ASSESSING FALL VELOCITY-MAXIMUM DIMENSION RELATIONSHIPS
AND PARTICLE SIZE DISTRIBUTIONS FOR SNOWFALL

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ABSTRACT

Snowfall is an atmospheric phenomenon that can cause significant impacts to many aspects of daily life in Missouri. Further, no two snowfall events are exactly the same, as even small differences in environmental characteristics can result in differing snow crystal types dominating the event, which in turn can result in differing impacts from event to event. Therefore, it is necessary to understand snowfall behavior so that better forecasts and in situ analyses may be made. In this study, snowflake maximum dimension and fall velocity measurements were recorded using the OTT Parsivel Laser Disdrometer. In conjunction with distribution of measured maximum dimensions, RAP Analysis soundings were used to determine snow crystal type. From there, the relationships between fall velocity and maximum dimension and the particle size distributions of snowflakes from many snowfall events were analyzed. Observed relationships between fall velocity and maximum dimension were compared with previously derived relationships, and it was found that, in most cases, no single curve represented the relationship in the observed data well, with discrepancies caused by instrumentation error and lack of a single dominant crystal type. To analyze particle size distributions, several distribution functions were fit to the observed distribution using a least-squares regression method in MATLAB. It was found that, overall, the triple Gaussian distribution function performed the best in modeling particle size distributions in snow, but there were some instances where the gamma function modeled the distribution best. Further study, especially with the inclusion of field observations in addition to instrument observations, is necessary to develop a better understanding of these snowfall events.

1. Introduction

1.1 Background

Snow is an atmospheric phenomenon that has necessitated much examination over the past century due to the various formations of crystals based on environmental characteristics (e.g. Nakaya and Iizima 1934, Nakaya and Hasikura 1934, Nakaya et al. 1938, Nakaya 1954, Locatelli and Hobbs 1974) and the impacts these events can have on normal day-to-day events (Rasmussen et al. 2003). Two primary foci in this greater body of research have been the determination of relationships between a snowflake's size and terminal fall velocity (e.g. Nakaya and Terada 1935, Locatelli and Hobbs 1974, Böhm 1989, Mitchell et al. 1990, Mitchell 1996, Heymsfield and Westbrook 2010, Pu et al. 2020) and the construction of particle size distributions (PSDs) for snowfall events (e.g. Nakaya and Sekido 1936, Langille and Thain 1951, Marshall and Gunn 1952, Gunn and Marshall 1958, Souverijns et al. 2017, Jia et al. 2019). These two developments have allowed for other advances, such as derivations of Radar Reflectivity-Snow Rate (Z-R) relationships (e.g. Marshall and Gunn 1952, Gunn and Marshall 1958, Sekhon and Srivastava 1970, Souverijns et al. 2017). Using snowfall data gathered by a disdrometer during the winters of 2019-20 and 2020-21, this research seeks to examine PSDs and evaluate calculated velocity-maximum dimension relationships for a variety of different snow crystal types.

In a broad sense, the intent of this research is to develop a better understanding of Missouri snowfall. Calculating both velocity-dimension relationships and PSDs is useful to determine in greater detail the character of snowfall events. Both velocity-dimension relationships and PSDs are useful in numerical weather prediction, as most accurate

parameterizations can be developed from gathered data and implemented, which will result in better model handling of snowfall events. This research, however, is only a first step toward the ultimate goal, which is to determine radar reflectivity-snow rate relationships for different snow crystal types. If this ultimate goal is attained, the results will be very useful operationally, which, given the degree of impacts some snowfall events can cause, may help increase public readiness during snowstorms.

Some of the earliest and most extensive studies on snow characteristics were carried out in Japan in the mid-20th century. Nakaya and Iizima (1934) conducted their research in Sapporo, Japan, which is a coastal city located at approximately 43°N latitude and experiences a number of different snow crystal types. In this research, snow crystals were gathered on a cold, dry, glass plate. Researchers would then select prime crystals using a magnifying glass. The selected crystals would finally be photographed under a microscope. Based on this data, Nakaya and Hasikura (1934) classified snow crystals into 28 types across six primary categories. Nakaya and Iizima (1934) also made note of the atmospheric temperature, relative humidity, wind speed, and wind direction at the surface as crystals were collected. They found that surface temperature and relative humidity had very little bearing on observed crystal type, but wind speed and direction did influence what crystal type fell. Nakaya and Sekido (1936) expanded on the previous work by calculating the observed frequency of different crystal types. One of their most significant findings was that typically, even in the most homogenous of snowfall events, multiple crystal types were observed at any given time during the snowfall.

Nakaya et al. (1938) then studied snow crystals by growing them in within four apparatuses built in a laboratory setting. The first three apparatuses were constructed in

roughly the same manner. The design consisted of a vertical cylinder with a shallow pool of water at the bottom and a filament (a strand of Rabbit hair) at the top. The shallow pool of water would be heated electrically as to cause evaporation. The main way these three apparatuses differed was in the construction of the upper part of the cylinder: apparatus two was shorter than apparatus one, and apparatus three was more narrow at the top than apparatus one. The fourth apparatus was different entirely in that it was constructed out of two cylinders of different temperatures to induce convection. The four apparatuses were used to grow snow crystals at temperatures between -15°C and -45°C . They found that, for room temperatures, crystal type was determined by its growth rate, with dendrites and broad-branched crystals growing the fastest ($1.3\text{-}4.6\text{ mm hr}^{-1}$), followed by plates (0.7 mm hr^{-1}), and needles (0.5 mm hr^{-1}). Growth rate itself was governed by the temperature of the water, the temperature of the apparatus, and the mode of convection.

Nakaya (1954) built on their 1938 work, continuing to grow snow crystals in the apparatuses previously explained. They discovered that the two key determining factors of expected crystal type were the environmental temperature and supersaturation with respect to ice. Their results are shown in Figure 1. It should be noted that the delineations on the diagram are not absolute, as this research documented crystal types that fell outside their expected domain of temperature and supersaturation values. Even though these results are derived from laboratory measurements, Nakaya (1955) found that they are valid for an environment full of aerosols, which is much more characteristic of Earth's atmosphere.

Figure 1 – Plot of supersaturation with respect to ice vs air temperature. Each symbol corresponds to the temperature and supersaturation at which a certain crystal formed. The thick black line marked “W” represents the supersaturation with respect to water. Taken from Nakaya (1954).

Under the premise that Nakaya (1954) classified symmetric crystals well, but further work was necessary for asymmetric and modified crystal types, Magono and Lee (1966) expanded the snow crystal classification scheme from 36 to 80 crystal types. To determine the corresponding temperature and supersaturation for each snow crystal type, they used a combination of ground measurements at different altitudes on a mountain slope and radiosonde data. From their research, they found that the plot of air temperature vs. supersaturation produced by their data was similar to that of Nakaya (1954).

Other methods of determining snow crystal type involved using Formvar solutions to develop snow crystal replicas (Schaeffer 1956). To create these replicas, snow crystals are collected on plates. The Formvar solution is then applied to the plate, where it hardens into the crystal's shape before the crystal itself is able to melt. This method has been used in numerous subsequent studies, with Power et al. (1964) utilizing this method to determine relationships between different crystal types and their densities. Hindman and Rinker (1967) improved on this method by developing an apparatus that would allow snowflakes to be gathered and replicated continuously. This resulted in a larger number of snowflakes accurately collected and replicated, as well as a larger range of snowflakes sizes replicated.

Nakaya's research projects also made the first attempt to construct PSDs for various forms of snow crystals (Nakaya and Sekido 1936). They gathered snow crystals in the same manner outlined by Nakaya and Iizima (1934), and once collected, the maximum dimension of the crystals (think diameters for plates and dendrites and lengths for needles and columns) was documented. They then plotted graphs of frequency vs maximum dimension for plane-type crystals (plates, sector crystals, broad-branched crystals, and dendrites), column-type crystals (columns and needles), and crystals composed of columns and planes. They found that plane-type crystals were observed more frequently than their columnar counterparts. They also found that PSDs for needles, columns, plates, sector crystals, stellar crystals, and broad-branched crystals were all skewed to the right. The most frequently observed maximum dimension for all of these crystals ranged between 0.25 mm (simple plates) and 2 mm (broad-branched crystals and needles), with the size range for all of these crystals between 0 and 4 mm. PSDs for

dendritic crystals weren't skewed as sharply to the right, had a much larger range (0-9 mm), and the most frequently observed maximum dimension for these crystals (2.5-4 mm) was much larger than any of the other crystal types.

Further examination of PSDs for snowfall were conducted in Canada in the 1950s. In the case of Langille and Thain (1951), snow crystals were collected on filter paper, where they were allowed to melt. The paper was then brushed with dye and spots on the filter paper were measured. In the case of Gunn and Marshall (1958), a shallow box with a piece of wool in the base was set in a courtyard surrounded by buildings on three sides. Once a sufficient number of crystals was collected, the wool would be brought inside, where the crystals would melt, but not seep into the wool. Dyed filter paper was then laid on the wool, absorbing the drops of melted snow. The spots left behind were then measured and recorded. From these observations, Marshall and Gunn (1952) and Gunn and Marshall (1958) concluded that the PSD for snowfall could be best described using the exponential distribution first developed to describe drop size distributions for rainfall events from Marshall and Palmer (1948):

$$N(D) = N_0 \exp(-\Lambda D) \quad (1)$$

Where $N(D)$ is the concentration of snowflakes as a function of diameter of the drop formed from the melted snowflake, N_0 is a concentration parameter, Λ is the slope parameter of the size distribution, and D is the diameter of the drop formed from the melted snowflake. Unlike for rain, where only Λ varies with rain rate, in the case of snow, both N_0 and Λ vary with equivalent rainfall rate, and thus, no singular PSD can be developed for a snowfall event without knowledge of the equivalent rainfall rate (Marshall and Gunn 1952, Gunn and Marshall 1958).

The above method for modeling drop size distributions in rainfall was modified by Ulbrich (1983), which added the term D^μ , where μ is a unitless scaling parameter. This, in turn, makes the gamma distribution a better descriptor of any given drop size distribution:

$$N(D) = N_t D^\mu \exp(-\Lambda D) \quad (2)$$

Later work analyzed the applicability of this relation in modeling PSDs for snowfall events. Research conducted in Colorado utilized a video disdrometer (Brandes et al. 2007) and vertical profiler (Newman et al. 2009) to calculate PSDs for snowfall events. From there, both publications fit gamma distributions to the calculated PSDs and recorded the generated values for μ . Brandes et al. (2007) plotted the frequency of μ values, and they found that the distribution was skewed to the right with a mode of +1. Around 56% of calculated μ values were between -2 and +2. Newman et al. (2009) found that 84% of μ values were within this range. Brandes et al. (2007) also noted 22% of μ values were negative, and both publications suggest negative values were most common in heavy snowfall.

Jia et al. (2019) used a Parsivel disdrometer to record PSDs for different precipitation types, including snow. They gathered data every ten seconds, and of the over 11000 samples they collected, 6785 of the samples were for snowfall. For each PSD, the skewness and kurtosis was calculated. Kurtosis vs skewness was then plotted for each PSD, with plots also displaying the expected curves relating skewness and kurtosis for lognormal, gamma, and Weibull distributions (Figure 2). For exponential distributions, the skewness is 2 and kurtosis is 6, while for Gaussian distributions, the skewness and kurtosis are both zero. In other words, for either of these distributions to be the best

descriptor of each PSD, the plotted points would cluster around those fixed points. As shown by this figure, both the Weibull and gamma curves fit the plotted data reasonably well, suggesting that gamma and Weibull distributions adequately describe any particular PSD. This conclusion was further supported by both Taylor diagrams, which showed that gamma and Weibull “model” distributions corresponded well with the measured PSDs, and calculations of relative Euclidean distance, where both the gamma and Weibull distributions had calculated values around 0.75, with the value for the gamma distribution slightly lower than that of the Weibull distribution. For reference, values closer to zero indicate better model fit, and the lognormal distribution had a relative Euclidean distance value greater than three. Therefore, both the gamma and Weibull distributions will be employed in analyzing this research.

Figure 2 – Kurtosis (K) vs Skewness (S) for all snowfall PSDs from Jia et al. (2019). The red, green, and blue lines represent the expected relationship between kurtosis and skewness for the lognormal, Weibull, and gamma distributions, respectively.

Despite Jia et al. (2019) concluding that the Gaussian distribution is not a good descriptor of snow PSDs, a method using a summation of multiple iterations of the Gaussian distribution will be employed in this research. This method was developed by Steward (2020) and used to develop more accurate drop size distributions based on data collected from a Micro Rain Radar and Parsivel disdrometer. This method worked very well in characterizing rain drop size distributions; therefore, it is worthwhile to determine its applicability to snow PSDs.

Velocity-size relationships are also explored in this research. Once again, the earliest work on developing these relationships was done in Japan (Nakaya and Terada 1935). In their experiments, they built a cylinder with two boxes on either end. The boxes at either end had windows on all sides so that the crystal could be seen. The time for the snow crystal to fall from the top box to the bottom box would be taken manually using a stopwatch. The cylinder was designed so that once the crystal was dropped into it, it would be sealed, resulting in negligible vertical motions within it. They found that velocity varied with maximum dimension only for needles. For plane-type snow, there was no discernable change in velocity as dimension increased. This finding was refuted by Locatelli and Hobbs (1974). Their method for measuring snow crystal velocity was much more sophisticated. In their experiment, they used an instrument wherein two beams of light were situated on top of one another. The beams shined through an open area, and on the other side of it was a sensor. A machine was attached to this end of the instrument. When precipitation would fall through the instrument, it would lessen the amount of light received by the sensor. The time difference between the disturbance in the upper and lower light beams was recorded by the machine, and from that value, a

velocity was calculated. Snow crystals were then collected and observed under a microscope to determine type and dimension. They found that for all snow crystals, increasing dimension corresponded with increasing velocity, with most velocities between 0.5 and 2 m/s. Fall speeds of aggregates of certain crystal types were also found to be greater than those of the individual types. This research was able to develop empirical velocity-size relationships from their data collected.

Further advancements in velocity-size relationships for snow crystals have been derived from the drag force equation (Böhm 1989, Mitchell et al. 1990, Mitchell 1996, Heymsfield and Westbrook 2010). Their methods in developing velocity-dimension relationships for each crystal type will be discussed in chapter 2.

1.2 Goals

In examining characteristics of snowfall, the following will be addressed:

1. Determine predominant ice crystal type based on environmental characteristics and maximum dimension data.
2. Examine particle size distributions from a variety of ice crystal types observed by the OTT Laser Disdrometer and characterize these distributions using machine learning techniques.
3. Evaluate fall velocity vs maximum dimension data gathered by the OTT and produce equations that relate the two variables for different ice crystal types, and assess variability within these datasets.
4. Compare the results from (3) with older published work, and assess variability between those results and the results produced by this work.

Using the OTT Parsivel disdrometer, the overarching purpose of this research is to analyze different potential ways of modeling PSDs and determine the accuracy of mathematically derived velocity-dimension relationships. The methodology for this research are described in chapter 2. Results are discussed in chapter 3. Conclusions are presented in chapter 4.

2. Methodology

2.1 The Laser Disdrometer

2.1.1 Overview

On a practical level, the most important area for assessing snowfall characteristics is at or just above ground level as the crystal type and PSD measured at this level will largely govern the magnitudes of resulting factors, namely snow accumulation; thus, a ground-based method of measurement is required. Methods have become more sophisticated over the past seventy years, evolving from manually collecting, measuring, and counting snowflakes (Marshall and Gunn 1952, Gunn and Marshall 1958) to using devices that record measurements based on the obstruction of light (Locatelli and Hobbs 1974) to using laser disdrometers (Battaglia et al. 2010, Jia et al. 2019, Pu et al. 2020) and photo disdrometers (Souverein et al. 2017), which currently serves as the most advanced method. Therefore, a disdrometer is utilized in this research.

The specific disdrometer used in this project is the OTT Particle size Velocity² (Parsivel²) disdrometer (Figure 3). This disdrometer functions by emitting, through a laser diode from one end, a laser with a wavelength of 650 nm and a peak power output of 0.2 mW. On the opposite end, spaced 18 cm away, there is an optical sensor that acts as the receiver for the emitted laser beam. The resulting plane through which precipitation falls is oriented parallel to the ground and has a surface area of 54 cm². The sensor can detect solid precipitation diameters from 0.2-25 mm and fall velocities from 0.2-20 ms⁻¹. The OTT system then sorts these measurements into 32 size and 32 velocity classes, or bins, with an accuracy of +/- 1 for size classes smaller than 2 mm and +/- 0.5 for sizes larger than 2 mm. Based on fall velocity and diameter ranges for different

precipitation types, the Parsivel² disdrometer can differentiate between eight hydrometeors/combinations of hydrometeors: drizzle, mixed drizzle and rain, rain, mixed rain and snow, snow, snow grains, sleet, and hail.

Figure 3 – The OTT Parsivel disdrometer (right). This instrument, along with the Micro-Rain Radar (left), is located at the Bradford Research Farm just east of Columbia, Missouri. Photo from Steward (2020).

This disdrometer was chosen to measure the PSD and velocity-dimension relationships of snowfall events. This particular disdrometer has been used in many previous works. Löffler-Mang and Joss (2000) used the Parsivel disdrometer to measure velocities and diameters of falling snowflakes, and they found that the sizes of snowflake observed by the Parsivel disdrometer matched well with a particle spectrometer, an accepted reliable tool for measuring hydrometeor sizes. Licznar and Krajewski (2016)

used the Parsivel disdrometer to derive a reflectivity-snowfall rate relationship in Warsaw, Poland. In using the common $Z = aR^b$ formulation for this relationship, where Z is reflectivity and R is liquid equivalent snowfall rate, they found values for a and b to both be significantly less for snow than for rain. There was notably a larger spread in the snowfall data than the rainfall data. Jia et al. (2019) used a Parsivel disdrometer in their determination of which probability functions can best be used in modeling PSDs, where they determined both the Gamma and Weibull distributions model PSDs well. Pu et al. (2020) was able to determine from disdrometer measurements that fall velocity vs dimension relationships differ by crystal type for snowfall, as do PSDs and any resulting relationships, such as that between reflectivity and snowfall rate.

There are some important assumptions made in the Parsivel disdrometer's firmware as it retrieves data (Battaglia et al. 2010). First, the disdrometer assumes all particles it observes are spheroids. Inherent to this assumption is that all particles are symmetric. Second, particles fall such that their major axis is oriented horizontally, parallel to the ground. Third, the vertical component of the fall velocity solely determines the particle signal duration, so horizontal velocity of a particle is neglected. Fourth, it is assumed that only one particle is in the beam area at a given time.

These assumptions indeed give rise to measurement errors. Battaglia et al. (2010) noted that the fall velocities measured by the disdrometer were underestimated for small snowflakes and overestimated for large snowflakes due to the assumed relationship between horizontal and vertical dimensions of snowflakes. With the Parsivel only retrieving a one-dimensional size parameter, the assumption inherent is that the size retrieved is representative of the maximum dimension of a symmetric particle. However,

this may not always be the case, especially for small particles, whose orientation is random, and large particles, which are likely asymmetrical (Pu et al. 2020). Furthermore, as noted by Battaglia et al. (2010), large snowflakes may fall through only part of the laser area in the disdrometer, further resulting in an underestimation in size of larger snowflakes. They also found that errors can be introduced as a result of horizontal wind, snowflake density, and the system interpreting one snowflake signal as a result of multiple. In the resulting PSD, the Parsivel overestimates the number of small particles and underestimates the number of large particles.

Moreover, the Parsivel disdrometer doesn't calculate values for diameter and fall velocity; instead, it counts and sorts these measurements into classes representing discrete ranges. The values themselves are represented only by the median value in each class, which leads to error (Pu et al. 2020). This is further compounded by the fact that the 32 binwidths for diameter are not uniform. Table 1 shows the drop sizes, bin-widths, and fall velocities for the 32 bins. As observed by Steward (2020) with rain, the variance in binwidth results in an erroneous spike in the snowflake concentration between the 1.187 and 1.375 mm bins. This is the first place in the classification system where the binwidth increases. This spike is most clearly evident when the difference between the number of small and large snowflakes observed by the disdrometer decreases. Figure 4 shows an example of how this spike appears in a drop concentration vs snowflake size framework, and figure 5 shows a contoured plot of fall velocity vs snowflake size, highlighting the increase in concentration between the 1.187 and 1.375 mm size bins. Nevertheless, this anomaly did not present a serious hindrance to this research, but it does seem to influence how different functions are fitted to the PSD.

Snowflake dimension (mm)	0.062	0.187	0.312	0.437	0.562	0.687	0.812	0.937
binwidth (mm)	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
Fall Velocity (m/s)	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75
Snowflake dimension (mm)	1.062	1.187	1.375	1.625	1.875	2.125	2.375	2.75
binwidth (mm)	0.125	0.125	0.25	0.25	0.25	0.25	0.25	0.5
Fall Velocity (m/s)	0.85	0.95	1.1	1.3	1.5	1.7	1.9	2.2
Snowflake dimension (mm)	3.25	3.75	4.25	4.75	5.5	6.5	7.5	8.5
binwidth (mm)	0.5	0.5	0.5	0.5	1	1	1	1
Fall Velocity (m/s)	2.6	3	3.4	3.8	4.4	5.2	6	6.8
Snowflake dimension (mm)	9.5	11	13	15	17	19	21.5	24.5
binwidth (mm)	1	2	2	2	2	2	3	3
Fall Velocity (m/s)	7.6	8.8	10.4	12	13.6	15.2	17.6	20.8

Table 1 – The values for snowflake dimension (mm), corresponding binwidth (mm), and fall velocity (ms^{-1}) into which the disdrometer software sorts its measurements. The highlighted area indicates the location of the erroneous data spike.

Figure 4 – Concentration spike as it appears on a particle size distribution plot. Plot constructed from disdrometer observations taken on 23 January 2020 from 2200 to 2215 UTC.

Figure 5 – Concentration spike as it appears on a plot of fall velocity vs snowflake dimension. Plot constructed from disdrometer observations taken on 23 January 2020 from 2200 to 2215 UTC.

2.1.2 Location and Event Selection

The disdrometer has been operational at Bradford Farm (Latitude 38.897236, Longitude -92.218070) since July 2019; thus, snowfall events from the winters of 2019-20 and 2020-21 are analyzed in this research. Further, 297 15-minute snowfall samples were examined. The 15-minute interval was chosen following the methods of Steward (2020). An interval of this length is required largely so that there is enough data to analyze. If shorter intervals are used, the data may be too sporadic, making it exceedingly difficult to analyze. If longer intervals are used, important features within the dataset may be smoothed out. The selection of the 297 samples was aided largely by the MRR. When

the MRR showed reflectivity values in the 10-30 dBZ range, it was assumed that snow was falling. Because the MRR viewing software plots MRR data in a time-height format, it is easy to determine start and end times for each snowfall event.

It should be noted that Bradford Farm is prone to technological issues and power outages that will cause the disdrometer to go offline. Therefore, data was not gathered for parts of or even entire snowfall events during the two winter seasons, and furthermore there were more than 297 15-minute periods where snowfall occurred at Bradford Farm. The case studies examined in greater detail were chosen most based on data availability, with events having reliable data throughout their duration being the most desirable. Moreover, the events chosen for further examination in this research are 23-24 January 2020, 14-15 February 2021, and 20 April 2021.

2.2 Determination of Crystal Type

In order to best assess velocity-dimension relationships and PSDs, a determination of expected dominant crystal type was required. Because there were no field measurements taken during these snowfall events, the dominant crystal type was not recorded. Upper-air soundings, which are useful in determining variations in temperature and moisture with height, were not launched during these snow events either. Therefore, it was decided that the best method to characterize the environment in which the crystals form and grow and determine the dominant snow crystal type would be to use analyzed soundings from the Rapid Refresh (RAP) model. The RAP has a grid spacing of 13 km and produces forecasts, and thus, analyses, every hour. RAP data were gathered from the National Centers for Environmental Information (NCEI) RAP analysis archive (<https://www.ncdc.noaa.gov/data-access/model-data/model-datasets/rapid-refresh-rap>).

The analysis soundings were viewed using NSHARP from the General Meteorological Package (GEMPAK). Because each forecast from the RAP ingests a large volume of atmospheric data from ground observations, observed soundings, aircraft measurements, and satellites, the 00-hour or analysis stage of the RAP should be consistent with the state of the atmosphere at that time.

To verify this, RAP analyzed soundings for case study dates were compared with observed soundings. The National Weather Service has a network of locations across the Continental United States where soundings are recorded at 0000 and 1200 UTC each day. Columbia, Missouri, is not one of these sites, so the closest site, Springfield, Missouri, which is approximately 225 km southwest of Columbia, was chosen to conduct this assessment. Observed soundings were gathered and viewed on the University of Wyoming upper air sounding archive online site.

Figure 6 – RAP analysis valid at 0000 UTC on 24 January 2020 for Springfield, MO (left) and 0000 UTC observed sounding from Springfield (right).

The first case analyzed in detail is 23-24 January 2020. The reference sounding used to verify the reliability of the RAP is the 0000 UTC sounding from Springfield (Figure 6). Overall, the RAP analysis matches the observed sounding well. The RAP analysis resolves well the two saturated layers, one from the surface to approximately 800 hPa and the other from 550 to 450 hPa, seen in the observed sounding. Both products put the mid-level dry layer at the same height with the same depth (approximately 775 to 600 hPa). The temperature profile in the RAP analysis is reasonably in agreement with the observed sounding, with surface temperatures just above 0°C and certain features, such as the minor inversion near 800 hPa, documented in both products. The wind profiles between the two products also agree, with wind speeds and directions throughout the depth of the column on the RAP analysis matching the observed sounding well.

The second case examined in detail is 15 February 2021. The observed sounding (Figure 7) used to check the accuracy of the RAP analysis in this case was the 1200 UTC sounding from that day. The RAP analysis lines up pretty well with the observed sounding with respect to the depth of the thermal inversion and the temperature difference between the bottom and the top of the inversion layer. The only difference is that the lowest 50 hPa of the RAP analysis sounding is about 2°C too warm. The RAP analysis also misses a dry layer between 650 and 550 hPa which is in the observed sounding. Other than these two minor discrepancies, the RAP analysis agrees reasonably well with the observed sounding taken at the same time. Wind speeds and directions throughout the column are well represented by the RAP analysis sounding, with the analysis picking up on the wind shift around 850 hPa.

Figure 7 – Same as figure 6, except data valid for 15 February 2021 at 1200 UTC.

The third and final case examined in detail is from the morning of 20 April 2021. The observed sounding used for comparison is the 1200 UTC sounding from Springfield (Figure 8). The RAP analysis once again does a good job representing the environment observed by the Springfield sounding. Temperature and moisture profiles in the RAP analysis match well with those in the observed sounding, with a saturated inversion layer in the lowest 100 hPa of the atmosphere. The dry layer just above that is also represented well by the RAP analysis sounding. The wind profiles in the column from the RAP analysis sounding match those of the observed sounding well, with a wind shift evident on both products around 900 hPa.

It should be noted that for all three of these analyses, the thermal and moisture profiles on the RAP analysis sounding are much smoother than those of the observed sounding. This is likely due to the observed sounding having more levels of measurements than the analysis sounding does levels of calculation.

Figure 8 – Same as figure 6, except with data from 1200 UTC on 20 April 2021.

Because RAP analysis soundings represent the actual state of the atmosphere well, these soundings can be used to determine the dominant crystal type. As mentioned in chapter 1, Nakaya (1954) determined that snow crystals will take on different shapes with differing temperature and supersaturation with respect to ice. Figure 1 shows the temperature and supersaturation percentage domains for different types of snow crystals. Unfortunately, there is no good way to calculate supersaturation with respect to ice from a sounding, as an environment can be supersaturated with respect to both ice and water, but supersaturation with respect to water cannot be determined solely from temperature and dewpoint measurements. Therefore, the degree of supersaturation with respect to ice was determined qualitatively.

2.3 Calculation of Expected Fall Speed from Measured Dimension

It has been well documented that snow crystals take a multitude of different forms based on the different environments in which they form. With each different form of crystal comes a difference in how each crystal type's mass and area changes with a

change in dimension (Nakaya and Terada 1935, Locatelli and Hobbs 1974, Mitchell et al. 1990, Mitchell 1996). This is in sharp contrast with rainfall. It is much easier to determine a fall velocity vs diameter relationship for raindrops because they are assumed to take roughly the same shape for any fixed diameter; thus, the mass and area vs diameter relationships can hold true for the population of raindrops in any given event. Steward (2020) shows an example of this in their research using measurements for the Parsivel disdrometer (Figure 9). It is noteworthy that the measured concentrations by diameter and fall velocity classes hug the curve, a modified form of the Gunn and Kinzer (1949) formulation modeling the fall velocity-diameter relationship for raindrops:

$$V(D) = 9.43(1 - \exp(-0.5195D^{1.147})) \quad (3)$$

Figure 9 – From Steward (2020). Measurements from the Parsivel disdrometer taken 30 August 2019 at 1900 UTC. The plot shows a 1-minute averaged observation for the number of drops counted in each of the diameter and fall velocity bins. The green curve represents the modified Gunn and Kinzer (1949) formulation of fall velocity vs diameter for raindrops.

The calculation of the fall velocity of a given snowflake based on its maximum dimension requires a much more involved process. This process begins with an understanding of the drag force exhibited on a falling particle. This expression is:

$$F_D = \frac{1}{2} \rho_{air} v_t^2 A C_d \quad (4)$$

Where F_D is the drag force on the particle, ρ_{air} is the air density, v_t is the terminal velocity of the falling particle, A is the surface area of the particle, and C_d is the drag coefficient. C_d itself presents a difficulty in the calculation of fall velocities; therefore, the Best number becomes helpful:

$$X = C_d Re^2 \quad (5)$$

Where X is the Best number and Re is the Reynolds number:

$$Re = \frac{\rho_{air} v_t D}{\eta} \quad (6)$$

Where D is the diameter of the particle and η is the dynamic viscosity of the air. Setting the drag force on the particle equal to the force of gravity acting on it, mg , where g is acceleration due to gravity (as the condition for terminal velocity), and solving for C_d in (4), then substituting that expression in for C_d in (5) and substituting (6) in for Re in (5) yields (Mitchell 1996):

$$X = \frac{\rho_{air}}{\eta^2} \frac{2mgD^2}{A} \quad (7)$$

Heymsfield and Westbrook (2010) and Mitchell (1996) include a discussion on the behavior of spherical particles and note that the drag on a non-spherical particle will follow the same form, just with some different parameters. Abraham (1970) evaluated drag on a spherical particle in terms of an assembly of the particle and the boundary layer attached to it. The drag was then computed as the inviscid drag on the entire assembly. Batchelor (1967) showed the depth of the boundary layer to be:

$$\delta = \frac{\delta_0 D}{\sqrt{Re}} \quad (8)$$

Where δ_0 is a dimensionless coefficient. This results in a factor of $(1 + \delta/D)^2$ increase in the effective area of the particle-boundary layer assembly, which produces a new expression for the drag coefficient:

$$C_d = C_0 \left(1 + \frac{\delta_0}{\sqrt{Re}}\right)^2 \quad (9)$$

Where C_0 is the inviscid drag coefficient of the assembly, which is assumed to be constant. This formulation can be substituted into (5) and a relationship between the Reynolds and Best numbers can be derived (Böhm 1989):

$$Re = \frac{\delta_0^2}{4} \left[\left(1 + \frac{4\sqrt{X}}{\delta_0^2 \sqrt{C_0}}\right)^{\frac{1}{2}} - 1 \right]^2 \quad (10)$$

Heymsfield and Westbrook (2010) developed a parameter named the area ratio, A_r , which they define as the ratio of the particle's projected area A to the area of the circle circumscribing the particle, and is calculated as:

$$A_r = \frac{A}{\left[\left(\frac{\pi}{4}\right)\left(\frac{D}{10}\right)^2\right]} \quad (11)$$

Applying (11) to (10) yields the Mitchell (1996) form of the Best number equation:

$$X = \frac{\rho_{air}}{\eta^2} \frac{8mg}{\pi A_r} \quad (12)$$

Heymsfield and Westbrook (2010) found that the resulting fall speeds calculated using (12) were too sensitive to A_r . They therefore developed a modified drag coefficient:

$$C_d^* = C_0 A_r^k \quad (13)$$

They also developed a corresponding modified Best number:

$$X^* = \frac{\rho_{air}}{\eta^2} \frac{8mg}{\pi A_r^{1-k}} \quad (14)$$

To determine the best value for k , Heymsfield and Westbrook (2010) calculated and plotted C_d vs. Re relationships until their points reasonably fit a curve. They found that the best results in this process came when they set k equal to 0.5.

Values for mass and area of snowflakes were not measured in this experiment; however, it has been determined that both quantities can be computed via power laws if the maximum dimension (D) of the snowflake is known:

$$m = \alpha \left(\frac{D}{10}\right)^\beta \quad (15)$$

$$A = \gamma \left(\frac{D}{10}\right)^\sigma \quad (16)$$

Where α , β , γ , and σ are all coefficients determined by the type of snow crystal, D is in mm, m is in kg, and A is in mm^2 . The values for these coefficients are shown in table 2.

Crystal Type	Mass		Area		References
	α	β	γ	σ	
Hexagonal plates	0.00739	2.45	0.65	2	1, 2
Hexagonal columns	0.000907	1.74	0.0512	1.414	1, 2, 3
Crystal with sector-like branches	0.00142	2.02	0.55	1.97	4
Broad-branched crystal	0.000516	1.8	0.21	1.76	4
Stellar crystal	0.00027	1.67	0.11	1.63	4
Aggregates of Plates	0.0033	2.2	0.2285	1.88	6
Aggregates of Columns	0.028	2.1	0.2285	1.88	6
Densely-rimed dendrites	0.003	2.3	0.21	1.76	5

Table 2 – Coefficients for mass and area power laws for different crystal types, partially adapted from Mitchell (1996). References are as follows:

- 1) Mitchell and Arnott (1994)
- 2) Auer and Veal (1970)
- 3) Heymsfield and Knollenburg (1972)
- 4) Pruppacher and Klett (1978)
- 5) Locatelli and Hobbs (1974)
- 6) Mitchell et al. (1990)

Thus, fall velocities of snowfall are calculated first by determining the snow crystal type. From that determination, coefficient values are applied to (15) and (16). The resulting expression from (16) will be inserted into (11) to get an expression for A_r . The resulting expressions for (15) and (11) will then be inserted into (14) to determine the modified Best number. The value for the modified Best number will be placed into a modified form of (10) to solve for the Reynolds number:

$$Re = \frac{\delta_0^2}{4} \left[\left(1 + \frac{4\sqrt{X^*}}{\delta_0^2 \sqrt{C_0}} \right)^{\frac{1}{2}} - 1 \right]^2 \quad (17)$$

Where $\delta_0 = 8.0$ and $C_0 = 0.35$, as prescribed by Heymsfield and Westbrook (2010). The calculated Reynolds number is then placed into (6) to solve for the terminal fall velocity.

2.4 Modeling PSDs using Known Distribution Functions

2.4.1 Overview

As mentioned earlier in section 2.1, the Parsivel counts and sorts observed hydrometeors into bins of fixed diameter ranges. In this research, the Parsivel disdrometer is used to determine concentrations of snowflakes of differing maximum dimension. This method follows Steward's (2020) method, where they calculate concentrations of raindrops by bin over a 15-minute interval. This value for concentration can be plotted vs the range of maximum dimensions to get a PSD. An example of this is shown in figure 10. Known distribution forms were then fit to the calculated PSDs to determine how well each distribution type models the data.

For all distribution types examined, the model distribution curves were fitted to the observed PSD using the non-linear least squares regression fitting method in MATLAB. The method takes the expression representing the probability density function of the desired distribution to model the observed data. The method then evaluates all

possible combinations of coefficients in the modeled distribution. The combination of coefficients which results in the lowest sum of squares error (SSE) when compared with the observed PSD will be applied to the modeled distribution to create the best fit curve of the given distribution.

Figure 10 – An example of a PSD relating concentrations of snowflakes with their maximum dimension, from 15 February 2021 at 2000 UTC.

There is a total of three distributions analyzed in this research with the goal being to determine if one distribution does a noticeably better job in modeling the data. The three distribution functions analyzed as the gamma, Weibull, and triple Gaussian.

2.4.2 The Gamma Distribution

The first distribution type examined is the gamma distribution. From Ulbrich (1983), the expression representing the probability density function of the gamma distribution is, as stated in chapter 1:

$$N(D) = N_0 D^\mu \exp(-\Lambda_g D) \quad (18)$$

Where $N(D)$ is the concentration of snowflakes as a function of maximum dimension, N_0 is a concentration parameter, D is the maximum dimension of snowflakes, μ is the scaling parameter, and Λ_g is the gamma distribution slope parameter, which is sensitive to larger flakes (Brandes et al. 2007). In their evaluation of different distribution types that may be used to model PSDs, gamma, Weibull, Gaussian, lognormal, and exponential, Jia et al. (2019) found that the gamma distribution is one of two distributions which performs well. Moreover, this distribution has been accepted as a good model for PSDs and has been used in many other works (Brandes et al. 2007, Newman et al. 2009, Pu et al. 2020).

Of particular interest to this research is the behavior of the fitted μ parameter over the different distributions. Therefore, the values for this parameter for all 297 15-minute PSDs were recorded. They were then placed into whole number ranges (i.e. 1-2, 2-3, etc) and plotted against the number of observations of μ values within each range (figure 11). The resulting plot is slightly skewed to the right with a mean value of 5.659 and a standard deviation of 1.58. This differs greatly from the findings of both Brandes et al. (2007) and Newman et al. (2009). The plot of the frequency of μ values from Brandes et al. (2007) exhibits a much more strongly right-skewed shape with the mean value around 1, but with some observed values above 20. 56% of their values were between -2 and 2,

and 22% of their values were negative. The values from Newman et al. (2009) are even more tightly clustered around zero, with no observations lower than -5 or higher than 5. Pu et al. (2020) stratified their observed μ values by whether the snow observed was wet or dry; their calculated mean μ values for wet and dry snow were 4.79 and 4.1, respectively. Both of those values are within one standard deviation of our observed mean. The standard deviation for wet snow was 4.49, while the standard deviation for dry snow was 5.38. Both of these standard deviations are larger than that calculated from our observations.

Figure 11 – The number of observed μ values in each whole number range from 297 15-minute PSDs.

There are a few reasons for the discrepancies between our distribution of fitted μ values and those calculated by other works. First, different instruments were used to measure snowflakes dimensions, with Brandes et al. (2007) using a video disdrometer,

Newman et al. (2009) using a vertical profiler, and Pu et al. (2020) using a Parsivel disdrometer. This may have resulted in different measurement accuracy between these three works and our own. Second, locations are different, with our study taking place in central Missouri, while both Brandes et al. (2007) and Newman et al. (2009) conducting their research along the Front Range of the Colorado Rocky Mountains, and Pu et al. (2020) conducting their research in Nanjing, China, which is located in the eastern part of the country. Differences in location may result in different favored microphysical characteristics of snowfall, which may in turn alter the PSDs gathered. Third, for at least Brandes et al. (2007) and Pu et al. (2020), the sample duration was shorter than ours-Brandes et al. 2007 sampled every 5 minutes, and Pu et al. (2020) sampled every one minute-and the sample size for each study was greater than ours-Brandes et al. (2007) had 916 samples, and Pu et al. (2020) had 2557 samples.

2.4.3 The Weibull Distribution

The second distribution examined in this research is the Weibull distribution. This is the other distribution Jia et al. (2019) found to perform well when modeling PSDs. The probability density function of the Weibull distribution is expressed in Jia et al. (2019) as:

$$N(D) = N_0 D^{q-1} \exp(-\Lambda_w D^q) \quad (19)$$

Where $N(D)$ is the concentration parameter of snowflakes as a function of their maximum dimension, N_0 is a concentration parameter, q is the shape parameter, akin to μ in the gamma distribution, and Λ_w is the slope parameter. As was done for the μ parameter in the gamma distribution, the fitted values for q were plotted (figure 12). These values are still somewhat skewed to the right, much like the fitted μ values, but they are much more tightly clustered, with a mean value of 2.904 and a standard

deviation of 0.37. Furthermore, the minimum fitted q value is 1.977, while the maximum fitted q value is 4.194.

Figure 12 – Plot of fitted q values from 297 15-minute PSDs. The range interval on this plot is 0.25.

2.4.4 The Triple Gaussian Distribution

The third and final distribution examined in this research is the triple Gaussian distribution. This distribution originates from the Gaussian distribution, whose probability density function is expressed in Jia et al. (2019) as:

$$N = \frac{N_T}{\sigma\sqrt{2\pi}} \frac{1}{D} \exp\left(-\frac{(D-D_m)^2}{2\sigma^2}\right) \quad (20)$$

It was determined by Jia et al. (2019) that the Gaussian distribution does not fit PSDs well. After all, the Gaussian distribution best models symmetric data distributions, but PSDs are typically skewed right. Steward (2020) noted this was the case for rain; however, their research found that multiple iterations of the Gaussian function to model

raindrop size distributions. Specifically, they found that three iterations of the Gaussian function reliably modeled any given raindrop size distribution. The form of this equation given by Steward (2020) is:

$$f_{g_3}(D) = \int_{D_0}^{D_{max}} [A_1 \exp\left(\frac{-(D-\alpha_1)^2}{\sigma_1^2}\right) + A_2 \exp\left(\frac{-(D-\alpha_2)^2}{\sigma_2^2}\right) + A_3 \exp\left(\frac{-(D-\alpha_3)^2}{\sigma_3^2}\right)] dD \quad (21)$$

(21) is the summation of the first three iterations of the Gaussian function (henceforth Gauss3 function). The coefficients A_1 , A_2 , and A_3 are given by $\frac{1}{\sqrt{2\pi\sigma_*^2}}$ where σ_* varies with each iteration of the model. D is the maximum dimension of the snowflake, α is the mean of the iteration of the distribution, and σ is the standard deviation of the iteration.

When it comes to best fitting the gamma, Weibull, and Gauss3 functions to the observed PSDs, the MATLAB regression method must solve for coefficients in all of these functions. In the gamma function, the MATLAB regression method solves for N_0 , Λ_g , and μ . In the Weibull function, the MATLAB regression method solves for N_0 , Λ_w , and q . In the Gauss3 function, the MATLAB regression method solves for A_1 , A_2 , A_3 , α_1 , α_2 , α_3 , σ_1 , σ_2 , and σ_3 . Table 3 shows the resulting goodness-of-fit statistics across all 297 15-minute PSD samples. Over all metrics, the Gauss3 function performs the best, followed by the gamma and then the Weibull.

Metric	Model		
	Gauss3	Gamma	Weibull
Mean SSE	134721.8	707572.4	2269989
Mean r^2	0.985488	0.964076	0.925855
Minimum r^2	0.7938	0.7019	0.6497
Maximum r^2	1	0.9993	0.9852
Standard deviation r^2	0.027715	0.039912	0.046757
Mean degrees of freedom of error	7	13	13
Mean adjusted r^2	0.968964	0.958546	0.914459
Minimum adjusted r^2	0.5581	0.6561	0.5958
Maximum adjusted r^2	0.9999	0.9992	0.9829
Standard deviation adjusted r^2	0.059338	0.046052	0.053931
Mean RMSE	102.455	159.0182	306.3108

Table 3 – Goodness-of-fit statistics for the Gauss3, gamma, and Weibull functions from 297 15-minute PSD samples. For r^2 and adjusted r^2 , values closer to 1 indicate better goodness of fit. For all other metrics, smaller values indicate better goodness-of-fit of the given function.

3. Results

3.1 Overview

In this chapter, the results of this research are presented. As mentioned in chapter 2, the three events examined in detail are 23-24 January 2020, 14-15 February 2021, and 20 April 2021, and the analyses for these cases are shown in sections 3.2, 3.3, and 3.4, respectively. Within each of these sections, an overview of the background meteorological conditions will be explained, followed by analysis of environment, fall velocity vs maximum dimension calculations, and PSDs for a multitude of times during the event. Finally, each section will have a summary explaining findings from each event.

3.2 Case Study I: 23-24 January 2020

3.2.1 Event Overview

This case is marked by an overall setup differing from what is typical for a wintertime snow event in Missouri. During the day on January 23, a closed center of low pressure developed over the northwestern part of the state and progressed quite slowly to the east/southeast from the evening of 23 January through the morning of 24 January. Figure 13 shows the surface analyses generated by the Weather Prediction Center (WPC) valid at 0000 and 1200 UTC on 24 January 2020. While the low pressure center was not particularly deep, the slow forward movement of the low pressure center allowed for a long duration of continuous snowfall at the disdrometer site, and further resulted in accumulations of snowfall of 10-15 cm in the areas near the disdrometer. Figure 14 shows a time-height of reflectivity from the MRR, positioned right next to the disdrometer, for the event. The particular times of interest analyzed are 0000, 0300, 0600, and 1000 UTC on 24 January.

Figure 13 – Surface analysis from the WPC valid at 0000 UTC (top) and 1200 UTC (bottom) on 24 January 2020, showing clearly the closed low located over central Missouri.

Figure 14 – Time-heights of reflectivity values from the Micro-Rain Radar (MRR) at Bradford Farm from 1800-2359 UTC 23 January (top) and 24 January. It appears as though the MRR experienced difficulty detecting snowfall intensity after 2300 UTC 23 January, and it shuts off abruptly around 1600 UTC on 24 January.

3.2.2 0000 UTC 24 January Analysis

The RAP analysis sounding valid at 0000 UTC on 24 January is shown in figure 15. The sounding shows that the 1000-700-hPa layer is very close to saturation with respect to water. Temperatures in this region of the sounding decrease rather slowly, with a surface temperature near 0°C and a temperature at the top of the layer around -7°C. Above this layer, the atmosphere is no longer saturated with respect to water, but with dewpoint depressions less than 3°C, the layer is likely supersaturated with respect to ice,

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In determining the dominant crystal type, it is important to note that whatever crystals have mainly developed above 700 hPa are likely to aggregate below that level, considering the temperatures and dewpoints in that layer. This is supported by the observed range of maximum dimensions. Figure 16 shows a plot of the log of snowflake concentration vs maximum dimension for the disdrometer data gathered between 0000 and 0015 UTC. The disdrometer observed several snowflakes over the 15-minute period from 0000-0015 UTC that had maximum dimensions greater than 4 mm, indicating there were likely aggregates falling at this time. As for the constituent crystal type of those aggregates, it appears that the layer above 700 hPa would favor the growth of a plate-type crystal, but the degree of supersaturation does not appear to be great enough to produce a large number of true dendrites. Therefore, the dominant snow type for this analysis is taken to be aggregates of plates.

Figure 16 – Plot showing the log of snowflake concentration vs each snowflake maximum dimension class, made from data gathered between 0000 and 0015 UTC 24 January. This plot is chosen to determine the range of sizes of snowflakes because it can resolve the lower concentrations of larger flakes best.

The selection of aggregates of plates as the dominant ice crystal falling at this time is then applied to calculate a theoretical curve modeling the relationship between sizes and fall velocities of these snowflakes using the Heymsfield and Westbrook (2010) method. The results are shown in figure 17. Overall, the theoretical relationship produced using this method generates expected values for velocity that are on the higher end of the range observed for each size class. The theoretical curve does match the overall shape of the observed velocity vs maximum dimension data, however.

Figure 17 – Plot showing the relationship between fall velocity and maximum dimension of snowflakes for the data gathered between 0000 and 0015 UTC on 24 January. The data are sorted by size and velocity class, with the shading indicating the concentration of snowflakes fitting each particular size and velocity class.

The PSD for this data, along with curves representing the Gauss3, gamma, and Weibull modeled functions, is plotted in figure 18. The PSD itself has a maximum at the 0.687 mm size class. It is skewed to the right, with non-negligible concentrations all the way up to 2.75 mm. The three model functions seem to model the PSD well up to the 1.187 mm size class, but after that, they diverge, with the Weibull and gamma model functions underestimating the concentrations of snowflakes in the tail region of the distribution, while the Gauss3 model overestimates the concentration of snowflakes in this region. Goodness-of-fit statistics of the three models for this period are shown in

table 4. By every statistic shown, it is the gamma distribution that models the PSD from this period well. Of the two other models, the Gauss3 had the better SSE and r^2 , while the Weibull distribution had the better adjusted r^2 and RMSE. It should be noted, however, that all of the goodness-of-fit statistics for each model are pretty close to one another, and r^2 and adjusted- r^2 values for each are greater than 0.94, indicating that all three models fit the observed PSD well.

Figure 18 – PSD of disdrometer observations from 0000 to 0015 UTC on 24 January (blue line), along with modeled fits of the Gauss3 (red), gamma (green), and Weibull (magenta) functions.

Metric	Model		
	Gauss3	Gamma	Weibull
SSE	115060	90855	160560
r^2	0.9747	0.98	0.9647
Adjusted r^2	0.9458	0.9769	0.9593
RMSE	128.2076	83.5994	111.1356

Table 4 – Goodness-of-fit statistics for the modeled Gauss3, gamma, and Weibull functions from the 0000-0015 UTC PSD.

3.2.3 0300 UTC 24 January Analysis

Between 0000 and 0300 UTC, very few discernable changes occur in the overall environment, as evidenced by the fact that the RAP analysis sounding from 0300 UTC (figure 19) looks almost identical to the 0000 UTC sounding. The 1000-700-hPa layer remains saturated with respect to water, and the overall temperature and moisture profiles above that layer strongly resemble those generated in the 0000 UTC analysis. If anything, the layer from 850-400 hPa has cooled slightly, and the magnitudes of omega in the column have decreased. With very few differences between the 0000 and 0300 UTC analyses, it is logical to maintain aggregates of plates as the selected crystal type. This is further supported by figure 20, which shows a similar range of maximum dimension values for the 0300 to 0315 UTC period as the range shown in figure 16. During this 15-minute period, however, the largest maximum dimension observed by the disdrometer was smaller than that shown in figure 16. Nevertheless, aggregates of plates are maintained as the dominant crystal type.

With the dominant crystal type remaining the same at 0300 UTC as 0000 UTC, the same theoretical curve modeling the relationship between fall velocity and maximum

dimension was applied to the disdrometer data gathered between 0300 and 0315 UTC. As with the 0000 UTC data, the model curve relating fall velocities and maximum dimensions of aggregates of plates falls on the upper end of most of the observed data (figure 21). It is noteworthy, though, that for the size classes >2 mm, the modeled curve transitions from being on the high end to being in the middle or even on the lower even of the observed range of fall velocity values. Based on the way the disdrometer observations are distributed, this may suggest that the rate of change of fall velocity with respect to maximum dimension for snowflakes may be too small.

Figure 19 – RAP analysis sounding valid at 0300 UTC on 24 January for Bradford Farm.

Figure 20 – Plot showing the log of snowflake concentration vs each snowflake maximum dimension class, made from data gathered between 0300 and 0315 on UTC 24 January.

The PSD for this data (figure 22) does exhibit different characteristics than the PSD from the 0000-0015 UTC data. First, the maximum concentration was observed at the 0.812 mm size class. Second, the overall PSD is more strongly skewed to the right for this period, with greater concentrations of larger snowflakes observed by the disdrometer. All three of the model functions seemed to have much more difficulty fitting this PSD, with all three underestimating the highest concentration values around the 0.812 mm size class and struggling to model the right tail region of the distribution. As such, goodness-of-fit statistics were lower for this PSD than the 0000 UTC PSD. The values for these statistics are shown in table 5. Still, the modeled gamma function had the best values

across all goodness-of-fit statistics, with the Gauss3 and Weibull modeled functions following the same pattern as with the 0000 UTC PSD.

Figure 21 – Plot showing the relationship between fall velocity and maximum dimension of snowflakes for the data gathered between 0300 and 0315 UTC on 24 January.

Figure 22 – PSD of disdrometer observations from 0300 to 0315 UTC on 24 January (blue line), along with modeled fits of the Gauss3 (red), gamma (green), and Weibull (magenta) functions.

Metric	Model		
	Gauss3	Gamma	Weibull
SSE	347250	255570	442240
r^2	0.9187	0.9402	0.8964
adjusted r^2	0.8258	0.931	0.8805
RMSE	222.7275	140.2107	184.4417

Table 5 – Goodness-of-fit statistics for the modeled Gauss3, gamma, and Weibull functions from the 0300-0315 UTC PSD.

3.2.4 0600 UTC 24 January Analysis

As was the case between 0000 and 0300 UTC, the environment appeared to change very little between 0300 and 0600 UTC. The RAP analysis sounding valid at 0600 UTC (figure 23) once again bears strong resemblance to the soundings previously analyzed. There continued to be a completely saturated layer from 1000 to roughly 700 hPa, with surface temperatures near 0°C. Because of the lack of change in environmental conditions shown by the RAP, the expected dominant crystal type remains aggregates of plates. This is supported by disdrometer observations, which indicate a significant number of flakes larger than 4 mm fell between 0600 and 0615 UTC (figure 24).

Figure 23 – RAP analysis sounding valid at 0600 UTC on 24 January for Bradford Farm.

Figure 24 – Plot showing the log of snowflake concentration vs each snowflake maximum dimension class, made from data gathered between 0600 and 0615 on UTC 24 January.

Once again, the same theoretical model relating maximum snowflake dimension and fall velocity was plotted over the disdrometer observations (figure 25). For size classes under 2 mm, this curve once again passes through the high end of the range of observed velocity values for each maximum dimension measurement. However, as observed with the 0300-0315 UTC plot, for size classes greater than 2 mm, the model curve passes through the middle of the range of observed velocities for each size class. It should be noted that on this plot, more so than the previous two of this type, there is a greater spread in velocities by size class, which may indicate that aggregates of plates weren't the only crystal type observed in large numbers.

Figure 25 – Plot showing the relationship between fall velocity and maximum dimension of snowflakes for the data gathered between 0600 and 0615 UTC on 24 January.

The PSD of this data is shown in figure 26. Overall, there were lower concentrations during this period than in the first two periods analyzed. The maximum concentration returned to the 0.687 mm size class, but the overall shape of the distribution remained skewed to the right. The three functions again did not model this distribution perfectly, with all three underestimating values in the tail, and the gamma and Weibull underestimating values in the peak region. When goodness-of-fit statistics were analyzed, the Gauss3 modeled fit had the best r^2 and SSE, while the gamma had the best adjusted- r^2 and RMSE (table 6). The Weibull had the worst values for all four statistics. It should be noted that, once again, with adjusted- r^2 values below 0.93 for all models, there is still some data in the PSD for which these models can't account.

Figure 26 – PSD of disdrometer observations from 0600 to 0615 UTC on 24 January (blue line), along with modeled fits of the Gauss3 (red), gamma (green), and Weibull (magenta) functions.

Metric	Model		
	Gauss3	Gamma	Weibull
SSE	67814	85050	153150
r^2	0.9483	0.9352	0.8832
Adjusted r^2	0.8892	0.9252	0.8653
RMSE	98.4262	80.8843	108.5405

Table 6 – Goodness-of-fit statistics for the modeled Gauss3, gamma, and Weibull functions from the 0600-0615 UTC PSD.

3.2.5 1000 UTC 24 January Analysis

The final time period analyzed for this event is 1000 UTC on 24 January. Much like the preceding soundings analyzed, this RAP analysis sounding valid at this time (figure 27) shows saturation with respect to water in the lower levels of the atmosphere, although the level where the atmosphere is no longer saturated with respect to water lowered from 700 to 750 hPa. Otherwise, most of the characteristics of this sounding are in line with previous soundings, save for perhaps a bit more cooling between 850 and 500 hPa. The temperature at the surface was still near 0°C. With most attributes remaining the same from previous soundings, it makes sense to continue analyzing this event with the dominant snow crystal being aggregates of plates.

Figure 27 – RAP analysis sounding valid at 1000 UTC on 24 January for Bradford Farm.

Figure 28 – Plot showing the log of snowflake concentration vs each snowflake maximum dimension class, made from data gathered between 1000 and 1015 on UTC 24 January.

An interesting point to note, though, is that there were fewer large aggregates observed by the disdrometer during the 1000-1015 UTC timeframe. As shown in figure 28, the largest observed flakes were around 3.75 mm in dimension, while most of the snowflakes observed had maximum dimensions less than 1 mm. This may suggest that fewer of the snowflakes were able to aggregate, and those that did were not able to aggregate as effectively as earlier in the event.

With aggregates of plates selected as the dominant crystal type, the same model curve applied to the previous velocity-dimension plots was applied to the plot showing the data gathered between 1000 and 1015 UTC on January 24 (figure 29). The model

curve fits the observed data in much the same way as the previous plots: it is on the upper end of the observed range of fall velocities for size classes below approximately 2 mm. For size classes above 2 mm, the curve goes right through the middle of the plotted velocity data.

Figure 29 – Plot showing the relationship between fall velocity and maximum dimension of snowflakes for the data gathered between 1000 and 1015 UTC on 24 January.

Because the overall range of dimension sizes observed by the disdrometer has decreased for this period, the PSD is dominated by smaller snowflakes sorted into the 0.437 and 0.562 mm size classes (figure 30). While the right tail is still apparent, it is not as pronounced as in previous PSDs during this event. When the three model functions are

fit to the PSD, they all fit the distribution quite well. The goodness-of-fit statistics for this period are shown in table 7. The most noteworthy change from previous distributions is the Gauss3 model has the best values for all four goodness-of-fit statistics, followed by the gamma and then the Weibull. This seems to suggest the Gauss3 model performs best when the concentration values in the right tail region are low.

Figure 30 – PSD of disdrometer observations from 1000 to 1015 UTC on 24 January (blue line), along with modeled fits of the Gauss3 (red), gamma (green), and Weibull (magenta) functions.

Metric	Model		
	Gauss3	Gamma	Weibull
SSE	53302	591350	1319000
r^2	0.9956	0.9517	0.8924
Adjusted r^2	0.9907	0.9443	0.8758
RMSE	87.2615	213.2797	318.5319

Table 7 – Goodness-of-fit statistics for the modeled Gauss3, gamma, and Weibull functions from the 1000-1015 UTC PSD.

3.2.6 Case I Summary

In this section, several different periods of time during this event were analyzed. For each period, the dominant crystal type appeared to remain consistent, as RAP analysis soundings and observed concentrations of snowflakes by maximum dimension changed very little throughout the course of the event. This allowed for the same theoretical velocity-dimension curve to be overlain on each plot analyzed. Overall, the theoretical curve tended to overestimate velocities for snowflakes with maximum dimensions less than 2 mm, while it estimated right in the middle of the observed range of velocities for snowflakes with maximum dimensions greater than 2 mm. It should be noted that, in all of the velocity-dimension plots, for any given maximum dimension class, there was a range of as many as 10 velocity classes observed. This indicates that there is quite a bit of spread in the observed data.

When it comes to the modeling of PSDs, the goodness-of-fit statistics for all 70 15-minute PSDs gathered during this event are in table 8. Despite the periods of time chosen for examination in this section, the Gauss3 function was, by both r^2 and SSE, the best function to model these PSDs overall. On the other hand, the gamma distribution

overall had the better values for adjusted r^2 and RMSE, making it the better model by these metrics. The gamma model especially did a better job handling PSDs whose concentrations of snowflakes for size classes in the tail region were higher. The reasoning for the gamma model performing best per adjusted r^2 and RMSE is that both of those statistics rely on the number of predictors in their calculations. Because the gamma model only depends on three predictors, or coefficients, while the Gauss3 model depends on nine, for equal r^2 values between the gamma and Gauss3 models, the gamma will have a higher adjusted r^2 values due to fewer coefficients. Moreover, the Weibull had worst values for all metrics, indicating it fit the data most poorly of the three models.

Metric	Model		
	Gauss3	Gamma	Weibull
Mean SSE	144699.8	248878.7	494407.9
Mean r^2	0.961093	0.940436	0.897869
Minimum r^2	0.7938	0.7401	0.6497
Maximum r^2	0.999	0.9915	0.9821
Standard deviation r^2	0.040253	0.047742	0.05893
Mean adjusted r^2	0.91662	0.931251	0.88216
Minimum adjusted r^2	0.5581	0.7001	0.5958
Maximum adjusted r^2	0.9979	0.9901	0.9794
Standard deviation adjusted r^2	0.086269	0.055078	0.067992
Mean RMSE	123.5264	122.9982	171.4843

Table 8 – Goodness-of-fit statistics across all 70 15-minute PSDs from 23-24 January 2020.

3.3 Case Study II: 14-15 February 2021

3.3.1 Event Overview

This case marks another interesting background setup for snowfall in central Missouri. During this event, a large arctic ridge was situated over the upper Midwest, while a strong cyclone developed over the Gulf of Mexico and moved into the southeastern United States. Figure 31 shows this progression. This cyclone was strong enough that it was able to draw moisture all the way into the Midwest, resulting in several waves of snowfall in very cold conditions. The reflectivity time-height from the MRR is shown in figure 32. An important note in the analysis of this event is that, at certain times, the MRR would observe precipitation but the Parsivel disdrometer would not. Therefore, there were less 15-minute data samples available for this event, despite the fact that it was a longer-duration event.

Figure 31 – WPC surface analyses valid for 1200 UTC 14 February (top left), 0000 UTC 15 February (top right), 1200 UTC 15 February (bottom left) and 0000 UTC 16 February (bottom right).

Figure 32 – Time heights of reflectivity values gathered by the MRR from 14 February (top) and 15 February (bottom).

3.3.2 1700 UTC 14 February Analysis

When examining the RAP analysis sounding valid for this time (figure 33), it is noteworthy that the temperature profile passes through the temperature range best suited for dendritic growth of ice crystals twice, once between 800 and 750 hPa, and again between 600 and 500 hPa. Unlike the previous case, it was exceptionally cold at the surface at this time, with an analyzed surface temperature of approximately -20°C. Considering the cold temperatures and lack of saturation with respect to water throughout, it is unlikely that a large number of aggregates fell. Considering a very

significant depth of the atmosphere was within the temperature range for dendritic growth, dendrites were selected as the primary crystal type for this period. It is conceivable, however, that plates and plate variants fell along with dendrites during this time due to much to the atmosphere not being saturated with respect to water.

Figure 33 – RAP analysis sounding valid at 1700 UTC on 14 February for Bradford Farm.

Furthermore, the observed range of sizes agrees with the assessment of little to no aggregation occurring, as the largest snowflakes observed by the disdrometer were in the 2.375 mm size class. The log of snowflake concentration vs maximum dimension for the period 1700-1715 UTC on 14 February is plotted in figure 34. Overall, it is dendrites that are selected as the dominant crystal type at this time.

Figure 34 – Plot showing the log of snowflake concentration vs each snowflake maximum dimension class, made from data gathered between 1700 and 1715 on UTC 14 February.

With dendrites being the preferred crystal type, it should be noted that mass- and area-dimensional relationships used in the Heymsfield and Westbrook (2010) method have only been derived for densely-rimed dendrites (Locatelli and Hobbs 1974). Being

that riming was not considered in this research, it is entirely possible the actual snowflakes that fell during this period were not rimed as heavily as those for which the mass- and area-dimensional power laws were derived. Nevertheless, the formulas for densely-rimed densities will be used in plotting the theoretical relationship between fall velocity and maximum dimension for all dendrite cases.

Upon examination of the fall velocity vs maximum dimension plot for this period (figure 35), there is large variability in fall velocity for any given size class. Specifically, for the size classes around 0.5 mm, there appears to be two preferred velocities-one just above 1 m/s and the other around 2.5 m/s. Given the environment, this could be an indicator of multiple crystal types dominating, or, considering the size of the snowflakes, this could be an issue of orientation as the flake passes through the laser of the disdrometer. In any event, because of the variability in velocity by size class is so large, the dendrite curve does not fit the data well.

Figure 35 – Plot showing the relationship between fall velocity and maximum dimension of snowflakes for the data gathered between 1700 and 1715 UTC on 14 February.

The PSD for this period (figure 36) exhibits a much less sharply right-skewed shape than the PSDs analyzed in case I. As such, all three functions model the PSD well, as evidenced by r^2 values less than 0.95 for each function. The Gauss3 model, though, performs the best by all goodness-of-fit metrics (table 9), followed by the gamma function and then the Weibull function.

Figure 36 – PSD of disdrometer observations from 1700 to 1715 UTC on 14 February (blue line), along with modeled fits of the Gauss3 (red), gamma (green), and Weibull (magenta) functions.

Metric	Model		
	Gauss3	Gamma	Weibull
SSE	126050	1087800	5469200
r^2	0.999	0.9911	0.9552
Adjusted r^2	0.9978	0.9897	0.9483
RMSE	134.1892	289.2758	648.6179

Table 9 – Goodness-of-fit statistics for the modeled Gauss3, gamma, and Weibull functions from the 1700-1715 UTC PSD.

3.3.3 1000 UTC 15 February Analysis

This period differs from the one previously analyzed because, per the RAP analysis sounding valid at this time (figure 37), most of the atmosphere between 900 and 500 hPa resides in the temperature range most suitable for the growth of dendrites. Despite the lack of saturation with respect to water shown on this sounding, dendrites remain the selected dominant snow crystal type because of the depth of the layer where temperatures are suitable for the growth of dendrites. This is further supported by the log concentration vs maximum dimension plot, which shows that snowflakes up to the 3.25 mm size class were recorded. Again, as stated for the previous period analyzed, it is entirely possible some plate variants were falling in addition to dendrites.

Figure 37 – RAP analysis sounding valid at 1000 UTC on 15 February for Bradford Farm.

Figure 38 – Plot showing the log of snowflake concentration vs each snowflake maximum dimension class, made from data gathered between 1000 and 1015 on UTC 15 February.

When examining the fall velocity vs maximum dimension data (figure 39), like the previous period, the period between 1000 and 1015 UTC showed a lot of variability in velocity values by size class. There were, once again, two preferred velocities for the size classes near 0.5 mm—one just over 1 m/s and one around 2.5 m/s. Therefore, as was the case with the plot from the first period of this event, the theoretical curve plotted over this data does not fit it well due to the large variability in velocity values.

Figure 39 – Plot showing the relationship between fall velocity and maximum dimension of snowflakes for the data gathered between 1000 and 1015 UTC on 15 February.

The PSD for this time period has a very similar shape to that from section 3.3.2. This PSD once again has low concentration values in the tail region of the distribution (>1.375 mm) when compared with those in the size classes around 0.5 mm. Much like in the previous analysis, all three functions model the PSD well, with each fit producing an r^2 value greater than 0.97. In this case, it was the gamma fit that performed slightly better than the Gauss3 fit per r^2 , adjusted r^2 and RMSE (table 10). The Gauss3 model produced the lowest value for SSE. It is important to note that all three models overestimated the maximum snowflake concentration.

Figure 40 – PSD of disdrometer observations from 1000 to 1015 UTC on 15 February (blue line), along with modeled fits of the Gauss3 (red), gamma (green), and Weibull (magenta) functions.

Metric	Model		
	Gauss3	Gamma	Weibull
SSE	126050	594480	3599800
r^2	0.9941	0.9951	0.9704
Adjusted r^2	0.9874	0.9944	0.9658
RMSE	319.8106	213.8448	526.2231

Table 10 – Goodness-of-fit statistics for the modeled Gauss3, gamma, and Weibull functions from the 1000-1015 UTC PSD.

3.3.4 2000 UTC 15 February Analysis

Much like the RAP analysis sounding from 1000 UTC on February 15, the RAP analysis sounding valid at 2000 UTC on 15 February 2021 (figure 41) shows much of the layer between 900 and 500 hPa residing in the prime temperature range for dendritic growth. As this sounding looks very similar to the 1000 UTC sounding, dendrites remain the primary crystal type for this analysis. This, once again, is supported by the log concentration vs dimension plot from this period, which is very similar to the one using data from 1000-1015 UTC, albeit there are a few snowflakes in the 3.75 mm size class.

Figure 41 – RAP analysis sounding valid at 2000 UTC on 15 February for Bradford Farm.

Figure 42 – Plot showing the log of snowflake concentration vs each snowflake maximum dimension class, made from data gathered between 2000 and 2015 on UTC 15 February.

As was the case for the other two time periods examined for this case, the velocity data for this period varied greatly for each size class. The data are plotted in figure 43. Once again, around 0.5 mm, there were two velocity values where drop concentrations were maximized, one just above 1 m/s and one around 2.5 m/s. Given the variability, the theoretical curve does not fit the plot well, as was the case for the other velocity vs dimension plots in this case.

Figure 44 shows the PSD from the disdrometer measurements made between 2000 and 2015 UTC on February 15. This PSD resembles the other PSDs analyzed in this case, with a peak in concentration near 0.5 mm, with very low concentrations of

snowflakes above the 1.375 mm size class. When the models are applied, the gamma and Weibull fits both estimate the peak of the PSD well, but underestimate the concentrations above 1 mm. The Gauss3 fit overestimates the peak of the PSD, but models the tail of the distribution fairly well. Goodness-of-fit statistics are shown in table 11. Overall, all three functions model the PSD well, as indicated by r^2 values of 0.94 for all three models. Examining the models against one another, the Gauss3 model was the best by all metrics, followed by the gamma and then the Weibull.

Figure 43 – Plot showing the relationship between fall velocity and maximum dimension of snowflakes for the data gathered between 2000 and 2015 UTC on 15 February.

Figure 44 – PSD of disdrometer observations from 2000 to 2015 UTC on 15 February (blue line), along with modeled fits of the Gauss3 (red), gamma (green), and Weibull (magenta) functions.

Metric	Model		
	Gauss3	Gamma	Weibull
SSE	264660	3603300	14193000
r^2	0.999	0.9865	0.9469
Adjusted r^2	0.9979	0.9844	0.9387
RMSE	194.4438	526.4792	1044.9

Table 11 – Goodness-of-fit statistics for the modeled Gauss3, gamma, and Weibull functions from the 2000-2015 UTC PSD.

3.3.5 Case II Summary

In this section, three different periods from this event were analyzed. Much like case I, the dominant snow crystal type did not seem to change much throughout the duration of the event. RAP analysis soundings and the range of sizes observed by the Parsivel disdrometer were roughly the same for each period analyzed, with the 1700-1715 UTC February 14 period showing the smallest maximum dimension observed of the three periods.

When plotting fall velocity vs maximum dimension for the periods analyzed in this case, there was large variability in the observed velocities for each size range. Therefore, the theoretical curve modeling the relationship between fall speed and maximum dimension for dendrites did not describe the observations well. The variability in velocity could be attributed to a couple of problems: either there were large numbers of multiple crystal types falling during the period, or, because the snowflakes were so small, they may have been randomly oriented, resulting in random fall velocities for any individual snowflake.

For the 56 15-minute PSDs collected during this case, the overall goodness-of-fit statistics are presented in table 12. For this case, by all statistics, it was the Gauss3 function that performed best overall in modeling these PSDs. The Gauss3 was followed by the gamma and then the Weibull functions. For both the gamma and Gauss3 models, no fit produced an r^2 value below 0.925 or an adjusted r^2 value of 0.91, indicating both of these functions modeled each PSD well.

Metric	Model		
	Gauss3	Gamma	Weibull
Mean SSE	121767.7	1553275	5633006
Mean r^2	0.998763	0.98365	0.948134
Minimum r^2	0.9898	0.925	0.8647
Maximum r^2	0.9999	0.9993	0.984
Standard deviation r^2	0.001647	0.01581	0.027213
Mean adjusted r^2	0.997354	0.981132	0.940154
Minimum adjusted r^2	0.9781	0.9135	0.8439
Maximum adjusted r^2	0.9997	0.9992	0.9815
Standard deviation adjusted r^2	0.003521	0.01823	0.031404
mean RMSE	104.9669	271.9166	547.7716

Table 12 – Goodness-of-fit statistics across all 56 15-minute PSDs from 14-15 February 2021.

3.4 Case Study III: 20 April 2021

3.4.1 Event overview

Unlike the first two case studies, this event was quite short in duration. The surface analysis from 1200 UTC on April 20 is shown in figure 45. The precipitation occurred behind a cold front that had passed through central Missouri a day earlier. Even though the event was short-lived, it snowed hard enough to allow for minor accumulations in areas where the soil temperature was 8-10°C. Precipitation observed by the MRR is shown in 46. It should be noted that the MRR and disdrometer went offline around 1630 UTC, so only the beginning of the snow event is analyzed.

Figure 45 – WPC surface analysis valid for 1200 UTC on April 20, 2021.

Figure 46 – MRR reflectivity from 1100-1700 UTC on April 20.

3.4.2 1400 UTC 20 April analysis

The RAP analysis sounding valid at this time indicates the atmosphere is saturated with respect to water from the surface almost all the way to 600 hPa. Temperatures throughout this layer remain between -10°C and 0°C, with the warmest temperatures at the surface. With this thermal profile, it would be expected for aggregates to form. When looking at the plot of log concentration vs maximum dimension observed by the disdrometer between 1400 and 1415 UTC, the largest flakes observed are in the 3.75 mm size class, so aggregates were not growing very large at this time. Given the temperature and moisture profiles above 600 hPa, it seems likely that plate-type crystals were forming in that region, aggregating as they descend. Therefore, aggregates of plates were selected as the dominant crystal type for this analysis.

Figure 47 – RAP analysis sounding valid at 1400 UTC on 20 April for Bradford Farm.

Figure 48 – Plot showing the log of snowflake concentration vs each snowflake maximum dimension class, made from data gathered between 1400 and 1415 on UTC 20 April.

When examining how the theoretical curve relating fall velocities with maximum dimensions of snowflakes, the same issue is experienced in this case as with case II. The curve was plotted over the observed data, as shown in figure 49. The theoretical curve itself passes right through the area of the plot with the greatest concentrations, but the variability in fall velocity for each size class, especially the smaller classes, is so large that the theoretical curve doesn't really describe the distribution.

Figure 49 – Plot showing the relationship between fall velocity and maximum dimension of snowflakes for the data gathered between 1400 and 1415 UTC on 20 April.

Examining the PSD for this period (figure 50), its shape most closely resembles the PSDs from case II, in that there is a concentration peak around 0.5 mm, and relatively low concentrations above 2 mm. Of the three models fit to this PSD, the Gauss3 performs best by all goodness-of-fit statistics, followed by the gamma and then the Weibull functions (table 13). The Gauss3 overestimates the peak slightly, but models the tail well; meanwhile, the gamma and Weibull functions both underestimate the peak and underestimate the size from about 0.8 mm on. Nevertheless, all three functions perform well in modeling the PSD.

Figure 50 – PSD of disdrometer observations from 1400 to 1415 UTC on 20 April (blue line), along with modeled fits of the Gauss3 (red), gamma (green), and Weibull (magenta) functions.

Metric	Model		
	Gauss3	Gamma	Weibull
SSE	320670	2390600	6744600
r^2	0.9964	0.9733	0.9246
Adjusted r^2	0.9923	0.9691	0.9129
RMSE	214.0327	428.8296	720.2896

Table 13 – Goodness-of-fit statistics for the modeled Gauss3, gamma, and Weibull functions from the 1400-1415 UTC PSD.

3.4.3 1600 UTC 20 April Analysis

The RAP analysis sounding valid at 1600 UTC on 20 April (figure 51) shows that all of the atmosphere but the lowest 150 hPa is no longer saturated with respect to water. With temperatures at the surface around 0°C, aggregation is likely to occur. The lack of saturation with respect to water above 850 hPa serves to restrict the ice crystal growth to plate types. This selection of aggregates as the dominant crystal type is well supported by figure 52, which shows significant concentrations of snowflakes above 5 mm in dimension, with some snowflakes measures as large as 9.5 mm.

Figure 51 – RAP analysis sounding valid at 1400 UTC on 20 April for Bradford Farm.

Figure 52 – Plot showing the log of snowflake concentration vs each snowflake maximum dimension class, made from data gathered between 1600 and 1615 on UTC 20 April.

For the fall velocity-dimension plot, the velocities once again show large variability for each dimension class. The theoretical curve plotted over this data seems to go right through the middle, but like so many of the plots before this one, because of the variability in the data, this single curve carries very little meaning, and it therefore cannot describe the whole distribution of values very well. Like the plots before this one, variability could come as a result of orientation issues and/or different crystal types accounting for large portions of the concentration values.

Figure 53 – Plot showing the relationship between fall velocity and maximum dimension of snowflakes for the data gathered between 1600 and 1615 UTC on 20 April.

The PSD from this period much more closely resembles those from case I, with higher concentration values for size classes greater than 1.375 mm and a low peak concentration value. The Gauss3 model completely underestimates concentrations for size classes greater than 1 mm, resulting in it performing poorly with regards to goodness-of-fit statistics. The gamma function performed best, as it was able to more accurately model the size classes up to around 2 mm than the Gauss3 or Weibull. The Weibull function, however, has better values of goodness-of-fit statistics than the Gauss3 (table 14).

Figure 54 – PSD of disdrometer observations from 1600 to 1615 UTC on 20 April (blue line), along with modeled fits of the Gauss3 (red), gamma (green), and Weibull (magenta) functions.

Metric	Model		
	Gauss3	Gamma	Weibull
SSE	1378200	605810	1236300
r^2	0.9114	0.961	0.9205
Adjusted r^2	0.8101	0.955	0.9083
RMSE	443.7165	215.8723	308.3773

Table 14 – Goodness-of-fit statistics for the modeled Gauss3, gamma, and Weibull functions from the 1600-1615 UTC PSD.

3.4.4 Case III Summary

Case III experienced the same problem as case II: there was a large variability in fall velocity data for each size class. Therefore, the plotted curve for both periods analyzed in closer detail did very little to describe the overall relationship between maximum dimension of a snowflake and its fall velocity. Again, as with case II, this could be attributed to multiple crystal types contributing significantly to the overall concentrations calculated.

As for PSD modeling, as with case study II, the Gauss3 function performed best in modeling the PSDs for this event, followed by the gamma function and then lastly the Weibull function. While the goodness-of-fit metrics were similar between the Gauss3 and gamma fits, the Weibull fit lagged significantly behind. Goodness-of-fit statistics for each of the three models from this case are presented in table 15.

Metric	Model		
	Gauss3	Gamma	Weibull
Mean SSE	575477.7	1736936	4438108
Mean r^2	0.985108	0.965775	0.918492
Minimum r^2	0.9114	0.9681	0.8902
Maximum r^2	0.9988	0.9817	0.9453
Standard deviation r^2	0.024081	0.013266	0.019282
Mean adjusted r^2	0.968075	0.960492	0.905933
Minimum adjusted r^2	0.8101	0.9345	0.8734
Maximum adjusted r^2	0.9974	0.9789	0.9368
Standard deviation adjusted r^2	0.051607	0.015329	0.022221
Mean RMSE	258.5017	348.4187	555.0618

Table 15 – Goodness-of-fit statistics across all 12 15-minute PSDs from 20 April 2021.

4. Conclusions

In this research, an attempt was made to both examine and describe fall velocity vs maximum dimension relationships for different crystal types. This research also evaluated the skill of different functions in modeling PSDs. Measurements were made by the OTT Parsivel disdrometer, and 15-minute datasets were analyzed to assess fall velocity vs dimension relationships and construct and model PSDs.

In this research, the dominant snow crystal type was assessed using RAP analysis soundings and the measured maximum dimension values observed by the disdrometer at that time. There are many flaws inherent in this method. First, as was noted in chapter 2, the RAP analysis tends to smooth out features on the soundings. This can lead to important details, like saturation in the dendritic growth zone, being missed entirely. Second, RAP analysis has a grid spacing of 13 km, so the features of the sounding are not specific to any given location, but instead they are averaged over that 13 km grid box. This too can result in the smoothing out of critical details that would be helpful in determining snow crystal type. Third, RAP analysis is generated every hour, while snow characteristics, including the makeup of crystal types, can change in a matter of minutes. Ultimately, it was found that determining predominant snow crystal type for a given snowfall event is exceedingly difficult without ground observations from that event. Even with the multitude of tools and products available for use, none of them will be able to determine definitively the form of the snow crystals.

Furthermore, as explained in section 2.1, the Parsivel disdrometer is a flawed instrument when measuring snowfall. Because of its flaws, it has a tendency to overestimate the number of small snowflakes and underestimate the number of large

snowflakes (Battaglia et al. 2010). Furthermore, fall velocities may be underestimated for small snowflakes and overestimated for large snowflakes (Battaglia et al. 2010). The disdrometer also does not calculate sizes and fall velocities of hydrometeors explicitly; instead, it sorts these values by bin, so the true size and fall velocity of any given hydrometeor remains unknown.

When fall velocity was plotted against maximum dimension, there are some relationships that don't appear to follow any singular theoretical curve. This differs from the almost parabolic shapes expected by Heymsfield and Westbrook (2010) and observed by Pu et al. (2020). In these instances, it is likely there were multiple different types of snow crystal falling, either mixed together, or switching between types in intervals. Either way, when the dominant crystal type is assumed, and the theoretical curve applied to the observed data, the curve does not describe the relationship well.

Furthermore, one of the key assumptions in the disdrometer measurement framework is that all hydrometeors fall oriented in such a way that their major axis, or maximum dimension in the case of snow, is parallel to the ground (Battaglia et al. 2010). In real situations, this may not be the case. If a snowflake is oriented in any way other than this, the area of the snowflake being acted on by the force of drag decreases, and thus, fall velocity should increase. Small snowflakes are especially prone to random orientation due to their relatively small masses. Larger snowflakes are more likely to oscillate horizontally as they descend, and this may result in inaccurate velocity observations from the disdrometer. All snowflakes may be subject to canting or tilting as they fall, too, which may result in the disdrometer recording maximum dimensions of

snowflakes that are smaller than true values, and may, in turn, lead to overestimations of velocity for snowflakes in smaller size classes.

The next main task of this research was to evaluate different functions that could be used to model PSDs. Jia et al. (2019) determined that, among the exponential, lognormal, Gaussian, gamma, and Weibull functions, the gamma and Weibull functions displayed the best skill in modeling PSDs. Steward (2020) found that the third-iteration Gaussian function models drop-size distributions for rainfall well. Therefore, in this research, the gamma, Weibull, and Gauss3 functions were evaluated to determine modeling skill. In MATLAB, these functions were fit to the PSD through a non-linear least-squares regression method, wherein the programming software determines the combination of coefficients which results in the lowest sum of squares error. A total of 297 15-minute PSDs were evaluated using this method. It was found that, overall, the Gauss3 function performed best, followed by the gamma and then the Weibull.

Moreover, the performance of the three distribution functions in modeling PSDs was assessed for three different snowfall cases. Model performance in case I, 23-24 January 2020, differed from the overall performance across all dates, with the gamma function performing comparably to or even better than the Gauss3 function in modeling PSDs. This is most clearly shown by metrics that take into account the number of predictors in the model, like adjusted r^2 , which was consistently closer to 1 for the gamma model than the Gauss3. Similar results are seen in case III, 20 April 2021, though the Gauss3 function overall performed slightly better than the gamma in this case. In both of these cases, it is likely that snowflakes were aggregating in large numbers, resulting in a smaller difference between the concentrations of large and small snowflakes. This

resulted in the Gauss3 being rather inconsistent in modeling PSDs across both events, as evidenced by it having the highest standard deviation in adjusted r^2 values in both cases, and it having the highest standard deviation in r^2 values in case III. For case II, 14-15 February 2021, a case where aggregation likely wasn't a big factor, the model performances matched the overall trends.

There are many ways this research can be expanded upon. First, the more time the disdrometer is in place at Bradford Farm, the more snow data it will collect. The more data amassed, the better the quality of results. This is especially true with snowfall, where each event differs greatly from the previous. Second, for any additional research, in situ field observations are practically necessary. Only relying on disdrometer-measured data with no contribution from field measurements makes it difficult to truly know what the disdrometer is observing. This is especially true in the case of many different crystal types falling at once. If field measurements can effectively determine the proportion of different crystal types, the disdrometer can then be used much more effectively.

As one final note, the MRR at Bradford Farm was not used in this research. The way the MRR operates is it assumes all hydrometeors are spherical. The MRR records fall velocities, and based on the data it records, it will then calculate the effective diameter of the sphere that would produce that fall velocity. This works perfectly well for rain, being that velocity increases appreciably with increasing diameter. For snowflakes, however, that increase in velocity with maximum dimension is not as large, so there won't be a large range of calculated diameters with observed fall velocity. This makes the MRR a very tricky instrument to use when examining snow.

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