On the Possibility of Paretian Egalitarianism*

1. Introduction

We here address the question of how, for a theory of justice, a concern for the promotion of equality can be combined with a concern for making people as well off as possible. Leximin, which requires making the worst off position as well off as possible, is one way of combining a concern for making people's lives go well with a special concern for those who are especially poorly off. Many egalitarians, however, reject its near-monomaniacal focus on the worst off position (to the exclusion of other poorly off persons). In this paper, we explore the possibility of combining a weak kind of egalitarianism with a weak kind of efficiency requirement in a way that avoids leximin's obsession with the worst off position. For example, one may consider solving all cases where efficiency is not at issue by choosing the alternative that is most equal according to the Gini-coefficient or some other well-established inequality measure. All standard inequality measures sometimes judge an alternative as more equal than another alternative even though the latter maximizes the benefits of the worst off. Thus it may seem like a promising way of avoiding the leximin approach within an egalitarian framework. Surprisingly, given certain generally accepted assumptions, this turns out to be impossible. The only possible way of combining weak egalitarianism with weak efficiency requires, we shall show, the rejection of a widely accepted—but perhaps dubious—contraction consistency condition on justice or the acceptance of some version of the leximin principle.

2. The General Framework

To fully specify an egalitarian theory one must specify the type of benefits that it seeks to equalize. Throughout, we leave open the relevant conception of benefit (resources, primary goods, brute luck well-being, etc.) References to a person being worse off than another should be understood in terms of the relevant benefits.

We shall assume, for the sake of argument, that benefits are fully measurable and interpersonally comparable. This may seem like a strong assumption, but in the present context it is a very weak assumption. The assumption that benefits are so measurable and comparable does not entail that such information is *relevant* for the moral assessment of options. The assumption is simply that such information is available. This ensures that no principle of justice is ruled out merely on the grounds that it presupposes that benefits are measurable or comparable in ways that they are not. One important qualification, however, is needed here: if benefits are not—as required by leximin—ordinally interpersonally comparable (i.e., if there are no facts about who is worse off than whom), then leximin is not possible. Because we are exploring the possibility of avoiding leximin (or something like it) where it is possible, the assumption that benefits are at least so measurable and comparable is essential to our results.

We shall be concerned with the assessment of the justice of alternatives, where alternatives are possible objects of choice (e.g., actions or social policies). Alternatives may have all kinds of features: they generate a certain distribution of benefits, satisfy or violate various rights, involve various intentions, and so on. In what follows, we shall assume that the only relevant information for the assessment of justice is the benefit distribution that an alternative generates. More formally, we shall assume:

Benefitism: Alternatives can be identified with (and thus their justice assessed solely on the basis of) their benefit distributions.

Benefitism is a generalization of welfarism. Although it does not assume that welfare is all that matters, it does assume that justice supervenes on individual benefits. If two alternatives generate the same distribution of benefits, then they have the same status with respect to justice.

Benefitism is a very strong assumption. It rules out the relevance, for example, of respecting the wills of agents (e.g., as reflected in their consent). It holds that there is no difference with respect to justice between forcing a person to go jogging against her will and merely successfully encouraging her to do so, if the benefits to her and everyone else are the same. Although we would reject this condition for this reason, we shall here grant it, since it is not relevant for the issue at hand. The issue of whether something like leximin is the only plausible egalitarian theory of justice arises even if equality promotion is limited, for example, to those alternatives that respect the wills of agents (e.g., violate no rights). Granting Benefitism will simplify the presentation. The results could, however, be recast as applying where the only relevant differences between alternatives concern the distributions of benefit that they generate.

Given Benefitism, we can identify an alternative with the benefit distribution that it generates, and in what follows we shall do so for simplicity. We shall further assume that the set of benefit distributions generated by the set of possible alternatives is *rich* in the following sense:

Domain Richness: For any logically possible benefit distribution there is an alternative that generates that distribution.

This condition rules out, for example, the possibility that, where there are just three people, the distribution <3,7,9> (3 to the first person, 7 to the second, 9 to the third) is not one of the alternatives. All logically possible benefit distributions are among the alternatives. This is not to say that all are part of any given *feasible* set (the alternatives that are open to an agent on a given occasion). Of course, there are lots of logically possible benefit distributions that are not feasible on a given occasion. The claim here is about the range of benefit distributions that can be assessed by justice, equality, or other normatively relevant perspectives. The condition holds that such judgements can be made for all logically possible distributions.

We believe that this is a highly plausible condition. Benefit distributions here play the role of test cases for a theory of justice. All logically possible test cases—assuming, as we shall, a finite population—are admissible.

Justice can be understood in *axiological* terms—what is at least as just as what—or in *deontic* terms—what is just (permitted by justice) relative to a set of feasible alternatives. Deontic justice in this sense need not be grounded in deontological properties. It might be, for example, that an option is deontically just if and only if it maximizes total wellbeing. We shall start by considering axiological justice and then turn to deontic justice.

Axiological justice is concerned with the *justice relation* of (one alternative) being-at-least-as-just-as (another). Following the standard definitions, (1) an alternative is *more just* than another if and only if it is at least as just and the other is not at least as just as it; and (2) an alternative is *equally as just* as another if and only if it is at least as just and that other is also at least as just as it.

3. Paretian Egalitarianism

Our general question is: What kinds of egalitarianism are compatible with the view that increasing everyone's benefits makes things more just? *Pure egalitarianism* holds that justice is concerned only with the equality of the distribution of benefits. Justice is sometimes understood as the *purely comparative* concern for ensuring that people get what they are owed, and, so understood, pure egalitarianism applied to brute luck benefits (benefits that were not deliberately produced by anyone) is not an implausible view. From a purely comparative perspective, it is worse when some get more brute luck benefits than others. Here, however, we are concerned with justice understood as the concern for people getting what they are owed in a *non-purely comparative manner*, and part of what individuals are so owed, we shall assume, is that their benefits be as high as possible, given the benefits that others receive. As a theory of justice so understood, pure egalitarianism is implausible, because it holds, for example, that perfect equality (e.g., <5,5,5>) is more just than everyone having more benefits but with an unequal distribution (e.g., <6,9,12>). Pure egalitarianism fails, that is, to recognize that justice is concerned in part with increasing the benefits that individuals obtain.

This problem with pure egalitarianism has lead many egalitarians to embrace *leximin*, which holds that justice is increased if and only if (1) the benefits to the worst off position are increased, or, (2) in case of ties, the benefits to the second worse off position are increased, and so on. Leximin adequately recognizes that justice is concerned with increasing people's benefits, but it suffers from a different problem. It gives *absolute* priority to the worst off position. It holds that justice can require giving a very small benefit to the worst off position rather than giving large benefits to many slightly better off people who would still have less than average benefits (e.g., that <2,2,2,2,400> is more just than <1,80,80,80,400>).

Leximin is but one of a class of principles that give strict priority to benefits to the worst off position. In what follows, we shall be concerned with *weak maximin*, which is the principle that an alternative is more just if it makes the worst off position better off. Weak maximin is silent about cases where the worst off position is equally well off (e.g., <2,3,5> vs. <2,4,4>), and is compatible with various ways of handling such cases. In what follows we shall examine the possibility of avoiding the strict priority of benefits to the worst off required by weak maximin.

A different—and seemingly more promising—way of combining a concern for the worse off with a concern for increasing people's benefits is (axiological) *Paretian Egalitarianism*, which is the conjunction of Strong Pareto and Weak Egalitarianism, as defined below.

Strong Pareto: For any two alternatives x and y, if each person has at least as much benefits in x as in y, then (1) x is as least as just as y, and (2) if there is at least one person that has more benefits in x than in y, then x is more just than y.

This is a weak efficiency condition on the promotion of benefits (much weaker than the utilitarian sum-total conception of efficiency). It requires, for example, that <2,4,6> be judged more just than <1,4,6> and also more just than <2,3,6>. It is silent about whether <2,4,6> is more just than <99,1,6>.

For the next condition, we appeal to the equality of distributions (which we will discuss in detail in the next section). We shall appeal to a relation that ranks distributions in terms of how equal they are (e.g., that ranks <2,2,2> as more equal than <1,2,3>). Moreover, we need to introduce the following definition. Call a distribution *anonymously Pareto incomparable* to another just in case it, and each of its benefit permutations (i.e., the same distribution of benefits except perhaps with people occupying different positions in the distribution), is Pareto incomparable (i.e., better for some and worse for some others) to the other distribution. This is a strong kind of Pareto incomparability. For example, <1,3,6> is Pareto incomparable with <2,1,6>

but it is not anonymously Pareto incomparable, since <1,2,6> is a permutation of the latter and it is Pareto inferior to <1,3,6>.

Consider now the following condition:

Weak Egalitarianism: For any two alternatives that are anonymously Pareto incomparable, (1) if their two benefit distributions are equally equal, then they are equally just, and (2) if the benefit distribution of one is more equal than that of the other, then the former is more just.

This is a very weak egalitarian condition. It does not require (as pure egalitarianism does) that equality be the only concern of justice. It only holds that equality is determinative of justice in the case where two alternatives are anonymously Pareto incomparable. It is a very weak egalitarian principle, because it is silent even in many cases of Pareto incomparability (viz., those that are not anonymously Pareto incomparable).

Weak Egalitarianism could be rejected for several possible reasons. One is the denial that equality is ever relevant to justice. A second reason allows that equality is relevant but only as a pro tanto consideration that can be outweighed by other considerations (e.g., sufficient increases in total benefits). A third reason holds that equality is indeed determinative of justice under certain conditions, but these conditions are limited to cases where the total is the same. Equality, on this view, is simply a tie-breaker when the totals are the same.

We believe that equality is more central to justice than these objections allow. In any case, in this paper we want to explore whether weak maximin is forced on a certain kind of weak egalitarianism, and hence we shall accept this condition. The conjunction of Strong Pareto and Weak Egalitarianism defines *Paretian Egalitarianism.* There are different forms of Paretian

Egalitarianism, because the conjunction of these two conditions leaves open how distributions are ranked that are Pareto incomparable but not anonymously Pareto incomparable (e.g., <8,10,12> vs. <9,8,12>). Different forms of Paretian Egalitarianism fill in these cases in different ways.

One version of Paretian Egalitarianism that we shall consider below is *Paretian Moderate Egalitarianism*, which is defined as the conjunction of Strong Pareto and the following stronger equality condition:

Moderate Egalitarianism: For any two alternatives that are Pareto incomparable, (1) if their two benefit distributions are equally equal, then they are equally just, and (2) if the benefit distribution of one is more equal than that of the other, then the former is more just.

This is just like Weak Egalitarianism except that it makes equality determinative of justice whenever two alternatives are Pareto incomparable, and not merely when they are anonymously Pareto incomparable. For example, if <9,8,8> is more equal than <8,10,12>, then it says that it is also more just, whereas Weak Egalitarianism is silent (since they are not anonymously Pareto incomparable). Moderate egalitarianism gives equality the strongest role possible while respecting Strong Pareto. It entails Weak Egalitarianism, but not vice versa.

Paretian Egalitarianism seems like a promising way of combining equality with the efficient promotion of benefits while avoiding weak maximin's obsession with the worst off individual. Paretian Egalitarians, for example, may wish to hold that <2,10,100> is less equal than <1,100,100>—and hence less just (given that they are anonymously Pareto incomparable). We shall now see, however, that if certain seemingly intuitively plausible assumptions are satisfied, then Paretian Moderate Egalitarianism is impossible, and Paretian Egalitarianism cannot disagree with weak maximin. It cannot, for example, hold that <2,10,100> is less just—or even less equal—than <1,100,100>. Thus, Paretian Egalitarianism appears not to be a real alternative to weak maximin.[1]

4. The Equality Relation

Paretian egalitarianism appeals to the *equality relation* of one distribution of benefits being at least as equal as that of another. Following the standard definitions, (1) an alternative is *more equal* than another if and only if it is at least as equal and the other is not at least as equal as it; and (2) an alternative is *equally as equal* as another if and only if it is at least as equal and that other is also at least as equal as it. We here identify some uncontroversial assumptions that we make about this relation.

Our framework assumption of Domain Richness is an assumption about the range of benefit distributions that can be assessed by justice, equality, or other normatively relevant perspectives, and thus applies for the equality relation. It thus entails that the domain of the equality relation is the set of all logically possible distributions of benefits. We do not assume that the equality relation is complete, nor, except for one result at the end, that it satisfies any consistency requirement (such as acyclicity).

We shall, however, make the following three assumptions about this relation, each of which is at least close to being a conceptual truth about equality.

Consider, first:

Perfect Equality: A distribution in which everyone gets the same benefits is more equal than one in which this is not so.

This is completely uncontroversial. It says, for example, that <3,3,3> is more equal than <2,3,4> and more equal than <5,6,6>.

A second condition is:

Equality Strong Anonymity: For any two alternatives, x and y, if x is at least as equal as y, then, for any alternative, x*, for which the benefit distribution is a permutation of the benefit distribution of x (i.e., has the same pattern of benefits, but perhaps with individuals receiving different allocations), x* is at least as equal as y.

This is a limited anonymity requirement on equality. It requires that, if <2,5,13> is more equal than <2,9,12>, then so is <5,2,13>. Equality Strong Anonymity is highly plausible, since equality is based solely on the pattern of benefits and is not concerned with who gets what.

A third plausible condition is:

Strong Contracting Extremes Equality: If (1a) the benefits of all the best off individuals are reduced without making them cease to the best off, and/or (1b) the benefits of all the worst off individuals are increased without making them cease to be the worst off, and (2) no one else is affected, then the result is more equal than the original distribution.[2]

This condition is also extremely plausible. It holds, for example, that <2,2,5,6,8,8> is more equal than each of <1,1,5,6,9,9>, <2,2,5,6,9,9>, and <1,1,5,6,8,8>.

We are almost ready to begin examining the possibility of Paretian Egalitarianism disagreeing with weak maximin. First, however, it will be useful to note that, *where there are only two people,* weak maximin and the equality relation are intimately related.

> **Result 1:** If (1) we have Benefitism and Domain Richness, (2) the equality relation satisfies Equality Strong Anonymity and Strong Contracting Extremes Equality, and (3) there are only two people, then for any two anonymously Pareto incomparable alternatives, the alternative that makes the worst off position better off is more equal.

The proofs of this and all following results are given in the appendix. We shall simply illustrate the core nature of each result. Consider here the anonymously Pareto incomparable pair <4,2> and <1,5>. The result establishes the intuitively obvious judgement that the former is more equal. Strong Contracting Extremes Equality entails that <2,4> is more equal than <1,5>, and hence Equality Strong Anonymity entails that <4,2> also is.

In the two person case, then, equality always favors the worst off when alternatives are anonymously Pareto incomparable. This ensures that, in the two person case, Weak Egalitarianism never conflicts with weak maximin. If two alternatives are not anonymously Pareto incomparable, then Weak Egalitarianism is silent and no conflict is possible. If the two alternatives are anonymously Pareto comparable, then equality agrees with weak maximin. This need not be the case, however, when there are more than two persons. The conjunction of Strong Contracting Extremes and Equality Strong Anonymity are silent, for example, about whether <6,8,100> is more equal than <100,5,100>. Nonetheless, as we shall now show, if certain additional assumptions are made, then Paretian Egalitarianism must hold that <6,8,100> is more just—and indeed, more equal—than <100,5,100>. More generally, even in the multi-person case, Paretian Egalitarianism, like weak maximin, must not judge an alternative as at least as just as another if it is worse for the worst off position.

5. Axiological Justice

Axiological justice is concerned with the justice relation, and hence we need to specify the structure of this relation. We shall assume that the justice relation satisfies the following consistency condition:

Acyclicity: If, for alternatives $x_1,\ldots x_n$, x_1 is more just than x_2, x_2 is more just than x_3, …. and x_{n-1} is more just than x_n, then x_n is not more just than x_1.

Acyclicity is much weaker than the better known requirement of transitivity. It applies only to chains of where each alternative is more just than its successor (as opposed to being at least as just) and allows the possibilities of silence (no ranking) and of holding the first alternative be judged equally just as the last (whereas transitivity requires that it be judged more just). It is about as close to being uncontroversial as one can get when it comes to consistency requirements on the justice relation.

For some of the results, we will strengthen this condition slightly in two ways. One is to invoke:

Consistency: If, for alternatives $x_1,\ldots x_n$, x_1 is at least as just as x_2, x_2 is at least as just as x_3, ….

and x_{n-1} is at least as just as x_n, then (1) x_n is not more just than x_1, and (2) if for some i inclusively between 1 and n-1, x_i is more just than x_{i+1}, then x_n is not at least as just as x_1.[3]

This strengthens Acyclicity by covering chains where each alternative is at least as just (as opposed to more just) as its successor. If the pairs are equally just, then it concludes that the last alternative is *not more just* than the first (as opposed to the silence of Acyclicity). Moreover, if all the relations in this chain are the relations of being more just, then Consistency strengthens Acyclicity by requiring that the last alternative *not be at least as just* as the first (as opposed to Acyclicity's requirement that it not be more just). Like Acyclicity, Consistency is accepted by almost everyone.

We will also consider the implications of the following strengthening of Acyclicity:

Quasi-Transitivity: If, for alternatives x_1, x_2, and x_3, x_1 is more just than x_2, x_2 is more just than x_3, then x_1 is more just than x_3.

This is strictly stronger than Acyclicity in that it requires that the first element be more just than the last (as opposed to Acyclicity's requirement that the last is not more just than the first). Quasi-Transitivity is strictly weaker than transitivity, since (like Acyclicity) it says nothing about chains in which one or more alternatives is only ranked as at least as just as its successor (as opposed to more just). It is also weaker than Consistency in this respect. It is, however, stronger than Consistency in another respect: for chains of alternatives that are each more just than their successors, it requires that the first be more just than the last, whereas Consistency allows silence.

Quasi-Transitivity is much more controversial than Consistency. Indeed, one of us would reject it on the ground that sometimes silence is appropriate concerning the ranking of the first and last alternatives. Fortunately, our core results do not depend on this assumption. We shall merely note how they can be strengthened if one assumes Quasi-Transitivity as opposed to Consistency.

It is important to note that we do not assume that the justice relation is complete (i.e., that for any two alternatives, at least one is at least as just as the other). Paretian Egalitarians, for example, may remain silent about the ranking of two alternatives which are Pareto incomparable (and hence not covered by Strong Pareto) but not anonymously so (and hence not covered by Weak Egalitarianism). Moreover, even in cases of anonymous Pareto incomparability, Weak Egalitarianism may be silent because the equality relation may be incomplete and be silent about which alternative is more equal. This possible incompleteness of the justice relation is unproblematic for our analysis. Our results only rely on statements about the justice relation and the equality relation in uncontroversial cases.

We are now ready to present our first result concerning Paretian Egalitarianism. Interestingly, given the weak and plausible conditions invoked, it turns out that Paretian Moderate Egalitarianism is impossible.[4]

> **Result 2**: If (1) we have Benefitism and Domain Richness, (2) the justice relation satisfies Acyclicity, and (3) the equality relation satisfies Equality Strong Anonymity and Strong Contracting Extremes Equality, then Paretian Moderate Egalitarianism is impossible.

The force of this result (and the structure of the proof) can be seen by considering <7,8,11>, <10,8,7>, and <7,8,9>. Note that Strong Contracting Extremes Equality and Equality Strong Anonymity jointly entail that <7,8,9> is more equal than <10,8,7>, which in turn is more equal than <7,8,11>. Paretian Moderate Egalitarianism makes the following judgements: (1) <7,8,11> is more just than <7,8,9> (by Strong Pareto), (2) <7,8,9> is more just than <10,8,7> (because the two are Pareto incomparable, <7,8,9> is more equal), and (3) <10,8,7> is more just than <7,8,11> (because the two are Pareto incomparable, <10,8,7> is more equal). These three judgements, however, violate Acyclicity, which, based on the first two judgements, requires that <10,8,7> *not* be judged more just than <7,8,11>. Paretian Moderate Egalitarianism violates Acyclicity, and thus is impossible if Acyclicity is required, as we believe it should be.

Assuming Acyclicity and the other conditions, Paretian Moderate Egalitarianism is impossible. Paretian Moderate Egalitarianism is not, however, the only form of Paretian Egalitarianism. The following result, however, establishes that no form of Paretian Egalitarianism can significantly disagree with weak maximin.

Result 3 : If (1) we have Benefitism and Domain Richness, (2) the justice relation satisfies Acyclicity, and (3) the equality relation satisfies Perfect Equality, then Paretian Egalitarianism judges that a distribution is *not less just* than another if it gives greater benefits to the worst off position. Moreover, (a) if the justice relation also satisfies Consistency, then Paretian Egalitarianism judges that a distribution is *not equally or less just*, if it gives greater benefits to the worst off position, and (b) if the justice relation also satisfies Quasi-Transitivity, then Paretian Egalitarianism judges that a distribution is *more just* than another if it gives greater benefits to the worst off position.

To see the force of this result, consider <3,3,99>, <2,4,4>, and <3,3,3>. Paretian Egalitarianism makes the following judgements: (1) <3,3,99> is more just than <3,3,3> (by Strong Pareto), and (2) <3,3,3> is more just than <2,4,4> (because they are anonymously Pareto incomparable and Perfect Equality entails that <3,3,3> is more equal). Acyclicity then entails that <3,3,99> is not less just than <2,4,4>. Moreover, Consistency entails that it is not equally or less just, and Quasi-Transitivity entails that it is more just. Thus, given the above conditions, Paretian Egalitarianism cannot judge <2,4,4> to be more equal than <3,3,99> (since if it did, given that they are anonymously Pareto incomparable, <2,4,4> would be judged more just, and that contradicts the results just obtained). Equality, that is, cannot be measured in a way that significantly diverges from weak maximin.

Given that Acyclicity is uncontroversial, the result establishes that Paretian Egalitarianism cannot judge an alternative to be *more just* than another if it is worse for the worst off position. This allows, however, that it might judge such an alternative to be *equally as just* as the other. Given that Consistency is also uncontroversial, the results also rule out this possibility. Thus, Paretian Egalitarianism cannot contradict weak maximin. It may be silent in certain cases where one alternative is better for the worst off than another, but it cannot affirm that the second is equally, or more, just than the first. This is enough, it seems, to eliminate any hope of Paretian Egalitarianism being significantly distinct from weak maximin. The much more controversial Quasi-Transitivity is not needed to establish this conclusion—although, if granted, it significantly strengthens the conclusion by ruling out the possibility of silence and thus requiring that Paretian Egalitarianism judge more just an alternative that is better for the worst off position.

It is worth noting that, assuming Consistency, the above result establishes that the measurement of *equality*—and not merely justice—must not contradict weak maximin *where the alternatives are anonymously Pareto incomparable*. This is because Paretian Egalitarianism ranks the justice of such distributions on the basis of their equality. The result does not require that equality agree with weak maximin in other cases. For example, it does not require that <2,9> is more equal than <8,1>, since the two options are not anonymously Pareto incomparable.

It is also worth noting that the conditions of the above result are compatible. It is possible, that is, to satisfy all of them. Leximin, for example, satisfies all of them. Indeed, this is true of all of the results of this paper, and thus we shall not bother to repeat the reminder that the conditions invoked are jointly compatible (and satisfied by leximin).

There is, however, a possibility that we have not yet explored. We have so far focused on the justice relation. We have, that is, focused on *axiological* justice (what is more just than what). A different approach is to formulate the conditions of justice in deontic terms, that is, in terms of what is just (i.e., permitted by justice) relative to set of feasible alternatives. This latter approach does not attempt to provide a global ranking of alternatives. Instead, it attempts simply to determine which of any given set of feasible alternatives are just. We shall now explore whether the above results remain valid in this context.[5]

6. Deontic Justice

To start, let us reformulate the above axiological conditions that were used to define Paretian Egalitarianism and the two specific versions. The following definitions will be used below. An alternative is *Pareto optimal*, relative to a given feasible set, if and only if no feasible alternative is Pareto superior (i.e., makes some better off and no one worse off). An alternative is *anonymously Pareto optimal*, relative to a given feasible set, if and only if it is *feasible* and neither it, *nor any of its permutations*, is Pareto inferior to some feasible option.

Consider, then, the following conditions:

Deontic Strong Pareto: For any two alternatives x and y, if each person has at least as much benefits in x as in y, then, whenever both are feasible: (1) x is just, if y is, and (2) if there is at least one person that has more benefits in x than in y, then y is not just.

Deontic Weak Egalitarianism: If two alternatives x and y are anonymously Pareto incomparable, then, whenever both are feasible: (1) if they are equally equal, one is just if and only if the other is, and (2) if x is more equal than y, then y is not just.

Deontic Moderate Egalitarianism: If two alternatives x and y are Pareto incomparable, then, whenever both are feasible: (1) if they are equally equal, one is just if and only if the other is, and (2) if x is more equal than y, then y is not just.

Deontic Paretian Egalitarianism is the conjunction of Deontic Strong Pareto and Deontic Weak Egalitarianism. The conjunction of Deontic Strong Pareto and Deontic Moderate Egalitarianism defines *Deontic Paretian Moderate Egalitarianism*. We shall starting by showing that, as in the axiological case, this latter theory of justice is impossible if one also accepts the following condition:

No Prohibition Dilemmas: For any feasible set of alternatives, at least one feasible alternative is just.

This condition is plausible from the perspective of *practical justice*, according to which justice is purely a matter of comparing favorably in the relevant respects with the feasible alternatives (e.g., being at least as good in the relevant respect as all (or 90%) of the feasible alternatives [which is always possible], as opposed to giving everyone an adequate level of benefits [which is not always possible]). Practical justice always satisfies No Prohibition Dilemmas. Of course, justice can be understood in a more ideal way that does not guarantee that at least one feasible alternative is just, but we here focus on practical justice.[6]

Before introducing our first deontic result, let us note that we are assuming (as is standard) that, with one qualification, any logically possible set of alternatives is a possible feasible set. The qualification is that we restrict feasible sets to those for which, for each individual, there is a maximum possible benefit. The qualification is needed because it is unclear what rationality and justice require when benefits can be increased without limit. Let us now note:

> **Result 4**: If (1) we have Benefitism and Domain Richness, (2) deontic justice satisfies No Prohibition Dilemmas, and (3) the equality relation satisfies Equality Strong Anonymity and Strong Contracting Extremes Equality, then Deontic Paretian Moderate Egalitarianism is impossible.

The force of this result (and the structure of the proof) can be seen by considering the feasible set {<11,9,8>, <10,9,8>, <9,10,8>}. Note that <10,9,8> is more equal than <11,9,8> by Strong Contracting Extremes Equality, and hence Equality Strong Anonymity entails that

<9,10,8> is also more equal. Deontic Paretian Moderate Egalitarianism thus makes the following judgements: (1) <11,9,8> is unjust (because it and <9,10,8> are Pareto incomparable and the latter is more equal), (2) <10,9,8> is unjust (by Deontic Strong Pareto), and (3) <9,10,8> is unjust because <10,9,8> is unjust and they are equally equal (by Deontic Moderate Egalitarianism). Thus, nothing is just, which violates No Prohibition Dilemmas.

Given the conditions of the result, Deontic Paretian Moderate Egalitarianism is impossible. It is not, however, the only form of Deontic Paretian Egalitarianism. Indeed, there are versions of Deontic Paretian Egalitarianism that, for some feasible sets, disagree with weak maximin. But this is not a particularly interesting observation, because, as we shall now show, if a feasibility set is rich in a certain sense, then the possibility of disagreement disappears.

Domain Richness guarantees that the set of all alternatives includes all logically possible distributions of benefits. This does not, however, guarantee that any particular feasibility set is rich in any interesting sense. We shall focus, therefore, on feasibility sets that are *minimally rich* in the sense that if alternative x is feasible and maximizes (relative to the feasibility set) the benefits to the worst off position, then so is an alternative that gives everyone those same benefits. Thus, the feasible set {<2,4,6> , <1,9,9>}, is not minimally rich because it does not contain <2,2,2>, whereas {<2,4,6> , <1,9,9>, <2,2,2>} is minimally rich.

We have the following result for feasible sets that are minimally rich:

Result 5: If (1) we have Benefitism and Domain Richness, (2) deontic justice satisfies No Prohibition Dilemmas, and (3) the equality relation satisfies Perfect Equality, then, *for any minimally rich set*, Deontic Paretian Egalitarianism (a) judges just some alternative that maximizes the benefits of the worst off position and (b) judges unjust all alternatives that do not maximize the benefits to the worst off position.

To see the force of this result, consider the minimally rich set {<2,4,6>, <3,3,6>, <3,3,3>}. Deontic Paretian Egalitarianism judges that (1) <2,4,6> is unjust (by Deontic Weak Egalitarianism, because it is less equal, by Perfect Equality, than the anonymously Pareto incomparable <3,3,3>), and (2) <3,3,3> is unjust (by Deontic Strong Pareto, because it is Pareto inferior to <3,3,6>). No Prohibition Dilemmas then entails that <3,3,6> is just. But <3,3,6> and <2,4,6> are anonymously Pareto incomparable, and thus it follows that Deontic Paretian Egalitarianism must judge <3,3,6> as more equal than <2,4,6> (since, if it did not, <2,4,6> would also be judged just, and that contradicts the first statement made in (1)). A suitable generalization of this shows that it is not possible for Deontic Paretian Egalitarianism to judge just some alternative that does not maximize the benefits to the worst off position, and has to judge just some alternative that does so if the set is minimally rich.

Thus, although Deontic Paretian Egalitarianism can disagree with weak maximin, it cannot do so for feasibility sets that are minimally rich. Given that minimal richness is a condition that is often satisfied in real life situations, this possibility of disagreement is not very significant. For most practical applications, Deontic Paretian Egalitarianism cannot contradict weak maximin.

It is worth emphasizing that this result does not depend on any assumption of Acyclicity or any similar deontic consistency condition. Below we shall establish some results that do so depend.

7. Deontic Justice Reconsidered

The results of the previous section establish that merely moving to a deontic framework is not enough to avoid the core of the two axiological results. One might argue, however, that the above formulations of the deontic egalitarian conditions are excessively strong. To see this, consider:

Restricted Deontic Weak Egalitarianism: If, for a given feasible set, two alternatives x and y are anonymously Pareto optimal, then (1) if they are equally equal, one is just if and only if the other is, and (2) if x is more equal than y, then y is not just.

Restricted Deontic Moderate Egalitarianism: If, for a given feasible set, two alternatives x and y are Pareto optimal, then (1) if they are equally equal, one is just if and only if the other is, and (2) if x is more equal than y, then y is not just.

These conditions are like their unrestricted counterparts, except that they apply, for a given feasible set, only to Pareto optimal alternatives (and, in the case of Restricted Deontic Weak Egalitarianism, only to those that are anonymously Pareto optimal)—rather than to all feasible alternatives. Suppose, for example, that the feasibility set is {<2,5>, <5,3>, <4,3>}. Deontic Weak Egalitarianism says that <2,5> is not just because it is anonymously Pareto incomparable to, and (by Equality Strong Anonymity) less equal than, <4,3>. Restricted Deontic Weak Egalitarianism, however, does not say this. It applies only to anonymously Pareto optimal options. Given that <4,3> is not anonymously Pareto optimal, the fact that it is more equal than <2,5> is not deemed relevant.

Restricted Deontic Paretian Egalitarianism is the conjunction of Deontic Strong Pareto and Restricted Deontic Weak Egalitarianism, and *Restricted Deontic Paretian Moderate Egalitarianism* is the conjunction of Deontic Strong Pareto and Restricted Deontic Moderate Egalitarianism.[7] Unlike the axiological and the unrestricted deontic case, in this restricted deontic case, Paretian moderate egalitarianism is, it seems, possible, as long as we assume that the equality relation satisfies the following condition:

Equality Acyclicity: If, for alternatives $x_1,\ldots x_n$, x_1 is more equal than x_2, x_2 is more equal than x_3, …. and x_{n-1} is more equal than x_n, then x_n is not more equal than x_1.

We have been assuming that the justice relation is acyclic, but so far we have not needed to assume that the equality relation is. Without this assumption, every Pareto optimal alternative could be less equal than some other Pareto optimal alternative, and Restricted Deontic Paretian Moderate Egalitarianism would judge all feasible alternatives unjust, which would violate No Prohibition Dilemmas. The assumption of acyclicity, however, is entirely uncontroversial, and we are merely explicitly noting it.

Consider then:

Result 6: Given (1) Benefitism and Domain Richness and (2) that the equality relation satisfies Equality Acyclicity, Perfect Equality, Equality Strong Anonymity, Strong Contracting Extremes Equality, there are versions of Restricted Deontic Paretian Moderate Egalitarianism (and hence Restricted Deontic Paretian Egalitarianism in general) that satisfy No Prohibition Dilemmas and that, even for a minimally rich feasible set, (a) judge a feasible alternative just even though it does not maximize the benefits to the worst off position, and (b) judge a feasible alternative unjust even though it does maximize the benefits to the worst off position.

To see this, consider the following version:

Full Restricted Deontic Paretian Moderate Egalitarianism: An alternative is just, relative to a feasible set, if and only if it is Pareto optimal and no Pareto optimal alternative is more equal.[8]

This theory judges all alternatives just except those that are not Pareto optimal and those that are less equal than some Pareto optimal alternative. Hence, it satisfies both Deontic Strong Pareto and Restricted Deontic Moderate Egalitarianism. Moreover, for any acyclic equality relation, it will also satisfy No Prohibition Dilemmas (since there will always be at least one Pareto optimal alternative that is not less equal than some other one). Hence, in order to see that this theory of justice constitutes a real alternative to weak maximin, we may simply consider an equality relation satisfying our minimal conditions that disagrees with weak maximin. By way of illustration, suppose that a distribution is more equal if the total shortfall from the mean is less. This equality relation is acyclic and satisfies our three conditions on equality (proof omitted). Now consider the feasible set {<2,2,2>, <2,3,52>, <1,40,46>}, which is minimally rich. Alternatives <2,3,52> and <1,40,46> are the only two Pareto optimal alternatives, and <2,3,52> (mean of 19, total shortfall of 33) is less equal than <1,40,46> (mean of 29, total shortfall of 28). Hence, given this equality relation, Full Restricted Deontic Paretian Moderate Egalitarianism judges <1,40,46> as just (because it is a most equal Pareto optimal alternative), judges <2,3,52>

as unjust (because it is less equal than some other Pareto optimal alternative than), and judges <2,2,2> as unjust (because it is not Pareto optimal). Thus, Restricted Deontic Paretian Moderate Egalitarianism can judge unjust an alternative that maximizes the benefits to the worst off position, and it can judge just an alternative that does not maximize these benefits. Now consider the feasible set {<2,2,2>, <2,3,43>, <1,40,46>}, which is minimally rich. Alternatives <2,3,43> and <1,40,46> are the only two Pareto optimal alternatives, and <2,3,43> (mean of 16, total shortfall of 27) is less equal than <1,40,46> (mean of 29, total shortfall of 28). Hence, given this equality relation, Full Restricted Deontic Paretian Moderate Egalitarianism judges <1,40,46> as just (because it is a most equal Pareto optimal alternative), judges <2,3,43> as unjust (because it is less equal than some other Pareto optimal alternative than), and judges <2,2,2> as unjust (because it is not Pareto optimal). Thus, Restricted Deontic Paretian Moderate Egalitarianism can judge unjust an alternative that maximizes the benefits to the worst off position, and it can judge just an alternative that does not maximize these benefits.

Finally, then, we have, it seems, the possibility of a form of Paretian egalitarianism that can disagree with weak maximin even in the case of minimal rich feasible sets. There may, however, be further plausible conditions that, if imposed, would rule out this apparent possibility. Before concluding, we shall consider one such candidate condition .

As noted above, we have not assumed any consistency conditions that correspond to Acyclicity (or related conditions). In a deontic context, however, the following condition is widely accepted:

Alpha (contraction consistency): If an alternative is judged just relative to a given feasible set, then it is also judged just from any subset containing it.

Alpha is concerned with the preservation of judgement of justness as a feasibility set is contracted while still containing the alternative originally judged just. It is violated, for example, if x is judged just relative to {x,y,z}, but not judged just relative to {x,y}. If it is a winner against x and y, why would not it also be a winner against y alone?

If we accept Alpha, the impossibility of Paretian Moderate Egalitarianism returns:

> **Result 7**: If (1) we have Benefitism and Domain Richness, (2) deontic justice satisfies No Prohibition Dilemmas and Alpha, and (3) the equality relation satisfies Equality Strong Anonymity and Strong Contracting Extremes Equality, then Restricted Deontic Paretian Moderate Egalitarianism is impossible.[9]

To see the force of this result, consider the feasible set consisting of <6,9,12>, <11,9, 6> and <100,9,6>. Note that <6,9,12> is more equal than <100,9,6> (by Strong Contracting Extremes Equality and Equality Strong Anonymity). Restricted Deontic Paretian Moderate Egalitarianism thus makes the following judgements: (1) <11,9,6> is not just (by Deontic Strong Pareto), (2) <100,9,6> is not just (by Restricted Deontic Moderate Egalitarianism, because it is less equal than the Pareto optimal <6,9,12>). No Prohibition Dilemmas then entails that <6,9,12> is just. We shall now show that this leads to a violation of Alpha by considering the feasible set {<6,9,12>, <11,9,6>}, a subset of the original set. Note that <6,9,12> is less equal than <11,9,6> (by Strong Contracting Extremes Equality and Equality Strong Anonymity). Restricted Deontic Moderate Egalitarianism here judges <6,9,12> as not just (by Restricted Deontic Moderate Egalitarianism). Thus, given that <6,9,12> was judged just from the first set, but not from this proper subset, Alpha is violated.

If we impose Alpha, then Restricted Deontic Paretian Moderate Egalitarianism is impossible. It is not, however, the only form of Restricted Deontic Paretian Egalitarianism and some of these other forms are possible. As in the previous cases we have considered, however, no form can disagree with weak maximin when applied to minimally rich sets.

> **Result 8:** If (1) we have Benefitism and Domain Richness, (2) deontic justice satisfies No Prohibition Dilemmas and Alpha, and (3) the equality relation satisfies Perfect Equality, then, for any feasible set, Restricted Deontic Paretian Egalitarianism must (a) judge just some alternative that maximizes the benefits to the worst off position, and (b), *if the feasible set is minimally rich,* judge unjust any alternative that does not maximize the benefits to the worst off position.

To see the force of this result, consider the minimally rich feasible set <2,4,6>, <3,3,3>, <3,3,6>. Restricted Deontic Paretian Egalitarianism judges <3,3,3> as unjust (by Deontic Strong Pareto). To see how it judges <2,4,6>, consider the following subset of the initial set, {<2,4,6>, <3,3,3>}. In this case, Restricted Deontic Paretian Egalitarianism judges <2,4,6> as unjust (by Restricted Deontic Weak Egalitarianism, because it is less equal than the anonymously Pareto optimal <3,3,3>). Alpha then entails that <2,4,6> is also unjust relative the expanded set. No Prohibition Dilemmas then entails that <3,3,6> is just relative to this expanded set. The proof generalizes this example to all minimally rich feasible sets. Notice moreover that, because <2,4,6> and <3,3,6> are each anonymously Pareto optimal, Restricted Deontic Paretian Egalitarianism must hold that <3,3,6> is more equal than <2,4,6>. More generally, a distribution that is maximin better cannot be judged less equal by this theory. There is, that is, no possibility of measuring equality in a way that significantly diverges from weak maximin.

Do these results show that even Restricted Deontic Paretian Egalitarianism must agree with weak maximin? That, of course, depends on the plausibility of Alpha. One of us is firmly inclined to reject it. We shall end with a discussion of the strengths and weaknesses of this condition.

If axiological justice is a complete ordering, and, for any given feasible set, the set of just alternatives is simply the set of best feasible alternatives, then Alpha will be satisfied. Alpha is clearly a desirable condition for justice in the sense that it would be nice if justice satisfied it. The question, however, is whether it is a *mandatory* condition—one that any minimally adequate conception of justice *must* satisfy.

Restricted Deontic Paretian Moderate Egalitarianism can violate Alpha because it requires that just options be no less good in some dimension (equality) than any alternative that is Pareto optimal. Removing an option can make an originally Pareto suboptimal option become Pareto optimal, and this option may be better (more equal) than some third option that was originally not less good than any Pareto optimal option. Thus, this third option may be just relative to the original set but not relative to the subset. Suppose for the sake of argument that <2,2,2> is more equal than <1,40,46>, and that the latter is more equal than <2,3,43>. In this case, Restricted Deontic Paretian Moderate Egalitarianism judges just <1,40,46> when the feasible alternatives are <2,2,2> and <2,3,43> (since it is Pareto optimal and more equal than <2,3,43> and <2,2,2> is not Pareto optimal), but it judges <1,40,46> unjust when the only feasible alternative is <2,2,2> (since the former is less equal and both are Pareto optimal). Why would this be incompatible with minimum adequacy in a theory of justice? It cannot be simply because there are two distinct dimensions of value. Justice can surely be pluralistic. Nor can it be the lexical priority given to one dimension (Pareto optimality). Minimally adequate theories of justice can surely invoke lexical priority. Nor, it seems, can it be that the lexically prior dimension has the feature that removing an option can make admissible a previously inadmissible option in this dimension. Thus, it is unclear why Alpha would be a mandatory condition on justice.

Several authors have argued against Alpha (or its counterparts formulated in terms of rationality and the like rather than justice). Amartya Sen, for example, has argued against any kind a priori requirement of internal consistency, and against Alpha in particular, for rational individual choice or social choice. There is nothing irrational, he argues, about having the goal of choosing the second largest piece of cake (e.g., so as not to appear greedy). This violates Alpha, however, since the originally second largest piece will not be rationally chosen if the largest piece is removed as a possible choice. Likewise, Sen argues, there is nothing irrational about choosing to fast when one has the alternatives of a full meal and of a very limited meal, but not choosing to fast when one's only alternative is the very limited meal. The symbolic significance of fasting is radically altered by the absence of the possibility of full meal.[10]

Moreover, there are many examples of conceptions of justice that violate Alpha, but are not wildly implausible. To start, suppose that an option is judged just, relative to a feasible set, if and only if it is in the top 10% of the feasible alternatives in terms of some complete order of the value of alternatives. This is a kind of satisficing theory. Consider an alternative that is barely in the top 10% and then remove many of the less valuable options. If enough are removed, the original alternative will cease to be in the top 10%, and thus cease to be just. This violation of Alpha does not, however, seem wildly implausible. It is not clearly inappropriate for justice to take into account the values of other feasible alternatives and to judge just those that are near the top in relative terms.

Consider next the bargaining solution (and view of justice), advocated by David Gauthier.[11] It requires that "relative benefit" be maximinned, where relative benefit for a given individual is the ratio of the benefit the individual receives to the maximum feasible benefit she could receive. Consider the feasible set {<3,1>, <2,4>, <10,0>}. The relative benefits of these three alternatives are respectively {< .33,.25>, <.2,1>, <1,0>}, and thus <3,1> alone is deemed just (since .25 is greater than .2 and 0). If <10,0> is removed, however, the relative benefits are respectively {<1,.25>, <.66,1>}, and thus <2,4> alone is judged just (since .66 is greater than .25). This violation of Alpha, however, does not seem wildly implausible. Moreover, it occurs even if the requirement to maximin relative benefits is replaced with the requirement to maximize the total relative benefits or the requirement to maximize some kind of prioritarian weighted total relative benefit. It is not clearly inappropriate to scale benefits relative to the most an individual could get in a situation.

Consider next the view of justice that requires leximaxing complaints (i.e., minimizing the maximum complaint, and in case of ties minimizing the second (third, etc.) maximum complaint), where a person's complaint is the absolute shortfall from the greatest benefits she could have gotten in the given choice situations. (This is the analogue of maximin regret in individual choice under uncertainty.) Consider the feasible set {<4,6>, <5,4>, <10,0>}. The individual complaints for these three alternatives are respectively {<6,0>, <5,2>, <0,6>}, and so <5,4> alone is just (since 5 is less than 6). If <10,0> is removed, however, the individual complaints are respectively {<1,0>, <0,2>}, and so <4,6> alone is judged just. Again, this violation of Alpha does not seem wildly implausible. Moreover, as above, this violation occurs even if the requirement to leximax complaints is replaced with the requirement to minimize the total complaints or the requirement to minimize some kind of prioritarian weighted total complaint. It is not clearly inappropriate to have justice depend on complaints in these ways.

Finally, consider a theory of justice that Isaac Levi views as not implausible.[12] It holds that (1) the weights assigned to each person's benefits are nonnegative and sum to one, (2) there may be more than one admissible way to weigh each person's benefits (perfectly equal weighting may not be required), and (3) an option is just if and only if it leximins benefits relative to the feasible options that maximize average benefits according to *at least one* admissible weighting scheme. Consider the feasible set consisting of <9,1>, <1,9>, and <2,2>. Option <9,1> maximizes average benefits when at least 50% of the weight is given to the first person, and <1,9> does so when at least 50% is given to the second person. Option <2,2>, however, does not maximize average benefits on any admissible weighting scheme. Thus, <9,1> and <1,9> are each just relative to this set, since each leximins benefits relative to the options that maximize average benefits relative to at least one weighting scheme. Consider now the feasible set consisting of just <9,1> and <2,2>. Here again <9,1> maximizes average benefits relative to some weighting schemes. In this case, however, so does <2,2> (e.g., for any weighting scheme that gives the second person at least 7/8 of the weight). Only <2,2>, however, leximins relative to the options that maximize the average on some weighting. Hence, <9,1> is not just relative to this subset, and that violates Alpha. Allowing some such indeterminacy in weightings (e.g., to allow for up to some specified extra weight to given to the chooser) is not clearly implausible. Nor is Levi's approach for dealing with such situations.[13]

In sum, if an alternative is just if and only if it is a best feasible alternative, where the goodness of alternatives is a complete ordering, then Alpha will be satisfied. Alpha need not be satisfied, however, if deontic justice is not *rationalizable* in the sense of there being some (context-free) ranking relation R, over alternatives, such that an alternative is just if and only if it is an R-best feasible alternative. Although rationalizability is a nice property, there seems to be little reason to require it of all minimally adequate conceptions of justice. As we have illustrated above, deontic justice may rely on normative concepts that are not compatible with a context-free rankling relation.

We therefore tentatively conclude that Restricted Deontic Paretian Egalitarianism is possible and can disagree with weak maximin.

8. Conclusion

To be plausible, egalitarian theories of justice cannot be pure egalitarian theories. Axiological versions (what is at least as just as what) must hold that 1) Pareto improvements (making some individuals better off and no one worse off) make things more just and deontic versions (what is just relative to the set of feasible alternatives) must hold that no Pareto suboptimal alternative is just. Pareto efficiency, that is, is lexically prior to the demands of equality. Paretian egalitarianism is committed to this view, and we have explored some of the forms that it can take.

Given the various plausible conditions that we identified, we have established that axiological Paretian egalitarianism (1) is impossible—if it makes equality dominant in all cases of Pareto incomparability, and (2) cannot disagree with weak maximin, if it limits the role of equality to cases of anonymous Pareto incomparability. It cannot, for example, hold that <2,10,100> is less just than <1,100,100>. Indeed, given that these two alternatives are anonymously Pareto incomparable (and thus ranked on the basis of equality), it cannot hold the former is less *equal* than the latter. This radically limits the admissible measures of equality available to Paretian Egalitarianism. Indeed, none of the standard measures of equality are available, since they all hold that at least sometimes a maximin worse distribution is more equal. (For example, above, the Gini coefficient of inequality is 0.58 for the first and 0.33 for the latter alternative.) Axiological Paretian egalitarianism thus seems quite unpromising as an alternative to leximin and other forms of weak maximin.

If we turn to deontic justice, roughly these results remain valid if the egalitarian conditions are taken to require that an alternative be judged unjust if some *feasible* alternative is more equal. If, however, the egalitarian requirements are restricted to require only that an alternative be judged unjust if some *Pareto optimal* alternative is more equal, then Paretian egalitarianism can disagree with weak maximin. This is the only promising form of Paretian egalitarianism. Even it, however, cannot disagree with weak maximin if Alpha (contraction consistency) is imposed. We have suggested that Alpha is not a mandatory condition for justice, but admittedly our discussion is not conclusive. The future of Paretian egalitarianism depends on this issue and thus further investigation is needed.

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Appendix

Note: We are assuming that the appendix will not be published, and are including it here just to be safe (since we would prefer that it be published). If it is included, then we need to modify the text immediately following Result 1 along with the accompanying footnote.

Result 1: If (1) we have Benefitism and Domain Richness, (2) the equality relation satisfies Equality Strong Anonymity and Strong Contracting Extremes Equality, and (3) there are only two people, then for any two anonymously Pareto incomparable alternatives, the alternative that makes the worst off position better off is more equal.

Proof: Consider any two anonymously Pareto incomparable alternatives. Because there are only two persons and they are anonymously Pareto incomparable, if the worst off position in one alternative, x, receives more benefits then the worst off position in the other alternative, y, then the best off position in x receives fewer benefits than the best off position in y. If the worst off person is the same in both alternatives (e.g., x=<6,8> and y=<5,100>), then it follows directly from Strong Contracting Extremes Equality that x is more equal than y. If the worst off person is not the same in both alternatives (e.g., x=<6,8> and y=<100,5>), then by Domain Richness, we may consider a permutation of x, x* (x*= <8,6>), and Strong Contracting Extremes Equality will judge x* more equal than y. Hence, by Equal Strong Anonymity, x is more equal than y.

Result 2: If (1) we have Benefitism and Domain Richness, (2) the justice relation satisfies Acyclicity, and (3) the equality relation satisfies Equality Strong Anonymity and Strong Contracting Extremes Equality, then Paretian Moderate Egalitarianism is impossible.

Proof: By Domain Richness, we may consider two alternatives x and y such that (a) they are Pareto incomparable but not anonymously so, (b) the benefits are the same in every position except for the best off position, and (c) the benefits in the best off position are higher in x than y (e.g., x = <7,8,11> and y = <10,8,7>). (1) By Domain Richness, there is an alternative, y* (<7,8,10>) that is a permutation of y and such that Strong Contracting Extremes Equality entails that y* is more equal than x. Equality Strong Anonymity then entails that y is more equal than x. Hence, Weak Egalitarianism judges that y is more just than x (because they are Pareto incomparable and y is more equal). (2) A contradictory judgement, however, can be obtained as follows. By Domain Richness, there is an alternative z (e.g., <7,8,9>) such that (a) x is Pareto superior to z, (b) Strong Contracting Extremes Equality judges a permutation of z, z* (e.g., <9,8,7>), more equal than y, and (c) z and y are Pareto incomparable. Paretian Moderate Egalitarianism thus makes the following judgements: x is more just than z (by Strong Pareto), and z is more just than y (z is more equal than y by Equality Strong Anonymity, and thus more just than y by Weak Egalitarianism). By Acyclicity, we have that y is *not* more just than x, which contradicts the conclusion obtained in (1).

Result 3 : If (1) we have Benefitism and Domain Richness, (2) the justice relation satisfies Acyclicity, and (3) the equality relation satisfies Perfect Equality, then Paretian Egalitarianism judges that a distribution is *not less just* than another if it gives greater benefits to the worst off position. Moreover, (a) if the justice relation also satisfies Consistency, then Paretian Egalitarianism judges that a distribution is *not equally or less just*, if it gives greater benefits to the worst off position, and (b) if the justice relation also satisfies Quasi-Transitivity, then

Paretian Egalitarianism judges that a distribution is *more just* than another if it gives greater benefits to the worst off position.

Proof: Consider any x and y, where the worst off position in x gets more benefits than the worst off position in y. (1) If no one is worse off in x than in y, then it follows from Strong Pareto that x is more just than y, as required. (2) If someone is worse off in x than in y, then the two are Pareto incomparable, and we consider two cases: (a) If there is perfect equality in x, then it follows that x and y are anonymously Pareto incomparable (e.g., x = <3,3,3> and y = <2,4,10>), and that x is more equal. Weak Egalitarianism then entails that x is more just than y, as required. (b) If there is inequality in x (e.g., x = <3,3,5> and y = <2,4,10>, then by Domain Richness there exists an alternative, z (e.g., <3,3,3>) such that everyone in z gets the same as the worst off in x. By Strong Pareto, x is more just than z. In this case, z and y are anonymously Pareto incomparable and Perfect Equality entails that z is more equal than y. Weak Egalitarianism then judges z more just than y. We thus have that x (<3,3,5>) is more just than z (<3,3,3>), which is more just than y (<2,4,10>). Acyclicity then entails that x is not less just than y, Consistency entails that x is not equally or less just than y, and Quasi-Transitivity entails that x is more just than y, as required.

Result 4: If (1) we have Benefitism and Domain Richness, (2) deontic justice satisfies No Prohibition Dilemmas, and (3) the equality relation satisfies Equality Strong Anonymity and Strong Contracting Extremes Equality, then Deontic Paretian Moderate Egalitarianism is impossible.

Proof: We will illustrate the proof with a three person example: By Domain Richness, we may consider the following feasible set {<11,9,8>, <10,9,8>, <9,10,8>}. Here, <10,9,8> is more equal than <11,9,8> by Strong Contracting Extremes Equality, and hence Equality Strong Anonymity entails that <9,10,8> is also more equal. Deontic Paretian Moderate Egalitarianism thus makes the following judgements: (1) by Deontic Weak Egalitarianism, <11,9,8> is unjust because it and <9,10,8> are Pareto incomparable and the latter is more equal, (2) by Deontic Strong Pareto, <10,9,8> is unjust because it is not Pareto optimal, and (3) by Deontic Weak Egalitarianism, <9,10,8> is unjust because <10,9,8> is unjust and they are equally equal (by Equal Strong Anonymity). Thus, nothing is just, which violates No Prohibition Dilemmas.

Result 5: If (1) we have Benefitism and Domain Richness, (2) deontic justice satisfies No Prohibition Dilemmas, and (3) the equality relation satisfies Perfect Equality, then, *for any minimally rich set*, Deontic Paretian Egalitarianism (a) judges just some alternative that maximizes the benefits of the worst off position and (b) judges unjust all alternatives that do not maximize the benefits to the worst off position.

Proof: Consider any minimally rich feasible set, and let x be any feasible alternative that does not maximize the benefits to the worst off position and let y be any feasible alternative that does maximize the benefits to the worst off position. We will now show that Deontic Paretian Egalitarianism has to judge x as unjust in this situation and that it must judge just some alternative that maximizes the benefits to the worst off position. (1) If no else gets more benefits in x than in y (e.g., x = <2,4,6> and y = <3,4,6>), then Deontic Strong Pareto entails that x is unjust, as required. (2) If someone else gets more benefits in x than in y (e.g., x = <2,4,6> and y

= <3,3,6>), then it follows that there is inequality in x. In this case, given that the feasible set is minimally rich, there exists an alternative, z (e.g., <3,3,3>), such that everyone in z gets the same benefits as the worst off position in y. Given that (a) alternative z is anonymously Pareto incomparable with x (since the benefits of the worst off position are higher in z than x, and the benefits of the best off position are lower in z than in x), and (b) Perfect Equality entails that z is more equal than x, Deontic Weak Egalitarianism entails that x is not just relative to this feasible set, as required. Given that x was an arbitrary feasible alternative that does not maximize the benefits to the worst off position, it follows that no such alternative is just. (3) By No Prohibition Dilemma, it follows immediately that some feasible alternative that does maximize the benefits to the worst off position is just, as required.

Result 6: Given (1) Benefitism and Domain Richness and (2) that the equality relation satisfies Equality Acyclicity, Perfect Equality, Equality Strong Anonymity, Strong Contracting Extremes Equality, there are versions of Deontic Paretian Moderate Egalitarianism (and hence Deontic Paretian Egalitarianism in general) that satisfy No Prohibition Dilemmas and that, even for a minimally rich feasible set, (a) judge a feasible alternative just even though it does not maximize the benefits to the worst off position, and (b) judge a feasible alternative unjust even though it does maximize the benefits to the worst off position.

Proof: Given in text.

Result 7: If (1) we have Benefitism and Domain Richness, (2) deontic justice satisfies No Prohibition Dilemmas and Alpha, and (3) the equality relation satisfies Equality Strong

Anonymity and Strong Contracting Extremes Equality, then Restricted Deontic Paretian Moderate Egalitarianism is impossible.

Proof: (1) By Domain Richness, we may consider the feasible set consisting of <6,9,12>, <11,9,6> and <100,9,6>. Here <6,9,12> and <100,9,6> are Pareto optimal, but <11,9,6> is not. (a) By Deontic Strong Pareto, <11,9,6> is not just. (b) By Equality Strong Anonymity and Strong Contracting Extremes Equality, <6,9,12> is more equal than <100,9,6>. Restricted Deontic Moderate Egalitarianism thus judges <100,9,6> not just. (c) By No Prohibition Dilemmas, <6,9,12> is thus just. (2) Consider now the feasible set consisting of just <6,9,12> and <11,9,6>, a subset of the original set. (a) By Equality Strong Anonymity and Strong Contracting Extremes Equality, <6,9,12> is less equal than <11,9,6>. Restricted Deontic Moderate Egalitarianism, thus judges <6,9,12> not just. (3) Given that <6,9,12> was judged just from the first set (in 1c), but not from this proper subset, Alpha is violated.

Result 8: If (1) we have Benefitism and Domain Richness, (2) deontic justice satisfies No Prohibition Dilemmas and Alpha, and (3) the equality relation satisfies Perfect Equality, then, for any feasible set, Restricted Deontic Paretian Egalitarianism must (a) judge just some alternative that maximizes the benefits to the worst off position, and (b), *if the feasible set is minimally rich,* judge unjust any alternative that does not maximize the benefits to the worst off position,.

Proof: Consider any feasible set for which some but not all alternatives maximize the benefits to the worst off position. Let y and x be arbitrary alternatives in this set that respectively maximize, and do not maximize, the benefits to the worst off position. We will first show that, if the feasible set is minimally rich, Restricted Deontic Paretian Egalitarianism has to judge x as unjust and has to judge just some alternative that maximizes the benefits of the worst off position. Following that, we show that it must make the second judgement even if the feasible set is not minimally rich. (1) Suppose, then, that the feasible set is minimally rich. (a) If no else gets more benefits in x than in y (e.g., x = <2,4,6> and y = <3,4,6>), then Deontic Strong Pareto entails that x is unjust. Given that x was an arbitrary alternative that does not maximize the benefits to the worst off position, No Prohibition Dilemmas then implies that some alternative that does maximize the benefits to worst off position is just. (b) If someone else gets more benefits in x than in y (e.g., x = <2,4,6> and y = <3,3,6>), then it follows that there is inequality in x. In this case, given that the feasible set is minimally rich, there exists an alternative, z (e.g., <3,3,3>), such that everyone in z gets the same benefits as the worst off position in y. Consider now the feasible set obtained from the original by removing all the alternatives other than z that maximize the benefits to the worst off position. In this restricted set, z is anonymously Pareto optimal and, by Perfect Equality, more equal than any other anonymously Pareto optimal alternative. Hence, by Restricted Deontic Weak Egalitarianism, x and all other alternatives except for z are unjust, and by No Prohibition Dilemmas z is just. By Alpha, x is also unjust in the original superset. Given that x was an arbitrary alternative that did not maximize the benefits to the worst off position, this shows that every alternative that fails to so maximize is unjust in the original set. No Prohibition Dilemmas then entails that some alternative that does maximize the benefits to the worst off position is just, as required. (2) Suppose now that the original set is not minimally rich. We may make this set minimally rich by adding an alternative that gives everyone the same benefits as the worst off position in an alternative that maximizes the benefits of the worst off. From (1), we know that for a minimally rich feasible set, there is an alternative, y, that maximizes the benefits of the worst off position, and is judged just, relative to this extended set. By Deontic Strong Pareto, it follows that y cannot be the alternative added to the initial set. Hence, y is in the initial feasible set. By Alpha, it follows that y is also just in the initial not minimally rich set.

Notes

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[1] We are here building upon Bertil Tungodden, “Egalitarianism: Is leximin the only option,” *Economics and Philosophy* 16 (2000): 229-45, “The value of equality,” *Economics and Philosophy* 19 (2003): 1-44, and “Hammond Equity: A generalization,” Discussion Paper, Norwegian School of Economics and Business Administration (2001). Our conditions are cast in terms of Paretian egalitarianism and the equality relation rather than in terms of a general justice relation (as Tungodden does). Tungodden’s work builds upon that of Peter Hammond, “Equity, Arrow’s condition, and Rawls difference principle, *Econometrica* 44 (1976): 793-804, and “Equality in two-person situations – some consequences”, *Econometrica* 44 (1979): 1127-35.

[2] This condition is stronger than that used by Tungodden in his cited papers. His version of the condition requires that antecedent clauses (1a) and (1b) both hold. The condition was first formulated by Peter Vallentyne, “Equality, Efficiency, and Priority for the Worse Off”, *Economics and Philosophy* 16 (2000): 1-19. It there takes (roughly) the stronger form that we invoke here. For brevity, we leave implicit the following antecedent clause: there is no natural zero for benefits (no non-arbitrary way of drawing the line between positive and negative levels), or all benefit levels are positive, or all are negative. This clause is needed so as not to require, for example, that <0,10> is less equal than <0,1>. Some measures of equality hold that these two distributions are equally equal on the grounds that each person has the same proportion of the total benefits under the first as under the second. Although we believe that Strong Contracting

Extremes Equality is plausible without being conditional on this antecedent clause, we do not presuppose this in what follows. Our proofs and examples are compatible with the weaker conditional version of the condition.

[3] This condition comes from Kotaro Suzumura, “Remarks on the Theory of Collective Choice,” *Economica* 43 (1976): 381-390. He there proves that Consistency is a necessary and sufficient condition for a reflexive ranking relation to be extendible to a complete, transitive, and reflexive relation.

[4] It is worth noting the connection of this result with the impossibility of combining Moderate Egalitarianism with the Suppes-Sen condition—according to which x is more just than y if it is anonymously Pareto superior to y. This latter impossibility is straightforward, given that Moderate Egalitarianism requires that x be less just than y, whenever (1) x is anonymously Pareto superior to y but not Pareto superior, and (2) x is less equal than y (e.g., x = <2,10> and y = <9,2>). Although our conditions do not entail Suppes-Sen (e.g., because we do not assume that the justice relation is anonymous), we do assume something close to it. Equality Strong Anonymity and Paretian Moderate Egalitarianism entail (assuming the reflexivity of equality) Justice Anonymity (a distribution and any permutation are equally just), and Justice Anonymity and Strong Pareto entail Suppes-Sen, if one assumes transitivity. We assume, however, only Acyclicity (which is not sufficient for the entailment).

[5] The axiological and deontic approaches to justice have been exhaustively studied in the social choice literature, where they are known as the social ordering and social choice function approaches. See, for example, Amartya Sen, “Social Choice Theory: A Re-Examination,” *Econometrica* 45 (1977): 53-89, reprinted in Amartya Sen, *Choice, Welfare, and Measurement*

(Oxford: Basil Blackwell, 1982), pp. 158-200.

[6] It is important to note that the choice function approach of social choice theory has No Prohibition Dilemmas implicitly built into it. This is because the concept of a choice function—which is a function that, for any given feasible set, selects the subset consisting of all and only the just alternatives relative to that feasible set—is defined to always select a non-empty set. We here posit No Prohibition Dilemmas for practical justice, but for an argument that prohibition dilemmas are conceptually possible (e.g., not ruled out by deontic logic as such), see Peter Vallentyne, "Prohibition Dilemmas and Deontic Logic," *Logique et Analyse* 18 (1987): 113-22, and "Two Types of Moral Dilemmas," *Erkenntnis* 30 (1989): 301-318.

[7] We have not formulated a restricted version Deontic Strong Pareto because it is equivalent to the unrestricted version.

[8] Roughly this view is defended in ch. 6 of Rex Martin, *Rawls and Rights* (University of Kansas Press, 1985). .

[9] The first to note the tension between Paretian egalitarianism and Acyclicity in the axiological case, and Alpha in the deontic case, was perhaps Albert Weale, "The Impossibility of Liberal Egalitarianism," *Analysis* 40 (1980): 13-19. The same point was made independently by Andrew Williams, "The Revisionist Difference Principle," *Canadian Journal of Philosophy* 25 (1995): 257-281. Each wrongly concludes, however, that Paretian egalitarianism is incompatible with these conditions, whereas the correct conclusion is that it is incompatible if it measures equality in ways that disagree with weak maximin.

[10] See Sen, "Internal Consistency of Choice," *Econometrica* 61 (1993): 495-521. Reprinted in Amartya Sen, *Rationality and Freedom* (Cambridge, MA: Harvard University Press, 2002), pp.

121-57.

[11] See David Gauthier, *Morals by Agreement* (London: Oxford University Press, 1986).

[12] See, ch. 9.6 of *Hard Choices* (Cambridge, MA: Cambridge University Press, 1986). Levi also defends violations of alpha for individual rational choice where there are indeterminate probabilities (ch. 7.9) and where there is ordinal indeterminacy in the value for an agent (ch. 6.9).

[13] For further criticisms of Alpha, see ch. 2 of Edward F. McClennen, *Rationality and Dynamic Choice* (Cambridge: Cambridge University Press, 1990); Piers Rawling, "Expected Utility, Ordering, and Context Freedom," *Economics and Philosophy* 13 (1997): 79-86; and ch. 4 of Paul Weirich, *Equilibrium and Rationality: Game Theory Revised by Decision Rules* (Cambridge: Cambridge University Press, 1998).