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Gauge-dependent cosmological ‘constant’

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Abstract

When the cosmological constant of spacetime is derived from the 5D induced-matter theory of gravity, we show that a simple gauge transformation changes it to a variable measure of the vacuum which is infinite at the big bang and decays to an astrophysically acceptable value at late epochs. We outline implications of this for cosmology and galaxy formation.

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1. Introduction

In Einstein’s theory of general relativity, the cosmological constant Λ is a fundamental parameter like the speed of light and the gravitational constant. It measures the energy density of the vacuum, and introduces a cosmological length scale of order 10^{28} cm based on current astrophysical data [1]. However, those same data imply that Λ could have been larger in the early universe, a possibility which has been the subject of numerous investigations; see, for example, [2] for recent reviews of some of the phenomenological as well as field-theoretical models involving variable cosmological ‘constants’. This possibility can be addressed as well using higher-dimensional gravitational theories. Extra dimensions have been employed by many authors in connection with issues involving the cosmological ‘constant’ (see, for example, [3] and the references cited therein). Such theories can also, in principle, help resolve the cosmological-constant problem, which is basically the mismatch between the small value of Λ derived from cosmological observations and the large value it should have as a measure of the vacuum fields of particle physics [4].

In this paper, we follow an approach based on the existence of an extra spacelike dimension, and we will use the canonical formalism of the induced-matter theory of gravity [5] to show that a simple gauge transformation involving the extra coordinate of 5D gravity changes Λ so that it is infinite at the big bang and decays to an astrophysically acceptable value at the present time. This suggests that Λ is a gauge-dependent measure of the energy

density of the vacuum, opening the way to a potential resolution of the cosmological-constant problem and helping with other astrophysical problems such as that of galaxy formation.

The two current versions of 5D gravity theory are membrane theory and induced-matter theory. In the former, gravity propagates freely (into the ‘bulk’), while the interactions of particle physics are confined to a hypersurface (the ‘brane’). Moreover, cosmological ‘constants’ may exist both in the bulk and on the brane. In the latter, there is no restriction on the dynamics except that provided by the geodesic equation, and matter is explained as a manifestation of the fifth dimension. The Ricci-flat condition is imposed on the 5D manifold; therefore, the only possible cosmological ‘constant’ is the one induced in 4D. Both theories involve conservation laws couched in 5D terms, which perforce means that the 4D laws are modified, resulting in a fifth force. The latter has been evaluated for the induced-matter approach [6] and the membrane approach [7], with compatible results. Also, it is now known that the field equations for these approaches are essentially the same [8]. However, if we wish to investigate the possibility that Λ is a measure of the energy density of a vacuum fluid, the most convenient formalism is the induced-matter one. We will therefore adopt this below, extending previous work on Λ which indicated a connection to particle mass [9] that has recently been the subject of renewed interest [10].

The induced-matter theory in its simplest form is the basic 5D Kaluza–Klein theory in which the fifth dimension is not compactified and the field equations of general relativity in 4D follow from the fact that the 5D manifold is Ricci-flat; the large extra dimension is thus responsible for the appearance of sources in 4D general relativity. In effect, the 4D world of general relativity is embedded in a five-dimensional Ricci-flat manifold; indeed, this is locally ensured by the Campbell theorem [11]. We assume in what follows that the extra dimension is spacelike.

An interesting result of the induced-matter theory is that if $ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$ is the 4D metric of any matter-free spacetime with a cosmological constant $\Lambda > 0$ in general relativity, then $dS^2 = (l/L)^2 ds^2 - dl^2$ with $L^2 = 3/\Lambda$ is the metric of a 5D manifold that is Ricci-flat [9]. Conversely, any 5D Ricci-flat metric of this canonical form corresponds to a matter-free spacetime with metric ds^2 and an effective cosmological constant $\Lambda = 3/L^2$. We are particularly interested in the question of the uniqueness of the latter correspondence. An important example is provided by the de Sitter solution of inflationary cosmology. In view of its basic significance, we first concentrate on this solution that is conformally flat in 4D and we write its metric tensor in the form $g_{\mu\nu}(x) = k(x)\eta_{\mu\nu}$. The explicit form of $k(x)$ is of no consequence for our discussion, since we are using the simple case of de Sitter spacetime to illustrate a rather general result. The question is then whether from the 5D standpoint the Ricci-flat metric $dS^2 = (l/L)^2 f(x, l)\eta_{\mu\nu} dx^\mu dx^\nu - dl^2$ has a unique solution for the function f . Clearly $f = k(x)$ is a possible solution, but it may not be the only solution. In fact, we find that in general $f = (1 - l_0/l)^2 k(x)$, where l_0 is an arbitrary constant. For $l_0 = 0$, we recover $f(x, l) = k(x)$; however, a novel situation arises in the generic case that $l_0 \neq 0$. This paper is devoted to a detailed derivation, interpretation and generalization of this result.

We work in 5D for the sake of simplicity; moreover, 5D theories are widely regarded as the low-energy limit of even higher-dimensional theories [12]. These include 10D supersymmetry, 11D supergravity and 26D string theory. These theories hold out the hope of unifying gravity with the interactions of particle physics, but our aim in what follows is to lay a solid 5D foundation.

The prospect of going from $\Lambda = \text{constant}$ to $\Lambda = \Lambda(\text{time, space})$ is an intriguing one [13]. However, it is also a fundamental shift from the way this parameter is viewed in general relativity. Therefore, in the next section we will not skimp the details of how we go from a constant Λ to a time-variable one; and we will be careful with our comments about

extending this to the space-variable case. A fundamental rethink of Λ can be justified for any higher-dimensional theory: the group of gauge changes (coordinate transformations) of the bigger space will necessarily affect the physics of the smaller space, if the change involves the higher coordinates. We will show how this works for a simple case in section 2. Those more interested in physics than mathematics will find a summary of our results in section 3.

2. A 5D gauge transformation that changes the 4D cosmological ‘constant’

We start with the 5D canonical metric of the induced-matter theory of gravity. Indeed, we will draw on previous work [5, 9] and use the same notation. (Lower-case Greek letters will run 0, 123 for time and space. Upper-case English letters will run 0, 123, 4 with $x^4 = l$ as the extra coordinate. Geometrical units will render the speed of light and the gravitational constant both unity.) Our aim is to show that the simple gauge transformation $l \rightarrow (l - l_0)$ changes the structure of the field equations significantly, taking the cosmological constant Λ from a true constant to an l -dependent parameter. The analysis will prove to be nontrivial (despite the simple nature of the gauge change). We will later confirm the result for $\Lambda \rightarrow \Lambda(l)$ by a less informative but quicker method.

The line element for the canonical metric can be written [9] in the form

$$dS^2 = \frac{l^2}{L^2} [g_{\alpha\beta}(x^\gamma, l) dx^\alpha dx^\beta] - dl^2. \quad (1)$$

This 5D element contains the 4D one $ds^2 = g_{\alpha\beta}(x^\gamma, l) dx^\alpha dx^\beta$. The l -dependence of the 4D metric tensor is necessary in order to preserve generality, since (1) uses all of the five available degrees of coordinate freedom to set the electromagnetic potentials ($g_{4\alpha}$) to zero and to set the scalar potential (g_{44}) to a constant. In general, our 4D physics takes place on a hypersurface of (1) specified by a value of l , about which particles do not wander freely but are constrained by the 5D geodesic equation (see below and [6, 7, 10]). The signature of (1) is (+ – – – –), since we have assumed a spacelike extra dimension. For this choice, the constant L in (1) is related to the cosmological constant Λ via $\Lambda = 3/L^2$; specifically, if $\partial g_{\alpha\beta}/\partial l = 0$, then the Ricci-flat requirement in 5D reduces to $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$ in 4D. This result has been known for a decade, and follows from the field equations. The latter, in terms of the Ricci tensor, are $R_{AB} = 0$ ($A, B = 0, 123, 4$). These 15 relations can always be written as one wave equation, four conservation equations and ten Einstein equations [5]. The latter, in terms of the Einstein tensor and the energy–momentum tensor, are $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ ($\alpha, \beta = 0, 123$). The source tensor may contain parts due to ‘ordinary’ matter and parts due to the ‘vacuum’, and we will see below that the second of these depends critically on the fifth dimension. That is, in the *general* case, the physics of the 4D vacuum which follows from (1) depends critically on the choice of the 5D gauge.

Here, we look at a *special* but physically instructive case of (1). That metric is general, so to make progress we need to apply some physical filter to it. Now, the physics of the early universe is commonly regarded as related to inflation; and the standard 4D metric for this is that of de Sitter, where $ds^2 = dt^2 - \exp[2\sqrt{\Lambda/3}t] d\sigma^2$. (Here $d\sigma^2 \equiv dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$ in spherical polar coordinates.) The physics flows essentially from the cosmological constant Λ . However, it is well known that the de Sitter metric is conformally flat. This suggests that physically relevant results in 4D may follow from metric (1) in 5D if the latter is restricted to the conformally flat form:

$$dS^2 = \frac{l^2}{L^2} [f(x^\gamma, l) \eta_{\alpha\beta} dx^\alpha dx^\beta] - dl^2. \quad (2)$$

Here $\eta_{\alpha\beta} = \text{diagonal } (+1 - 1 - 1 - 1)$ is the metric for flat Minkowski space. We are particularly interested in the l -dependence of $f(x^\gamma, l)$. To determine the latter, we need to solve the field equations.

The components of the 5D Ricci tensor for the general metric (1) are

$${}^{(5)}R_{44} = -\frac{\partial A^\alpha_\alpha}{\partial l} - \frac{2}{l}A^\alpha_\alpha - A_{\alpha\beta}A^{\alpha\beta}, \quad (3a)$$

$${}^{(5)}R_{\mu 4} = A_{\mu;\alpha}^\alpha - \frac{\partial \Gamma_{\mu\alpha}^\alpha}{\partial l}, \quad (3b)$$

$${}^{(5)}R_{\mu\nu} = {}^{(4)}R_{\mu\nu} - S_{\mu\nu}, \quad (3c)$$

where $S_{\mu\nu}$ is a symmetric tensor given by

$$S_{\mu\nu} \equiv \frac{l^2}{L^2} \left[\frac{\partial A_{\mu\nu}}{\partial l} + \left(\frac{4}{l} + A^\alpha_\alpha \right) A_{\mu\nu} - 2A_{\mu}^\alpha A_{\nu\alpha} \right] + \frac{1}{L^2} (3 + lA^\alpha_\alpha) g_{\mu\nu}. \quad (4)$$

Here ${}^{(4)}R_{\mu\nu}$ and $\Gamma_{\nu\rho}^\mu$ are, respectively, the 4D Ricci tensor and the connection coefficients constructed from $g_{\alpha\beta}$. Moreover

$$A_{\alpha\beta} \equiv \frac{1}{2} \frac{\partial g_{\alpha\beta}}{\partial l}, \quad (5)$$

where $A_\alpha^\beta = g^{\beta\delta} A_{\alpha\delta}$, and the semicolon in equation (3b) represents the usual 4D covariant derivative. We need to solve (3) in the form $R_{AB} = 0$, subject to putting $g_{\mu\nu}(x^\gamma, l) = f(x^\gamma, l)\eta_{\mu\nu}$ as in (2), which ensures (4D) conformal flatness. We note that $g^{\mu\nu} = \eta^{\mu\nu}/f$ and $A_{\mu\nu} = f'\eta_{\mu\nu}/2$, where $f' \equiv \partial f(x^\gamma, l)/\partial l$. Also, $A^{\alpha\beta} = f'\eta^{\alpha\beta}/(2f^2)$, $A^\alpha_\alpha = 2f'/f$ and $A^\alpha_\beta = f'\eta^\alpha_\beta/(2f)$. Then the scalar component of the field equation (3a) becomes

$$2\frac{\partial}{\partial l} \left(\frac{f'}{f} \right) + \left(\frac{f'}{f} \right)^2 + \frac{4}{l} \left(\frac{f'}{f} \right) = 0. \quad (6)$$

To solve this, we define $U \equiv f'/f + 2/l$. Then (6) is equivalent to $2U' + U^2 = 0$, or $\partial(U^{-1})/\partial l = 1/2$, so on introducing an arbitrary function of integration $l_0 = l_0(x^\gamma)$ we obtain $U^{-1} = [l - l_0(x^\gamma)]/2$. This, in terms of the original function f , means that $f'/f + 2/l = U = 2/[l - l_0(x^\gamma)]$, or $\partial[\ln(l^2 f)]/\partial l = \partial\{\ln[l - l_0(x^\gamma)]^2\}/\partial l$. This gives $l^2 f/[l - l_0(x^\gamma)]^2 = k(x^\gamma)$, where $k = k(x^\gamma)$ is another arbitrary function of integration. We have noted this working to illustrate that the solution of the scalar component of the field equations (3a) or (6) involves an arbitrary length $l_0(x^\gamma)$ and an arbitrary dimensionless function $k(x^\gamma)$. Here, the solution for the conformal factor in the metric $g_{\mu\nu}(x^\gamma, l) = f(x^\gamma, l)\eta_{\mu\nu}$ is

$$f(x^\gamma, l) = \left[1 - \frac{l_0(x^\gamma)}{l} \right]^2 k(x^\gamma) \quad (7)$$

and involves both arbitrary functions.

However, one of these is actually constrained by the vector component of the field equations (3b). To see this we note that $A_{\mu\nu}$ of (5) is symmetric, and it is a theorem that then

$$A_{\nu;\mu}^\mu = A^\mu_{\nu;\mu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} A^\mu_\nu) - \frac{A^{\alpha\beta}}{2} \frac{\partial g_{\alpha\beta}}{\partial x^\nu}. \quad (8)$$

Here g is the determinant of the 4D metric, so since $g_{\mu\nu} = f\eta_{\mu\nu}$ we have $\sqrt{-g} = f^2$. Then using (8), equation (3b) becomes

$$\frac{1}{2f^2} \frac{\partial}{\partial x^\mu} (f f' \delta^\mu_\nu) - \frac{f'}{f^2} \frac{\partial f}{\partial x^\nu} = \frac{\partial}{\partial l} (\Gamma_{\nu\alpha}^\alpha). \quad (9)$$

The right-hand side of this can be re-expressed using the identity $\Gamma_{\nu\alpha}^\alpha \equiv (\sqrt{-g})^{-1}\partial(\sqrt{-g})/\partial x^\nu$, whence (9) becomes

$$\frac{1}{2f^2} \frac{\partial}{\partial x^\nu} (ff') - \frac{f'}{f^2} \frac{\partial f}{\partial x^\nu} = 2 \frac{\partial}{\partial l} \left[\frac{1}{f} \frac{\partial f}{\partial x^\nu} \right]. \quad (10)$$

In this form, we can multiply by $2f^2$ and rearrange to obtain

$$f \frac{\partial f'}{\partial x^\nu} = f' \frac{\partial f}{\partial x^\nu}. \quad (11)$$

Dividing by $ff' \neq 0$, we find

$$\frac{\partial}{\partial x^\nu} \left(\frac{f'}{f} \right) = 0. \quad (12)$$

But the term in parentheses here, by (7), is $f'/f = 2l_0(x^\gamma)/\{l[l - l_0(x^\gamma)]\}$. Thus (12) implies that $l_0(x^\gamma) = l_0$ and is constant. We have noted this working to illustrate that the scalar and vector components of the field equations (3a) and (3b) together yield the conformal factor

$$f(x^\gamma, l) = \left(1 - \frac{l_0}{l} \right)^2 k(x^\gamma), \quad (13)$$

which involves only one ‘arbitrary’ function that is easy to identify: if the constant parameter l_0 vanishes, then $k\eta_{\mu\nu}$ is simply our original de Sitter metric tensor.

The tensor component of the field equations (3c) does *not* further constrain the function $k(x^\gamma)$. However, we need to work through this component in order to isolate the 4D Ricci tensor ${}^{(4)}R_{\mu\nu}$ and so obtain the effective cosmological constant. To do this, we need to evaluate $S_{\mu\nu}$ of (4). The working for this is straightforward but tedious. The result is simple, however:

$$S_{\mu\nu} = \frac{3}{L^2} k(x^\gamma) \eta_{\mu\nu}. \quad (14)$$

By the field equations (3c) in the form ${}^{(5)}R_{\mu\nu} = 0$, this means that the 4D Ricci tensor is also equal to the right-hand side of (14). We recall that our (4D) conformally flat spaces (2) have $g_{\mu\nu} = f(x^\gamma, l)\eta_{\mu\nu} = (1 - l_0/l)^2 k(x^\gamma)\eta_{\mu\nu}$ using (13). Thus $k(x^\gamma)\eta_{\mu\nu} = l^2 g_{\mu\nu}/(l - l_0)^2$ and

$${}^{(4)}R_{\mu\nu} = \frac{3}{L^2} \frac{l^2}{(l - l_0)^2} g_{\mu\nu}. \quad (15)$$

This is equivalent to the Einstein field equation for the de Sitter metric tensor $k\eta_{\mu\nu}$, since under a constant conformal scaling of a metric tensor, the corresponding Ricci tensor remains invariant. Nonetheless, (15) defines an Einstein space ${}^{(4)}R_{\mu\nu} = \Lambda g_{\mu\nu}$ with an effective cosmological constant given by

$$\Lambda = \frac{3}{L^2} \left(\frac{l}{l - l_0} \right)^2. \quad (16)$$

This is our main result; for $l_0 = 0$, Λ reduces to the de Sitter cosmological constant.

It differs from the ‘standard’ one $\Lambda = 3/L^2$, which is obtained by reducing the 5D field equations to the 4D Einstein equations for a pure-canonical metric in which the 4D metric tensor does not depend on the extra coordinate l [9]. The difference between the results is mathematically modest, but can be physically profound, because (16) admits the possibility that $\Lambda \rightarrow \infty$ for $l \rightarrow l_0$. We will return to this below, but here we wish to make some comments about the nature of (16).

The (4D) conformally flat metric we are considering here and the pure-canonical metric considered by other workers [5, 6, 8–10] have 5D line elements given respectively by

$$dS^2 = \frac{(l - l_0)^2}{L^2} k(x^\gamma) \eta_{\alpha\beta} dx^\alpha dx^\beta - dl^2, \quad (17a)$$

$$dS^2 = \frac{l^2}{L^2} g_{\alpha\beta}(x^\gamma) dx^\alpha dx^\beta - dl^2. \quad (17b)$$

Clearly the two are compatible, and the second implies the first if we shift $l \rightarrow (l - l_0)$ and write $g_{\alpha\beta}(x^\gamma) = k(x^\gamma) \eta_{\alpha\beta}$. Of course, we can always make the (apparently trivial) coordinate transformation or gauge change $l \rightarrow (l - l_0)$. This leaves the extra part of the canonical metric (17b) unchanged, while the prefactor on the 4D part changes from l^2/L^2 to $(l - l_0)^2/L^2 = (l^2/L^2) [(l - l_0)/l]^2$.

Let us now replace $k(x^\gamma) \eta_{\alpha\beta}$ in (17a) by a generic metric tensor $g_{\alpha\beta}(x^\gamma)$ and write $\bar{g}_{\alpha\beta} = [(l - l_0)/l]^2 g_{\alpha\beta}$. Then we obtain a line element which looks like (17b) except that $g_{\alpha\beta}$ has been replaced by $\bar{g}_{\alpha\beta}$. Now it is a theorem that solutions of the source-free 5D field equations $R_{AB} = 0$ with metric (17b) satisfy the source-free 4D field equations $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$ with $\Lambda = 3/L^2$ [9]. Therefore, the same must hold with $R_{\alpha\beta} = R_{\alpha\beta}(\bar{g}_{\alpha\beta}) = \bar{\Lambda} \bar{g}_{\alpha\beta}$ and $\bar{\Lambda} = (3/L^2) l^2 / (l - l_0)^2$, since a constant conformal transformation of the metric leaves the Ricci tensor invariant. This is identical to (16) above. Put another way: a translation along the l -axis preserves the form of the canonical metric, and since the 5D field equations are covariant we obtain again the 4D field equations, but with a different cosmological constant.

This is an elementary example of a situation that has been alluded to before in the literature [5 p 125]: 5D quantities $Q = Q(x^A)$ are preserved under $x^A \rightarrow \bar{x}^A(x^B)$, but 4D quantities $q = q(x^\gamma, l)$ will in general not be if the gauge change involves $x^4 = l$. The situation is analogous to that in quantum field theory, where a choice of gauge (in some cases even a non-covariant one) is necessary in order to calculate physical quantities. In the present case, we have two mathematically acceptable metrics which have physically different cosmological constants: (17a) has $\Lambda = (3/L^2) l^2 / (l - l_0)^2$ and (17b) has $\Lambda = 3/L^2$. The latter is standard, insofar as Λ is a true constant, which with its astrophysically indicated size implies $L \simeq 1 \times 10^{28}$ cm [1]. The former is non-standard, because Λ is expected to change as l changes, and can indeed be unbounded for a certain value of the extra coordinate. On the other hand, if physics takes place on a hypersurface of constant l , then Λ is constant in any case; however, in the induced-matter theory the observed value of Λ depends on the evolution in 5D as determined by the geodesic equation.

To investigate this in more detail, we will adopt the approach used elsewhere, in which $l = l(s)$ is given by a solution of the 5D geodesic equation [5, 9, 10]. To shorten the present discussion, we note that 5D geodesic motion generally implies departures from 4D geodesic motion (the pure-canonical metric is an exception), and that 5D null paths can correspond to 4D timelike paths (so a higher-dimensional ‘photon’ can appear as a massive particle in spacetime). To proceed, we return to the general form of metric (1), for which the 5D geodesic equation splits naturally into a 4D part and an extra part:

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = f^\mu, \quad f^\mu \equiv \left(-g^{\mu\alpha} + \frac{1}{2} \frac{dx^\mu}{ds} \frac{dx^\alpha}{ds} \right) \frac{dl}{ds} \frac{dx^\beta}{ds} \frac{\partial g_{\alpha\beta}}{\partial l}, \quad (18a)$$

$$\frac{d^2 l}{ds^2} - \frac{2}{l} \left(\frac{dl}{ds} \right)^2 + \frac{l}{L^2} = -\frac{1}{2} \left[\frac{l^2}{L^2} - \left(\frac{dl}{ds} \right)^2 \right] \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} \frac{\partial g_{\alpha\beta}}{\partial l}. \quad (18b)$$

In these, following (17a) and the preceding discussion of metrics, we substitute

$$g_{\alpha\beta}(x^\gamma, l) = \left(\frac{l-l_0}{l}\right)^2 k_{\alpha\beta}(x^\gamma), \quad (19)$$

where $k_{\alpha\beta}$ is any admissible 4D vacuum metric of general relativity with a cosmological constant $3/L^2$. Furthermore we assume a null 5D path as noted above, and rewrite the line element as

$$dS^2 = \left[\frac{l^2}{L^2} - \left(\frac{dl}{ds}\right)^2 \right] ds^2 = 0. \quad (20)$$

Since a massive particle in spacetime has $ds^2 \neq 0$, we have that the velocity in the extra dimension is given by $(dl/ds)^2 = (l/L)^2$. Then the right-hand side of (18b) disappears, and to obtain the l -motion we need to solve

$$\frac{d^2l}{ds^2} - \frac{2}{l} \left(\frac{dl}{ds}\right)^2 + \frac{l}{L^2} = 0 \quad (21)$$

and $(dl/ds)^2 = (l/L)^2$ simultaneously. Substituting the latter into (21), we find that l is a superposition of simple hyperbolic functions. There will be two arbitrary constants of integration involved in this solution, which can be written as

$$l = A \cosh\left(\frac{s}{L}\right) + B \sinh\left(\frac{s}{L}\right). \quad (22)$$

Moreover, $(dl/ds)^2 = (l/L)^2$ implies that $A^2 = B^2$. To fix the constants A and B here, it is necessary to make a choice of boundary conditions. It seems most natural to us to locate the big bang at the zero point of proper time and to choose $l = l_0(s = 0)$. Then $A = l_0$ and, $B = \pm l_0$ in (22), which thus reads

$$l = l_0 e^{\pm s/L}. \quad (23)$$

The sign choice here is trivial from the mathematical perspective, and merely reflects the fact that the motion is reversible. However, it is not trivial from the physical perspective, because it changes the behaviour of the cosmological constant.

This is given by (16), which with (23) yields

$$\Lambda = \frac{3}{L^2} \frac{1}{(1 - e^{\mp s/L})^2}. \quad (24)$$

In the first case (upper sign), Λ decays from an unbounded value at the big bang ($s = 0$) to its asymptotic value of $3/L^2$ ($s \rightarrow \infty$). In the second case (lower sign), Λ decays from an unbounded value ($s = 0$) and approaches zero ($s \rightarrow \infty$). We infer from astrophysical data [1] that the first case is the one that corresponds to our universe.

To investigate the physics further, let us now leave the last component of the 5D geodesic (18b) and consider its spacetime part (18a). We are especially interested in evaluating the anomalous force per unit mass f^μ of that equation, using our metric tensor (19). The latter gives $\partial g_{\alpha\beta}/\partial l = 2(l-l_0)(l_0/l^3)k_{\alpha\beta}(x^\gamma) = 2l_0[l(l-l_0)]^{-1}g_{\alpha\beta}$ in terms of itself. We can substitute this into (18a), and note that the 4-velocities are normalized as usual via $g_{\alpha\beta}(dx^\alpha/ds)(dx^\beta/ds) = 1$. The result is

$$f^\mu = -\frac{l_0}{l(l-l_0)} \frac{dl}{ds} \frac{dx^\mu}{ds}. \quad (25)$$

This is a remarkable result. It describes an acceleration in spacetime which depends on the 4-velocity of the particle and whose magnitude (with the choice of boundary conditions noted

above) is infinite at the big bang. It is typical of the non-geodesic motion found in other applications of induced-matter and membrane theory [6, 7]. It follows from (23) that

$$f^\mu = \mp \frac{1}{L} \frac{dx^\mu}{ds} \frac{1}{(e^{\pm s/L} - 1)}. \quad (26)$$

In the first case (upper sign), $f^\mu \rightarrow (-1/s)(dx^\mu/ds)$ for $s \rightarrow 0$ and $f^\mu \rightarrow 0$ for $s \rightarrow \infty$. In the second case (lower sign), $f^\mu \rightarrow (-1/s)(dx^\mu/ds)$ for $s \rightarrow 0$ and $f^\mu \rightarrow (-1/L)(dx^\mu/ds)$ for $s \rightarrow \infty$. Thus both cases have a divergent, attractive nature near the big bang. However, at late times the acceleration disappears in the first case, but persists (though is small if L is large) in the second case. As in our preceding discussion of Λ , we infer from astrophysical data on the dynamics of galaxies [1] that the first case is the one that corresponds to our universe.

As regards anomalous accelerations, let us consider the implications of (26) above. That equation shows that there is an extra force (per unit mass) which is proportional to both the velocity in the extra dimension (dl/ds) and the velocity in spacetime (dx^μ/ds). The first dependency shows that the extra force arises from motion with respect to the extended coordinate frame, so it is inertial in the Einstein sense (like centrifugal force). The second dependency shows that the acceleration is coupled to the dynamics in 4D. In fact, there is a kind of restoring force towards the rest state. This is of importance for the dynamics of galaxies, for it shows that the comoving frame used in standard 4D cosmological models is actually an equilibrium state. This agrees with the data which show that the Hubble flow is smooth and that the peculiar velocities of galaxies are small at the present epoch [1, 5]. However, while there are constraints from data on the 3K microwave background, some dynamical departures must have been present at early epochs. This with (26) opens a new perspective on the problem of the formation of structure in the early universe. The standard theory, wherein a small statistical perturbation of the density is supposed to grow by gravitational instability into a galaxy, has long been known to suffer from a timescale problem. The basis of this is that a small perturbation does not have enough gravitational pull to counteract the rate at which matter is being diluted by the Hubble expansion, thus limiting the rate at which it can grow. To explain the properties of galaxies as they are observed, the process needs to happen faster. The anomalous acceleration (26) of our model may resolve this problem, since it augments gravity and thereby assists galaxy formation.

3. Conclusion and discussion

It is apparent from the contents of the preceding section that the cosmological ‘constant’ may not be what it appears to be. In this section, we review the foregoing algebraic results, and then summarize the physical consequences of what we have found.

Metric (1) of general relativity extended to five dimensions can in principle handle all of 5D physics. However, it is instructive to look at the restricted case of (4D) matter-free conformally flat metrics (2), as they are the analogues of the inflationary de Sitter cosmology. The field equations for apparently empty 5D space are given by (3), and these are known by Campbell’s theorem to contain all solutions of the 4D Einstein equations [5]. The latter involve the cosmological ‘constant’, which can either be regarded as related to an extra force in addition to gravity, or as a measure of the energy density of the vacuum. The pure-canonical 5D metric ($\partial g_{\alpha\beta}/\partial l = 0$) yields $\Lambda = 3/L^2$, where L is a length that scales the 4D part of the metric, and which is known from astrophysical data to be $L \simeq 1 \times 10^{28}$ cm [1]. A simple gauge transformation, wherein the extra coordinate is shifted by a constant, revalues Λ to (16), which is variable if there is motion in the extra dimension. This is constrained by the 5D geodesic equation, which splits naturally into a 4D part involving an extra force (18a) and an

extra part in $x^4 = l$ (18b). The motion in the extra dimension can easily be found (23), which enables $\Lambda = \Lambda(s)$ to be evaluated (24). The motion in the regular dimensions of spacetime can likewise be evaluated, but involves an extra force (per unit mass) or acceleration (26).

The main physical result of our working is that in 5D, the cosmological constant is changed from $\Lambda = 3/L^2$ to $\Lambda = (3/L^2)l^2(l - l_0)^{-2}$, where l is the value of the extra coordinate. This is the result of merely changing l to $(l - l_0)$. Such a result may appear at first sight to be surprising, but in retrospect it could have been foreseen: if we extend general relativity from four to five dimensions, *any* change in the extra coordinate will preserve the 5D formalism but alter the 4D one. Covariance is powerful, and if applied in $N(\geq 5)$ D will alter our view of 4D physics. In other words, if the world has more than four physically significant dimensions, what we perceive in spacetime depends on how we choose the gauge (coordinate frame) in five dimensions. The results we have found can be viewed as a test of whether or not there are more than four dimensions: the decay of the cosmological ‘constant’ as in (24) and the existence of a fifth force as in (26) are both, in principle, open to test. These are small effects as measured at the present epoch, and in conformity with current astrophysical data [1]. But both of these effects must, according to the present model, have drastically influenced the early universe. Galaxy formation, in particular, must have been influenced by an anomalous acceleration that complements the decay of Λ and augments gravity.

If we can obtain such significant physical effects from a mere shift along the axis of a minimally extended version of general relativity, it is permissible to wonder about more complicated gauge changes. Phenomenological arguments have recently been made, which indicate that a particle may have its ‘own’ associated Λ , connected to its mass (see, e.g., [13]). Our results are the consequence of a particularly simple change of gauge.

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