Estimating Estate-Specific Price-to-Rent Ratios in Shanghai and Shenzhen: A Bayesian Approach

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Abstract

The price-to-rent ratio, a common yardstick for the value of housing, is difficult to estimate when rental properties are poor substitutes of owner-occupied homes. In this study we estimate price-to-rent ratios of residential properties in two major cities in China, where urban high-rises (estates) comprise both rental and owner-occupied units. We conduct Bayesian inference on estate-specific parameters, using information of rental units to elicit priors of the unobserved rents of units sold in the same estate. We find that the price-to-rent ratios tend to be higher for low-end properties. We discuss economic explanations for the phenomenon and the policy implications.

Keywords: Housing price, rents, heterogeneity, Bayesian analysis

JEL Codes: C11, R21, R31, G00

Acknowledgements: We are grateful for the Shanghai and Shenzhen real estate data provided by the Centaline Property Consultants Ltd. China.

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1. Introduction

Unlike most investment or consumption goods, housing units differ in attributes that are difficult to measure. Establishing the relationship between housing price and the service flow of housing is an empirical challenge because the transaction price and rent of the same property are rarely simultaneously observed and rental properties are usually qualitatively different from owner-occupied housing. This study is an empirical analysis on cross-sectional comparison of price-to-rent ratios, taking advantage of housing market data of two major Chinese cities.

In China, an urban complex of high-rises (we call estate) typically comprises rental units and owner-occupied units of a similar quality. Using over twelve thousand observations on price and over nine thousand observations on rental transactions in Shanghai and Shenzhen, we develop a Bayesian approach to estimating the relationship between prices of housing units and their unobserved rents. Our analysis on latent rents of owner-occupied units sheds new lights on the pricing of housing, complementing the existing approach of the present value model, hedonic pricing, and pricing based on repeated sales.

A voluminous literature has been devoted to the time series behavior of price-to-rent ratio (for a survey of the literature see Himmelberg & Sinai (2005).) A number of authors (e.g., Mankiw & Weil (1989), Clark (1995), Meese & Wallace (1994), Clayton (1996)) find that the present value of aggregate housing price indexes in North America is sensitive to the rent index data. The researchers also recognize the difficulty in measuring the rent index because rental properties may differ in quality from the owner-occupied homes. [2] As an

[2] There are a number of novel approaches to imputing rent index. Meese & Wallace (1994) use generated hedonic price indices and two different rent indices. One is the rent component of the U.S. Consumer Price Index, and the other index is a rent series based on "asking rents for two-bedroom condos pulled from local newspapers". Clark (1995) uses neighborhood data instead of city data to minimize measurement errors. Clayton (1996) proxies imputed rents with a function of housing market fundamentals such as net

alternative to the rent-based pricing model, hedonic pricing method is often used to estimate the market value of real estate features such as lot size, number and size of rooms, number of bathrooms, house age, and environmental characteristics. [3] A limitation of the hedonic pricing approach is that it does not directly link price to the latent rent of the property and does not reflect unobservable quality difference in the hedonic features for different estates. Case & Shiller (1989) focus on price changes of repeated home sales to circumvent the need for estimating the effect of unobserved heterogeneity on real estate pricing. This approach is effective in capturing the time series of housing price but does not reveal cross-sectional differences in housing values. Our research complements these approaches by combining hedonic features with rent and price to study how the price of a housing unit relates to its latent rent.

Our empirical model consists of price and rent equations. In the rent equation, the property fixed effects on rent are estimated from data of rental units. The price equation links the price of a unit sold to its latent rent and other factors. A prominent feature of our rent-based pricing model lies in its cross-estate heterogeneity, which requires inference for thousands of parameters. Because of the limited observations of the price and rent data (about ten on average for each estate), the asymptotic distribution can be misleading for our problem. In this study, we conduct finite sample inference through a Bayesian approach, which produces probability distribution of parameters conditioning on the observed sample. The Bayesian approach is particularly effective for models with heterogeneity where the number of parameters is large relatively to the number of observations. In a Bayesian

immigration of households and the inventory of newly completed but unsold homes.

[3]Some examples of hedonic pricing studies on data of various countries include Goodman (1988), Can (1990), Downes & Zabel (2002), Vanderford et al. (2005), Bao & Wan (2004), Mats (2002), Witte et al. (1979), Ridker & Henning (1967), Kim (1992)), Schwartz et al. (2003), Hughes & Sirmans (1992), Basu & Thibodeau (1998), Coulson & Bond (1990), among others.

framework, we can use information of the housing market for eliciting priors of parameters for each estate to produce sharper inference. The Bayesian approach is often used for drawing finite sample inference of latent parameters and missing data. We use the information obtained from of the rental units to elicit priors of the latent rent of a unit sold. The posterior of the latent rent is simulated jointly with other parameters in the model. We use Bayes factors as the criterion to select from competing models. We find strong evidence supporting heterogeneous (estate-specific) models of price and rent.

Recently, an innovative study by Hui et al. (2010) applies Bayesian method to pricing of Hong Kong's real estate market. Similar to the markets of Shanghai and Shenzhen, the Hong Kong residential market mainly consists of high-rise estates with multiple units of similar features in each estate. Hui et al. (2010) develop a Bayesian hierarchical model that makes use of information of units in the same estate for efficient estimation of pricing factors of each unit. Hui et al. (2010) show the Bayesian approach is effective in drawing inference of a large number of parameters, as we do in this study. However, our study has a different research objective from that by Hui et al. (2010). Hui et al. (2010) focus on advancing the literature of hedonic pricing by using the transaction price data only, we focus on cross-sectional comparison of the price-rent ratios by using both the transaction price and rental data.

We address two questions concerning the cross-sectional distribution of the price of housing units relative to their fundamentals. First, "How large is the dispersion in the price-to-rent ratio across housing units?" We find that the cross-estate standard deviation of estimated price-to-rent ratio is substantial: about forty percent for Shanghai and twenty three percent for Shenzhen. The second empirical question is "How is the price-to-rent ratio correlated with features of the property?" We find that the price-to-rent ratio is higher for low-end housing. The cross estate correlation of estate-fixed effects of price-to-rent ratio and that of rent is -0.87 for Shanghai and -0.63 for Shenzhen. Economic explanations for the high price-to-rent ratio of low-end housing considered in the paper include better growth prospects of estates in newly developed locations and stronger demand for low-end housing due to a variety of housing market features and government subsidies.

The rest of the paper is organized as follows. Section 2 lays out the models for latent rent and the pricing models. It also includes a Markov chain Monte Carlo (MCMC) algorithm for posterior simulation. Section 3 presents empirical results on Bayesian model selection and estimation. Section 4 offers economic explanations to the phenomenon that low-end estates tend to carry a high price-to-rent ratio. Section 5 concludes.

2. A Bayesian Framework of Real Estate Pricing

2.1. Pricing Real Estate Based on Latent Rent

For estate i $(i = 1, \cdots, I)$ apartment unit sold j, $(j = 1, \cdots, J_i)$, the price equation is

$$P_{ij} = c_i + \alpha_i \hat{R}_{ij} + \mathbf{y}'_{ij}\boldsymbol{\beta} + \epsilon_{ij}, \tag{1}$$

where P_{ij} is the logarithm of the price of apartment j in estate i, $\hat{R}_{ij}$ is the logarithm of the latent rent of unit j in estate i, $\mathbf{y}'_{ij}$ is a row-vector of factors that influence real estate pricing, for instance macroeconomic variables such as mortgage rates. $\boldsymbol{\beta}$ is a column vector of unknown parameters. The pricing error ϵ_{ij} is assumed to be normal $N(0, \sigma_i^2)$.

The rent equation is given by

$$R_{ik} = \mu_i + \mathbf{x}'_{ik}\boldsymbol{\theta} + \nu_{ik}, \tag{2}$$

where R_{ik} is the logarithm of the observed rent of rental unit k $(k = 1, \cdots, K_i)$ in estate i $(i = 1, \cdots, I)$, $\mathbf{x}'_{ik}$ is a row-vector of seasonal and unit-specific factors that influence rent (e.g. the size of the unit, the number of bedrooms and bathrooms, the condition of the unit, etc.) We assume that the error term ν_{ik} is normal $N(0, \tau_i^2)$.

The price and rent equations contain three types of heterogeneity in regression coefficients: μ_i, c_i, and α_i. The estate-fixed effect μ_i captures the location, environmental characteristics, the quality of property management service, and other intangibles beyond the hedonic factors included in the rent equation. The parameter c_i reflects the estate-fixed effect on price conditioning on the latent rent and other factors. The parameter α_i concerns heterogeneity in the price elasticity with respect to the latent rent. There are eight combinations of model restrictions to be compared. For instance, a heterogenous pricing model without estate-fixed effect for rent is defined by $\mu_i = \mu$ while c_i and α_i are estate-specific for $(i = 1, \cdots, I)$. In the most restrictive homogenous pricing model, the estate-fixed effect is absent and all parameters are constant across estates. We will show that the empirical result strongly favors the presence of estate-specific fixed effects.

We noted earlier that we observe rental and sale prices of units in the same estate but few units are sold and rented at the same time. We need to develop methodologies for estimation of the latent rents of units sold using the information from rent data from the same estate, for which we conduct a Bayesian analysis.

We assume that the unobserved rent of unit j sold in estate i follows the same distribution as rental units given in equation (2):

$$\hat{R}_{ij} = \mu_i + \hat{\mathbf{x}}'_{ij}\boldsymbol{\theta} + \nu_{ij}. \tag{3}$$

An alternative way of viewing the assumption is that we set the prior for the unobserved rent of a unit sold conditioning on its hedonic features and model parameters as $\hat{R}_{ij} \sim N(\mu_i + \hat{\mathbf{x}}'_{ij}\boldsymbol{\theta}, \tau_i^2)$. The price equation (1) depicts the likelihood function of parameters including the unobserved rent. Vector $\hat{\mathbf{x}}_{ij}$ captures observable hedonic features of the unit j sold in estate i. The posterior of the latent rent $\hat{R}_{ij}$ will be simulated along with other parameters.

A common frequentist solution to unobservable regressors is the instrumental variables approach. By the IV approach, we can treat the rent equation as the first-stage regression and the pricing equation as the second stage regression, based on the instruments of the observed rents of units rented in the same estate and hedonic factors of the units sold. In contrast to the IV approach, the Bayesian approach conducts joint inference for the parameters in the rent and price equations. The information in the price data contributes to the posterior of the imputed rent. Zellner (1970) argues that Bayesian analysis for regression models is preferable when the sample size is small. Although some advances have been made in finite sample hypothesis testing in error-in-variable regressions with homogenous parameters (e.g., Dufour & Jasiak (2001)), the frequentist finite sample distributions of heterogenous parameters are complicated. Bayesian approach is convenient for dealing with heterogenous parameters and measurement errors. Zellner (1970) uses non-informative priors for Bayesian analysis in a regression model with unobservable independent variables. In the present study, we follow the strategy taken by Rossi & Allenby (1993) and elicit informative priors for a large number of estate-specific parameters using information borrowed from data of the whole sample. This approach is designed for sharper inference of estate-specific parameters.

2.2 Prior setting

We assume the following prior setting: $c_i \sim N(\bar{c}, \bar{v_c}^2)$, $\alpha_i \sim N(\bar{\alpha}, \bar{v_\alpha}^2)$, $\boldsymbol{\beta} \sim N(\bar{\beta}, \mathbf{B})$, $\sigma_i^2 \sim IG(s_\sigma, v_\sigma)$, $\mu_i \sim N(\bar{\mu}, \bar{v_\mu}^2)$, $\boldsymbol{\theta} \sim N(\bar{\boldsymbol{\theta}}, \boldsymbol{\Theta})$, $\tau_i^2 \sim IG(s_\tau, v_\tau)$. The prior setting for parameters in heterogenous parameter model is as follows. Following Rossi & Allenby (1993), we elicit priors for estate-specific parameters using the posterior of the homogenous (constant) parameter model (where no parameter is estate specific) under flat prior. Normal prior means of parameter (c_i, α_i, μ_i) are set as their posterior means in the constant parameter model. Posterior variances are set as the number of estates (733 for Shanghai and 125 for

Shenzhen) times the posterior variances of the corresponding parameters in the homogenous model. Inverse Gamma priors for σ_i^2 and τ^2 are set as $IG(3, 0.3)$ and $IG(3, 0.2)$, which have means of 0.15 and 0.1 and standard deviations of 0.15 and 0.1. We consider alternative sets of hyperparameters and find that the reported empirical results are robust to the prior setting.

2.3 Decomposing the price-to-rent ratio by factors

To explore the determinants of the estimated price-to-rent ratio, we decompose the price-to-rent ratio into estate-specific, unit-specific, and macroeconomic factors.

Taking the difference of the logarithm of price in (1) and logarithm of imputed rent in (3), we have the logarithm of the price-to-rent ratio

$$P_{ij} - \hat{R}_{ij} = [c_i + (\alpha_i - 1)\mu_i] + (\alpha_i - 1)\hat{\mathbf{x}}'_{ij}\boldsymbol{\theta} + \mathbf{y}'_{ij}\boldsymbol{\beta} + (\epsilon_{ij} + (\alpha_i - 1)\nu_{ij}). \tag{4}$$

Note that since the latent rent is unobserved, so is the price-to-rent ratio. In (4) the price-to-rent ratio is explicitly decomposed. The term $c_i + (\alpha_i - 1)\mu_i \equiv f_i$, is the estate-specific factor. Empirically, this turns out to be the dominating factor of the price-rent ratio. The second term $(\alpha_i - 1)\mathbf{x}_{ij}'\boldsymbol{\theta} \equiv u_{ij}$ is a unit-specific factor that captures the effect of hedonic features of the unit on the price-to-rent ratio.

One important factor of housing price is the value of land user right. In China, land is state owned but land user right is tradable and transferable. According to the government regulation, owners of commercialized residential housing has a limited time of land use (usually 70 years from the date of the initial commercialization of the real estate or the initial development of the land). When the lease on the land expiries, its owner-occupier is expected to either extend the lease by paying a renewal fee, or revert the housing to the state. The effect of the land use policy on housing price is largely unexplored empirically.

The Shenzhen data contain the year each estate became commercialized (from which we can calculate the remaining years of the user right). The land user effect is measured by $e_{ij} = \delta l_{ij}$, where l_{ij} is the remaining years of the land user right at the time unit j in estate i is sold. $\mathbf{y}'_{ij}\boldsymbol{\beta}$ is the sum of the macroeconomic and seasonal factors and the user right factor. The seasonal and macroeconomic effect is measured by part of the unit-specific effect $m_{ij} = \mathbf{y}'_{ij}\boldsymbol{\beta} - e_{ij}$.

The remaining portion of the price-to-rent ratio, $\xi_{ij} = \epsilon_{ij} + (\alpha_i - 1)\nu_{ij}$, is a pricing error that can not be attributed to factors mentioned above. With this notion, from (4), the price-rent ratio $\lambda_{ij} = P_{ij} - \hat{R}_{ij}$ can be written as

$$\lambda_{ij} = f_i + u_{ij} + m_{ij} + e_{ij} + \xi_{ij}. \tag{5}$$

In the following we will discuss how to simulate numerical distributions of the posterior of model parameters. The posteriors of price-to-rent ratio and its components, $(\lambda_{ij}, f_i, u_{ij}, e_{ij}, m_{ij})$, can be computed from the simulated posteriors of model parameters.

2.4. Posterior simulation

Let

$$\mathbf{P}_i = \begin{pmatrix} P_{i,1} \\ \vdots \\ P_{i,J_i} \end{pmatrix}, \ \hat{\mathbf{R}}_i = \begin{pmatrix} \hat{R}_{i,1} \\ \vdots \\ \hat{R}_{i,J_i} \end{pmatrix}, \ \mathbf{R}_i = \begin{pmatrix} R_{i,1} \\ \vdots \\ R_{i,K_i} \end{pmatrix}, \ \mathbf{y}_i = \begin{pmatrix} \mathbf{y}'_{i,1} \\ \vdots \\ \mathbf{y}'_{i,J_i} \end{pmatrix}, \ \mathbf{x}_i = \begin{pmatrix} \mathbf{x}'_{i,1} \\ \vdots \\ \mathbf{x}'_{i,K_i} \end{pmatrix} \ \hat{\mathbf{x}}_i = \begin{pmatrix} \hat{\mathbf{x}}'_{i,1} \\ \vdots \\ \hat{\mathbf{x}}'_{i,J_i} \end{pmatrix}. \tag{6}$$

The variables with 'hat' ($\hat{\mathbf{R}}_i$, $\hat{\mathbf{x}}_i$) pertain to the units sold, while those without 'hat' pertain to rental units in estate i. Denote data $\mathbf{D} = \{\mathbf{P}_i, \mathbf{R}_i, \mathbf{y}_i, \mathbf{x}_i\}, (i = 1, \cdots, N)$, and the collection of μ_i, c_i, and α_i for all $i = 1, \cdots, I$ by $\{\boldsymbol{\mu}, \boldsymbol{\alpha}, \mathbf{c}\}$. The joint posterior is

$$\begin{aligned} &\pi(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\tau}, \mathbf{c} \mid \mathbf{D}) \\ &\propto L(\mathbf{P}|\hat{\mathbf{R}}, \boldsymbol{\alpha}, \mathbf{c}, \boldsymbol{\beta}, \boldsymbol{\sigma})\pi(\hat{\mathbf{R}}|\boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\tau}, \hat{\mathbf{x}})L(\mathbf{R}|\boldsymbol{\theta}, \boldsymbol{\mu}, \boldsymbol{\tau})\pi(\boldsymbol{\beta})\pi(\boldsymbol{\theta})\pi(\boldsymbol{\sigma})\pi(\boldsymbol{\tau})\pi(\boldsymbol{\alpha})\pi(\boldsymbol{\mu})\pi(\mathbf{c}). \end{aligned}$$

Unlike the conventional regression model, the presence of the endogenous latent rent $\hat{R}_{ij}$ in the price equation implies that the marginal posteriors of the regression coefficients are not standard distributions. In Bayesian analysis, when the posterior does not follow a standard distribution, researchers usually use numerical draws from the posterior for computing quantities of interest. A common approach to numerical draws is MCMC Gibbs sampling, based on the posterior of each parameter conditional on the data and other parameters in the model.

In the appendix, we present the conditional posteriors and a Gibbs sampling algorithm. We focus on the simulation of the posterior for Model 1, the most general model in Table 1. The algorithms for more restricted models such as Models 2 to 8 are similar and are omitted (we only need to replace vector parameters c_i or α_i for $i = 1, , \cdots, N$ by scalar parameters c or α.)

Using the simulated posteriors, the eight combinations of specifications on (μ, c, α) will be compared through Bayesian model selection.

2.5 Bayesian model selection

We compare competing models on the basis of posterior probability of the model given the data $\mathbf{D}$, $pr(M|D)$. $pr(D|M)$ is the product of prior probability $pr(M)$ and the marginal likelihood of the data given the model. The marginal likelihood is obtained by integrating out model parameters in the posterior. The choice between two competing models M_1 and M_2 depends on the ratio $\frac{pr(M1|D)}{pr(M2|D)} = \frac{pr(M1)}{pr(M2)} \times \frac{pr(D|M1)}{pr(D|M2)}$, that is, posterior odds=prior odds $\times$ Bayes factor. When the prior probabilities of the competing models are equal ($pr(M1) = pr(M2) = 0.5$), the Bayes factor $B_{12} = \frac{pr(D|M_1)}{pr(D|M_2)}$ greater than unity suggests the data are in favor of Model 1 over Model 2. The strength of evidence is given by the size of the Bayes factor. Kass & Raftery (1995) suggest a guideline in interpreting Bayes factors: If the natural logarithm of the Bayes factor is between 1 and 3, the evidence is “positive”; between 3 and 5, the evidence is “strong”; and above 5, the evidence is “very strong.” In each model, the posterior is simulated by 10000 MCMC cycles after 10000 “burn in” runs. We report the model selection result in Table 1 and conduct empirical analysis of the price-rent ratio based on the selected model. We calculate the marginal likelihood of each model by integrating parameters analytically whenever possible and then using the harmonic mean for numerical approximation of the remaining portion of the integration.

3. Empirical Results

3.1 Data

The data of two major cities in China, Shanghai and Shenzhen, from January 2003 to December 2005, were provided by Centaline Property Consultants Ltd. China. The data for Shanghai include 7303 rental transactions (each with an observation of monthly rent) and 10335 resale transactions of 733 estates. The data for Shenzhen consist of 2061 rental transactions and 2416 resale transactions of 125 estates. These transactions are made on secondary market so there are considerable variations in the time of transactions across apartment units within each estate.

In this study, transaction price and monthly rent are in RMB Yuan per square meter. The Shanghai data include the following hedonic features: the floor level of the apartment, the number of bedrooms, the number of living rooms, the number of restrooms, and the size of the apartment. In addition, measurements of the quality of interior decoration and furniture, kitchen cabinets, and accessories, are grouped into three categories: none, simple, or luxurious. The Shenzhen data are more limited, the size of the apartment is the only observable feature for all units. For both markets, in the rent equation we control for month and year dummies. Besides the latent rent, control variables in the price equation include mortgage rate at the time of purchase for Shanghai; and mortgage rate and the remaining years of the land user right for Shenzhen.

3.2. Results of model selection

Table 1 lists the log marginal likelihoods of eight competing models. For both Shenzhen and Shanghai's residential housing markets, Bayes factors favor Model 4 over other models. The constant parameter Model 8 is decisively rejected by data. This suggests that estate-specific heterogeneity plays an important role in explaining the price-to-rent ratio. The empirical results for the rest of the paper pertain to Model 4 ($\alpha_i = \alpha$ for all estates).

3.3. Posterior properties the latent rent

Tables 2 and 3 report the cross-estate average of the posterior mean and standard deviations of parameters in Model 4. Posterior mean, our benchmark estimator, is the Bayesian estimator under quadratic loss. The posterior mean of estate specific intercept μ_i ranges from 2.5 to 5.5, indicating a substantial quality difference in the neighborhood characteristics, environmental features, and accessability of various estates.

Structural attributes identified in the previous studies of hedonic pricing, such as the floor level and the number of bedrooms, are available in Shanghai. The number of floor level of the apartment has an ambiguous effect on price. This may be because for apartments in a high-rise housing complex, the higher the floor level the better air quality and view. On the other hand, because most of low-rise apartment buildings are not equipped with elevators, high floor level may be a negative factor. Tables 2 and 3 also report the estimates of other parameters in the model.

The number of bedrooms is negatively correlated with the rent (per square meter) while the number of living rooms and restrooms is positively correlated with rent. In addition, we find that compared with a unfurnished apartment, a plainly furnished apartment does not raise rent by much, while a luxuriously decorated one on average raises rent by about 12 percent. Real mortgage rate is found to be negatively correlated with housing prices. An one percent increase in real mortgage rate drives the housing price down by 9.3 percent in Shanghai and by 5.2 percent in Shenzhen. On average, the number of observations for each estate of Shenzhen is about 1.5 times of that of Shanghai. Hence under similar priors, the posterior for Shenzhen should be tighter. On the other hand, the data set for Shanghai contains more hedonic features of the units sold and rented. The overall fit of the model for the two markets are comparable. The posterior means of the variance of the pricing error σ_i^2 are similar for Shanghai and Shenzhen.

Tables 4 and 5 report the sample average and standard deviations of the posterior means and posterior standard deviations of the factors that influence the price-rent ratio. The tables illustrate the prominent role of the estate-fixed effect. The unit factor and mortgage rate play more significant roles in Shanghai than in Shenzhen. The average estimate of 1.925 for land-user right (which is only observed for Shenzhen) accounts for roughly thirty-six percent of the price-to-rent ratio for units sold in Shenzhen. The cross-estate variation in the land user right is small because the housing reform only started in late 1990s and the transactions in the sample occurred within a short period of time. One may incorporate the land-user right into the estate fixed effect in price-to-rent ratio f_i. The correlation between f_i and estate fixed effect in rent c_i is not significant altered if we do so.

Figures 1 and 2 plot the posterior mean and the posterior 10 and 90 percentiles of the price-to-rent ratio of each unit sold and fixed effect μ_i of each estate of Shanghai and Shenzhen. The horizontal axis corresponds to estates of each city sorted by the posterior mean of μ_i. These figures exhibit two distinct features. First, the posterior distributions of the price-to-rent rent ratio and that of estate quality (μ_i) are quite tight, i.e., the estate-specific fixed effects are estimated with high precision relative to the cross sectional difference. Second, the price-to-rent ratio of low-end housing is higher than that of high-end housing, especially for Shanghai.

Figures 3 and 4 show a positive cross-estate correlation between the fixed effect in rent, μ_i, with the fixed effect in price, c_i. There is a substantial portion of the cross-estate variation in the pricing factor c_i that are not explained by μ_i. Across estates, c_i are more strongly correlated with μ_i in the Shanghai market than the Shenzhen market. This is because the data of Shanghai include richer hedonic features. For the Shenzhen sample, most of the variation in rent is explained by the estate-fixed effect rather than unit-specific features. Consequently the estimate of α (price elasticity with respect to rent) is larger for the Shanghai sample.

3.4. 'Within' estates and 'between' estates decomposition of price-to-rent ratios

In the following, we will examine the 'within' estates and 'between' estate variations of the posterior means and posterior standard deviations of $(\lambda_{ij}, f_i, u_{ij}, e_{ij}, m_{ij})$, which characterize the cross-unit and cross-estate distributions of the price-rent ratio and its determinants. Comparisons of the patterns of these variations shed new light on the housing markets of the two cities.

An essential difference between the cities of Shanghai and Shenzhen lies in their histories. Shanghai is a city of several hundred of years old, with well developed historic districts and recently developed suburban districts. Shenzhen, a major city with more than eight million residents, on the other hand, was a small fishing town three decades ago and was developed when it was designated as the Special Economic Zone in 1979. One would expect larger dispersion in estate-fixed effect in Shanghai than in Shenzhen.

Tables 6 and 7 show the cross-unit ('within'-estate) and 'between'-estate distributions of the point estimate of the price-rent ratio. Consistent with the cities history of development, the magnitude of cross-estate ('between') price-rent ratios and estate fixed factors are much larger for Shanghai than for Shenzhen. In contrast, the cross-unit ('within') estate distributions are similar for Shanghai and Shenzhen. We also consider the price-rent ratio and its determinants from a different perspective, by examining the posterior means and standard deviations of 'within' and 'between' variations of $(\lambda_{ij}, f_i, u_{ij}, e_{ij}, m_{ij})$, the point estimate and posterior uncertainty (precision) of cross-unit and cross-estate variations of the price-rent ratio and its determinants. These unreported estimates are similar to those in Tables 6 and 7.

4. Explain the Negative Correlation between Rent and Price-to-Rent Ratio

The negative correlation between the estate-specific fixed effect in the price-rent ratio, f_i, and the fixed effect of rent, μ_i, warrants further discussion. We consider three types of explanations in subsections 4.1 to 4.3 for the negative correlation.

4.1 Estimation errors in rent fixed effect (μ_i)

First, note that the negative cross-estate correlation between $f_i = c_i + (\alpha - 1)\mu_i$ and μ_i may be partly due to the estimation errors in μ_i. Suppose the 'true' parameter is $f_i^* = c_i + (\alpha - 1)\mu_i^*$ and μ_i^*, and suppose $\mu_i = \mu_i^* + v_i$, where the error v is uncorrelated with μ or c. Then the cross-estate covariance

$$\begin{aligned} cov(f_i, \mu_i) &= E(f_i - \bar{f})(\mu_i - \bar{\mu}) \\ &= E\{[c_i + (\alpha - 1)(\mu_i^* + v_i) - \bar{c} - (\alpha - 1)\bar{\mu}](\mu_i^* - \bar{\mu} + v_i)\} \\ &= E\{(f_i^* - \bar{f}_i)(\mu_i^* - \bar{\mu})\} + (\alpha - 1)Ev_i^2. \end{aligned}$$

Since the estimate of α is smaller than unity, the covariance has a downward bias $(\alpha - 1)Ev_i^2$.

We will show that for two reasons the magnitude of this bias is small for the present problem.

First, if we use the posterior variance of μ_i to proxy that of the estimation error v_i, then based on the statistics reported in Tables 2 and 4, and the fact that the cross-estate standard deviation μ_i is 0.53 for the Shanghai data (not reported in the tables), $\frac{(\alpha-1)Ev_i^2}{sd(f_i)sd(\mu_i)}$ is roughly $\frac{(0.29-1)\times 0.11^2}{0.39\times 0.53} \approx -0.04$, much lower than the sample correlation -0.87 (for Shanghai). For the Shenzhen data, the cross-estate standard deviation μ_i is 0.39. Given the numbers reported in Tables 2 and 4, the contribution from estimation error in the estate-fixed effects of rent is approximately $\frac{(\alpha-1)Ev_i^2}{sd(f_i)sd(\mu_i)} = \frac{(0.148-1)\times 0.08^2}{0.24\times 0.39} \approx -0.06$, negligible compared to -0.63, the negative correlation between the the posterior mean of f_i and that of μ_i. Note that because the lack of hedonic features of housing units in Shenzhen data, the unit-specific rent plays a smaller role in explaining price in Shenzhen than in Shanghai. In particular, the estimate of α (0.148) for the Shenzhen sample is half the magnitude of that of Shanghai (0.290). For a given estimation error in the estate-fixed effect μ_i, the downward bias in the cross-estate correlation between f_i and μ_i for Shenzhen should be larger than that for Shanghai. Yet the obtained sample correlation for Shenzhen (-0.63) is higher than that of Shanghai (-0.87). This suggests that besides estimation error in μ_i, there are fundamental reasons that the price-to-rent ratio is negatively correlated with latent rent.

Second, estimation error in μ_i leads to negative bias in the correlation between f_i and μ_i when α is less than unity. To shut down this source of bias, we consider a more restrictive pricing model by setting $\alpha_i = 1$ in (1), with the rent equation unchanged. The corresponding price-to-rent ratio becomes

$$P_{ij} - \hat{R}_{ij} = c_i + \mathbf{y}'_{ij}\boldsymbol{\beta} + \epsilon_{ij}. \tag{7}$$

Now the estate specific factor $f_i = c_i$. The resultant cross-estate correlation between the posterior mean of f_i and μ_i is -0.86 for Shanghai and -0.56 for Shenzhen, not much different from the values reported earlier. We conclude that the negative cross-estate correlation between the price-to-rent ratio and latent rent is not a statistical artifact and warrants economic explanations.

4.2 Growth potentials of low-rent high price-to-rent ratio estates

One explanation for the negative correlation between latent rent and price-to-rent ratio is that low-rent estates tend to have higher potential for future growth and commend a higher price-to-rent ratio. The expected return to housing is $\mathbb{E}(\frac{future\ price}{current\ price}) + \frac{rent}{current\ price}$. Hence in an economy consists of risk-neutral investors, equalization of the expected return to all estates implies that owners of high price-to-rent properties should expect fast appreciation in housing price. Using MSA level data, Capozza & Seguin (1996) show that the price-to-rent ratio is useful in predicting long-run housing price appreciation, but only if the differences in the quality of rental versus owner-occupied housing are controlled.

We find ample evidence in Shanghai data that supports this theory. Specifically, we discover that estates in newly developed locations that are away from city center of Shanghai tend to command low rent, high price-to-rent ratio, and better growth potential. There are thirteen administrative districts of Shanghai. Sorting the districts by the average price reveals a pattern that the high-price districts located in the old city of Shanghai with high population density and with established shopping areas and restaurants. In comparison, estates in newly developed suburban districts tend to be cheaper in price and rent, but with higher price-to-rent ratio. The premium in housing price in the low price districts in part reflects the growth potential of the expanding suburban districts in the Shanghai. From 2003 to 2005, the population density of the suburban districts increased while that of the historic downtown districts remained unchanged or even decreased. Growth in allocation of public goods is faster in Shanghai's suburban districts. From 2003 to 2005, the teacher-student ratio increased in suburban districts more than it did in the old city districts. This trend in the improvement of public education in the low rent-high price/rent ratio districts point to their better prospects of price appreciation. We argue that the difference between Shanghai and Shenzhen lends support to the location-dependent growth potential theory. The negative correlation between f_i and μ_i is stronger in Shanghai than in Shenzhen partly because in Shanghai the low-rent estates are more concentrated in the lightly populated suburban districts while districts in Shenzhen the districts are similar and correlation between rent and growth potential is weaker.

The higher growth of remote districts explains a portion of the negative correlation between f_i and μ_i, but it is unlikely to be the only factor. The within-district correlations between f_i and μ_i are also negative (about -0.7 on average within districts of Shanghai and -0.3 within districts of Shenzhen). This suggests that the presence of other factors for the negative correlation between f_i and μ_i. These factors are considered in the following.

4.3. Strong demand of low-end units induced by factors in housing market and government policy

The notion that low quality housing is more expensive (relative to the fundamentals) than high quality housing is not new. Sweeney (1974) presents a theoretical model on equilibrium of indivisible goods with quality hierarchy. He shows that replacing a low quality unit by a high quality one creates a chain reaction that lowers the prices of all units with quality higher than a threshold level and raises the prices of all units with quality below that level. The accumulation of the new constructions then make the low-end housing more expensive relative to the high-end housing.

The price of housing may be systematically related to factors other than the service flow of the housing for various reasons. Genesove & Mayer (1997) show that the price of housing is affected by the owner's equity. Housing units with higher loan-to-equity ratio tend to be more expensive and take longer to sell. Another possible explanation is that low-end housing may carry higher risk premium. For the U.S. market, Sinai & Souleles (2005) show that regional prices of real estates are positively related with the volatility of risk of rents in the region. In our sample, if renters of low-end housing are more averse of unexpected increase in rent than renters of high-end housing, or the growth rate in the rent of low-end housing is more uncertain, then owners of low-end housing are willing to pay a higher price. In addition, the low-end housing units is more expensive given its hedonic features because they tend to be traded more frequently. The transaction cost, commissions, transaction tax are paid more times on low-end units. Case et al. (1997) find that properties that transact more frequently tend to be at the lower-end (starters or fix-uppers) but experience faster price appreciation. Although their finding does not directly relate the frequency of transactions to price-to-rent ratio, it is consistent with the scenario that starter units are more frequently traded at higher price-to-rent ratios.

Stein (1995) studies the effect of down-payment requirement on housing price dynamics. He shows that the requirement of down-payment amplifies the magnitude of price fluctuations and creates a positive correlation between price movement and trading volume. The down-payment requirement also have cross-section effect on prices. The high housing price (relative to income) of the two Chinese cities coupled with the thirty percent down-payment requirement for standard mortgages make housing unaffordable for a large fraction of residents. In markets where fast appreciation in housing price is anticipated, households often own maximum amount of housing permitted by the ability to pay. As the income and saving of potential buyers rise above a threshold, the majority of the first-time home buyers purchase low-end units. This elevates demand of low-end housing and makes it relatively more expensive than high-end housing. The theory also predicts that the larger the dispersion in quality the more expensive the low-end housing is. The evidence in the two cities confirms this prediction. The estate-specific factor is much more diverse in Shanghai than in Shenzhen. The negative correlation of the price-to-rent ratio and latent rent is also stronger in Shanghai.

Finally, the demand for low-end housing maybe disproportionately raised by government policies. Studies on similar policies in other countries yield the same conclusion. Susin (2002) finds evidence that rent vouchers for low income households in the U.S. elevate the price of low-end housing. Vigdor (2006) finds empirical evidence that U.S. government policy that relaxes liquidity constraint for veterans pushes up housing price. Mortgage subsidies for low-end housing raises its demand for low-end housing in Shanghai and Shenzhen. According to the lending policy of China People's Bank, low income households that purchase low-end housing (with its size and price below a threshold) are eligible for subsided mortgage loans for 60 months. Empirical estimation of the factors discussed above will be left for future research.

4.4 Policy implications of high price of low-end housing

An adverse welfare implication of high price-to-rent ratio of low-end housing is that low-income households pay a relatively high price for the service of housing they own. The policy remedy to the resultant inequity is far from being obvious. Numerous studies (e.g., Susin (2002) and Vigdor (2006) discussed above) show that housing subsides to low-income families in the form of mortgage or rent assistance likely make the low-end housing more expensive, which at least partially offset the intended policy objective of housing affordablity. Owning the low-end housing can be rationalized by a higher expected price appreciation or higher risk premium for hedging rent growth. However, the ownership of low-end housing makes low- and middle-income families more vulnerable to a downturn in housing market when they invest a large portion of household wealth in housing. Because at the low-end it is more expensive to own than to rent, government policies promoting ownership of low-end housing may reduce the welfare of its owner ex-post.

The large difference in the price-to-rent ratio between high-end and low-end housing found in this study also implies that different segments of housing should be treated as different asset classes from the investment perspective. Owner-occupied housing is the most valuable investment for many households. Higher price-to-rent ratio of low-end housing means that its owner holds an asset resembles to a stock with a high price-to-dividend ratio. Because the owner-occupied housing is sorted by income, unlike investment in financial assets that are available to all investors, low-income households unwittingly hold a class of housing investment with different risk characteristics from the those held by high-income households. It is useful to analyze investment strategies for households with different income levels while taking into account the difference in risk of owner-occupied housing. For the purpose of diversification, it is useful to construct housing indices of low-end housing as a separate asset class from high-end housing.

5. Concluding Remarks

In this paper, we investigate how prices of housing units relate to their latent rents. Our cross-sectional analysis of the housing price-to-rent ratio adds to the literature of real-estate pricing based on the present value of rent, hedonic features, and repeated sales. The model consisting of price and rent equations contains several thousands of parameters. Bayesian model selection indicates strong heterogeneity in the parameters across estates. The estimated price-to-rent ratios differ substantially across estates (e.g., the cross-estate standard deviation of the logarithm of price-to-rent ratio is about forty percent in Shanghai.) We analyze the factors that influence the price-to-rent ratio. We find that the estate fixed-effect of the price-rent ratio is negatively correlated with the estate fixed-effect of rent. This finding agrees with several economic theories and affords us new perspectives in investment and public policy in housing.

Appendix

The joint posterior is

$$\pi(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\tau}, \mathbf{c} \mid \mathbf{D})$$

$$\begin{aligned}
&\propto L(\mathbf{P}|\hat{\mathbf{R}}, \boldsymbol{\alpha}, \mathbf{c}, \boldsymbol{\beta}, \boldsymbol{\sigma})\pi(\hat{\mathbf{R}}|\boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\tau}, \hat{\mathbf{x}})L(\mathbf{R}|\boldsymbol{\theta}, \boldsymbol{\mu}, \boldsymbol{\tau})\pi(\boldsymbol{\beta})\pi(\boldsymbol{\theta})\pi(\boldsymbol{\sigma})\pi(\boldsymbol{\tau})\pi(\boldsymbol{\alpha})\pi(\boldsymbol{\mu})\pi(\mathbf{c}) \\
&\propto \prod_{i=1}^{I} exp\left\{-\frac{\sum_{j=1}^{J_i}[P_{ij} - (c_i + \alpha_i \hat{R}_{ij} + \mathbf{y}'_{ij}\boldsymbol{\beta})]^2}{2\sigma_i^2} - \frac{\sum_{j=1}^{J_i}[\hat{R}_{ij} - (\mu_i + \hat{\mathbf{x}}'_{ij}\boldsymbol{\theta})]^2}{2\tau_i^2}\right\} \\
&\times \prod_{i=1}^{I} exp\left\{-\frac{\sum_{k=1}^{K_i}[R_{ik} - (\mu_i + \mathbf{x}'_{ik}\boldsymbol{\theta})]^2}{2\tau_i^2}\right\} exp\left\{-\frac{1}{2}(\boldsymbol{\beta} - \bar{\boldsymbol{\beta}})'\mathbf{B}^{-1}(\boldsymbol{\beta} - \bar{\boldsymbol{\beta}})\right\} \\
&\times exp\left\{-\frac{1}{2}(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}})'\boldsymbol{\Omega}^{-1}(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}})\right\} \prod_{i=1}^{I}\left\{\sigma_i^{-2(s_\sigma+1)} \exp(-\frac{\upsilon_\sigma}{\sigma_i^2})\sigma_i^{-J_i}\tau_i^{-2(s_\tau+1)} exp(-\frac{\upsilon_\tau}{\tau_i^2})\tau_i^{-K_i-J_i}\right\} \\
&\times \prod_{i=1}^{I}\left\{exp[-\frac{(\alpha_i - \bar{\alpha})^2}{2\bar{\upsilon_\alpha}^2}]exp[-\frac{(\mu_i - \bar{\mu})^2}{2\bar{\upsilon}_\mu^2}]exp[-\frac{(c_i - \bar{c})^2}{2\bar{\upsilon}_c^2}]\right\}.
\end{aligned}$$

The following conditional posteriors are used for Gibbs sampling:

(i)

$$\begin{aligned}
&\pi(\mu_i \mid \tau_i, \boldsymbol{\theta}, \hat{\mathbf{R}}_i, \mathbf{D}) \\
&\propto exp\left\{-\frac{(\mu_i - \bar{\mu})^2}{2\bar{\upsilon_\mu}^2}\right\} exp\left\{-\frac{\sum_{k=1}^{K_i}(\mu_i - \tilde{\mu_{ik}})^2}{2\tau_i^2}\right\} exp\left\{-\frac{\sum_{j=1}^{J_i}(\mu_i - \hat{\mu_{ij}})^2}{2\tau_i^2}\right\} \\
&\propto N(\frac{\frac{\bar{\mu}}{\bar{\upsilon_\mu}^2} + \frac{\sum_{k=1}^{K_i}\tilde{\mu}_{ik} + \sum_{j=1}^{J_i}\hat{\mu}_{ij}}{\tau_i^2}}{\frac{1}{\bar{\upsilon_\mu}^2} + \frac{K_i + J_i}{\tau_i^2}}, \frac{1}{\frac{1}{\bar{\upsilon_\mu}^2} + \frac{K_i + J_i}{\tau_i^2}}),
\end{aligned}$$

where $\tilde{\mu}_{ik} = R_{ik} - \mathbf{x}'_{ik}\boldsymbol{\theta}, \hat{\mu}_{ij} = \hat{R_{ij}} - \mathbf{x}'_{ij}\boldsymbol{\theta}$.

(ii)

$$\begin{aligned}
&\pi(c_i \mid \alpha_i, \boldsymbol{\beta}, \boldsymbol{\theta}, \sigma_i, \hat{\mathbf{R}}_i, \mathbf{D}) \\
&\propto exp\left\{-\frac{(c_i - \bar{c})^2}{2\bar{\upsilon_c}^2}\right\} exp\left\{-\frac{\sum_{j=1}^{J_i}(c_i - \hat{c_{ij}})^2}{2\sigma_i^2}\right\} \propto N(\frac{\frac{\bar{c}}{\bar{\upsilon_c}^2} + \frac{\sum_{j=1}^{J_i}\hat{c}_{ij}}{\sigma_i^2}}{\frac{1}{\bar{\upsilon_c}^2} + \frac{J_i}{\sigma_i^2}}, \frac{1}{\frac{1}{\bar{\upsilon_c}^2} + \frac{J_i}{\sigma_i^2}}),
\end{aligned}$$

where $\hat{c}_{ij} = P_{ij} - \alpha_i \hat{R}_{ij} - \mathbf{y}'_{ij}\boldsymbol{\beta}$.

(iii)

$$\pi(\alpha_i \mid c_i, \boldsymbol{\beta}, \sigma_i, \hat{\mathbf{R}}_i, \mathbf{D}) \propto exp\left\{-\frac{(\alpha_i - \bar{\alpha})^2}{2\bar{\upsilon_\alpha}^2}\right\} exp\left\{-\frac{\sum_{j=1}^{J_i}(\alpha_i \hat{R}_{ij} - w_{ij})^2}{2\sigma_i^2}\right\}$$

$$\propto N\left(\frac{\frac{\bar{\alpha}}{\bar{v}_\alpha^2} + \frac{\sum_{j=1}^{J_i} \hat{R}_{ij} w_{ij}}{\sigma_i^2}}{\frac{1}{\bar{v_\alpha}^2} + \frac{\sum_{j=1}^{J_i} \hat{R}_{i,j}^2}{\sigma_i^2}}, \frac{1}{\frac{1}{\bar{v}_\alpha^2} + \frac{\sum_{j=1}^{J_i} \hat{R}_{i,j}^2}{\sigma_i^2}}\right),$$

where $w_{ij} = P_{ij} - c_i - \mathbf{y}'_{ij}\boldsymbol{\beta}$.

(iv)

$$\begin{aligned}
&\pi(\boldsymbol{\beta} \mid \mathbf{c}, \boldsymbol{\alpha}, \boldsymbol{\sigma}, \hat{\mathbf{R}}_i, \mathbf{D}) \\
&\propto exp\left\{-\frac{1}{2}(\boldsymbol{\beta} - \bar{\boldsymbol{\beta}})'\mathbf{B}^{-1}(\boldsymbol{\beta} - \bar{\boldsymbol{\beta}})\right\} exp\left\{-\sum_{i=1}^{I} \frac{(\mathbf{g}_i - \mathbf{y}_i\boldsymbol{\beta})'(\mathbf{g}_i - \mathbf{y}_i\boldsymbol{\beta})}{2\sigma_i^2}\right\} \\
&\propto N([\mathbf{B}^{-1} + \sum_{i=1}^{I} \sigma_i^{-2}\mathbf{y}'_i\mathbf{y}_i]^{-1}[\mathbf{B}^{-1}\bar{\boldsymbol{\beta}} + \sum_{i=1}^{I} \sigma_i^{-2}\mathbf{y}'_i\mathbf{g}_i],\ [\mathbf{B}^{-1} + \sum_{i=1}^{I} \sigma_i^{-2}\mathbf{y}'_i\mathbf{y}_i]^{-1}),
\end{aligned}$$

where $\mathbf{g}_i = \mathbf{P}_i - c_i\mathbf{1} - \alpha_\mathbf{i}\hat{\mathbf{R}}_\mathbf{i}$.

(v)

$$\begin{aligned}
&\pi(\boldsymbol{\theta} \mid \boldsymbol{\mu}, \boldsymbol{\tau}, \hat{\mathbf{R}}_i, \mathbf{D}) \\
&\propto exp\left\{-\frac{1}{2}(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}})'\boldsymbol{\Theta}^{-1}(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}})\right\} exp\left\{-\sum_{i=1}^{I} \frac{(\mathbf{q}_i - \mathbf{x}_i\boldsymbol{\theta})'(\mathbf{q}_i - \mathbf{x}_i\boldsymbol{\theta}) + (\hat{\mathbf{q}}_i - \hat{\mathbf{x}}_i\boldsymbol{\theta})'(\hat{\mathbf{q}}_i - \hat{\mathbf{x}}_i\boldsymbol{\theta})}{2\tau_i^2}\right\} \\
&\propto N([\boldsymbol{\Theta}^{-1} + \sum_{i=1}^{I} \tau_i^{-2}(\mathbf{x}'_i\mathbf{x}_i + \hat{\mathbf{x}}_i{}'\hat{\mathbf{x}}_i)]^{-1}[\boldsymbol{\Theta}^{-1}\bar{\boldsymbol{\theta}} + \sum_{i=1}^{I} \tau_i^{-2}(\mathbf{x}'_i\mathbf{q}_i + \hat{\mathbf{x}}'_i\hat{\mathbf{q}}_i)],\ [\boldsymbol{\Theta}^{-1} + \sum_{i=1}^{I} \tau_i^{-2}(\mathbf{x}'_i\mathbf{x}_i + \hat{\mathbf{x}}'_i\hat{\mathbf{x}}_i)]^{-1}),
\end{aligned}$$

where $\mathbf{q}_i = \mathbf{R}_i - \mu_i\mathbf{1}$, $\hat{\mathbf{q}}_i = \hat{\mathbf{R}}_i - \mu_i\mathbf{1}$.

(vi)

$$\begin{aligned}
&\pi(\sigma_i^2 \mid c_i, \alpha_i, \boldsymbol{\beta}, \hat{\mathbf{R}}_i, \mathbf{D}) \\
&\propto (\sigma_i^2)^{-(s_\sigma+1)} exp(-\frac{v_\sigma}{\sigma_i^2})(\sigma_i^2)^{-\frac{Ji}{2}} exp\left\{-\frac{\hat{v}_i}{2\sigma_i^2}\right\} \\
&\propto IG(s_\sigma + \frac{J_i}{2}, v_\sigma + \frac{\hat{v}_i}{2}),
\end{aligned}$$

where $\hat{v}_i = \sum_{j=1}^{J_i}[P_{ij} - (c_i + \alpha_i\hat{R_{ij}} + \mathbf{y}'_{ij}\boldsymbol{\beta})]^2$.

(vii)

$$\pi(\tau_i^2 \mid \mu_i, \boldsymbol{\theta}, \hat{\mathbf{R}}_i, \mathbf{D})$$

$$\propto (\tau_i^2)^{-(s_\tau+1)} exp(-\frac{v_\tau}{\tau_i^2})(\tau_i^2)^{-\frac{K_i}{2}}(\tau_i^2)^{-\frac{J_i}{2}} exp\left\{-\frac{\tilde{h}_i}{2\tau_i^2}\right\} exp\left\{-\frac{\hat{h}_i}{2\tau_i^2}\right\}$$
$$\propto IG(s_\tau + \frac{K_i + J_i}{2}, v_\tau + \frac{\tilde{h}_i + \hat{h}_i}{2}),$$

where $\tilde{h}_i = \sum_{k=1}^{K_i}[R_{ik} - (\mu_i + \mathbf{x}_{ik}'\boldsymbol{\theta})]^2, \hat{h}_i = \sum_{j=1}^{J_i}[\hat{R}_{ij} - (\mu_i + \mathbf{x}_{ij}'\boldsymbol{\theta})]^2$.

(viii)

$$\pi(\hat{R}_{ij} \mid \mu_i, \alpha_i, c_i, \boldsymbol{\beta}, \boldsymbol{\theta}, \tau_i, \sigma_i, \mathbf{D})$$
$$\propto exp\left\{-\frac{[P_{ij} - (c_i + \alpha_i \hat{R_{ij}} + \mathbf{y}_{ij}'\boldsymbol{\beta})]^2}{2\sigma_i^2}\right\} exp\left\{-\frac{[\hat{R_{ij}} - (\mu_i + \mathbf{x}_{ij}'\boldsymbol{\theta})]^2}{2\tau_i^2}\right\}$$
$$\propto N(\frac{\frac{\alpha_i w_{ij}}{\sigma_i^2} + \frac{\mu_i + x_{ij}'\boldsymbol{\theta}}{\tau_i^2}}{\frac{1}{\tau_i^2} + \frac{\alpha_i^2}{\sigma_i^2}}, \frac{1}{\frac{1}{\tau_i^2} + \frac{\alpha_i^2}{\sigma_i^2}}),$$

where $w_{ij} = P_{ij} - c_i - \mathbf{y}_{ij}'\boldsymbol{\beta}$.

The conditional posteriors suggest a Gibbs sampling MCMC algorithm: In cycle k, with $(c_i^{(k-1)}, \mu_i^{(k-1)}, \alpha_i^{(k-1)}, \sigma_i^{(k-1)}, \tau_i^{(k-1)}, \boldsymbol{\theta}^{(k-1)}, \boldsymbol{\beta}^{(k-1)})$ drawn already,

(1) Draw $(\mu_i^{(k)} \mid \tau_i^{(k-1)}, \boldsymbol{\theta}^{(k-1)}, \hat{\mathbf{R}}_i^{(k-1)}, \mathbf{D})$ (for all $i = 1, \cdots, I$) from the normal distribution in (i).

(2) Draw $(c_i^{(k)} \mid \alpha_i^{(k-1)}, \boldsymbol{\beta}^{(k-1)}, \sigma_i^{(k-1)}, \hat{\mathbf{R}}_i^{(k-1)}, \mathbf{D}$ (for all $i = 1, \cdots, I$) from the normal distribution in (ii).

(3) Draw $(\alpha_i^{(k)} \mid c_i^{(k)}, \boldsymbol{\beta}^{(k-1)}, \sigma_i^{(k-1)}, \hat{\mathbf{R}}_i^{(k-1)}, \mathbf{D})$ (for all $i = 1, \cdots, I$) from the normal distribution in (iii).

(4) Draw $(\boldsymbol{\beta}^{(k)} \mid c_i^{(k)}, \alpha_i^{(k)}, \sigma_i^{(k-1)}, \hat{\mathbf{R}}_i^{(k-1)}, \mathbf{D})$ from the normal distribution in (iv).

(5) Draw $(\boldsymbol{\theta}^{(k)} \mid \mu_i^{(k)}, \tau_i^{(k-1)}, \hat{\mathbf{R}}_i^{(k-1)}, \mathbf{D})$ from the normal distribution in (v).

(6) Draw $(\sigma_i^{(k)} \mid c_i^{(k)}, \alpha_i^{(k)}, \boldsymbol{\beta}^{(k)}, \hat{\mathbf{R}}_i^{(k-1)}, \mathbf{D})$ (for all $i = 1, \cdots, I$) from the IG distribution in (vi) (for all $i = 1, \cdots, I$).

(7) Draw $(\tau_i^{(k)} \mid \mu_i^{(k)}, \boldsymbol{\theta}^{(k)}, \mathbf{D})$ (for all $i = 1, \cdots, N$) from the IG distribution in (vii) (for all $i = 1, \cdots, I$).

(8) Draw $(\hat{R}_{ij}^{(k)} \mid c_i^{(k)}, \alpha_i^{(k)}, \mu_i^{(k)}, \tau_i^{(k)}, \sigma_i^{(k)}, \boldsymbol{\beta}^{(k)}, \boldsymbol{\theta}^{(k)}, \mathbf{D})$ (for all $i = 1, \cdots, I$) from the normal distribution in (viii).

Some of the Gibbs sampling steps can be combined to simulate a larger block of parameters, for example, steps (2) and (3) can be combined to simulate (c, α). We present the algorithm based on the conditional posterior for each parameter vector because it is most transparent and is applicable to different models (for example, the four combinations that c and/or α may be estate-specific or constant across estates).

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Table 1: Bayesian model selection, log(marginal likelihoods) of Shanghai and Shenzhen data

	c_i	μ_i	α_i	log-m.l (Shanghai)	log-m.l (Shenzhen)
Model 1	estate-specific	estate-specific	estate-specific	-3068.93	-73.75
Model 2	constant	estate-specific	estate-specific	-1623.71	6.32
Model 3	estate-specific	constant	estate-specific	-8379.62	-1945.48
Model 4	estate-specific	estate-specific	constant	**-1519.32**	**315.73**
Model 5	constant	constant	estate-specific	-8208.22	-1821.96
Model 6	constant	estate-specific	constant	-1821.83	-146.12
Model 7	estate-specific	constant	constant	-7508.37	-1839.41
Model 8	constant	constant	constant	-8471.34	-2151.23

Note: A parameter is labeled 'estate-specific' ('constant') when it is assumed to be different (the same) across estates. The marginal likelihood of Model i ($i = 1, \cdots, 8$) is computed by integrating out all parameters in the posterior under Model i. A difference of 5 between the log marginal likelihoods of two models is considered a strong evidence in favor of the model with the larger marginal likelihood. By this criterion, Model 4 is the best model.

Table 2: Bayesian estimates averaged over estates, Shanghai

parameter	Average of Posterior Mean	Average of Posterior STD
price model		
c_i	8.5193	0.0889
α(constant)	0.2903	0.0068
β_i: real mortgage rate	-0.0934	0.0017
σ_i^2	0.0330	0.0181
rent model		
μ_i	3.7280	0.1126
θ		
floor level	0.0050	0.0005
number of bedroom	-0.1068	0.0085
number of livingroom	-0.0659	0.0090
number of restroom	0.0081	0.0052
decoration: simple	0.0003	0.0067
decoration:luxury	0.1213	0.0079
size	-0.0001	0.0001
τ_i^2	0.0657	0.0305

Note: The results pertain to Model 4 (selected based on the Bayes factor.) The posterior of parameter of each estate is simulated using the MCMC algorithm stated in the paper. From the numerical distribution we compute the posterior mean and posterior standard deviation for each parameter. The first column is obtained by averaging the posterior mean over all estates. The second column is obtained by averaging the posterior standard deviation over all estates. The estimates of seasonal and year dummies are not reported.

Table 3: Bayesian estimates averaged over estates, Shenzhen

parameter	Average of Posterior Mean	Average of Posterior STD
price model		
c_i	6.0685	0.1265
α (constant)	0.1482	0.0314
β_{1i} : real mortgage rate	-0.0524	0.0035
β_{2i}: land user right	0.0308	0.0014
σ_i^2	0.0361	0.0168
rent model		
μ_i	3.5822	0.0808
θ		
Size	-0.0012	0.0001
τ_i^2	0.0579	0.0233

Note: See the note of Table 2.

Table 4: Decomposition of the price-to-rent ratio, Shanghai

parameter	Posterior Mean Average	Posterior Std Average
λ_{ij}	5.8867(0.4346)	0.2471(0.0571)
f_i	5.9025(0.3911)	0.0987(0.0455)
u_{ij}	0.2385(0.1195)	0.0278(0.0053)
m_{ij}	-0.2543(0.1211)	0.0047(0.0022)

Table 5: Decomposition of the price-to-rent ratio, Shenzhen

parameter	Posterior Mean Average	Posterior Std Average
λ_{ij}	5.3184 (0.2451)	0.2516 (0.0966)
f_i	3.3551 (0.2156)	0.1221 (0.0290)
u_{ij}	0.0926 (0.0534)	0.0162 (0.0027)
m_{ij}	-0.0548 (0.0630)	0.0039 (0.0041)
e_{ij}	1.9251 (0.1072)	0.0889 (0.0050)

Note for Tables 4 and 5: The posterior of parameter of each estate is simulated using the MCMC algorithm stated in the paper. From the numerical distribution we compute the posterior mean and posterior standard deviation for each term in equation (5). The first column is obtained by averaging the posterior mean of these terms over all estates. The second column is obtained by averaging the posterior standard deviation of the terms over all estates. Numbers in parenthesis in the first (second) column are standard deviations of the posterior mean (posterior standard deviation) across all estates.

Table 6: 'Within' and 'between' variations in the posterior means of price-to-rent ratios, Shanghai

	Between-Estate Variation of Posterior Mean	Within Estate Variation of Posterior Mean
λ_{ij}	0.1607	0.0299
f_i	0.1538	NA
u_{ij}	0.0046	0.0098
m_{ij}	0.0038	0.0108

Table 7: 'Within' and 'between' variations in the posterior means of price-to-rent ratios, Shenzhen

	Between-Estate Variation of Posterior Mean	Within Estate Variation of Posterior Mean
λ_{ij}	0.0520	0.0315
f_i	0.0309	NA
u_{ij}	0.0010	0.0019
m_{ij}	0.0009	0.0031
l_{ij}	0.0116	0.0004

Notes for Tables 6 and 7: The tables report the 'within estates' and 'between estates' variations of the posterior mean of each term in equation (5). Let z_{ij} be a generic notation for a quantity of interest of estate $i\ (= 1, \cdots, N)$ and unit sold $j\ (j = 1, \cdots, J_i)$. Denote the within estate average $z_{i.} = \frac{1}{J_i}\sum_j z_{ij}$ and the whole sample average $z_{.} = \frac{1}{N}\sum_i z_{i.}$. The 'within' and 'between' variations of quantity of z_{ij} are given by $\frac{1}{N}\sum_i \frac{1}{J_i}\sum_j (z_{ij} - z_{i.})^2$ and $\frac{1}{N}\sum_i (z_{i.} - z_{.})^2$. The posterior of parameter of each estate is simulated using the MCMC algorithm stated in the paper.

Figure 1: Price-to-rent ratio λ_{ij} and estate fixed effect μ_i (Shanghai)

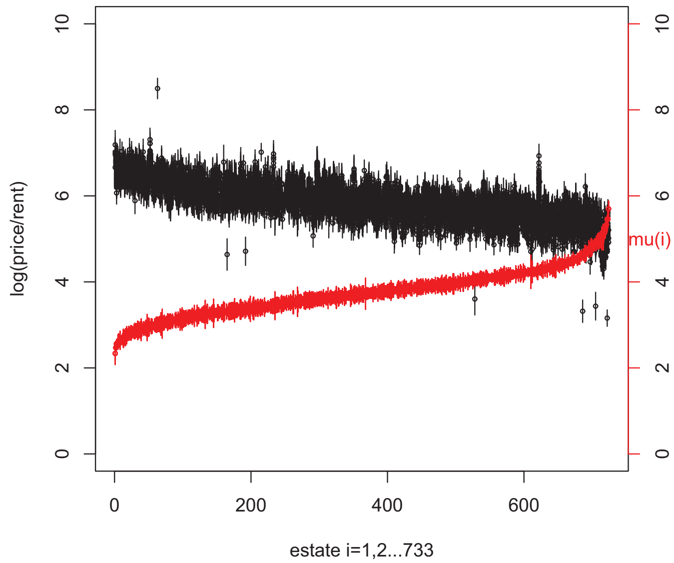
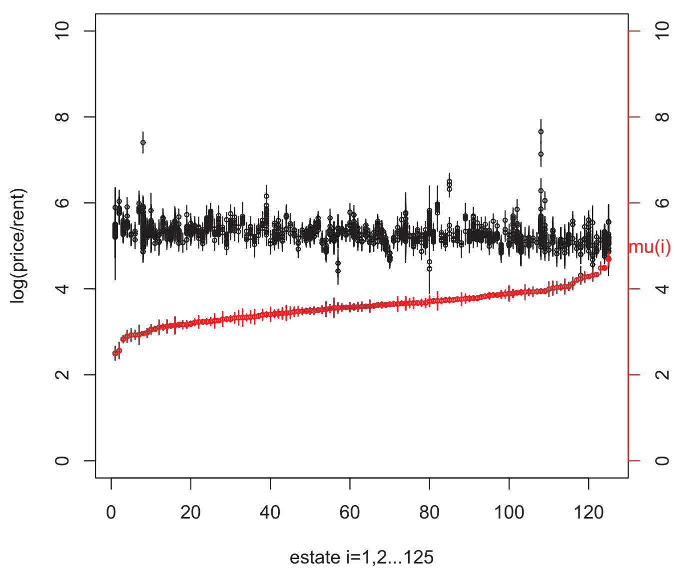


Figure 2: Price-to-rent ratio λ_{ij} and estate fixed effect μ_i (Shenzhen)



Note: Figure 1 (Shanghai) and Figure 2 (Shenzhen) plot the posterior mean and the posterior 10 and 90 percentiles of the price-to-rent ratio of each unit sold λ_{ij} (in black) and those of the estate-fixed effect μ_i in rent (in red). The estates on the horizonal axis are sorted by the posterior mean of μ_i.

Figure 3: Correlation between estate fixed effect in pricing c_i and estate fixed effect in rent μ_i (Shanghai)

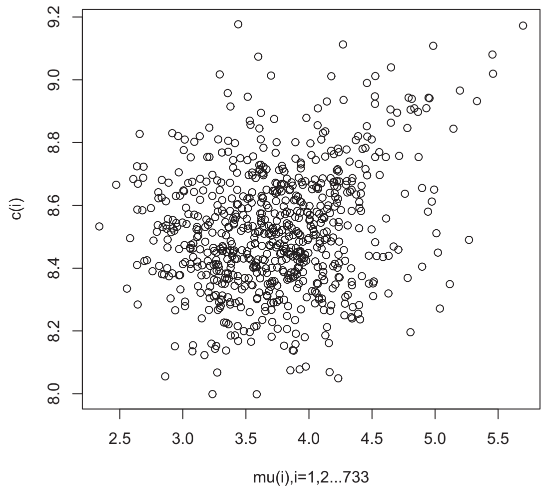
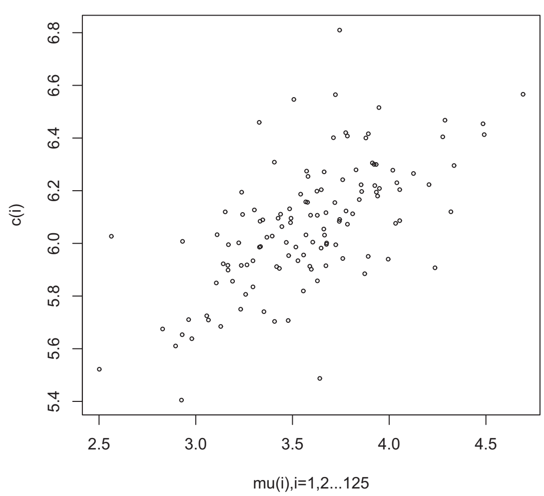


Figure 4: Correlation between estate fixed effect in pricing c_i and estate fixed effect in rent μ_i (Shenzhen)



Note: Figures 3 and 4 plot the posterior mean of c_i for estate i against μ_i. c_i appears in the price equation (1) and μ_i appears in the rent equation (2). The posterior of parameter of each estate is simulated using the MCMC algorithm stated in the paper. The cross-estate correlation between the posterior means of c_i and μ_i is 0.2265 for Shanghai in Figure 3 and is 0.6315 for Shenzhen in Figure 4.

Figure 5: Correlation between estate fixed effect in price-to-rent ratio f_i and estate fixed effect in rent μ_i (Shanghai)

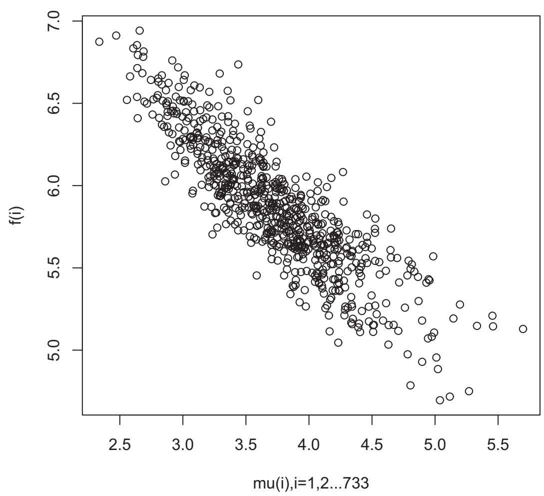
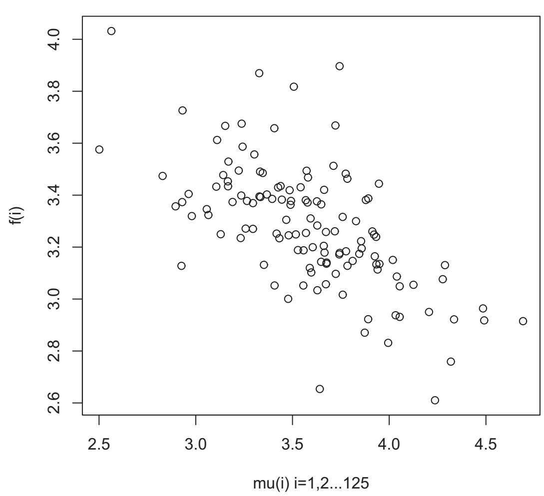


Figure 6: Correlation between estate fixed effect in price-to-rent ratio f_i and estate fixed effect in rent μ_i (Shenzhen)



Note: Figures 5 and 6 plot the posterior mean of f_i for estate i against μ_i. f_i is the fixed effect of price-to-rent ratio in equation (5) and μ_i appears in the rent equation (2). The posterior of parameter of each estate is simulated using the MCMC algorithm stated in the paper. The cross-estate correlation between the posterior means of f_i and μ_i is -0.8671 in Figure 5 (Shanghai data) and -0.6257 (Shenzhen data).