THE DYNAMICS OF SPIRAL BANDING IN A
SYMMETRIC VORTEX WITH VERTICAL SHEAR

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TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
II. THE THEORY	5
The Equations of Motion and Continuity	5
Perturbation Theory	5
The Problem	7
III. SUMMARY AND SUGGESTIONS	18
Summary	18
Suggestions	19
LIST OF REFERENCES	20

LIST OF FIGURES

FIGURE	PAGE
1. Spiral Rainbands in Hurricane Donna	2
2. Cylindrical Coordinates	8
3. Radial Form of Vertical Component of Perturbation Velocity .	15
4. Spiral Nature of Isolines of Constant Phase	17

CHAPTER I

INTRODUCTION

Spiral rain bands in tropical cyclones were revealed by Wexler (1947) through analysis of radar observations, and many investigators have directed their attention to them. The bands may now be easily observed through the use of radar (figure 1) and Tiros photographs. However, the mechanism responsible for this frequently observed phenomena is not as yet fully understood.

As a possible mechanism for the formation of the spiral bands, Haurwitz (1947), Wexler (1947) and Kuettner (1959) have suggested elongation of convection cells in the tangential direction by vertical wind shear. Further studies are needed to establish this hypothesis, since there is as yet no clear evidence for the existence of Bernard's cells in tropical cyclones.

Tepper (1958) proposed a hypothesis of internal gravity waves. However, a necessary condition for the existence of gravity waves is an inversion layer which has not been observed in tropical cyclones, except in the eye.

Krishnamurti (1961, 1962) solved numerically the Navier-Stokes equation for steady symmetric and moving asymmetric cyclones and obtained a few ring-shaped or spiral-shaped regions of vertical motion. He attributed the vertical motion in a steady symmetric cyclone to friction.

By observing that the mean crossing angle of radar rain bands

FIGURE 1

SPIRAL RAIN BANDS IN HURRICANE DONNA
(COURTESY U. S. DEPT. OF COMMERCE, WEATHER BUREAU)

with the isobars was the same as the wind flow at about 800 meters, Senn et al. (1957) suggested that it is the wind flow at this level which is responsible for the generation and the shape of the spirals.

Axi-symmetric motion propagating only along the rotational axis has been given the name elastoid waves by Bjerknes et al. (1933), while gravoid waves are those which propagate in the tangential direction. Yamamoto (1963) superimposed elastoid-gravoid perturbations upon a basic flow of a horizontal circular symmetric vortex and demonstrated the possibility for a banded or spiral region of upward motion by considering the steady state case with no vertical shear.

Stern (1960) has examined, theoretically, the stability of Ekman layer flows in rotating systems. The Ekman or friction layer is that layer of a fluid in which the motion is significantly influenced by surface friction. In the atmosphere, the Ekman layer extends 500 meters to 2000 meters above the surface. Stern has shown that the vertical shear in the Ekman layer is the driving mechanism or the source of energy for an increase in the kinetic energy of disturbances superimposed upon a basic rotational motion field.

Observations of axi-symmetric fluid flow by Arons et al. (1961) revealed a highly organized pattern of instability in the laminar Ekman flow, the principal features of which were predicted qualitatively by Stern. As described by Arons,

The instability manifests itself in the form of almost perfectly concentric sheets or curtains of fluid which rise as sharply defined vertical jets from the Ekman layer and penetrate the entire depth of the fluid forming circulation cells with less sharply defined downward motions in the region between the upward rising curtains.

Thus, although the entire depth of the fluid may be subject to banded disturbances, the origin and energy source for these disturbances is restricted to those layers with marked vertical shear of the undisturbed flow, principally the Ekman layer. In the case of the atmosphere, moist air in the Ekman layer may be lifted by the periodic vertical motion field to its lifting condensation level becoming visible upon condensation in the form of spiral clouds and rain bands. The release of latent heat in the regions of maximum lifting should serve to increase the vertical motion in these bands and accentuate the upward propagation of the disturbances into the otherwise undisturbed layers above.

Arons' observations and Stern's theoretical findings strongly suggest that the vertical shear in the Ekman layer is the dynamical mechanism for instability in the vertical motion field, which may manifest itself in the form of periodic motion in the radial direction. One might even suggest that without the presence of shear the banded structure might not arise.

It was these findings and considerations that led the present author to re-examine Yamamoto's work and attempt to retain the vertical shear terms rather than consider only the case of zero shear as did Yamamoto.

CHAPTER II

THE THEORY

I. THE EQUATIONS OF MOTION AND CONTINUITY

The equation of motion for a parcel of air in a frictionless atmosphere is

$$\frac{d\vec{v}}{dt} = -\alpha \vec{\nabla} p - 2\Omega \sin \phi \vec{k} \times \vec{v} - g \vec{k}, \quad (1)$$

where

$\vec{v}$ = velocity of parcel,

p = pressure,

t = time,

α = specific volume,

Ω = angular rotational velocity of the earth,

g = gravitational acceleration,

$\vec{k}$ = unit vector in the direction of the local vertical,

ϕ = latitude,

$\alpha \vec{\nabla} p$ = pressure gradient force,

$2\Omega \sin \phi \vec{k} \times \vec{v}$ = coriolis force.

The equation of continuity is

$$\frac{1}{\rho} \frac{d\rho}{dt} = -\vec{\nabla} \cdot \vec{v}, \quad (2)$$

where

ρ = density.

II. PERTURBATION THEORY

The above fundamental equations are nonlinear, i.e., they contain products of a dependent variable and a derivative, e.g., $(\vec{v} \cdot \vec{\nabla}) \vec{v}$

and $\vec{v} \cdot \vec{\nabla} \rho$. Moreover, the equation of motion is non-homogeneous due to the gravitational term.

The method of perturbations transforms nonlinear differential equations into linear homogeneous equations which, in principle, are more easily solved. Perturbation theory accomplishes this transformation by assuming that:

1. The total motion consists of a basic steady state flow upon which is superimposed small perturbations.
2. The basic flow and the total flow satisfy the fundamental equations, i.e., the equations of motion and continuity.
3. Products of perturbation quantities are small and may be neglected.

Let the total flow quantities be represented by starred letters, the basic flow quantities by capital letters or barred letters in the cases of α and ρ , and the perturbation quantities by small letters.

Then $\vec{v}^* = \vec{V} + \vec{v},$

$$p^* = P + p,$$

$$\alpha^* = \bar{\alpha} + \alpha,$$

and $\rho^* = \bar{\rho} + \rho.$

Substituting the total quantities into the fundamental equations and the basic quantities into the fundamental equations yields two sets of equations. Subtracting like equations and neglecting products of perturbation quantities gives the following perturbation equations:

$$\begin{aligned} \frac{\partial \vec{v}}{\partial t} + (\vec{V} \cdot \vec{\nabla}) \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{V} \\ = -\alpha \vec{\nabla} P - \bar{\alpha} \vec{\nabla} p - 2\vec{\Omega} \times \vec{v}, \end{aligned} \quad (3)$$

and $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\bar{\rho} \vec{v}) + \vec{\nabla} \cdot (\rho \vec{V}) = 0. \quad (4)$

As the basic field quantities are predetermined, the above equations are linear and homogeneous.

III. THE PROBLEM

In cylindrical coordinates (figure 2), the three components of the equation of motion for a frictionless fluid are the r-component,

$$\begin{aligned} \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \\ = -\alpha \frac{\partial p}{\partial r} + f v_\theta, \end{aligned} \quad (5)$$

the θ -component,

$$\begin{aligned} \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \\ = -\frac{\alpha}{r} \frac{\partial p}{\partial \theta} - f v_r, \end{aligned} \quad (6)$$

and the z-component,

$$\begin{aligned} \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \\ = -\alpha \frac{\partial p}{\partial z} - g, \end{aligned} \quad (7)$$

where

$$\begin{aligned} v_r &= \frac{dr}{dt} = \text{radial velocity component,} \\ v_\theta &= r \frac{d\theta}{dt} = \text{tangential velocity component,} \\ v_z &= \frac{dz}{dt} = \text{vertical velocity component,} \\ f &= 2\Omega \sin \phi = \text{coriolis parameter.} \end{aligned}$$

In the same notation, the equation of continuity in cylindrical coordinates is

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = \frac{1}{\rho} \frac{d\rho}{dt}. \quad (8)$$

FIGURE 2
CYLINDRICAL COORDINATES

Perturbation theory is now applied using the following model for the basic flow:

1. $V_n = 0$, $V_z = 0$ and $V_T = V_T(n, z)$.
2. Incompressible fluid or $\alpha = \rho = 0$ and $\bar{\alpha} = \frac{1}{\bar{\rho}} = \text{constant}$.
3. Hydrostatic equilibrium or $\frac{\partial p}{\partial z} = -\bar{\rho}g$.
4. Coriolis force is negligible compared with other terms since the model will be restricted to low latitudes (less than twenty degrees) and small radii (less than 150 kilometers).

With the above model, the resulting perturbation equations in cylindrical coordinates are the r -component of the equation of motion,

$$\frac{\partial v_n}{\partial t} + \frac{V_T}{n} \frac{\partial v_n}{\partial \theta} - 2 \frac{v_T V_T}{n} = -\bar{\alpha} \frac{\partial p}{\partial n}, \quad (9)$$

the θ -component of the equation of motion,

$$\frac{\partial v_T}{\partial t} + h v_n + \frac{V_T}{n} \frac{\partial v_T}{\partial \theta} + \eta v_z = -\frac{\bar{\alpha}}{n} \frac{\partial p}{\partial \theta}, \quad (10)$$

the z -component of the equation of motion,

$$\frac{\partial v_z}{\partial t} + \frac{V_T}{n} \frac{\partial v_z}{\partial \theta} = -\bar{\alpha} \frac{\partial p}{\partial z}, \quad (11)$$

and the perturbation equation of continuity,

$$\frac{\partial v_n}{\partial n} + \frac{v_n}{n} + \frac{1}{n} \frac{\partial v_T}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0, \quad (12)$$

where again the lower case letters refer to perturbation quantities, the upper case and barred letters refer to the basic flow components and where

$$h = \frac{\partial V_T}{\partial n} + \frac{V_T}{n} = \text{the vertical component of the vorticity,}$$

and
$$\eta = \frac{\partial V_T}{\partial z} = \text{the vertical shear of the basic tangential velocity field.}$$

In order to solve these equations, it is now assumed that the perturbations are elastoid-gravoid waves, i.e.,

$$v_n = A(n) e^{imz} e^{il\theta} e^{i\sigma t},$$

$$v_r = B(n) e^{imz} e^{il\theta} e^{i\sigma t},$$

$$v_z = C(n) e^{imz} e^{il\theta} e^{i\sigma t},$$

$$p = D(n) e^{imz} e^{il\theta} e^{i\sigma t},$$

where m , l , and σ are constants and l is an integer.

Then equations (9) - (12) become

$$i\sigma v_n + \frac{V_T}{n} i l v_n - 2 \frac{v_r v_r}{n} = -\bar{\alpha} \frac{\partial p}{\partial n}, \quad (13)$$

$$i\sigma v_r + \frac{h}{n} v_n + \frac{V_T}{n} i l v_r + \eta v_z = -\frac{\bar{\alpha}}{n} i l p, \quad (14)$$

$$i\sigma v_z + \frac{V_T}{n} i l v_z = -\bar{\alpha} i m p, \quad (15)$$

$$\frac{\partial v_n}{\partial n} + \frac{v_n}{n} + \frac{1}{n} i l v_r + i m v_z = 0. \quad (16)$$

From equation (15),

$$-\bar{\alpha} p = \frac{\sigma}{m} v_z + \frac{l}{m} \frac{V_T}{n} v_z. \quad (17)$$

From equation (16),

$$\frac{l}{n} v_r = -m v_z + i \frac{\partial v_n}{\partial n} + i \frac{v_n}{n}. \quad (18)$$

Substituting equations (17) and (18) into equation (14),

$$\begin{aligned} i\sigma \left(\frac{n}{il} \right) \left(-im v_z - \frac{\partial v_n}{\partial n} - \frac{v_n}{n} \right) + \frac{h}{n} v_n \\ + V_T \left(-im v_z - \frac{\partial v_n}{\partial n} - \frac{v_n}{n} \right) + \eta v_z \\ = \frac{il}{n} \left(\frac{\sigma}{m} v_z + \frac{l}{m} \frac{V_T}{n} v_z \right). \end{aligned} \quad (19)$$

Multiplying equation (19) by $-\frac{\ell}{n}$ and rearranging terms gives

$$\begin{aligned} \frac{\partial v_n}{\partial n} \left(\sigma + \frac{\ell V_T}{n} \right) + \frac{v_n}{n} \left(\sigma + \frac{\ell V_T}{n} - \ell h \right) \\ = -v_z \left[-\frac{\ell n}{n} + im \left(\sigma + \frac{\ell V_T}{n} \right) \left(1 + \frac{\ell^2}{m^2 n^2} \right) \right]. \quad (20) \end{aligned}$$

From equation (18),

$$\begin{aligned} -\bar{\alpha} \frac{\partial p}{\partial n} = \frac{\sigma}{m} \frac{\partial v_z}{\partial n} + \frac{\ell V_T}{mn} \frac{\partial v_z}{\partial n} \\ + \frac{\ell v_z}{mn} \frac{\partial V_T}{\partial n} - \frac{\ell V_T}{mn^2} v_z. \quad (21) \end{aligned}$$

Substituting equations (18) and (21) into equation (13) gives

$$\begin{aligned} i\sigma v_n + \frac{V_T}{n} i\ell v_n + \frac{2V_T}{i\ell} \left(imv_z + \frac{\partial v_n}{\partial n} + \frac{v_n}{n} \right) \\ = \frac{\sigma}{m} \frac{\partial v_z}{\partial n} + \frac{\ell V_T}{mn} \frac{\partial v_z}{\partial n} + \frac{\ell v_z}{mn} \frac{\partial V_T}{\partial n} - \frac{\ell V_T}{mn^2} v_z. \quad (22) \end{aligned}$$

Multiplying equation (22) by $\frac{i\ell}{n}$ and rearranging terms gives

$$\begin{aligned} \frac{\partial v_n}{\partial n} \left(\frac{2V_T}{n} \right) + \frac{v_n}{n} \left(2\frac{V_T}{n} - \ell\sigma - \frac{\ell^2 V_T}{n} \right) \\ = \frac{i\ell}{mn} \left(\sigma + \frac{\ell V_T}{n} \right) \frac{\partial v_z}{\partial n} \\ + i v_z \left[\frac{\ell^2}{mn^2} \left(\frac{\partial V_T}{\partial n} - \frac{V_T}{n} \right) - 2\frac{mV_T}{n} \right]. \quad (23) \end{aligned}$$

The previous equations may be simplified by confining ourselves to the case of steady state motion ($\sigma=0$) and assuming that the basic tangential wind profile is of the form

$$V_T = k(z) n^{-\frac{1}{2}}.$$

This profile of V_T has been observed to exist in several hurricanes, especially the more intense steady state variety, by Riehl (1963). In this case, the vertical component of the vorticity is

$$\zeta = \frac{\partial V_T}{\partial n} + \frac{V_T}{n} = \frac{V_T}{2n} ,$$

and
$$\frac{\partial V_T}{\partial n} - \frac{V_T}{n} = - \frac{3V_T}{2n} .$$

Hence, equations (20) and (23) become respectively

$$\frac{\partial v_n}{\partial n} + \frac{v_n}{n} \left(1 - \frac{1}{2}\right) = -v_z \left[-\frac{\eta}{V_T} + im \left(1 + \frac{\ell^2}{m^2 n^2}\right) \right] , \quad (24)$$

and
$$\frac{\partial v_n}{\partial n} + \frac{v_n}{n} \left(1 - \frac{\ell^2}{2}\right) = \frac{i\ell^2}{2mn} \frac{\partial v_z}{\partial n} + i v_z \left[-\frac{3\ell^2}{4mn^2} - m \right] , \quad (25)$$

while equations (17) and (18) become respectively

$$\bar{\alpha} p = - \frac{\ell V_T}{mn} v_z , \quad (26)$$

and
$$\frac{\ell}{n} v_T = -m v_z + i \left(\frac{\partial v_n}{\partial n} + \frac{v_n}{n} \right) . \quad (27)$$

In order to solve the set of equations (24) - (27) for the vertical motion field, the assumption is now made that $\ell = 1$. This limits us to a single band. Senn and Hiser (1959) give support to this assumption. They reported that frequently only one band from a given hurricane was found to be measurable for periods exceeding twenty to thirty minutes. Although several bands may be present at any one time, their lifetimes are of the order of ten to twenty minutes while there

is usually only a single band in the hurricane that may exist for periods of hours. It is this single band that should appear in the steady state solution.

With the above assumption, equations (24) and (25) may be equated to give the following:

$$\begin{aligned}
 & -v_z \left[-\frac{\eta}{V_T} + im \left(1 + \frac{1}{m^2 r^2} \right) \right] \\
 & = \frac{i}{2m\eta} \frac{\partial v_z}{\partial r} + iv_z \left(-\frac{3}{4m\eta^2} - m \right). \quad (28)
 \end{aligned}$$

Upon rearranging terms, we get the following differential equation in v_z :

$$\frac{\partial v_z}{\partial r} + v_z \left[\frac{1}{2r} + 2im\eta \frac{\eta}{V_T} \right] = 0. \quad (29)$$

It should be noted that $\frac{\eta}{V_T} = \frac{1}{V_T} \frac{\partial V_T}{\partial z}$ is independent of r since it has been assumed previously that $V_T = k(z) r^{-\frac{1}{2}}$. The solution to equation (29) is then

$$v_z = C r^{-\frac{1}{2}} e^{-im \frac{\eta}{V_T} r^2} e^{i\theta} e^{imz}, \quad (30)$$

where C is a constant since it has been assumed that v_z is of the form $v_z = C(r) e^{i\theta} e^{imz}$.

$\frac{\eta}{V_T}$ must also be equal to a constant in order for the final solution to agree with the initially assumed form. But $\frac{\eta}{V_T}$ is constant only if V_T is of the following specific form:

$$V_T = b r^{-\frac{1}{2}} e^{az}, \quad (31)$$

where a and b are real constants in order for V_T to have a physical meaning.

Imposing the boundary condition $v_z = 0$ at $z = 0$ then gives the following solution:

$$v_z = C \pi^{-\frac{1}{2}} e^{i(\theta - a m \pi^2)} \sin m z . \quad (32)$$

Substituting equation (32) into equation (24) or (25) and solving the differential equation for v_r gives

$$v_r = C \pi^{-\frac{1}{2}} e^{i\theta} \sin m z \left[A + (a - im) \int e^{-iam\pi^2} d\pi - \frac{i}{m} \int \pi^{-2} e^{-iam\pi^2} d\pi \right] . \quad (33)$$

Substitution of equation (32) into equation (26) gives

$$p = -\frac{Cb}{am} \pi^{-1} e^{az} e^{i(\theta - am\pi^2)} \sin m z . \quad (34)$$

Substitution of equations (24), (32) and (33) into equation (27) gives

$$v_r = C \pi^{-\frac{1}{2}} e^{i\theta} \sin m z \left\{ (ia\pi + \frac{1}{m\pi}) e^{-iam\pi^2} + \frac{i}{2} \left[A + (a - im) \int e^{-iam\pi^2} d\pi + \frac{i}{m} \int \pi^{-2} e^{-iam\pi^2} d\pi \right] \right\} , \quad (35)$$

where A is an arbitrary constant.

As an illustration of the form of v_z , figure 3 shows a plot of the real part of v_z , for real values of C , versus π for constant θ and z and for $am = \pm \frac{1}{L^2}$. The abscissa is in non-dimensional units of $R = \frac{\pi}{L}$ where L is the radius of maximum tangential velocity, i.e., the minimum radius for which the assumed radial form of

FIGURE 3
RADIAL FORM OF VERTICAL COMPONENT OF PERTURBATION VELOCITY

V_T may be expected to be accurate (for most hurricanes, L is approximately twenty to thirty kilometers).

The equation for phase isolines is

$$\Theta - am\pi^2 = k. \quad (36)$$

Figure 4 illustrates the spiral nature of the regions of equal phase. Four spirals are shown for different values of the constant k . The constant am has been chosen to be equal to $-\frac{1}{4L^2}$. It should be noted that there are a family of spirals with parameters am and k . The value of k for the line of maximum vertical motion depends upon the signs of the real and imaginary parts of C and upon whether the real or imaginary part of v_z applies to the particular physical situation. The parameter am determines the tightness of spiraling.

From equation (32), it is evident that if there is no vertical shear of the basic tangential wind, then $a = 0$ and

$$v_z = C \pi^{-\frac{1}{2}} e^{i\theta} \sin m\pi z. \quad (37)$$

For this case, there is no spiral banding present.

FIGURE 4

SPIRAL NATURE OF ISOLINES OF CONSTANT PHASE

CHAPTER III

SUMMARY AND SUGGESTIONS

I. SUMMARY

In summarization, perturbation theory applied to a circular symmetric vortex has been studied to reveal spiral banding of the vertical motion field in the presence of elastoid-gravoid waves. The vertical shear has been shown to play an essential role in the spiral banding feature of the present model. The radial form of V_r was chosen in agreement with the majority of profiles observed by Riehl (1963), but this choice of $r^{-\frac{1}{2}}$, together with the assumption of the elastoid wave, forced the limitation of the z -dependence of V_r to e^{az} . Although this form may not exist throughout the entire Ekman layer, it may be a good approximation in certain portions of the layer. The work of Stern (1960) and Arons (1961) has strongly suggested that the vertical shear in the Ekman layer is the energy source of wave instability and the present solutions admit no waves in the radial direction unless there is non-zero vertical shear.

The reader is reminded that the present solutions are for the steady state, and it is yet to be shown that this model represents the unstable waves, although the agreement with Arons' and Stern's work suggests that this is the case.

II. SUGGESTIONS

As a guide to future work, it is suggested that one or more of the following improvements to the present model may give a more general solution to the problem: (1) Retain the V_r terms with a more realistic form than a zero value; (2) Keep the coriolis terms in the equations so that the solution may be expected to hold at higher latitudes and larger radii; (3) Generalize the $r^{-\frac{1}{2}}$ dependency of V_r and attempt assumed forms of the perturbations other than elastoid-gravoids waves in order that one will not be forced to assume an exponential z -dependency of V_r ; (4) Attempt to solve the non-steady state case in order to determine the stability characteristics of the waves; and (5) Make more detailed observations of the velocity profiles in the tropical cyclone as well as additional quantitative measurements of the spiral bands in order to more accurately compare the theory to reality.

LIST OF REFERENCES

- Arons, A. B., A. P. Ingersoll, and T. Green III, 1961; "Experimentally Observed Instability of a Laminar Ekman Flow in a Rotating Basin," Tellus, Vol. 13, pp. 31-39.
- Bjerknes, V., et al., 1933; Physikalische Hydrodynamik, Springer, Berlin, pp. 429-450.
- Haurwitz, B., 1947; "Internal Waves in the Atmosphere and Convective Patterns," Ann. N. Y. Acad. Sci., Vol. 48, pp. 727-748.
- Krishnamurti, T. N., 1961; "On the Vertical Velocity Field in a Steady Symmetric Hurricane," Tellus, Vol. 13, pp. 171-180.
- Krishnamurti, T. N., 1962; "Some Numerical Calculations of the Vertical Velocity Field in Hurricanes," Tellus, Vol. 14, pp. 195-211.
- Kuettner, J., 1959; "The Banded Structure of the Atmosphere," Tellus, Vol. 11, pp. 267-294.
- Riehl, H., 1963; "Some Relations Between Wind and Thermal Structure of Steady State Hurricanes," J. atmos. Sci., Vol. 20, pp. 276-287.
- Senn, H. V. and H. W. Hiser, 1959; "A Note on the Origin of Hurricane Radar Spiral Bands and the Echoes Which Form Them," National Hurricane Project Pre-print Report No. 26, U. S. Dept. of Commerce, Weather Bureau.
- Senn, H. V., H. W. Hiser, and R. C. Bouvret, 1957; "Studies of Hurricane Spiral Bands as Observed on Radar," Final Report, Contract Cwb-9066, Marine and Radar Lab., Univ. of Miami, Fla., cited by Kuettner, J., 1959; "The Banded Structure of the Atmosphere," Tellus, Vol. 11, p. 280.
- Stern, M. E., 1960; "Instability of Ekman Flows at Large Taylor Numbers," Tellus, Vol. 12, pp. 399-417.
- Tepper, M., 1958; "A Theoretical Model for Hurricane Radar Bands," Proc. 7th Radar Conf., Miami Beach, Fla., Nov. 1958, cited by Yamamoto, R., 1963; "A Dynamical Theory of Spiral Rain Band in Tropical Cyclones," Tellus, Vol. 15, p. 155.
- Wexler, H., 1947; "Structure of Hurricanes as Determined by Radar," Ann. N. Y. Acad. Sci., Vol. 48, pp. 821-844.
- Yamamoto, R., 1963; "A Dynamical Theory of Spiral Rain Band in Tropical Cyclones," Tellus, Vol. 15, pp. 155-161.

The undersigned, appointed by the Dean of the Graduate Faculty, have
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