DESIGN AND SIMULATION ANALYSIS OF MEMS PARALLEL PLATE CAPACITOR MODELS FOR VOLTAGE CONVERSION AND POWER HARVESTING

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and hereby certify that in their opinion it is worthy of acceptance.

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DESIGN AND SIMULATION ANALYSIS OF MEMS PARALLEL PLATE CAPACITOR’s TWO MODELS FOR VOLTAGE CONVERSION AND POWER HARVESTING

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ABSTRACT

In environment, unwanted and undamped vibrations are abundantly available which can be converted into electrical energy and used for energy harvesting. This paper contains the design, modeling and simulation results of MicroElectroMechanical System’s (MEMS) variable parallel plate capacitor which is used for stepping up the voltage and power harvesting using forced vibration. Basic design, electric circuit and simulation results for model with single cavity and model with two cavities of parallel plate variable capacitor are presented. This is first time, study of parallel plate with two cavities conducted. Forced vibration is used as activation force and dynamics of models are tested for different combination of forcing frequencies and amplitude of vibration. Performance of both models is analyzed by computing average current and power. Different trials are conducted by changing various input parameters.
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CHAPTER 1

INTRODUCTION

1.1 Motivation for Research

In today’s world use of energy is going exponentially and prices of energy from sources like coal, oil and natural gases are increasing in market. Conventional energy sources like crude oil, coal and natural gas are major energy sources in our day to day life but conventional energy takes thousands of years to reproduce. So production rate of conventional energy sources is smaller than the consumption rate and will become extinct after a few hundred years. Our future generation will not have any conventional energy sources to use. That is why; the focus on other options of energy sources, i.e. nonconventional energy sources has increased in last few years. Non-conventional energy sources include solar, wind, tide etc. as energy sources but there is one more important energy source available in nature i.e. vibration energy. In environment, undamped and unwanted vibrations are abundantly available like in industry, construction site, machines produces lot of unwanted vibration when they are in use. In such case vibration based energy scavengers are considered to be ideal power sources for low power devices. Now a day’s, use of mobile and electronic devices increased and charging of these devices consumes lot of electric energy. Imagine, you are travelling in car and we have device which converts vibrations produce from car into electric energy. Then we can charge all these electronic devices for free while travelling.
1.2 Literature Review

MicroElectroMechanical System (MEMS) is a combination of electrical and mechanical systems and size of this system is in micrometers. Converting vibration energy into electrical energy can be done using three methods 1. Inductive, 2. Piezoelectric and 3. Capacitive. The variable capacitive method is considered as easy and capable of miniaturization but it needs a voltage bias for conversion process [5].

The work related to variable parallel plate capacitor was more focused on single cavity model but in this work we analyzed the simulation of model with single cavity and two cavities which leads to one variable capacitor and two variable capacitors respectively. This is the first time we did study of MEMS parallel plate capacitor Model with two cavities. The model design and arrangement, electric circuit and numerical solution for both models are discussed in this paper. We will compare the dynamics and simulation output for single cavity and two cavity model. Also we analyses the performance of both models to generate output current and power.

The activation force for these models is the vibration force so we tested both models for different combination of forced frequencies and base amplitude and different results are presented in this paper.
1.3 Concept of Basic Design

The concept of basic design for MEMS parallel plate capacitor model is based on mass-spring-dashpot system (Figure 1.3.1)

![Mass-spring-dashpot system arrangement](image)

**Figure 1.3.1. Mass-spring-dashpot system arrangement**

In spring-mass-dashpot system, mass is suspended on spring and mass moves with vibrations provided to mass as force of excitation. Dashpot works as damping force to the system.

Here,

Mass = \( m \), Spring constant = \( k \), Damping constant = \( c \) and Displacement of mass = \( x \)

In this system, mass is attached to Hook’s law of spring,

So,

\[
\text{Spring force} = -kx
\]

\[
\text{Damping force} = -c \frac{dx}{dt}
\]
Now, From Newton’s second law, the acceleration ‘a’ of a body is parallel and
directly proportional to the net force ‘f’ and inversely proportional to the mass ‘m’ [11]
i.e.

\[ \sum f = ma \]  \hspace{1cm} (3)

For given system (Figure 1),

\[ m\ddot{x} = -c\dot{x} - kx \]  \hspace{1cm} (4)

By rearranging and divided by m,

\[ \ddot{x} + \frac{c}{m} \dot{x} + \frac{k}{m} x = 0 \]  \hspace{1cm} (5)

This gives the governing equation for Mass spring dashpot system. Similarly,
Governing equation for MEMS parallel plate capacitor models with single cavity and two
cavities are computed and numerical solutions are computed using Matlab program.
CHAPTER 2

MATHEMATICAL MODEL OF DEVICE

2.1 Mechanical Model with Single Cavity

MEMS parallel plate capacitor model with single cavity consists of one movable and one fixed plate electrode. Fixed plate electrode is placed on base plate. Movable plate is supported by four serpentine springs at particular distance from fixed plate to create one cavity. Four stopper springs are placed on fixed plate (Figure 2.1.1) to avoid collision of movable plate with fixed one. All these arrangement is placed on base plate to which vibration force is applied. This arrangement creates single cavity MEMS parallel plate capacitor.

Figure 2.1.1. Single cavity variable parallel plate capacitor model
To avoid the collision of movable plate with fixed one, we placed four stopper springs on fixed plate at equal distance from four corners of plate. The length of stopper spring is considered as 0.2 times the gap between the two plates (Figure 2.1.2).

![Figure 2.1.2. Stopper spring arrangement](image)

2.2 Mechanical Model with Two Cavities

This model consists of two fixed plate electrodes and one movable plate electrode placed in between two fixed one. Movable plate is supported by four serpentine springs. Four stopper springs are placed on each fixed plate (Figure 2.2.1) to avoid collision of movable plate with fixed plates. All these arrangement is placed on base plate and vibration force is applied to base plate. This model leads to two variable capacitors with gap1 and gap2 (Figure 2.3.2).
2.3 Circuit Arrangement for Device

Circuit arrangement for electric circuit for MEMS single cavity parallel plate capacitor is shown in Figure 2.3.1.

Figure 2.2.1. Variable parallel plate capacitor model with two cavities

Figure 2.3.1. Electric circuit of single cavity model
Input DC power source and output batteries are connected to movable plate electrode and fixed plate electrode is made ground. (Figure 2.3.1) Output battery voltage is more than input power source. Charge across the plates is controlled by diodes.

**Figure 2.3.2. Electric circuit of two cavity model**

Input and output DC voltage batteries are connected to fixed plates electrode and movable plate electrode is made ground (Figure 2.3.2). Charge across the plates is controlled by diodes.

2.4 Working Principle

Input DC power supply, output battery and diodes are attached as per respective electric circuit of models. Vibration force is supplied to base plate so that all the forces will act on movable plate as mentioned earlier.
We assume that the package containing the capacitors is subject to sinusoidal displacement. Movable plate will start moving up and down due to vibration force and because of the motion the gap between the movable plate electrode and fixed plate electrode will change which leads to change in charge across plates. As distance between two plates decreases charge across the plate decreases and as distance increases charge increases almost instantaneously.

This leads to change in voltage and after reaching to pull down voltage, plate will start striking on stopper spring to avoid collision between electrodes. This cycle will continue to repeat until the forced vibrations are applied. This dynamics effect we can check in our simulation results (Figure 3.2.1, 3.2.2).

The charges are controlled by diodes to achieve the pumping effect from low voltage source to high voltage battery and ultimately to charge the output battery.

So using variable parallel plate capacitor we converted vibration energy into electrical energy.

Through charging and discharging, the capacitive plates exchange energy with the electrical components both upstream and downstream; mechanical energy is dispersed in the process.

2.5 Symbols and Terminology

In this system, following symbols are used

\[ V = \text{Voltage across plate} \]
\( \varepsilon = \) permittivity,

\( A = \) c/s area of plate

\( d_0 = \) gap between plates at equilibrium, \( d_1 = \) gap between plates at time \( t \)

\( m = \) mass of movable plate,

\( k = \) total spring force

\( c = \) Air damping coefficient

\( F_e = \) Electrostatic force, \( F_s = \) spring force, \( F_i = \) Inertia force

2.6 Forces acting on Movable Plate and Governing Equations of Motion for Model with Single Cavity

**Figure 2.6.1. Free body diagram of movable plate of single cavity model**

The moving plate is subjected to inertia force from vibrating base plate, spring force from serpentine springs, air damping force and the electrostatic force due to charge in capacitor (Figure 2.6.1). Equations for all these forces are given as follows,
a. Electrostatic force:

The electrostatic force $F_e$ refers to the electrostatic force over two capacitive plates. The plate capacitance is given by

$$C = \frac{\varepsilon A}{d}$$

Where $\varepsilon$ is electrical permittivity, $A$ the plate area, and $d$ the plate separation. For plates with varying gap, the electrostatic force caused by a voltage difference for capacitor is calculated by using following equation,

$$F_e = \frac{1}{2} \frac{\varepsilon A V^2}{d^2}$$

Electrostatic force can also be calculated using charge and potential energy equations as follows,

In capacitor, external influence is used to move charge between capacitor plates i.e. work is done and energy is stored. When charge is allowed to return to its equilibrium position energy is released. The work done and amount of energy stored is given by,

$$W = \int_{q=0}^{Q} V dq = \frac{1}{2} V Q = \frac{1}{2} \frac{Q^2}{C}$$

Consider two cavity model where we have two charges in each cavity i.e. $Q_1, C_1$ and $Q_2, C_2$

For One cavity model, $Q_2, C_2 = 0$

So Potential energy is given by,
\[ U_E = \frac{1}{2} \left( \frac{Q_1^2}{c_1} + \frac{Q_2^2}{c_2} \right) \] ...........................................(8)

Where \( Q_1 \) and \( Q_2 \) are constants

Now, Electrostatic force is given by,

\[ F_e = -\frac{\partial U_E}{\partial x} = -\frac{1}{2} \left[ Q_1^2 \frac{\partial}{\partial x} \left( \frac{1}{c_1} \right) + Q_2^2 \frac{\partial}{\partial x} \left( \frac{1}{c_2} \right) \right] \] .................(9)

Since

\[ C_1 = \frac{\varepsilon A}{d_0 + x} \quad \text{and} \quad C_2 = \frac{\varepsilon A}{d_0 - x} \]

Here \( x \) is non-dimension ratio which is explained in equation (16)

Taking derivative on \( \frac{1}{C_1} \) and \( \frac{1}{C_2} \)

\[ \frac{\partial}{\partial x} \left( \frac{1}{C_1} \right) = \frac{\partial}{\partial x} \left( \frac{d_0 + x}{\varepsilon A} \right) = \frac{1}{\varepsilon A} \]

\[ \frac{\partial}{\partial x} \left( \frac{1}{C_2} \right) = \frac{\partial}{\partial x} \left( \frac{d_0 - x}{\varepsilon A} \right) = -\frac{1}{\varepsilon A} \]

\[ F_e = -\frac{1}{2} \left[ \frac{Q_1^2}{\varepsilon A} - \frac{Q_2^2}{\varepsilon A} \right] = \frac{1}{2\varepsilon A} \left( -Q_1^2 + Q_2^2 \right) \] .................(10)

Positive sign of \( F_e \) means the force is in the same direction as \( x \) but as per above equation magnitude of \( F_e \) is independent of \( x \).

b. Inertia force:

\[ y = \text{Displacement of whole assembly due to base motion; } y = -A\cos\omega t \]
**Base acceleration**

\[ F_i = \ddot{y} = mb\omega^2 \cos \omega t \] .................. (11)

c. Spring force

\[ F_s = k(d_1 - d_0) \] .................. (12)

In spring force, additional stopper spring force will be added

To avoid the collision between the plates, we placed four stoppers on the fixed plate. The moving plate makes contact with the stopper when its displacement is 80% of the initial gap. The stopper is modeled as a very stiff spring whose spring constant is 1000 times of the plate suspension. Therefore, the stopper acts like a nonlinear spring and its force can be written as shown below.

In this case we considered stopper length is 0.2 times the gap between two plates and the stiffness is 2000 times stiffness of serpentine spring ‘k’

\[ \text{Stopper spring force} F_{ss} = 1000k*(\max((d_1 - d_0) - 0.8)) \]

d. Air damping force

\[ \text{Air damping force} = c\dot{d}_1 \] .................. (13)

If the device is sealed in a vacuum, the mechanical damping can be negligible.

Using the Newton’s second law of motion, the equation for system is given as below,

\[ m(\ddot{d}_1 + \ddot{y}) = -c\dot{d}_1 - k(d_1 - d_0) - F_e \] .................. (14)

Now, to simplify this equation, we introduce some terms.
Figure 2.6.2. Non dimensional ratio for gap between two plates

e. Non-dimensional ratio:

\[ d_1 = d_0(1 + x) \] ...........................................(15)

\[ x = \frac{d_1}{d_0} - 1 \] ...........................................(16)

So the equation becomes:

\[ m d_0 \ddot{x} = -c d_0 \dot{x} - k d_0 x - F_e - F_i \] ...........................................(17)

\[ \ddot{x} = -\frac{c}{m} \dot{x} - \frac{k}{m} x - \frac{1}{m d_0} (F_e + F_i) \] ...........................................(18)

Also to simplify this equation further we introduce some terms as follows.

f. Damping Ratio \( \zeta \):

The damping coefficient \( c \) represents the loss of energy associated with the plate motion; the damping by the air surrounding the plate is the main contributor to the damping. We introduce the damping ratio \( \zeta \), in the following equation:

\[ \zeta = \frac{c}{\text{critical damping coefficient } c_c} = \frac{c}{2\sqrt{mk}} \] ...........................................(19)
Substituting this term in equation (18) we get,

\[
\ddot{x} = -2\zeta \sqrt{\frac{k}{m}} \dot{x} - \frac{k}{m} x - \frac{1}{md_0}(F_c + F_t) \quad \text{(21)}
\]

g. Pull down voltage \(V_{pd}\):

Since the gap is varying, this force is dependent on the plate displacement \(x\). When the gap decreases, the electrostatic force increases. This force is nonlinear, i.e. the increase is not proportional to the displacement. The increase may overwhelm the restoring force of the elastic support to cause the gap to collapse. When this is caused by the gradual increase of the voltage, a critical voltage, called the pull down voltage, is known to exist:

\[
V_{pd} = \sqrt{\frac{8kd_0^2}{27C_0}}, \quad \text{where} \quad C_0 = \frac{\varepsilon A}{d_0} \quad \text{(22)}
\]

Where \(d_0\) and \(C_0\) are the gap and capacitance at static equilibrium when no voltage is applied.

The electrostatic force is determined by the voltage on the capacitor. The voltage is calculated from the charge on the plate: \(V_c = \frac{Q}{C}\)

The charge on the plate can be determined by keeping track of the current flowing in and out the capacitor. However, since the plate capacitance is very small, charging and discharging time is very short. Keeping track of the current flow would require additional differential equation. The numerical solution of this additional equation would require extremely small integration time steps to prevent numerical instability. Since the short
charging and discharging time can be ignored, we assume that the charge on the plate is constant when no charging or discharging takes place. That is, $Q$ is constant if $V_{in} < V_c < V_{out}$. By ignoring the charging and discharging time, we set up the following limitations on the voltage on the capacitor:

$$\min V_c = V_{in} \quad \text{And} \quad \max V_c = V_{out}.$$ 

The above limitations are imposed at the end of each time integration step. If the voltage exceeds $V_{out}$, we calculate the excess electric charge which is moved to the battery $V_{out}$. If the voltage is below $V_{in}$, $V_c$ is reset to $V_{in}$.

h. Calculating spring constant using $V_{pd}$

$$k = \frac{27C_0V_{pd}^2}{8d_0^2} \tag{23}$$

We consider the pull down voltage as fixed value and from that we calculate the spring constant.

Now,

$$\sqrt{\frac{k}{m}} = \omega_0 \text{And}, F_s = k \times x, \text{ Stopper spring force} = F_{ss}$$

Putting in equation (21), we get:

$$\ddot{x} = -2\zeta \omega_0 \dot{x} - \frac{1}{m} (F_s - \frac{1}{d_0} (F_e + F_l)) \tag{24}$$

So simplified equation becomes,

$$\ddot{x} = -2\zeta \omega_0 \dot{x} - \omega_0^2 x - \frac{1}{m} (F_{ss} - \frac{1}{d_0} (F_e + F_l)) \tag{25}$$
2.7 Forces acting on Movable Plate and Governing Equations of Motion for Model with Two Cavities

![Free body diagram of movable plate in two cavity model](image)

**Figure 2.7.1. Free body diagram of movable plate in two cavity model**

Forces acting on movable plate of two cavity model is similar to one cavity model but one additional force is introduce i.e. electrostatic force $F_{e2}$ due to second cavity capacitor.

So the equation for system becomes,

$$md'1 = -cd_1 - k(d_1 - d_0) + F_{e2} - F_{e1} + F_l \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (26)$$

Now, simplifying the equation similar as for one cavity model we get,

$$md_0\ddot{x} = -cd_0\dot{x} - kd_0x + F_{e2} - F_{e1} + F_l \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (27)$$

$$\ddot{x} = - \frac{c}{m}\dot{x} - \frac{k}{m}x + \frac{1}{md_0}(F_{e2} - F_{e1} + F_l) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (28)$$

Simplifying more this equation similarly to one cavity equation we get,

$$\dddot{x} = -2\zeta\omega_0\dot{x} - \omega_0^2 x - \frac{1}{m}(F_{s2} - \frac{1}{d_0}(F_{e2} - F_{e1} + F_l)) \ldots \ldots \ldots \ldots (29)$$
CHAPTER 3

NUMERICAL SOLUTION OF THE EQUATIONS OF MOTION

3.1 Dimensions of Model

Matlab programming is used to find the numerical solutions of governing equations for MEMS parallel plate capacitor models with single and two cavities. To run matlab programming, some input parameter values are need to be consider as fixed as follows,

Permittivity constant = 8.85*10^-12

Mass of moving plate = Density of material * Volume of plate

Density of Material= 8912kg per cubic meter for Nickel

Volume of plate= Area of plate (Length *width) * thickness of plate

Pull down voltage = 20V

Capacitance = Permittivity constant* area of plate / gap between plates

Input Potential = 15V

Output Potential = 45V

Natural Frequency = 164.8223 hertz
3.2 Implementation of Numerical Equation

Using the equations of forces for movable plate we developed Matlab program pump.m and pumpsub2.m (Single cavity -Appendix II, Two cavities- Appendix I) to simulate the system and we got following results.

For single cavity model, movable plate will move up and down and after some time it will start striking on stopper spring at bottom only. It does not have any limit in up direction because of stopper springs.

From Figure 3.2.1 a, we can observe the motion and position of movable plate and in figure 3.2.1b, we can observe the change in voltage across the plates as position of plates changes. As plate moves down, voltage decreases and as plate moves in up direction after striking to stopper springs voltage increases almost instantaneously.

Using the plate position we have plotted charge and change in charge across both plates.
Figure 3.2.1. Simulation result for single cavity model at Forcing frequency = 3.5\* 
Natural frequency, amplitude = 175micrometer. 
a. Displacement of movable plate wrt time 
b. Voltage across two plates wrt time 
c. Change in charge across two plates wrt time 
d. Charge across two plates wrt time
In two cavity model, stopper springs are placed on both fixed plate electrodes. So as voltage across plates reaches to pull down voltage, plate start striking on stopper springs. We can observe this effect in figure 3.2.2.a.

Two cavity models have two gaps which lead to two capacitors. From figure 3.2.2.b we can observe the change in voltage across the gaps, (Figure 3.2.2.b only, blue color- bottom gap voltage, and green color- upper gap voltage). As plate moves down, the voltage across the bottom gap decreases but on the other hand this leads to increase in distance between electrodes of upper gap which increases the voltage for upper gap capacitor and vice versa.
Figure 3.2.2. Simulation result for two cavity model at Forcing frequency = 2.2*

Natural frequency, amplitude= 150micrometer.  

a. Displacement of movable plate wrt time       b. Voltage across two plates wrt time  
c. Change in charge across two plates wrt Time       d. Charge across two plates wrt time
3.3 Problems Encountered in Numerical Solutions

Average Current and Average power calculation for Single cavity model:

Table 3.3.1. Average current computed for various combination of forcing frequency and amplitude for single cavity model (Highlighted values shows integration error in program)

<table>
<thead>
<tr>
<th>Forcing frequency multiplier ( \times ) (Dimensionless)</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
<th>2</th>
<th>2.25</th>
<th>2.5</th>
<th>2.75</th>
<th>3</th>
<th>3.5</th>
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<tbody>
<tr>
<td>250</td>
<td>0</td>
<td>1.56E-08</td>
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<td>1.79E-06</td>
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<td>1.43E-07</td>
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<td>2.34E-11</td>
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<td>2.77E-10</td>
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<tr>
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<td>1.43E-07</td>
<td>7.91E-10</td>
<td>7.91E-12</td>
<td>0</td>
<td>7.22E-08</td>
<td>5.24E-08</td>
<td>3.47E-08</td>
<td>7.22E-08</td>
<td>2.23E-07</td>
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<td>8.34E-08</td>
<td>5.30E-07</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
</tr>
</tbody>
</table>

The highlighted area in table 3.3.1 shows integration error in Matlab programming. After particular threshold value, it shows some error in program

Program is modified to get rid of this problem with following modifications.
Concept of Impact and Coefficient of Restitution:

In our model, moving plate will collide on stopper spring but as moving plate will pushes stopper spring, Stopper spring will become stiffer and will stop further compression.

So, here spring force due to serpentine springs = \( F_s = k(d_1 - d_0) = kx \)

And stopper spring force is given by,

\[ \text{Stiffness} = k_2 = 1000k \]

In our model, Stopper spring is at distance of 80% from original position of movable plate. So stopper force will be as follows,

Nonlinear spring equation represents the stopper.

If \( x < -0.8 \);

\[ F_{ss} = \frac{k_2}{|1+x|} |x(1) - 0.8| \]

else if \( x > 0.8 \);

\[ F_{ss} = -\frac{k_2}{|1+x|} |x(1) - 0.8| \]

else,

\[ F_{ss} = 0 \]

For one cavity only equation 30 and 32 were used.
In initial programming, we considered simulation of stopper spring through the duration when the stopper spring is acting but we considered this movement as instantaneously.

In this concept we are adding coefficient of restitution (COR). COR of an object is a frictional value representing the ratio of velocities after and before an impact. For a COR = 0, the object effectively "stops" at the surface with which it collides, not bouncing at all. For COR=1, object collides elastically and an object with a COR < 1 collides inelastically.

\[ \text{COR} = \frac{v}{u} \]

Where, \( v \) = scalar velocity of object after impact

\( u \) = scalar velocity of object before impact

Finally, by reducing step size i.e. from 10^-5 step size to 10^-6 step size.

Results for modified program with concept of impact, coefficient of restitution and step size computed.
CHAPTER 4

RESULTS FOR SINGLE CAVITY MODEL

4.1 Calculation method of Average Current and Power

Using Matlab program developed for movable plate in MEM system and graphs are plotted for charge and change in charge across both plates.

Using this data we have calculated average current using following formula.

\[
\text{Current (I)} = \frac{\text{change in charge}}{\text{time period}} = \frac{dQ}{dt} \quad \text{………………………………………………(33)}
\]

And Power (P) = current * voltage = VI \quad \text{………………………………………………(34)}

In Matlab program this equations are used and Average current and average power are calculated at different combination of forcing frequency and amplitudes.

4.2 Average Current and Power for Model with Single Cavity

Average Current and Average power calculation for Single cavity model:
Table 4.2.1. Average current computed for various combination of forcing frequency and amplitude for single cavity Model

<table>
<thead>
<tr>
<th>Forcing frequency multiplier 'x'(Dimensionless)= Forcing frequency / natural frequency</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
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</thead>
<tbody>
<tr>
<td>Amplitude (Microns)</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>3.65E-08</td>
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<td>0</td>
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<td>2.98E-08</td>
<td>2.45E-08</td>
<td>2.25E-08</td>
</tr>
</tbody>
</table>

Table 4.2.2. Average power computed for various combination of forcing frequency and amplitude for single cavity model

<table>
<thead>
<tr>
<th>Forcing frequency multiplier 'x'(Dimensionless)= Forcing frequency / natural frequency</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
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<tbody>
<tr>
<td>Amplitude (Microns)</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>250</td>
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<td>0</td>
<td>7.83E-07</td>
<td>1.64E-06</td>
<td>1.7E-06</td>
<td>1.57E-06</td>
<td>1.11E-06</td>
<td>9.86E-07</td>
</tr>
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<td>9.78E-07</td>
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<td>1.56E-06</td>
<td>1.27E-06</td>
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<td>1.03E-06</td>
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</table>
Figure 4.2.1 shows Average current and power for single cavity model is mostly increases as we increases amplitude and frequency but it is not uniform, it shows some decline in results after some values of amplitude and frequency. This is observed because in single cavity model, movable plate has stopper spring only at one side and on other side it does not have any restriction on motion other than spring force.
CHAPTER 5

RESULTS FOR TWO CAVITY MODEL

5.1 Calculation method of Average Current and Power

In two cavity model, average current is calculated using the same equations for single cavity model but the change in charge is observed in both gaps of model. So we modified program accordingly to calculate both change in charge calculations.

Average power is calculated by multiplying average current and voltage.

5.2 Average Current and Power for Model with Two Cavities

Table 5.2.1. Average current computed for various combination of forcing frequency and amplitude for two cavity model

<table>
<thead>
<tr>
<th>Amplitude (Microns)</th>
<th>Forcing frequency multiplier $X'$ (Dimensionless)= Forcing frequency / natural frequency</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
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<td>4.25E-07</td>
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</table>
Table 5.2.2. Average power computed for various combination of forcing frequency and amplitude for two cavity model

<table>
<thead>
<tr>
<th>Forcing frequency multiplier ( 'x' ) (Dimensionless) = Forcing frequency / natural frequency</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
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<tr>
<td>Amplitude (Microns)</td>
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</tr>
</tbody>
</table>

Figure 5.2.1. Average current vs. amplitude vs. forcing frequency multiplier graph for two cavities Model
From graphs for average current and power, it is observed that as forcing frequency and/or base amplitude increases, average current and power also increases.

It is observed that, model with two cavities gives better results than model with single cavity. In single cavity model average current and power do not increase uniformly as it does in model with two cavities. The reason behind this is that in model with two cavities, movement of movable plate is restricted on both sides using stopper springs. So as forcing frequency and/or amplitude increases, movement of plate increases. Movable plate strikes more frequently in one cycle of sinusoidal wave of forcing frequency. In model with one cavity, movement of movable plate is restricted only on one side. So as forcing frequency and/or amplitude increases, plate strikes on one side but keep flying on other side. So after particular values of amplitude and frequency it does not show any improvement in results.
CHAPTER 6

TRIALS WITH DIFFERENT INPUT PARAMETERS

6.1 Different Curve Patterns Observed

Different simulation curves observed for different combination for dynamic forces. To check the effect of vibration force we computed average current and power for different combination of forced frequencies and amplitude.

Different trails were conducted for various combination of forcing frequency and amplitude and different waveforms were observed for displacement of movable plate. Because of different plate displacement the average current and power also changes. So its effect on average current were studied and it is observed that at some particular displacement waveform of plate we get more average current compare to others.

Different waveforms observed at different combination of frequency- amplitude and average current computed for particular waveform are mentioned below.
Figure 6.1.1. Graph when no vibration force provided

i.e. at 0 amplitude and forcing frequency is 0 times natural frequency. $V_{in} = 5$, $V_{out} = 25$. Simulation shows no change and hence no energy harvesting.

Figure 6.1.2 Graph when small vibration force provided
i.e. at 5 micron amplitude and forcing frequency is 1.75 times natural frequency. $V_{in} = 5$, $V_{out} = 25$. Simulation shows change but not significant hence no energy harvesting.

![Graph when vibration force provided is increased](image)

**Figure 6.1.3. Graph when vibration force provided is increased**
i.e. at 20 micron amplitude and forcing frequency is 1.75 times natural frequency. $V_{in} = 5$, $V_{out} = 25$. Simulation shows significant change power is harvested.
Figure 6.1.4. Movable plate stuck to fixed plate

In some cases movable plate get stuck at either of stopper spring location. The attraction between two plates is more and vibration force in not significant enough to move movable plate against it. At 10 micron amplitude and forcing frequency is 1.75 times natural frequency. $V_{in} = 15$, $V_{out} = 25$. Simulation shows movable plate get stuck at lower stopper spring plate and hence no energy harvesting possible.

In Initial programming, many waveform patterns are found common as follows,
1. For forcing frequency = 2.2*Natural frequency, Amplitude= 150 microns

Average Current = 1.4943e-007

Figure 6.1.5. Regular repetitive pattern of plate movement

2. For forcing frequency = 3* Natural frequency, Amplitude= 75 microns

Average Current = 7.58E-12

Figure 6.1.6. Random pattern of plate movement
3. For forcing frequency =1.75* Natural frequency, \[ \text{Amplitude}= 100 \text{ microns} \]

Average Current = 1.1284e-007

**Figure 6.1.7. Upside double bounce repetitive pattern of plate movement**

4. For forcing frequency = 2*natural frequency, \[ \text{Amplitude}=150 \text{ microns} \]

Average Current =1.3973e-007

**Figure 6.1.8. Downside double bounce repetitive pattern of plate movement**
5. For forcing frequency = 1.5* natural frequency, Amplitude= 225 microns

Average Current =1.0802e-007

Figure 6.1.9. Upside-downside double bounce repetitive pattern of plate movement

6. For forcing frequency =0.75*natural frequency, Amplitude= 150 microns

Average Current =3.6274e-008

Figure 6.1.10. Multiple bounce on both sides, repetitive pattern of plate movement

It is observed that for displacement motion of movable plate observed in first waveform (Figure 6.1.5) gives more output average current compare to others.
We use different symbols for different displacement curve as in Figure 6.1.11 and took trials for different combination of forcing frequency and base amplitude. We plotted all symbols of chart of various combination of frequency-amplitude Figure 6.1.12.

**Figure 6.1.11.** Different symbols used for different displacement waveforms

**Figure 6.1.12.** Pattern of different displacement waveforms for different base amplitude and forcing frequency combinations (This result is before using COR)
It is observed that the some combinations of forcing frequency and base amplitude gives more average current compare to others and this patterns found in particular region of forcing frequency and amplitude.

6.2 Different Input Parameters of System

MEMS parallel plate capacitor model with single cavity and two cavities, numerical solutions are found out by solving governing equations using Matlab programming. These equations has many variables, so to solve these equations, some parameters needs to be consider as fixed values and only few parameters as consider as variable like forcing frequency and amplitude values.

To check the effect of other parameters like plate size, input and out voltages, gap between two plates etc. by considering as variable, different trials were conducted.

6.3 Trials with Different Plate Sizes and Gap between Plates

In original programming, plate size is considered as 5mm*5mm, to see the effect of plate size; plate size is reduced to 2mm*2mm.

Also Initial gap between two plates is considered as a fixed value of 50micrometer but to check the effect, it is changed to 10 micrometers.
Table 6.3.1. Average power computed for various combination of forcing frequency and amplitude, single cavity model with Plate dimension 2mm by2mm and gap 10 microns

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Table 6.3.2. Average power computed for various combination of forcing frequency and amplitude for single cavity model with Plate dimensions 5mm by 5mm and gap 50 microns

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Table 6.3.3. Average power computed for various combination of forcing frequency and amplitude for model with two cavities, Plate dimensions 2mm by 2mm, gap 10 microns

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Table 6.3.4. Average power computed for various combination of forcing frequency and amplitude for model with two cavities, Plate dimensions 5mm by 5mm, gap 50 microns

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From all these results, it is observed that as we reduces size of plate and reduces gap between plates, average current and power values increases.
6.4 Changes in Input and Output Potentials

In main program, input potential is 15V and output potential is 45V, various trials are conducted with different Input and output potential values.

Table 6.4.1.a. Average current for various values of frequencies at two fixed amplitude values for two cavities Model by changing input and output potentials

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Table 6.4.1.b. Average current for various values of frequencies at two fixed amplitude values for two cavities Model by changing input and output potentials

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Table 6.4.1.c. Average current for various values of frequencies at two fixed amplitude values for two cavities Model by changing input and output potentials

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Table 6.4.1.d. Average current for various values of frequencies at two fixed amplitude values for two cavities Model by changing input and output potentials

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Table 6.4.2.a. Average current computed for various values of Amplitude at two fixed values of frequencies for two cavity model by changing input and output potentials

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Table 6.4.2.b Average current computed for various values of Amplitude at two fixed values of frequencies for two cavity model by changing input and output potentials

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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Vin=15, Vout=20

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Forcing frequency=x* natural frequency 0.75</th>
<th>1.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0</td>
<td>3.89E-09</td>
</tr>
<tr>
<td>225</td>
<td>0</td>
<td>3.07E-10</td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>8.18E-10</td>
</tr>
<tr>
<td>175</td>
<td>0</td>
<td>3.17E-10</td>
</tr>
<tr>
<td>150</td>
<td>0</td>
<td>3.22E-10</td>
</tr>
<tr>
<td>125</td>
<td>0</td>
<td>3.87E-10</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>1.30E-11</td>
</tr>
<tr>
<td>75</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Vin=5, Vout=25
So from above tables it is observed that, if difference between input and output potential is more, at lower forcing frequency average current is zero at all amplitudes. If difference is small, average current can be calculated at lower forcing frequency. As we reduces the value of output voltage, average current increases.

If we reduces input potential, at lower frequencies average current is zero and average current is lower at higher frequency. When difference between input and output potential is more, average current at lower frequencies cannot be calculated but as we reduces that difference, we can compute the average current at lower frequencies too.

Keeping input potential same and just by reducing input potential i.e. reducing the difference between input and output potential we get better result.
CHAPTER 7

CONCLUSION AND RECOMMENDATIONS

7.1 Conclusion

Two different models for variable parallel plate capacitor were proposed and workings of both models were discussed for stepping up the voltage and power harvesting. Simulation for movable plate electrode estimated and average current and power computed for various combination of forcing frequencies and amplitude. Different trials with various input parameters conducted and compared. MEMS Model with two cavities gives better results than single cavity model. From different trials, we can conclude that as we increase forcing frequency and/or amplitude of vibration we get better results. Also better results can be generated by reducing the difference between input and output potential, by reducing the plate size and by reducing the gap between the two plates.

7.2 Future Work Recommendations

In future, trials with other input parameters should be conducted. Generalized equation between input parameter and output needs to be generated. In this study motion of plate is considered in one axis of direction but in future tilting effect of plate due to uneven forces needs to be studied.
REFERENCES


Appendix 1: Matlab program for Model with Two Cavities

1.1: Main Program for Model with Two Cavities

clear all
global epsilon m A R d0 vin vout omega0 k zeta amp cap_p omega Q1 Q2
coeff=0.8; % coefficient of restitution
epsilon=8.85*10^(-12); % constant permittivity
A=25e-6; % plate area 5um by 5um
m=8912.0*A*10e-6; % thickness is assumed to be 10 um, % Nickel density
8912 kg per cubic meter
d0=50e-6; % gap assumed to be 50um
epsilon*A/d0; % capacitance
cap_p=1.*cap0; % parasitic capacitance
V_pd=20.; % pull down voltage is assumed to be 20 volts
k=27.*cap0*V_pd^2/(8.*d0^2); % Spring constant using pull down voltage
omega0=sqrt(k/m);% natural frequency rad /sec
omega=3.5*omega0;% forcing frequency
freq=omega0/(2*pi)% frequency in Hz
amp=250e-6; % base excitation amplitude, only about 50 um if use sine.
R=1e7;% resistance
zeta=0.1;% damping coefficient
vin=15.0;% Input Voltage
vout=45.0;% Downstream voltage
z=[0.0 0.0]; % initial condition
v1=vin;
v2=vin;
X=z(1);
gap1=d0*(1.0+X); % gap1 calculation using dimensionless term
cap1=epsilon*A/gap1+cap_p;% Capacitance in gap1 calculation
gap2=d0*(1.0-X);% gap2 calculation using dimensionless term
cap2=epsilon*A/gap2+cap_p;% Capacitance in gap2 calculation
Q1=v1*cap1;% Charge in gap 1
Q2=v2*cap2;% Charge in gap 2
trans=1000; % transient steps not saved
steps=1000; % number of steps saved after transient steps
totalsteps=trans+steps;
tstep=0.00001;% time steps
a=0;% counter for charge time
b=0;% counter for charge time
count1=0;% counter for average current cal
count2=0;% counter for average current cal
for i=1:totalsteps
t=i*tstep;
dQ1=0.0; % Initial change in charge across gap1
dQ2=0.0; % Initial change in charge across gap2
tspan = [(i-1)*tstep t];
[ttemp, sol] = ode23s('pumpsub', tspan, z); % Run subroutine
len=length(ttemp);
z=sol(len,:);
X=z(1);
gap1=d0*(1.0+X); % gap1 calculation using dimensionless term
cap1=epsilon*A/gap1+cap_p;% Capacitance in gap1 calculation
gap2=d0*(1.0-X); % gap2 calculation using dimensionless term

cap2=epsilon*A/gap2+cap_p; % Capacitance in gap1 calculation
v1=Q1/cap1; % Voltage across gap1
v2=Q2/cap2; % Voltage across gap2

if v1<vin % charging capacitor
    v1=vin;
    dQ1=0; % change in charge during charging is not counted
    Q1=cap1*v1;
elseif v1>vout % discharging to the downstream battery
    Q1new=cap1*vout;
    dQ1=Q1-Q1new;
    v1=vout;
    Q1=Q1new;
else
    dQ1=0;
end

if v2<vin % charging capacitor
    v2=vin;
    dQ2=0; % change in charge during charging is not counted
    Q2=cap2*v2;
elseif v2>vout % discharging to the downstream battery
    Q2new=cap2*vout;
    dQ2=Q2-Q2new;
    v2=vout;
    Q2=Q2new;
else
    dQ2=0;
end

if cap2<0 % to see if the capacitance become negative
    print=i
end

if i>trans
    dcharge1(i-trans)=dQ1;
    dcharge2(i-trans)=dQ2;
    count1=count1+abs(dcharge1(i-trans));
    count2=count2+abs(dcharge2(i-trans));
    T(i-trans)=t;
    X=z1;
    dimlessdisp(i-trans)=X;
    V1(i-trans)=v1;
    V2(i-trans)=v2;

    if V1(i-trans)>=vout
        a=a+1;
    end
    if V2(i-trans)>=vout
        b=b+1;
    end

    force_i(i-trans)=500*amp*omega0^2*m*cos(omega*t);
end
% check if collision has occurred
end

chargeTime=(a+b)*tstep % Calculate charge time

AverageCurrent=((count1+count2)/(2*steps*tstep))% Calculate average current

subplot(3,1,1); plot(T,dimlessDisp,T,force_i);
xlabel('Time(t)-sec','FontSize',10)
ylabel('Displacement ','FontSize',10)
subplot(3,1,2); plot(T,V1,T,V2);
xlabel('Time(t)-sec','FontSize',10)
ylabel('Voltage ','FontSize',10)
subplot(3,1,3); plot(T,dcharge1,T,dcharge2);
xlabel('Time(t)-sec','FontSize',10)
ylabel('Change in charge - Coulomb ','FontSize',10)

% subplot(4,1,4); plot(T,Q1,T,Q2);
% xlabel('Time(t)-sec','FontSize',10)
% ylabel('Charge across gaps - Coulomb ','FontSize',10)

1.2: Subroutine Program for Model with Two Cavities

function xdot = pumpsub(t,x); % with two capacitors

global epsilon m A R d0 vin vout omega0 k zeta amp cap_p omega Q1 Q2

Q2=0.;% for single cavity only

d1=d0*(1.+x(1));
d2=d0*(1.-x(1));

xdot(1,1)=x(2);
k2=1000*k; % stopper spring stiffness is considered to be 1000 times of total stiffness of serpentine spring

force_e=-epsilon*A*(v1^2/(2.0*d1^2)-v2^2/(2.0*d2^2));

force_i=-amp*omega^2*m*cos(omega*t); %bigger transient with sin.

if x(1)<-0.8
    force_s = (k2/abs(1+x(1)))*abs(-x(1)-0.8);
elseif x(1)>0.8
    force_s = -(k2/abs(1-x(1)))*abs(x(1)-0.8);
else
    force_s=0.0;
end

xdot(2,1)=-2.0*omega0*zeta*x(2)-
omega0^2*x(1)+(force_s+(force_e+force_i)/d0)/m;
Appendix 2: Matlab program for Model with Single Cavities

2.1: Main Program for single cavity model

clear all
global epsilon m A R d0 vin vout omega0 k zeta amp cap_p omega Q1 Q2
coeff=0.8; % coefficient of restitution
epsilon=8.85*10^(-12); % constant permittivity
A=25e-6; % plate area 5um by 5um
m=8912.0*A*10e-6; % thickness is assumed to be 10 um, % Nickel density
8912 kg per cubic meter
d0=50e-6; % gap assumed to be 50um
cap0=epsilon*A/d0; % capacitance
cap_p=1.*cap0; % parasitic capacitance
V_pd=20.; % pull down voltage is assumed to be 20 volts
k=27.*cap0*V_pd^2/(8.*d0^2); % Spring constant using pull down voltage
omega0=sqrt(k/m);% natural frequency rad /sec
omega=3.5*omega0;% forcing frequency
freq=omega0/(2*pi)% frequency in Hz
amp=250e-6;% base excitation amplitude, only about 50 um if use sine.
% R=1e7;% resistance
zeta=0.1;%Damping coefficient
vin=15.0;%Input Voltage
vout=45;% Downstream voltage
z=[0.0 0.0]; % initial condition
v1=vin;
v2=vin;
X=z(1);
gap1=d0*(1.0+X); % gap1 calculation using dimensionless term
cap1=epsilon*A/gap1+cap_p;% Capacitance in gap1 calculation
gap2=d0*(1.0-X); % gap2 calculation using dimensionless term
cap2=epsilon*A/gap2+cap_p;% Capacitance in gap2 calculation
Q1=v1*cap1;% Charge in gap 1
Q2=v2*cap2;% Charge in gap 2
trans=1000; %transient steps not saved
steps=1000; % number of steps saved after transient steps
totalsteps=trans+steps;
tstep=0.000001;% time steps
a=0;%counter for charge time
b=0;%counter for charge time
count1=0;%counter for average current cal
count2=0;%counter for average current cal
for i=1:totalsteps
    t=i*tstep;
dQ1=0.0; % Initial change in charge across gap1
dQ2=0.0; % Initial change in charge across gap2
tspan = [(i-1)*tstep t];
[ttemp,sol] = ode23s('pumpsub',tspan,z); % Run subroutine
len=length(ttemp);
z=sol(len,:);
X=z(1);
    vel=z(2);
    eps=-tstep*vel;
if \( \text{abs}(X+0.8) < \frac{\varepsilon}{2} \)
    \[
    z(1) = -0.8; \\
    z(2) = -\text{coeff} \times \text{vel}; 
    \]
end

X = z(1);

gap1 = d0 \times (1.0 + X); % gap1 calculation using dimensionless term

\[
\text{cap1} = \varepsilon / \text{gap1} + \text{cap_p}; 
\]

\[
\text{gap2} = d0 \times (1.0 - X); % gap2 calculation using dimensionless term 
\]

\[
\text{cap2} = \varepsilon / \text{gap2} + \text{cap_p}; 
\]

\[
\text{v1} = Q1 / \text{cap1}; % Voltage across gap1 \]

\[
\text{v2} = Q2 / \text{cap2}; % Voltage across gap2 \]

if \( v1 < \text{vin} \) % charging capacitor

\[
\text{v1} = \text{vin}; \\
\text{dQ1} = 0; % change in charge during charging is not counted \\
\text{Q1} = \text{cap1} \times \text{v1}; 
\]
elseif \( v1 > \text{vout} \) % discharging to the downstream battery

\[
\text{Q1new} = \text{cap1} \times \text{vout}; \\
\text{dQ1} = \text{Q1} - \text{Q1new}; \\
\text{v1} = \text{vout}; \\
\text{Q1} = \text{Q1new}; 
\]
else

\[
\text{dQ1} = 0; 
\]
end

if \( v2 < \text{vin} \) % charging capacitor

\[
\text{v2} = \text{vin}; \\
\text{dQ2} = 0; % change in charge during charging is not counted \\
\text{Q2} = \text{cap2} \times \text{v2}; 
\]
elseif \( v2 > \text{vout} \) % discharging to the downstream battery

\[
\text{Q2new} = \text{cap2} \times \text{vout}; \\
\text{dQ2} = \text{Q2} - \text{Q2new}; \\
\text{v2} = \text{vout}; \\
\text{Q2} = \text{Q2new}; 
\]
else

\[
\text{dQ2} = 0; 
\]
end

if \( \text{cap2} < 0 \) % to see if the capacitance become negative
    \text{print=}1 
end

if \( i > \text{trans} \)

\[
\text{dcharge1}(i-\text{trans}) = \text{dQ1}; \\
\text{dcharge2}(i-\text{trans}) = \text{dQ2}; \\
\text{count1} = \text{count1} + \text{abs}(\text{dcharge1}(i-\text{trans})); \\
\text{count2} = \text{count2} + \text{abs}(\text{dcharge2}(i-\text{trans})); \\
\text{T}(i-\text{trans}) = \text{t}; \\
X = z(1); \\
\text{dimlessdisp}(i-\text{trans}) = X; \\
\text{V1}(i-\text{trans}) = \text{v1}; \\
\text{V2}(i-\text{trans}) = \text{v2}; 
\]
if \( \text{V1}(i-\text{trans}) \geq 45 \)
    \text{a} = \text{a} + 1; 
end
if V2(i-trans)>=45
    b=b+1;
end

force_i(i-trans)=50*amp*omega0^2*m*cos(omega*t);
end
% check if collision has occurred
end

chargetime=(a+b)* tstep % Calculate charge time

Averagecurrent=((count1+count2)/(2*steps*tstep))% Calculate average current

subplot(3,1,1); plot(T,dimlessdisp,T,force_i);
xlabel('Time(t)-sec','FontSize',10)
ylabel('Displacement ','FontSize',10)
subplot(3,1,2); plot(T,V1,T,V2);
xlabel('Time(t)-sec','FontSize',10)
ylabel('Voltage ','FontSize',10)
subplot(3,1,3); plot(T,dcharge1,T,dcharge2);
xlabel('Time(t)-sec','FontSize',10)
ylabel('Change in charge - Coulomb ','FontSize',10)
% subplot(4,1,4); plot(T,Q1,T,Q2);
% xlabel('Time(t)-sec','FontSize',10)
% ylabel('Charge across gaps - Coulomb ','FontSize',10)

2.2: Subroutine Program for single cavity model

function xdot = pumpsub(t,x); % with two capacitors
    global epsilon m A R d0 vin vout omega0 k zeta amp cap_p omega Q1 Q2
    Q2=0.;% for single cavity only
    d1=d0*(1.+x(1));
    d2=d0*(1.-x(1));
    cap1=epsilon*A/d1+cap_p;
    cap2=epsilon*A/d2+cap_p;
    v1=Q1/cap1;
    v2=Q2/cap2;
    xdot(1,1)=x(2);
    k2=1000*k; % stopper spring stiffness is considered to be 1000 times of total stiffness of serpentine spring
    % force_e=epsilon*A*(v1^2/(2.0*d1^2)-v2^2/(2.0*d2^2));
    force_e=(-Q1^2+Q2^2)/(epsilon*A);
    force_i=-amp*omega^2*m*cos(omega*t); %bigger transient with sin.
    if x(1)<-0.8
        force_s = (k2/abs(1+x(1)))*abs(-x(1)-0.8)*d0;
    elseif x(1)>0.8
        force_s= -(k2/abs(1-x(1)))*abs(x(1)-0.8);
    else
        force_s=0.;
    end
    xdot(2,1)=-2.0*omega0*zeta*x(2)-omega0^2*x(1)+(force_s+(force_e+force_i)/d0)/m;