

SPECIALIZED MATHEMATICAL CONTENT KNOWLEDGE OF PRESERVICE  
ELEMENTARY TEACHERS:  
THE EFFECT OF MATHEMATICS TEACHER EFFICACY

A DISSERTATION IN  
Curriculum and Instruction  
and  
Urban Leadership and Policy Studies in Education

Presented to the Faculty of the University  
of Missouri-Kansas City in partial fulfillment of  
the requirements for the degree

DOCTOR OF PHILOSOPHY

by  
ANN C. MCCOY

B.S.E., Central Missouri State University, 1983  
M.S.E., Central Missouri State University, 1988

Kansas City, MO  
2011

© 2011

ANN C. MCCOY

ALL RIGHTS RESERVED

SPECIALIZED MATHEMATICAL CONTENT KNOWLEDGE OF PRESERVICE  
ELEMENTARY TEACHERS:  
THE EFFECT OF MATHEMATICS TEACHER EFFICACY

Ann C. McCoy, Candidate for the Doctor of Philosophy Degree

University of Missouri–Kansas City, 2011

ABSTRACT

In 2000, the National Council of Teachers of Mathematics ([NCTM], 2000) described a compelling vision for mathematics education in the United States calling for high-quality instruction, knowledgeable teachers, ambitious expectations, and engaging curriculum. One aspect of this vision, knowledgeable teachers, has been the subject of many studies as researchers attempt to determine what types of teacher knowledge are important in the development of effective teachers. In addition, factors that impact the development of teacher knowledge have also been investigated. The beliefs of teachers, including teaching efficacy, are one such factor.

This dissertation presents findings from a study that examined the relationship between mathematics teacher efficacy and the growth in specialized mathematical content knowledge of preservice elementary teachers. The participants in this study were 101 elementary education majors enrolled in a two-course mathematics content and methods sequence at a mid-sized, mid-western university located in a small city. Two dimensions of

mathematics teacher efficacy, personal mathematics teacher efficacy and mathematics teaching outcome expectancy, were measured using the Mathematics Teaching Efficacy Beliefs Instrument (Enochs, Smith, and Huinker, 2000). Specialized mathematical content knowledge was measured using items developed by the Learning Mathematics for Teaching (LMT) project, and common mathematical content knowledge was measured using an instrument developed and used at another university.

The findings of this study indicate that the level of specialized mathematics content knowledge of preservice teachers increased significantly during the mathematics methods/content course. Personal mathematics teacher efficacy, mathematics teaching outcome expectancy, and common mathematical content knowledge also increased significantly. Significant correlations were found among several of the variables assessed in the study, including personal mathematics teacher efficacy and specialized mathematical content knowledge. However, neither dimension of mathematics teacher efficacy significantly predicted growth in specialized mathematical content knowledge. A supplementary analysis revealed that the initial level specialized content knowledge did significantly predict growth in personal mathematics teacher efficacy of female students.

## APPROVAL PAGE

The faculty listed below, appointed by the Dean of the School of Graduate Studies, have examined a dissertation titled “Specialized Mathematical Content Knowledge of Preservice Elementary Teachers: The Effect of Mathematics Teacher Efficacy” presented by Ann C. McCoy, candidate for the Doctor of Philosophy degree, and certify that in their opinion it is worthy of acceptance.

### Supervisory Committee

Rita Barger, Ph.D., Committee Chair  
Department of Curriculum and Instruction

Sue C. Thompson, Ph.D.  
Department of Urban Leadership and Policy Studies in Education

Tamara B. Murdock, Ph.D.  
Department of Psychology

Susan A. Adler, Ph.D.  
Department of Curriculum and Instruction

Sue Vartuli, Ph.D.  
Department of Curriculum and Instruction

## CONTENTS

ABSTRACT .....	iii
LIST OF ILLUSTRATIONS.....	xi
LIST OF TABLES .....	xii
ACKNOWLEDGMENTS .....	xiv
Chapter	
1. INTRODUCTION .....	1
The Context of the Study .....	1
Teacher Knowledge .....	5
Teacher Efficacy .....	7
Preservice Programs .....	8
Purpose of the Study .....	11
Research Questions .....	12
Significance of the Study .....	13
Definition of Terms .....	14
Delimitations of the Study .....	16
Limitations of the Study .....	16
Organization of Remaining Chapters .....	17
2. REVIEW OF LITERATURE .....	19
Characteristics of Elementary Teachers and Teaching .....	21
Implications for Mathematics Teacher Education Programs .....	23

Content and Nature of Elementary Mathematics Teacher Education Programs....	25
Beneficial Impact of Mathematics Teacher Education Courses.....	26
Limitations of Impact of Teacher Education Programs.....	27
Knowledge for Teaching .....	29
Pedagogical Content Knowledge .....	29
Mathematical Knowledge for Teaching .....	36
Measures of Mathematical Knowledge for Teaching .....	41
Studies of Mathematical Knowledge for Teaching .....	43
Developing Mathematical Knowledge for Teaching .....	45
Mathematics Teacher Efficacy .....	47
Teacher Beliefs and Attitudes.....	47
Beliefs Regarding the Teaching of Mathematics .....	49
Beliefs of Preservice Teachers .....	50
Beliefs of Preservice Teachers Regarding Teaching Mathematics .....	52
Self-Efficacy .....	53
Sources of Self-Efficacy .....	54
Teacher Efficacy .....	57
Dimensions of Teacher Efficacy .....	58
Measures of Teacher Efficacy .....	60
Teaching Efficacy – Findings and Implications .....	63
Implications for Mathematics Teacher Education .....	67

Previous Research .....	70
Conclusion .....	71
3. METHODOLOGY.....	72
Participants .....	73
Procedures .....	75
Consideration of Ethical Concerns .....	77
Pilot Study .....	78
Data Measurement and Collection .....	78
Instruments .....	78
Demographic Instrument .....	80
Mathematic Teaching Efficacy Beliefs Instrument .....	80
Common Content Knowledge Instrument .....	81
Learning Mathematics for Teaching: Measures of Mathematical Knowledge for Teaching .....	82
Data Analysis Plan .....	83
Preliminary Data Analysis .....	83
Data Analysis .....	84
Research Question One .....	84
Research Question Two .....	84
Research Question Three .....	85
Research Question Four .....	85
Research Question Five .....	85

4. RESULTS	87
Demographics	89
Descriptive Statistics	92
Hypothesis Testing	108
Research Question One	108
Research Question Two	110
Research Question Three	111
Research Question Four	113
Research Question Five	115
Supplementary Analysis	116
Conclusion	122
5. DISCUSSION	123
Review of Purpose of Study	123
Conclusions	125
Research Question One	125
Summary of Results	125
Discussion	125
Research Question Two	128
Summary of Results	128
Discussion	129
Research Question Three	131

Summary of Results .....	131
Discussion .....	131
Research Question Four .....	132
Summary of Results .....	132
Discussion .....	132
Research Question Five .....	132
Summary of Results .....	132
Discussion .....	132
Supplementary Analysis .....	132
Summary of Results .....	133
Discussion .....	133
Limitations of the Study .....	135
Suggestions for Future Research .....	139
Conclusion .....	142
Appendix	
A. INSTRUMENT.....	144
B. COURSE SYLLABI .....	155
C. STUDY APPROVAL DOCUMENTATION .....	166
REFERENCE LIST.....	172
VITA .....	190

## ILLUSTRATIONS

Figure	Page
1. Dimensions of Mathematical Knowledge for Teaching .....	6
2. Improvement of Elementary Mathematics Education .....	10

## TABLES

Table	Page
1. Number of Preservice Elementary Teacher Participants by Section .....	88
2. Demographics .....	91
3. Means, Standard Deviations, Skewness, and Kurtosis .....	92
4. Class Section 1: Means, Standard Deviations, Skewness, and Kurtosis ....	93
5. Class Section 2: Means, Standard Deviations, Skewness, and Kurtosis ....	94
6. Class Section 3: Means, Standard Deviations, Skewness, and Kurtosis ....	94
7. Class Section 4: Means, Standard Deviations, Skewness, and Kurtosis ....	95
8. Class Section 5: Means, Standard Deviations, Skewness, and Kurtosis ....	95
9. Results of Independent Samples <i>t</i> -tests for Variables by Semester .....	97
10. Analysis of Variance for Assessed Variables by Class Section .....	98
11. Order of Sections of Forms of Instrument .....	99
12. Analysis of Variance for Assessed Variables by Pretest Form .....	100
13. Analysis of Variance for Assessed Variables by Posttest Form .....	101
14. Analysis of Variance for Assessed Variables by Status .....	102
15. Analysis of Variance for Assessed Variables by High School Attended ...	103
16. Analysis of Variance for Assessed Variables by Mathematics Courses Completed .....	105
17. Means and Standard Deviations for Posttest Personal Mathematics Teacher Efficacy and Common Content Knowledge by Mathematics Courses Completed .....	106
18. Results of Independent Samples <i>t</i> -tests for Variables by Gender .....	107

19. Correlations Among Variables .....	108
20. Paired Samples t-tests for Variables Comparing Pretest and Posttest.....	109
21. Results of Multiple Regression for Research Question 2 .....	111
22. Results of Multiple Regression for Research Question 3 .....	113
23. Results of Multiple Regression for Research Question 4 .....	114
24. Results of Multiple Regression for Research Question 5 .....	116
25. Correlations Among Variables for Females .....	118
26. Results of Multiple Regression for Supplementary Analysis – Specialized Content Knowledge .....	120
27. Results of Multiple Regression for Supplementary Analysis – Common Content Knowledge .....	121

## ACKNOWLEDGMENTS

If it truly takes a village to raise a child, then it surely took a “small mid-western city” to help me get to this point. To all those who have helped me achieve this long-term goal – thank you for your inspiration, support, encouragement, and friendship.

I would like to thank my committee for their expertise, wisdom, and patience. I appreciate the wise suggestions from Dr. Susan Adler and Dr. Sue Vartuli, and the shared love of middle school students of Dr. Sue Thompson. I owe a big thank you to Dr. Tamera Murdock for making me think and for encouraging me to see the story behind the numbers. Rita, thank you for everything – for your support and guidance as my advisor, for the wonderful teaching example you set, and, most of all, for your friendship. I look forward to many more years of working together.

Above all, I would like to thank the men in my life – my husband and three sons. Thank you for allowing me to pursue my dream even though at times that has meant that I haven’t always been there when you needed me. You are the most important people in my life and I’m incredibly proud of you.

## CHAPTER 1

### INTRODUCTION

#### **The Context of the Study**

An ambitious vision for mathematics education is proposed by the National Council of Teachers of Mathematics (NCTM) in its 2000 document, *Principles and Standards for School Mathematics* (PSSM):

Imagine a classroom, a school or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodations for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding (NCTM, 2000, p. 3).

In reality, the mathematics education described by NCTM is quite different from the mathematics education experienced by many, if not most, of our children. In fact, Ball, Hill, and Bass (2005) comment that “We are simply failing to reach reasonable standards of mathematical proficiency with most of our students” (p. 14). Concern about the mathematics achievement of students in the United States grows each time results of international mathematics assessments are released (Ball, Hill, & Bass, 2005). According to Silver and Kenny (2000), domestic and international assessments of mathematics

consistently indicate that students in the United States are learning less mathematics, less deeply than they could. One such assessment, the Program for International Student Assessment (PISA), is a system of international assessments that focus on 15-year-olds' capabilities in a variety of areas including mathematics literacy. PISA is coordinated by the Organization for Economic Cooperation and Development (OECD), an organization of developed countries, and is given every three years. Results from the 2009 PISA indicated the average score in mathematics literacy of students in the United States was lower than the average score of OECD countries. Twenty-seven percent of students in the United States scored at or above the proficient level compared to 32% of students in OECD countries (National Center for Education Statistics, 2010).

Additionally, the 1995 Third International Mathematics and Science Study (TIMSS), showed that the mathematics experienced by students in the United States was less challenging than that experienced by students in other countries. In addition, lessons in the United States featured an emphasis on procedures and correct answers to a much greater extent than the lessons from other countries (Hiebert & Stigler, 2004). These assessments indicate that while our students demonstrate a moderate level of procedural knowledge, they demonstrate a much lower level of conceptual knowledge (Vinson, 2001).

In a discussion of the results of TIMSS, Forgione (1998) offers three possible explanations for the lack of achievement demonstrated by students in the United States including the curriculum utilized, student course-taking patterns, and ineffective teaching and teacher preparation. The mathematics curricula available in the United States have

been characterized as being shallow and undemanding when compared to that used in countries achieving at a significantly higher level on international assessments (McKnight, et al., 1987). The mathematics textbooks used in the United States cover many topics but do so superficially, and instructional materials continue to emphasize paper-and-pencil computation and repeated practice (National Research Council, 2001). However, the curricula used cannot alone explain the differences seen in mathematics achievement between students in the United States and those in other countries. In fact, a study by Banilower, Boyd, Pasley, and Weiss (2006) found that U.S. teachers often reduced the challenging nature of the tasks in the curricula and these findings are supported by studies by other researchers (Arbaugh, Lannin, Jones, & Park-Rogers, 2006; Tarr, Chàvez, Reys, & Reys, 2006; Hiebert & Stigler, 2004).

Student course-taking patterns may limit U.S. students' exposure to challenging mathematics content and, thus, contribute to their low performance in mathematics. Forgione (1998) comments that while some increases in academic coursetaking have occurred, less than ten percent of students in the United States take calculus in high school. Almost one-third of college bound U.S. students take fewer than four years of mathematics in high school.

The third factor suggested by Forgione (1998), ineffective teaching and teacher preparation, must be addressed if we are to improve mathematics education and achieve NCTM's vision. According to NCTM, effective mathematics teaching "requires understanding what students know and need to learn and then challenging and supporting

them to learn it well” (NCTM, 2000, p. 16). Effective teachers must understand their students as learners of mathematics and be deeply committed to helping students learn. In addition, effective teachers are reflective and continually seek to improve and grow professionally. These teachers understand that the decisions they make dramatically influence student learning and shape the attitudes students possess about mathematics and learning mathematics (NCTM, 2000).

Sadly, many students continue to be taught by teachers who have a limited understanding of mathematics and appropriate instructional methods. While research shows that increases in subject matter knowledge lead to increases in the ability of teachers to connect mathematical topics and teach in a way that emphasizes conceptual understanding (Brown and Borko, 1992), the mathematics content knowledge of U.S. teachers, especially those at the elementary level, is weak and procedural in nature (Ma, 1989; Weiss, 1995; Vinson, 2001; da Ponte & Chapman, 2008).

Data regarding instructional practices utilized by teachers is provided by the TIMSS video study (Hiebert & Stigler, 2004). Randomly selected mathematics classes were video-taped and these tapes were viewed and summarized in an attempt to paint a picture of the mathematics instruction taking place. The videotaped classrooms included some from countries that scored well in the 1995 TIMSS study and others that scored poorly on this assessment. The researchers who conducted this study found that no one single teaching method was evident in all of the countries who scored well, but some similarities in these countries distinguished them from the United States. Unlike teachers in

the countries scoring well, teachers in the United States tended to ignore the conceptual aspects of problems and were quick to step in and do the work for students. In addition, U.S. teachers spent little time allowing students to explore and discuss mathematical relationships and connections (Hiebert & Stigler, 2004).

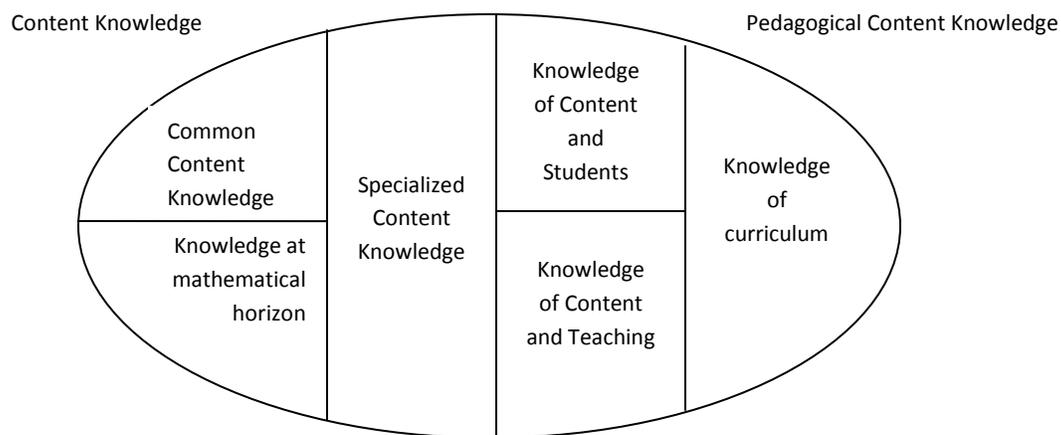
This issue of elementary teachers possessing inadequate content knowledge and limited appropriate instructional strategies may be made more problematic by the beliefs these teachers hold regarding mathematics and the teaching of mathematics. For example, many elementary teachers believe that mathematics is simply a collection of unrelated facts and that some people are good at mathematics while others are not (Barlow & Reddish, 2006). In addition, The 2000 National Survey of Science and Mathematics Education (Weiss, Banilower, McMahon, & Smith, 2001) found that only 60% of the elementary teachers surveyed believed they were qualified to teach mathematics. Obviously beliefs such as these will have a profound impact on the instructional practices of these teachers.

The current state of mathematics education in the United States, particularly at the elementary level, indicates the importance of considering both the content knowledge of elementary teachers as well as their beliefs related to mathematics. These two important ideas are the primary foci of this study, and prior work on these ideas provides a theoretical framework for this study.

### **Teacher Knowledge**

The knowledge needed for the teaching of elementary mathematics has been the subject of an increasing amount of research as mathematics educators seek to determine

the specific knowledge that is needed to teach effectively. Recently, the term mathematical knowledge for teaching (MKT) has been used to describe the knowledge that is needed by teachers, and research indicates this knowledge is multidimensional. Viewed as an extension of Shulman’s (1986) theory of pedagogical content knowledge, MKT is currently believed to be composed of two dimensions each composed of several components (Thames, Sleep, Bass, & Ball, 2008). The first dimension, content knowledge, represents the knowledge of mathematics needed by teacher, and is composed of three components including common content knowledge, specialized content knowledge, and knowledge at the mathematical horizon. Pedagogical content knowledge, the second dimension of mathematical knowledge for teaching, is composed of knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum. Thames et al. envision the dimensions of MKT as shown below:



*Figure 1: Dimensions of Mathematical Knowledge for Teaching, (Thames, Sleep, Bass, & Ball, 2008)*

The study described in this document was designed to investigate the development of specialized content knowledge, described as the knowledge of mathematics needed uniquely by teachers (Ball, Hill, & Bass, 2005; Ball, Hill, Rowan, & Schilling, 2002; Ball, Thames, & Phelps, 2008). While specialized mathematics content knowledge is related to the common mathematics content knowledge needed by those in other professions using mathematics, the two are not the same. For instance, the ability to multiply two-digit numbers is an example of common content knowledge needed by those in many professions. Related to this, but needed uniquely by teachers, is the ability to examine the nonstandard algorithms used by children and determine if these algorithms will generalize. The existence of specialized content knowledge (SCK) is supported by research, but additional research is needed regarding the development of this specialized content knowledge and the factors that impact its development in preservice teachers. Since SCK is a type of mathematics content knowledge, it is not dependent on knowledge of students and teaching (Morris, Hiebert, & Spitzer, 2009); thus, it is an appropriate choice for consideration with preservice teachers who usually have limited opportunities to gain knowledge of students and classroom teaching. A description of the remaining dimensions of MKT is provided in Chapter 2.

### **Teacher Efficacy**

The second idea of importance in this study, the beliefs of elementary teachers related to mathematics, also plays a role in determining the mathematics education our students receive. One such belief, mathematics teacher efficacy, has been found to be

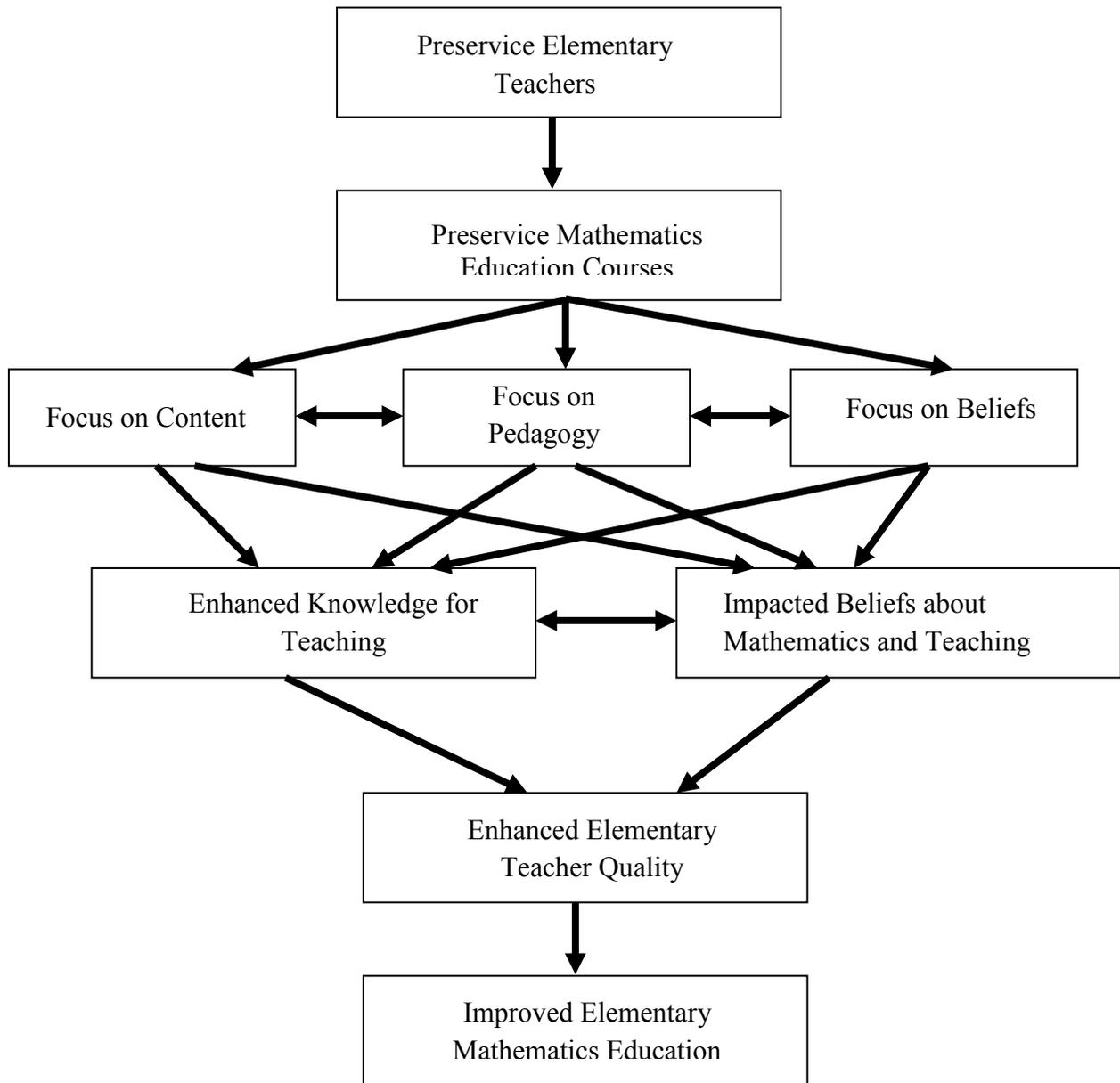
related to teacher behavior and, in general, high levels of teaching efficacy are related to positive teaching behaviors and student outcomes (Ashton, Webb, & Doda, 1983; Brand & Wilkins, 2007, Gibson & Dembo, 1984; Pajares, 1992). Research has indicated that mathematics teacher efficacy is composed of two dimensions that are similar in nature to the dimensions of self-efficacy first described by Bandura (1977). The first dimension, personal mathematics teacher efficacy, represents an individual's belief in his or her ability to effectively teach mathematics (Swars, 2005). Mathematics teaching outcome expectancy, the second dimension of teacher efficacy, represents an individual's belief that effective teaching of mathematics will bring about student learning (Swars, 2005). Knowing that efficacy is related to teaching practices indicates the need to consider the efficacy level of preservice teachers, how it develops in teacher education courses, and how it impacts other aspects of teacher education. One goal of this study was to determine if positive perceptions regarding the ability to teach mathematics result in greater increases in specialized content knowledge.

### **Preservice Programs**

Findings such as those reported above have important implications for mathematics teacher education programs. Ma (1999) contends that preservice teacher education programs play a vital role in determining the quality of the mathematics teaching that occurs in elementary schools. Research has supported the importance of preservice mathematics education programs in improving the content knowledge of prospective

elementary teachers and in influencing their beliefs regarding mathematics and the teaching of mathematics (Ball, 1990; Battista, 1986; Quinn, 1997).

Since teacher education programs play an important role in promoting teacher quality, many important questions are raised for those involved in the mathematics teacher education of preservice elementary teachers. What mathematics do elementary teachers need to know and be able to do? In what ways do elementary teachers need to know mathematics in order to be effective teachers and does this knowledge differ from the content knowledge needed by other professionals? What experiences will best prepare preservice teachers to become effective practicing elementary teachers? How do beliefs and attitudes regarding mathematics and mathematics teaching impact the preparation of elementary teachers? This study was designed to investigate some of these questions (further explained later in this chapter) and contribute to the literature regarding preparing preservice elementary teachers to teach mathematics. The study is based on a conceptual framework, as illustrated in Figure 2, in which content knowledge, pedagogy, and beliefs all impact teacher development and quality. Thus, if preservice mathematics education courses focus on each of these, teacher knowledge will be enhanced and teacher beliefs will be positively impacted. As a result, teacher quality will be improved as will mathematics education in general.



*Figure 2:* Diagram representing the improvement of elementary mathematics education through a three-pronged focus in preservice programs on content, pedagogy, and beliefs of prospective teachers.

### **Purpose of the Study**

The intent of this study was to investigate the relationship between two of the areas from the conceptual framework believed to be important foci of preservice elementary mathematics education. Within these two areas, teacher mathematics knowledge and teacher beliefs, two specific aspects were identified for exploration. Thus, the purpose of this study was to explore the relationship between the mathematics teacher efficacy and increases in specialized mathematical content knowledge (SCK) of preservice elementary teachers during a university mathematics methods/content course. Specifically, the study was designed to determine if increases in SCK experienced by preservice elementary teachers can be predicted by their levels of mathematics teacher efficacy. Since SCK and the common mathematics content knowledge (CCK) needed by many professions have been identified as two components of teacher content knowledge, the relationship between the two was considered in designing this study. In addition, since research identifies two dimensions of teacher efficacy, personal mathematics teacher efficacy and mathematics teaching outcome expectancy, this study examined the relationship between each dimension and specialized content knowledge. Personal mathematics teacher efficacy, mathematics teaching outcome expectancy, SCK, and CCK were measured through pre- and posttests and the relationships among them were investigated.

## **Research Questions**

The mathematical knowledge needed for teaching is a multidimensional construct that includes both content and pedagogical knowledge. The content knowledge dimension is itself multidimensional involving both the mathematical knowledge that is needed by those in all professions as well as the specialized mathematical content knowledge needed uniquely by teachers. The specialized content knowledge of preservice teachers is of particular interest to mathematics teacher educators due to its potential to impact their own learning as well as the learning of their future students. Therefore, the first research question of this study is:

- (1) Does the specialized mathematical content knowledge of preservice elementary teachers increase during a university mathematics methods/content course?

Teacher efficacy is also multidimensional and is composed of an individual's belief in his or her ability to effectively teach mathematics as well as a belief that effective teaching will result in increased learning. Each of these dimensions of self-efficacy is of interest in exploring the development of specialized content knowledge. Therefore, the proposed research was designed to investigate the following questions:

- (2) Is there a relationship between preservice elementary teachers' sense of personal mathematics teacher efficacy (PMTE) and growth in specialized mathematics content knowledge during a university mathematics methods course?
- (3) Is there a relationship between preservice elementary teachers' sense of

mathematics teaching outcome expectancy (MTOE) and growth in specialized mathematics content knowledge during a university mathematics methods course?

While the specialized mathematical content knowledge needed by teachers differs from the common content knowledge needed in all professions, certainly they are related. The level of common mathematical content knowledge a preservice teacher possesses may alter the relationship between his or her mathematics teacher efficacy and the growth he or she experiences in specialized content knowledge. Therefore, this study also explored the following questions:

- (4) Does the relationship between PMTE and growth in specialized content knowledge during a university mathematics methods course vary as a function of common content knowledge?
- (5) Does the relationship between MTOE and growth in specialized content knowledge during a university mathematics methods course vary as a function of common content knowledge?

### **Significance of the Study**

The educational significance of this study is to advance the literature in the field of mathematics education on the topic of mathematics teacher efficacy and its impact on the development of mathematical knowledge for teaching in preservice elementary teachers. The study sought to improve the understanding of the role that self-efficacy plays in the growth of specialized content knowledge in prospective elementary teachers. Findings

from this study may provide guidance for the development of preservice mathematics education courses and increased understanding of the specific types of activities that will enhance the development of each dimension of mathematical knowledge for teaching.

### **Definition of Terms**

For the purpose of this study, the following definitions are used:

**Common content knowledge.** Common content knowledge is the mathematical knowledge and skills used in all professions and settings (Ball, Thames, & Phelps, 2008). In this study, common content knowledge was measured using items from an instrument used at a large university located in a major city as a means of determining if students would be allowed to test out of the Number and Operations course for preservice elementary teachers (Barger, 1998).

**Mathematical knowledge for teaching.** Mathematical knowledge for teaching is a multidimensional construct that represents the professional knowledge of mathematics needed by teachers (Ball and Bass, 2000).

**Mathematics methods/content course.** Mathematics methods/content courses are university courses that are part of a program to prepare university students to become elementary teachers. The courses focus on developing the mathematical content and teaching methods used to deliver elementary mathematics content to elementary students. A full description of the courses involved in this study is included in chapter 3. In addition, the syllabi used for these courses are included in Appendix B.

**Mathematics teaching outcome expectancy.** Mathematics teaching outcome expectancy is a teacher's belief that effective mathematics teaching will bring about mathematics learning regardless of outside factors (Swars, 2005). For this study, the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) was used to measure mathematics teacher outcome expectancy (Enochs, Smith, & Huinker, 2000).

**Personal mathematics teacher efficacy.** Personal mathematics teacher efficacy is a teacher's beliefs in his or her ability to be an effective teacher of mathematics (Swars, 2005). For this study, personal mathematics teacher efficacy was measured using the Mathematics Teaching Efficacy Beliefs Instrument (Enochs, Smith, & Huinker, 2000).

**Preservice elementary teachers.** Preservice elementary teachers are students enrolled in university courses designed to prepare them to teach in an elementary school. These courses are part of a program leading to elementary teacher certification

**Self-efficacy.** Self-efficacy is the judgment a person makes regarding his or her ability to accomplish a task (Bandura, 1977).

**Specialized content knowledge.** Specialized content knowledge is the mathematical knowledge that is not used in professions or settings other than teaching (Ball, Thames, & Phelps, 2008). Looking for patterns in student errors, determining if nonstandard or student invented algorithms will work in general, presenting mathematical ideas, and recognizing the benefits of various representations are examples of specialized content knowledge. For this study, specialized content knowledge was measured using items from the Learning Mathematics for Teaching project (Hill, Schilling, & Ball, 2004).

**Teacher beliefs.** Teacher beliefs are teachers' assumptions about students, learning, classrooms, and the subject matter to be taught (Kagan, 1992).

**Teacher efficacy.** Teacher efficacy is a teacher's belief in his or her skills and abilities to be an effective teacher as well as a belief that effective teaching will bring about student learning regardless of outside factors (Swars, 2005).

### **Delimitations of the Study**

The researcher chose to focus only on prospective elementary teachers enrolled in mathematics methods courses at a single mid-western, mid-sized university located in a small city. Although other universities of various sizes and geographical locations could have been included, the researcher chose to limit the study to one university. This decision was made so that all participants would have the same instructor for the mathematics methods course thus limiting the impact of the instructor and the curriculum on the findings of the study. Additional research could expand this study to other settings. While the measures used for this study were designed to assess a wide range of mathematical strands, the researcher chose to focus solely on number and operations and to include only items that assess this strand since it is a primary focus in the elementary grades as well as in mathematics methods courses.

### **Limitations of the Study**

A limitation of this study is that participants completed an instrument on a voluntary basis. The participants self-reported their answers meaning that interpretations of various items may have varied among participants. Based on knowledge gained in teaching

other groups of students in elementary mathematics methods courses, the researcher acknowledges that some preservice elementary teachers may have prior negative experiences with mathematics and that these experiences may have influenced their participation and their responses to some items.

The participants of this study were students in courses taught by the researcher and while efforts were made to deal with this ethical concern, the responses of some of the participants may have been influenced by this relationship. Measures taken to address this concern are described fully in Chapter Three.

Because all of the participants in this study attended the same mid-sized, mid-western university, the sample created is distinct. Therefore, the generalizability of the findings is limited to prospective elementary teachers with the same characteristics as the prospective teachers in the sample.

### **Organization of Remaining Chapters**

Chapter Two contains a literature review providing a theoretical framework for this study. Mathematical knowledge for teaching, specialized content knowledge, teacher beliefs, and self-efficacy are discussed in the review of literature. In addition, the content knowledge and beliefs of prospective elementary teachers are described as is the importance of preservice teacher education programs. Measures of mathematical knowledge for teaching, self-efficacy, and mathematics teacher efficacy are identified and described. Chapter Three explains the methodology used in this study. Specifically, the recruitment of participants, data collection procedures, instrumentation, data analysis, and

ethical considerations are described. The findings of the study are presented in Chapter Four. The final chapter, Chapter 5, discusses the findings of the study, highlights the contributions and implications of these findings, and provides suggestions for further research. The instrument to be used in this study is included in the Appendix A of this dissertation.

## CHAPTER 2

### REVIEW OF LITERATURE

“We live in a time of extraordinary and accelerating change” [National Council of Teachers of Mathematics, (NCTM), 2000] and because of this, the need for our students to become mathematically literate citizens is vital. “The globalization of markets, the spread of information technologies, and the premium being paid for workforce skills all emphasize the mounting need for proficiency in mathematics” [National Research Council, (NRC), 2001, xiii]. Consequently, a continued focus on improving mathematics education by all is of great importance.

One aspect of improving mathematics education lies in improving the effectiveness of mathematics teaching and to do so requires identifying the factors that contribute to effective mathematics teaching. Knowledge of these factors is especially important to those responsible for the mathematics teacher education of preservice teachers. One such factor, mathematics content knowledge, has long been accepted as an important aspect of effective mathematics teaching. More recently, the existence of a dimension of mathematics content knowledge needed uniquely by teachers, specialized content knowledge, has emerged as a factor impacting teacher effectiveness. In addition, the beliefs of teachers regarding mathematics and mathematics teaching have also been of interest in exploring teacher effectiveness. Preservice teacher education programs have been shown to be instrumental in developing content knowledge as well as in impacting the beliefs of prospective teachers. The relationships among these factors and how these

relationships impact the development of effective teachers in the context of preservice education must be explored in order to improve mathematics education in the United States. Therefore, this research study examined the relationship between growth in specialized content knowledge and the mathematics teacher efficacy of elementary preservice teachers during an university mathematics methods/content course.

This literature review begins by looking at what is known about preservice and inservice elementary teachers. The content knowledge and beliefs held by these teachers will be considered as will the implications these hold for student achievement.

The idea of mathematical knowledge for teaching as an extension of pedagogical content knowledge will be explored next. Mathematical knowledge for teaching represents the special way teachers must know mathematics content in order to effectively teach the content. The difference between common content knowledge and specialized content knowledge will be described as will the development of such knowledge.

Mathematics teacher efficacy will be discussed by looking first at teacher beliefs in general and then by examining mathematics anxiety and its relationship to beliefs. Self-efficacy, a special form of belief, will be examined next and the sources of self-efficacy will be discussed. Teacher efficacy and, more specifically, mathematics teacher efficacy are described as are the classroom implications of differing levels of mathematics teacher efficacy.

## **Characteristics of Elementary Teachers and Teaching**

Many factors impact and influence the instructional decisions and practices of elementary teachers. In *Adding It Up*, the NRC (2001) suggests a model for the proficient teaching of mathematics that consists of five interrelated components. These components include conceptual understanding of the core knowledge required in the practice of teaching, fluency in carrying out basic instructional routines, strategic competence in planning effective instruction, adaptive reasoning in justifying and explaining instructional practices, and a productive disposition toward mathematics, teaching, and learning. Research provides insight into how these proficiencies are currently reflected in elementary preservice and inservice teachers.

In a comparative study of Chinese and United States elementary teachers, Ma (1999) found that while Chinese teachers have less formal schooling than teachers in the United States, they begin their teaching careers with a better understanding of elementary mathematics than most U.S. elementary teachers. She indicates that the mathematical knowledge of Chinese teachers is “clearly coherent while that of the U. S. teachers is clearly fragmented” (p. 107). Ma writes of the need to view elementary mathematics as more than a collection of disconnected number facts and calculations. Rather, teachers must see elementary mathematics as an “intellectually demanding, challenging, and exciting field – a foundation on which much can be built” (p. 116). Obviously, teaching from such a perspective requires a sophisticated and coherent conceptual understanding of elementary mathematics. Similar findings were reported by Weiss (1995) from a study of

1250 schools and approximately 6000 teachers. These findings indicate little evidence of conceptual teaching in the elementary grades, relatively limited use of manipulatives, and few connections made between instructional decisions and the development of conceptual understanding.

In a summary of multiple studies, an extensive list of areas in which the mathematics content knowledge of elementary preservice and inservice teachers is of concern is provided by da Ponte and Chapman (2008). Elementary teachers have “procedural attachments that inhibit the development of a deeper understanding of the concepts related to the multiplicative structure of whole numbers” (p. 227) and are influenced by primitive models for multiplication and division. While these teachers may have adequate procedural knowledge, they have inadequate conceptual knowledge and see only very limited connections between the two. An incomplete understanding of fractions and their representations as well as incorrect definitions and images of rational numbers are characteristic of elementary teachers. In addition, these teachers lack the ability to connect real-world situations with symbolic computations and possess inadequate logical reasoning. Finally, elementary teachers have difficulties with algebra and a lack of basic knowledge and skills in geometry (da Pointe & Chapman).

Recent reports regarding mathematics teacher quality and the preparation of teachers raise similar issues and concerns regarding the content knowledge and preparation of elementary teachers. The National Mathematics Advisory Panel (2008) suggests that teachers must know the mathematical content they are teaching and how it connects to

mathematics both prior to and beyond the level being taught. The results of a study of 77 schools of education by the National Council on Teacher Quality (2008) indicate many concerns regarding elementary teachers' knowledge of mathematics, and the group states that the reform of current elementary teacher preparation programs cannot wait until research provides a definitive answer on how best to prepare elementary teachers to teach mathematics.

### **Implications for Mathematics Teacher Education Programs**

NCTM's ambitious vision for mathematics education and the teachers who provide this education has direct implications for teacher education programs, especially those for elementary teachers. As described earlier, in a comparison of elementary teachers in the United States and China, Ma (1999) noted that while Chinese elementary teachers have completed fewer years of school, they possess more mathematics content knowledge than their counterparts in the United States. She contends, "teacher education is a strategically critical period during which change can be made" (p. 149). She adds that teacher preparation programs may be the force that breaks the cycle of low-quality mathematics education and low-quality teacher knowledge of mathematics.

Acknowledging the importance of preservice education programs, NCTM (2000) cautions that while mathematics teachers must know and understand the big ideas of mathematics, this kind of knowledge is beyond what most teachers experience in standard preservice mathematics courses. Supporting the need for change in preservice education,

the authors of *The Mathematical Education of Teachers*, a report from the Conference Board of the Mathematical Sciences (CBMS, 2001) state:

Those who prepare prospective teachers need to recognize how intellectually rich elementary level mathematics is. At the same time, they cannot assume that these aspiring teachers have ever been exposed to evidence that this is so. Indeed, among the obstacles to improved learning at the elementary level, not the least is that many teachers were convinced by their own schooling that mathematics is a succession of disparate facts, definitions, and computational procedures to be memorized piecemeal. As a consequence, they are ill-prepared to offer a different more thoughtful kind of mathematics instruction to their students (p. 17)

In general, preservice elementary teachers with weak mathematics backgrounds must “rekindle their own powers of mathematics thought” (CBMS, 2001, p. 17) in university mathematics education courses. This may be done by providing classroom experiences in which these prospective teachers’ “ideas for solving problems are elicited and taken seriously, their sound reasoning affirmed, and their missteps challenged in ways that help them make sense of their own errors” (p. 17). Successful mathematics education programs will work from what students know and utilize the “mathematical ideas they hold, the skills they possess, and the context in which these are understood” (CBMS, p. 17) in order to prepare future teachers.

## **Content and Nature of Elementary Mathematics Teacher Education Programs**

The role of mathematics teacher education programs is made more complex by differing opinions about what future elementary teachers need to know and be able to do. In addition, successfully preparing teachers today is much different than in the past. As Even and Lappan, (1994) state:

To teach the arithmetic-driven curriculum of the past, one needed little more than computation skill with the standard algorithms and a textbook to provide practice. That is not longer so. To prepare a teacher dedicated to helping children think mathematically requires a very different experience with mathematics from the traditional college course for elementary school teachers (p. 128).

Most researchers agree that mathematics content knowledge is a vital component in the preparation of elementary teachers. Ball, Hill, and Bass (2005) state that how well teachers know mathematics is central to their ability to use instructional materials, assess student progress, and make sound instructional decisions. However, they argue United States teachers lack mathematical understanding and skill since they are products of the very educational system educators and researchers seek to improve. Brown and Borko (1992) note increases in teachers' subject matter knowledge lead to increases in abilities to connect mathematical topics. Frykholm (2005) acknowledges the need for content knowledge development but suggests broadening the scope of content knowledge to include the ability to make connections and utilize insights without relying only on a knowledge base gained previously.

The link between content knowledge and classroom practice is acknowledged by many. When they “possess explicit and well-integrated content knowledge, teachers feel free to teach dynamically with many representations of the same concept. Teachers with more limited content knowledge may depend too heavily on textbooks for explanation of mathematical principles” (Sutton & Krueger, 2002, p. 15). On the other hand, Beckman et al. (2004) caution that taking more mathematics content courses moves a prospective teacher farther away from the curriculum they will actually be teaching.

Researchers also propose that courses for elementary education students must include a development of pedagogical skills. This pedagogical knowledge will help future teachers understand how students learn mathematics and will equip them with a range of teaching techniques and practices (NCTM, 2000). Some proponents of the importance of teaching pedagogy assume there is a general set of pedagogical practices which are instructionally effective no matter what the subject or grade level and without regard for the content knowledge possessed by the teacher (Rowan, Schilling, Ball, & Miller, 2001). In the past, this perspective has, at times, resulted in a separation between a prospective teacher’s learning of subject matter knowledge and knowledge of general pedagogical principles into separate university courses (Rowan et al, 2001).

### **Beneficial Impact of Mathematics Teacher Education Courses**

The potential of university mathematics education courses to positively impact future teachers and their classroom practice is documented in research. Ball (1990) found that mathematics methods courses can change preservice teachers’ knowledge,

assumptions, and feelings about mathematics as well as their beliefs regarding their role as mathematics teachers in the classroom. In a study of 38 preservice elementary teachers that examined mathematics anxiety, Battista (1986) reports that those prospective teachers who initially had an above average level of mathematics anxiety experienced a significant decrease in level of mathematics anxiety during a mathematics methods course.

A study by Quinn (1997) revealed that preservice elementary teachers enrolled in a mathematics methods course experienced significant improvement in both attitudes towards mathematics and in meaningful knowledge of mathematics. The researcher comments that it is clear that elementary mathematics methods courses are beneficial and that it seems reasonable to conclude spending more time in courses combining mathematical content knowledge and pedagogical strategies will benefit future elementary teachers.

### **Limitations of Impact of Teacher Education**

While teacher education courses have the potential to positively impact the beliefs and attitudes of preservice teachers, research has indicated that teacher socialization may limit the lasting impact of changes brought about by teacher education courses. Teacher socialization has been described as the process by which an individual becomes a member of the society of teachers (Zeichner & Gore, 1990). Zeichner and Gore describe three distinct periods during which teacher socialization impacts teacher beliefs and attitudes.

The first period of socialization occurs prior to formal teacher education. Preservice teachers come to the university with deeply held ideas and beliefs about teaching and

learning constructed through observing teachers at work. Lortie (1975) suggests that these predispositions exert a more powerful socializing effect than preservice training or workplace socialization. This is suggested as an explanation for the lack of success of school reform measures, professional development, and preservice teacher education in altering the beliefs of teachers (Zeichner & Gore, 1990).

Formal teacher education is a second time during which teacher socialization affects the beliefs and attitudes of teachers. Sadly, research indicates that the knowledge, skills, and dispositions teachers are exposed to during teacher education courses have very little impact on their later teaching practice (Zeichner & Gore, 1990). In fact, some researchers contend that not only do teacher education courses fail to alter the values, beliefs, and attitudes preservice teachers bring with them, they may instead actually reinforce them (Bullough, 1989; Ginsburg & Newman, 1985; Ross, 1987). Adler (1991) suggests the “taken-for-granted wisdom is that the liberalizing influences of the university context are canceled by the generally conservative context of schools” (p. 218). However, university courses may do more to promote a conservative approach to teaching than experiences before and after the courses do.

The third period of teacher socialization occurs as novice teachers begin working in the classroom. Frustration, anger, and bewilderment are common emotions experienced by new teachers as their idealistic views of teaching are confronted by the realities of teaching and pressure from schools to teach in a traditional way (Cole & Knowles, 1993). New teachers may feel as if they were not fully prepared by their teacher education programs

and, as a result, come to view colleagues in their schools as more knowledgeable about teaching than are their university professors (Wideen, Mayer-Smith, & Moon, 1998). Additional research is needed regarding specific components of teacher education, both preservice and inservice, that will ensure teacher competence increases.

## **Knowledge for Teaching**

### **Pedagogical Content Knowledge**

The importance of understanding what mathematics teachers need to know has been a part of discussions regarding mathematics education for many years. A significant turning point in these discussions occurred in 1924 when Felix Klein, a German mathematician, issued a textbook intended specifically for training prospective teachers and called for special courses for these future teachers. In so doing, he indicated the content knowledge needed by teachers differs to some degree from that needed in other professions (Guberman & Gorev, 2008). This viewpoint marked a change in the traditional practice of educating future mathematics teachers through the same courses as those taken by individuals pursuing careers as mathematicians.

Traditionally, the degree earned, the certification held, and the number of mathematics courses completed were used as measures of teacher content knowledge (Ball, Thames, & Phelps, 2008). A variety of studies were conducted to determine the relationship between student achievement and teacher knowledge using these measures of teacher content knowledge (Begle, 1979). The findings from these studies revealed little relationship between these traditional measures of teacher content knowledge and student

achievement and, in fact, some indicated a negative relationship. An interesting finding from a study by Begle revealed that teachers taking advanced mathematics courses produced positive main effects on student achievement in only 10% of the cases and actually had a negative effect on student achievement in 8% of the cases (Begle, 1979; Ball & Bass, 2000). The lack of significant findings failed to deter researchers, and interest in determining the connection between teacher knowledge and student achievement did not diminish (Hill & Ball, 2004). However, the focus of this research began to shift away from exploring how teacher characteristics influence learning, to an approach that suggested that teacher characteristics influence how teachers teach which ultimately will impact student learning (Wilkins, 2008).

In the 1980s, researchers began to see the need to think about teacher knowledge in new ways and, in particular, to think about the types of teacher knowledge most related to teaching (Hill & Ball, 2004; Fennema & Franke, 1992). As a result, the idea of pedagogical content knowledge (PCK) began to emerge as a model for describing the relationship between content knowledge and pedagogical knowledge. First described by Shulman (1986) in response to the existing emphasis on general pedagogical skills as a measure of effective teaching, PCK represents the belief that content knowledge and pedagogy cannot and should not be treated separately. In fact, Shulman comments:

Mere content knowledge is likely to be as useless pedagogically as content-free skill. But to blend properly the two aspects of a teacher's capacities requires that

we pay as much attention to the content aspects of teaching as we have recently devoted to the elements of the teaching process (p. 8).

Shulman described this intersection of subject matter knowledge and pedagogy as the “missing paradigm” (p. 7).

A supporting perspective is provided by Even (1993) who comments that good subject matter preparation is necessary but not sufficient because teachers have a tendency to teach in the same manner they were taught unless they develop a different repertoire of teaching skills. She adds that a “powerful content-specific pedagogical preparation based on meaningful and comprehensive subject-matter knowledge” (p. 114) would enable teachers to create effective learning environments for their students.

The concept of PCK is based on this belief that teachers need more than subject matter knowledge and general pedagogical techniques to be successful. Rather, prospective teachers must know how to structure the content in order to best teach it to students, what makes learning specific topics challenging, what conceptions and misconceptions students will experience, and what specific teaching strategies can be used to address learning needs in a variety of classroom situations (Rowan, Schilling, Ball, & Miller, 2001; Shulman, 1986). Hill and Ball (2004) suggest PCK supports the belief that “at least in mathematics, how teachers hold knowledge may matter more than how much knowledge they hold” (p. 332). They add that teaching quality might be more related to whether a teacher’s knowledge is “procedural or conceptual, whether it is connected to big ideas or isolated

into small bits, or whether it is compressed or conceptually unpacked” (p. 332) than to the amount of acquired knowledge.

Shulman (1987) envisioned a theoretical framework of teacher knowledge that included several categories of this knowledge. The first four of these categories were those widely in use in determining a teacher’s effectiveness into 1980s and included: (1) general pedagogical knowledge with reference to those broad principles and strategies of classroom management and organization; (2) knowledge of learners and their characteristics; (3) knowledge of educational contexts; and (4) knowledge of educational ends, purposes, and values, and their philosophical and historical grounds.

The remaining categories represent three dimensions of content knowledge, each of which, according to Shulman (1987), is essential for effective teaching. The first dimension, content knowledge, refers to the amount and organization of knowledge the teacher possesses. Shulman suggests teachers are expected to have at least as much content knowledge as subject matter majors but that teachers must also know more. Teachers must not only understand that something is true but they must fully understand why it is true. The teacher must understand why particular topics and concepts are of central importance to a discipline while others are less important.

Curricular knowledge is a second content knowledge dimension of Shulman’s (1987) theoretical framework and, according to him, one often neglected by teacher education programs. Curricular knowledge includes knowledge of the full range of programs designed and available for teaching particular subjects and topics, the variety of

instructional materials available, and the set of characteristics that guide educators in deciding whether or not to include particular materials as part of the curriculum. Curricular knowledge also includes lateral curriculum knowledge, described by Shulman as the ability to relate the content of specific courses or lessons to topics from other subject areas. In addition, teachers must have vertical curriculum knowledge or an understanding of the topics that have been and will be taught in the same subject area during preceding and later years of school.

The final of the three content-related dimensions, pedagogical content knowledge (PCK), goes beyond knowledge of subject matter and focuses on the aspects of content knowledge that are related to teaching. According to Shulman (1987), PCK includes knowledge of the topics most regularly taught and the most useful forms of representation of these topics including the most powerful analogies, illustrations, examples, explanations, and demonstrations. PCK includes an understanding of what makes learning certain topics more difficult than learning other topics as well as knowledge of the conceptions and misconceptions that students of differing ages and backgrounds commonly possess.

Shulman's work emphasized the importance of content knowledge in teaching and also identified ways in which content knowledge for teaching is distinct from disciplinary content knowledge (Ball, Thames, & Phelps, 2008). In addition, new conceptions of teacher knowledge created as a result of Shulman's work are affecting teacher assessment practices and licensing examinations in education (Rowan, Schilling, Ball, & Miller,

2001). In summary, Shulman's work "had important implications for informing an emerging argument that teaching is professional work with its own unique professional knowledge base" (Ball, et al., p. 392).

Included in this professional knowledge base is the work of other researchers who have proposed models to explain PCK in slightly different manners. A two component model to represent the core areas of knowledge for teachers is described by Leinhardt and Smith (1985). One of these components, lesson structure knowledge, is similar to Shulman's pedagogical content knowledge and includes the skills needed to plan and run a lesson smoothly, to move easily from one segment of the lesson to another, and to explain material clearly. Subject matter knowledge, the second component, supports lesson structure and serves as a resource for examples, formulations of explanations, and demonstrations.

Grossman (1990) proposes a four category model to represent the knowledge needed by teachers. These categories include subject-matter knowledge, general pedagogical knowledge, PCK, and knowledge of context. Grossman's description of PCK includes knowledge of students' understanding, curriculum, and instructional strategies.

Gess-Newsome (1999) describes two models, integrative and transformative, for PCK. The integrative model suggests that the knowledge bases used in teaching develop separately and are then integrated in the teaching process. In this model, PCK does not exist separately as a domain of knowledge. According to Gess-Newsome, "The task of the teacher is to selectively draw upon the independent knowledge bases of subject matter,

pedagogy, and context and integrate them as needed to create effective learning opportunities” (p. 11). Since students may leave teacher preparation programs with limited mathematical understanding and an inability to effectively integrate their knowledge bases, this model is problematic (Carpenter, Fennema, Peterson, & Carey, 1988).

On the other hand, Gess-Newsome’s (1999) transformative model describes PCK as the result of a transformation of knowledge and the creation of new knowledge. This new knowledge may be similar to existing mathematical and pedagogical knowledge but still possesses unique characteristics not present in the original form. Teacher education programs reflecting a transformative model are distinctly different from those based on an integrative model in that the transformative model requires the purposeful integration of experiences to provide teachers with opportunities to extend and connect existing mathematical and pedagogical knowledge to create new knowledge (Silverman & Thompson, 2008).

A cautionary viewpoint is articulated by Deng (2007) who states that the key to PCK is the idea of transforming the subject matter of an academic discipline into a school subject. This process is more than the pedagogical process described by Shulman and his colleagues, according to Deng. Rather, transforming subject matter is also a complex curricular task requiring the participation of experts including curriculum theorists or specialists, subject matter experts, and classroom teachers. This contradicts Shulman’s view that the transformation of subject matter is accomplished in the classroom by individual teachers (Deng).

Research suggests a link between pedagogical content knowledge and student learning. Sutton and Kruger (2002) comment, “As teachers’ pedagogical content knowledge increases within the context of a strong knowledge of mathematical content, their ability to impact student learning also increases” (p. 16). However a clear definition of PCK is lacking. In addition, while the impact of PCK has been studied in a variety of ways across fields, since “researchers tend to specialize in a single subject, much of the work has unfolded in roughly parallel but independent strands” (Ball, Thames, & Phelps, 2008, p. 394). As a result, little is known about how findings in one subject area are related to those of other subject areas. The importance of the special type of knowledge represented as PCK is worthy of additional research in order to better understand the construct and its implications for teacher education and professional development.

### **Mathematical Knowledge for Teaching**

A great deal of research in mathematics education has been devoted to understanding the role of PCK in mathematics teaching. Ball and Bass (2000) utilize the term *mathematical knowledge for teaching* (MKT) to represent the professional knowledge of mathematics needed by teachers and assert that such knowledge is different from that needed by other occupations. Rather than describing what teachers need to know based on what they need to teach or the curriculum they will use, MKT is derived from an explicit focus on the work of teachers. From this focus, the researchers conclude that teaching is a complex endeavor that involves many different tasks that are specific to the teaching profession. For example, teachers must be able to interpret the work of students and

analyze errors students make. They must be able to choose the best model or representation for a given situation but be able to utilize other models and representations as needed. Teachers must have the ability to explain ideas in a way that makes sense to a student and they must develop fluency with mathematical language.

In a continually evolving model of MKT, Thames, Sleep, Bass, & Ball (2008) propose a refinement of Shulman's categories. They suggest that MKT "elaborates pedagogical content knowledge, rather than replaces it" (p. 5) and indicate teacher knowledge consists of four domains that fall into two categories. In this new model of mathematical knowledge for teaching, two of the four domains are envisioned as components of subject matter knowledge and each of these are related to knowledge about mathematics. The first of these domains, common content knowledge, represents the mathematical knowledge and skills used in professions and settings other than teaching. This domain indicates the importance of teachers knowing the material they teach and the ability to know when students provide wrong answers or textbooks give inaccurate definitions. In addition, teachers must know and be able to use vocabulary and notation correctly. However, the need for this type of knowledge is not unique; rather, it is used in a wide variety of settings and in a diverse array of professions.

A second dimension, specialized content knowledge, captures the mathematical knowledge and skills that are unique to teaching and includes the mathematical knowledge that is not used in areas other than teaching (Ball, Thames, & Phelps, 2008). For example, teachers must routinely look for patterns in student errors, understand the subtle

differences in different interpretations of operations, and determine if nonstandard or student invented algorithms will work in general. The unique mathematical tasks associated with teaching also include presenting mathematical ideas, responding to student questions, finding appropriate examples, recognizing the benefits of various representations, connecting a variety of representations, connecting topics, appraising and adapting the mathematical content of textbooks, modifying tasks for a variety of student abilities and needs, quickly evaluating the accuracy of students' claims, and asking productive mathematical questions. (Ball, et al., 2008).

The two remaining domains of MKT are described as components of pedagogical content knowledge. The first of these, knowledge of content and students, is described as “content knowledge intertwined with knowledge of how students think about, know, or learn this particular content” (Hill, Ball, & Schilling, 2008). This domain involves the ability to anticipate what students will find confusing, to select examples that will motivate and interest students, and to hear and interpret the incomplete thinking of students. An important aspect of each of these abilities is knowledge of common student conceptions and misconceptions about particular mathematical content (Ball, Thames, & Phelps, 2008).

Knowledge of content and teaching is a second domain of this portion of the model of MKT and it combines knowing about teaching and knowing about mathematics. Sequencing content for instruction, evaluating the instructional advantages and disadvantages of particular representations to teach specific ideas, and determining the best examples to use first and those to use to take students deeper into the content being taught

are all components of this dimension. Each of these involves an interaction between mathematical understanding and an understanding of pedagogical issues (Ball, Thames, & Phelps, 2008).

Ball, Thames, and Phelps (2008) suggest two additional, provisional domains to this model but caution that due to a lack of research on these domains, they may not actually be unique but may instead be part of each of the accepted domains. The first of these is envisioned as being a third domain in the subject matter knowledge portion of MKT. Referred to as horizon content knowledge, this domain reflects an awareness of how mathematical topics are related throughout grade levels and an understanding of how certain mathematics provides the foundation for what will be studied in later grades. Horizon content knowledge is necessary to prepare teachers to make effective instructional decisions and to avoid choices that may distort the later development of mathematical ideas. The second provisional domain, similar to Shulman's curricular knowledge, is referred to as knowledge of content and curriculum and is currently believed to be part of the pedagogical content portion of the model.

Other models of the concept of MKT have been suggested by researchers. One such model, proposed by Rowland, Huckstep, and Thwaites (2005), resulted from the use of a grounded approach to data analysis that led these researchers to identify four broad dimensions of teacher knowledge referred to as the "knowledge quartet" (Rowland, et al., p. 258). The foundation dimension involves knowledge and understanding of mathematics, knowledge of significant literature which has resulted from inquiry into the

teaching and learning of mathematics, and beliefs about mathematics and how it is learned. A second dimension, transformation, represents knowledge-in-action and includes such behaviors as use of analogies and explanations, choosing examples, and choosing representations. Connection is the third dimension suggested by these researchers and involves connecting concepts and recognizing alternative ways of representing concepts, making decisions about sequencing, anticipating complexity, and recognizing conceptual appropriateness. Finally, the contingency dimension represents the ability of a teacher to respond quickly and appropriately to students' responses and questions.

A transformative model is reflected in the framework for MKT suggested by Silverman and Thompson (2008). According to these researchers, a person's MKT is grounded in a "personally powerful understanding of particular mathematical concepts that is created through the transformation of the concept from an understanding having pedagogical potential to an understanding that does have pedagogical power" (Silverman and Thompson, p. 502). They suggest that when a teacher has developed a level of MKT that supports conceptual teaching, he or she will have experienced a change in how they think about and perceive mathematical relationships related to the concept. In addition, the teacher will have constructed models of the variety of ways students may understand the material and will have developed an image of how someone else may think of the mathematical idea in a similar way. He or she will have an image of the kinds of activities and conversations that might support a learner's understanding of the idea and an image of how students may be empowered to learn other, related mathematical ideas.

## **Measures of Mathematical Knowledge for Teaching**

The theoretical work on the mathematical content knowledge needed for teaching described above has contributed a great deal to the field of mathematics education.

However, the lack of measures of this knowledge made it difficult for researchers to track the development of MKT and its relationship to student achievement (Hill, Schilling, & Ball, 2004).

Spurred by the need for such measures in order to evaluate comprehensive school reform projects, beginning in 2000, researchers in the *Study of Instructional Improvement* (SII) and the *Learning Mathematics for Teaching* (LMT) projects began to work on the development of a set of items intended to measure the mathematical knowledge used in teaching elementary mathematics. Specifically, the items were developed to measure teachers' knowledge of representing numbers, interpreting unusual student answers, interpreting student algorithms, and anticipating student difficulties with content material.

Analysis of the results of piloting several different sets of these items covering different mathematics content led the researchers to draw conclusions regarding the existence of specialized content knowledge as well as the success with which their items could measure this. They note that repeated analyses across the various forms provide evidence of multidimensionality in their measure thus suggesting that teachers' knowledge of mathematics is "at least partly domain specific rather than simply related to a general factor such as overall intelligence, mathematical ability, or teaching ability" (Hill, Schilling, & Ball, 2004, p. 26). They suggest that the items they intended to reflect the

specialized mathematical knowledge needed by teachers did, in fact, indicate the existence of this special knowledge. The researchers conclude that while common content knowledge is an important element of mathematical knowledge for teaching, elementary teachers must have a deeper understanding of mathematics than that required to complete the mathematics problems in elementary textbooks.

To establish content validity, the items were mapped for congruence with standards established by NCTM (Siedel & Hill, 2003). In conducting additional work to validate the measures, Hill, Dean, and Goffney (2007) conducted a study with teachers, non-teachers, and mathematicians in which participants completed the measures described above. As they worked, the participants were asked to track their thinking processes. After completing the measures, the participants were asked to report on how they determined their answers and researchers determined if the answer provided was consistent with the reasoning described. Low inconsistencies in responses led the researchers to conclude that the items were indeed measuring the content knowledge of participants. Interestingly, the descriptions of the reasoning used by the teachers often included evidence of knowledge of how students would respond to items (Hill, Dean, & Goffney).

In a similar study, Delaney, Sleep, Ball, Bass, Hill, & Dean (2005) investigated the extent to which specialized content knowledge exists by studying whether or not mathematicians had difficulty answering any of the items designed to measure specialized content knowledge. They found that there were a few items on which the mathematicians struggled and these items tended to be related to non-standard solution methods. They also

noted that the mathematicians fluently used equivalent representations and were unaware that a simple substitution might impact the teaching of mathematics to children and that what seems like a logical step to mathematicians might not be obvious to students (Delaney, et al., 2005).

### **Studies of Mathematical Knowledge for Teaching**

The relationship between MKT and other factors involved in teaching has been explored by a variety of researchers. Hill, Rowan, and Ball (2005) investigated the extent to which MKT contributes to gains in first and third grade students' mathematics achievement. They found that teachers' mathematical knowledge for teaching was significantly related to student achievement gains in both first and third grades. Other variables including teacher certification, number of mathematics courses completed, and years of teaching experience were not found to be significant predictors of student achievement gains.

In an analysis of several case studies, Hill, Blunk, et al. (2008) found strong links between teachers' knowledge and the mathematical quality of their teaching practices. Teachers with high levels of MKT were found to avoid errors more frequently, to utilize more rigorous mathematics in their instruction, and to more skillfully respond to students than were teachers with lower levels of MKT. The researchers found that mathematical errors, including errors in language, were the most closely related to teacher knowledge. Interestingly, for the majority of the teachers in this study, the use of supplemental activities and materials actually lowered the quality of mathematics instruction. An

additional finding of the study was that beliefs about the nature of mathematics mediated the relationship between MKT and the quality of mathematics instruction.

The relationship between teachers' MKT and the population of students in their school was investigated by Hill and Lubenski (2007). A non-random sample of California teachers who volunteered to participate in a professional development program served as the data source. This study revealed that schools enrolling larger numbers of low-income and minority students employed teachers with slightly lower levels of MKT than those teachers employed by more affluent schools. The researchers suggest this may be due to more affluent schools being able to attract and retain more knowledgeable teachers as well as their ability to transfer teachers with lower levels of mathematical knowledge.

An experimental study by Philipp et al. (2007) investigated the content knowledge and beliefs of prospective elementary teachers enrolled in a mathematics education course. A focus on the mathematical thinking of children through video study and actual work with children served as the intervention for this study and three variations of the intervention were utilized. One treatment group worked exclusively in classrooms, one group exclusively utilized video study, and the final experimental group both worked in classrooms and viewed videos to learn about the mathematical thinking of children. The researchers found that the content knowledge of those students in the experimental groups improved more than the content knowledge of the students in the control group. Interestingly, beliefs regarding mathematics and the teaching of mathematics of those in the experimental groups also changed significantly as compared to the control group.

Hill and Ball (2004) explored the development of MKT of teachers participating in a professional development program. The researchers comment that many educators view the allocation of extra time for professional development and extending the length of the professional development experience as being important in increasing teachers' levels of MKT. However, their study indicates that the opportunity to engage in mathematical analysis, reasoning, and communication are effective in improving teacher knowledge and, as a result, curricular variables may be of importance in the quality and impact of professional development.

### **Developing Mathematical Knowledge for Teaching**

While studies have demonstrated that increased levels of MKT help support increased student achievement, the ways in which this knowledge can be developed and the extent to which it can be developed in preservice education programs is not well researched. Ball, et al. (2008) suggest that a practice-based approach may be effective in helping prospective teachers develop higher levels of MKT and indicate this approach initially involves the development of tasks that create opportunities for learning MKT. These tasks make explicit the mathematical ideas that are central to the school curriculum and open opportunities to build connections among mathematical ideas. The tasks intentionally provoke a “stumble” (Ball, et al., p. 22) due to superficial understanding of the mathematical idea being examined. Alternative and multiple representations and solution methods are suggested by the task and opportunities are provided to engage in the mathematical practices that are central to teaching. These tasks are then implemented in a

way that situates the teachers' opportunities to learn in the context of use and in ways that maintain the focus on developing MKT and the ability to use it in teaching.

When these tasks are presented to prospective or practicing teachers, a variety of questions may be used to support the development of MKT. Asking the teachers to explain their solutions, explain what is confusing, and to explain someone else's thinking are provided by Ball, et al. (2008) as strategies that may aid in increasing MKT. Additionally, asking the teachers to make connections between solutions and representations, providing them with the opportunity to talk about mathematics, and provoking common errors as a springboard for discussion are also suggested as being beneficial in the development of MKT.

While a practice-based approach is more common currently in methods courses and professional development settings, extending this philosophy to all mathematics courses in which prospective teachers enroll is suggested as a way to ensure that development of MKT occurs (Thames, Sleep, Bass, & Ball, 2008). This approach suggests that the mathematics taught in courses for teachers should "be the mathematics required for the work of teaching, and that this mathematical knowledge for teaching should be integrated with and learned in the context of practice" (Thames, et al., 2008, p. 6). Thus, teaching is placed in the foreground, and the mathematics that is learned, is learned through situations related to teaching practice (Thames, et al. 2008).

The idea that the mathematics needed by teachers differs from that needed by other professions and that teaching mathematics involves a unique set of abilities is relatively

new in mathematics education. A great deal of research has been and is being conducted to learn about the nature of this knowledge, how it develops, and how it impacts student learning; however, many questions linger regarding this important component of teacher preparation.

### **Mathematics Teacher Efficacy**

Mathematics teacher efficacy is a construct investigated in this study. Since mathematics teacher efficacy is a type of teacher belief, this section of the literature review begins with a general discussion of teacher beliefs. This is followed by a discussion of teacher efficacy, and specifically, mathematics teacher efficacy.

### **Teacher Beliefs and Attitudes**

While mathematics content knowledge, pedagogy, and pedagogical content knowledge are certainly important aspects of teaching and teacher education, some researchers caution that there are other factors that play a role in shaping teacher practice. Pajares (1992) suggests that “another perspective is required from which to better understand teacher behaviors, a perspective focusing on the things and ways that teachers believe” (p. 307).

Although a great deal of research on teacher beliefs has been conducted, a single definition of beliefs is not evident in the research (Pajares, 1992). Kagan (1992) describes teacher beliefs as a “form of personal knowledge that is generally defined as pre- or inservice teachers’ implicit assumptions about students, learning, classrooms, and the subject matter to be taught” (p. 66). Beswick (2006) makes a distinction between beliefs

and attitudes by describing beliefs as non-evaluative ideas a person regards as being true while attitudes are evaluative in nature. In addition, attitudes are the “consequence of belief but there is not a one-to-one correspondence between beliefs and attitudes” (Beswick, 2006, p. 37).

According to Pajares (1992), attempts to distinguish between beliefs and knowledge contribute to the confusion regarding an operational definition of beliefs. The difference between beliefs and knowledge is described by Snider and Roehl (2007) who indicate that beliefs are based on judgment and evaluation. Beliefs, unlike knowledge, are personal and do not require validation (Snider & Roehl, 2007; Orton, 1996). Nespor (1987) indicates that beliefs are, for the most part, unchanging and are not open to critical examination while knowledge is open to such evaluation.

Kagan (1992) classifies research regarding teacher beliefs into two broad categories, each with direct implications for teacher education programs. One category, self-efficacy, will be addressed later in this literature review. The second category involves content-specific beliefs involving a teacher’s “orientation to specific academic content” (Kagan, p. 67). Content specific beliefs include beliefs about appropriate instructional activities, goals, assessment and evaluation, and student learning (Kagan, 1992).

The need to consider the beliefs of inservice teachers is supported by many researchers. According to Snider and Roehl (2007), the importance of teacher beliefs is increased due to a lack of consensus regarding “empirically based teaching practices” (p. 873). Liljedahl, Rolka, and Rösken (2007) suggest the importance of considering the

beliefs that are held by new teachers and caution that if beliefs are not addressed, novice teachers are likely to revert to a “method of teaching that is more reflective of their own experiences as students than of their experiences as prospective teachers (p. 319). Barlow and Reddish (2006) agree with this idea and state, “Beliefs impact practices because beliefs affect how teachers see their students, how they view the practices of other teachers, and how they accept the ideas given to them to develop practice” (p. 145). Importantly, attempts to change the practice of teachers must involve change in their beliefs (Beswick, 2006).

### **Beliefs Regarding the Teaching of Mathematics**

The relationship between teachers’ beliefs and their mathematics teaching practices has been the topic of a great deal of research. Thompson (1982) contends that the ways teachers deal with problems associated with the teaching of mathematics are closely connected to their beliefs about mathematics, mathematics teaching, and mathematics learning. In researching problem solving, Thompson (1988) found that both teachers and students held the same beliefs regarding problem solving. These beliefs included the idea that getting the right answer is what counts, the belief that the answer to a problem is usually a number, and the reliance on a set procedure to arrive at an answer. She suggests this indicates that teachers are, unknowingly, communicating their own beliefs to their students.

Four distinct approaches to mathematics teaching and the beliefs associated with these approaches are described by Kuhs and Ball (1986). A learner-focused approach

indicates a belief that mathematics is a dynamic discipline that utilizes problem-solving extensively. The belief that mathematics is the practice of mathematicians and that mathematics should make sense is reflected in a content-focused approach that emphasizes understanding. A third approach, content-focused emphasizing performance, is indicative of a belief that mathematics is composed of a set of rules to be memorized and mastered. Finally, a classroom-focused approach reflects a belief that mathematics is whatever is specified in the curriculum.

Research provides evidence of the impact of beliefs about mathematics and mathematics teaching. Ernest (1989) found that even if two teachers have similar knowledge of mathematics, they may teach in very different ways due to their individual beliefs. A similar finding by Brown and Cooney (1982) indicates mathematics teachers often use beliefs rather than knowledge to guide classroom practices.

### **Beliefs of Preservice Teachers**

Because of the potential impact of beliefs and attitudes on future practice, identifying the beliefs held by prospective teachers is of importance and concern to teacher educators. In a very general sense, prospective teachers may have simplistic beliefs about what it takes to be a successful teacher and may erroneously believe that teaching is a merely a process of transmitting knowledge and dispensing information (Stuart & Thurlow, 2000). These students often view knowledge as “absolute and certain” (Cady, Meier, & Lubinski, 2006, p. 296) and believe that others, particularly their instructors, have all of the answers. Preservice teachers may have a narrow view of teaching and

classroom practice, according to Castro (2006), and may bring preconceptions about teaching to their university courses. These prospective teachers have developed ideas about good teaching from watching their own teachers, but have little understanding about the challenges and decisions faced by teachers.

The stability of beliefs is supported in the results of a study by Calderhead and Robson (1991) that followed students during their first year in an elementary education program. The researchers found that the students entered the program with clear images of good teaching derived from their own experiences as students. These images remained unchanged across classroom contexts and the prospective teachers seemed unable to adapt their conceptions of teachers and lessons to different situations. Another study by McDaniel (1991) yielded similar findings. Students in an education course related their content of the course to their own beliefs and past experiences and the content of the course failed to affect these beliefs.

In a summary of several studies, Kagan (1992) writes that in general, preservice teachers enter education programs with “personal beliefs about teaching, images of good teaching, images of self as teacher, and memories of themselves as pupils in classrooms.” These beliefs remain largely unchanged through the education program. However, according to Kagan, for professional growth to occur, prior beliefs must be modified. Complicating the issue of changing beliefs is the fact that beliefs regarding education are well-established by the time students enter college (Pajares, 1992) and that teacher

preparation programs have only a limited amount of time to change these beliefs (Swar, Hart, Smith, Smith, & Tolar, 2007).

### **Beliefs of Preservice Teachers Regarding Teaching Mathematics**

In regard to mathematics, preservice teachers come to universities with “deep-seated beliefs about mathematics and what it means to learn and teach mathematics” (Liljedahl, Rolka, and Rösken, 2007, p. 320). These prospective teachers’ firmly established beliefs, attitudes, and perceptions about mathematics teaching and learning are “born out of and nurtured by their previous experiences in school” (Minor, Onwuegbuzie, Witcher & James, 2002).

Barlow and Reddish (2006) describe a series of mathematical myths commonly believed to be true by future elementary teachers. Among these myths are beliefs that some people have a math mind and some don’t, that there is a best way to do a math problem, men are better than women in math, and math is not creative. Obviously, beliefs in myths such as these will drastically affect the practice of these future teachers.

In addition, such beliefs and attitudes may result in mathematics anxiety, which is more than a dislike of mathematics (Vinson, 2001). Vinson characterizes math anxiety as uneasiness when asked to complete a mathematical task, avoidance of math courses, feelings of physical illness or panic, inability to perform on a mathematics tests, and unsuccessful utilization of tutoring sessions. Interestingly, a study by Kelly and Tomhave (1982) indicates that elementary education majors have the highest mathematics anxiety level when compared to four other math-anxious college groups.

Mathematics anxiety has direct implications for teacher education programs since teachers with high mathematics anxiety tend to use more traditional teaching methods and focus on teaching basic skills rather than mathematics concepts. These teachers tend to have teacher-centered classrooms and devote more time to seatwork and whole-class instruction. In addition, teachers with high mathematics anxiety may avoid teaching math and may pass negative attitudes regarding mathematics on to their students (Swars, Daane, & Giesen, 2006). Myers (2007) comments that every teacher affects the attitudes of hundreds of children and can “nurture or negate the innate curiosity young children bring with them into the elementary school classroom” (p. 692).

### **Self-Efficacy**

Beliefs and attitudes towards mathematics, as well as mathematics anxiety, may impact the teaching self-efficacy of prospective teachers. Teaching self-efficacy is a form of self-efficacy, a construct first described by Bandura (1977) as a judgment of one’s capability to accomplish a task. The concept of self-efficacy is an important aspect of Bandura’s social cognitive theory which reflects his belief that direct reinforcement does not account for all learning, and stresses the importance of learning through observation of others. He indicates that perceived self-efficacy, significant in determining performance, operates at least partially independently of underlying skills and knowledge. In fact, knowledge, skills, and previous performance may be poor predictors of later performance because of the powerful influence of beliefs individuals hold about their abilities and the results of their efforts (Bandura, 1986).

Bandura (1977) comments that self-efficacy affects how individuals think feel, and motivate themselves. As a result, “a strong sense of self-efficacy enhances an individual’s sense of accomplishment and determines whether or not an individual perceives a task as a reachable or an unreachable goal” (Brand & Wilkins, 2007, p. 298). Those with higher self-efficacy are more likely to attempt new tasks, to work harder, and persist longer in the face of difficulty. Bandura states that perceived efficacy could mediate performance because it affects whether people make good or poor use of their capabilities and, in fact, self-doubt may overrule even the best of skills. Tschannen-Moran, Woolfolk Hoy and Hoy (1998) comment that since self-efficacy involves perception of ability rather than actual ability, the overestimation or underestimation of ability may influence the course of action taken, the degree of effort exerted, and the successful utilization of these skills.

### **Sources of self-efficacy**

Bandura believes that there are four sources of information that contribute to self-efficacy. These are mastery experiences gained through past successful performance, vicarious experiences from observing the performance of others, social persuasion by others that one is capable, and stress reduction. Knowledge of these sources is important for mathematics educators and mathematics teacher educators because if educators accept the idea that efficacy is important in learning, steps may be taken by teachers to improve the self-efficacy of learners. In fact, teachers “can design instructional presentations and interactions that capitalize on the influence of these sources” (Siegle & McCoach, 2007, p. 282) of self-efficacy.

Bandura (1994) proposes that the most effective method of building self-efficacy is through mastery experiences. Successes will strengthen self-efficacy while failures will undermine it, especially if these failures occur prior to the establishment of a strong sense of efficacy (Bandura). He cautions that if only easy success is experienced, individuals begin to expect quick results and failure leads easily to discouragement. Conversely, experience in overcoming obstacles through perseverance leads to a resilient sense of self-efficacy and the belief that success requires sustained effort (Bandura).

The second most influential source of self-efficacy beliefs, according to Bandura (1994), is found in the vicarious experiences provided by observing the performance of others. Viewing the success through effort of those an individual views as being similar to himself or herself increases the observer's belief that he or she is also capable of success. Likewise, observance of failure even though effort is expended lowers the efficacy of the observer. Bandura comments that the impact of this modeling is strongly affected by the perceived degree to which the observer feels he or she is similar to the model.

Consequently, if an individual views the model as being very different from himself or herself, the influence of the model's behavior on the individual's perceived self-efficacy is lessened. Self-efficacy that develops as a result of watching the success of others is less stable than self-efficacy derived from personal success (Siegle & McCoach, 2007).

Schunk (1989) comments that when strong self-efficacy develops based on successful past experiences, occasional failure may not lessen self-efficacy. However, self-efficacy based solely on the observation of others decreases rapidly when failure is experienced.

Social persuasion is a third factor in strengthening the self-efficacy of individuals. Those who are verbally persuaded by others that they are capable of performing certain tasks are more likely to expend greater and more sustained effort than those who possess self-doubt and focus on personal deficiencies (Bandura, 1994). Bandura states that it is more difficult to increase self-efficacy through social persuasion alone than it is to undermine it. In other words, unrealistic increases in self-efficacy brought about by social persuasion are easily dismissed when an individual's efforts lead to disappointing results. However, those who have been persuaded by others that they lack capability tend to avoid challenging activities and to give up quickly when difficulty arises. Greater increases in self-efficacy through persuasion occurs when the persuader is viewed as credible and trustworthy (Schunk, 1989). Successful efficacy building through social persuasion involves more than simply conveying positive appraisals but also involves the structuring of situations that encourage success. In addition, successful efficacy builders avoid placing individuals in situations that will likely result in frequent failure (Bandura).

Finally, individuals partly judge their ability to successfully perform a task on their emotional state (Bandura, 1994). Feelings of stress and tension are seen as signs of vulnerability to failure and so a positive mood results in increased perceived self-efficacy while a more negative mood decreases self-efficacy. Therefore, another way to enhance self-efficacy is to reduce stress levels and alter any negative emotions an individual possesses.

## **Teacher Efficacy**

Teacher efficacy involves a teacher's belief in his or her skills and abilities to be an effective teacher as well as a belief that effective teaching will bring about student learning regardless of outside factors (Swars, 2005). Teacher efficacy is of interest when looking at teacher effectiveness because it has been linked to measures of effectiveness (Benz, Bradley, Flowers, & Alderman, 1992). However, while teacher efficacy has long been of interest to researchers, two "separate but intertwined conceptual strands" (Tschannen-Moran, Woolfolk Hoy & Hoy, 1998, p. 203) have contributed to a "lack of clarity about the nature of teacher efficacy" (Tschannen-Moran, Woolfolk Hoy & Hoy, 1998, p. 203).

Teacher efficacy was first conceived by researchers from the RAND organization (Armor, et al., 1976) as being a measure of the "extent to which teachers believed that they could control the reinforcement of their action, that is, whether control of reinforcement lay within themselves or in the environment" (Tschannen-Moran, Woolfolk Hoy & Hoy, 1998, p. 202). The RAND measure actually consisted of only two items within an extensive questionnaire and, interestingly, one item measured a general form of teacher efficacy while the other item measured a more personal sense of efficacy. The theoretical base for this view of teacher efficacy was the social learning theory of Rotter (1966). In this conceptualization, it was assumed that student motivation and performance reinforced teacher actions and behaviors and, as a result, teachers with high levels of efficacy believed they could influence student achievement and motivation (Tschannen-Moran, Woolfolk Hoy & Hoy).

Bandura's social cognitive theory provided the theoretical basis for the second conceptual strand of teacher efficacy that describes teacher efficacy as a type of self-efficacy. As noted earlier, self-efficacy is defined as a future-oriented belief about the level of competence a person expects he or she will display in a given situation" (Tschannen-Moran, Woolfork Hoy & Hoy, 1998, p. 210). Bandura's work provides a theoretical basis for the current study.

Tschannen-Moran, Woolfork Hoy and Hoy (1998) propose an integrated model for teacher efficacy that weaves together both of these conceptual strands and emphasizes the cyclical nature of teacher efficacy. The model identifies task analysis and assessment of personal teaching competence as two dimensions of teacher efficacy. While this model acknowledges the importance of Bandura's four sources of self-efficacy, the researchers believe that the interpretation of these sources is what is critical. "Cognitive processing determines how these sources of information will be weighed and how they will influence the analysis of the teaching task and the assessment of personal teaching competence" (Tschannen-Moran, Woolfork Hoy & Hoy, 1998, p. 230 ). Teacher efficacy is shaped through the interaction of tasks analysis and assessment of personal competence.

### **Dimensions of Teacher Efficacy**

Many researchers see teacher efficacy as being composed of two dimensions although they do not all agree on the meaning of these dimensions (Tschannen-Moran, Woolfork Hoy & Hoy, 1998; Gibson & Dembo, 1984). The first dimension, often called personal teaching efficacy (PTE), is generally agreed upon as having to do with one's own

feeling of competence as a teacher and the belief in one's ability to cause student learning (Gibson & Dembo, 1984). Swars (2005) describes PTE as a teacher's belief in his or her own skills and abilities to be an effective teacher. Zambo and Zambo (2008) comment that teachers with high levels of PTE "believe they can and do make a difference in the lives of their students and that their students can and will achieve" (p. 160).

The second dimension is often described as a more general teaching efficacy but the label used, as well as the meaning of the dimension, varies among researchers. Usually this dimension is referred to as general teaching efficacy (GTE) and represents the belief that effective teaching can bring about student learning regardless of other factors such as "home environment, family background, and parental influences (Swars, 2005). Enochs, Smith, and Huinker (2000) suggest this dimension be called outcome expectancy to reflect the idea that teachers expect specific teaching behaviors to result in desirable outcomes. This second dimension is referred to by Emmer and Hickman (1990) as external influences, a term in keeping with Rotter's construct of external control. A similar distinction is made by Guskey and Passaro (1994) who propose that the two dimensions do not correspond to a personal versus general teaching efficacy but rather to an internal versus external control distinction. These researchers question whether describing the dimension as outcome expectancy actually captures Bandura's view of this aspect of self-efficacy.

Regardless of the label or description used, these two dimensions of teacher efficacy are distinct and, therefore, an individual teacher may have a high level of personal

teaching efficacy with regard to a particular content area but a low level of teaching outcome expectancy (Allinder, 1995; Aston & Webb, 1986). In a study of teachers in Kenya, Onderi and Croll (2009) found that teachers were able to think of themselves as “effective professionals without necessarily thinking that they could have a substantial impact on student performance” (p. 106). Interestingly, these teachers’ sense of themselves as competent professionals was much higher than their sense of being effective in producing student outcomes.

### **Measures of Teacher Efficacy**

The concept of teacher efficacy has been of interest to researchers for over thirty years. However, although numerous researchers have conducted studies to gain insight about teacher efficacy, many questions about the concept and its implications for the classroom exist. A variety of efficacy measures, differing in theoretical base and context specificity, have been developed in an attempt to find answers to these questions (Rohs, 2007).

The first measure of teacher efficacy occurred when the RAND organization (Armor et al, 1976) added two questions dealing with efficacy to a questionnaire being administered to teachers participating in reading innovations. The questions reflected Rotter’s (1966) social learning theory and responses on the items were found to be strongly related to variations in reading achievement. A second RAND study found teacher efficacy to be a predictor of the continuation of federally funded projects once funding ended (Tschannen-Moran, Woolfork Hoy, & Hoy, 1998).

Other longer, more comprehensive measures, also based on Rotter's theory, were developed in an attempt to measure efficacy. These included the Teacher Locus of Control (Rose and Medway, 1981), Responsibility for Student Achievement (Guskey, 1981), and the Webb Efficacy Scale (Ashton, Olejnik, Crocker, & McAuliffe, 1982). Each of these measures was intended to "capture a global measure of teacher efficacy" (Rohs, 2007, p. 37).

The second strand of research on efficacy, based on Bandura's (1977) social cognitive theory, has also spurred the development of a variety of global measures of teacher efficacy. These include the Teacher Efficacy Scale (Gibson & Dembo, 1984), Ashton vignettes (Ashton, Buhr, & Crocker, 1984), and Bandura's Teacher Self-Efficacy Scale (1977). In addition, researchers have combined items from several different existing instruments to create new instruments (Midgley, Feldlaufer, & Ecces, 1989; Raudenbush, Rowen, & Cheong, 1992).

Gibson and Dembo's Teacher Efficacy Scale (TES) instrument reflects Bandura's work on self-efficacy and assumes that the two items on the RAND questionnaire mirrored the two expectancies in Bandura's theory, self-efficacy and outcome expectancy (Tschannen-Moran, Woolfork Hoy & Hoy, 1998). Their TES was initiated as a pilot study with 90 teachers who were presented 53 sample items. These items were written based on interviews with teachers identified as having high levels of teacher efficacy and from analyzing the existing literature on teacher efficacy. Preliminary data analysis resulted in the initial 53 items being reduced to 30 items.

Factor analysis on the 30-item measure confirmed the existence of two dimensions of teacher efficacy. Gibson and Dembo (1984) called the first factor *personal teaching efficacy*, assumed to reflect self-efficacy, and the second factor *general teaching efficacy*, assumed to reflect outcome expectancy. Studies of preservice and inservice teachers have indicated these two factors explain from 18% to 30% of the variance between teachers.

According to Tschannen-Moran, Woolfork Hoy and Hoy (1998), further research on the TES has revealed some inconsistencies in how items load on the two factors. Sixteen of the items load uniquely on one factor only and many researchers have chosen to administer only these items. Additional concerns have been raised regarding whether the general teaching efficacy actually reflects Bandura's outcome expectancy. Regardless, TES has "continued to serve as a reference point for the creation of several other teacher efficacy instruments" (Rohs, 2007).

Teacher efficacy has been defined as being both context and subject-matter specific and, as a result, some researchers believe that many of the global efficacy instruments overlook the specific teaching context (Tschannen-Moran, Woolfork Hoy & Hoy, 1998). Riggs and Enochs (1990) point out that teacher efficacy studies have tended to focus on general teacher efficacy beliefs when, especially for elementary teachers, a subject specific instrument would be more informative. They indicate that teacher efficacy beliefs are dependent on the specific teaching situation and a teacher's overall level of teacher efficacy may not reflect his or her beliefs about their ability to effectively teach a particular subject.

As a result, Riggs and Enochs modified items on the TES (Gibson & Dembo, 1984) to reflect an elementary science setting. The resulting instrument was named the Science Teaching Efficacy Beliefs Instrument (STEBI) and included two scales to measure personal science teaching efficacy and science teaching outcome expectancy. Factor analysis of the items resulted in a final measure with 25 items. In addition, a version for preservice teachers, the STEBI-B, has been developed with the wording of the items indicating future behavior. Riggs and Enochs' (1990) measure has been utilized by several science researchers and has been adapted to measure specific subjects in science (Ross, 1994).

The instrument used in this study, the Mathematics Teaching Efficacy Belief Instrument (MTEBI, Enochs, Smith, & Huinker, 2000) for preservice teachers, was created by modifying the STEBI-B. MTEBI consists of 21 items designed to measure both personal mathematics teacher efficacy (PMTE) and mathematics teaching outcome expectancy (MTOE). Originally, the MTEBI consisted of 23 items but subsequent analysis resulted in two items being omitted. In the final version, 13 of the 21 items measure PMTE and 8 of the items measure MTOE.

### **Teaching Efficacy – Findings and Implications**

Research has demonstrated a strong relationship between teaching efficacy and a variety of factors of interest to educators. Classroom environment, student achievement, teaching behaviors, and professional commitment are examples of such factors. Research

regarding each of these factors indicates the importance of considering the teaching efficacy of both preservice and inservice teachers.

The establishment of a productive learning environment is believed by many to be an important consideration for educators. A study by Ashton, Webb, and Doda (1983) found that teachers with high personal teaching efficacy were more likely to establish and maintain warm, accepting classroom climates. These teachers worked to build positive relationships with their low-achieving students while teachers with low efficacy tended to sort classes according to student ability and give preference to high-ability students. The teachers' use of harsh control tactics was found to be negatively related to their sense of efficacy. Additionally, teachers with high levels of teacher efficacy tended to create classroom environments that are responsive to the needs of students and to use and get better results from using small cooperative groups (Brand & Wilkins, 2007, Ashton & Webb, 1986; Gibson and Dembo, 1984). Pajares (1992) found high levels of teacher efficacy were related to the creation of classrooms where rigor and intellectual challenge are accompanied by emotional support and encouragement.

Many teacher behaviors have been found to be related to efficacy. For example, Hoy and Woolfork (1990) found that teachers with high levels of personal teaching efficacy tend to spend more time planning, designing, and organizing what they teach. They are more open to new ideas and more likely to persist through setbacks and times of difficulties. High personal teaching efficacy has been shown to be related to the use of a wider variety of teaching strategies, the use of more student-centered teaching strategies,

and a willingness to take risks in the classroom (Swars, 2005; Riggs & Enoch , 1990).

Teachers with high levels of both personal and general teaching efficacy are less likely to criticize a student for an incorrect response and more likely to persist with a student experiencing failure (Gibson and Dembo, 1984).

In a study of 19 special education teachers, Allinder (1995) measured the teachers on both dimensions of teacher efficacy as well as on several factors regarding implementation of formative evaluation in the classroom. The findings of this study indicated that teachers with high personal teaching efficacy and high teaching outcome expectancy set goals for their students that were more ambitious than those teachers with lower levels of efficacy. She suggests that this willingness of teachers to set and to strive for higher goals for their students may reflect a belief in their ability to teach students and a belief that students can benefit from what education can offer.

Inadequate mathematical background knowledge and practical experiences contribute to low teacher efficacy according to Chang and Wu (2007). Their study of beginning teachers with varying levels of mathematical backgrounds found that teachers with low personal teaching efficacy tended to utilize insufficient instructional strategies and relied on lecturing and other teacher-centered methods while teaching. These teachers made mistakes in the teaching process, demonstrated poor student-teacher interactions, and had difficulty posing and answering questions. In addition, teachers with low scores on both personal and general teaching efficacy tend to be flustered by interruptions to their

routines while teachers with higher levels of efficacy are better at conducting whole group discussions and at managing multiple small groups (Gibson & Dembo, 1984).

In reviewing research regarding teacher efficacy, Ross (1994) found that one of the most consistently reported findings is that “teachers with higher teaching efficacy are more willing and likely to implement new instructional programs” (p. 20). One such finding indicates that high levels of teaching efficacy are related to more positive attitudes regarding curriculum implementation (Guskey, 1988). Similarly, Ghaith and Yaghi (1996) investigated the relationship among teaching experience, efficacy, and attitude toward the implementation of instructional innovation. Their results indicated that experience was negatively correlated, personal teaching efficacy was positively correlated, and general teaching efficacy was not correlated with teachers’ attitude toward implementing new instructional practices.

General teaching efficacy and personal teaching efficacy were the two strongest predictors of teaching commitment according to the results of a study by Coladarci (1992). The study of 170 teachers found that greater teaching commitment tended to be expressed by those teachers with higher levels of both general and personal teacher efficacy. Other factors such as teacher-student ratio, school climate and gender also predicted commitment to teaching.

Teacher efficacy has also been found to be related to student achievement, and in general, teachers with a high level of personal teaching efficacy tend to have students with higher levels of achievement (Brand & Wilkins, 2007). This may be partly due to the fact

that teachers with higher levels of teaching efficacy are more likely to maintain high standards, concentrate on academic instruction, and monitor on-task behavior than are those teachers with a lower sense of efficacy (Ashton, Webb, & Doda, 1983). Studies have found that achievement on standardized tests is related to teacher efficacy with students of teachers with high levels of efficacy scoring higher (Ross, 1992; Moore & Esselman, 1992; Watson, 1991).

Student attitudes have also been shown to be related to teaching efficacy. Woolfork, Rosoff, and Hoy (1990) found that teachers with stronger general teaching efficacy tend to have students with a greater degree of interest in school. These students had a more positive attitude towards school and the subject being taught. In addition, higher levels of personal teaching efficacy have been found to be related to more positive teacher evaluations by students.

### **Implications for Mathematics Teacher Education**

The findings from studies of teacher efficacy have created an interest in learning more about how teaching efficacy develops. Consequently, studies have been conducted to investigate the relationship between teaching efficacy and the beliefs and actions of preservice teachers. Prospective teachers with low levels of personal teaching efficacy tend to rely on “strict classroom regulations, extrinsic rewards, and punishments to make students study” and to have a less optimistic view of students’ motivation (Hoy, 2000). In a study of student teachers, Saklofske, Michaluk, and Randhawa (1988) found that student teachers with higher personal teaching efficacy tended to be rated more positively on

lesson presentations, classroom management, and questioning by their supervising teachers.

Hoy (2004) suggests that Bandura's theory implies that efficacy beliefs are most malleable early in learning. Therefore, according to Hoy, the first few years of teacher development, including preservice education, are critical to the long-term development of teaching efficacy. Once efficacy beliefs are established they are very resistant to change but experiences at the university level can impact efficacy. Hoy and Woolfork (1990) found that personal teacher self-efficacy increases during university coursework and continues to increase during student teaching. On the other hand, teaching outcome efficacy increases during college coursework but declines during student teaching. The researchers suggest this is because preservice teachers often hold an inflated, optimistic view of what teachers can accomplish. A slightly different finding by Swars, Hart, Smith, Smith, and Tolar (2007) indicates that PMTE increases during student teaching while MTOE remained stable.

The development of mathematics teaching efficacy is of particular interest because research has indicated preservice teachers are concerned about their ability to effectively teach mathematics (Buss, 2010; Swars, 2005; Swars, Daane, & Giesen, 2006). In an attempt to discover how beliefs about teaching mathematics and mathematics teacher self-efficacy are related, Swars, Hart, Smith, Smith, and Tolar (2007) studied preservice teachers over a two course mathematics methods sequence. The researchers found that throughout the program, students with higher levels of PMTE have more cognitively-

oriented beliefs about the teaching and learning of mathematics. A similar result was found for those preservice teachers with high levels of MTOE.

A study by Swars, Danne, and Giesen (2006) suggests that mathematics anxiety, in general, has a negative impact on a preservice teacher's belief in his or her ability to effectively teach mathematics. The prospective teachers with low self-efficacy and high mathematics anxiety emphasized negative previous experiences in school mathematics. For example, in interviews with the researcher, preservice teachers with low mathematics teaching self-efficacy mentioned timed tests and pop quizzes which imply a focus on procedural mathematics. Conversely, preservice teachers with high levels of self-efficacy focused on experiences that implied processes such as problem-solving, reasoning, and communication. The researchers note that high levels of mathematics anxiety did not adversely affect the preservice teachers' belief that effective teaching can result in student learning of mathematics. An interesting related idea is expressed by Cruikshank and Sheffield (1992) who state that they are unconvinced that elementary students suffer from math anxiety. Rather, teachers who fail to implement positive practices cause students to develop this anxiety.

Swars, Daane, and Giesen (2006) state that previous studies have indicated that mathematics methods courses are effective in reducing mathematics anxiety and building mathematics teacher efficacy among elementary preservice teachers. They suggest that consideration of Bandura's sources of efficacy in developing preservice courses may be beneficial in increasing teacher efficacy and reducing mathematics anxiety. Such courses

should “allow preservice teachers to have mastery experiences through actual mathematics teaching experiences as well as vicarious experiences of observing role models teach mathematics”(p. 313). Bandura assertion that emotional states must be addressed in order to build efficacy indicates the importance of reducing mathematics anxiety in preservice mathematics methods courses. Preservice teachers, therefore, need experiences within methods course that address their past experiences with mathematics.

### **Previous Research**

As this literature review shows, a great deal of research has been conducted regarding self-efficacy and teacher efficacy in general, but less research has been content area specific. In addition, the research base for mathematical knowledge for teaching continues to grow. However, little research has been conducted to explore how these two constructs are related.

A 2007 study by Swars, Hart, Smith, Smith, and Tolar investigated the mathematics beliefs and content knowledge of elementary preservice teachers. Participants were measured on mathematics beliefs, mathematics teacher efficacy, and mathematical knowledge for teaching. The instruments used by these researchers are the same as those proposed for use in this study. Analysis of the data showed no significant relationships among personal mathematics teacher efficacy, mathematics teacher outcome expectancy, and specialized content knowledge. Interestingly, although the instrument used to measure specialized content knowledge also measures common content knowledge, these researchers chose not to address this in the study. In addition, the researchers measured

participants on all strands of mathematics and only assessed the preservice teachers at the end of student teaching.

The proposed study will build on the findings of this research and contribute new knowledge in a variety of ways. First, a measure of common content knowledge will be administered so that common content knowledge may be statistically controlled. Second, this study will focus only on number and operations since this is a primary focus of elementary mathematics. Finally, the proposed study will examine growth in specialized content knowledge during a preservice mathematics methods course.

### **Conclusion**

This chapter presents a review of the literature that forms the foundation for this proposed study. The review began with a description of the disconnect between the vision of quality mathematics education and the reality of mathematics education in our elementary schools. The content knowledge and beliefs of elementary teachers and the impact of these on the mathematics learning of our students provides a rationale for this study. Shulman's work on pedagogical content knowledge led to the idea of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) and a discussion of these presents one theoretical context for this study. A discussion of Bandura's conceptualization of self-efficacy, a part of his social cognitive theory, provides another theoretical context for this study. The chapter summarizes literature related to measures of these constructs, the relationships between these constructs and teacher behaviors, and implications for student achievement.

## CHAPTER 3

### METHODOLOGY

This quantitative study was conducted to explore the relationship between the mathematics teacher efficacy and growth in mathematical knowledge for teaching of preservice teachers during a five-hour university mathematics methods/content course sequence. Both teacher efficacy and mathematical knowledge for teaching are multidimensional with each construct playing a role in teacher behaviors. An individual's belief in his or her ability to be an effective teacher as well as his or her belief that effective teaching results in learning are both components of teacher efficacy. Mathematical knowledge for teaching involves constructs of both content knowledge and pedagogical knowledge with each of these constructs being multidimensional as well. The mathematical content knowledge needed by teachers includes both the common content knowledge needed by all and the specialized content knowledge needed only by teachers. Specialized content knowledge and common content knowledge are distinct but related aspects of knowledge.

As mathematics teacher educators work to increase the level of specialized content knowledge of preservice elementary teachers, factors impacting this knowledge are of interest with teacher efficacy being one such potential factor. Therefore, this study was designed to answer the following research questions: (1) Does the specialized mathematics content knowledge of preservice elementary teachers increase during a university mathematics methods/content course?; (2) Is there a relationship between preservice

elementary teachers' sense of personal mathematics teacher efficacy (PMTE) and growth in specialized mathematics content knowledge during a university mathematics methods/content courses?; (3) Is there a relationship between preservice elementary teachers' sense of mathematics teaching outcome expectancy (MTOE) and growth in specialized mathematics content knowledge during a university mathematics methods/content courses?; (4) Does the relationship between PMTE and growth in specialized content knowledge during a university mathematics methods/content courses vary as a function of common content knowledge?; (5) Does the relationship between MTOE and growth in specialized content knowledge during a university mathematics methods/content course vary as a function of common content knowledge? This chapter describes the sample, instrumentation, data collection, and data analysis plan for the study.

### **Participants**

The participants in this study were elementary education majors enrolled in a two-course mathematics content and methods sequence at a mid-sized, mid-western university located in a small city. Both courses are intended to be taken during the same semester for a total of five semester hours of credit. The first course in the sequence is taught during the first six weeks of the semester and is designed to provide instruction on the content and methods for teaching the mathematics that is common to the elementary school. Specifically, the course is intended to develop the basic concepts, skills, and techniques for teaching number, number theory, and fractions with a focus on developing models for teaching the appropriate mathematical content, relevant learning theories, and alternative

teaching strategies. The second course in the sequence, taught during the last ten weeks of the semester, extends work on number and operation to include all rational numbers and also includes the geometry, probability and statistics concepts commonly taught in the elementary school. Throughout this dissertation, this two course sequence is referred to as a “university mathematics methods/content course”. Course syllabi may be found in Appendix B. All potential participants had the same instructor for both courses.

Data were collected across a two semester period from two different groups of students. All students enrolled in these two courses during one of the two semesters of the study were invited to participate. A total of 130 students were invited to participate in this study. As was assumed, some students chose not to participate and others were lost to attrition; however, an accessible population of 130 ensured an adequate sample for this study.

As is typical of elementary preservice teachers at the university being studied, the sample was primarily Caucasian and female. The majority of the students were sophomores or juniors, and all had successfully completed a general studies mathematics survey course at the university being studied or a comparable course at another institution. Additional demographic data are found in Chapter 4. Quantitative data were collected from this sample to examine the relationship between mathematics teacher self-efficacy and growth in specialized content knowledge. Results of this data collection and analysis may be found in Chapter 4.

## **Procedures**

The researcher provided information about the study to potential participants during the first week of the semester. This information was provided both verbally and in writing, and was presented during a regular class meeting. The purpose of the study, procedures to be utilized, description of the instruments to be used, expected time commitment, potential benefits of the study, and any risks of the study were included in the information provided to potential participants. In addition, measures taken to protect confidentiality were described. Potential participants were informed that participation was voluntary and that a participant was free to choose to not complete any measure or to withdraw from the study at any time. Finally, potential participants were assured that choosing not to participate in the study would in no way affect grades in the mathematics methods/content course. Potential participants were informed that administration of the first instrument would occur during the class meeting immediately following the day that initial information was provided and should a student choose not to participate, he or she should not attend class the day of instrument administration.

On the day that initial information was provided, potential participants were given two copies of a consent document, one to keep and one to submit. The submitted consent documents were collected by an assistant to the researcher on the day the instrument was first administered. The assistant sealed the forms in an envelope to ensure the confidentiality of the participants of the study. Copies of the informational letter and the consent document may be found in Appendix C.

Participants were asked to complete an instrument during the first week of the semester and again two weeks prior to the end of the semester. Both instruments were administered during regular class times by an assistant to the researcher and the researcher was not present during administration of either instrument. The instrument used at the beginning of the semester included demographic questions, questions concerning mathematics teacher efficacy, questions designed to measure common mathematical content knowledge, and items measuring specialized mathematical content knowledge. Participants needed approximately 45 minutes to complete this instrument. The instrument used at the end of the semester included questions designed to measure mathematics teacher efficacy, common mathematics content knowledge, and specialized content knowledge. Participants needed approximately 30 minutes to complete this posttest. The items used for the posttest were identical to the items used in the pretest. The instrument is described fully later in this chapter and copy of the instrument used in the study may be found in Appendix A.

Participation in the study was voluntary, and codes were used to protect the identity of the participants. The codes were created by the participants and consisted of the participant's mother's maiden name and city of birth, information unknown to the researcher. Each participant completing the survey received a \$5 food coupon as a token of appreciation for participation; participants received one coupon upon completion of the pretest and another coupon upon completion of the posttest. Because of the risk that

negative prior experiences in mathematics courses might influence potential participants, the researcher believed such measures were needed to ensure a sample of adequate size.

Following completion of the data collection, data analysis was conducted to determine the significance of results. An explanation of this analysis is found later in this chapter.

### **Consideration of Ethical Concerns**

Since the participants in this study were students of the researcher, careful consideration was given to addressing ethical concerns. As stated above, participants in this study used a code consisting of the participant's mother's maiden name and city of birth. All measures were administered by an assistant to the researcher, the researcher was not present during administration, and no list of names of participants was created. To further protect the identity of individual students, in presenting the results of this study all data are reported as aggregates. In addition, the name of the university the participants attend will not be given and will be referred to as a mid-sized, small city, mid-western university (MSCMU).

A research proposal for this study was approved by the Human Subjects Review Committee of MSCMU and a proposal was approved by the Social Sciences Institutional Review Board (SSIRB) at the University of Missouri-Kansas City. Approval documents are included in Appendix C of this dissertation.

## **Pilot Study**

Prior to the start of this study, a pilot study using the instruments proposed in this study was conducted at the same university of this study. Knowledge gained from the pilot study was utilized to inform the development of this study. For example, apprehension expressed by some participants regarding participation in a study involving mathematics indicated the need for extensive information being provided to participants as well as the use of an incentive for participation. In addition, based on the pilot study, it was determined that the measures used were of an appropriate length and the time needed to complete them was not extreme. Finally, in the pilot study, grades received by participants in previously completed general studies mathematics courses were used as a measure of common content knowledge. Since much of the mathematics content from these courses does not reflect the mathematics content taught in the elementary grades, the researcher determined that these grades were not satisfactory as a measure of common content knowledge for this study. As a result, an instrument utilized by a nearby university to measure content knowledge in mathematics methods/content courses was obtained for use in the proposed study. This instrument is described later in this chapter.

## **Data Measurement and Collection**

### **Instruments**

Items from each of the instruments described below were combined into a single instrument administered to participants in one session at the beginning of the semester and one session at the end of the semester. The instrument used at the beginning of the

semester included demographic items, common content knowledge items, teaching efficacy items, and items designed to measure mathematical knowledge for teaching. The instrument used at the end of the semester included items designed to measure mathematics teacher efficacy, common mathematical knowledge, and mathematical knowledge for teaching. Since demographic information did not need to be collected again, the posttest required a shorter amount of time to complete than the pretest.

Six different forms of the instrument were created and administered. As described above, this instrument is composed of items taken from four individual measures; each of these individual measures is discussed below. The first section of each of the six forms contained the same set of demographic questions. This section is labeled Section A in the form of the instrument included in Appendix A. The remaining three sections are designed to measure mathematics teacher efficacy, common mathematics content knowledge, and specialized mathematics content knowledge. These sections are labeled as sections B, C, and D respectively in the form of the instrument included in Appendix A. Items used to measure specialized content knowledge (section D in the instrument in the appendix) are copyrighted. Permission to use these items is granted to researchers who complete training, but these items may not be included in papers or presentations so that testing integrity is not compromised. However, released items may be included. Section D of the instrument in Appendix A includes examples of these released items to provide an illustration of the types of items participants were asked to complete. To minimize the impact of testing fatigue on any particular variable, these three sections described above were included in six

different orders for the six forms of the instrument. The six forms were distributed randomly to participants.

**Demographic Instrument.** In this section, participants were asked to provide their gender, class designation (freshman, sophomore, junior, senior), and classification of high school attended (rural, suburban, urban). They were also asked to indicate the mathematics courses they had completed while in high school.

**Mathematics Teaching Efficacy Beliefs Instrument (Section B of instrument in Appendix A).** The Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) was administered to measure the level of mathematics teacher efficacy of the participants. Created by Enochs, Smith, and Huinker (2000), MTEBI is a modification of the Science Teaching Efficacy Beliefs Instrument (STEBI-B, Riggs and Enochs, 1990) for preservice teachers. STEBI-B, and thus MTEBI, is a modification of the Teacher Efficacy Scale created by Gibson and Dembo (1984).

The MTEBI is a 21-item self-reporting instrument with two significant subscales. The first subscale, the personal mathematics teacher efficacy (PMTE) subscale, has 13 items while the second subscale, mathematics teaching outcome expectancy (MTOE) subscale, has eight items. Each item has five response categories (strongly agree, agree, uncertain, disagree, strongly disagree) and scores on each item range from one to five points. This means that assuming all items are answered, total scores on the PMTE subscale range from 13 to 65 points while total scores on the MTOE subscale range from 8 to 40 points.

Enochs, Smith, and Huinker (2000), creators of MTEBI, sampled 324 preservice elementary teachers in a study designed to determine the reliability and validity of the instrument. Reliability analysis produced an alpha coefficient of .88 for the PMTE subscale of the MTEBI while the MTOE subscale showed an alpha coefficient of .75. Confirmatory factor analysis (CFA), a process more rigorous than exploratory factor analysis (Enochs, Smith, & Huinker, 2000), was used to examine the validity of MTEBI. Unlike exploratory factor analysis that seeks to determine the number of factors needed to explain relationships in a set of items, CFA instead is based on an expected factor structure and seeks to confirm its presence (Enochs, Smith, & Huinker, 2000). CFA indicated the PMTE and MTOE scales are independent thus adding to the construct validity of the MTEBI. Therefore, the creators conclude that MTEBI is a valid and reliable assessment of personal mathematics teacher efficacy and mathematics teacher outcome expectancy (Enochs, Smith, & Huinker, 2000).

**Common Content Knowledge Instrument (Section C of instrument in Appendix A).** To assess participants' level of common content knowledge, items were selected from a test used by a nearby university located in a large city to determine if preservice teachers possess sufficient content knowledge in the area of number and operations to test out of the required Numbers and Operations course. This test has been used at that university for more than ten years and is felt by professors of mathematics education at the university to be an effective instrument to measure common content

knowledge of number and operations; thus, it has face validity. The researcher could not correlate scores on the test with course grades because test data were not archived.

**Learning Mathematics for Teaching: Measures of Mathematical Knowledge for Teaching (Section D of instrument in Appendix A).** The Learning Mathematics for Teaching (LMT) instrument is designed to measure teachers' mathematical content knowledge including both common content knowledge and specialized content knowledge. This instrument reflects the work of Deborah Ball and her colleagues at the University of Michigan. The instrument consists of 14 mathematical tasks that reflect situations teachers may encounter in the classroom such as assessing students' work, representing mathematical ideas, and explaining mathematical rules and procedures (Swars, Hart, Smith, Smith, & Tolar, 2007). While scales are available for several different strands of mathematics, only items from the number and operation scale were included on the instrument to be used in this study. An analysis of reliability, conducted through the testing of 3 parallel forms of the instrument with a total sample of 1552 participants, indicated alpha coefficients ranging from 0.81 to 0.84 for the number and operations scale on the three forms (Hill, Schilling, & Ball, 2004). Validity of these measures was established by the creators of the instrument and the process used to establish validity is described fully in Chapter 2. They comment, "Our elemental assumption, that these measures represent teachers' mathematical knowledge for teaching, is supported by evidence for the content knowledge items" (Hill, Dean, & Goffney, 2007, p. 92).

## **Data Analysis Plan**

Multiple regression analyses were used to investigate the hypotheses framing this study. These analyses were used to determine if growth in specialized mathematics content knowledge experienced by preservice elementary teachers could be predicted by their level of mathematics teacher efficacy or by one of its subscales.

### **Preliminary Data Analysis**

When data collection was complete, preliminary data analysis was used to determine if assumptions for multiple regression were met. A missing data analysis was conducted and one case was eliminated because the participant chose not to complete one of the instruments. Histograms were created and utilized to determine if the assumption of normal distribution of data for each variable was met, and each set of data was examined for outliers. Scatter plots were created to examine the relationships between pairs of variables and to determine if the relationships were linear as assumed in multiple regression. In addition, correlations between variables were calculated to determine if multicollinearity was an issue. Descriptive statistics were used to characterize the sample and include sample size, means of each measure, and standard deviations for each measure. Results of this preliminary data analysis may be found in Chapter 4.

Since participants were enrolled in five different sections of the course being studied, data were examined by course section to determine if significant differences existed among course sections. In addition, since six different forms of the instruments were administered to reduce testing fatigue on any one variable, data were examined by

pretest form and posstest form to determine if significant differences existed due to the instrument form completed. Results of this analysis may be found in Chapter 4.

### **Data Analysis**

Following preliminary data analysis, each research question of the study was analyzed. Description of the analyses used follow and results of these analyses may be found in Chapter 4.

**Research Question One.** The first research question was: Does the specialized mathematical content knowledge of preservice elementary teachers increase during a university mathematics methods/content course? To examine this question, a t-test on the pre- and post-test scores of participants on the specialized mathematics content knowledge items was conducted.

**Research Question Two.** The second research question was: Is there a relationship between preservice elementary teachers' sense of personal mathematics teacher efficacy (PMTE) and growth in specialized mathematics content knowledge during a university mathematics methods course? Multiple regression analysis was used to test this hypothesis with scores on the section of the pretest designed to measure specialized content knowledge being entered first as a statistical control. Scores on the PMTE subscale of the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) were used as the independent variable and scores on the section of the posttest designed to measure specialized mathematical content knowledge were the dependent variable.

**Research Question Three.** The third research question of this study was: Is there a relationship between preservice elementary teachers' sense of mathematics teaching outcome expectancy (MTOE) and growth in specialized mathematics content knowledge during a university mathematics methods course? Multiple regression analysis was used to test this hypothesis with scores on the section of the pretest designed to measure specialized content knowledge being entered first as a statistical control. Scores on the MTOE subscale of the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) were used as the independent variable and scores on the section of the posttest designed to measure mathematical content knowledge were the dependent variable.

**Research Question Four.** The fourth research question of this study was: Does the relationship between PMTE and growth in specialized content knowledge (SCK) during a university mathematics methods course vary as a function of common content knowledge? To determine if common mathematics content knowledge and PMTE interact to predict specialized mathematics content knowledge, the predictor variables were centered and an interaction term was computed. Multiple regression analysis was performed with pretest specialized content knowledge, the centered variables, and the interaction term used as predictors, and with posttest scores on specialized mathematical content knowledge used as the dependent variable.

**Research Question Five.** The fifth research question of this study was: Does the relationship between MTOE and growth in specialized content knowledge (SCK) during a university mathematics methods course vary as a function of common content knowledge?

To determine if common mathematics content knowledge and MTOE interact to predict specialized mathematics content knowledge, the predictor variables were centered and an interaction term was computed. Multiple regression analysis was performed with pretest specialized content knowledge, the centered variables, and the interaction term used as predictors, and with posttest scores on specialized mathematical content knowledge used as the dependent variable.

## CHAPTER 4

### RESULTS

The purpose of this study was to investigate the relationship between the mathematics teacher efficacy of preservice elementary teachers and their growth in specialized mathematics content knowledge. The study was conducted over a two-semester period of time at a mid-western university in a small city with students enrolled in a five-hour mathematics methods/content course. During each semester, data were collected during the first week of the semester as well as at the end of the semester. The instrument used in the study as both a pretest and posttest included sections designed to measure common mathematics content knowledge, mathematics teacher efficacy, and specialized mathematics content knowledge. Demographic data were collected at the beginning of each semester.

Five sections of the mathematics methods/content course investigated in this study were offered during the two semesters of the study. Three of these sections were taught during the spring 2010 semester and the remaining two sections were taught during the fall 2010 semester. A total of 130 preservice elementary teachers were invited to participate in this study and of these, 101 participants completed the study. The number of participants from each section ranged from 8 to 29 (See Table 1). The number of participants from section 2 is quite low when compared to that of other sections; however, the total enrollment of this section was twelve students so the rate of participation is similar to that

of other sections. Each of the five sections was taught by the researcher, and all sections were conducted similarly.

Table 1  
*Number of Preservice Elementary Teacher Participants  
by Section (N = 101)*

Section	Semester	Number of participants
1	Spring 2010	23
2	Spring 2010	8
3	Spring 2010	21
4	Fall 2010	20
5	Fall 2010	29

Following data collection, data analyses were conducted to investigate the following research questions:

- (1) Does the specialized mathematical content knowledge of preservice elementary teachers increase during a university mathematics methods/content course?
- (2) Is there a relationship between preservice elementary teachers' sense of personal mathematics teacher efficacy (PMTE) and growth in specialized mathematics content knowledge during a university mathematics methods course?
- (3) Is there a relationship between preservice elementary teachers' sense of

mathematics teaching outcome expectancy (MTOE) and growth in specialized mathematics content knowledge during a university mathematics methods course?

(4) Does the relationship between PMTE and growth in specialized content knowledge during a university mathematics methods course vary as a function of common content knowledge?

(5) Does the relationship between MTOE and growth in specialized content knowledge during a university mathematics methods course vary as a function of common content knowledge?

The remainder of this chapter is organized in three sections. The first section provides demographic data. Descriptive statistics are provided in the second section. The third and final section gives the results of the hypotheses testing.

### **Demographics**

Demographic data on a variety of variables including gender, location of high school attended (rural, suburban, urban), status (freshman, sophomore, junior, senior), and number of completed high school mathematics courses past algebra I were collected. These data were analyzed to paint a picture of the participants. As is typical of elementary education majors at the university being studied, the participants, were primarily female with 95 of the 101 participants (94.1%) being female. Forty-nine of the 101 participants (48.5%) were juniors, while 10 (9.9%) were freshmen, 20 (19.8%) were sophomores and 22 (21.8%) were seniors. Most of the participants attended either rural (41.6%) or

suburban (50.5%) high schools and a majority of the participants (79.2%) completed at least two mathematics courses beyond algebra I while in high school (See Table 2). All of the participants had previously completed a general studies mathematics course at the university being studied or an equivalent course at another institution. As is typical of the university being studied, the majority of the participants were Caucasian. Ethnicity information was not gathered in the demographic section of the instrument because of concern that the very small number of non-Caucasian students might make it possible to match a completed instrument to an individual student.

Table 2

*Demographics (N = 101)*

	Section 1		Section 2		Section 3		Section 4		Section 5		Total	
	<i>N</i>	%	<i>N</i>	%	<i>N</i>	%	<i>N</i>	%	<i>N</i>	%	<i>N</i>	%
Gender												
Male	2	8.7	0	0.0	0	0.0	1	5.0	3	10.3	6	5.9
Female	21	91.3	8	100.0	21	100.0	19	95.0	26	89.7	95	94.1
Number of high school math courses past algebra I												
0	1	4.3	0	0.0	1	4.8	4	20.0	1	3.5	7	6.9
1	3	13.1	1	12.5	4	19.0	4	20.0	2	6.9	14	13.9
2	6	26.1	2	25.0	6	28.6	5	25.0	7	24.1	26	25.7
3	7	30.4	2	25.0	6	28.6	3	15.0	11	37.9	29	28.7
4	6	26.1	3	37.5	4	19.0	4	20.0	8	27.6	25	24.8
Location of high school												
Rural	9	39.1	4	50.0	7	33.3	9	45.0	13	44.8	42	41.6
Suburban	11	47.8	3	37.5	13	61.9	9	45.0	15	51.7	51	50.5
Urban	3	13.1	1	12.5	1	4.8	2	10.0	1	3.5	8	7.9
Status												
Freshman	6	26.1	1	12.5	0	0.0	2	10.0	1	3.4	10	9.9
Sophomore	7	30.4	3	37.5	0	0.0	2	10.0	8	27.6	20	19.8
Junior	7	30.4	3	37.5	11	52.4	12	60.0	16	55.2	49	48.5
Senior	3	13.1	1	12.5	10	47.6	4	20.0	4	13.8	22	21.8

## Descriptive Statistics

The data were examined to determine the normality of distribution of scores on each of the variables for which data were collected. First, the data for all participants were examined and then the data was aggregated by class section. As seen in Table 3, the data is acceptable in terms of skewness (skewness < +/-2.0) and kurtosis (kurtosis < +/-2.0)

Table 3

<i>Means, Standard Deviations, Skewness, and Kurtosis for Assessed Variables (N = 101)</i>				
	Mean	SD	Skewness	Kurtosis
Common Content Knowledge				
Pretest	13.39 <sup>1</sup>	3.59	-0.39	-0.49
Posttest	17.55 <sup>1</sup>	2.61	-1.04	1.46
Specialized Content Knowledge				
Pretest	10.06 <sup>2</sup>	3.66	0.38	0.12
Posttest	12.76 <sup>2</sup>	4.10	0.09	-0.62
Personal Mathematics Teacher Efficacy				
Pretest	45.92 <sup>3</sup>	6.35	-0.12	1.48
Posttest	54.51 <sup>3</sup>	5.40	-0.65	1.06
Mathematics Teaching Outcome Expectancy				
Pretest	29.46 <sup>4</sup>	3.66	0.46	0.19
Posttest	31.41 <sup>4</sup>	3.77	-0.15	0.52

*Note: Higher scores represent greater levels of the variable.*

<sup>1</sup>Maximum score = 22. <sup>2</sup>Maximum score = 26. <sup>3</sup>Maximum score = 65.

<sup>4</sup>Maximum score = 40.

Examining the data aggregated by class section reveals that in class section 5, the data is slightly kurtotic (kurtosis = 2.09) for the Personal Mathematics Teacher Efficacy posttest variable (See Table 8). Since this value is relatively close to the value used to determine acceptable levels of kurtosis (kurtosis < +/-2) and scores on all other variables are acceptable, the researcher decided to include the data from class section 5. Scores on

all variables in other class sections were acceptable in terms of kurtosis and skewness (See Tables 4, 5, 6, and 7).

Table 4

*Class Section 1: Means, Standard Deviations, Skewness, and Kurtosis for Assessed Variables (N = 23)*

	Mean	SD	Skewness	Kurtosis
Common Content Knowledge				
Pretest	12.70 <sup>1</sup>	3.35	-0.45	-0.51
Posttest	17.09 <sup>1</sup>	3.03	-1.54	2.51
Specialized Content Knowledge				
Pretest	10.13 <sup>2</sup>	4.39	0.31	-0.87
Posttest	13.43 <sup>2</sup>	4.52	-0.54	-0.31
Personal Mathematics Teacher Efficacy				
Pretest	46.09 <sup>3</sup>	7.11	0.39	0.48
Posttest	53.39 <sup>3</sup>	5.47	-0.44	-0.70
Mathematics Teaching Outcome Expectancy				
Pretest	29.30 <sup>4</sup>	3.14	0.33	-0.36
Posttest	31.09 <sup>4</sup>	3.85	0.46	-0.12

*Note: Higher scores represent greater levels of the variable.*

<sup>1</sup>Maximum score = 22. <sup>2</sup>Maximum score = 26. <sup>3</sup>Maximum score = 65.

<sup>4</sup>Maximum score = 40.

Table 5

*Class Section 2: Means, Standard Deviations, Skewness, and Kurtosis for Assessed Variables (N = 8)*

	Mean	SD	Skewness	Kurtosis
Common Content Knowledge				
Pretest	14.00 <sup>1</sup>	4.75	-0.34	-1.06
Posttest	17.25 <sup>1</sup>	3.65	-0.17	-1.34
Specialized Content Knowledge				
Pretest	10.75 <sup>2</sup>	5.47	0.85	0.43
Posttest	13.75 <sup>2</sup>	4.06	1.20	1.77
Personal Mathematics Teacher Efficacy				
Pretest	45.38 <sup>3</sup>	4.07	0.32	0.20
Posttest	57.25 <sup>3</sup>	5.23	-0.44	0.38
Mathematics Teaching Outcome Expectancy				
Pretest	32.63 <sup>4</sup>	3.02	0.44	0.63
Posttest	32.88 <sup>4</sup>	3.68	1.0	-0.12

*Note: Higher scores represent greater levels of the variable.*

<sup>1</sup>Maximum score = 22. <sup>2</sup>Maximum score = 26. <sup>3</sup>Maximum score = 65.

<sup>4</sup>Maximum score = 40.

Table 6

*Class Section 3: Means, Standard Deviations, Skewness, and Kurtosis for Assessed Variables (N = 21)*

	Mean	SD	Skewness	Kurtosis
Common Content Knowledge				
Pretest	12.29 <sup>1</sup>	3.05	-0.84	0.13
Posttest	17.71 <sup>1</sup>	2.03	-0.17	0.14
Specialized Content Knowledge				
Pretest	9.57 <sup>2</sup>	2.86	0.65	-0.20
Posttest	12.38 <sup>2</sup>	4.07	0.09	-0.61
Personal Mathematics Teacher Efficacy				
Pretest	45.24 <sup>3</sup>	4.56	-0.32	-0.10
Posttest	54.62 <sup>3</sup>	3.63	0.77	0.50
Mathematics Teaching Outcome Expectancy				
Pretest	28.57 <sup>4</sup>	2.77	0.68	1.09
Posttest	30.90 <sup>4</sup>	4.44	-0.52	0.57

*Note: Higher scores represent greater levels of the variable.*

<sup>1</sup>Maximum possible score = 22. <sup>2</sup>Maximum possible score = 26.

<sup>3</sup>Maximum possible score = 65. <sup>4</sup>Maximum possible score = 40.

Table 7

*Class Section 4: Means, Standard Deviations, Skewness, and Kurtosis for Assessed Variables (N = 20)*

	Mean	SD	Skewness	Kurtosis
Common Content Knowledge				
Pretest	14.10 <sup>1</sup>	4.02	-0.84	-0.32
Posttest	17.80 <sup>1</sup>	2.26	-0.82	1.01
Specialized Content Knowledge				
Pretest	10.65 <sup>2</sup>	3.79	0.19	0.22
Posttest	12.60 <sup>2</sup>	4.20	0.52	-0.90
Personal Mathematics Teacher Efficacy				
Pretest	47.35 <sup>3</sup>	6.60	0.58	1.26
Posttest	55.15 <sup>3</sup>	5.30	-0.31	-1.06
Mathematics Teaching Outcome Expectancy				
Pretest	29.25 <sup>3</sup>	5.13	0.50	-0.53
Posttest	31.25 <sup>4</sup>	4.04	-0.49	1.24

*Note: Higher scores represent greater levels of the variable.*

<sup>1</sup>Maximum score = 22. <sup>2</sup>Maximum score = 26. <sup>3</sup>Maximum score = 65.

<sup>4</sup>Maximum score = 40.

Table 8

*Class Section 5: Means, Standard Deviations, Skewness, and Kurtosis for Assessed Variables (N = 29)*

	Mean	SD	Skewness	Kurtosis
Common Content Knowledge				
Pretest	14.07 <sup>1</sup>	3.41	-0.35	-0.29
Posttest	17.72 <sup>1</sup>	2.64	-1.15	1.56
Specialized Content Knowledge				
Pretest	9.76 <sup>2</sup>	3.01	-0.57	0.08
Posttest	12.34 <sup>2</sup>	3.88	0.13	-0.49
Personal Mathematics Teacher Efficacy				
Pretest	45.45 <sup>3</sup>	7.32	-0.90	1.72
Posttest	54.14 <sup>3</sup>	6.45	-1.05	2.09
Mathematics Teaching Outcome Expectancy				
Pretest	29.48 <sup>4</sup>	3.32	0.47	1.01
Posttest	31.72 <sup>4</sup>	3.10	0.2.	0.08

*Note: Higher scores represent greater levels of the variable.*

<sup>1</sup>Maximum score = 22. <sup>2</sup>Maximum score = 26. <sup>3</sup>Maximum score = 65.

<sup>4</sup>Maximum score = 40.

Data from individual participants were examined to determine if any outliers existed. The scores of one participant were found to be more than three standard deviations from the mean on both the Personal Mathematics Teacher Efficacy pretest ( $z = -3.45$ ) and posttest ( $z = -3.62$ ). The score of one participant on the Common Content Knowledge posttest ( $z = -3.66$ ) also exceeded three standard deviations from the mean. Because the scores of both of these participants on other variables were within the acceptable range, the researcher decided to include these data.

Since the data came from participants in five different class sections over two semesters, the data were examined to determine if differences in scores on assessed variables differed by semester or class section. In addition, six different forms of the instrument were used and so determining if the form used resulted in results that differed significantly was also of importance.

The data were first split by semester. Independent sample  $t$  tests were performed on each of the assessed variables to determine if significant differences existed between semesters. As Table 9 indicates, there were no significant differences found between the semesters on any of the assessed variables.

Table 9

*Results of Independent Samples t tests for Assessed Variables by Semester (N = 101)*

	<i>df</i>	<i>t</i>	<i>p(.05)</i>
Common Content Knowledge Pretest	99	1.92	0.97
Common Content Knowledge Posttest	99	-0.75	0.66
Specialized Content Knowledge Pretest	99	-0.17	0.15
Specialized Content Knowledge Posttest	99	0.74	0.72
Personal Mathematics Teacher Efficacy Pretest	99	-0.47	0.35
Personal Mathematics Teacher Efficacy Posttest	99	-0.70	0.36
Mathematics Teaching Outcome Expectancy Pretest	99	0.18	0.90
Mathematics Teaching Outcome Expectancy Posttest	99	-0.32	0.35

The data were next split by class section. One-way Analysis of Variance (ANOVA) was performed to determine if significant differences existed among classes. As Table 10 indicates, there were no significant differences found among the class sections on any of the assessed variables.

Table 10

*Analysis of Variance for Assessed Variables by Class Section (N = 101)*

	<i>df</i>	<i>F</i>	<i>p(0.5)</i>
Common Content Knowledge Pretest			
Between Groups	4	1.237	0.30
Within Groups	96		
Common Content Knowledge Posttest			
Between Groups	4	0.30	0.88
Within Groups	96		
Specialized Content Knowledge Pretest			
Between Groups	4	0.34	0.85
Within Groups	96		
Specialized Content Knowledge Posttest			
Between Groups	4	0.39	0.82
Within Groups	96		
Personal Mathematics Teacher Efficacy Pretest			
Between Groups	4	0.36	0.82
Within Groups	96		
Personal Mathematics Teacher Efficacy Posttest			
Between Groups	4	0.87	0.49
Within Groups	96		
Mathematics Teaching Outcome Expectancy Pretest			
Between Groups	4	1.90	0.12
Within Groups	96		
Mathematics Teaching Outcome Expectancy Posttest			
Between Groups	4	0.49	0.75
Within Groups	96		

Six different forms of the instrument used in this study were used for both the pretest and the posttest. While the sections of each form were identical, the order in which the sections were arranged differed. This was done to reduce the effects of testing fatigue on any one particular variable. Table 11 provides the order of the sections for each form of the pretest and posttest. The Mathematics Teaching Efficacy portion of the instrument included items to assess both Personal Mathematics Teacher Efficacy and Mathematics Teaching Outcome Expectancy.

Table 11

*Order of Sections of Forms of Instrument (N = 101)*

Pretest and Posttest Form	Section 1 <sup>1</sup>	Section 2	Section 3	Section 4
A	Demographics	MTE <sup>2</sup>	SCK <sup>3</sup>	CCK <sup>4</sup>
B	Demographics	CCK	SCK	MTE
C	Demographics	CCK	MTE	SCK
D	Demographics	SCK	MTE	CCK
E	Demographics	SCK	CCK	MTE
F	Demographics	MTE	CCK	SCK

<sup>1</sup>The Posttest did not include a demographics section.

<sup>2</sup>MTE represents Mathematics Teacher Efficacy

<sup>3</sup>SCK represents Specialized Content Knowledge

<sup>4</sup>CCK represents Common Content Knowledge

The data were next split by the pretest form used by the participants and one-way ANOVA was performed to determine if significant differences existed among these forms. As Table 12 indicates, there were no significant differences found among the pretest forms on any of variables assessed during the pretest.

Table 12

*Analysis of Variance for Assessed Variables by Pretest Form (N = 101)*

	<i>df</i>	<i>F</i>	<i>p(.05)</i>
Common Content Knowledge Pretest			
Between Groups	5	0.72	0.62
Within Groups	95		
Specialized Content Knowledge Pretest			
Between Groups	5	0.96	0.45
Within Groups	95		
Personal Mathematics Teacher Efficacy Pretest			
Between Groups	5	1.03	0.41
Within Groups	95		
Mathematics Teaching Outcome Expectancy Pretest			
Between Groups	5	1.16	0.34
Within Groups	95		

To determine if significant differences existed due to the posttest form used by participants, the data were split by the posttest forms and one-way ANOVA was performed. As Table 13 indicates, there were no significant differences found among the pretest forms on any of variables assessed during the pretest.

Table 13

*Analysis of Variance for Assessed Variables by Posttest Form (N = 101)*

	<i>df</i>	<i>F</i>	<i>p(.05)</i>
<b>Common Content Knowledge Posttest</b>			
Between Groups	5	1.37	0.24
Within Groups	95		
<b>Specialized Content Knowledge Posttest</b>			
Between Groups	5	0.50	0.78
Within Groups	95		
<b>Personal Mathematics Teacher Efficacy Posttest</b>			
Between Groups	5	0.28	0.93
Within Groups	95		
<b>Mathematics Teaching Outcome Expectancy Posttest</b>			
Between Groups	5	0.26	0.93
Within Groups	95		

The data were also split using the groups identified through demographic data to determine if differences existed due to any of the demographic variables. First, a one-way ANOVA was performed on the data split by status to determine if significant differences existed among freshmen, sophomores, juniors, and seniors. No significant differences were found as is shown in Table 14.

Table 14

*Analysis of Variance for Assessed Variables by Status (Freshman, Sophomore, Junior, Senior) (N = 101)*

	<i>df</i>	<i>F</i>	<i>p</i> (.05)
Common Content Knowledge Pretest			
Between Groups	3	1.10	0.35
Within Groups	97		
Common Content Knowledge Posttest			
Between Groups	3	1.12	0.32
Within Groups	97		
Specialized Content Knowledge Pretest			
Between Groups	3	1.25	0.29
Within Groups	97		
Specialized Content Knowledge Posttest			
Between Groups	3	1.87	0.14
Within Groups	97		
Personal Mathematics Teacher Efficacy Pretest			
Between Groups	3	1.72	0.17
Within Groups	97		
Personal Mathematics Teacher Efficacy Posttest			
Between Groups	3	0.90	0.44
Within Groups	97		
Mathematics Teaching Outcome Expectancy Pretest			
Between Groups	3	1.31	0.28
Within Groups	97		
Mathematics Teaching Outcome Expectancy Posttest			
Between Groups	3	0.28	0.84
Within Groups	97		

The data were next split according to the location of the high school attended by the participants (rural, suburban, urban). A one-way ANOVA was performed to determine if significant differences existed by location of the high school attended and, as shown in Table 15, no such differences were found. However, the pretest of specialized content knowledge was borderline,  $F(2, 92) = 3.003, p = .054$ .

Table 15

*Analysis of Variance for Assessed Variables by Location of High School Attended (Rural, Suburban, Urban) (N = 101)*

	<i>df</i>	<i>F</i>	<i>p(.05)</i>
Common Content Knowledge Pretest			
Between Groups	2	0.61	0.55
Within Groups	98		
Common Content Knowledge Posttest			
Between Groups	2	0.23	0.80
Within Groups	98		
Specialized Content Knowledge Pretest			
Between Groups	2	3.00	0.05
Within Groups	98		
Specialized Content Knowledge Posttest			
Between Groups	2	2.86	0.06
Within Groups	98		
Personal Mathematics Teacher Efficacy Pretest			
Between Groups	2	0.63	0.53
Within Groups	98		
Personal Mathematics Teacher Efficacy Posttest			
Between Groups	2	0.89	0.41
Within Groups	98		
Mathematics Teaching Outcome Expectancy Pretest			
Between Groups	2	0.68	0.51
Within Groups	98		
Mathematics Teaching Outcome Expectancy Posttest			
Between Groups	2	0.59	0.56
Within Groups	98		

Splitting the data by the number of high school mathematics classes past algebra I completed by the participants and performing a one-way ANOVA on the split data revealed two significant differences and one borderline difference among groups. One significant difference was found in the posttest scores,  $F(4, 96) = 3.972, p = 0.005$ . The second significant difference was found in posttest scores on common content knowledge,  $F(4, 96) = 2.601, p = 0.041$ . The pretest of specialized content knowledge was borderline,  $F(4, 96) = 2.458, p = 0.051$ . The researcher decided to accept these differences. Table 16 provides a complete picture of the results of these analyses. Table 17 provides means and standard deviations for scores on posttest personal mathematics teacher efficacy and posttest common content knowledge by number of high school mathematics classes past algebra I.

Table 16

*Analysis of Variance for Assessed Variables by Number of High School Mathematics Courses Past Algebra I (N = 101)*

	<i>df</i>	<i>F</i>	<i>p</i> (.05)
Common Content Knowledge Pretest			
Between Groups	4	0.93	0.45
Within Groups	96		
Common Content Knowledge Posttest			
Between Groups	4	2.61	0.04*
Within Groups	96		
Specialized Content Knowledge Pretest			
Between Groups	4	2.46	0.05
Within Groups	96		
Specialized Content Knowledge Posttest			
Between Groups	4	1.76	0.14
Within Groups	96		
Personal Mathematics Teacher Efficacy Pretest			
Between Groups	4	1.93	0.11
Within Groups	96		
Personal Mathematics Teacher Efficacy Posttest			
Between Groups	4	3.97	0.01*
Within Groups	96		
Mathematics Teaching Outcome Expectancy Pretest			
Between Groups	4	0.61	0.66
Within Groups	96		
Mathematics Teaching Outcome Expectancy Posttest			
Between Groups	4	0.33	0.86
Within Groups	96		

\*Significant at the 0.05 level.

Table 17

*Means and Standard Deviations for Posttest Personal mathematics teacher efficacy and Posttest Common Content Knowledge by Number of High School Mathematics Courses Past Algebra I (N = 101)*

Number of Courses Past Algebra I	<i>n</i>	Posttest Personal mathematics teacher efficacy <sup>1</sup>	Posttest Common Content Knowledge <sup>2</sup>
		<i>M (SD)</i>	<i>M (SD)</i>
0	7	50.29 (7.74)	17.86 (1.68)
1	14	51.57 (5.29)	16.29 (3.75)
2	26	55.15 (4.57)	17.31 (1.95)
3	29	54.24 (5.16)	17.24 (2.96)
4	25	57.00 (4.61)	18.80 (1.73)

<sup>1</sup>Maximum score possible = 65.

<sup>2</sup>Maximum score possible = 22.

Finally, the data were split by gender to determine if significant differences exist between males and females on the assessed variables. Independent samples *t*-tests indicated a significant difference on the pretest for Mathematics Teaching Outcome Expectancy variable  $t(99) = 2.384, p = 0.019$ . The number of males included in this study is very low ( $n = 6$ ) so conclusions cannot be drawn from this finding. However, further research regarding the impact of gender on Mathematics Teaching Outcome Expectancy would be of interest. Table 18 provides the complete results of these analyses by gender.

Table 18

*Results of Independent Samples t tests for Assessed Variables by Gender*

	<i>df</i>	<i>t</i>	<i>p(.05)</i>
Common Content Knowledge Pretest	99	-0.78	0.44
Common Content Knowledge Posttest	99	1.08	0.28
Specialized Content Knowledge Pretest	99	0.04	0.97
Specialized Content Knowledge Posttest	99	-0.76	0.45
Personal Mathematics Teacher Efficacy Pretest	99	1.03	0.31
Personal Mathematics Teacher Efficacy Posttest	99	-0.30	0.76
Mathematics Teaching Outcome Expectancy Pretest	99	2.38	0.02*
Mathematics Teaching Outcome Expectancy Posttest	99	-0.51	0.61

\*Significant at the 0.05 level.

Data were then analyzed to determine correlations among the variables assessed in the study. Several pairs of variables were significantly correlated as is shown in Table 19; however, the correlations were not so high that multicollinearity, or the redundancy of predictors, is a concern.

Table 19

*Correlations Among Variables (N = 101)*

		2	3	4	5	6	7	8
1.	CCK <sup>1</sup> Pretest	0.47*	0.52*	0.43*	0.19	0.27*	-0.06	0.02
2.	CCK Posttest	--	0.52*	0.42*	0.40*	0.30*	-0.10	-0.02
3.	SCK <sup>2</sup> Pretest		--	0.68*	0.27*	0.30*	-0.01	0.14
4.	SCK Posttest			--	0.19	0.26*	0.05	0.11
5.	PMTE <sup>3</sup> Pretest				--	0.59*	0.13	0.08
6.	PMTE Posttest					--	0.01	0.31*
7.	MTOE <sup>4</sup> Pretest						--	0.46*
8.	MTOE Posttest							--

\*Correlation is significant at the 0.01 level (2-tailed).

<sup>1</sup>CCK represents Common Content Knowledge.

<sup>2</sup>SCK represents Specialized Content Knowledge.

<sup>3</sup>PMTE represents Personal Mathematics Teacher Efficacy.

<sup>4</sup>MTOE represents Mathematics Teaching Outcome Expectancy.

### Hypothesis Testing

**Research Question 1.** The first research question of this study was: Does the specialized mathematical content knowledge of preservice elementary teachers increase during a university mathematics methods/content course?

To test this question, a paired samples *t*-test was conducted on the pre- and posttest scores of participants on the measure of specialized content knowledge. Scores on

specialized content knowledge increased significantly,  $t(100) = 8.664, p = .000, d = 0.924$ , from the pretest ( $M = 10.059, SD = 3.66$ ) to the posttest ( $M = 12.762, SD = 4.099$ ).

Although not part of the research questions of this study, t- tests were performed on the other assessed variables to provide additional information. A significant increase from pretest to posttest was noted in each of these variables. Scores on common content knowledge also showed a significant increase,  $t(100) = 12.746, p = .000, d = 1.342$ , from the pretest ( $M = 13.386, SD = 3.589$ ) to the posttest ( $M = 17.554, SD = 2.606$ ). A significant increase  $t(100) = 15.977, p = .000, d = 1.277$ , was found between the pretest scores ( $M = 45.921, SD = 6.351$ ) and posttest scores ( $M = 54.515, SD = 5.397$ ) of participants for Personal Mathematics Teacher Efficacy. Finally, scores for Mathematics Teaching Outcome Expectancy increased significantly,  $t(100) = 5.030, p = .000, d = 0.489$ , from the pretest ( $M = 19.455, SD = 3.662$ ) to the posttest ( $M = 31.406, SD = 3.771$ ). Table 20 provides a summary of these results.

Table 20

*Paired Sample T-Tests for All Assessed Variables Comparing Pretest and Posttest Scores (N = 101)*

	<i>t</i>	<i>df</i>	<i>p</i> (.05)
Common Content Knowledge	12.75	100	0.00*
Specialized Content Knowledge	8.66	100	0.00*
Personal Mathematics Teacher Efficacy	15.98	100	0.00*
Mathematics Teaching Outcome Expectancy	5.03	100	0.00*

\*Significant at the 0.01 level.

**Research Question 2.** The second research question of this study was: Is there a relationship between preservice elementary teachers' sense of personal mathematics teacher efficacy (PMTE) and growth in specialized mathematics content knowledge (SCK) during a university mathematics methods course?

Multiple regression analysis was used to investigate this question with posttest scores on SCK being the dependent variable. Pretest scores on SCK were entered as a predictor as a means of determining growth in specialized content knowledge. Pretest scores on PMTE were then entered as a predictor variable. The overall regression model to predict posttest scores on SCK using pretest SCK and pretest PMTE was statistically significant,  $F(2, 98) = 41.927, p < .001$ . For this overall model,  $R = 0.679, R^2 = .461$ . In other words, when pretest SCK and pretest PMTE are used as predictors, approximately 46% of the variance in posttest SCK could be explained.

Pretest SCK was significantly predictive of posttest SCK when the variable pretest PMTE was statistically controlled:  $t(98) = 8.789, p < 0.001$ . The positive slope for pretest SCK as a predictor of posttest SCK was 0.756 indicating that there was approximately a three-quarters of a point increase in posttest SCK scores for every increase of one in pretest SCK scores. The squared semi-partial correlation to determine the amount of variance in posttest SCK uniquely predictable from pretest SCK was  $sr^2 = 0.43$  meaning that pretest SCK accounted for approximately 43% of the variance in posttest SCK when pretest PMTE was statistically controlled.

Pretest PMTE was not a significant predictor of posttest SCK when pretest SCK was statistically controlled:  $t(98) = 0.149, p = 0.882$ . Based on the analysis results for the second research question of this study, the null hypothesis is accepted. Complete results for this analysis are found in Table 21.

Table 21

*Results of Standard Multiple Regression to Predict Posttest Specialized Content Knowledge from Pretest Specialized Content Knowledge and Pretest Personal Mathematics Teacher Efficacy (N = 101)*

Variables	$R^2$	Adj. $R^2$	$R$	$F$	$b$	$\beta$	$t$	$sr^2$
Overall model	0.46	0.45	0.68	41.93*				
SCK <sup>1</sup> Pretest					0.76*	0.68	8.79	0.43
PMTE <sup>2</sup> Pretest					0.01	0.01	0.15	0.00

\* $p < .001$

<sup>1</sup>SCK represents Specialized Content Knowledge

<sup>2</sup>PMTE represents Personal Mathematics Teacher Efficacy

**Research Question 3.** The third research question of this study was: Is there a relationship between preservice elementary teachers' sense of mathematics teaching outcome expectancy (MTOE) and growth in specialized mathematics content knowledge (SCK) during a university mathematics methods course?

Multiple regression analysis was used to investigate this question with posttest scores on SCK being the dependent variable. Pretest scores on SCK were entered as a

predictor as a means of determining growth in specialized content knowledge. Pretest scores on MTOE were then entered as a predictor variable. The overall regression model to predict posttest scores on SCK using pretest SCK and pretest MTOE was statistically significant,  $F(2, 98) = 42.375, p < .001$ . For this overall model,  $R = 0.681, R^2 = .464$ . In other words, when pretest SCK and pretest MTOE are used as predictors, approximately 46% of the variance in posttest SCK could be explained.

Pretest SCK was significantly predictive of posttest SCK when the variable pretest MTOE was statistically controlled:  $t(98) = 9.186, p < 0.001$ . The positive slope for pretest SCK as a predictor of posttest SCK was 0.761 indicating that there was approximately a three-quarters of a point increase in posttest SCK scores for every increase of one in pretest SCK scores. The squared semi-partial correlation to determine the amount of variance in posttest SCK uniquely predictable from pretest SCK was  $sr^2 = 0.46$  meaning that pretest SCK accounted for approximately 46% of the variance in posttest SCK when pretest MTOE was statistically controlled.

Pretest MTOE was not a significant predictor of posttest SCK when pretest SCK was statistically controlled:  $t(98) = 0.710, p = 0.479$ . Based on the analysis results for the second research question of this study, the null hypothesis is accepted. Complete results for this analysis are found in Table 22.

Table 22

*Results of Standard Multiple Regression to Predict Posttest Specialized Content Knowledge from Pretest Specialized Content Knowledge and Pretest Mathematics Teaching Outcome Expectancy (N = 101)*

Variables	$R^2$	Adj. $R^2$	$R$	$F$	$b$	$\beta$	$t$	$sr^2$
Overall model	0.46	0.45	0.68	42.38*				
SCK <sup>1</sup> Pretest					0.76*	0.68	9.19	0.46
MTOE <sup>2</sup> Pretest					0.06	0.05	0.71	0.00

\* $p < .001$

<sup>1</sup>SCK represents Specialized Content Knowledge

<sup>2</sup>MTOE represents Mathematics Teaching Outcome Expectancy

**Research Questions 4.** The fourth research question of this study was: Does the relationship between PMTE and growth in specialized content knowledge during a university mathematics methods/content courses vary as a function of common content knowledge?

To determine if common content knowledge (CCK) and personal mathematics teacher efficacy (PMTE) interact to predict growth in specialized content knowledge (SCK), the predictor variables were centered and an interaction term was computed. Multiple regression analysis was used with pretest scores on SCK being entered first as a means of determining growth in SCK. The centered predictor variables were entered next, and the interaction term was entered last.

The overall regression model to predict posttest scores on SCK using pretest SCK, centered pretest CCK, and centered pretest PMTE was statistically significant,  $F(4, 96) =$

21.188,  $p < .001$ . For this overall model,  $R = 0.685$ ,  $R^2 = .469$ . In other words, when pretest SCK, centered pretest CCK, and centered pretest PMTE are used as predictors, approximately 47% of the variance in posttest SCK could be explained.

Including the interaction term did not result in a significant change in the amount of variance explained,  $F_{change}(1, 96) = 0.010$ ,  $p = 0.922$ . The interaction term was not significant,  $t(96) = -0.099$ ,  $p = 0.922$ . Complete results of this analysis are found in Table 23.

Table 23

*Results of Standard Multiple Regression to Predict Posttest Specialized Content Knowledge from Pretest Specialized Content Knowledge, Pretest Personal Mathematics Teaching Efficacy, Pretest Common Content Knowledge Including Interaction Term (N = 101)*

Variables	$R^2$	Adj. $R^2$	$R$	$F$	$b$	$\beta$	$t$	$sr^2$
Overall model	0.47	0.45	0.69	21.19*				
SCK <sup>1</sup> Pretest					0.70*	0.62	7.02	0.27
Centered CCK <sup>2</sup> Pretest					0.12	0.10	1.18	0.01
Centered PMTE <sup>3</sup> Pretest					0.00	0.01	0.08	0.00
Interaction – CCK and PMTE					0.00	0.00	-0.10	0.00

\* $p < .001$

<sup>1</sup>SCK represents Specialized Content Knowledge

<sup>2</sup>CCK represents Common Content Knowledge

<sup>3</sup>PMTE represents Personal Mathematics Teacher Efficacy

**Research Questions 5.** The fifth research question of this study was: Does the relationship between MTOE and growth in specialized content knowledge during a university mathematics methods/content courses vary as a function of common content knowledge?

To determine if common content knowledge (CCK) and mathematics teaching outcome expectancy (MTOE) interact to predict growth in specialized content knowledge (SCK), the predictor variables were centered and an interaction term was computed. Multiple regression analysis was used with pretest scores on SCK being entered first as a means of determining growth in SCK. The centered predictor variables were entered next, and the interaction term was entered last.

The overall regression model to predict posttest scores on SCK using pretest SCK, centered pretest CCK, and centered pretest MTOE was statistically significant,  $F(4, 96) = 22.606, p < .001$ . For this overall model,  $R = 0.696, R^2 = .485$ . In other words, when pretest SCK, centered pretest CCK, and centered pretest PMTE are used as predictors, 48.5% of the variance in posttest SCK could be explained.

Including the interaction term did not result in a significant change in the amount of variance explained,  $F_{change}(1, 96) = 2.397, p = 0.125$ . The interaction term was not significant,  $t(96) = 1.548, p = 0.125$ . Complete results of this analysis are found in Table 24.

Table 24

*Results of Standard Multiple Regression to Predict Posttest Specialized Content Knowledge from Pretest Specialized Content Knowledge, Pretest Mathematics Teaching Outcome Expectancy, Pretest Common Content Knowledge Including Interaction Term (N = 101)*

Variables	$R^2$	Adj. $R^2$	$R$	$F$	$b$	$\beta$	$t$	$sr^2$
Overall model	0.49	0.46	0.70	22.61*				
SCK <sup>1</sup> Pretest					0.71*	0.64	7.39	0.29
Centered CCK <sup>2</sup> Pretest					0.11	0.10	1.16	0.01
Centered MTOE <sup>3</sup> Pretest					0.07	0.06	0.82	0.00
Interaction – CCK and MTOE					0.03	0.11	1.55	0.01

\* $p < .001$

<sup>1</sup>SCK represents Specialized Content Knowledge

<sup>2</sup>CCK represents Common Content Knowledge

<sup>3</sup>MTOE represents Mathematics Teaching Outcome Expectancy

### Supplementary Analysis

Although research questions two and three indicated that neither dimension of mathematics teacher efficacy predicted posttest scores on specialized mathematics content knowledge, the researcher was interested in further exploring the relationship between the two variables. In thinking more about this relationship, since previous research has shown high levels of mathematics teacher efficacy are desirable in creating positive outcomes in the classroom, the researcher wondered if any of the variables investigated in this study are related to growth in personal mathematics teacher efficacy. Such knowledge would be useful in determining the types of courses and experiences students need prior to enrolling

in mathematics methods/content courses in order to maximize their growth in personal mathematics teacher efficacy during the methods/content course. In addition, although the number of males in the sample was quite small ( $n = 6$ ), the researcher was especially interested in the results for only the female participants since they compose the majority of students enrolled in the course investigated in this study. Results of these analyses could be used to gain information about the need for further research.

To complete this additional analysis, the data were first split by gender and correlations were found. The number of males was too small for any conclusions to be drawn regarding this subgroup of the sample. For female participants alone, the entry level of personal mathematics teacher efficacy was found to be significantly correlated with the entry level of specialized content knowledge ( $r = 0.239, p = 0.020$ ) but not with the entry level of common content knowledge ( $r = 0.161, p = 0.118$ ). Scores of the female participants on the posttest of personal mathematics teacher efficacy were found to be significantly correlated with pretest common content knowledge ( $r = 0.251, p = 0.014$ ), posttest common content knowledge ( $r = 0.317, p = 0.002$ ), pretest specialized content knowledge ( $r = 0.322, p = 0.001$ ), and posttest specialized content knowledge ( $r = 0.253, p = 0.013$ ). Table 25 provides a summary of these findings.

Table 25

*Correlations Among Variables for Females (N = 95)*

		2	3	4	5	6
1.	CCK <sup>1</sup> Pretest	0.40**	0.57**	0.41**	0.16	0.25*
2.	CCK Posttest	--	0.51**	0.42**	0.38**	0.32**
3.	SCK <sup>2</sup> Pretest		--	0.69**	0.24*	0.32**
4.	SCK Posttest			--	0.16	0.25*
5.	PMTE <sup>3</sup> Pretest				--	0.60**
6.	PMTE Posttest					--

\*Correlation is significant at the 0.05 level (2-tailed).

\*\*Correlation is significant at the 0.01 level (2-tailed).

<sup>1</sup>CCK represents Common Content Knowledge.

<sup>2</sup>SCK represents Specialized Content Knowledge.

<sup>3</sup>PMTE represents Personal Mathematics Teacher Efficacy.

To determine if the entry level of specialized mathematics content knowledge (SCK) of the females in the sample significantly predicted growth in personal mathematics teacher efficacy (PMTE), multiple regression analysis was conducted with posttest scores on PMTE being the dependent variable. Pretest scores on PMTE were entered as a predictor as a means of determining growth in personal mathematics teacher efficacy. Pretest scores on SCK were then entered as a predictor variable. The overall regression model to predict posttest scores on PMTE using pretest PMTE and pretest SCK was statistically significant,  $F(2,92) = 30.222, p < .001$ . For this overall model,  $R = 0.630, R^2 =$

0.393. In other words, when pretest SCK and pretest PMTE are used as predictors, approximately 40% of the variance in posttest PMTE could be explained.

For the females of the sample, pretest PMTE was significantly predictive of posttest PMTE when the variable pretest SCK was statistically controlled:  $t(92) = 6.683, p < 0.001$ . The positive slope for pretest SCK as a predictor of posttest PMTE was 0.482 indicating that there was almost a one-half point increase in posttest PMTE scores for every increase of one in pretest PMTE scores. The squared semi-partial correlation to determine the amount of variance in posttest PMTE uniquely predictable from pretest PMTE was  $sr^2 = 0.293$  meaning that pretest PMTE accounted for approximately 29% of the variance in posttest PMTE when pretest SCK was statistically controlled.

For the females of the sample, pretest SCK was significantly predictive of posttest PMTE when the variable pretest PMTE was statistically controlled:  $t(92) = 2.262, p = 0.026$ . The positive slope for pretest SCK as a predictor of posttest PMTE was 0.284 indicating that there was slightly more than a one-fourth of a point increase in posttest PMTE scores for every increase of one in pretest SCK scores. The squared semi-partial correlation to determine the amount of variance in posttest PMTE uniquely predictable from pretest SCK was  $sr^2 = 0.033$  meaning that pretest SCK accounted for approximately 3% of the variance in posttest PMTE when pretest PMTE was statistically controlled. Complete results for this analysis are found in Table 26.

Table 26

*Results of Standard Multiple Regression to Predict Posttest Personal mathematics teacher efficacy from Pretest Specialized Content Knowledge and Pretest Personal Mathematics Teacher Efficacy of Females (N = 95)*

Variables	$R^2$	Adj. $R^2$	$R$	$F$	$b$	$\beta$	$t$	$sr^2$
Overall model	0.40	0.38	0.63	30.22*				
SCK <sup>1</sup> Pretest					0.28*	0.19	2.26	0.03
PMTE <sup>2</sup> Pretest					0.48**	0.56	6.68	0.29

\* $p < .05$

\*\* $p < .001$

<sup>1</sup>SCK represents Specialized Content Knowledge

<sup>2</sup>PMTE represents Personal mathematics teacher efficacy

To determine if the entry level of common mathematics content knowledge (CCK) of the females in the sample significantly predicted growth in personal mathematics teacher efficacy (PMTE), multiple regression analysis was conducted with posttest scores on PMTE being the dependent variable. Pretest scores on PMTE were entered as a predictor as a means of determining growth in personal mathematics teacher efficacy. Pretest scores on CCK were then entered as a predictor variable. The overall regression model to predict posttest scores on PMTE using pretest PMTE and pretest CCK was statistically significant,  $F(2,92) = 29.084, p < .001$ . For this overall model,  $R = 0.622, R^2 = 0.387$ . In other words, when pretest SCK and pretest PMTE are used as predictors, approximately 39% of the variance in posttest PMTE could be explained.

For the females of the sample, pretest PMTE was significantly predictive of posttest PMTE when the variable pretest CCK was statistically controlled:  $t(92) = 6.977, p < 0.001$ . The positive slope for pretest SCK as a predictor of posttest PMTE was 0.499 indicating that there was almost a one-half point increase in posttest PMTE scores for every increase of one in pretest PMTE scores. The squared semi-partial correlation to determine the amount of variance in posttest PMTE uniquely predictable from pretest PMTE was  $sr^2 = 0.324$  meaning that pretest PMTE accounted for approximately 32% of the variance in posttest PMTE when pretest CCK was statistically controlled.

For females, pretest CCK was not a significant predictor of posttest PMTE when pretest PMTE was statistically controlled:  $t(92) = 1.915, p = 0.059$ . Complete results for this analysis are found in Table 27.

Table 27

*Results of Standard Multiple Regression to Predict Posttest Personal mathematics teacher efficacy from Pretest Common Content Knowledge and Pretest Personal mathematics teacher efficacy of Females*

Variables	$R^2$	Adj. $R^2$	$R$	$F$	$b$	$\beta$	$t$	$sr^2$
Overall model	0.39	0.37	0.62	29.08*				
CCK <sup>1</sup> Pretest					0.24	0.16	1.92	0.02
PMTE <sup>2</sup> Pretest					0.50*	0.58	6.98	0.32

\* $p < .001$

<sup>1</sup>CCK represents Common Content Knowledge

<sup>2</sup>PMTE represents Personal mathematics teacher efficacy

## **Conclusion**

Through the use of multiple regression analyses, the researcher found that the specialized mathematics content knowledge (SCK) of preservice elementary teachers significantly increased during a methods/content course. Additional analyses indicated that common mathematics content knowledge (CCK), personal mathematics teacher efficacy (PMTE), and mathematics teaching outcome expectancy (MTOE) also increased during this course. While PMTE was significantly correlated with SCK, the level of PMTE of the participants did not significantly predict the growth in SCK experienced during the semester. Likewise, the MTOE of participants did not significantly predict their growth in SCK. In addition, the relationship between PMTE and growth in SCK and the relationship between MTOE and growth in SCK were not moderated by CCK. Supplementary analysis revealed that for the female participants, the initial level of SCK of a participant significantly predicted growth in PMTE. However, the initial level of CCK was not a significant predictor of growth in PMTE. Chapter five includes further discussion of these results, a discussion of the limitations of the study, and suggestions for future studies.

## CHAPTER 5

### DISCUSSION

This chapter is divided into three sections. The first section provides a review of the purpose of the study. A discussion of the findings of the study and the limitations of the study follows in the second section. The concluding section of the chapter includes suggestions for future research studies.

#### **Review of Purpose of Study**

The National Council of Teachers of Mathematics ([NCTM], 2000) provides a compelling vision for mathematics education in the United States calling for high-quality instruction, knowledgeable teachers, ambitious expectations, and engaging curriculum. However, many students in the United States do not have the opportunity to experience the type of mathematics education envisioned by NCTM. Further, concern about the mathematics education of our students increases with the release of results from international assessments that indicate students in the United States are not performing as well as students in many other countries.

The effectiveness of the teacher is often described as one of the most important factors in improving student achievement in mathematics (Darling-Hammond, 2000). Identifying the factors that contribute to the development of effective teachers has been the focus of a great deal of research in mathematics education. Teacher knowledge, pedagogical skills, and beliefs have each been the subject of many studies and the results of these studies provide a research base and foundation for other studies. Since preservice

education programs play a vital role in preparing teachers, preservice teachers and their characteristics are important in education research.

Therefore, the purpose of this study was to explore the relationship between the mathematics teacher efficacy of preservice elementary teachers and the growth they experienced in specialized mathematics content knowledge during a university mathematics methods/content course. The intent of the study was to contribute to the knowledge base regarding preparing elementary teachers to effectively teach mathematics to their elementary students.

The participants of this study were prospective elementary teachers enrolled in a mathematics methods/content course at a midwestern university in a small city. A total of 101 students participated in this study and all did so on a voluntary basis. The participants completed a pretest that included demographic information, items designed to measure common mathematics content knowledge, items designed to measure specialized mathematics content knowledge, and items designed to measure mathematics teacher efficacy. They also completed a posttest that omitted demographic items but included all other sections of the pretest. The instrument used in this study may be found in Appendix A.

Multiple regression was the primary method of analysis used in this study in an attempt to determine if mathematics teacher efficacy predicted growth in the specialized mathematics content knowledge of preservice elementary teachers during a university mathematics methods/content course. A discussion of the results of these analyses follows.

## Conclusions

**Research Question 1.** Does the specialized mathematical content knowledge of preservice elementary teachers increase during a university mathematics methods/content course?

**Summary of results.** A paired samples *t*-test indicated that the level of specialized mathematics content knowledge of preservice elementary teachers significantly increased during the mathematics methods/content course.

**Discussion.** The existence of a special type of mathematical content knowledge needed uniquely by teachers has been of increasing interest to researchers. This specialized content knowledge (SCK) has been found to be related to, but distinct from, the common mathematical content knowledge needed by teachers as well as nonteachers (Ball, Thames & Phelps, 2008). The information gained from this study supports the existence of SCK and its distinctness from common content knowledge. As expected, specialized mathematical content knowledge and common content knowledge were found to be significantly correlated ( $r = .52$ ).

Researchers are increasingly of the belief that SCK is critical for effective teaching (Morris, Hiebert, & Spitzer, 2009). The development of SCK is appropriate for preservice teacher education programs because it is content knowledge that does not depend directly on knowledge of students and teaching. It can be difficult for prospective teachers to develop classroom-based knowledge since their opportunities to build such knowledge may be limited until they begin teaching. The findings of this study indicating that SCK can and does grow during a university methods/content course support the idea that a focus

on SCK is appropriate for preservice programs. This finding lends support to the restructuring of elementary methods/content courses, as well as methods-only courses, to include a more explicit focus on the development of SCK. In addition, consideration of this finding could impact the development of curriculum for these courses and the selection of instructional materials and resources to be used.

The importance of the development of SCK and the fact that it increases during methods/content courses lends support to a reconsideration of the types of mathematics courses elementary education students are expected to complete at the university level. Many universities, including the one from this study, require elementary education majors to successfully complete one or more general studies mathematics courses that focus entirely on common mathematical content knowledge. Ball, Thames, and Phelps (2008) comment that “subject matter courses in teacher preparation programs tend to be academic in both the best and worst sense of the word, scholarly and irrelevant, either way remote from classroom teaching” (p. 404). Obviously knowledge of mathematics is essential for elementary teachers, but elementary education programs contain only a limited number of hours for mathematics courses. Given the very limited time mathematics teacher educators have to work with preservice elementary teachers, perhaps these hours could be better utilized in courses focusing on the development of SCK.

An additional finding of this study was that scores on both subscales of mathematics teacher efficacy, personal mathematics teacher efficacy and mathematics teaching outcome expectancy, increased from the pretest to posttest. This is a somewhat

different finding from other studies. Many studies that have looked at teacher efficacy during methods courses have found that while personal teacher efficacy increases from pretest to posttest, teaching outcome expectancy tends to remain stable (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998). This finding has remained constant regardless of the content area being examined (Buss, 2010; Enochs, Smith, & Huinker, 2000; Swars, 2005; Wingfield, Nath, Freeman, & Cohen, 2000).

In this study, while a much greater increase was noted in personal mathematics teacher efficacy ( $t = 15.98$ ) than in mathematics teaching outcome expectancy ( $t = 5.03$ ), both increases were statistically significant. This difference from previous findings could be due to the fact that the most recent mathematics course the participants of this study had completed prior to enrolling in the mathematics methods/content course investigated in this study was a general studies mathematics course. These courses are often taught by mathematics faculty members who tend to focus on content over pedagogy. This lack of focus on pedagogy may result in limited consideration of instructional factors such as use of a variety of instructional strategies, active student engagement, and effective questioning. Consequently, participants may place less value on the importance of effective mathematics teaching in promoting student learning and, therefore, lower pretest scores on mathematics teaching outcome expectancy will result.

The nature of the course being studied may have contributed to the increase in mathematics teaching outcome expectancy as well. Research has shown that mathematics teaching outcome expectancy drops during student teaching when student teachers are

faced with the complexity of teaching (Tschannen-Moran, Woolfork Hoy, & Hoy, 1998). Tschannen-Moran, Woolfork Hoy, and Hoy suggest a greater emphasis on field experiences prior to student teaching might provide a more realistic belief about teaching. At the university being studied, all students must successfully complete the mathematics methods/content course described in this study prior to their enrollment in a professional semester. The professional semester occurs at least one semester prior to student teaching, involves extensive field experiences, and is the first real opportunity students have to work in classrooms. Therefore, even senior level students enrolled in the mathematics methods/content course have had quite limited opportunities to work in the schools. Consequently, since many of the participants in this course have very little field experience work prior to enrolling in this course, they may have a less than realistic view of teaching resulting in high scores in mathematics teaching outcome expectancy.

**Research Question 2.** The second research question of this study was: Is there a relationship between preservice elementary teachers' sense of personal mathematics teacher efficacy (PMTE) and growth in specialized mathematics content knowledge (SCK) during a university mathematics methods course?

**Summary of results:** Multiple regression analysis indicated pretest scores on SCK and PMTE together significantly predicted posttest SCK scores and explained 46% of the variance in posttest SKC scores. Pretest SCK was found to be a significant predictor of posttest SCK and uniquely predicted 43% of the variance in posttest SCK when pretest

PMTE was statistically controlled. However pretest PMTE did not significantly predict posttest SCK when pretest SCK was statistically controlled.

**Discussion:** The findings of this study were similar to those of Swars, Hart, Smith, Smith, and Tolar (2007) who investigated the interrelatedness of mathematics teacher efficacy and specialized content knowledge. Their study explored both dimensions of mathematics teacher efficacy, personal mathematics teacher efficacy and mathematics teaching outcome expectancy, and used a measure of specialized mathematics content efficacy that included all strands of mathematics. This previous study indicated no relationship between the two as evidenced by the lack of significant correlations between the variables. The findings of the current study are similar in that neither subscale of mathematics teacher efficacy as measured by the Mathematics Teaching Efficacy Beliefs Instrument significantly predicted growth in specialized mathematics content knowledge. However, specialized content knowledge and personal mathematics teacher efficacy were found to have a weak positive correlation in this study. Specialized content knowledge and mathematics teaching outcome expectancy were not found to be correlated in the current study.

The findings of the current study may be impacted in part by an overestimation of personal mathematics teacher efficacy by the participants. In a discussion of the findings of their study, Swars, Hart, Smith, Smith, and Tolar (2007) describe the disconnect between preservice teachers' level of specialized content knowledge and the beliefs in their ability to teach mathematics effectively. They comment:

It appears that preservice teachers can be quite efficacious about their teaching and not have developed strong specialized content knowledge for teaching mathematics. This naïve perspective is not surprising and is consistent with the human condition of not being aware of what you do not know (Swars, Hart, Smith, Smith, and Tolar, 2007, p. 333).

If an overestimation of efficacy occurred, while the lack of a significant relationship between personal mathematics teacher efficacy and specialized content knowledge is not completely surprising, it is also not discouraging. Given the level of mathematics anxiety and the negative prior experiences in mathematics of many preservice elementary teachers, perhaps an overestimation of self-efficacy is indeed positive. Bandura (1989) suggests:

Optimistic self-appraisals are a benefit rather than a cognitive failing to be eradicated. If self-efficacy beliefs always reflected only what people can do routinely, people would rarely fail but neither would they mount the extra effort needed to surpass their ordinary performances ( p. 732).

Perhaps a high level of personal mathematics teaching self-efficacy will be what is needed to ensure that some of the preservice teachers with relatively few prior positive experiences with mathematics will continue to learn mathematics and how to best teach mathematics to their students. The results of a study by Buss (2010) lend support to the idea that a higher level of personal mathematics teaching efficacy is positive. Three hundred twenty-five preservice teachers were included in this study which explored their

teaching efficacy for several content areas. Buss found these prospective teachers scores on personal teaching efficacy for mathematics and science were significantly lower than those for other content areas.

**Research Question 3.** The third research question of this study was: Is there a relationship between preservice elementary teachers' sense of mathematics teaching outcome expectancy (MTOE) and growth in specialized mathematics content knowledge (SCK) during a university mathematics methods course?

**Summary of results:** Similar to the results found for research question two, multiple regression analysis indicated pretest scores on SCK and MTOE together significantly predicted posttest SCK scores and explained 46% of the variance in posttest SKC scores. Pretest SCK was found to be a significant predictor of posttest SCK and uniquely predicted 46% of the variance in posttest SCK when pretest MTOE was statistically controlled. However pretest MTOE did not significantly predict posttest SCK when pretest SCK was statistically controlled.

**Discussion:** The results described above were as expected and consistent with the findings from previous research. Similar to the work of Swars, Hart, Smith, Smith, and Tolar (2007), the current study found no significant relationship between mathematics teaching outcome expectancy and specialized content knowledge. This is not surprising as mathematics teaching outcome expectancy involves an overall view of the power of teaching in general to bring about learning and is not a reflection of the belief a person has in his or her own ability to teach effectively.

**Research Questions 4.** The fourth research question of this study was: Does the relationship between Personal Mathematics Teacher Efficacy (PMTE) and growth in specialized content knowledge (SCK) during a university mathematics methods course vary as a function of common content knowledge (CCK)?

**Summary of results:** Multiple regression analysis revealed that the relationship between PMTE and growth in SCK is not moderated by CCK.

**Discussion:** Given that no main effect was found in the analysis of research question 2, the finding that the relationship between PMTE and growth in SCK is not moderated by CCK is not unexpected.

**Research Questions 5.** The fifth research question of this study was: Does the relationship between Mathematics Teacher Outcome Expectancy (MTOE) and growth in specialized content knowledge (SCK) during a university mathematics methods course vary as a function of common content knowledge (CCK)?

**Summary of results:** Multiple regression analysis revealed that the relationship between MTOE and growth in SCK is not moderated by CCK.

**Discussion:** Given that no main effect was found in the analysis of research question 2, the finding that the relationship between PMTE and growth in SCK is not moderated by CCK is not unexpected.

**Supplementary Analyses.** Supplementary analyses were conducted to further examine the relationship between specialized content knowledge (SCK) and personal mathematics teacher efficacy (PMTE) and to look specifically at the female participants. The purpose of

this additional analysis was to gather information that could provide information regarding the types of mathematics courses and experiences prospective elementary teachers need prior to enrolling in a university methods/content course in order to maximize their growth in mathematics teacher efficacy. As stated previously in this dissertation, efficacy has been shown to be related to a variety of teaching behaviors and student outcomes and, thus, is an important consideration for preservice educators. Looking specifically at females is important as the majority of prospective elementary teachers are female.

**Summary of results:** Supplementary analysis revealed that for the female participants, the initial level of specialized mathematics content knowledge (SCK) of a participant significantly predicted growth in personal mathematics teacher efficacy (PMTE). However, the initial level of common mathematics content knowledge (CCK) was not a significant predictor of growth in PMTE.

**Discussion:** The need for elementary teachers to know more about mathematics is well documented in research as shown earlier in this dissertation. The nature of this knowledge has been a topic of a great deal of research as has how this knowledge can best be developed. Many universities, including the one involved in this study, require prospective elementary teachers to complete a general studies mathematics course prior to enrolling in specialized mathematics methods/content courses. Some universities require successful completion of College Algebra while others require a more general, survey of mathematics course as a prerequisite to methods/content courses. The nature of these

prerequisite courses and, in fact, the need for such courses is an important consideration for mathematics teacher educators.

The findings from the supplementary analysis provide support for those concerned with increasing the mathematics teaching efficacy of elementary teachers. Since the initial level of specialized content knowledge of these prospective teachers significantly predicted their growth in personal mathematics teacher efficacy, additional courses in which developing this specialized content knowledge is an emphasis should be considered. With only a limited number of semester hours dedicated to the preparation of future elementary teachers in the area of mathematics, careful consideration must be given to how those hours are utilized. Perhaps additional courses that integrate the development of specialized content knowledge with teaching methods would be the most effective in developing the teaching efficacy needed to enhance teacher performance.

Other studies have looked at the varying impact of specialized content courses and more general mathematics courses on the development of mathematics knowledge for teaching. For example, Matthews, Rech, and Grandgenett (2010) compared preservice elementary teachers who had completed mathematics content courses specifically focusing on the mathematics taught at the elementary level to students who had completed a more general mathematics course such as College Algebra. Their findings indicated that students who took the specialized content courses had significantly higher mathematical content knowledge than those who completed the more general course. Additional research

regarding the relationship between these courses and mathematics teacher efficacy is needed.

Finally, the lasting impact of teacher education on teachers' practice in the classroom has been the topic of a great deal of research. Many authors claim the effects of teacher education programs on teacher practice are minimal, and that teacher socialization often diminishes the educational ideas promoted in teacher education programs (Veenman, 1984; Wideen, Mayer-Smith, & Moon, 1998; Zeichner & Tabachnik, 1981). New teachers may feel their teacher education programs did not sufficiently prepare them for the classroom, and that their new colleagues are more reliable sources of information than were their teacher educators (Wideen, Mayer-Smith & Moon, 1998). However, Brouwer and Korthagen (2005) point out that relatively little is known about the degree to which specific strategies utilized in teacher education courses can overcome the socialization of new teachers into the established practices of their schools. If, as this study indicates, the mathematics teacher efficacy of prospective elementary teachers does grow during a university mathematics methods/content course, knowledge of the types of strategies that will ensure this growth is maintained as these novice teachers begin their careers. Additional research regarding these strategies is warranted.

### **Limitations of the Study**

The limitations of the study will be discussed in terms of study participants, the researcher as course instructor, the instrument used to measure specialized content knowledge, and the measurement of self-efficacy.

One limitation of this study was in the sample selection. A convenience sampling method was utilized in which all students enrolled in the methods/content course involved in this study were invited to participate. Those who chose to participate did so on a volunteer basis. Because the sample consisted of those students who volunteered to participate, the sample may have consisted of students with higher mathematics teacher efficacy or higher levels of mathematics content knowledge than those who did not choose to participate. This may have resulted in a restriction of the range of scores for the study. This is particularly likely given that many prospective elementary teachers come to the university with prior negative experiences in mathematics or with high levels of mathematics anxiety. The level of mathematics anxiety of participants was not measured as part of this study. In addition, the sample was drawn entirely from the student population of a single university in a small city in the Midwest and, therefore, the results of this study will not generalize to other situations.

Another limitation of the study is that the researcher was also the instructor for the courses investigated in the study. While efforts, as described in Chapter 3, were taken to limit the impact of this, students may have felt compelled to participate. Furthermore, the responses of some participants may have been affected by the fact that the researcher was also the instructor. The researcher chose this method to minimize the impact of the instructor on the results of the study. Another method of doing so would have been to collect a very large sample from many different universities. However, this would have introduced additional variables such as course content and impact of required prerequisite

courses. For this particular study, the impact of the instructor as teacher may be most noticeably seen in scores on both subscales of the Mathematics Teaching Efficacy Beliefs Instrument. The personal beliefs of the researcher regarding the importance of considering mathematics teacher efficacy and of explicitly focusing on developing it in preservice elementary teachers may have affected classroom experiences and activities and, as a result, scores on the efficacy subscales.

A third limitation is found in the instrument used in the study. First, the instrument was relatively lengthy and testing fatigue could have impacted results. As described in Chapter 3, six different arrangements of the sections of the instrument were utilized in an attempt to minimize testing fatigue. However, scores on portions of the instrument may have been influenced by the length of the instrument as a whole. In addition, the instrument used to measure specialized content knowledge was developed for use with inservice teachers. Although the developers themselves have used the instrument with preservice teachers, some other researchers have raised questions about this practice as well as with the instrument itself. Matthews, Rech, and Grandgenett (2010) comment that the instrument is limited in the scope of material covered as evidenced by the fact that several topics have multiple questions associated with the topic while other concepts vital to elementary mathematics are not included. In a study designed to establish the reliability of the instrument when used by preservice teachers, Gleason (2010) concludes that some items on the instrument are not appropriate for preservice teachers.

The instrument used to measure mathematics teacher efficacy was a self-report tool. Since the participants were made aware that the study was designed to investigate the mathematics teaching efficacy of preservice elementary teachers, this knowledge may have impacted their responses. As noted earlier, it is not uncommon for prospective teachers to have an inflated sense of efficacy, and while this is not particularly troubling, it may have affected the findings of the study. Further research that includes multiple methods of measuring self-efficacy may be needed.

The decision by the researcher to only assess specialized content knowledge within the number and operations strand may have influenced the results of this study. While number and operations is heavily emphasized in elementary grades, and also in the course examined in this study, certainly other strands of mathematics are important as well. In addition, preservice elementary teachers typically feel the most capable and well-prepared in the number and operations strand. Therefore, looking only at number and operations may have provided a restricted view of the growth in specialized content knowledge of the preservice teachers and, as a result, limited this study.

A final limitation of the study may be in the selection of self-efficacy as the construct used as a predictor for growth in specialized mathematics content knowledge. As described earlier, it is not uncommon for preservice teachers to overestimate their teaching efficacy and this overestimation could have impacted the findings of this study. In thinking more about the questions posed in this study and the experiences that led to posing these questions, another construct of interest came to the attention of the researcher. This

construct, professional mathematics teacher identity, is distinct but related to self-efficacy. In simple terms, self-efficacy is a belief of “I can” while identity reflects a belief of “I am”.

The idea of teacher identity is of increasing importance in teacher education research. Identity is important because teachers “are engaged in practice not just with their knowledge but with all their being” (da Ponte & Chapman, 2008, p. 241). Da Ponte and Chapman suggest teachers project their identity, or who they are, onto their students, the subject itself, and the classroom environment. Wenger (1998) suggests that teacher identity includes experiences and knowledge, perceptions of self, and perceptions of others. Other researchers describe identity as a lens through which preservice teachers perceive teacher education curriculum and give meaning to experiences in teacher education (Bullough, 1997). Therefore, perhaps the level of development and the nature of the mathematics teacher identity of preservice elementary teachers play a role in their acquisition of specialized content knowledge. Exploring this relationship would be an interesting follow-up to this study.

### **Suggestions for Future Research**

The purpose of this study was to investigate mathematics teacher efficacy as a factor that might impact the growth of preservice elementary teachers in the area of specialized mathematics content knowledge. Future research regarding mathematics teacher efficacy and specialized content knowledge individually as well as further research regarding the relationship between the two would provide additional information to those interested in mathematics education.

First, mathematics teacher efficacy was not found to be significantly related to growth in specialized content knowledge but the supplementary analysis revealed that for female students, initial levels of specialized content knowledge did predict levels of personal mathematics teacher efficacy at the end of the course. Future research could look more closely at the type of mathematics content knowledge that is most likely to result in increases in mathematics teacher efficacy. If higher levels of mathematics teacher efficacy are believed to result in more positive classroom practices, the findings of such research would be of use to those charged with determining the nature of the mathematics courses offered to preservice elementary teachers.

In addition, research regarding the relationship between mathematics teacher efficacy and the nature of experiences preservice teachers are afforded is in order. Bandura's sources of self-efficacy could be used as a means of classifying the experiences and research could examine the relationship between these categories of experiences and increases in mathematics teacher efficacy. Further, these categories of experiences could be investigated to determine the relationship between these experiences and growth in the specialized content knowledge of preservice teachers. For example, research could be conducted that would address the question of whether providing additional mastery experiences results in growth in specialized content knowledge.

As described earlier, the teaching efficacy of preservice elementary teachers is generally lower in mathematics and science than in other content areas; yet, elementary education programs are heavy in language arts courses. Additional research could explore

this idea to determine if a more balanced approach in terms of hours allotted for content areas would result in increased efficacy in mathematics and science teaching.

Future studies could follow prospective elementary teachers through the first few years of their careers. These studies could explore the role of mathematics teacher efficacy on classroom practice and on the continual development of specialized mathematics content knowledge. Research could investigate specific strategies utilized by mathematics teacher educators on the maintenance of mathematics teacher efficacy during the first few years of teaching. The impact of teacher socialization on the mathematics teacher efficacy of novice teachers could be explored as well.

The relationship between teacher knowledge and student achievement has been explored in a variety of ways in the past. The categories of mathematics teacher knowledge described in Chapter 2 could be used to further explore the relationship between various types of teacher knowledge and student achievement. For example, one category of teacher knowledge as described by Ball, Thames, and Phelps (2008) is horizon content knowledge which reflects, in part, an awareness of how mathematical topics are related throughout grade levels. The relationship of increased acquisition of such knowledge and student achievement could be the subject of future research. In addition, investigating relationships that exist among these specific types of mathematics content knowledge and mathematics teacher efficacy would be of interest.

Finally, the idea of mathematics teacher identity and its relationship to mathematics teacher self-efficacy could be the subject of future research. Researchers could explore the

question of the impact of self-efficacy on the development of the teacher identity of preservice teachers. In addition, the role of mathematics teacher identity in the development of specialized content knowledge could be explored. Since teacher identity is a relatively new construct of importance in teacher education, ways to assess teacher identity are somewhat limited. Exploring ways to do so could provide valuable information to mathematics teacher educators.

### **Conclusion**

This study investigated the relationship between the mathematics teacher efficacy of preservice elementary teachers and the growth they experienced in specialized mathematics content knowledge during a university mathematics methods/content course. The intent of the study was to contribute to the knowledge base regarding preparing elementary teachers to effectively teach mathematics to their elementary students. Significant increases were found in specialized mathematical content knowledge, common content knowledge, personal mathematics teacher efficacy, and mathematics teaching outcome expectancy. Neither dimension of mathematics teaching efficacy significantly predicted growth in specialized mathematical content knowledge; however, supplementary analyses revealed that for female students, initial levels of specialized mathematical content knowledge did significantly predict growth in personal mathematics teacher efficacy. The findings of the study provide many ideas for future research in the areas of mathematics teacher knowledge and mathematics teaching efficacy. Further research regarding the mathematics teacher identity of prospective elementary teachers is also

warranted. Knowledge gained through such studies will enable mathematics teacher educators to better prepare elementary teachers and, as a result, the mathematics education of our students

APPENDIX A  
INSTRUMENT USED IN STUDY

**On the line below, please write your mother's maiden name followed by the city of your birth. This information will be used for data collection purposes only.**

---

Copyright © 2006 The Regents of the University of Michigan. For information, questions, or permission requests please contact Merrie Blunk, Learning Mathematics for Teaching, 734-615-7632. Not for reproduction or use without written consent of LMT. Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.



Learning Mathematics for Teaching  
University of Michigan  
School of Education  
610 E. University #1600  
Ann Arbor, MI 48109-1259

**NOTE: THE ACTUAL ITEMS USED FROM THE LMT PROJECT ARE NOT INCLUDED DUE TO AN AGREEMENT BY RESEARCHERS TO NOT INCLUDE THEM IN PAPERS. SECTION D OF THE INSTRUMENT INSTEAD INCLUDES RELEASED ITEMS FROM THE LMT PROJECT**

## Section A

Please complete the following questions by circling the appropriate response.

**GENDER:**                      Male              Female

**STUDENT STATUS:**              Freshman      Sophomore      Junior      Senior

### MATH COURSES COMPLETED IN HIGH SCHOOL

Please check all that apply.

\_\_\_\_\_ Algebra I

\_\_\_\_\_ Geometry

\_\_\_\_\_ Algebra II

\_\_\_\_\_ College Algebra

\_\_\_\_\_ Trigonometry

\_\_\_\_\_ Pre-Calculus

\_\_\_\_\_ Calculus

Other (please list) \_\_\_\_\_

### HIGH SCHOOL LOCATION

Please check.

\_\_\_\_\_ Rural

\_\_\_\_\_ Suburban

\_\_\_\_\_ Urban

### Section B – Instructions

Please indicate the degree to which you agree or disagree with each statement below by circling the appropriate letters to the right of each statement.

**SA – Strongly Agree**

**A – Agree**

**UN – Uncertain**

**D – Disagree**

**SD – Strongly Disagree**

	<b>SA Strongly Agree</b>	<b>A Agree</b>	<b>U Uncertain</b>	<b>D Disagree</b>	<b>SD Strongly Disagree</b>
1. When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.	SA	A	U	D	SD
2. I will continually find better ways to teach mathematics.	SA	A	U	D	SD
3. Even if I try very hard, I will not teach mathematics as well as I will most subjects.	SA	A	U	D	SD
4. When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.	SA	A	U	D	SD
5. I know how to teach mathematics concepts effectively.	SA	A	U	D	SD
6. I will not be very effective in monitoring mathematics activities.	SA	A	U	D	SD
7. If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.	SA	A	U	D	SD
8. I will generally teach mathematics ineffectively.	SA	A	U	D	SD
9. The inadequacy of a student's mathematics background can be overcome by good teaching.	SA	A	U	D	SD

10. When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.	SA	A	U	D	SD
11. I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	SA	A	U	D	SD
12. The teacher is generally responsible for the achievement of students in mathematics.	SA	A	U	D	SD
13. Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.	SA	A	U	D	SD
14. If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.	SA	A	U	D	SD
15. I will find it difficult to use manipulatives to explain to students why mathematics works.	SA	A	U	D	SD
16. I will typically be able to answer students' mathematics questions.	SA	A	U	D	SD
17. I wonder if I will have the necessary skills to teach mathematics.	SA	A	U	D	SD
18. Given a choice, I will not invite the principal to evaluate my mathematics teaching.	SA	A	U	D	SD
19. When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.	SA	A	U	D	SD
20. When teaching mathematics, I will usually welcome student questions.	SA	A	U	D	SD
21. I do not know what to do to turn students on to mathematics.	SA	A	U	D	SD

## Section C - Instructions

### Part 1: True or False

Write "True" if the statement is true. Write "False" if the statement is false.

- \_\_\_\_\_ 1. 13,579,246,710,470 is divisible by 4.
- \_\_\_\_\_ 2. 13,579,246,710,470 is divisible by 3.
- \_\_\_\_\_ 3.  $\frac{5}{8} > \frac{3}{4}$
- \_\_\_\_\_ 4. 1 is a prime number.
- \_\_\_\_\_ 5. The sum of a positive number and a negative number is a negative number.
- \_\_\_\_\_ 6. The whole numbers are closed for subtraction and multiplication.
- \_\_\_\_\_ 7.  $0 \div \frac{3}{4} = 0$
- \_\_\_\_\_ 8.  $4x + 3x = 3x + 4x$  is an example of the commutative property.
- \_\_\_\_\_ 9.  $-10 \times 1 = -10$  is an example of the identity property.
- \_\_\_\_\_ 10.  $-20 > -25$

### Part 2: Short Answer

For 11 – 18, write your answer on the line provided.

- \_\_\_\_\_ 11. Write a number that is between 2.5 and 2.49.
- \_\_\_\_\_ 12. Write a fraction between  $\frac{1}{4}$  and  $\frac{2}{9}$ .
- \_\_\_\_\_ 13. Write an algebraic expression for 3 subtracted from twice x.
- \_\_\_\_\_ 14. Write 0.9 as a fraction in simplest form.
- \_\_\_\_\_ 15. Round \$14.957 to the nearest cent.

\_\_\_\_\_ 16. Compute:  $6 - 18 \div 6 \times 10 - 5 + 1$

\_\_\_\_\_ 17. Solve:  $\frac{x}{15} = \frac{7}{45}$

\_\_\_\_\_ 18. Paul takes  $3\frac{1}{2}$  hours to type 14 pages. How long will it take him to type 44 pages?

19-20. Complete the chart. Be sure all fractions are in simplest form.

<b>Fraction</b>	<b>Decimal</b>	<b>Percent</b>
		17%
$\frac{2}{5}$		

## Section D - Instructions

- Answer questions by circling your choice, e.g.

1. During a unit on functions, Ms. Lopez asks her students to write journal entries on exponential growth. Which of the following journal entries illustrate exponential growth? (For each item below, circle EXPONENTIAL, NOT EXPONENTIAL or I'M NOT SURE.)

	Exponential	Not exponential	I'm not sure
a) An example of exponential growth would be if you got a 1% raise each year.	①	2	3
b) An example of exponential growth would be if a car increases in speed by 10 miles per hour every second.	1	②	3
c) Exponential growth is when the y-axis increases faster than the x-axis. For example, if each time the x-coordinate goes up by 2, the y-coordinate goes up by 3.	①	2	3

- In completing this questionnaire, you should not spend more than 1-2 minutes on any question. Imagine you are responding to real classroom situations, and select the answer that most closely matches what you would do, say, or answer at that moment.
- Your responses are voluntary and confidential. If you come to a question you do not wish to answer, simply skip it. We hope that you will answer as many questions as possible.

**NOTE: THE ACTUAL ITEMS USED FROM THE LMT PROJECT ARE NOT INCLUDED DUE TO AN AGREEMENT BY RESEARCHERS TO NOT INCLUDE THEM IN PAPERS. THE FOLLOWING ARE RELEASED ITEMS INCLUDED TO SHOW THE TYPES OF ITEMS ACTUALLY ON THE INSTRUMENT.**

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

2. Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

- a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
- b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
- c) Check to see whether 371 is divisible by any prime number less than 20.
- d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ +75 \phantom{0} \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ +700 \phantom{0} \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \phantom{0} \\ 100 \phantom{00} \\ +600 \phantom{00} \\ \hline 875 \end{array}$

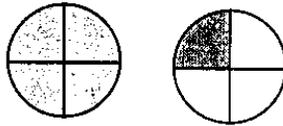
Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

4. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

- a) Four is an even number, and odd numbers are not divisible by even numbers.
- b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).
- c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.
- d) It only works when the sum of the last two digits is an even number.

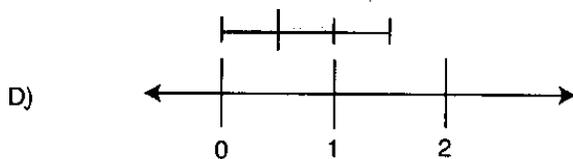
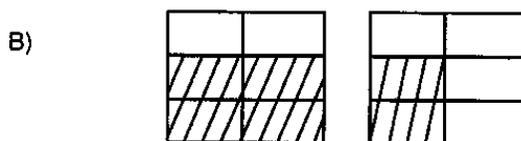
5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



- a)  $\frac{5}{4}$
- b)  $\frac{5}{3}$
- c)  $\frac{5}{8}$
- d)  $\frac{1}{4}$

6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that  $1\frac{1}{2} \times \frac{2}{3} = 1$ ? (Mark ONE answer.)



7. Which of the following story problems could be used to illustrate  $1\frac{1}{4}$  divided by  $\frac{1}{2}$ ? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have \$1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

$$\begin{array}{r} 983 \\ \times 6 \\ \hline 488 \\ +5410 \\ \hline 5898 \end{array}$$

What is Todd doing here? (Mark ONE answer.)

- a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.
- b) Todd is using the traditional multiplication algorithm but working from left to right.
- c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.
- d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

9. Ms. James' class was investigating patterns in whole-number addition. Her students noticed that whenever they added an even number and an odd number the sum was an odd number. Ms. James asked her students to explain why this claim is true for all whole numbers.

After giving the class time to work, she asked Susan to present her explanation:

I can split the even number into two equal groups, and I can split the odd number into two equal groups with one left over. When I add them together I get an odd number, which means I can split the sum into two equal groups with one left over.

Which of the following best characterizes Susan's explanation? (Circle ONE answer.)

- a) It provides a general and efficient basis for the claim.
- b) It is correct, but it would be more efficient to examine the units digit of the sum to see if it is 1, 3, 5, 7, or 9.
- c) It only shows that the claim is true for one example, rather than establishing that it is true in general.
- d) It assumes what it is trying to show, rather than establishing why the sum is odd.

APPENDIX B  
COURSE SYLLABI

A Course Syllabus

for

Math 1800

**Introduction to Teaching Elementary and Middle School Mathematics**

(Two semester hours credit)

in the

Department of Mathematics and Computer Science

of the

COLLEGE OF SCIENCE AND TECHNOLOGY

Catalog Description:

A six week course on the concepts and methods of teaching mathematics in both elementary and middle school. Prerequisite: Math 1620

CENTRAL MISSOURI STATE UNIVERSITY

Warrensburg, Missouri

## I. Purpose of the Course

This course provides instruction on the content and methods for teaching the mathematics that is common to elementary and middle school. Specifically, this course is designed to:

- A. Develop the basic concepts, skills, and techniques for teaching sets, number, number theory, and fractions in elementary and middle school
- B. Develop models for teaching which focus on the appropriate mathematical content, relevant learning theories, and alternative teaching strategies.

## II. Objectives and Desired Student Competencies

Upon completion of this course, the student should be able to:

- A. Define, identify and describe the fundamental concepts of rational number systems.
- B. Develop a variety of mathematical skills involving the four fundamental operations.
- C. Describe teaching strategies, activities, and materials useful in teaching the rational number system, number theory, sets, and numeration systems.
- D. Communicate mathematical ideas in both written and oral form.
- E. Describe connections that exist between various areas; for example, place value and different number systems or sets and number theory.
- F. Distinguish three types of computation- mental, paper and pencil algorithms, and calculators- can use all three to solve problems and to teach mathematics, and can tell which is more appropriate for a given situation.

## II Course Content Outline

Text: [A Problem Solving Approach to Mathematics](#), 10<sup>th</sup> ed., by R. Billstein, S. Libeskind, and J. Lott, Addison-Wesley Publishing Company, 2010.

- A. General Framework for Teaching- to be incorporated and modeled throughout the course
  - 1. Relevant learning theories
  - 2. NCTM *Curriculum and Evaluation Standards* and Missouri *Show Me Standards*
  - 3. Assessment techniques
  - 4. Identification of resources and resource materials
  - 5. Use of technology

- B. Sets
  - 1. Review properties and operations on sets
  - 2. Discussion of relevance of topic and activities for teaching sets
- C. Number and Numeration Systems
  - 1. Place value and use of base 10 pieces
  - 2. Standard and non-standard algorithms for whole number operations
  - 3. Non-decimal numeration systems
  - 4. Operations on Non-decimal numeration systems
- D. Number Theory
  - 1. Multiples and divisibility
  - 2. Primes, composites, and factorization
  - 3. Greatest common divisor and least common multiple
  - 4. Properties of mathematical systems
- E. Fractions
  - 1. Concrete and semi-concrete models for representing fractions and performing operations with fractions (fraction strips, fraction table and fraction circles)
  - 2. Operations with fractions using various algorithms

#### IV. Evaluation

Grade will be based on homework assignments, two hour-long tests, review of an article from an NCTM journal, and a comprehensive departmental final.

A sample grading scale:

- A 90-100%
- B 80-89%
- C 70-79%
- D 60-69%
- F 0-59%

A Course Syllabus

for

Math 2801

**Concepts and Methods for Elementary School Mathematics**

(Three semester hours credit)

in the

Department of Mathematics and Computer Science

of the

COLLEGE OF SCIENCE AND TECHNOLOGY

Catalog Description:

A nine-week course focusing on the concepts and methods of teaching mathematics in grades K-5. Prerequisite: Math 1800

CENTRAL MISSOURI STATE UNIVERSITY

Warrensburg, MO

## I. Purpose of the Course

This three-hour course provides instruction on the content and methods of teaching the mathematics for elementary school K-5. Specifically, the course is designed to:

- A. Develop basic concepts, skills, and techniques for teaching sets and counting numbers, numeration systems, rational numbers, measurement, geometry, probability, and statistics for the elementary school.
- B. Develop models for teaching which focus on the appropriate mathematical content, relevant learning theories, and alternative teaching strategies.

## II. Objectives and Desired Student Competencies

Upon completion of this course, the student should be able to:

- A. Define, describe, and identify the sets of numbers (counting, whole, rational, integer) and their properties.
- B. Use a variety of mathematical skills including the four fundamental operations and problem solving.
- C. Apply mathematical concepts/skills to solve routine and non-routine problems.
- D. Communicate mathematical ideas in both written and oral form.
- E. Determine when and where to use the three types of computation --- mental, paper and pencil, and calculators B and use all three to solve problems as well as in a teaching activity.
- F. Recognize and analyze basic geometric shapes as well as describe their properties and other basic geometric figures (points, lines, planes, etc.).
- G. Develop activities appropriate for elementary children to illustrate the basic concepts for probability and statistics.

## III. Course Content Outline

Text: A Problem Solving Approach to Mathematics, 10<sup>th</sup> ed., by R. Billstein, S. Libeskind, and J. Lott, Addison-Wesley Publishing Company, 2010.

- A. Problem Solving
  - 2. Teaching/learning strategies to problem solving
  - 3. Critical thinking, inductive/deductive reasoning, looking for a pattern, and finding the general case.
  
- B. Number and Numeration Systems
  - 1. Use of concrete models for representing place value and operations on 2-digit numbers ; for example the use of bean sticks or popsicle sticks)
  - 2. Estimation skills
  
- C. Rational Numbers
  - 1. Percent
  - 2. Introduction to proportion
  - 4. Integers (concrete models for work with integers)
  - 5. Operations with integers
  
- D. Measurement
  - 1. Standard and non-standard measurements with teaching strategies
  - 2. Conversions within the metric system
  - 3. Use of concrete models for perimeter and area of triangles, quadrilaterals, and circles
  - 4. Use of concrete models for surface area and volume for prisms and cylinders
  
- E. Geometry
  - 1. Recognizing and analyzing basic shapes
  - 2. Properties of basic shapes
  - 3. Reflective and rotational symmetry
  - 4. Points, lines, planes, and angles
  - 5. Rigid transformations and tessellations
  - 6. Activities to develop spatial abilities
  
- F. Probability
  - 1. Review definitions and computations of simple probability, combinations, and permutations
  - 2. Review sample spaces (lists, trees, etc.)
  - 3. Fundamental principle of counting
  - 4. Teaching strategies/activities for probability

G. Statistics

1. Graphs for representing data
  - a. stem and leaf
  - b. box and whisker
  - c. histograms
6. Review measures of central tendency
7. Teaching/learning activities for statistic

IV Grading and Evaluation

Student evaluation uses a criterion-referenced model on group and individual assignments, special projects, regular examinations, and a comprehensive final exam.

A sample grading scale:

- A 90-100%
- B 80-89%
- C 70-79%
- D 60-69%
- F 0-59%

APPENDIX C  
STUDY APPROVAL DOCUMENTATION

**McCoy, Ann C. (UMKC-Student)**

**From:** hughesge@umkc.edu [hughesge@umkc.edu] **Sent:** Mon 1/4/2010 1:15 PM  
**To:** McCoy, Ann C. (UMKC-Student)  
**Cc:** Hughes, Germaine; Anderman, Shella H.; Barger, Rita  
**Subject:** Study 091115: Investigating the Effect of Efficacy on the Mathematical Knowledge for Teaching of Elementary Pre-Service Teachers

**Attachments:**

January 4, 2010

Ann McCoy  
101 Stayton Drive  
Clinton, MO 64735

Approval Date: December 11, 2009

RE: SSIRB Protocol #: 091115: Investigating the Effect of Efficacy on the Mathematical Knowledge for Teaching of Elementary Pre-Service Teachers

Dear Ann McCoy:

Your request for approval of the research study # 091115 - Investigating the Effect of Efficacy on the Mathematical Knowledge for Teaching of Elementary Pre-Service Teachers was reviewed on December 11, 2009 through the Social Sciences Institutional Review Board's exempt review process.

You have met the requirements of the restrictions.

Your study has been reviewed and it has been determined that under § 46.101 of the Department of Health and Human Services' regulations pertaining to the protection of human subjects of research 45 CFR 46.101(b)(2), it is exempt from review by UMKC's Social Sciences Institutional Review Board (SSIRB).

You have full approval of the consent form SSIRB date stamped 1/4/2010 thru 12/10/2010 (which will follow in a separate email).

Therefore you may proceed with your work.

Should you want to make any changes to the approved study, you will need to obtain prior SSIRB permission. The exempt status of your study ends on 12/10/2010.

An SSIRB progress report/amendment request/continuation request/completion report form can be downloaded from our website. You should complete and return this form to the SSIRB at the earlier of any request to amend your study, completion of your work, or on 12/10/2010.

Best wishes for a successful study. If we can be of further assistance, please don't hesitate to call the SSIRB office at 816-235-1764.

**PLEASE NOTE:**

If you are using a signed consent form you must use the copy of the consent form that has been stamped and approved by the SSIRB, which is attached, before you begin consenting subjects. All subjects must be consented on a copy of the approved consent form with the SSIRB Stamp. If requested, a hard copy of the stamped consent can be mailed to you.

Thanks,

Ms. Germaine Hughes  
Administrator  
Social Sciences Institutional Review Board  
University of Missouri - Kansas City  
5319 Rockhill Road  
Kansas City, MO 64110-2499  
Office: 816-235-1764  
Fax: 816-235-5602  
hughesge@umkc.edu

**CONSENT TO PARTICIPATE IN A RESEARCH STUDY**  
**Mathematics Teacher Efficacy**

You are being invited to participate in a research study.

This study will be conducted by Ann McCoy, a faculty member in the Department of Mathematics and Computer Science at the University of Central Missouri and an I.Ph.D. student at the University of Missouri-Kansas City.

Approximately 120 preservice elementary teachers who are enrolled in Math 1800/2801 during the spring 2010 semester or the fall 2010 semester will be invited to participate in this study.

The purpose of this study is to learn how the level of mathematics teacher efficacy of preservice teachers impacts the growth they experience in mathematical knowledge for teaching during mathematics methods/content courses.

You will be completing two paper and pencil instruments – one at the beginning of the semester and one at the end of the semester. The instruments will be administered during a regularly scheduled class meeting by an assistant to the researcher. The instrument given at the beginning of the semester is made up of four sections – demographic questions, questions designed to measure common mathematical content knowledge, questions designed to measure specialized mathematical content knowledge needed for teaching, and mathematics teacher efficacy. This instrument will take no more than 45 minutes to complete. The instrument administered at the end of the semester will be made up of three sections – questions designed to measure specialized mathematical content knowledge, common content knowledge, and mathematics teacher efficacy. This instrument will take no more than 40 minutes to complete.

You will be asked to create a code consisting of your mothers' maiden name and the city of your birth – information unknown to the researcher. This code will be used on both instruments. You may choose to skip any items that you wish and you may withdraw from the study at any point. An assistant to the researcher will administer all measures and the researcher will not be present during administration. No list of participants will be created. All data will be reported as aggregates when findings are presented and the name of this university will not be used in reporting findings.

Your participation in this research is voluntary. You may choose to not participate or to withdraw your participation at any time. If you choose not to participate, your grade in Math 1800/2801 will not be affected in any manner.

You are not responsible for any costs or expenses associated with this study.

You will receive a \$5 coupon to be used at a local restaurant upon completion of each of the two instruments.

UMKC SOCIAL SCIENCES  
INSTITUTIONAL REVIEW BOARD  
INIT ~~5/11~~ APPRVD from: 4/6/10 to: 12/10/10

There are no anticipated risks due to participation in this study.

Although there are no foreseeable direct benefits to you, it is hoped that the research will benefit future preservice teachers. Your participation in this study will provide the university with data needed to make informed decisions about the mathematics courses offered for elementary education majors.

While every effort will be made to keep confidential all of the information you provide, it cannot be completely guaranteed. Individuals from the University of Missouri-Kansas City Institutional Review Board or the Human Subjects Review Board of the University of Central Missouri (committees that reviews and approves research studies) may look at records related to this study. Remember that you will be creating and using a code on all study instruments. In addition, completed instruments will be kept in a locked cabinet in the researcher's office and will be destroyed when the study is completed.

The University of Missouri-Kansas City appreciates the participation of people who help it carry out its function of developing knowledge through research. If you have questions about the study, you are encouraged to call Ann McCoy, the investigator, at 660-543-4386 or by email at [mccoy@ucmo.edu](mailto:mccoy@ucmo.edu).

Although it is not the University's policy to compensate or provide medical treatment for persons who participate in studies, if you think you have been injured as a result of participating in this study, please call the IRB Administrator of UMKC's Social Sciences Institutional Review Board at 816-235-1764. You may also contact the Human Subjects Protection Program of the University of Central Missouri at (660) 543-4621.

By signing your name below, you are indicating that (1) you have read this form, (2) you agree to participate in this study, (3) you have received a copy of this consent form, and (4) you agree to have the information you share in this study be used for the stated research purposes. If you choose to participate, please complete and sign one copy of this consent form and retain the other for your records. You will give the signed form to the researcher's assistant who will seal the forms in an envelope.

Printed Name of Participant \_\_\_\_\_

Participant Signature \_\_\_\_\_ Date \_\_\_\_\_

Printed Name of Witness \_\_\_\_\_

Witness Signature \_\_\_\_\_ Date \_\_\_\_\_

UMKC SOCIAL SCIENCES  
INSTITUTIONAL REVIEW BOARD  
INITIALS APPROVED from: *Handwritten initials*



Human Subjects Protection Program  
Ward Edwards 1800  
Warrensburg, MO 64093  
Office 660-543-4621  
FAX 660-543-4778  
[www.ucmo.edu](http://www.ucmo.edu)

7/21/2009

Ann C. McCoy  
WCM 115  
UCM  
Warrensburg, MO/64093

Dear Ms. Ann C. McCoy,

Your research project, 'Investigating the Effect on Efficacy on the Mathematical Knowledge for Teaching of Elementary Pre-Service Teachers', was approved by the Human Subjects Review Committee on 7/14/2009. This approval is valid through 7/14/2010.

Please note that you are required to notify the committee in writing of any changes in your research project and that you may not implement changes without prior approval of the committee. You must also notify the committee in writing of any change in the nature or the status of the risks of participating in this research project.

Should any adverse events occur in the course of your research (such as harm to a research participant), you must notify the committee in writing immediately. In the case of any adverse event, you are required to stop the research immediately unless stopping the research would cause more harm to the participants than continuing with it.

At the conclusion of your project, you will need to submit a completed Project Status Form to this office. You must also submit the Project Status Form if you wish to continue your research project beyond its initial expiration date.

If you have any questions, please feel free to contact me at the number above.

Sincerely,

Joseph Vaughn, Ph.D.  
Assistant Provost for Research  
and Dean of the School of  
Graduate and Extended Studies



Human Subjects Protection Program  
Ward Edwards 1800  
Warrensburg, MO 64093  
Office 660-543-4621  
FAX 660-543-4778  
www.ucmo.edu

5/26/2010

Ann C. McCoy  
WCM 115  
UCM  
Warrensburg, MO/64093

Dear Ms. Ann C. McCoy,

Your research project, 'Investigating the Effect on Efficacy on the Mathematical Knowledge for Teaching of Elementary Pre-Service Teachers', was approved by the Human Subjects Review Committee on 5/26/2010. This approval is valid through 5/26/2011. Your informed consent is also approved until 5/26/2011.

Please note that you are required to notify the committee in writing of any changes in your research project and that you may not implement changes without prior approval of the committee. You must also notify the committee in writing of any change in the nature or the status of the risks of participating in this research project.

Should any adverse events occur in the course of your research (such as harm to a research participant), you must notify the committee in writing immediately. In the case of any adverse event, you are required to stop the research immediately unless stopping the research would cause more harm to the participants than continuing with it.

At the conclusion of your project, you will need to submit a completed Project Status Form to this office. You must also submit the Project Status Form if you wish to continue your research project beyond its initial expiration date.

If you have any questions, please feel free to contact me at the number above.

Sincerely,

Wendy Geiger, Ph.D.  
Associate Dean of The Graduate School  
geiger@ucmo.edu

## REFERENCES

- Adler, S. A. (1991). The education of social studies teachers. In J. P. Shaver (Ed.), *Handbook of research on social studies teaching and learning*, (pp. 210-221). New York: Macmillan.
- Allinder, R. M. (1995). An examination of the relationship between teacher efficacy and curriculum-based measurement and student achievement. *Remedial and Special Education, 16*(4), 247-254.
- Arbaugh, F., Lannin, J.K., Jones, D.L., & Park-Rogers, M.A. (2006). Examining instructional practices in Core-Plus lessons: Implications for professional development. *Journal of Mathematics Teacher Education 9*(6), 517-550.
- Armor, D., Conroy-Oseguera, P., Cox, M., King, N., McDonnell, L., Pascal, A., Pauly, E., & Zellman, G. (1976). *Analysis of the school preferred reading programs in Selected Los Angeles minority schools* (Rep. No R-2008-LAUSD). Santa Monica, CA: RAND. (ERIC Document Reproduction Service No, 130 243)
- Ashton, P. T., Olejnik, S., Crocker, L., & McAuliffe, M. (1982, April). *Measurement problems in the study of teachers' sense of efficacy*. Paper presented at the Annual Meeting of the American Educational Research Association, New York.
- Ashton, P., Buhr, D., & Crocker, L. (1984). Teachers' sense of efficacy: A self- or norm-referenced construct? *Florida Journal of Educational Research, 26*(1), 29-41.
- Ashton, P. T., & Webb, R. B. (1986). *Making a difference: Teachers' sense of efficacy and student achievement*. New York: Longman.

- Ashton, P. T., Webb, R. B., & Doda, C. (1983). *A study of teachers' sense of efficacy*.  
Gainesville, FL: University of Florida.
- Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal of Research in Mathematics Education, 21*, 132-144.
- Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics*, (pp. 83 – 104).  
Westport, CT: Ablex.
- Ball, D. L., Bass, H., Boerst, T., Cole, Y., Jacobs, J., Kim, Y., et al. (2008, October). *Developing Teachers' Mathematical Knowledge for Teaching*. Paper presented at the Mathematics Focus Session, University of North Carolina, Chapel Hill, NC.
- Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching. *American Educator*, Fall 2005, 14-46.
- Ball, D. L., Hill, H.C., Rowan, B., & Schilling, S. (2002) Measuring teachers' content knowledge for teaching: Elementary mathematics release items, 2002. Ann Arbor, Michigan: Study of Instructional Improvement.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.
- Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review, 84*(2), 191-215.
- Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory*.

Englewood Cliffs, NJ: Prentice Hall.

Bandura, A. (1989). Regulation of cognitive processes through perceived self-efficacy.

*Developmental Psychology*, 25, 729-735.

Bandura, A. (1994). Self-efficacy. In V. S. Ramachaudran (Ed.), *Encyclopedia of human behavior* (Vol. 4, pp. 71-81). New York: Academic Press. (Reprinted in H.

Friedman [Ed.], *Encyclopedia of mental health*. San Diego: Academic Press, 1998).

Banilower, E. R., Boyd, S. E., Pasley, J. D., & Weiss, I. R. (2006). *Lessons from a decade of mathematics and science reform: A capstone report for the local systemic change through teacher enhancement initiative*. Chapel Hill, NC: Horizon Research, Inc.

Barger, R. (1998). University of Missouri-Kansas City: Number and operations instrument. Unpublished instrument.

Barlow, A. T., & Reddish, J. M. (2006). Mathematical myths: Teacher candidates' beliefs and the implications for teacher education. *The Teacher Educator*, 41(3), 145-157.

Battista, M. T. (1986). The relationship of mathematics anxiety and mathematical knowledge to the learning of mathematical pedagogy by preservice elementary teachers. *School Science and Mathematics*, 86, 10-19.

Beckman, C. E., Wells, P. J., Gabrosek, J., Billings, E.M., Aboufadel, E. F., Curtiss, P. (2004). Enhancing the mathematical understanding of prospective teachers using standards-based grades K-12 activities. In R. N. Rubenstein & G. W. Brights

- (Eds.), *Perspectives on the teaching of mathematics, sixty-sixth yearbook*, (pp. 151-163). Reston, VA: National Council of Teachers of Mathematics.
- Begle, E. G. (1979). *Critical variables in mathematics education: Findings from a survey of the empirical literature*. Washington, D. C: Mathematical Association of America and National Council of Teachers of Mathematics.
- Benz, C. R., Bradley, L., Flowers, M. A., & Alderman, M. K., (1992). Personal teaching efficacy: Developmental relationships in education. *Journal of Educational Research*, 85(5), 274-285.
- Beswick, K. (2006). Changes in preservice teachers' attitudes and beliefs: The net impact of two mathematics education units and intervening experiences. *School Science and Mathematics*, 106(1), 36-47.
- Brand, B. R., & Wilkins, J. L. (2007). Using self-efficacy as a construct for evaluating Science and mathematics methods courses. *Journal of Science Teacher Education*, 18(2), 297-317.
- Brouwer, N., & Korthagen, F. (2005). Can teacher education make a difference? *American Educational Research Journal*, 42(1), 153-224.
- Brown, C. A. & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. (pp. 209-242). NewYork: Macmillan.
- Brown, C. A., & Cooney, T. J. (1982). Research on teacher education: A philosophical orientation. *Journal of Research and Development in Education*, 15(4), 13-18.
- Bullough, R. V. (1989). *First-year teacher: A case study*. New York: Teachers College

Press.

- Bullough, R. V. (1997). Practicing theory and theorizing practice. In J. Loughran & T. Russell (Eds.), *Purpose, passion, and pedagogy in teacher education*. (pp. 13-31). London, Washington D.C.: Falmer Press.
- Buss, R. R. (2010). Efficacy for teaching elementary science and mathematics compared to other content. *School Science and Mathematics, 110*(6), 290-297.
- Cady, J., Meier, S. L., Lubinski, C. A. (2006). Developing mathematics teachers: The transition from preservice to experienced teacher. *The Journal of Educational Research, 99*(5), 295-305.
- Calderhead, J. & Robson, M. (1991). Images of teaching: Student teachers' early conceptions of classroom practice. *Teaching and Teacher Education, 7*, 1-8.
- Carpenter, T., Fennema, E., Peterson, P. L., & Carey, D. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education, 19*, 385-401.
- Castro, A. M. (2006). Preparing elementary preservice teachers to use mathematics curriculum materials. *The Mathematics Educator, 16*(2), 14-24.
- Chang, Y. L. & Wu, S. C. (2006). Teacher efficacy and elementary teacher education. *Academic Exchange Quarterly, 10*(3), 75-79.
- Coladarci, T. (1992). Teachers' Sense of Efficacy and Commitment to Teaching. *Journal of Experimental Education, 60*(4), 323-327.
- Cole, A. L., & Knowles, J. G. (1993). Shattered images: Understanding expectations and realities of field experiences. *Teaching and Teacher Education, 9*, 457-471.

- Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers*. Providence, RI: American Mathematical Association.
- Cruikshank, D. E., & Sheffield, L. J., (1992). *Teaching and learning elementary and middle school mathematics*. Columbus, OH: Merrill.
- da Ponte, J. P., & Chapman, O. (2008). *Preservice mathematics teachers' knowledge and development*. In L. D. English (Ed.), *Handbook of International Research in Mathematics Education*, 2<sup>nd</sup> edition, New York: Routledge.
- Darling-Hammond, L. (2000). Teacher quality and student achievement: A review of state policy evidence. *Education Policy Analysis Archives*, 8(1).
- Delany, S. F., Sleep, L., Ball, D. L., Bass, H., Hill, J. C., & Dean, C. (2005, April). *Validating "Specialized Mathematics Knowledge for Teaching": Evidence from Mathematicians*. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Canada.
- Deng, Z. (2007). Transforming the subject matter: Examining the intellectual roots of pedagogical content knowledge. *Curriculum Inquiry*, 37(3), 279-295.
- Emmer, E., & Hickman, J. (1990, April). *Teacher decision making as a function of efficacy, attribution, and reasoned action*. Paper presented at the Annual Meeting of the American Educational Research Association, Boston. (Revised June, 1990).
- Enochs, L. G., Smith, P. L., & Huinker, D. (2000). Establishing factorial validity of the mathematics teaching efficacy beliefs instrument. *School Science and Mathematics*, 100, 194-203.

- Ernest, P. (1989). The knowledge, beliefs, and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, 15, 13-34.
- Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, 24(2), 94-116.
- Even, R., & Lappan, G. (1994). Constructing meaningful understanding of mathematics content. In D. B. Aichele & A. F. Coxford (Eds.), *Professional Development for Teacher of Mathematics*, (pp. 128-143). Reston, VA: National Council of Teachers of Mathematics.
- Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: MacMillan Publishing Co.
- Forgione, P. D. (1998). *Achievement in the United States: Progress since a nation at risk?* Washington, D. C.: National Center for Education Statistics.
- Frykholm, J. (2005). Innovative curricula: Catalysts for reform in mathematics teacher education. *Action in Teacher Education*, 26(4), 20-36.
- Gess-Newsome, J. (1999). Introduction and orientation to examining pedagogical content knowledge. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge*, (pp. 3-20). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Ghaith, G., & Yaghi, H. Relationships among experience, teacher efficacy, and attitudes toward the implementation of instructional innovation. *Teaching and Teacher*

- Education*, (13), 4, pages 451-458.
- Gibson, S. & Dembo, M. H. (1984). Teacher efficacy: A construct validation. *Journal of Educational Psychology*, 76, 769-582.
- Ginsburg, M., & Newman, K. (1985). Social inequalities, schooling, and teacher education. *Journal of Teacher Education*, 26, 49-54.
- Gleason, J. (2010). Reliability of the content knowledge for teaching – mathematics instrument for preservice teachers. *Issues in Undergraduate Mathematics Preparation of School Teachers*, 1.  
Retrieved from <http://www.k-12prep.math.ttu.edu>
- Grossman, P. L. (1990). *The makings of a teacher: Teacher knowledge and teacher education*. New York: Teachers College Press.
- Guberman, R. & Gorev, D. (2008, July). *Learning mathematics for teaching: Sources of subject-matter knowledge*. Paper presented at the meeting of the International Congress on Mathematics Education, Montérrey, Mexico.
- Guskey, T. R. (1981). Measurement of responsibility teachers assume for academic successes and failures in the classroom. *Journal of Teacher Education*, 32, 44-51.
- Guskey, T. R. (1988) *Teaching and Teacher Education: An International Journal of Research and Studies*, 4(1), 63-69.
- Guskey, T. R., & Passaro, P. D. (1994). Teacher efficacy: A study of construct dimensions. *American Educational Research Journal*, 31,627-643.
- Hiebert, J., & Stigler, J. (2004). A world of difference. *Journal of Staff Development (JSD)*, 25(4), 10- 15.

- Hill, H. C. & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's Mathematics ) Professional Development Institutes. *Journal for Research in Mathematics Education*, 35(5), 330-352.
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372 – 400.
- Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., et al. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, 26, 430 – 511.
- Hill, H. C., Dean, C., & Goffney, I. M. (2007). Assessing elemental and structural validity: Data from teachers, non-teachers, and mathematicians. *Measurement: Interdisciplinary Research and Perspective*, 5(2), 81-92.  
doi: 10.1080/15366360701486999
- Hill, H. C., & Lubienski, S. T. (2007). Teachers' mathematics knowledge for teaching and school context: A study of California teachers. *Educational Policy*, 21(5), 747 – 768.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 45(2), 371 - 406.
- Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers'

- mathematics knowledge for teaching. *The Elementary School Journal*, 105(1), 11- 30.
- Hoy, A. W. (2000). *Changes in teacher efficacy during the early years of teaching*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Hoy, A. W. (2004). The educational psychology of teacher efficacy. *Educational Psychology Review*, 15, 153-176.
- Hoy, A. W., & Woolfork, A. E. (1990). Socialization of student teachers. *American Educational Research Journal*, 27, 279-300.
- Kagan, D. M. (1992). Professional growth among preservice and beginning teachers. *Review of Educational Research*, 62(2), 129-169.
- Kelly, W. P., & Tomhave, W. (1982). A study of math anxiety and math avoidance in preservice elementary teachers. *Arithmetic Teacher*, 32, (51-53).
- Kuhs, T. M. & Ball, D. L. (1986). Approaches to teaching mathematics: Mapping the domains of knowledge, skills, and dispositions. East Lansing: Michigan State University, Center on Teacher Education.
- Leinhardt, G. & Smith, D. A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology*, 77(3), 247-271.
- Liljedahl, P., Rolka, K., & Rösken, B. (2007). Affecting affect: The reeducation of preservice teachers' beliefs about mathematics and mathematics learning and teaching. In W. G. Martin, M. E. Strutchens, & P.C. Elliott (Eds.), *The learning and teaching of mathematics, sixty-ninth yearbook* (pp. 319-330). Reston,

VA: National Council of Teachers of Mathematics.

Lortie, D. (1975). *Schoolteacher: A sociological study*. Chicago: University of Chicago Press.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Matthews, M., Rech, J., & Grandgenett, N. (2010). The impact of content courses on preservice elementary teachers' mathematical content knowledge. *Issues in Undergraduate Mathematics Preparation of School Teachers, 1*. Retrieved from <http://www.k-12prep.math.ttu.edu>

McDaniel, J. E. (1991, April). Close encounters: How do student teachers make sense of the social foundations? Paper presented at the annual meeting of the American Educational Research Association, Chicago.

McKnight, C. C., Crosswhite, F. J., Dossey, J. A., Kifer, E., Swafford, J. O., Travers, K. J., & Cooney, T. J. (1987). *The underachieving curriculum: Assessing U. S. school mathematics from an international perspective*. Champaign, IL: Stipes.

Midgley, C., Feldaufer, H., & Eccles, J. (1989). Change in teacher efficacy and student self- and task-related beliefs in mathematics during the transition to junior high school. *Journal of Educational Psychology, 81*, 247-258.

Minor, L. C., Onwuegbuzie, L. J., Witcher, A. W., & James, T. L. (2002). Preservice teachers' educational beliefs and their perceptions of characteristics of effective teachers. *Journal of Educational Research, 96*(2), 116-127.

Moore, W., & Esselman, M. (1992, April). *Teacher efficacy, power, school climate and*

- achievement: A desegregating district's experience*. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco.
- Morris, A. K., Hiebert, J., & Spitzer, S. M. (2009). Mathematical knowledge for teaching in planning and evaluating instruction: What can preservice teachers learn? *Journal for Research in Mathematics Education*, 40(5), 491-529.
- Myers, P. (2007). Why? Why? Why? Future teachers discover mathematical depth. *Phi Delta Kappan*, 88(9), 691-696.
- National Center for Education Statistics. (2010). Mathematics literacy performance of 15-year-olds. Retrieved from [http://nces.ed.gov/surveys/pisa/pisa2009highlights\\_3.asp](http://nces.ed.gov/surveys/pisa/pisa2009highlights_3.asp)
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*, Reston, VA: Author.
- National Council on Teacher Quality. (2008). No common denominator: The preparation of elementary teachers in mathematics by American education schools. Retrieved from [http://www.nctq.org/p/publications/docs/nctq\\_ttmath\\_fullreport\\_2008071708174.pdf](http://www.nctq.org/p/publications/docs/nctq_ttmath_fullreport_2008071708174.pdf)
- National Mathematics Advisory Panel. (2008). Foundations of success: Final report of the National Mathematics Advisory Panel. Retrieved from <http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>
- National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Study Committee,

Center for Education, Division of Behavioral and Social Sciences and Education.  
Washington, D. C.: National Academy Press.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies*, 19(4), 317-328.

Onderi, H., & Croll, P. (2009). Teacher self-perception of effectiveness: A study in a district of Kenya. *Educational Research*, 51(1), 97-107.

Orton, R. E. (1996). How can teacher beliefs about student learning be justified?  
*Curriculum Inquiry*, 26(2), 133-146.

Pajares, M. F. (1992). Teacher's beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62(3), 307-332.

Philipp, R. A., Ambrose, R., Lamb, L. L., Sowder, J., Schappell, B. P., Sowder, L., et al. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education*, 38(5), 438-476.

Quinn, R. J. (1997). Effects of mathematics methods courses on the mathematical attitudes and content knowledge of preservice teachers. *Journal of Educational Research*, 91(2). 108-113.

Raudenbush, S., Rowen, B., & Cheong, Y. (1992). Contextual effects on the self-perceived efficacy of high school teachers. *Sociology of Education*, 65, 150-167.

Riggs, I. M., & Enochs, L. G. (1990). Toward the development of an elementary teacher's science teaching efficacy belief instrument. *Science Education*, 74, 625-637.

Rohs, J. M. (2007). *Child outcomes in Head Start classrooms: The impact of teacher*

- beliefs and interactions*. Unpublished doctoral dissertation, University of Missouri, Kansas City.
- Rose, J. A., & Medway, F. J. (1981). Measurement of teachers' beliefs in their control over student outcome. *Journal of Educational Research, 74*, 185-190.
- Ross, E. W. (1987). Teacher perspective development: A study of preservice social studies teachers. *Theory and Research in Social Education, 15*, 225-243.
- Ross, J. A. (1992). Teacher efficacy and the effect of coaching on student achievement. *Canadian Journal of Education, 17*, 51-65.
- Ross, J. A. (1994). The impact of an inservice to promote cooperative learning on the stability of teacher efficacy. *Teaching and Teacher Education, 10*(4), 381-394.
- Rotter, J. B. (1966). Generalized expectancies for internal versus external control of reinforcement. *Psychological Monographs, 80*, 1-28.
- Rowan, B., Schilling, S. G., Ball, D. L., & Miller, R. (2001). *Measuring teachers' pedagogical content knowledge in surveys: An exploratory study*. Ann Arbor: Consortium for Policy Research in Education, University of Pennsylvania.
- Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*, 255-281.
- Saklofske, D., Michaluk, B., & Randhawa, B. (1988). Teachers' efficacy and teaching behaviors. *Psychological Report, 63*, 407-414.
- Schunk, D. H. (1989). Self-efficacy and cognitive skill learning. In C. Ames & R. Ames

- (Eds.) *Research on motivation in education: Vol. 3. Goals and cognitions*  
(pp. 13-44) San Diego, CA: Academic.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching.  
*Educational Researcher*, 15(2), 4-14.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform.  
*Harvard Educational Review*, 57, 1-22.
- Siedel, H., & Hill, H. (2003). *Content validity: Mapping SII/LMT mathematical items  
Onto NCTM and California standards*. Unpublished manuscript, University of  
Michigan, Ann Arbor.
- Siegle, D., & McCoach, D. B. (2007). Increasing students mathematics self-efficacy  
through teacher training. *Journal of Advanced Academics*, 18(2), 278-312.
- Silver, E. A., & Kenney, P. A. (Eds.). (2000). *Results from the seventh mathematics  
assessment of the National Assessment of Educational Progress*. Reston, VA:  
National Council of Teachers of Mathematics.
- Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of  
mathematical knowledge for teaching. *Journal for Mathematics Teacher  
Education*, 11, 499 – 511.
- Snider, V. E., & Roehl, R. (2007). Teachers' beliefs about pedagogy and related issues.  
*Psychology in the Schools*, 44(8), 873-886.
- Stuart, C., & Thurlow, D. (2000). Making it their own: Preservice teachers' experiences,  
beliefs, and classroom practices. *Journal of Teacher Education*, 51(2), 113-121.

- Sutton, J., & Krueger, A. (Eds.). (2002). *EDThoughts: What we know about mathematics teaching and learning*. Aurora, CO: Mid-continent Research for Education and Learning.
- Swars, S. L. (2005). Examining perceptions of mathematics teaching effectiveness among elementary preservice teachers with differing levels of mathematics teacher efficacy. *Journal of Instructional Psychology, 32*(2), 139-147.
- Swars, S. L., Daane, C. J., & Giesen, J. (2006). Mathematics anxiety and mathematics teacher efficacy: What is the relationship in elementary preservice teachers? *School Science and Mathematics, 106*(7), 306-315.
- Swars, S. L., Hart, L. C., Smith, S. Z., Smith, M. E., & Tolar, T. (2007). A longitudinal study of elementary preservice teachers' mathematics beliefs and content knowledge. *School Science and Mathematics, 107*(8), 325-335.
- Tarr, J. E., Chavez, O., Reys, R. E. & Reys, B. J. (2006). From the written to enacted curriculum: The intermediary role of middle school mathematics teachers in shaping students' opportunities to learn. *School Science and Mathematics, 106*(4), 191-201.
- Thames, M. H., Sleep, L. Bass, H., & Ball, D. L. (2008, July). *Mathematical knowledge for teaching (K-8): Empirical, theoretical, and practical foundations*. Paper presented at the meeting of the International Congress on Mathematics Education, Montérrey, Mexico.
- Thompson, A. G. (1982). Teachers' conceptions of mathematics: Three case studies.

- Unpublished doctoral dissertation. University of Georgia: Athens.
- Thompson, A. G. (1988). Learning to teach mathematical problem solving: Changes in teachers' conceptions and beliefs. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving, Vol. 3*. Reston, VA: National Council of Teachers of Mathematics.
- Tschannen-Moran, M., Woolfolk Hoy, A., & Hoy, W. K. (1998). Teacher efficacy: It's meaning and measure. *Review of Educational Research, 68*(2), 202-248.
- Veenman, S. A. (1984). Perceived problems of beginning teachers. *Review of Educational Research, 54*, 143-178.
- Vinson, B. M. (2001). A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives. *Early Childhood Education Journal, 29*(2), 89-94
- Watson, S. (1991). *A study of the effects of teacher efficacy on the academic achievement of third-grade students in selected elementary schools in South Carolina*. Unpublished doctoral dissertation, South Carolina State College, Orangeburg. (University Microfilms No. UMI 9230552)
- Weiss, I. R. (1995). A profile of science and mathematics education in the United States. Chapel Hill, NC: Horizon Research.
- Weiss, I. R., Banilower, E. R., McMahon, K. C., & Smith, P. S. (2001). Report of the 2000 survey of science and mathematics education. Chapel Hill, NC: Horizon Research.
- Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.

- Wideen, M., Mayer-Smith, J., & Moon, B. (1998). A critical analysis of the research on learning to teach: Making the case for an ecological perspective on inquiry. *Review of Educational Research, 68*, 130-178.
- Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*, 139-164. doi: 10.1007/s10857-007-9068-2
- Wingfield, M., Nath, J. L., Freeman, L., & Cohen, M. (2000, April). *The effect of site-based preservice experiences on elementary social studies, language arts, and mathematics teaching self-efficacy beliefs*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Woolfork, A.E., Rosoff, B., & Hoy, W. K. (1990). Teachers' sense of efficacy and their beliefs about managing students. *Teaching and Teacher Education, 6*, 81-91.
- Zambo, R., & Zambo, D. (2008). The impact of professional development in mathematics on teachers' individual and collective efficacy: The stigma of underperforming. *Teacher Education Quarterly, 35*(1), 159-168.
- Zeichner, K. M., & Tabachnick, B. R. (1985). Are the effects of university teacher education "washed out" by school experience? *Journal of Education for Teaching, 32*, 7-13.
- Zeichner, K., & Gore, J. (1990). Teacher socialization. In W. R. Houston (Ed.), *Handbook of research on teacher education*, (pp. 329-348). New York: Macmillan.

## VITA

Ann Zumbrunnen McCoy was born on November 1, 1961 in Clinton, Missouri. She attended elementary, middle, and high school in Clinton and graduated in 1980 from Clinton High School. She graduated summa cum laude from Central Missouri State University in 1983 with a B.S.E. in elementary education with a concentration in mathematics. Upon completion of her undergraduate work, Mrs. McCoy taught middle school mathematics at Clinton Middle school. She earned an M.S.E. in curriculum and instruction with an emphasis in mathematics education from Central Missouri State University in 1988. She continued working in the Clinton School District teaching at both the elementary and middle schools until 2004 when she began working as a consultant for the Missouri Mathematics Academy.

In 2006, Mrs. McCoy began working towards her Interdisciplinary Ph.D. at the University of Missouri-Kansas City with co-disciplines in Education and Urban Leadership and Policy Studies in Education. While working towards this degree, she has had the opportunity to teach at the University of Central Missouri (UCM) in Warrensburg, Missouri, first as a visiting lecturer, and most recently as an assistant professor of mathematics education. At UCM, she teaches a variety of courses for preservice teachers as well as general studies mathematics courses.

Upon completion of degree requirements, Mrs. McCoy plans to continue her career as a mathematics teacher educator and to pursue her research interests in the preparation and professional development of elementary teachers in the area of mathematics. Mrs.

McCoy is a member of the National Council of Teachers, the Association of Mathematics Teacher Educators, the Missouri Council of Teachers of Mathematics, the Missouri Mathematics Association for the Advancement of Teacher Training, and the Kansas City Area Teachers of Mathematics. She is currently serving as vice-president/president elect of the Missouri Mathematics Association for the Advancement of Teacher Training.