonlinear static equilibrium problems, arc length method has been used for unstable solutions in which all load magnitudes are controlled by single parameter factor. Since it is an automatic load step method, appropriate sub-steps are required. The same

geometrical properties in Table 3.1 and node coordinates in Table 3.2 have been used in Ansys.

Figure 3.3. Load deflection curves for truss with two elements and initial angle $\alpha_0 = 15^\circ$ obtained with different methods.

Figure 3.3 shows the results obtained from Gesa in MATLAB for truss with angle ($\alpha_0 = 15^\circ$) and from Ansys. The results both agree with the analytical solution. The nonlinear geometric result in the saddle-node bifurcation occurs at the maximum load, no equilibrium exists above the maximum load except when the bottom two bars snap over to bear the load in tension signifying the collapse of the arch. The coordinate of saddle

bifurcation point is (0.11, 7.04 e-3) by using Gesa and the coordinate in Ansys is (0.11, 7.1e-3) which are almost the same values. In conclusion, Gesa and Ansys are capable of nonlinear bifurcation analysis. They detect the ultimate limit load of the symmetric structure. Moreover, for load that is below the ultimate value, both programs detect stable as well as unstable equilibriums.

3.2.4 Bifurcation and stability phenomena in Von Mises arch

The previous saddle bifurcation due to the geometric nonlinearity has been well-known. But the same structure also exhibits symmetry-breaking bifurcation. If there is a small perturbation causing the center node to have a small displacement along the horizontal and vertical directions away from the equilibrium position, we can formulate the fully nonlinear problem again by treating the two bars as springs whose spring constants are defined by the unstressed length to obtain the lengths as in the following:

$$l_1 = \sqrt{(L \cos \alpha_0 + x)^2 + (L \sin \alpha_0 - \Delta + y)^2} \quad (3.8)$$

$$l_2 = \sqrt{(L \cos \alpha_0 - x)^2 + (L \sin \alpha_0 - \Delta + y)^2} \quad (3.9)$$

Use the potential energy approach for the conservative structure. The potential energy of the structure under a static load is the summation of stored energy in the system and the work done by external forces and that can be given as:

$$V = \frac{k}{2} (L - l_1)^2 + \frac{k}{2} (L - l_2)^2 - P(\Delta + y) \quad (3.10)$$

This can be written in the following form:

$$V = \frac{kL^2}{2} \left[\left(1 - \frac{l_1}{L}\right)^2 + \left(1 - \frac{l_2}{L}\right)^2 - \frac{P}{kL} \left(\frac{\Delta}{L} + \frac{y}{L}\right) \right] \quad (3.11)$$

This energy is at a stationary position so, in this notation, the two equations of static equilibriums are given by:

$$\frac{\partial V}{\partial x} = 0, \quad \frac{\partial V}{\partial y} = 0 \quad (3.12)$$

The fully nonlinear forms of the two equations are algebraically very cumbersome. Since the purpose of the study is stability to infinitesimal perturbation from the equilibrium, knowledge of second variation is required; the stiffness matrix is defined as following:

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 V}{\partial x^2} & \frac{\partial^2 V}{\partial x \partial y} \\ \frac{\partial^2 V}{\partial x \partial y} & \frac{\partial^2 V}{\partial y^2} \end{bmatrix} \quad (x = y = 0), \quad (3.13)$$

$$k_{11} = k[-1 - 2\delta^2 + \cos 2\alpha_0 + 2\rho + 2\delta^2\rho - 4\delta \sin\alpha_0 (\rho - 1)]/\rho^3 \quad (3.14)$$

$$k_{12} = k_{21} = 0 \quad (3.15)$$

$$k_{22} = -k(1 + \cos 2\alpha_0 - 2\rho - 2\delta^2\rho + 4\delta \sin \alpha_0 \rho)/\rho^3 \quad (3.16)$$

$$\text{where: } \delta = \frac{\Delta}{L} \quad (3.17)$$

$$\rho = \sqrt{1 + \delta^2 - 2\delta \sin\alpha_0} \quad (3.18)$$

we will consider the sign determinant of the matrix in Equation 3.13 to check the stability of the system if it is positive or negative value. Since the stiffness matrix is a diagonal, the stability of the equilibrium requires both k_{11} and k_{22} to be positive

(a)

(b)

Figure 3.4. The values of $\frac{k_{11}}{k}$ and $\frac{k_{22}}{k}$ for (a) $\alpha_0 = 11\pi/30$ and (b) $\alpha_0 = 2\pi/5$

By plotting normalized k_{11} given in Equation 3.14, we can see that the structure is stable for initial $\alpha_0 = 11 \pi/30$ because of positive values of k_{11} as in Figure 4a. For larger initial angle $\alpha_0 = \frac{2\pi}{5}$, the structure becomes unstable when the dimensionless displacement is just above (0.13) because of negative k_{11} and negative determinant of the matrix. k_{22} is positive in both cases of initial angles as shown in Figure 3.4a-b. The symmetry breaking bifurcation in the two-member arch can occur only when the two bars are steep. Although this bifurcation is interesting, it is less relevant to those arches whose angle α_0 is usually far below a right angle starting from angle 72° or $\alpha_0 = 2\pi/5$.

Based on Equation 3.12, from the solution of the first derivative of the potential energy, we can find the ultimate load at the bifurcation point and the downward displacement (y/L) of the top node of the arch as in Table 3.3 starting from 72 to 85 degrees. Larger angle results in smaller displacement as seen from the table and Figure 3.5a. The bifurcation point in (b) for $\alpha_0 = 72^\circ$ shows this point is one of the critical points that the behavior of structure will change from stable (solid line) to unstable state (dotted line).

(a)

(b)

Figure 3.5. (a) The relation between the load and displacement (y/L) in bifurcation point, (b) The bifurcation point with $\alpha_0 = 72^\circ$

Table 3.3

The values of the load and downward displacement at bifurcation point

α_0_{deg}	72	76	77	78	79	80	81	82	83	84	85
$P / (KL)$	0.233	0.130	0.110	0.093	0.077	0.063	0.051	0.040	0.030	0.025	0.016
y/L	- 0.132	- 0.069	- 0.058	- 0.050	- 0.040	- 0.032	- 0.023	- 0.020	- 0.017	- 0.010	- 0.0090

3.2.5 Symmetry breaking for arch with Gesa

Symmetric arch truss means symmetric in geometry and loading. Based on that, we have applied small disturbance in the active node (the top one) to break the symmetry. The disturbance is introduced in two ways: by applying a small horizontal force in x-axis and by a deviation in the initial geometry along the horizontal axis. The analysis is done for the arch with different angles (66° , 72° and 80°) to show the bifurcation point using Gesa program.

The structure behavior shows saddle bifurcation point when the initial angle for the truss is $\alpha_0 = 11 \pi/30$ or $\alpha_0 = 66^\circ$. Two kinds of disturbance, small horizontal force or small horizontal displacement are applied in the top node as shown Figure 3.1(b). An appropriate initial applied vertical load P is chosen to control the step size while following the equilibrium path. The equilibrium path does not show pitchfork-bifurcation point which was considered symmetry breaking bifurcation point since k_{11} and k_{22} are

both positive and the determinant of the stiffness structure is positive as illustrated from Figure 3. 4.

Gesa shows that the pitchfork-bifurcation can happen when the angle for arch is above 66° . The equilibrium path between the dimensionless load $\left(\frac{P}{kL}\right)$ and dimensionless displacement $\left(\frac{\Delta}{L}\right)$ bifurcates at a point of coordinates (0.135, 0.232) when $\alpha_0 = 72^\circ$ as shown in Figure 3.6(a)-(b). Figure 3.6a corresponds to the case when a small horizontal force (578.269 N) is applied at node 3. The equilibrium path bifurcates at the end of the curve presented with numbered arrows. The computer program will miss the bifurcation if it had not followed the equilibrium path to the end. When the disturbance is a small displacement in the initial geometry ($5.4864\text{e-}4$ m), the symmetry breaking bifurcation is detected in the beginning of the path as shown in Figure 3.6(b). The bifurcation point is independent of perturbation sources as indicated by the same value in (a) and (b). The values agree with the dimensional displacement $\left(\frac{\Delta}{L}\right)$ from stability analysis as in Figure 3.4 and with the bifurcation in Table 3.3. The path in the bifurcation point (symmetry bifurcation point) goes into two branches. For both types of perturbations, an asymmetric equilibrium path bifurcates from the symmetric path. However, the bifurcation was detected in different sequences as shown by the arrows in Figure 3.6 (a) and Figure 3.6(b). Since the path starting at zero load is stable, the path after the bifurcation corresponds to unstable equilibrium. Magnified area in Figure 3.6(c)-(d) shows how the truss behaves corresponding to some points on the asymmetric paths.

(a)

(b)

(c)

(d)

Figure 3.6. Load deflection curve for truss initial angle $\alpha_0 = 72^\circ$ (a) With horizontal force (black line for symmetric bifurcation and red line for asymmetric bifurcation (b) Small deviation on geometry (black line for symmetric bifurcation and red line for asymmetric bifurcation (c) Some of asymmetric steps loading (d) The asymmetric loading steps (5, 7, 11, 12 and 13) on the geometry of truss

The load corresponding to the the symmetry breaking bifurcation is much lower for larger initial angles. Figure 3.7 shows that the coordinate of the bifurcation point is (0.035, 0.065) using Gesa which agrees with the values in Table3. 3. The limit load at bifurcation point is smaller with larger initial angle $\alpha_0 = 80^\circ$. Both disturbances give the same bifurcation point, which means that the structure bifurcates with the same bifurcation coordinates point value for different disturbances. The horizontal disturbance force is (569.372 N) and the deviation in geometry is $3.047\text{e-}4$ m. The initial applied vertical force is $P = (-12 \text{ e}5 \text{ N})$. As we see in this case, when the initial angle gets closer to 90° , the symmetric equilibrium becomes unstable at a much lower load and at a strain which is well within the linear elastic range of many alloys.

Figure 3.7. The load deflection curve for truss angle $\alpha_0 = 80^\circ$ with horizontal force using Gesa.

3.3 Symmetry breaking bifurcation in shallow arches with seven bars

The previous section has shown the symmetry breaking bifurcation in a two-member truss. The symmetric equilibrium becomes unstable to asymmetric perturbations at a very low load for very steep initial angles. In this section, we consider the symmetry breaking bifurcation in a shallow truss structure with seven bars as shown in Figure 3.8.

3.3.1. Theoretical analysis

Figure 3.8. Symmetric pinned truss structure with a roller at the node 3

The truss is symmetric in geometry and loading, the deformation is symmetric. However, if node 3 in the figure can have one degree of freedom, the structure can become asymmetric under asymmetric perturbation. Our interest is to determine the stability to small perturbations in general. We first determine the potential energy of the system.

$$V = \frac{k_1}{2} [\sqrt{(L \cos \alpha_0 + x_1)^2 + (L \sin \alpha_0 - \Delta + y_1)^2} - L]^2 + \quad (3.19)$$

$$\begin{aligned}
& \frac{k_1}{2} [\sqrt{(L \cos \alpha_0 - x_1 + x_3)^2 + (L \sin \alpha_0 - \Delta + y_1)^2} - L]^2 + \\
& \frac{k_1}{2} [\sqrt{(L \cos \alpha_0 - x_2 + x_3)^2 + (L \sin \alpha_0 - \Delta + y_2)^2} - L]^2 + \\
& \frac{k_1}{2} [\sqrt{(L \cos \alpha_0 - x_2)^2 + (L \sin \alpha_0 - \Delta + y_1)^2} - L]^2 + \\
& \frac{k_2}{2} [\sqrt{(2L \cos \alpha_0 - x_2)^2 + (y_2 - y_1)^2} - 2L_0 \cos \alpha_0]^2 + \\
& 2 \frac{k_3}{2} x_3^2 + P(-\Delta + y_1) + P(-\Delta + y_2)
\end{aligned}$$

To simplify the analysis, we consider a special case when the top horizontal member is absent by letting $k_2 = 0$. Consequently, we have $x_1 = x_2 = x_3/2$. We also introduce change of variables:

$$y_1 = \frac{1}{2} (Y_1 + Y_2) \quad (3.20)$$

$$y_2 = \frac{1}{2} (Y_1 - Y_2) \quad (3.21)$$

The stability of the equilibrium is determined by the matrix

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 V}{\partial Y_1^2} & \frac{\partial^2 V}{\partial Y_1 \partial Y_2} & \frac{\partial^2 V}{\partial Y_1 \partial x_3} \\ \frac{\partial^2 V}{\partial Y_1 \partial Y_2} & \frac{\partial^2 V}{\partial Y_2^2} & \frac{\partial^2 V}{\partial Y_2 \partial x_3} \\ \frac{\partial^2 V}{\partial Y_1 \partial x_3} & \frac{\partial^2 V}{\partial Y_2 \partial x_3} & \frac{\partial^2 V}{\partial x_3^2} \end{bmatrix}; Y_1 = Y_2 = x_3 = 0 \quad (3.22)$$

We deliberately introduced Y_1 and Y_2 to achieve the effect that $k_{12} = k_{13} = 0$ and

$$k_{11} = -k_1 [1 + \cos 2 \alpha_0 - 2\rho - 2\delta^2 \rho + 4\delta \sin \alpha_0 \rho] / (2\rho^3) \quad (3.23)$$

$$k_{22} = k_1 + k_2 - \frac{3k_1}{4\rho^3} + \sec \alpha_0 \left[-k_2 \sqrt{\cos^2 \alpha_0} - k_1 \cos 3\alpha_0 / (4\rho^3) \right] \quad (3.24)$$

$$k_{23} = k_1 \cos \alpha_0 (-\delta + \sin \alpha_0) / \rho^3 \quad (3.25)$$

$$k_{33} = k_1 \left[-1 - 2\delta^2 + \cos 2\alpha_0 + 4\delta \sin \alpha_0 + 2\rho^3 + \frac{4k_3}{k_1} \rho^3 \right] / (2\rho^3) \quad (3.26)$$

Because of the block diagonal property of the stiffness matrix, the stability is not determined by k_{22} or k_{33} alone. Instead it is determined by the determinant (Det):

$$Det = k_{11} (k_{22}k_{33} - k_{23}^2) = k_{11} \det \quad (3.27)$$

where $\det = (k_{22}k_{33} - k_{23}^2)$

Bifurcation occurs when Det in Equation 3.27 is zero. That is when k_{11} is zero or $\det = (k_{22}k_{33} - k_{23}^2)$ is zero. Figure 3.9(a)-(c) shows $\det$ as function of the normalized displacement. For initial angle $\alpha_0 = \frac{\pi}{12}$ or 15° , $\det$ changes from positive to negative values at Δ/L near 0.07 when the bottom bars are assumed to have the same cross section as other bars, and this point is the bifurcation point where the maximum load occurs. Symmetry breaking bifurcation occurs at a lower displacement and a lower load when k_3 is changed from $(\cos \pi/12)/2 k_1$ to $\frac{1}{10}(\cos \pi/12)/2 k_1$. The term k_{11} , which is independent of k_2 or k_3 , changes from positive to negatives values at Δ/L near 0.11 as in Figure 3.9 (c). This bifurcation corresponds to the snap-through, i.e. the saddle-node bifurcation.

(a)

(b)

(c)

Figure 3.9. Change of stability as function of Δ/L for $\alpha_0 = \frac{\pi}{12}$, $k_2 = 0$ (a) for $k_3 = (\cos \pi/12)/2 k_1$

(b) For $k_3 = \frac{1}{10}(\cos \pi/12)/2 k_1$, (c) k_{11} , $\alpha_0 = \frac{\pi}{12}$

Although the structure shown in Figure 3. 8 is relatively simple, the algebraic work required to determine the nonlinear equilibrium based on the potential energy function is too cumbersome. We thus proceed to using nonlinear analysis capabilities of FEM packages for the bifurcation study.

3.3.2 Bifurcation study by Gesa

The symmetric shallow truss in Figure 3.8 has been modeled using finite element method coded in Gesa undergoing large static deformation to show the bifurcation analysis for geometrically nonlinear problems. The material properties for symmetric truss with seven elements with initial angle 15° are the same in Table 3.1 with symmetric loading in nodes 3 and 4 in z direction with nodes= [0, 0, 0; c, 0, 0; 2*c, 0, 0; 0.5*c, 0, h; 1.5*c, 0, h], where $c=2*0.2944$ m and $h=0.0789$ m.

(a)

(b)

Figure 3.10. (a) Symmetric load deflection curve for truss loading in node 4 and 5

with $\alpha_0 = 15^\circ$ (b) A truss with symmetric loading with $\alpha_0 = 15^\circ$

Figure 3.10 (a) shows the symmetric load deflection curve for truss with seven elements and initial angle $\alpha_0 = 15^\circ$ when loading is symmetric in node 4 and 5 by R. It agrees with the diagram from analysis in Figure 3.2 and verifies Figure 3.3 for symmetric arch with two elements. The symmetry can be broken by adding disturbance in node 2 as in Figure 3.10-(b). In this work we used two kinds of disturbance, one by introducing small horizontal force in the active node 2 and second by adding small deviation in node 2 in the initial geometry condition as a small imperfection. However, the truss is under symmetric loading with $R = (-35585.77 \text{ N})$ at node 4 and 5.

(a)

(b)

Figure 3. 11. Asymmetric load deflection curve (a) Small axial force perturbation (b) Small horizontal deviation perturbation

Figure 3.11 shows the symmetric load deflection curve with black color and the asymmetric one with green and red colors for truss with seven elements and angle $\alpha_0 = 15^\circ$. The equilibrium paths of node 4 and 5 are no longer the same as perturbations at node 2 are introduced. The perturbations are either a horizontal force of (404.788 N) in node 2 or a horizontal displacement of (5.4864e-4) at node 2. The bifurcation coordinate is at (0.07, 6.2e-3) for horizontal force perturbation and at (0.07, 6.17e-3) for initial displacement. The loading paths and the ultimate load are nearly the same for both types of perturbations.

The analysis above indicates that symmetry bifurcation leads to a lower critical load than the ultimate load predicted for the symmetric response. However, the reduction in critical load due to symmetry breaking is not significant as a result. Since the two bottom bars carry no load in symmetry response. The designer is tempted to reduce the cross-sectional area of them to reduce the cost. We carried out numerical study when the cross-sectional areas for bars 1 and 2 are changed to $A/10$. Obviously, this reduction in the cross-sectional areas has no effect on the symmetric loading paths as seen in Figure 3.12 with black color for symmetric response. However, when asymmetric disturbance at node 2 is introduced, significant reduction in ultimate load can be seen at the bifurcation point as illustrated in Figure 3.12. The equilibrium path bifurcates into two branches. Node 4 in green color behaves differently from node 5 in red color. Therefore, the seemingly redundant two bottom bars greatly affect the load carrying ability of this shallow truss.

Figure 3.12. Symmetric and asymmetric truss angle $\alpha_0 = 15^\circ$ loading with R, at node 4 node 5 and small axial displacement in node 2 when changing cross section area to $A/10$ for bar 1 and 2.

3.3.3 Bifurcation study by Ansys

The shallow truss in Figure 3.8 has been analyzed in Ansys version 16.0 (Moaveni, 2011), to verify the theoretical results and the results from Gesa. The bifurcation study is done by using the nonlinear analysis tools of Ansys. Each bar is treated as an element of type defined as (link 180). The material properties for linear elastic isotropic materials with static analysis are listed in Table 1. The nodal coordinates are listed in Table 3.4.

Table 3.4

The node coordinates for truss with seven elements

Node	X	Y	Z
1	0	0	0
2	0.588	0	0
3	1.177	0	0
4	0.294	0.259	0
5	0.883	0.0789	0

We carried out the study in Ansys of the cases studied in the previous subsection. The results are given in Figure 3.13. (a)- (b) agree with those obtained from using Gesa shown in Figure 3.11 and Figure 3.12. The symmetric loading paths are the same as given by the analytical solution in Figures 3.2 and 3.3.

(a)

(b)

Figure 3.13. The symmetric and asymmetric curves in nodes4 &5 for arch angle $\alpha_0 = 15^\circ$ with 7 elements using Ansys data (a) All bars with same cross section area (b) Changing cross section area to A/10 in the bottom bars.

3.4 Conclusion:

In this study, we have presented new tool for the researchers to use to detect bifurcation points by using two programs Gesa and Ansys in addition to the analytical tools. Simple truss with two element and another more complicated truss with seven bars are used in this study to show the robustness and limitation when conduct the bifurcation study in Gesa and in Ansys.

For the symmetric structures, the bifurcation point is identified as a saddle point in the load deflection curve of nonlinear geometric analysis. Both Ansys and Gesa can detect the saddle bifurcation point and follow the loading path to the unstable branch of the load deflection curves. However, they failed to detect symmetry breaking bifurcation unless small asymmetric perturbations in loading or in geometry are introduced. Therefore, care must be taken when these programs are employed to conduct stability analysis.

The load at the bifurcation point in the presence of perturbations is much lower than the ultimate load for the symmetric response. Missing the symmetry breaking bifurcations can dangerously overestimate the loading carrying capacity of the structure.

To the best of our knowledge, the result that symmetry breaking bifurcation occurs in a steep Von Mises Arch has not been noted in the literature.

The results for the shallow symmetric truss are more significant to structural design. In engineering design, we take advantage of symmetry to simplify the mathematical model. We must carefully consider the structural stability in the presence of imperfections and asymmetric disturbances. Although nonlinear finite element analysis packages can detect some bifurcation points, detections of symmetry breaking bifurcations require additional

work in setting up the problem under asymmetric perturbations. The seemingly redundant members of a truss can significantly affect the load carrying capacity of the whole structure.

CHAPTER 4

DAMAGE DETECTION USING RESIDUAL ERROR METHOD

4.1 Introduction

Damage detection at the early stages of any structural design is a crucial issue to prevent unpredicted failures. Based on vibration analysis, many different methods have been developed to detect damage in the structure. The residual error method which is based on dynamic structural characteristic changes for free vibration analysis has been used. By observing the residual error in the equation of motion, the damage in one and two-dimensional structures can be detected and located.

4.2 Residual error method

We adopted residual error method for damage detection and location in plates. This method has been used by Brasiliano (2004) to identify damage on a continuous beam. In general, for free vibration with undamped system, the equation of motion expressed by matrix form as:

$$M\ddot{x} + Kx = 0 \quad (4.1)$$

where M and K are mass and stiffness matrices, respectively. $\ddot{x}$, x are acceleration and displacement vectors. Assuming:

$$x = X e^{i\omega t} \quad (4.2)$$

Then from eigenvalue problem solution we get

$$K X = M X \Lambda \quad (4.3)$$

The damage can be identified in structure by observing the error in the equation of motion as in (Brasiliano, 2004).

$$E_r = K_u X^* - (M_u X^*) \Lambda^* \quad (4.4)$$

in which K_u and M_u represent stiffness and mass matrices of the undamaged structure, respectively. Λ^* and X^* are the diagonal matrix of natural frequencies and mode shapes matrix of damaged structure, respectively. K_u and M_u are obtained from the FEM model of the undamaged structure. Λ^* and X^* are obtained from experimental measurements. We use FEM results that are obtained from damaged structure as data for damage detection.

$$\Lambda^* = \begin{bmatrix} \omega_1^{*2} & 0 & \cdots & 0 \\ 0 & \omega_2^{*2} & \cdots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_n^{*2} \end{bmatrix} \quad (4.5)$$

$$X^* = [X_1^* \ X_2^* \ X_3^* \ X_4^* \ \cdots \ X_n^*] \quad (4.6)$$

where E_r is the matrix represents the error, caused by damage in the equation of motion.

$$E_r = [e_1 \ e_2 \ e_3 \ \cdots \ e_n] \quad (4.7)$$

The method depends on the observation to the values of E_r matrix which contains several vector columns. Each column in E_r matrix represents one mode shape. The highest value

in E_r indicates the error present in some positions of the structure and gives an indication of the damage. The damage will be considered as stiffness reduction only as a factor multiplied by the undamaged stiffness matrix γ is between (0 and 1) It means the damaged element is $K_d = (1-\gamma)K_u$ of the undamaged element (Alnefaie, 2004). The residual error technique is applied on three kinds of structures: bar, beam, and plate.

4.2.1 Application on a bar (one dimensional structure)

The free vibration analysis of a bar with and without damage was performed. The modal quantities of the bar were numerically generated using finite element models without and with damage cases, with routines coded in MATLAB. Our study will consider applying residual error method on one dimension and two dimensional structures to detect and quantify damage. To implement the residual error, a numerical model with FEM needs to generate mass and stiffness matrices for the structure. After that, we can find eigen values (natural frequencies) and eigen vectors (displacement mode shape) from eigen value problem. The stiffness and mass matrices for the bar are (Pai, 2007):

$$\begin{bmatrix} m^i \end{bmatrix} = \frac{me_i}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} k^i \end{bmatrix} = \frac{EA}{e_i} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, i = 1 : n_e \quad (4.8)$$

e_i is the element length to subscript i which is the number of elements and m^i is the elemental mass with the assumption of constant mass m , k^i is the elemental stiffness matrix where the EA is constant. E is modulus of elasticity and A is the cross-section area. n_e is the number of elements (200 elements). The global matrix for mass and stiffness can be obtained by assembling the elemental mass and stiffness matrices.

Figure 4.1. Fixed-free ends bar geometry with its element

Table 4.1

Boundary conditions for bar

Bar Boundary Conditions		
End conditions	Fixed end	Free end
Along x-axis	$u(0, t) = 0$	$\frac{\partial u}{\partial x}(l, t) = 0$
Displacement= $u(x, t)$	$u(l, t) = 0$	$\frac{\partial u}{\partial x}(0, t) = 0$
Strain= $\frac{\partial u}{\partial x}(x, t)$	No displacement	No constrain
		No force

The exact natural frequency for the fixed-free bar can be determined by the equation below (Rao, 1995),

$$\omega_n = \frac{(2N+1)\pi}{2L} \sqrt{\frac{E}{\rho}} \quad (\text{rad/sec}), N = 1, 2, 3 \dots \infty, f = \frac{\omega_n}{2\pi} \text{ (Hz)} \quad (4.9)$$

4.2.2 Application on a beam (one dimensional structure)

The stiffness and mass matrices for Euler beam theory are (Pai, 2007), as:

$$[m^{(i)}] = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22l_i & 54 & -13l_i \\ 22l_i & 4l_i^2 & 13l_i & -3l_i^2 \\ 54 & 13l_i & 156 & -22l_i \\ -13l_i & -3l_i^2 & -22l_i & 4l_i^2 \end{bmatrix} \quad (4.10)$$

$$[k^{(i)}] = \frac{EI}{l_i^3} \begin{bmatrix} 12 & 6l_i & -12 & 6l_i \\ 6l_i & 4l_i^2 & -6l_i & 2l_i^2 \\ -12 & -6l_i & 12 & -6l_i \\ 6l_i & 2l_i^2 & -6l_i & 4l_i^2 \end{bmatrix} \quad (4.11)$$

where the mass m and EI are constant. The global matrix for mass and stiffness for beam can be obtained by assembling the elemental mass and stiffness matrices. The number of element is (80 elements).

Table 4.2

Boundary conditions for beam

Beam Boundary Conditions			
End conditions along x-axis	Simply-supported (pinned) end	Fixed(clamped) end	Free end
$Deflection = w$ $slope = \theta = \frac{\partial w}{\partial x}$ $Bending\ moment = EI \frac{\partial^2 w}{\partial x^2}$ $Shear\ force = \frac{\partial}{\partial x} \left(EI \frac{\partial^2 w}{\partial x^2} \right)$	$w = 0$ $EI \frac{\partial^2 w}{\partial x^2} = 0$	$w = 0$ $\theta = 0$	$\frac{\partial}{\partial x} \left(EI \frac{\partial^2 w}{\partial x^2} \right) = 0$ $EI \frac{\partial^2 w}{\partial x^2} = 0$

The exact natural frequencies ω_n in (Hz) for a cantilever (fixed free ends) beam can be calculated (Rao, 1995) as:

$$\omega_n = \sqrt{\frac{EI}{A\rho}} \frac{1}{2\pi} \frac{1}{l^2} Z^2 \quad (4.12)$$

where $Z = [1.875104, 4.694091, 7.854757, 10.99554, 14.13717, 17.279]$ for the first six modes of cantilever beam.

4.2.3 Application on plate (two dimensional structures)

In this study, we consider a rectangular perfectly flat thin plate. The plate is divided into 400 elements. Each element has 4 nodes with three degrees of freedom. The plate is isotropic material. The nodal degree (d.o.f) is $\{w, \alpha, \beta\}$ in each node as in Figure

4.2 (b), where $\alpha = w_y = \frac{\partial w}{\partial y}$ is rotation around x-axis and $\beta = -w_x = -\frac{\partial w}{\partial x}$ is rotation around y- axis. The derivation for mass and stiffness matrices is as in appendix C.

Figure 4.2 (a) Rectangular plate (b) Plate element

Table 4.3

Boundary conditions for plate

Boundary Conditions for simply supported plate				
w disp. in Z -axis	Edge conditions	Simply supported end	Clamped end	Free end
$\theta_y = -\frac{\partial w}{\partial x} = \beta$	Along x-axis	$w = \theta_y = 0$	$w = \theta_x = \theta_y = 0$	No constrain
θ_y rotation around y	Along y-axis	$w = \theta_x = 0$	$w = \theta_x = \theta_y = 0$	No constrain
$\theta_x = \frac{\partial w}{\partial y} = \alpha$				
θ_x rotation around x				

Exact natural frequency for simply supported plate (Nayfeh & Pai, 2008) is

$$\omega_n = \left[\frac{D\pi^4}{\rho h} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 + \frac{N_1 m^2 n^2}{\rho h a^2} \right]^{\frac{1}{2}} \quad (4.13)$$

where N_1 is the in-plane axial load in x-direction

$$\omega_n = \left[\frac{D\pi^4}{\rho h} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 \right]^{\frac{1}{2}} \quad (4.14)$$

For free vibration N_1 is neglected. a is the length in x direction and b is the length in y direction, m and n are the total numbers of half mode waves in x and y direction, respectively. D is the plate flexural rigidity.

$$D = \frac{Eh^3}{12(1-\mu^2)} \quad (4.15)$$

4.3 Results of residual error method

4.3.1 Result for the bar

Six damage scenarios are listed in Table 4.4. Case1 and case 2 were simulated for single damage by reducing the stiffness of an element near to 0.075L from the left end of the bar. The locations of damage in cases 3 and 4 are close to the mid-span of the bar. The cases 5 and 6 represent the multiple damage scenarios at two locations. The material and dimensions for an Aluminum 2024-T851 bar are listed in the Table 4.5. The six damage scenarios investigated are summarized in Table 4.6.

Table 4.4

Damage cases for bar

Damage Cases	Damaged Element	Damaged location	Damage d ratio
Case 1	15	0.075 L	0.1
Case 2	15	0.075 L	0.2
Case 3	99	0.495 L	0.1
Case 4	99	0.495 L	0.2
Case 5	15 & 99	0.075L & 0.495L	0.1
Case 6	15 & 99	0.075L & 0.495L	0.2

Table 4.5

Material and dimensions for bar

Modulus of elasticity E	$72.4e9 \text{ N} / \text{m}^2$
Poisson's ratio μ	0.33
Mass density ρ	$2780 \text{ Kg} / \text{m}^3$
Length L	1.5 m
Width b	0.04 m
Height h	0.02 m

Table 4.6

Six natural frequencies of fixed bar with different damage scenario

Mode number	Exact Hz	FEM Hz						
	No damage	No damage	Case1	Case2	Case3	Case4	Case4	Case6
	0	0	0.1	0.2	0.1	0.2	0.1	0.2
Mode 1	850.54	850.54	850.08	849.5	850.3	850	849.84	848.95
Mode 2	2551.6	2551.7	2550.4	2548.9	2551	2550.2	2549.8	2547.4
Mode 3	4252.7	4253	4251.3	4249.2	4251.7	4250	4250	4246.3
Mode 4	5953.8	5954.5	5952.9	5950.9	5953.2	5951.4	5951.5	5947.8
Mode 5	7654.9	7656.5	7655.3	7653.9	7653.9	7650.7	7652.8	7648.1
Mode 6	9356	9358.9	9358.4	9357.7	9356.9	9354.5	9356.4	9353.4

For each damage scenario, the dynamic characteristics (frequencies and mode shapes) before and after the damage were numerically evaluated with program coded in MATLAB. According to the results in the Table 4.6 with simulated damage scenarios, the natural frequency decreases in all cases and this reduction increases with the damage severity increases in the damaged elements of the bar. Changes in natural frequencies cannot provide information about the location of structural damage. This conclusion agrees with the observations in the studies carried out by (Kim et al. 2003).

(a)

(b)

(c)

(d)

Figure 4.3 Residual error detection for fixed-free bar: (a) 1st residual error mode with 10% damage at element 15 (b) 2nd residual error mode with 20% damage at element 99 in fixed bar (c) 1st residual error mode with 10% damage at elements 15 and 99 (d) 2nd residual error mode with 20% at elements 15 and 99.

The results in Figure 4.3 (a)-(d), show that the residual error method can detect and locate the damage for fixed- free ends bar. The damage detection is represented by a peak in the damage location. The residual error method can detect single damage at element 15 with 10% damage with respect to first mode as in Figure 4.3(a). The second residual error mode can detect the single damage at element 99 with 20% damage ratio. The method can detect multiple damages in two locations in 15 and 99 with different mode and damage severity level as in Figure 4.3 (c)-(d).

4.3.2 Results for the Beam

Four damage scenarios are investigated, as summarized in Table 4.7. The damage scenarios named C1 to C 2 were simulated for single damage by reducing the stiffness of an element near to 0.125 L from the left of the beam. The cases C3 and C4 correspond to a multiple damage scenario at two locations, one at 0.125 L and the second at the mid-span of the beam 0.5L. The material and dimensions for the cantilever beam are listed in the Table 4.8.

Table 4.7

Damage cases for beam

Damage cases	Damaged element	Damaged location	Damaged ratio
C 1	10	0.125 L	0.1
C 2	10	0.125 L	0.2
C 3	10 & 40	0.125 L& 0.5L	0.1
C 4	10 & 40	0.125 L& 0.5L	0.2

Table 4.9 shows that the natural frequency decreases with the damage severity in the beam of all damage scenarios in six modes. The results are compared with the undamaged FEM with the exact one. The reduction in natural frequency of beam with multiple damages is larger than single location as seen in cases C3 and C4 when damage simulated in elements 10 and 40 with 10% and 20% respectively. These results give good indication to detect damage even though the damage position cannot be located.

Table 4.8

Material and dimensions for beam

Modulus of elasticity E	$72.4e9 \text{ N} / \text{m}^2$
Poisson's ratio μ	0.33
Mass density ρ	$2780 \text{ Kg} / \text{m}^3$
Length L	2 m
Width b	0.05 m
Height h	0.05 m

Table 4.9

Six natural frequencies for cantilever beam with different damage scenarios

Mode number	Exact with no damage	FEM with no damage	C1	C2	C3	C4
	0	0	0.1	0.2	0.1	0.2
Mode1	10.305	10.305	10.285	10.26	10.281	10.252
Mode2	64.579	64.579	64.545	64.503	64.454	64.3
Mode3	180.82	180.82	180.82	180.81	180.82	180.81
Mode4	354.34	354.34	354.29	354.24	353.81	353.15
Mode5	585.75	585.75	585.41	584.99	585.4	584.97
Mode6	875.03	875.01	874.11	873	872.92	870.36

(a)

(b)

(c)

(d)

Figure 4.4 Residual error detection for cantilever beam: (a) 1st residual error with 10% damage at element 10(b) 1st residual error mode with 10% damage at elements 10 & 40(c) 2nd residual error mode with 10% damage at elements 10 and element 40 (d) 3rd residual error with 20% at elements 10 and 40.

As shown in Figure 4.4, the residual error method can detect and locate a single damage at element 10 with 10% for cantilever beam with respect to the first mode (first residual error). The first residual error method can detect multiple damages with 10% damage in elements number 10 and 40. Although the peak in element 40 is small in Figure 4.4(c), the damage can be detected with second residual error mode. Furthermore, the third residual error mode is better than the second error to display large peaks in the two locations as in Figure 4.4(d). In other words, the residual error method can detect and locate damage in beam using residual error of different modes, different locations and damage severity.

4.3.3 Results for the Plate

Six damage scenarios are investigated, as summarized in Table 4.10. The damage scenarios CP1 to CP2 were simulated for single damage by reducing the stiffness of an element in location $(0.2L, 0.1b)$ with respect to length and width of the plate. Case CP3 and CP4 simulated for damage in two locations. The cases CP5 and CP6 correspond to a multiple damage scenario at three locations. The cases listed with severity from 10% to 20% in the table. The material and dimensions for the simply supported plate listed in Table 4.11.

Table 4.10

Damage cases for plate

Cases	Damaged element	Damaged location	Damage ratio
CP 1	24	(0.2L,0.1 b)	0.1
CP 2	24	(0.2L,0.1 b)	0.2
CP 3	24 & 98	(0.2L,0.1b), (0.9L,0.25b)	0.1
CP 4	24 & 98	(0.2L,0.1b), (0.9L,0.25b)	0.2
CP 5	24,98 & 324	(0.2L,0.1b),(0.9L,0.25b)&(0.2L,0.85b)	0.1
CP 6	24,98 & 324	(0.2L,0.1b),(0.9L,0.25b)&(0.2L,0.85b)	0.2

Table 4.11

Material and dimensions for plate

Modulus of elasticity E	$72.4e9 \text{ N} / \text{m}^2$
Poisson's ratio μ	0.33
Mass density ρ	$2780 \text{ Kg} / \text{m}^3$
Length L	1 m
Width b	0.8 m
Height h	0.02 m

Figure 4.5. The geometry for undamaged rectangular meshed plate and damaged elements 24, 98 and 324.

Table 4.12

Six natural frequencies for simply supported plate with different damage scenarios

Mode- Shape	Exact natural frequency	FEM natural frequency	CP1	CP2	CP3	CP4	CP5	CP6
1	125.63	125.46	125.44	125.43	125.43	125.4	125.42	125.37
2	272.72	272.07	272.06	272.05	272.03	271.98	272	271.92
3	355.45	354.72	354.69	354.67	354.67	354.62	354.63	354.54
4	502.54	499.77	499.73	499.68	499.64	499.49	499.52	499.25
5	517.86	516.47	516.46	516.45	516.38	516.27	516.32	516.15
6	738.48	736.84	736.79	736.73	736.74	736.63	736.65	736.44

Table 4.12 shows the values of natural frequencies for six mode shapes for simply supported plate. The plate is simulated numerically using FEM with respect to six cases of damage scenarios. CP1 and CP2 are single damage scenarios in elements number 24 with damage severity 10% and 20% respectively. Multiple damages in two elements in 24 and 98 with damage severity 10% and 20% are in cases scenarios CP3 and CP4 and three element damages in 24, 98 and 324 with same amount of damages as in CP5 and CP6. Based on the results in the table, the natural frequencies decrease as the damage severity increase from 10% to 20% in all cases. The decreasing of the natural frequencies

with triple damages is more than that with single or two damages in the same damage severity.

The changes in natural frequencies for six mode shapes with six cases of damages are listed in Table 4.13. The damage in the plate causes the natural frequencies shift in values and they are different from mode to mode. The changes in natural frequencies in cases CP1 and CP2 with single damage location in the first mode shape is higher than the other modes. The cases CP3, CP4, CP5, and CP6 for multiple damage locations the higher modal frequencies changes occur in the fourth mode. This information about the changes in natural frequencies is not enough to provide information about the location of the damage in structures.

Table 4.13

Natural frequencies change ratio for six modes

Element number	Dama-mag Ratio %	Dama-mag. Cases	Natural frequency change ratio = $(\omega_h - \omega_d) / \omega_h \times 100$					
			Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
24	0.1	CP1	0.01594	0.00367	0.00845	0.00800	0.00193	0.00678
	0.2	CP2	0.02388	0.00735	0.01409	0.0180	0.00387	0.0149
24 & 98	0.1	CP3	0.02391	0.01470	0.01409	0.02601	0.01742	0.0135
	0.2	CP4	0.04782	0.03307	0.02819	0.04802	0.03872	0.02850
24,98& 324	0.1	CP5	0.031883	0.025729	0.025372	0.05002	0.02904	0.0257
	0.2	CP6	0.071730	0.055128	0.050744	0.10404	0.06195	0.0542

Figure 4.6 Damaged and undamaged mode shape for simply supported plate with 30% damage at elements 24, 98 and 324(a) first mode shape (b) second mode shape.

Figure 4.6 shows the undamaged and damaged first and second mode shape for simply supported beam with 30% of damage severity. The multiple damages are in the elements 24, 98 and 324. The difference between the undamaged and damaged mode shape is hardly noticeable in Figure 4.6 (a)-(b). The small changes cannot be used directly to locate the damage even with 30% in the plate structure.

We present the results of six natural frequencies for bar, beam and plate structures to show that when there is damage in the structure, the natural frequency decreases, and this reduction increases with the damage severity. This reduction gives indication for damage, but it is not enough to detect the location of the damage. In addition, we cannot depend on the shifting in the mode shapes to locate the damage. Both of natural frequencies reduction and mode shape shifting alone cannot detect the location of the damage.

(a)

(b)

Fig 4.7 Residual error detection on plate with damage in elements 24,98 and 324 (a) 1st residual mode with 5% damage(b) 2nd residual with 5% damage (c) 1st residual mode with 10%(d) 2nd residual mode with 10% damage (e) 1st residual mode with 20% damage (f) 2nd residual mode with 20% damage.

Figure 4.7 illustrates the results of residual error method on plate with multiple damages in elements 24, 98 and 324. This result shows that the first mode of residual error can detect and locate the damage precisely. It shows the ability of the method to detect even with small damage severity of 5% as in Figure 4.7(a) with respect to first residual error. Second residual error can detect the damage in the simulated location as a peak at the damaged elements even it is a small peak as in Figure 4.7 (b). As we can see in Figure 4.7(c)-(d), increasing the damage amount to 10% gives bigger peaks comparing with 5% damage. With 20% of damage in the simulated elements the peaks are clearer than 10% damage with respect to first and second residual error.

4.4 Comparative study by using mode shape curvature

4.4.1 Mode shape curvature for one dimension

Mode shape curvature method is one of the robustness methods that has been used to detect damage in structure (Pandey et al 1991; Brasiliano et al. 2004; Wahab et al 1999; Frans & Parung, 2017). The damage causes stiffness reduction in the local damaged place and consequently the curvature will increase. The method is based on the difference between the intact curvature mode shape and the damaged curvature mode to detect and locate the damage. The limitation of this method is the modal data (natural frequencies and mode shape) of the healthy structure should be available, and this is a hard task in some cases. There is a relation between the curvature and the flexural stiffness of the structure. The curvature and stiffness of a beam at any point are related by:

$$w''(x) = \frac{d^2 w(x)}{dx^2} = -\frac{M_b}{EI(x)} \quad (4.16)$$

where $w''(x)$ is the one-dimensional curvature mode shape which can be used for beam, $w(x)$ is the displacement mode shape E is the Young's modulus, I is the moment of inertia and M_b is the bending moment at section. $EI(x)$ is the bending stiffness. Numerically, the curvature can be found for beam from central finite difference approximation as:

$$w''_i = \frac{w_{i+1} - 2w_i + w_{i-1}}{h^2} \quad (4.17)$$

where w''_i is the curvature at point i , h is the distance between two successive nodes or measured points and w_i is the mode shape displacement at point i .

4.4.2 Mode shape curvature for two dimensions

The mode shape curvature for two dimensions can be represented for plate (Xu et al, 2015; Yang et al, 2016) as:

$$w''(x, y) = w''_x + w''_y = \frac{\partial^2 w(x, y)}{\partial x^2} + \frac{\partial^2 w(x, y)}{\partial y^2} \quad (4.18)$$

$$\text{where } \frac{\partial^2 w(x, y)}{\partial x^2} = -\frac{M_x(x, y)}{D}, \frac{\partial^2 w(x, y)}{\partial y^2} = -\frac{M_y(x, y)}{D} \quad (4.19)$$

$$D = \frac{Eh^3}{12(1 - \mu^2)} \quad (4.20)$$

where D is the bending stiffness of the plate and μ is the Poisson's ratio, $M_x(x, y)$ and $M_y(x, y)$ are the moments. Numerically the mode shape curvature for two dimensions can be written as:

$$w''(x, y) = \frac{w(x)_{i+1} - 2w(x)_i + w(x)_{(i-1)}}{h_x^2} + \frac{w(y)_{i+1} - 2w(y)_i + w(y)_{i-1}}{h_y^2} \quad (4.21)$$

$\Delta w_i''$ can be obtained by taking the absolute difference between the damage mode shape curvature $w_i''^d$ and the healthy mode shape curvature $w_i''^h$.

$$\Delta w_i'' = \left| w_i''^d - w_i''^h \right| \quad (4.22)$$

The damage detection will consider the absolute difference mode shape curvature ADMSC to detect and locate the damage in the plate.

4.5. Effect of noise

In any experimental testing, there would be some results deviation due to the measurements noise. The amount of noise depends on the complexity and accuracy of the instruments and devices. To evaluate the robustness of the damage detection methods in this study under effect of measurements noise. An artificial random noise is added to the simulated mode shape data to contaminate the data. The contaminated mode shapes are represented as k th component of the l th w_{lk} noisy eigenvector obtained from l th w_{lk}^0 free noise eigenvector (Rahai, 2007), as:

$$w_{lk} = w_{lk}^0 (1 + \psi_{lk}) \quad (4.23)$$

where ψ_{lk} is a Gaussian random number normal distribution with zero mean and standard deviation (1) to investigate the method proposed to sensitivity of noise level P , the equation above will be modified to:

$$w_{lk} = w_{lk}^0 (1 + P\psi_{lk}) \quad (4.24)$$

We insert two levels of noise p on simulated data of the two detection methods: residual error and (ADMSC) to check the capability of the residual error method detection compare with (ADMSC) method.

We use two noise levels, the first level $P1 = (2 \times 10^{-4})$ and the second level $P2 = (2 \times 10^{-3})$. The two levels of noise have been inserted on simulated data of the two detection methods in plate: residual error and ADMSC to check the capability of the residual error method detection compare with ADMSC method.

Figure 4. 8 10% damage at elements 24,98and 324 (a) first residual error with noise level 1(b) first (ADMSC) with noise level 1(c) first residual error with noise level 1(d) first (ADMSC) with noise level 2.

Figure 4.8 shows the comparative results between residual error method and the (ADMSC) method. The simulated data in the two methods are contaminated with noise in level 1(P1) and level 2 (P2). The first residual error can visibly detect and locate the damage in the three damage elements (24, 98 and 324) with noise in level 1. As a comparison to ADMSC method which gives good damage detection. However, both methods almost miss the detection when there is a high noise with level 2 as in Figure 4.8 (c)-(d).

When we use second mode to examine the comparative detection between the two methods, the second residual error and the second (ADMSC) can find the location of damage clearly with noise in level 1 as in Figure 4.9 (a)-(b).

The second residual error can detect damaged locations as the peaks appear clearer than the first residual error even with high level of noise (P2) as seen in Figure 4.9 (c)-d).

Figure 4.9 10% damage at elements 24,98and 324(a) second residual error with noise level 1(b) second (ADMSC) with noise level 1(c) second residual error with noise level 1(d) second (ADMSC) with noise level 2.

4.6. Applying the residual error method on plate with hole.

We assumed four elements with numbers (190, 191, 210 and 211) completely without stiffness and mass which means zero mass and stiffness in the center of the rectangular plate to be considered as a hole in the plate as in figure 4.10. We applied residual error method on plate with hole and with damage element number (65) with different two ratios of damage as shown in and figure 4.11 and figure 4.12. These figures show that the residual error can detect damage in the plate even though there is a hole in the plate with respect to two damage severity (10% and 20% damage).

Figure 4.10. Geometry for rectangular plate with hole in the center

Figure 4.11. Residual error with 10% damage at element (65) and four elements in center with zero mass and stiffness.

Figure 4.12. Residual error with 20% damage at element (65) and four elements in center with zero mass and stiffness.

4.7 Conclusion

The residual error method has been used to detect and locate damage in structures. The method is tested with different damage scenarios in bar, beam, and plate. The performance of the method is compared with ADMSC method for two dimensions to evaluate the robustness and limitation of the method. Noise has been added to the simulated mode shape to contaminate the data to represent the measurements errors.

From the results shown in this study, we conclude the residual error method can locate the damage in three structures for different damage scenarios. The method can detect and locate the damage in single and multiple locations while, in literature many methods can just detect single damage location.

In reality and experimental test the residual error method does not need the history data for natural frequency and mode shape; we only need the damaged values of natural frequencies and mode shapes. In contrary, ADMSC needs to know the history data of undamaged mode shapes and natural frequencies. Thus, the computation of residual error is much simpler.

The method can measure the evolution of the damage in the structure if there is continuous monitoring to the structure in its service duty based on the changes of damaged modal parameters.

The performance of the method works best when the damage severity is moderate (10% and 20%) or high severity of damage even with noise contamination. When there is a small severity of damage (5%), the method can detect damage in three locations if there

is no noise. When the detection fails due to the high noise level, the residual error of higher order mode can provide better detection.

The detection quality of method increases if we control and reduce the measurements noise. To get better results we recommend testing the structures with higher mode order when the lower mode do not works well.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

We present the analysis and simulation for bifurcation detection in symmetric structure specified by Von Mises and arch truss structure. The analysis is done with Gesa program and Ansys software. Gesa is coded in Matlab which is basically depending on using nonlinear finite element with static state analysis. In addition, we present damage detection methods specified by residual error method. The analysis is done with linear finite element method in dynamic state. The damage detection simulated in three kinds of structures, namely by bar, beam and plate. The following conclusions are supported by the analysis, results, and discussion of this dissertation.

5.1 Conclusions

1. The bifurcation point is identified as a saddle point in the load deflection curve of nonlinear geometric analysis in symmetric structures. Both Ansys and Gesa can detect the saddle bifurcation point and follow the loading path to the unstable branch of the load deflection curves.
2. Gesa and Ansys cannot detect symmetry breaking bifurcation unless small asymmetric perturbations in loading or in geometry are inserted in active node.
3. The load at the bifurcation point in the presence of perturbations is much lower than the ultimate load for the symmetric response. Missing the symmetry breaking

bifurcations can dangerously overestimate the loading carrying capacity of the structure.

4. The results for the shallow symmetric truss are more significant to structural design. We must carefully consider the structural stability in the presence of imperfections and asymmetric disturbances.
5. Although nonlinear finite element analysis packages can detect some bifurcation points, detections of symmetry breaking bifurcations require additional work in setting up the problem under asymmetric perturbations.
6. The seemingly redundant members of a truss can significantly affect the load carrying capacity of the whole structure.
7. The residual error method can locate the damage in three structures for different damage scenarios. The method can identify the damage in single or multiple locations.
8. Based on the comparison with the absolute difference mode shape curvature method (ADMSC) with noisy data, the residual error method has the detection capability in two levels of noise.
9. The performance of the method works best when the damage severity is moderate (10% and 20%) or high severity of damage even with noise contamination.
10. The residual error method is a good method, it can be used to detect and locate damage in one and two-dimensional structures. The detection quality of the method increases if we control and reduce the measurements noise.
11. To get better results we recommend testing the method with lower and higher mode order.

5.2 Recommendation for Future Work

Several recommendations for further work can be summarized in the following points:

1. The bifurcation analysis can be done for more complicated structures design.
2. Residual error method can be used as comparative method with other invented method to test the validation of the new method.
3. Experimental study need to be done with the theory of residual error method for future work.

REFERENCES

1. Worden, K., Farrar, C. R., Manson, G., & Park, G. (2007, June). The fundamental axioms of structural health monitoring. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* (Vol. 463, No. 2082, pp. 1639-1664). The Royal Society.
2. Mohan, S. C., Maiti, D. K., & Maity, D. (2013). Structural damage assessment using FRF employing particle swarm optimization. *Applied Mathematics and Computation*, 219(20), 10387-10400.
3. Pai, P. F. (2007). *Highly flexible structures: modeling, computation, and experimentation*. American Institute of Aeronautics and Astronautics.
4. Kala, Z. (2016). Computation of equilibrium paths in nonlinear finite element models. In *MATEC Web of Conferences* (Vol. 76, p. 04026). EDP Sciences.
5. Thompson, J. M. T. (1963). Basic principles in the general theory of elastic stability. *Journal of the Mechanics and Physics of Solids*, 11(1), 13-20.
6. Kim, N. H. (2014). *Introduction to nonlinear finite element analysis*. Springer Science & Business Media.
7. Pintea, A. (2012). Comparison between the Linear and Nonlinear Responses of Cable Structures I–Static Loading. *Civil Engineering and Architecture*, 55(2), 182-188.
8. Pecknold, D. A., Ghaboussi, J., & Healey, T. J. (1985). Snap-through and bifurcation in a simple structure. *Journal of Engineering Mechanics*, 111(7), 909-922.
9. Greco, M., & Vicente, C. E. R. (2009). Analytical solutions for geometrically nonlinear trusses. *Rem: Revista Escola de Minas*, 62(2), 205-214.
10. Jansen, M., Lombaert, G., & Schevenels, M. (2015). Robust topology optimization of structures with imperfect geometry based on geometric nonlinear analysis. *Computer Methods in Applied Mechanics and Engineering*, 285, 452-467.

11. Bazzucchi, F., Manuello, A., & Carpinteri, A. (2017 a). Interaction between snap-through and Eulerian instability in shallow structures. *International Journal of Non-Linear Mechanics*, 88, 11-20.
12. Wang, T., Xu, F., Huo, Y., & Potier-Ferry, M. (2018). Snap-through instabilities of pressurized balloons: Pear-shaped bifurcation and localized bulging. *International Journal of Non-Linear Mechanics*, 98, 137-144.
13. Wu, J., Frangopol, D. M., & Soliman, M. (2015). Geometry control simulation for long-span steel cable-stayed bridges based on geometrically nonlinear analysis. *Engineering Structures*, 90, 71-82.
14. Liu, S. T., & Long, Q. L. (2012). A new method tracing load-deflection equilibrium path of a doubly nonlinear truss. In *Applied Mechanics and Materials* (Vol. 166, pp. 68-72). Trans Tech Publications.
15. Liu, S. T., & Long, Q. L. (2012). A new method tracing load-deflection equilibrium path of a doubly nonlinear truss. In *Applied Mechanics and Materials* (Vol. 166, pp. 68-72). Trans Tech Publications.
16. Nistor, M., Wiebe, R., & Stanciulescu, I. (2017). Relationship between Euler buckling and unstable equilibria of buckled beams. *International Journal of Non-Linear Mechanics*, 95, 151-161.
17. Kala, Z. (2012). Geometrically non-linear finite element reliability analysis of steel plane frames with initial imperfections. *Journal of Civil Engineering and Management*, 18(1), 81-90.
18. Hrinda, G. A. (2007). Geometrically nonlinear static analysis of 3D trusses using the arc-length method. In *13th International Conference on Computational Methods and Experimental Measurements* (pp. 243-252).
19. Tsiatas, G. C., & Babouskos, N. G. (2017). Linear and geometrically nonlinear analysis of non-uniform shallow arches under a central concentrated force. *International Journal of Non-Linear Mechanics*, 92, 92-101.
20. Virgin, L. N., Guan, Y., & Plaut, R. H. (2017). On the geometric conditions for multiple stable equilibria in clamped arches. *International Journal of Non-Linear Mechanics*, 92, 8-14.

21. Gantes, C. J., Connor, J. J., Logcher, R. D., & Rosenfeld, Y. (1989). Structural analysis and design of deployable structures. *Computers & Structures*, 32(3-4), 661-669.
22. Degertekin, S. O., Saka, M. P., & Hayalioglu, M. S. (2008). Optimal load and resistance factor design of geometrically nonlinear steel space frames via tabu search and genetic algorithm. *Engineering Structures*, 30(1), 197-205.
23. Danso, L. A., & Karpov, E. G. (2017). Cusp singularity-based bistability criterion for geometrically nonlinear structures. *Extreme Mechanics Letters*, 13, 135-140.
24. Gioncu, V. (1994). General theory of coupled instabilities. *Thin-Walled Structures*, 19(2-4), 81-127.
25. Fu, C. J. (2013). Nonlinear Solution Formulation for Complex Space Truss. In *Advanced Materials Research* (Vol. 671, pp. 505-508). Trans Tech Publications.
26. Hu, N., & Burgueño, R. (2015). Buckling-induced smart applications: recent advances and trends. *Smart Materials and Structures*, 24(6), 063001.
27. Batra, R. C., Porfiri, M., & Spinello, D. (2007). Review of modeling electrostatically actuated microelectromechanical systems. *Smart Materials and Structures*, 16(6), R23.
28. Bazzucchi, F., Manuello, A., & Carpinteri, A. (2017 b). Instability load evaluation of shallow imperfection-sensitive structures by form and interaction parameters. *European Journal of Mechanics-A/Solids*, 66, 201-211.
29. Doherty, J. E. (1987). Nondestructive evaluation. *Handbook on experimental mechanics*, 1.
30. Zoidis, N., Tatsis, E., Vlachopoulos, C., Gotzamanis, A., Clausen, J. S., Aggelis, D. G., & Matikas, T. E. (2013). Inspection, evaluation and repair monitoring of cracked concrete floor using NDT methods. *Construction and Building Materials*, 48, 1302-1308.
31. Doebling, S. W., Farrar, C. R., & Prime, M. B. (1998). A summary review of vibration-based damage identification methods. *Shock and vibration digest*, 30(2), 91-105.

32. Koh, C. G., & Perry, M. J. (2009). *Structural Identification and Damage Detection using Genetic Algorithms: Structures and Infrastructures Book Series, Vol. 6*. Crc Press.
33. Heckman, V. M. (2014). *Damage detection in civil structures using high-frequency seismograms* (Doctoral dissertation, California Institute of Technology).
34. Rytter, A. (1993). *Vibrational based inspection of civil engineering structures* (Doctoral dissertation, Dept. of Building Technology and Structural Engineering, Aalborg University).
35. Kim, J. T., Ryu, Y. S., Cho, H. M., & Stubbs, N. (2003). Damage identification in beam-type structures: frequency-based method vs mode-shape-based method. *Engineering structures*, 25(1), 57-67.
36. Mousavi, M., & Gandomi, A. H. (2016). A hybrid damage detection method using dynamic-reduction transformation matrix and modal force error. *Engineering Structures*, 111, 425-434.
37. Yang, C., & Oyadiji, S. O. (2017). Damage detection using modal frequency curve and squared residual wavelet coefficients-based damage indicator. *Mechanical Systems and Signal Processing*, 83, 385-405.
38. Abdo, M. A. B. (2012). Damage detection in plate-like structures using High-Order mode shape derivatives. *International Journal of Civil and Structural Engineering*, 2(3), 792.
39. Abdo, M. A. B. (2014). Reliability of using curvature techniques in structural damage detection. *Journal of Engineering and Applied Sciences*, 1(1), 1-12.
40. Rucevskis, S., Akishin, P., & Chate, A. (2015). Numerical and Experimental Study on the Application of Mode Shape Curvature for Damage Detection in Plate-Like Structures. In *Solid State Phenomena* (Vol. 220, pp. 264-270). Trans Tech Publications.
41. Kumar, K. A., & Reddy, D. M. (2016, September). Application of frequency response curvature method for damage detection in beam and plate like structures. In *IOP Conference Series: Materials Science and Engineering* (Vol. 149, No. 1, p. 012160). IOP Publishing.

42. Rucevskis, S., Janeliukstis, R., Akishin, P., & Chate, A. (2016). Mode shape-based damage detection in plate structure without baseline data. *Structural Control and Health Monitoring*, 23(9), 1180-1193.
43. Bayissa, W. L., & Haritos, N. (2007). Structural damage identification in plates using spectral strain energy analysis. *Journal of Sound and Vibration*, 307(1-2), 226-249.
44. Radzieński, M., Doliński, Ł., Krawczuk, M., & Palacz, M. (2013). Damage localisation in a stiffened plate structure using a propagating wave. *Mechanical Systems and Signal Processing*, 39(1-2), 388-395.
45. Ghodrati, A., Seyed, R., & Bagheri, A. (2011). Damage detection in plates based on pattern search and genetic algorithms. *Smart Structures and Systems*, 7(2), 117-132.
46. Genovese, M. (2000). Localização e quantificação de danos em estruturas por meio de suas características dinâmicas. *MSc. Diss., PECC, Universidade de Brasília, Brasília, Brasil*.
47. Brasiliano, A., Doz, G. N., & de Brito, J. L. V. (2004). Damage identification in continuous beams and frame structures using the residual error method in the movement equation. *Nuclear Engineering and Design*, 227(1), 1-17.
48. Ligaro, S. S., & Valvo, P. S. (2006). Large displacement analysis of elastic pyramidal trusses. *International journal of solids and structures*, 43(16), 4867-4887.
49. <http://www.aerospacemetals.com>
50. Moaveni, S. (2011). *Finite element analysis theory and application with ANSYS*, 3/e. Pearson Education India.
51. ALNEFAIE, K. A. (2004). Damage Factor Assessment in Plate Structures Using Finite Element Method. *Engineering Sciences*, 15(2).
52. Rao, S. S., & Yap, F. F. (1995). Mechanical vibrations, vol. 4. *Reading: Addison-Wesley*.

53. Nayfeh, A. H., & Pai, P. F. (2008). *Linear and nonlinear structural mechanics*. John Wiley & Sons.
54. Kim, J. T., Ryu, Y. S., Cho, H. M., & Stubbs, N. (2003). Damage identification in beam-type structures: frequency-based method vs mode-shape-based method. *Engineering structures*, 25(1), 57-67.
55. Pandey, A. K., Biswas, M., & Samman, M. M. (1991). Damage detection from changes in curvature mode shapes. *Journal of sound and vibration*, 145(2), 321-332.
56. Wahab, M. A., & De Roeck, G. (1999). Damage detection in bridges using modal curvatures: application to a real damage scenario. *Journal of Sound and vibration*, 226(2), 217-235.
57. Frans, R., Arfiadi, Y., & Parung, H. (2017). Comparative study of mode shapes curvature and damage locating vector methods for damage detection of structures. *Procedia engineering*, 171, 1263-1271.
58. Xu, W., Cao, M., Ostachowicz, W., Radzieński, M., & Xia, N. (2015). Two-dimensional curvature mode shape method based on wavelets and Teager energy for damage detection in plates. *Journal of Sound and Vibration*, 347, 266-278.
59. Yang, Z. B., Radzienski, M., Kudela, P., & Ostachowicz, W. (2016). Two-dimensional modal curvature estimation via Fourier spectral method for damage detection. *Composite Structures*, 148, 155-167.
60. Rahai, A., Bakhtiari-Nejad, F., & Esfandiari, A. (2007). Damage assessment of structure using incomplete measured mode shapes. *Structural Control and Health Monitoring: The Official Journal of the International Association for Structural Control and Monitoring and of the European Association for the Control of Structures*, 14(5), 808-829.
61. Cook, R. D. (1994). *Finite element modeling for stress analysis*. Wiley.

Appendix A

Calculation the difference between Cauchy strain and Green strain

Based on our results in Figures 3.2 and 3.3, we found that, the coordinates of the ultimate point is (0.11, 6.98 e-3) which means the dimensionless displacement ($\frac{\Delta}{L}$) at the maximum load is equal to (0.11) when the original angle of Von Mises arch is $\alpha_0 = 15^\circ$ as in Figure A.1 and A.2 Based on reference (Danso and Eduard, 2017; Ligaro and Valvo, 2006). The assumption is Large nodal deformation and moderate strain should be < 0.05 with high Euler load to eliminate the buckling instability.

$$\text{Cauchy strain} = \varepsilon_c = \frac{l - L}{L} \quad (\text{A.1})$$

where L is the original length link and l is the deformed length link.

$$\text{Green strain} = \varepsilon_G = \varepsilon_c + \frac{1}{2} \varepsilon_c^2 \quad (\text{A.2})$$

$$\varepsilon_G = \frac{l - L}{L} + \frac{(l - L)^2}{L^2} \quad (\text{A.3})$$

$$= \frac{l - L}{L} + \frac{l^2 - 2lL + L^2}{2L^2} \quad (\text{A.4})$$

$$\Rightarrow \varepsilon_G = \frac{l^2 - L^2}{2L^2} \quad (\text{A.5})$$

Where the values of $L = 1$, $\Delta = 0.11$ and $\alpha_0 = 15^\circ$

$$y = L \sin \alpha_0 - \Delta \quad (\text{A.6})$$

$$l = \sqrt{y^2 + x^2} \quad (\text{A.7})$$

$$l = \sqrt{(L \sin \alpha_0 - \Delta)^2 + (L \cos \alpha_0)^2} \quad (\text{A.8})$$

$$\Rightarrow l = 0.9773227768$$

$$\varepsilon_G = \frac{l^2 - L^2}{2L^2} = \frac{(0.9773227768)^2 - (1)^2}{2(1)^2} = -0.0224$$

$$\text{and } \varepsilon_c = \frac{l - L}{L} = -0.02267$$

So the difference between the two strains (Cauchy strain and Green strain) is less than 0.3% and this is insignificant results on assumption of moderate strain.

Figure A.1 the geometry for Von Mises arch

Figure A.2 The geometry of half deformed Von Mises

Appendix B

Procedure to use Ansys for bifurcation analysis

For example, to obtain the results for symmetric curve in Fig B.1 and Fig B.2 in the dissertation, the following procedure should be considered in Ansys.

Figure. B.1

Figure. B.2

Ansys Methodology

1-Build up the model:

Assume the model contains from 7 element bars with five nodes. Using the steps in Ansys to build up the model starting from choosing nodes and the elements. Since we consider nonlinear geometry analysis alone, so the modeling for material is structural linear and isotropic. The element type is link180.

2- Apply loads and constrain on nodes.

Following the steps in Ansys, how to apply load on the model. Symmetric load is on node 4 and 5 respectively, as in Figure B.3.

Fig.B.3

3- From main Menu> Analysis type- new analysis-static.

4- To do nonlinear analysis should follow: Main menu> solution> Analysis type> sol'n controls-Basic.

The analysis options > large Displacement Static. The number of sub steps that chosen with (trial and error) in basic options is 50 for this specific analysis as in Figure B.4

Fig.B.4

From Sol'n options choose restart control (-1) and chose (write every Nth sub-step) with N=1000 as Figure.B.5

Fig.B.5

5- Main menu> solution> solve> current LS.

Fig.B.6

Choose for Equilibrium iterations, maximum number of iteration is (15) as in Fig.B.6

Fig.B.7

Activate the Arc length method options with maximum multiplier (1) and minimum multiplier (0.1), do not terminate the analysis as in Figure B. 7. The status for analysis as in Figure B.8.

Fig. B.8

The results for accumulative iteration number like in Figure B.9

Fig. B.9

6- Main Menu>TimeHist Postpro-click on green bottom in the screen as in Figure B.10

and then choose nodal solution> DOF solution >Y- Component as in Figure B. 11.

Fig. B. 10

Fig.B.11

Then click ok and then pick node 4 to plot and pick node 5. Transfer the results data to excel sheet to plot or to MATLAB as in Figure B.12 and Figure B.13, where the time is proportional to the load steps.

Fig B.12

Fig.B. 13

Appendix C

Derivation of mass and stiffness matrices for plate

Rectangular Plate element

Linear classical plate theory assumption:

1. The deflection of the mid plane is small compare to the thickness of the plate.
2. The mid plane of the plate stays unchanged and not deformed during bending.
3. Plain sections before and after bending keep normal to the mid plane and as a result the transverse normal strain ($\varepsilon_{33} = 0$) can be ignored.
4. The normal stress vanished in the transverse direction, i.e. ($\sigma_{33} = 0$), (Nayfeh and

Pai, 2008). In the case of plate flexure in which the deflections are small, the state of displacement for un arbitrary point p within element can be represented by three components: direction $u_1 = -z(\partial w / \partial x)$ in x direction and

$u_2 = -z(\partial w / \partial y)$ in y direction and lateral displacement $u_3 = w(x, y)$ in z

(Cook,1994) Hence the strain as:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} = \begin{Bmatrix} u_{1x} \\ u_{2y} \\ u_{1y} + u_{2x} \end{Bmatrix} = \begin{Bmatrix} -zw_{xx} \\ -zw_{yy} \\ -2zw_{xy} \end{Bmatrix} - z \begin{Bmatrix} \partial^2 / \partial x^2 \\ \partial^2 / \partial y^2 \\ 2\partial^2 / \partial x \partial y \end{Bmatrix} w \quad (C.1)$$

$$\varepsilon_{13} = \varepsilon_{23} = \varepsilon_{33} = 0$$

and the stress -strain relation for an isotropic material with plane stress as

$$[\sigma] = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} \quad (C.2)$$

we will use rectangular element with four nodes at each corner in thin plate flexure case, the element has three degrees of freedom at each node. Namely one is transverse deflection and two are rotations. The transverse displacement w in z -axis, rotation about x - axis is $\theta_x = \frac{\partial w}{\partial y} = \alpha$ and the rotation about y -axis is $\theta_y = -\frac{\partial w}{\partial x} = \beta$ as shown in Figure 1.b, the negative singe of θ_x corresponds to positive rotation about x - axis .The nodal displacement at each node (i) can be represented by $\{d\} = \{w_i \ \alpha_i \ \beta_i\}^T$ and each element has twelve degrees of freedom.

Fig.C.1 (a) Geometry plate element (b) Degree of freedom at each node in the element

Assume lower order polynomial function to represent

$$w(x, y, t) = w(\xi, \eta, t) = c_1 + c_2\xi + c_3\eta + c_4\xi^2 + c_5\xi\eta + c_6\eta^2 + c_7\xi^3 + c_8\xi^2\eta + c_9\xi\eta^2 + c_{10}\eta^3 + c_{11}\xi\eta^3 + c_{12}\xi^3\eta \quad (C.3)$$

The relation between Natural coordinates (ξ, η) and (x, y) Cartesian coordinates as:

$$\xi = \frac{x - (x_1 + x_2)/2}{a/2}, \eta = \frac{y - (y_2 + y_3)/2}{b/2} \Rightarrow dx = \frac{a}{2} d\xi, dy = \frac{b}{2} d\eta \quad (C.4)$$

$$w_x = \frac{2}{a} w_\xi \Rightarrow w_\xi = -\frac{a}{2} \beta \quad \text{and in the same manner} \quad w_y = \frac{2}{b} w_\eta \Rightarrow w_\eta = \frac{b}{2} w_y = \frac{b}{2} \alpha \quad (C.5)$$

Substitute the values of $w(\xi, \eta, t)$, $w_\xi(\xi, \eta, t)$ and $w_\eta(\xi, \eta, t)$ in each corner node with corresponding values and writing those in matrix form to get:

$$\begin{bmatrix} w(-1, -1, t) = w_1 \\ w_\eta(-1, -1, t) = \alpha_1 b / 2 \\ w_\xi(-1, -1, t) = -\beta_1 a / 2 \\ w(1, -1) = w_2 \\ w_\eta(1, -1, t) = \alpha_2 b / 2 \\ w_\xi(1, -1, t) = -\beta_2 a / 2 \\ w(1, 1) = w_3 \\ w_\eta(1, 1, t) = \alpha_3 b / 2 \\ w_\xi(1, 1, t) = -\beta_3 a / 2 \\ w(-1, 1) = w_4 \\ w_\eta(-1, 1, t) = \alpha_4 b / 2 \\ w_\xi(-1, 1, t) = -\beta_4 a / 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & -1 & -2 & 0 & 1 & 2 & 3 & 1 & -3 \\ 0 & 1 & 0 & -2 & -1 & 0 & 3 & 2 & 1 & 0 & -3 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 & -2 & 0 & 1 & -2 & 3 & 1 & 3 \\ 0 & 1 & 0 & 2 & -1 & 0 & 3 & -2 & 1 & 0 & -3 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 2 & 0 & 1 & 2 & 3 & 1 & 3 \\ 0 & 1 & 0 & 2 & 1 & 0 & 3 & 2 & 1 & 0 & 3 & 1 \\ 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 2 & 0 & 1 & -2 & 3 & 1 & -3 \\ 0 & 1 & 0 & -2 & 1 & 0 & 3 & -2 & 1 & 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} \quad (C.6)$$

$$\Leftrightarrow \{\bar{d}\} = [A]\{c\} \quad (C.7)$$

$$\{c\} = [A]^{-1} \{\bar{d}\} \quad (C.8)$$

$$\Rightarrow w(\xi, \eta, t) = \{1, \xi, \eta, \xi^2, \xi\eta, \eta^2, \xi^3, \xi^2\eta, \xi\eta^2, \eta^3, \xi^3\eta, \xi\eta^3\} \{c\} \quad (C.9)$$

Substitute Eq.(C.8) into Eq.(C.9) to get

$$\Rightarrow w(\xi, \eta, t) = \{1, \xi, \eta, \xi^2, \xi\eta, \eta^2, \xi^3, \xi^2\eta, \xi\eta^2, \eta^3, \xi^3\eta, \xi\eta^3\} [A]^{-1} \{\bar{d}\} \quad (C.10)$$

$$\{N\} = \{1, \xi, \eta, \xi^2, \xi\eta, \eta^2, \xi^3, \xi^2\eta, \xi\eta^2, \eta^3, \xi^3\eta, \xi\eta^3\} [A]^{-1} \quad (C.11)$$

where $\{N\}$ is the shape function

$$\Rightarrow w(\xi, \eta, t) = \{N_1, N_2, N_3, \dots, N_{12}\} \{\bar{d}\} \quad (C.12)$$

$$\equiv \{N\}^T \{\bar{d}\} = \{\bar{d}\}^T \{N\} \quad (C.13)$$

$$\text{where } \{\bar{d}\} = \text{diag}[1, b/2, -a/2, \dots, 1, b/2, -a/2] \{w_1, \alpha_1, \beta_1, \dots, w_4, \alpha_4, \beta_4\}^T \quad (C.14)$$

$$\equiv [S] \{d\} \quad (C.15)$$

$$\text{where } \{d\} = \{w_1, \alpha_1, \beta_1, \dots, w_4, \alpha_4, \beta_4\}^T \quad (C.16)$$

$$\text{and } [S] = \text{diag}[1, b/2, -a/2, \dots, 1, b/2, -a/2] \quad (C.17)$$

using the extended Hamilton principle, Pai (2007).

$\int_{t_1}^{t_2} (\delta T - \delta \Pi + \delta W_{nc}) dt = 0$ where T is the kinetic energy for the system and δT is the variation of the kinetic energy. Π is the elastic energy and $\delta \Pi$ is the variation of elastic energy. W_{nc} is the non-conservative due to external and damping forces and δW_{nc} is the variation to the non-conservative forces. t is the time.

The variation of kinetic energy is as:

$$\int_{t_1}^{t_2} \delta T dt = \int_{t_1}^{t_2} \delta \int_A \frac{1}{2} \rho \dot{w}^2 dx dy dt \quad (C.18)$$

$$= - \int_{t_1}^{t_2} \int_A \rho \ddot{w} \delta w dx dy dt \quad (C.19)$$

$$\delta T = - \int_A \rho \ddot{w} \delta w dx dy \quad (C.20)$$

$$= - \sum_{i=1}^{Ne} \int_{Ai} \{ \delta \bar{d} \}^T \{ N \} \rho \{ N \}^T \{ \ddot{d} \} J d\xi d\eta, \text{ where } dxdy = J d\xi d\eta, J = \frac{ab}{4} \quad (C.21)$$

where N_e is the number of element

$$= - \sum_{i=1}^{Ne} \int_{Ai} \{ \delta d \}^T [S]^T \{ N \} \rho \{ N \}^T [S] \{ \ddot{d} \} J d\xi d\eta \quad (C.22)$$

$$= - \sum_{i=1}^{Ne} \{ \delta d \}^T [m^{(i)}] \{ \ddot{d} \} \quad (C.23)$$

$$\text{where } [m^{(i)}] = [S]^T \int_{Ai} \{ N \} \rho \{ N \}^T [S] J d\xi d\eta \quad (C.24)$$

$[m^{(i)}]$ is the elemental mass matrix

$$= - \{ \delta D \}^T [M] \{ \ddot{D} \} \text{ where } [M] \text{ is the total mass matrix for the plate.} \quad (C.25)$$

To drive the stiffness matrix:

The variation of strain energy is as:

$$\delta \Pi = \int_V (\sigma_{11} \delta \varepsilon_{11} + \sigma_{12} \delta \varepsilon_{12} + \sigma_{12} \delta \varepsilon_{12}) dxdydz \text{ this is for thin plate} \quad (C.26)$$

$$= \int_V \{ \delta \varepsilon \}^T \{ \sigma \} dxdydz \quad (C.27)$$

$$= \int_V \{ \delta \varepsilon \}^T [Q] \{ \varepsilon \} dxdydz \quad (C.28)$$

$$\text{where } \{ \sigma \} = [Q] \{ \varepsilon \} \text{ and } [Q] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \quad (C.29)$$

$$w_{xx} = \frac{4}{a^2} w_{\xi\xi} = \frac{4}{a^2} \{N_{\xi\xi}\}^T \{\bar{d}\} = \{N_{xx}\}^T \{\bar{d}\} \quad (C.30)$$

$$w_{yy} = \frac{4}{b^2} w_{\eta\eta} = \frac{4}{b^2} \{N_{\eta\eta}\}^T \{\bar{d}\} = \{N_{yy}\}^T \{\bar{d}\} \quad (C.31)$$

$$w_{xy} = \frac{4}{ab} w_{\xi\eta} = \frac{4}{ab} \{N_{\xi\eta}\}^T \{\bar{d}\} = \{N_{xy}\}^T \{\bar{d}\} \quad (C.32)$$

$$\therefore \{\varepsilon\} = \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} = -z \begin{Bmatrix} \partial^2 / \partial x^2 \\ \partial^2 / \partial y^2 \\ 2\partial^2 / \partial x \partial y \end{Bmatrix} w = z \begin{Bmatrix} -w_{xx} \\ -w_{yy} \\ -2w_{xy} \end{Bmatrix} = \begin{Bmatrix} -\{N_{xx}\}^T \\ -\{N_{yy}\}^T \\ -2\{N_{xy}\}^T \end{Bmatrix} \{\bar{d}\} z \equiv [B][S][d]z \quad (C.33)$$

where $[B] = \begin{Bmatrix} -\{N_{xx}\}^T \\ -\{N_{yy}\}^T \\ -2\{N_{xy}\}^T \end{Bmatrix}$ is the derivative for the shape function

$$\delta \Pi = \sum_{i=1}^{Ne} \int_{Ai} \int_{-h/2}^{h/2} \{\delta d\}^T [S]^T [B]^T z^2 [Q][B][S] \{d\} dx dy dz \quad (C.34)$$

$$\delta \Pi = \sum_{i=1}^{Ne} \{\delta d\}^T [k^{(i)}] \{d\} = \{\delta D\}^T [K][D] \quad (C.35)$$

where $[K]$ is global stiffens matrix for the plate

$$[k^{(i)}] = \int_{Ai} [S]^T [B]^T [\Phi][B][S] dx dy \text{ is the elemental stiffens matrix} \quad (C.36)$$

$$= \int_{-1}^1 \int_{-1}^1 [S]^T [B]^T [\Phi][B][S] J d\xi d\eta \quad (C.37)$$

$$\text{where } [\Phi] \equiv \int_{-h/2}^{h/2} z^2 [Q] dz \quad (\text{C.38})$$

Since the case is free vibration so we do not need the variation of non-conservative energy

to drive δW_{nc} for the system.

$$\because \int_{t_1}^{t_2} (\delta T - \delta \Pi + \delta W_{nc}) dt = 0 \Rightarrow \int_{t_1}^{t_2} (\delta T - \delta \Pi) dt = 0 \quad (\text{C.39})$$

$$\int_{t_1}^{t_2} (\delta T - \delta \Pi) dt = 0 \Rightarrow - \int_{t_1}^{t_2} \{ \delta D \}^T ([M] \{ \ddot{D} \} + [K] \{ D \}) dt = 0 \quad (\text{C.40})$$

$$\Rightarrow \int_{t_1}^{t_2} (\delta T - \delta \Pi) dt = 0 \Rightarrow - \int_{t_1}^{t_2} \{ \delta D \}^T ([M] \{ \ddot{D} \} + [K] \{ D \}) dt = 0 \quad (\text{C.41})$$

$\Rightarrow [M] \{ \ddot{D} \} + [K] \{ D \} = 0$ is the equation of motion for the system with free vibration.

Appendix D

Matlab code for beam with residual error and curvature methods

```
% in case of free vibration

format short g; clear all; close all; clc;

%Input information for the beam -----

E=72.4e9; nu=0.33; rho=2780; %Young's modulus, Poisson's ratio, mass density of
Aluminum %2024-T851 (metric units) % from http://www.aerospacemetals.com

L=2; b=0.05; h=0.05; A=b*h; % beam length; width, thickness, and area of the cross
section

I=b*h^3/12; EI=E*I; m=rho*A; % area moment of inertia, bending rigidity, mass per unit
length

ne=80; % number of elements

xyz=[0:L/ne:L].'; % nodal x-coordinates

nn=size(xyz,1); % total number of nodes

ndf=2; % number of DOFs (degrees of freedom) per node (w, w')

ntt=nn*ndf; % total DOFs of the finite element model for beam

% Damage location element-----

ND=10;%the first damage element, ( ND =10)

NDD=45;%the second element damage, NDD =45( second element damage) as example

% Damage ratio at the element damage-----

RTO=0.2; RTOO=0.2;%%% damage ratio ( RTO with ND);(RTOO with NDD)

% Construct global [K], [M]-----

K=zeros(ntt,ntt); M=zeros(ntt,ntt); F=zeros(ntt,1); Kbar=zeros(ntt,ntt);

for i=1:ne

    e=xyz(i+1,1)-xyz(i,1); % elemental length

    e2=e^2;

    ke=(EI/e^3)*[12, 6*e, -12, 6*e; 6*e, 4*e2, -6*e, 2*e2; -12, -6*e, 12, -6*e; 6*e, 2*e2, -
6*e, 4*e2]; % elemental stiffness matrix
```



```

W=[0; 0; B]; % add the known nodal displacements (i.e., geometric BCs) back
Wu=[0; 0; Bu]; % add the known nodal displacements (i.e., geometric BCs) back
% To estimate the exact natural frequencies-----
z=[1.875104, 4.694091, 7.854757, 10.99554, 14.13717, 17.279]; % constant values %for
%cantilever beam
NFexact=(EI/m)^0.5/(2*pi)/L^2*z.^2; % exact natural frequencies (Hz)
NFexact=NFexact(1:6)'
% To insert noise for the mode shape data-----
N=size(W,1);
S=0.5*10^-4;scale=0.01;
rng(5); % to ensure the same random data in each run
noise1=S*randn(N,1);
noise2= S*randn(N,1);
w=W+noise1 .*W*scale ;wu=Wu+noise2 .*Wu*scale ;
xx= [0:ne]*e;
%----- plotting the mode shape
figure (1)
subplot(2,1,1) % mode shape without noise
plot(xx,W(1:2:ntt),'r-.',xx,Wu(1:2:ntt),'-k','linewidth',1.5)
hold ('on'), xlabel('Beam length'), ylabel('Third mode shape'),hold on
legend ('Damaged ','Undamaged ');hold on,

subplot (2,1,2) % modeshape with noise
plot(xx,w(1:2:ntt),'r-.',xx,wu(1:2:ntt),'-k','linewidth',1.5)
hold ('on'), xlabel('Beam length'), ylabel('Third mode shape'),hold on
legend ('Damaged with noise','Undamaged with noise'); hold on,
% applying the curvature mode shape-----
% applying finite difference to estimate Curvature mode shape
BB=W (1:2: ntt);BBu=Wu(1:2:ntt);BBn=w(1:2:ntt);BBun=wu(1:2:ntt);

```

```

VC=(BB(3: end)-2*(BB(2: end-1))+BB(1: end-2))/e^2;% damage curvature without
noise
VCu=(BBu(3: end)-2*(BBu(2: end-1))+BBu(1: end-2))/e^2;% undamage curvatre without
noise
VCn=(BBn(3: end)-2*(BBn(2: end-1))+BBn(1: end-2))/e^2;% damage curvature with
noise
VCun=(BBun(3: end)-2*(BBun(2: end-1))+BBun(1: end-2))/e^2;% undamage curvature
with %noise
VCC=[VC',0,0]; % No noise damaged curvature
VCCu=[VCu',0,0]; % No noise undamaged curvature
VCCn=[VCn',0,0]; % noise damaged curvature
VCCun=[VCun',0,0]; % noise undamged curvature
figure (2)
subplot (2,1,1)
plot (xx, VCC,'k-.',xx,VCCu,'r','linewidth',1.5);
hold ('on'), xlabel('Beam length'), ylabel('Curvature 3rd mode shape');hold on
legend ('Damaged 3rd curvature','Undamaged curvature'); hold on,
subplot (2,1,2)
plot (xx, VCCn,'k-.',xx,VCCun,'r','linewidth',1.5);
hold ('on'), xlabel('Beam length'), ylabel('Curvature 3rd mode shape');hold on
legend ('Damaged 3rd with noise ','Undamaged with noise'); hold on,
Cu1=abs ((VCCu)- (VCC)); % absolute difference curvature without noise
Cu1n=abs ((VCCun)- (VCCn)); % absolute difference curvature with noise
figure (3)
plot (xx, Cu1,'k','linewidth',1.5)% absolute curvature NO noise
hold on, xlabel('Beam length');ylabel('Absolute curvature');hold on , xlabel('Beam
length');ylabel('Absolute curvature 3rd')
title ('0.2 element damage 10& 0.5 element damage 40')
legend (' No noise')
figure (4)

```

```

plot (xx, Cu1n,'k','linewidth',1.5);xlabel('Beam length');
hold on, xlabel('Beam length');ylabel('Absolute curvature 3rd')
legend (' With noise')
% Applying residual error NO noise-----
figure (5);
Abar=D;phibar=V;
e= K*phibar- M*phibar*Abar;
ee=e(:,3);
E=ee(1:2:end-3);EE=[0;0 ;E] ;% take just the translation dof
plot (xx, EE,'k','linewidth',1.5);
hold on, legend ('10% damage C1');xlabel('Beam Length'),ylabel('Third Residual Error ')
title ('0.2 element damage 10& 0.5 element damage 40')

N= size(V,1);
S=0.5*10^-4; scale=0.01;
rng(5);
noise1=S*randn(N, N);
phibar0=V+ noise1.*V*scale;
e0= K*phibar0- M*phibar0*Abar;
ee0=e0(:,3);
Et=ee0(1:2: end-3); Et=[0;0; Et]; %just take translation degrees
figure (6)
plot (xx, Et,'k','linewidth',1.5);
xlabel('Beam length'), ylabel('Third residual error 3rd');legend (' With noise ')

```

VITA

Iqbal Abdullah Alshalal is from Iraq. She graduated from the University of Baghdad and earned B.S in mechanical engineering. After graduating, she worked in ministry of industry. During the time of writing this dissertation, she is PHD student in Mechanical and Aerospace Engineering in University of Missouri-Columbia. She earned master's degree in engineering in university of Missouri-Colombia in May 2017. She is already earned master's degree in mechanical engineering from University of Baghdad in Iraq in 2007. She is one of the faculties of University of Technology in Iraq. Her interest is on damage detection methods based on vibration analysis and bifurcation detection based on nonlinear analysis.