

## 2008 Interface B

*Mathematics and Science: Your Ticket to the Future*

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### Dynamic Geometry: GeoGebra

#### Why use technology?

The use of technology in mathematics education has probably not lived up to its promise. Technology is too often described as a “solution,” even when the problem is not clearly defined, or even stated.

In 2000, the NCTM's Principles and Standards for School Mathematics devoted one of its six overarching principles wholly to technology. The *Technology Principle* states, “Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.”

At least, we may say, has the *potential* to enhance student learning. How is that technology can accomplish this goal?

People who specialize in applications of technology to education tend to be fascinated with their own gizmos. They are interested in getting teachers to “adopt” and “implement” particular instructional technologies that they believe will exercise a profoundly transformative effect on instructional practice and student learning... By adopting this posture, they fall into the “implementation trap.” That is, the results they promise depend on teachers' faithful implementation of the design specifications of their gizmos... [I]nstructional models, no matter how well-designed, never dictate practice; individual knowledge and skill and organizational culture always trump externally initiated models. (Elmore, 2004, p. xiii-xiv)

Why then, spend the next 90 minutes learning to use yet another computer program that does geometry? If for no other reason, to explore what possibilities it offers for our own, and our students', mathematical learning.

#### GeoGebra

GeoGebra is a “dynamic mathematics software” for learning and teaching in schools. There are many dynamic geometry software programs. Some are commercially available (Geometer's Sketchpad, Cabri, Cinderella) and some are open source (e.g., GeoView, C.a.R., among others). GeoGebra is freely available (licensed under the GNU General Public License). It uses Java, which makes it truly platform independent: you can run GeoGebra on any computer (MacOS X, Windows, Linux). As other dynamic geometry programs, GeoGebra can enhance mathematics learning by enabling students to *visualize, use different representations, and experiment*.

It enables teachers to create accurate diagrams and illustrations, create interactive demonstrations, create construction files for students to use, create dynamic worksheets (as web pages).

GeoGebra combines characteristics of dynamic geometry programs (such as Geometer's Sketchpad, Cabri, Cinderella, Euklid Dynageo, Geonext, Ruler and Compass,...) —drag mode (dynamic) and graphical representation— with some characteristics of Computer Algebra Systems (such as Derive, Mathematica, Maple, MuPad, MathCAD,...) —algebraic and numerical computations and symbolic representation.

GeoGebra can be downloaded from <http://www.geogebra.org>. GeoGebra's web site has also links to teaching materials, users forums, and online help.

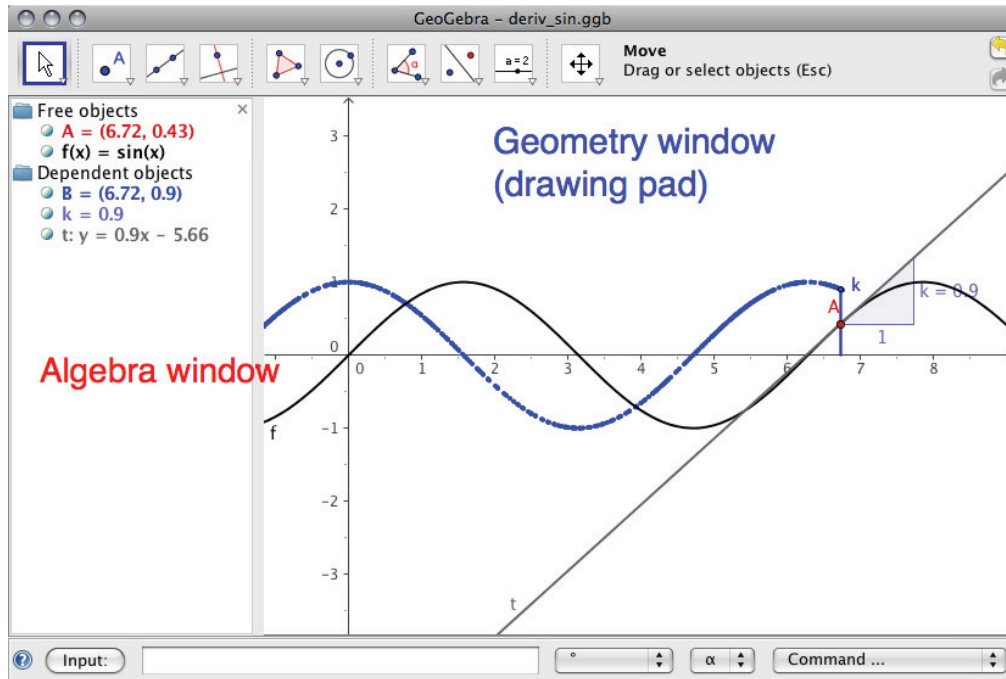








Figure 1: GeoGebra

## GeoGebra basics

Most constructions can be done by selecting the appropriate *mode* in the toolbar. By default, this toolbar shows 10 buttons. Clicking on any of these changes the tool. Clicking on the tiny triangle on the lower left opens a pop-up menu where other tools can be found. It is important to note that there are more than one way to do things in GeoGebra, so play and experiment as needed. Whenever you click on a tool, indications on how to use it appear at the top right of the toolbar.

### Construct a triangle's nine-point circle

1. Open a new GeoGebra window.
2. Open the View menu to uncheck Axes, Grid, and Algebra Window, so that we have a blank drawing pad.
3.  Use the New point mode and construct three points on the plane.
4.  Use the Segment between two points mode to construct the sides of the triangle.
5.  Use the Midpoint or center mode to construct the midpoint of each one of the three sides.
6.  Use the Circle through three points mode to construct the circle through the three midpoints. This is the 9-point circle of our original triangle. (Why 9?)

Whenever you're done using a tool, change the mode back to  Move. This will prevent you to construct unwanted segments, circles, or other things by accident. Also, whenever you want to move an object, or edit it, the pointer should be in the  Move mode.

Under the View menu you will find an option, Navigation bar for construction steps. Checking this option gives you just that: navigation buttons that let you re-play your construction step by step, or as a slideshow. Also, under the View menu you can find a Construction Protocol, a list of all your construction steps.

### Investigating the absolute value function

1. Open a new GeoGebra window. Make sure your drawing pad shows a grid and the axes. Make sure your algebra window is visible.
2. We will explore the function  $f(x) = a|x - b| + c$  (why is this function of any interest to us?). We will create *sliders* for  $a$ ,  $b$ , and  $c$ , so that we can play with these parameters and see how the function's graph changes.
3. Use the slider button to create a slider for  $a$ . Leave the default options unchanged. Create sliders for  $b$  and  $c$ .
4. In the Input window, type  $f(x) = a * \text{abs}(x - b) + c$ . What happens?
5. Play with your sliders. Can you describe the effect of the parameters  $a$ ,  $b$ , and  $c$ ? (Of course you can.)
6. Kick it up a notch: on the algebra window, double click on  $f(x)$  and edit it. Change `abs` to `sin` and click on the Apply button. What just happened?

### Investigating derivatives

1. Open a new GeoGebra window. Make sure your drawing pad shows a grid and the axes. Make sure your algebra window is visible.
2. In the Input window, type  $f(x) = \sin(x)$ .
3. Right-click on the graph of the function. In the contextual menu that appears select Properties. Change the appearance of the graph (paint it blue, make it thicker).
4. Use the New point mode, click on the graph to select a point on the graph.
5. Use the Tangents mode to construct the tangent to the graph through the point you just defined (which I hope you labeled  $A$ ). Change the appearance of this tangent line (right-click, properties, make it gray).
6. Use the Slope tool to measure the slope of this tangent. Drag point  $A$  along the sine curve and see what happens to the slope of the tangent through  $A$ .
7. Now you will define a point whose  $x$  coordinate is the same as  $A$ 's and whose  $y$  coordinate is  $m$ , the slope of the tangent line through  $A$ . To do this, type in the Input window:  $B = (x(A), m)$ . Drag point  $A$  along the sine curve and see what happens to point  $B$ .
8. To see what kind of graph is  $B$  describing, let's make it leave a trace: right-click on point  $B$  and select Trace on. Drag  $A$  along the sine curve and see what happens.
9. Another way to see what kind of graph  $B$  is describing is to find the locus of  $B$ : select the Locus tool, then click on  $B$  and on  $A$ .
10. Drag  $A$  close to the origin. Under the View menu, select Refresh views to erase all the trace of  $B$ .

11. In the algebra window, double-click on  $f(x)$  and redefine it as  $f(x) = x^3 + x^2 - 2x$  (type  $x^3 + x^2 - 2x$ ).
12. After playing a little with this new function, change it back to  $f(x) = \sin(x)$ .
13. Export it as Dynamic Worksheet as Webpage (html):
  - (a) Give it a title, write some text that will go before and after the construction.
  - (b) Click on the Advanced button.
  - (c) Check all options, so that students can open GeoGebra and modify the sketch (see Fig. 2).
  - (d) Make the window  $800 \times 600$ .

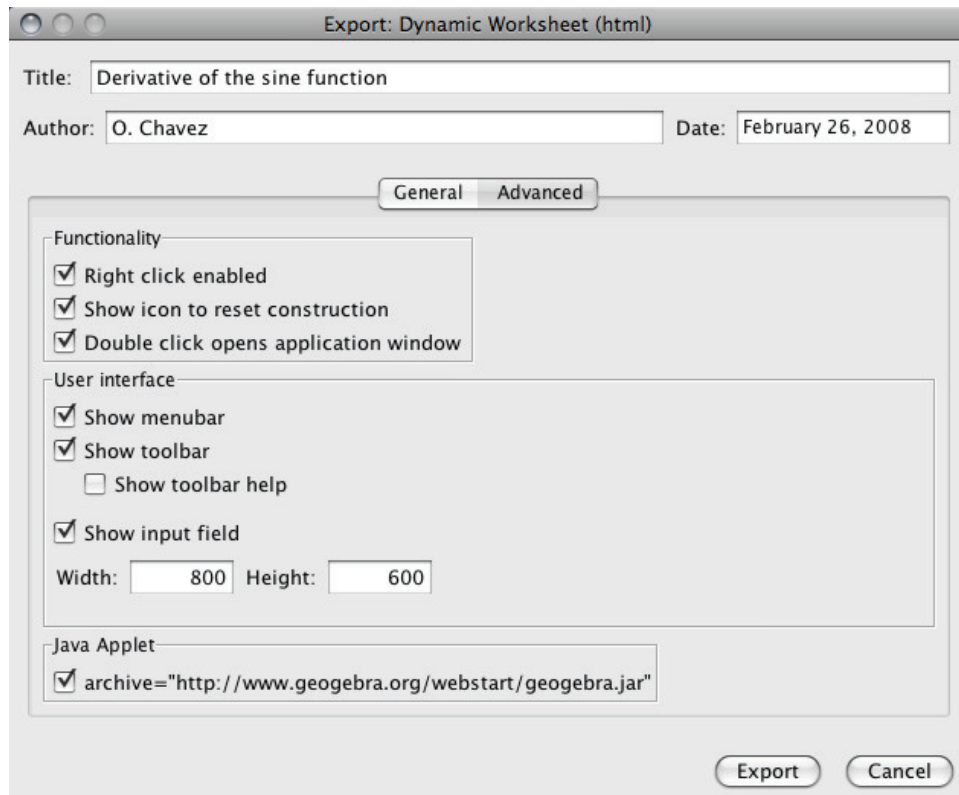


Figure 2: Creating a dynamic worksheet

### Your turn

Choose any of the following tasks:

1. Construct a parabola as the locus of all the points that are equidistant to a given point (*focus*) and a given a line (*directrix*).
2. Construct a quadrilateral with the property that it has two congruent opposite sides.

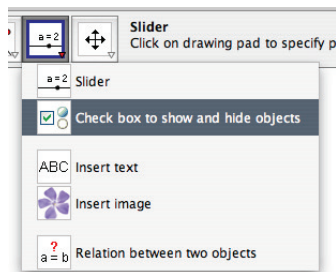
## A problem about triangles

The following problem was taken from Yaglom, I. M. (1962). *Geometric transformations*. New York: Random House (Chapter 1 prob. 4, p. 17):

Let  $D$ ,  $E$ , and  $F$  be the midpoints of sides  $AB$ ,  $BC$ , and  $CA$ , respectively, of triangle  $ABC$ . Let  $O_1$ ,  $O_2$ , and  $O_3$  denote the centers of the circles circumscribed about triangles  $ADF$ ,  $BDE$ , and  $CEF$ , respectively, and let  $Q_1$ ,  $Q_2$ , and  $Q_3$  be the centers of the circles inscribed in these same triangles. Show that the triangles  $O_1O_2O_3$  and  $Q_1Q_2Q_3$  are congruent.

If students were to attempt this problem, it would take them a lot of time to make a diagram, by hand or using software. It is also a problem that would be difficult to solve without visualizing what's happening. A sketch illustrating this problem will serve two purposes: create an opening for students to engage with the problem and providing evidence that the proposition is true.

The construction is not particularly complicated, but there are a lot of auxiliary objects that must be created (perpendicular bisectors of at least two sides of three triangles to find the centers of the circumscribed circles, angle bisectors of at least two angles of three triangles to find the centers of the inscribed circles, and all the segments that form the triangles involved). Most of these objects are not even part of the problem, and should be hidden in the final sketch. However, leaving the entire final construction visible takes a little away from the problem, and also makes it difficult to see the different parts of the problem. Ideally, we'd like to see, in order:  $\triangle ABC$ , the initial triangle;  $\triangle DEF$ , the midpoints triangle, and finally the triangles in question:  $\triangle O_1O_2O_3$ , and  $\triangle Q_1Q_2Q_3$ . We would also like to be able to hide or show at will the triangles involved. The feature that we can use for this purpose is the tool "Check box to show and hide objects:"



The final result can be seen at <http://web.missouri.edu/~chavez/geogebra/yaglom.html>. Notice that the sketch is fully functional, and that double clicking on the drawing pad opens the sketch in a GeoGebra window. A user (e.g., a student or fellow teacher) can even create new sketches and use GeoGebra, as long as the web page is open.

Copies of this handout, files used in this presentation, and links to GeoGebra materials: <http://web.missouri.edu/~chavez/interface2008>.

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