Spreadsheets and algebra
Promoting algebraic thinking in the middle grades using spreadsheets

What is Algebra? For most people, algebra *means using and manipulating (algebraic) symbols, and solving equations. The Algebra Standard, as it is stated in the* Principles and Standards for School Mathematics, *expects more than that from students and teachers. In this workshop we will address some of the ways in which spreadsheets can help to promote algebraic thinking in the middle school mathematics classroom.*

Algebra Standard

— Understand patterns, relations, and functions

— Represent and analyze mathematical situations and structures using algebraic symbols

— Use mathematical models to represent and understand quantitative relationships

— Analyze change in various contexts

Expectations for Grades 6–8

Among the expectations for this standard, the following will be addressed in this workshop.

— Understand patterns, relations, and functions

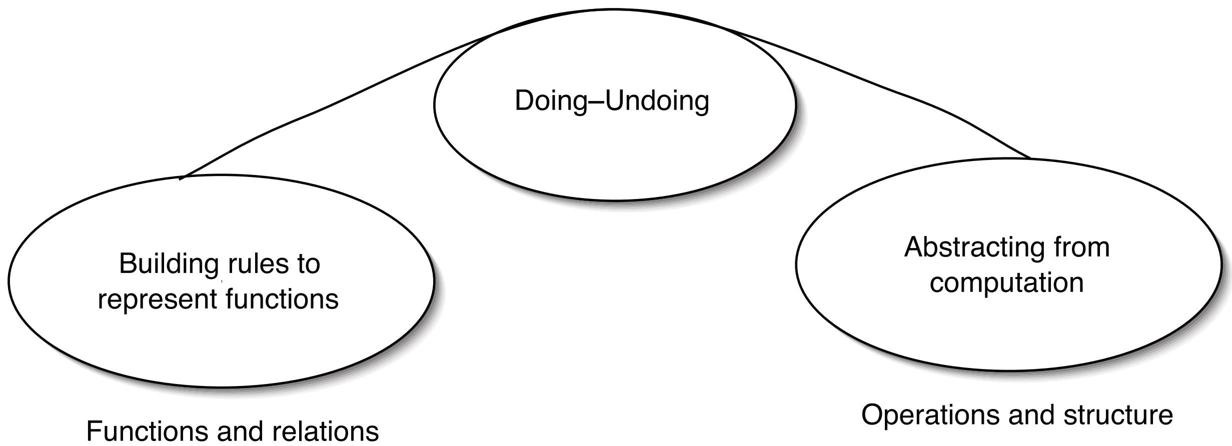
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- relate and compare different forms of representation for a relationship

— Represent and analyze mathematical situations and structures using algebraic symbols

- develop an initial conceptual understanding of different uses of variables

Algebraic Thinking

The following *habits of mind* help promote algebraic thinking (Driscoll, 1999):



Doing-Undoing. Effective algebraic thinking sometimes involves reversibility, being able to undo mathematical processes as well as do them, understanding the process well enough to work backward from the answer to the starting point.

Building Rules to Represent Functions. Critical to algebraic thinking is the capacity to recognize patterns and organize data to represent situations in which input is related to output by well-defined functional rules.

Abstracting from Computation. This is the capacity to think about computations independently of particular numbers that are used. Thinking algebraically involves being able to think about computations freed from the particular numbers they are tied to in arithmetic, that is, abstracting system regularities from computation.

In this workshop we will mostly be concerned with *Building Rules to Represent Functions*.

Why Spreadsheets?

— Dynamic nature.

— Ability to see immediate results of calculations.

— Ability to perform many operations at once (or the same operation over a range of values).

— Students seem to like spreadsheets!

— Spreadsheets can be a natural environment for the introduction of the concept of variable. Students learn to be explicit about what are they doing: instead of "Multiply by 3" or "Multiply A1 by 3" say "Multiply **the number in A1** by 3."

— Spreadsheet applications haven't changed much in many years, there are versions for every kind of computer and operating system.

Practical Considerations

— Use a big size font! (Select all cells and change the font size to 24 pt)

— Plan in advance, adjust the software preferences accordingly.

— Make sure students have pencil and paper.

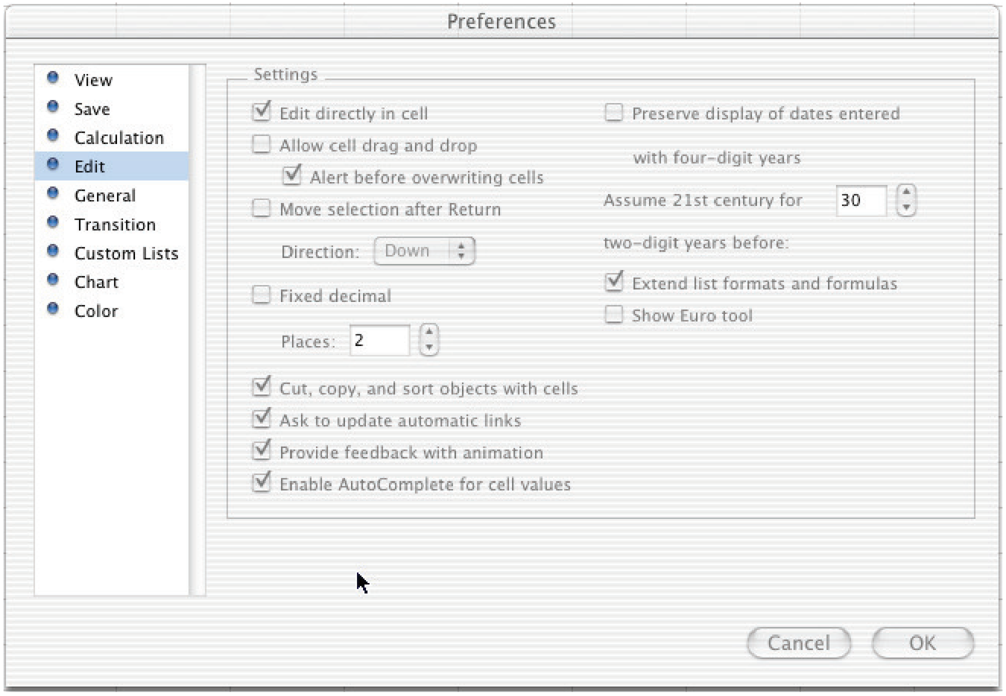
How to Get Ready

Adjust the preferences of your spreadsheet application. In MS Excel, go to Preferences, select Edit. My own preference is:

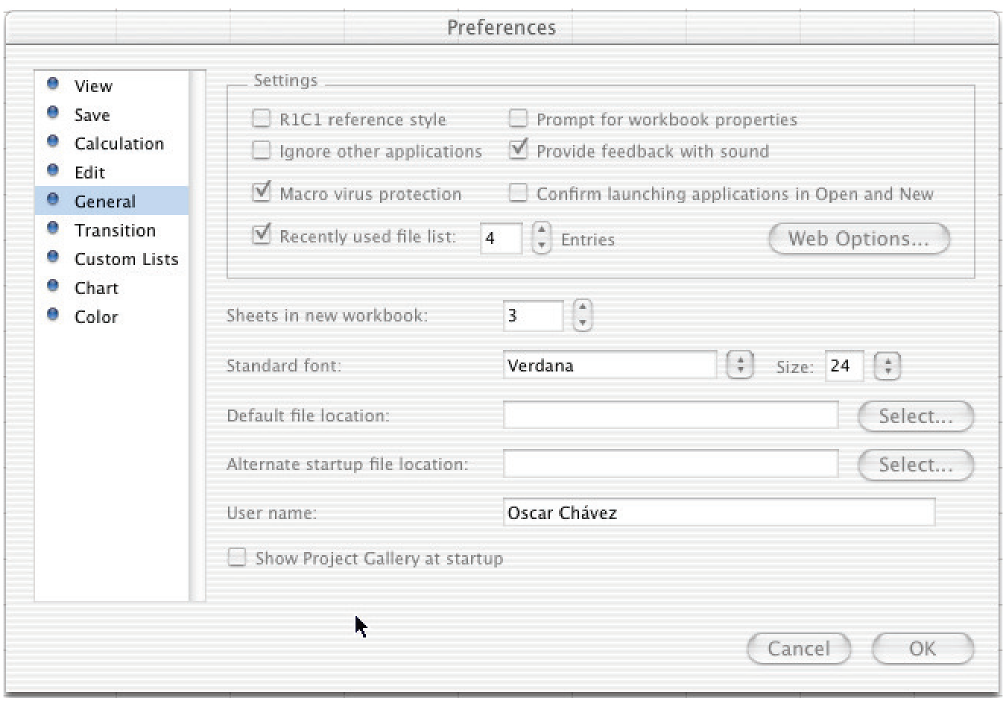
— Edit directly in cell → checked,

— Allow cell drag and drop → unchecked, and

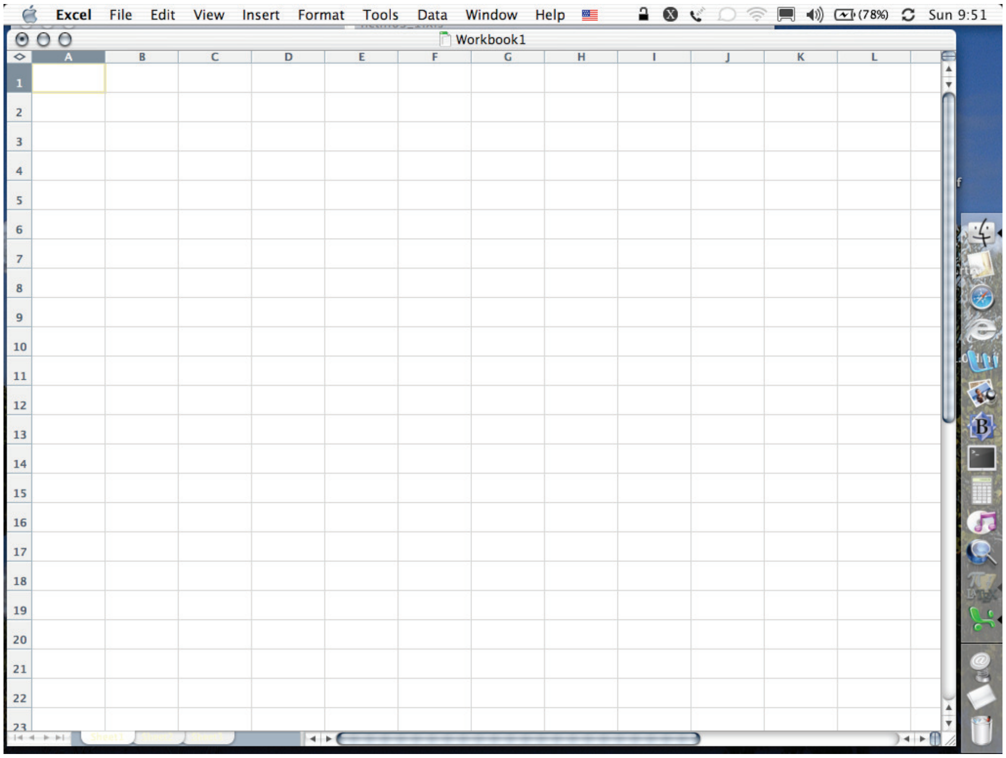
— Move selection after return → unchecked.



Select General and change the standard font to a 24 pt font, so the new worksheets that your students create are already in a large font.



I like to see only the worksheet, so I close **all** the toolbars (Standard and Formula bar included).



Try different things, and choose what is more convenient for you and your students.

The Basics

For most of the time, you'll want your students working in pairs at the computer. One of the things you want to encourage is that they communicate their ideas, that they explain their thinking, that they convince others.

Play Around

Show students how to make calculations. Ask them to always start with a $=$ sign. Try examples such as $= 14-2$, $= 8*3$, $= 30/4$, etc. Now you have just turned a $1200 computer and a $300 piece of software into a $2 calculator!

Show students how to write in a cell a formula that uses a number in other cell. Although they can type `=A1+4` it is easier to get them used to type = then click on cell A1 and then type `+4`. Ask them to change the number repeatedly, so they get used top the idea of the dynamic nature of this environment. Ask your students to write a formula, and have their partners to find what the formula is, by trying different input numbers. Ask them to write a second formula that undoes what the first did.

Write formulas that use the result of another formula (i.e. two step formulas), such as

$$4 \xrightarrow{+7} 11 \xrightarrow{*2} 22$$

Or, in general,

$$\mathsf{A3} \xrightarrow{+7} \mathsf{A4} \xrightarrow{*2} \mathsf{A5}$$

Of course, this notation is only to simplify this handout. In your classroom, you would probably prefer to give verbal indications, or demonstrate how to do it before asking them to do it.

It is not really important if they use contiguous cells or not, but it is better for them to get used to certain conventions in presentation. It will be useful later, when they make tables, for example.

Ask your students to *condense* one of these two step formulas. For example,

$$4 \xrightarrow{+7} 11 \xrightarrow{*2} 22 \quad \text{becomes} \quad 4 \longrightarrow 22$$

Questions to ask:

— How do you write the formula?

— Is the order of operations important?

Let them see how parentheses can help to make the formula work, and let them (or help them to) make sense of the parentheses and how they relate to the original sequence of operations.

Copying Formulas

It is preferably to ask them to copy formulas using the menu, or a shortcut, rather than dragging and dropping, or filling down. The action of actually copying a formula helps to make them aware of what **they** are doing (i.e. it is not some ”magic” that Excel or AppleWorks do). In any case, it is better not to push them to use the capabilities that Excel or AppleWorks has to offer, but better let them try different ways, until they see the need for making something in a more efficient manner. For example, they can make a table with consecutive numbers in one column, and they'd do it by typing the numbers one by one. That's fine. Later they'll find useful to use a formula, or to use Fill Down or Fill Series. Show them how to copy from one cell to another one, and later from one cell to many.

The Problems

These are sample problems that will give your students a context to find patterns and to propose rules that describe those patterns. it is very important that the students have pencil and paper available at all times. They might need to make diagrams, to try formulas, etc.

1. Consider the sequence 13, 17, 21, 25,. . . What number is next?

   While students shouldn't have much trouble finding the next terms in the sequence, the question is more interesting if I want to know what is, for example, the 29th term. Since your students are (a little) familiar now with pasting formulas, arranging the sequence as a table would be a sensible suggestion.

<>	A	B
1	1	13
2	2	17
3	3	21
4	4	25
5		
6		
7		
8		

   Students will very likely approach this problem in a recursive way, finding all previous terms before finding the one they want.

2. What is the 72nd term?

   This question should prompt them to look for other ways of finding the answer, going from a recursive relationship to an explicit rule. The important questions to ask at this point are questions such as:

— How are things changing?

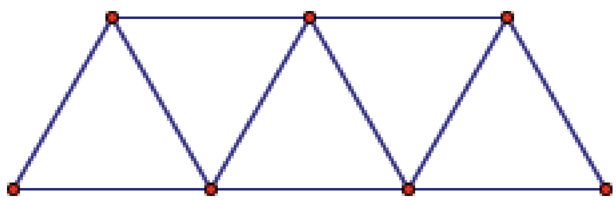
— What steps are you doing over and over?

— Can you write down a mechanical rule that will do this job once and for all?

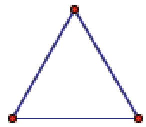
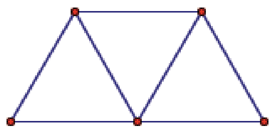
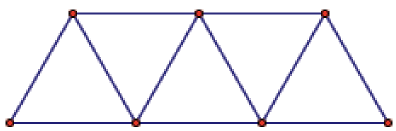
There is a problem with the previous example. It could be argued, and rightly so, that simply posing a sequence of numbers shouldn't create in the students the assumption that a rule should describe it appropriately. In our example it does, because it is, in some way, a contrived example. This kind of examples should help the kids to feel comfortable using the spreadsheet, but only work under the assumption that there is a rule and that their job is to find it. It is a better approach to use a realistic problem, and let them see that the regularity in the pattern is given naturally by the context.

The following example was taken from *Building formulas*, a 7th grade unit from *Mathematics in Context*.

A structure is built with beams such as the following:



We will say that this is a beam of length 3. As you can see, this beam is made of 11 rods. How many rods would a beam of length 1 need? And one of length 2? We can organize this information in the following table:

	Length of beam	Number of rods
	1	
	2	
	3	
	4	

1. How many rods would I need for a beam of length 4? Of length 12?
2. How many rods would I need for a beam of length 56?
3. Find a rule that counts the rods for a beam of any given length.

Again, our purpose is to go from a recursive relationship to an explicit rule. We should be asking the same questions as above:

— How are things changing?

— What steps are you doing over and over?

— Can you write down a mechanical rule that will do this job once and for all?

Once students find a rule, the following questions are appropriate:

— Will this rule work for all cases?

— How does the rule work?

— Now that you've found a rule, how do the numbers (parameters) in the formula relate to the problem context?

A diagram should help to answer this question. And this is what makes this kind of problems more useful.

For example, a student might find the rule

$$B = L \times 3 + (L - 1),$$

which in the spreadsheet notation will look like

$$\boxed{= \mathsf{A5} * 3 + (\mathsf{A5} - 1)}.$$

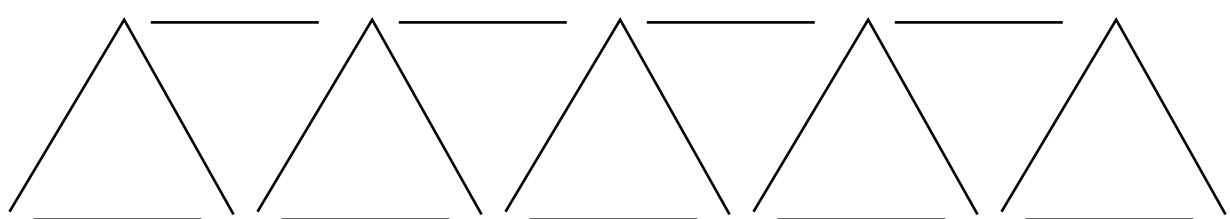
Another student might use the rule

$$B = L + (L - 1) + 2L,$$

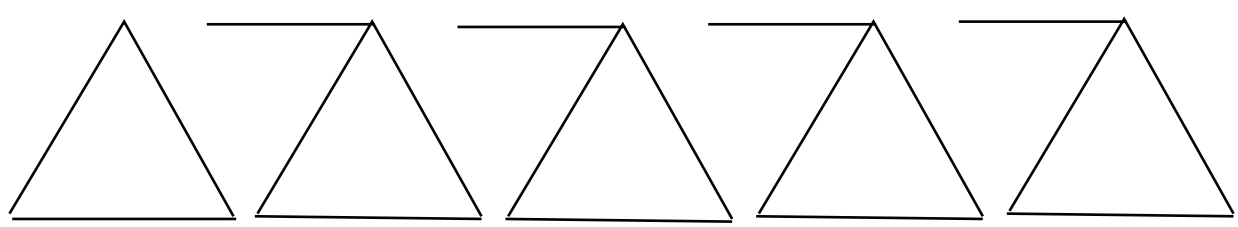
or

$$\boxed{= \mathsf{A5} + (\mathsf{A5} - 1) + 2 * \mathsf{A5}},$$

which could be explained by the diagram:

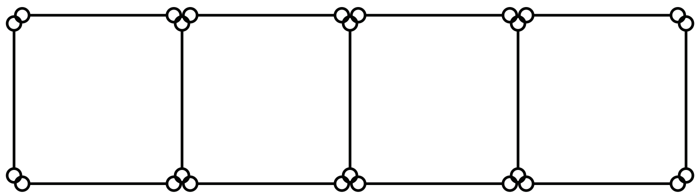


What formula would correspond to the following diagram?



Try this!

1. The following structure is made with rods:



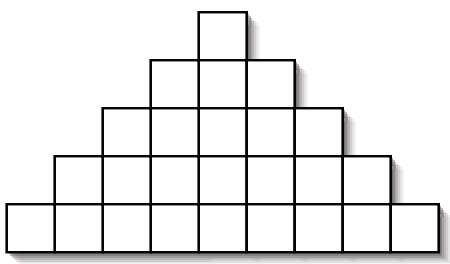
   (a) How many rods do I need to make one square? Two squares? Three squares?

   (b) How many rods do I need to make 23 squares?

   (c) How many rods do I need to make 189 squares?

   (d) Is there a rule that counts how many rods do I need for any given number of squares?

2. There are five rows in the following tower.



   (a) How many bricks are there in the 5th row?

   (b) If I want to build a tower that has 25 rows using the same design, how many bricks do I need for the 25th row?

   (c) I saw a tower with a longest row of 299 bricks, how many rows of bricks did the tower have?

   (d) Is there a rule that tells me how many bricks are in the longest row of any given tower? Is there a rule that tells me how many rows are there in a tower that has any given number of bricks in the longest row?

Students should acquire the habit of asking themselves:

— How are things changing?

— What steps am I doing over and over?

— Can I write down a mechanical rule that will do this job once and for all?

— How is this calculating situation like/unlike that one?

— When I do the same thing with different objects (numbers), what still holds true? What changes?

— Will this rule work for all cases?

— How does the rule work?

— Now that I've found my rule, how do the numbers (parameters) in the formula relate to the problem context?

It is important to bear in mind what are the difficulties of generalization in this kind of activities:

— Students may generalize too quickly

— Pattern spotting can remain trivial

— Students can generalize about the wrong properties

Students must give convincing arguments for the rules, to justify their generalizations. Many patterning activities are difficult to justify, and are not helpful to encourage students to build explicit rules from recursive relationships.

Levels of sophistication in procedural thinking

Battista and van Auken (1998) suggest the following *Levels of sophistication in procedural thinking*

— Students' knowledge of a procedure is restricted to performing it

— Students see a procedure as applicable to numerous instances rather than one particular case

— Students can reflect on, decompose, and analyze a numerical procedure

The purpose of the following activity is to create spreadsheets that will help students to develop an algebraic approach to some problems. We will create spreadsheets that perform repeated calculations. Through this activity, we expect students to reflect, decompose and analyze a numerical procedure.

Number Shifters

A *number shifter* will be a rule or set of rules that performs certain calculation. It takes an input number, and after a certain sequence of steps, returns an output number.

First, we'll create a number shifter that calculates long-distance charges. The rate is 30 cents per minute, plus 75 cents per call. So, this number shifter will take an input number (the number of minutes), multiply it by 30, and add 75 to the result.

	A	B	C
1	**1**	**Input number**	
2	**Step 1**	Multiply the input by 30	
3	**Step 2**	Add 75 to the result from step 1	
4		Output number	
5			

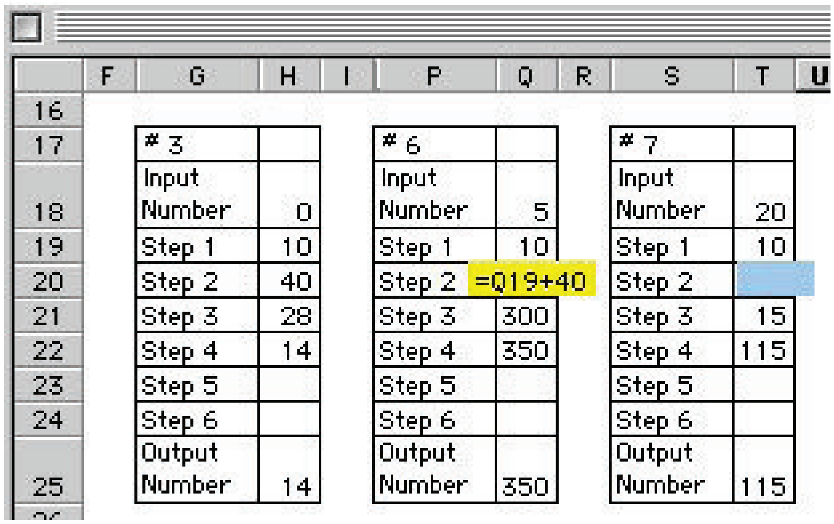
We need to write in cell C2 the formula that will multiply the number in cell C1 by 30, and in cell C3 we will write a formula that adds 75 to the result in cell C2. Once we've done this, we can change at will the input number to get different outputs.

Now we will experiment with other number shifters, e.g. what happens if we invert the order of the operations, what happens if we make longer rules (with more steps). To do this, click on the tab labeled "1" on the file `examples.xls`. We can see the formulas in the worksheet labeled "1 (formulas)".

Now we will try to find what input number is needed to get a given output, following given rules. Click on the tab labeled "2." For each problem, find the input that gives the desired output.

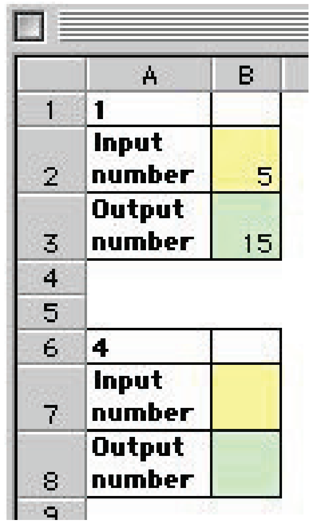
Mystery Number Shifters

Click on the tab labeled "3." In these number shifters the steps are not described (don't peek at the formulas!). In problems 1–3, the student has to find the rule, but the results of all steps are visible. In problems 4–6, the result from a step is concealed, but not the formula. In problems 7–9, the result and the formula for a step are not known. In all cases, the student has to find the rule. He or she will need to explore what happens to different input numbers. The empty templates above the given examples can be used to try different solutions (or they can use pencil and paper).



	F	G	H	I	P	Q	R	S	T	U
16										
17		# 3			# 6			# 7		
18		Input Number	0		Input Number	5		Input Number	20	
19		Step 1	10		Step 1	10		Step 1	10	
20		Step 2	40		Step 2	=Q19+40		Step 2		
21		Step 3	28		Step 3	300		Step 3	15	
22		Step 4	14		Step 4	350		Step 4	115	
23		Step 5			Step 5			Step 5		
24		Step 6			Step 6			Step 6		
25		Output Number	14		Output Number	350		Output Number	115	

In the worksheet labeled "4," the number shifters are *condensed*, the rules are not separated in steps. Try to find the rule for the first three. Invent one or two condensed number shifters, and ask a partner to find the rule. Now trade places, and try to find the rule invented by your partner (keep it linear!).



	A	B
1	1	
2	Input number	5
3	Output number	15
4		
5		
6	4	
7	Input number	
8	Output number	
9		

References

Algebraic thinking, grades K–12: Readings from NCTM's school-based journals and other publications (1999). Moses, Barbara (Ed.). Reston, VA: NCTM.

Battista, M. T. & Van Auken Borrow, C. (1998). Using spreadsheets to promote algebraic thinking. *Teaching Children Mathematics, 4*, 470–478.

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Sutherland, R. & Rojano, T. (1993). A spreadsheet approach to solving algebra problems. *Journal of Mathematical Behavior, 12*, 353–383.

Wijers, M., Roodhart, A., van Reeuwijk, M., Burrill, G., Cole, B. R., & Pligge, M. A. (1998). Building formulas. In National Center for Research in Mathematical Sciences Education & Freudenthal Institute (Eds.), *Mathematics in Context.* Chicago: Encyclopædia Britannica.

Copies of the handout and the Excel files can be found at:

`http://www.missouri.edu/~oc918/nctm03`

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