MODELING A HYDRAULIC HYBRID DRIVETRAIN:
EFFICIENCY CONSIDERATIONS

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by
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MODELING A HYDRAULIC HYBRID DRIVETRAIN:
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ABSTRACT

Increasing petroleum prices and environmental concerns have caused a demand for more fuel efficient vehicles. Hybrid vehicles provide a solution to this demand, and hydraulic hybrid vehicles are shown to have a cheaper cost of ownership compared to electric hybrid vehicles. A hybrid drivetrain with a hydraulic continuously variable transmission (CVT) is modeled to include efficiency information of the engine and hydraulic components. Since the expressions comprising the model can be set as functions of a control input, which is related to the swashplate angle of the hydraulic motor in the CVT, an optimization algorithm can determine a control input that maximizes the overall vehicle efficiency allowing the vehicle to increase its fuel economy. Simulations are conducted using two driving schedules: one to represent city driving and the other to represent highway driving. Based on the results of these simulations, the hybrid vehicle produces a 1.28% increase in fuel economy over a similar conventional vehicle in city driving and a 22.62% increase in fuel economy for highway driving.
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\( m \)  Vehicle Mass
\( v_x \)  Vehicle Velocity
\( \dot{v}_x \)  Vehicle Acceleration
\( F_x \)  Total Wheel Propulsive Force
\( F_d \)  Aerodynamic Drag Force
\( F_r \)  Rolling Resistance Force
\( g \)  Gravitational Constant
\( \beta \)  Driving Surface Incline
\( l \)  Number of Axles
\( n \)  Number of Wheels per Axle
\( J \)  Wheel Moment of Inertia
\( r \)  Wheel Radius
\( \omega_w \)  Wheel Angular Speed
\( \dot{\omega}_w \)  Wheel Angular Acceleration
\( T_w \)  Wheel Torque
\( T_{w_{\text{max}}} \)  Maximum Wheel Torque
\( \rho \)  Air Density
\( C_d \)  Drag Coefficient
\( A_d \)  Reference Area
\( C_r \)  Rolling Resistance Coefficient
\( \omega_e \)  Engine Angular Speed
\( T_e \) Engine Torque

\( \omega_s \) Sun Gear Angular Speed

\( T_s \) Sun Gear Torque

\( \Xi_t \) Mechanical Transmission Gear Ratio

\( r_h \) Hydraulic Path Gear Radius

\( r_R \) Ring Gear Outer Radius

\( r_r \) Ring Gear Inner Radius

\( r_A \) Arm Gear Radius

\( r_a \) Arm Length

\( r_s \) Sun Gear Radius

\( r_p \) Planet Gear Radius

\( r_m \) Mechanical Path Gear Radius

\( F_s \) Sun Gear Reaction Force

\( F_a \) Arm Reaction Force

\( \omega_r \) Ring Gear Angular Speed

\( T_r \) Ring Gear Torque

\( \omega_a \) Arm Gear Angular Speed

\( T_a \) Arm Gear Torque

\( \xi_a \) Arm Gear Gear Ratio

\( \omega_r \) Ring Gear Angular Speed

\( T_r \) Ring Gear Torque

\( \xi_r \) Ring Gear Gear Ratio

\( \omega_1 \) Mechanical Path Angular Speed
$T_1$  Mechanical Path Torque
$
\xi_m$
Mechanical Path Gear Ratio

$\omega_h$
Hydraulic Path Angular Speed

$T_h$
Hydraulic Path Torque

$\xi_h$
Hydraulic Path Gear Gear Ratio

$P$
CVT Working Pressure

$V'_{p}$
Pump Instantaneous Volumetric Displacement

$V'_{m}$
Motor Instantaneous Volumetric Displacement

$\eta_p$
Pump Torque Efficiency

$\eta_{v_p}$
Pump Volumetric Efficiency

$\eta_{m}$
Motor Torque Efficiency

$\eta_{v_m}$
Motor Volumetric Efficiency

$\eta_{f}$
CVT Friction Efficiency

$\eta_s$
CVT Speed Efficiency

$Q$
Volumetric Flow Rate from Pump to Motor

$\Xi_v$
CVT Gear Ratio

$\omega_2$
CVT Output Angular Speed

$T_2$
CVT Output Torque

$\xi_2$
CVT-to-Spur Gearbox Gear Ratio

$V'_{p_{\text{max}}}$
Pump Maximum Volumetric Displacement

$\alpha_p$
Pump Swashplate Angle

$\alpha_{p_{\text{max}}}$
Pump Maximum Swashplate Angle
\( \hat{\alpha}_p \)  Nondimensionalized Pump Swashplate Angle

\( V_{m_{\text{max}}} \)  Motor Maximum Volumetric Displacement

\( \alpha_m \)  Motor Swashplate Angle

\( \alpha_{m_{\text{max}}} \)  Motor Maximum Swashplate Angle

\( \hat{\alpha}_m \)  Nondimensionalized Motor Swashplate Angle

\( \omega_3 \)  Compound Spur Gear Angular Speed

\( T_3 \)  Compound Spur Gear Torque

\( r_1 \)  Spur Gearbox Mechanical Path Gear Radius

\( r_2 \)  Spur Gearbox Hydraulic Path Gear Radius

\( r_{3_1} \)  Compound Spur Gear Outer Radius

\( r_{3_2} \)  Compound Spur Gear Inner Radius

\( \xi_d \)  Differential Gear Ratio

\( \xi_{\text{eff}} \)  Drivetrain Effective Gear Ratio

\( \eta_{\text{eff}} \)  Drivetrain Effective Efficiency

\( \psi \)  CVT Torque Ratio

\( \Pi_e \)  Engine Power

\( \eta_e \)  Engine Efficiency

\( \Pi_f \)  Fuel Combustion Power

\( \Pi_w \)  Wheel Power Demand

\( \Pi'_{e} \)  Maximum Available Engine Power

\( \Pi_{e_{\text{max}}} \)  Absolute Maximum Engine Power Capability

\( \hat{\Pi}_e \)  Nondimensionalized Maximum Available Engine Power
Engine Speed at Maximum Power

Nondimensionalized Engine Speed

Minimum Engine Speed

Maximum Engine Speed

Maximum Available Engine Torque

Fuel Mass Flow Rate

Fuel Lower Heating Value

Fuel Density

Vehicle Efficiency

Brake Specific Fuel Consumption

\( K_i \)  
\( i^{th} \) Brake Specific Fuel Consumption Coefficient

\( A \)  
Component Torque Efficiency Coefficient

\( B \)  
Component Torque Efficiency Coefficient

\( C \)  
Component Torque Efficiency Coefficient

\( D \)  
Component Torque Efficiency Coefficient

\( K_i \)  
\( i^{th} \) Component Volumetric Efficiency Coefficient

Iteration Count Variable

Scaled Engine Speed

Scaled Engine Torque

\( g_i \)  
\( i^{th} \) Inequality Constraint

Equality Constraint
1.1 Introduction

In modern times, rising fuel costs and environmental concerns have increased demand for fuel efficient vehicles with lower emission levels. Several vehicle configurations exist to provide a solution to this need, one being hybrid vehicles; other configurations include pure electric vehicles and vehicles designed to reduce aerodynamic drag on the vehicle. This chapter focuses on the emergence of alternative vehicles (alternative to pure gasoline-powered vehicles), in particular, hydraulic hybrid vehicles. Section 1.2 discusses the need for more fuel efficient vehicles, and Section 1.3 provides a review of studies undertaken to understand the nature of hybrid vehicles. Section 1.4 describes the objectives and goals for this thesis, and Section 1.5 outlines this document.

1.2 Background

For much of the history of automobiles, liquid fossil fuels have been the main source of energy. However, in the late 19\textsuperscript{th} to early 20\textsuperscript{th} centuries, vehicles were frequency designed to run on electricity or steam as well as fossil fuels. In 1913, the introduction of the self-starter made starting gasoline-powered vehicle much easier, and as a result, sales of gasoline-powered vehicles overshadowed all other competing vehicle types and continue to do so. In modern times, motivating factors have caused consumers to reconsider the dominance of gasoline-powered vehicles and revisit alternative power generation technology [1].
The first motivating factor is the cost of fuel. The price of fossil fuels is influenced by a variety of factors. The US Energy Information Administration (EIA) lists the following as the main components of the retail price of gasoline [2]: 1) the cost of crude oil, from which gasoline is produced, 2) refining costs and profits, 3) distribution and market costs and profits, and 4) taxes. As shown in Figure 1-1, the major factor of retail gasoline cost is the price of crude oil. Crude oil prices are determined by both supply and demand factors. World economy growth is the biggest factor on the demand side. Inherently, crude oil is a non-renewable resource so as long as a demand exists, prices will increase as less crude oil becomes available. Furthermore, as more countries become more developed, such as Brazil and China, and begin to compete with developed countries, such as the United States, on the world market, overall demand for fossil fuels increases. As such, oil has become a very political issue, which lends to one of the major factors on the supply side: the Organization of the Petroleum Exporting Countries (OPEC). OPEC influences gasoline prices by setting limits on oil production of its
members, which control all of the world’s spare oil production capacity and possess two-thirds of the world’s crude oil reserves. Furthermore, the countries making up OPEC are in the most unstable regions in the world. Figure 1-2 shows the geographic locations of OPEC members. The instability in these regions causes instability in the production of crude oil that is seen in the cost of gasoline. Figure 1-3 show the instabilities and overall change in gasoline price over the past nine years. As such, many countries are exploring options to decrease dependence on OPEC and other foreign oil as much as possible; these options will be discussed later.

Figure 1-2. Geographic location of OPEC members [3].

The second motivating factor for the switch away from pure gasoline-power vehicles is environmental concerns. The byproducts of the combustion process occurring in the vehicle’s engine are mostly nitrogen, water, and carbon dioxide (CO₂); these are not toxic or noxious, but CO₂ is recognized as a greenhouse gas that contributes to global warming. As discussed, more countries are developing their infrastructures, and the number of vehicles in operation is increasing causing more CO₂ to be released into the
atmosphere. Over the past 20 years, the burning of fossil fuels has been blamed for about 75% of the atmosphere’s increase in CO$_2$ from human activity. Figure 1-4 shows the increase of CO$_2$ concentration measured at Mauna Loa, HI. Although global warming is denounced by some as a manufactured crisis, governments are becoming more proactive towards the environment with the creation of regulations such as emission standards encouraging industries like the automobile industry to design products with reduced emission levels.

Figure 1-4. Increase in CO$_2$ concentration from 1959 to 2009 at Mauna Loa, HI [5].
One realization of an alternative vehicle is the hybrid vehicle. However, hybrid vehicles still utilize internal combustion engines (ICE) unlike alternative fuel powered vehicles such as electric and fuel-cell powered automobiles. Hybrid vehicles, as defined in this thesis, are vehicles that use a power storage device in conjunction with the combustion engine and a continuously variable transmission (CVT) path; both concepts are designed to improve engine operating performance, thus increasing the fuel efficiency of the vehicle. Figure 1-5 shows the power losses through different components of a conventional vehicle. Power released from the power storage device supplements power produced by the ICE and usually takes the form of a battery or hydraulic accumulator. The purpose of the CVT is to allow the ICE to run in a more efficient manner, thus reducing engine power losses. The CVT speed ratio is allowed to vary continuously unlike a mechanical transmission, which is made up of a discrete set of speed ratios that are changes either manually or automatically. Allowing the ICE to run at a more efficient level increases fuel economy as more power is produced with less fuel.

Figure 1-5. Energy requirements for combined city/highway driving [6].
There are several configurations for hybrid vehicles based on the placement of the CVT and secondary fuel source. Figure 1-6 shows two of these configurations. In a series configuration, the engine feeds power directly to the CVT, which effectively replaces the mechanical transmission of conventional vehicles. In a parallel configuration, power from the ICE is split along two paths, one of which contains the CVT. Typical CVTs used in hybrid vehicles are electric and hydraulic CVTs, both comprising similar components; Figure 1-7 displays a typical CVT layout. In the case of an electric CVT, an electric generator and motor are used in conjunction with a battery storage device; in the case of a hydraulic CVT, a hydraulic pump and motor are used in conjunction with an accumulator storage device. In both, the operating conditions of the generator and motor are altered to change the overall CVT speed ratio. This change of speed ratio allows the CVT to control how the ICE operates, the desired result of which is to maximize efficiency.

A) Engine CVT Wheels

B) Engine CVT Wheels

Figure 1-6. Two hybrid vehicle configuration types: A) series and B) parallel.
Several electric hybrid vehicles are on the market in the United States; one such vehicle, the 2011 Honda Insight, can achieve a fuel economy of 40 miles-per-gallon in city driving and 43 miles-per-gallon highway. However, the associated cost of electric hybrid vehicles is much higher than that of hydraulic hybrid vehicles. The Environmental Protection Agency (EPA) estimates that electric hybrid vehicles require 14 years for a payback, i.e. fuel savings that offset the initial cost of the vehicle, whereas a hydraulic hybrid vehicle only has an estimated 3 year payback offset [7]. Furthermore, the size of the hydraulic components is much smaller than the electric equivalent requiring less alteration to an existing vehicle frame. These benefits over electric hybrid vehicles are why the EPA is heavily invested in the research of hydraulic hybrid vehicles.

1.3 Literature Review

Historically in the study of hybrid vehicles, the goal is to bring the engine to a more efficiency operating point. In [8], Pfiffner et al. continues this trend by defining a line of best efficiency on an engine efficiency map; this line is constructed by finding the most efficient engine torque for every engine speed. The CVT chosen has the same specifications as the RHVF 154 CVT manufactured by P.I.V. Reimers and has no
secondary power source. In running a simulation of an electric hybrid vehicle with the CVT in series, see type A in Figure 1-6, with the DIRCOL computational package developed by von Stryk in 1999, the efficiency of the transmission is assumed one and the engine is only allowed to run on the line of best efficiency. The engine operating point is modulated by calculating for a CVT gear ratio that results in an engine speed and torque combination that lies on the line of best efficiency and still meets the power output demanded by the wheels. This research concludes with a fuel economy improvement between 2.5% and 5% in the hybrid vehicle over a conventional vehicle with the same engine and body type.

On the other hand, Mapelli et al. also study an electric hybrid vehicle of the same type as Pfiffner et al. but focus on reducing power losses through the electric CVT only, i.e. maximize efficiency of the CVT and the inverter component specifically [9]. In modeling the vehicle, Mapelli et al. derive in depth expressions for the power distribution between the components of the CVT, including these components’ efficiencies, but use a simple representation for the engine itself. To control the power flux through the CVT and the losses through the inverter, Mapelli et al. use a direct self control method, which is based on how electrical current moves through the CVT; the inverter efficiency is increased by modulating the frequency of the current. No results pertaining to a comparison between the hybrid and a conventional vehicle are made, however.

Kessels et al. provide a much more detailed description of their electric hybrid under study in [10]. Their CVT consists of two motor/generators (MG), an inner MG which receives power from the engine and an external MG which directly applies a propulsion torque to the driveline. Power electronics are connected to both MGs to
regulate battery power input and output. Together, the MGs alter the transmission ratio thereby bringing the internal combustion engine (ICE) to a desired operating point. Theoretically, the desired operating point lies on the line of best efficiency described in [8]; however, Kessels et al. included in their models efficiency information on the MGs and discovered that running at the line of best efficiency is not optimal for the entire vehicle as the efficiencies of the MGs could be low here. The control scheme used by Kessels et al. aims to account for transmission efficiency to achieve global optimization. Whenever the CVT has poor efficiency, to be defined by Kessels et al., power is not sent to the CVT; however, if the CVT has a high enough efficiency, the transmission ratio could be changed via the MGs to bring the engine towards the economy line. Kessels et al. conclude that the transmission losses are too great for the most part, and power is sent around the CVT, thus perfect tracking of the line of best efficiency is discouraged.

In [11], Dirck analyzes a hydraulic hybrid vehicle. The hybrid under study is a power split configuration with the CVT in one path and the mechanical transmission in the other. The CVT comprises of a hydraulic pump and motor; the pump has a variable displacement, and the motor is fixed. This CVT does not include a power storage device to store excess engine power. To alter the gear ratio of the CVT and by extension, the entire drivetrain itself, the variable displacement pump changes its swashplate angle to regulate the pressure and flow rate it outputs. In modeling the vehicle, drivetrain, and engine, Dirck takes no efficiency information into account. The control system Dirck takes is to have the engine running at its wide open throttle (WOT) line; this line represents the maximum torque the engine can output at any given engine speed. This notion is similar to the method of tracking the line of best efficiency; however, the WOT
line is not necessary the same as the line of best efficiency. In running a simulation for both city and highway driving, Dirck computes the engine power demanded by the vehicle and finds an operating point on the WOT line that meets this demand. Dirck reports fuel economy increases of 37% and 14% for city and highway driving, respectively, over those of a conventional vehicle with similar body type and engine. However, the models for the vehicle are idealized such that all efficiencies are one or a nominal constant, so the results are highly idealized as well.

Kumar and Ivantysynova, like Dirck, analyze a power split, hydraulic hybrid vehicle [12]. However, the configuration of the CVT is different. In this case, the CVT includes an accumulator to store excess engine power. Also, both the pump and motor are variable displacement; the pump is controlled to maintain a constant pressure within the system, and the motor is controlled to determine the power output of the CVT. Since the pump is automatically controlled to maintain a constant pressure in the hydraulic lines, the swashplate of the motor is used to control the gear ratio of the CVT. A control scheme for the motor swashplate angle is used in conjunction with a control scheme for the release of power from the accumulator; the accumulator charges when the vehicle is decelerating, or braking. When decelerating, the swashplate angle of the motor becomes negative, i.e. the motor becomes a pump, and the pressure build up from the two pumps is stored in the accumulator. The gear ratio of the CVT is determined through some optimization algorithm to find a balance between the efficiency of the drivetrain and the efficiency of the engine. They find that at low speeds, the hydraulic components have a low efficiency and the CVT is disadvantageous to use, so all power is sent through the mechanical path from the engine to the wheels. At these low speeds is where the
accumulator is activated to release pressure supplementing engine power; this way the CVT can still contribute to increasing the fuel economy even though the CVT cannot force the engine into a more efficient operating point. Simulations of a Prius equipped with the CVT increases fuel economy about 16% over a standard Prius.

1.4 Research Objective

The objective of this thesis is to expand upon the research done in [11] by including efficiency information of the engine and hydraulic components in the CVT. However, the CVT configuration is slightly different, replacing the fixed motor used in [11] with a variable displacement motor, similar to the CVT analyzed in [12]. The difference between the CVT analyzed in [12] and the CVT studied in this thesis is the mechanical transmission is retained and operates as a conventional manual or automatic transmission found in conventional vehicles. The system is modeled including component efficiency information and expressed in a form as to be dependent on the swashplate angle of the CVT variable displacement motor, which defines the gear ratio of the CVT. Since the model equations, including the efficiencies, are designed to be functions of the motor swashplate angle, a control measure can be applied to the swashplate angle as to optimize the overall vehicle efficiency and theoretically improve fuel economy. Simulations using a virtual test vehicle show the effect of altering this swashplate angle on the entire vehicle and determine if sending power through the CVT will indeed maximize fuel economy. Since the CVT configuration is similar to [12], results will be compared to results attained in that paper to confirm any conclusions.
1.5 Thesis Outline

This thesis is divided into six chapters. The first chapter gives an overview on the state of alternative vehicles, in particular hybrid vehicles, and states the need for hydraulic hybrid vehicles. A brief survey of prior work similar to the research conduct in this thesis is presented along with the research objects of this thesis.

Chapter 2 analyzes the system and provides models of the various components of the hybrid vehicle including efficiency information. An expression for the engine power demand by the vehicle in this chapter connects the desired operation of the vehicle with the operating conditions of the engine. Furthermore, the relationship between vehicle efficiency and fuel economy is presented as to provide a means to optimize for greater fuel economy.

Chapter 3 describes the simulations necessary to test the effectiveness of the models and optimization algorithm. Two driving schedules are used to test the CVT operating while under city and highway driving conditions. The simulations are programmed using the MATLAB environment, and these numerical codes are provided in the Appendices.

Results obtained from the simulations described in Chapter 3 are presented in Chapter 4. A comparison between the virtual test vehicle without a CVT and it with a CVT is made for the two driving schedules. Results here are also compared against the papers reviews in Section 1.3 to find similarities in data.

Chapter 5 concludes this thesis by providing conclusions made from of the results obtained in Chapter 4. These conclusions include the comparison between the conventional and hybrid vehicles as well as trends found in the optimization for greatest
efficiency. Furthermore, recommendations for future work are made to add to the conclusions reached in this thesis to progress the development of the hybrid drivetrain studied hereinafter.
CHAPTER 2. SYSTEM ANALYSIS

2.1 Introduction

This chapter focuses on deriving equations that describe power conveyance from the vehicle’s intake of fuel to the tires as well as the vehicle’s overall efficiency. The chapter consists of four sections for modeling the vehicle dynamics, drivetrain, engine, and vehicle efficiency. Newton’s Second Law is used to derive the vehicle dynamics and the drivetrain and engine models are derived with steady state conditions. In Section 2.3, a schematic of the drivetrain under study is provided while all other models are derived as generally as possible to be flexible enough to apply to all sorts of vehicle and engine models. These models are then used to determine the overall vehicle efficiency, a crucial equation for the purpose of this thesis.

2.2 Vehicle Dynamics

Modeling a hybrid vehicle of any sort begins with deriving vehicle dynamics. The vehicle dynamics connect the engine performance to the environment in which the vehicle operations. Deriving the vehicle’s equation of motion begins with considering the forces acting on the vehicle body, which tend to oppose the motion of the vehicle, and the force generated by the wheels for propulsion. Figure 2-1 is a free body diagram of a generic vehicle of mass $m$ traveling with longitudinal velocity $v_x$. Using Newton’s Second Law of Motion, the forces acting about the vehicle can be summed such that

$$m v_x = F_x - F_d - F_r - mg \sin(\beta), \quad (2.1)$$
where \( \dot{v}_x \) is the time derivative of velocity \( v_x \), \( F_x \) is the total propulsive force generated by all powered wheels, \( F_d \) is aerodynamic drag, \( F_r \) is rolling resistance, \( g \) is the gravitational constant, and \( \beta \) is the driving surface incline.

![Vehicle free body diagram with emphasized typical wheel.](image)

The propulsion force, \( F_x \), is the sum of forces generated by each wheel. Assuming the vehicle travels on a straight path and has \( l \) axles, the propulsion force can be written as

\[
F_x = n \sum_{i=1}^{l} F_{x_i},
\]

where \( n \) is the number of wheels per axle and \( F_{x_i} \) is the propulsion force generated by a wheel on the \( i^{th} \) axle; note that \( F_{x_i} = 0 \) for unpowered axles. Furthermore, by using
Newton’s Second Law for rotational motion, a relationship between the torque, $T_{w_i}$, in the $i^{th}$ axle and its propulsion force such that

$$J_i \omega_{w_i} = T_{w_i} - F_{s_i} r_i,$$  \hspace{1cm} (2.3)

where $J_i$ is the moment of inertia for a tire on the $i^{th}$ axle and $\omega_{w_i}$ is its angular acceleration. With the propulsion force now related to axle torque, vehicle dynamics can be related to engine performance, as discussed in subsequent sections of this chapter.

As the vehicle moves, it must displace air. This air creates aerodynamic drag on the vehicle opposing the vehicle’s direction of motion. Drag force is a function of the vehicle’s absolute velocity, atmospheric density, $\rho$, and projected area perpendicular to motion, $A_{d_i}$, such that

$$F_d = \frac{1}{2} \rho C_d A_{d_i} v_i^2$$  \hspace{1cm} (2.4)

The drag coefficient, $C_d$, is an experimentally determined value based on the geometry of the vehicle.

Rolling resistance is a property of the tires that opposes motion of the vehicle. Rolling resistance stems from the weight of the vehicle deforming the air-filled tires and is dependent on the tire and road surface materials. This resistive force can be approximated by

$$F_r = C_r mg,$$  \hspace{1cm} (2.5)

where $C_r$ is the rolling resistance coefficient; this coefficient can assume a value typical to automobiles with standard tires on an asphalt road.

The vehicle configuration being studied assumes only one powered axle, the rear axle, with two wheels, in other words, $l = 1$ and $n = 2$ for Equations (2.2) and (2.3), and
since \( l = 1 \) only, the \( i \) subscript is dropped. Furthermore, the angular dynamics are related to linear dynamics as follows:

\[
\begin{align*}
\dot{v}_x &= r\omega_w, \\
\ddot{v}_x &= r\dot{\omega}_w. 
\end{align*}
\] (2.6)

Using this relation and Equations (2.2) through (2.5) with Equation (2.1), the torque on a single tire may be written as

\[
T_w = \left( \frac{1}{2} mr^2 + J \right) \dot{\omega}_w + \frac{1}{4} \rho C_d A_d r^3 \alpha_w^2 + \frac{1}{2} mgr \left( C_r + \sin(\beta) \right). 
\] (2.7)

Equations (2.6) and (2.7) will be used to connect the vehicle dynamics with engine performance by determining a relationship between axle torque and speed with engine torque and speed.

2.3 Drivetrain Model

The transmission configuration studied, shown in Figure 2-2, is a combination of both series and parallel hybrid designs; series in the fact that the mechanical and hydraulic transmissions are in series, but parallel in that a parallel path stems from the mechanical transmission via a planetary gear to circumvent the hydraulic transmission all together. In this design, power is produced by an internal combustion engine and transmitted to a mechanical transmission then split with a planetary gear to two paths, one leading directly to the spur gear box and one through a hydraulic continuously variable transmission (CVT). The spur gear box sends power through a differential out to the wheels.
2.3.1 Mechanical Transmission

A mechanical transmission, both manual and automatic, can be described as adjustable between a discrete set of speed ratio; typically, automotive transmissions vary between four and six different speed ratios. Power is transmitted through the mechanical transmission via the relations

\[ T_s = -\Xi T_e, \]
\[ \omega_s = -\frac{\omega_e}{\Xi}, \]

(2.8)

where \( T_e \) and \( \omega_e \) are the engine torque and speed, and \( T_s \) and \( \omega_s \) are the torque and speed sent to the input sun gear of a planetary gear set. The rule governing how the gear ratio is changed is usually determined by vehicle velocity, i.e. between certain speeds, a certain gear ratio is used. Note that torque and speed efficiencies are purposefully disregarded since they are high and non-variable; all efficiencies seen later are of a variable nature.

Figure 2-2. Hybrid transmission schematic.
2.3.2 Planetary Gearbox

A planetary gearbox is used to split power from the engine and mechanical transmission down two paths. Figure 2-3 shows this gearbox with the hydraulic path on the left, mechanical path on the right, and the planetary gear set in the center. The hydraulic path gear is meshed with the outer ring of the planetary gear set with pitch radii $r_h$ and $r_R$, respectively. Likewise, the mechanical path gear is meshed to a gear attached to the planetary arm, shown in Figure 2-3 as four spokes with a planet gear at each end, and is represented in Figure 2-3 by a dashed pitch-circle with radius $r_A$; the mechanical path gear has a pitch radius of $r_m$.

![Planetary Gearbox Schematic](image)

Figure 2-3. Planetary gearbox schematic.

As with the vehicle dynamics model, a free-body diagram approach is used to understand the power conveyance of the planetary gearbox. Figure 2-4 displays a simplified planetary gear set with a free-body-diagram of each component. Summing the
moments about the center of the sun gear, the first component considered, the torque experienced, $T_s$, is

$$T_s = F_s r_s,$$

(2.9)

where $F_s$ is the reaction force between the sun gear and planet gear and $r_s$ is the pitch.

Figure 2-4. Partial free-body diagrams for components within a planetary gear set.
radius of the sun gear. Note that the moments of inertia and angular accelerations for all
the gears discussed are absent when assuming steady state conditions, i.e. acceleration is
zero. Likewise, performing the same operation for the moments about the arm yields the
relation

\[ T_a = -F_a r_a, \]  

(2.10)

where \( T_a \) is the applied torque to the arm, \( F_a \) is the reaction force at the pin joint between
the arm and planet gear, and \( r_a \) is the arm radius. Next, the planet gears do not actually
transmit power since they undergo no rotational torque, but they do provide a relation
between the reaction force of the sun gear, \( F_s \), and the reaction force of the arm, \( F_a \), such
that

\[ F_s = \frac{F_a}{2}. \]  

(2.11)

Finally, performing the same summing of moments on the ring gear produces

\[ T_r = F_r r_r, \]  

(2.12)

where \( T_r \) is the applied torque to the ring, \( F_r \) is the reaction force between the planet gear
and ring gear, and \( r_r \) is the internal pitch radius of the ring. Since the applied torque to
the sun gear is known from Equation (2.8), the arm and sun reaction forces are expressed
in terms of \( T_s \) using Equations (2.9) through (2.12) such that

\[ F_a = \frac{2T_r}{r_s}, \]  

\[ F_s = \frac{T_r}{r_s}. \]  

(2.13)

Likewise, the same can be done to relate the applied torques of the arm and ring:

\[ T_a = -\xi_a T_s, \]  

\[ T_r = \xi_r T_s. \]  

(2.14)
where the speed ratios for the arm and ring are given respectively as

\[ \xi_a = 2 \frac{r_a}{r_s}, \]
\[ \xi_r = \frac{r_r}{r_s}. \]  

(2.15)

Another property to determine is the speed outputs of the arm and ring gears. The conservation of power within the gear set is written as

\[ T_a \omega_a + T_s \omega_s + T_r \omega_r = 0. \]  

(2.16)

Note that again, efficiencies are purposefully ignored since they are constant. Using the torque relations from Equation (2.14) in conjunction with Equation (2.16), a speed relation between the input and outputs of the planetary gear set is

\[ \omega_s = \xi_a \omega_a - \xi_r \omega_r. \]  

(2.17)

In summary, Equations (2.14) and (2.17) will be used to describe the planetary gear set in the transmission.

As mentioned earlier, the mechanical path gear is meshed to the arm gear. Considering static equilibrium between the mechanical path gear and arm gear and conservation of power, torque and speed relations are given as

\[ T_1 = \xi_m T_a, \]
\[ \omega_1 = -\frac{\omega_a}{\xi_m}, \]  

(2.18)

where \( T_1 \) is the torque delivered to the mechanical path, \( \omega_1 \) is the angular velocity of the mechanical path shaft, and the speed ratio between the arm and mechanical path gears is given as

\[ \xi_m = \frac{r_m}{r_A}, \]  

(2.19)
where \( r_m \) and \( r_A \) are shown in Figure 2-3. Furthermore, the hydraulic path gear is meshed to the outer ring of the planetary gear set. Considering static equilibrium between these gears and conservation of power yields

\[
T_h = \xi_h T_r, \\
\omega_h = -\frac{\omega_r}{\xi_h},
\]

(2.20)

where \( T_h \) is the torque delivered to the hydraulic transmission, \( \omega_h \) is the angular velocity of the input shaft to the hydraulic transmission, and the speed ratio between the ring and hydraulic path gears is given as

\[
\xi_h = \frac{r_h}{r_R},
\]

(2.21)

where \( r_h \) and \( r_R \) are shown in Figure 2-3.

2.3.3 Hydraulic Transmission

The hydraulic CVT shown in Figure 2-5 is comprised primarily of a pressure controlled pump and a displacement controlled motor, as shown in Figure 2-5. The novel aspect of this CVT is that the pump is controlled to maintain a constant pressure, allowing only one input, the swashplate angle of the motor, to affect the gear ratio of the

Figure 2-5. Hydraulic CVT schematic.
CVT. From the definition in [13] of torque efficiency for these two devices, the input, \( T_h \), and output, \( T_2 \), torques of the CVT are given by

\[
T_h = \frac{PV_p'}{\eta_{tp}},
\]

\[
T_2 = -PV_m'\eta_{tm},
\]

(2.22)

where \( P \) is the working pressure of the transmission, \( V_p' \) is the instantaneous volumetric displacement of the pump, \( V_m' \) is the volumetric displacement of the motor, and \( \eta_{tp} \) and \( \eta_{tm} \) are the torque efficiencies of the pump and motor, respectively. The instantaneous volumetric displacement of the pump automatically changes to hold the working pressure constant, and the instantaneous volumetric displacement of the motor is controlled to optimize the overall vehicle efficiency, as discussed in the next chapter. From the definition in [13] of volumetric efficiency for a pump and motor, the input, \( \omega_h \), and output, \( \omega_2 \), speeds for the CVT given as

\[
\omega_h = \frac{Q}{V_p'\eta_{tp}},
\]

\[
\omega_2 = -\frac{Q\eta_{tm}}{V_m'},
\]

(2.23)

where \( Q \) is the volumetric flow rate from the pump to the motor, and \( \eta_{tp} \) and \( \eta_{tm} \) are the volumetric efficiencies for the pump and motor, respectively. Algebraically eliminating \( P \) and \( Q \) from Equations (2.22) and (2.23), respectively, yields the power conveyance through the CVT, written as

\[
T_2 = -\eta_f\Xi_v T_h,
\]

\[
\omega_2 = -\frac{\eta_f\omega_h}{\Xi_v},
\]

(2.24)
where the friction and speed efficiencies are given respectively as

\[
\eta_f = \eta_p \eta_{fa}, \quad \eta_s = \eta_{v_p} \eta_{va},
\]  

(2.25)

and the speed ratio for the CVT is given as

\[
\Xi_v = \frac{V_m'}{V_p'}. 
\]  

(2.26)

Rearranging Equation (2.22) for the volumetric displacement of the pump shows that

\[
V_p' = \frac{T_h \eta_p}{P},
\]  

(2.27)

which illustrates the point that the volumetric displacement of the pump automatically changes in response to a changing input torque, \(T_h\). Furthermore, it will become important in a later section to know the swashplate angle the pump uses to maintain the working pressure. The volumetric displacement of the pump can be written as

\[
V_p' = V_{p_{\text{max}}} \hat{\alpha}_p,
\]  

(2.28)

where \(V_{p_{\text{max}}}\) is the maximum volumetric displacement of the pump and \(\hat{\alpha}_p\) is the control input to the pump. Physically, the pump control input is its nondimensionalized swashplate angle such that

\[
\hat{\alpha}_p \equiv \frac{\alpha_p}{\alpha_{p_{\text{max}}}},
\]  

(2.29)

where \(\alpha_p\) is the pump swashplate angle and \(\alpha_{p_{\text{max}}}\) is the maximum pump swashplate angle; the control input can take a value between 0 and 1. Using Equations (2.27) and (2.28), the pump control input necessary to maintain the working pressure, \(P\), can be written as
\[ \hat{\alpha}_p = \frac{T_\theta \eta_p}{V_{\max} P}. \]  

(2.30)

Similar to Equation (2.28), a convenient way to describe the adjustment of the volumetric displacement for the motor is given as

\[ V'_m = V_{\max} \hat{\alpha}_m, \]  

(2.31)

where \( V_{\max} \) is the maximum volumetric displacement of the motor and \( \hat{\alpha}_m \) is the motor control input. The motor control input is the result of nondimensionalizing its swashplate angle using the same method as in Equation (2.29). A new expression for the CVT speed ratio, \( \Xi_v \), is produced by substituting Equations (2.27) and (2.31) into Equation (2.26) such that

\[ \Xi_v = \frac{PV_{\max} \hat{\alpha}_m}{T_\theta \eta_p}. \]  

(2.32)

Likewise, Equation (2.24) can be updated with Equation (2.32) such that

\[ T_2 = -\eta_m PV_{\max} \hat{\alpha}_m, \]

\[ \omega_2 = -\frac{\eta_p \eta_{\theta_2} T_h}{PV_{\max} \hat{\alpha}_m}. \]  

(2.33)

2.3.4 Spur Gearbox

The spur gearbox closes the parallel mechanical and hydraulic paths of the transmission to convey power to the differential gearbox; Figure 2-6 shows its schematic. Conservation of energy and kinematic relations applied here show that

\[ T_3 = \xi_1 T_1 + \xi_2 T_2, \]

\[ \omega_3 = -\frac{\omega_1}{\xi_1} = -\frac{\omega_2}{\xi_2}. \]  

(2.34)
where $T_3$ is the applied torque in the compound spur gear, $\omega_3$ is its angular velocity, and the speed ratios are given by

$$\xi_1 = \frac{r_3}{r_1}, \quad \xi_2 = \frac{r_3}{r_2}. \quad (2.35)$$

The pitch radii $r_1$, $r_2$, $r_3_1$, and $r_3_2$ are shown in Figure 2-5.

Figure 2-6. Compound spur gearbox schematic.

2.3.5 Differential Gearbox

A differential gearbox is used to split power from the spur gearbox to each of the rear wheels. Power distribution through the differential is described with

$$T_w = \frac{\xi_d T_3}{2},$$

$$\omega_w = -\frac{\omega_3}{\xi_d}. \quad (2.36)$$
Recall $T_w$ and $\omega_w$ are the wheel torque and speed, respectively, described by Equations (2.7) and (2.6) in Section 2.2.

To summarize the entire drivetrain model, the ratios between input and output speed and torque are useful as they contain both the drivetrain’s effective efficiency, $\eta_{\text{eff}}$, and effective speed ratio, $\xi_{\text{eff}}$. Using Equations (2.8), (2.14), (2.17), (2.18), (2.20), (2.33), (2.34), and (2.36) that each describe the conveyance of power from one component to the next, the relationship between the input and output speed and torque is given by the following:

\[
\frac{\omega_c}{\omega_w} = \frac{\xi_{\text{eff}}}{\eta_{\text{eff}}} = \Xi_{1} \Xi_{2} \Xi_{3} \Xi_{4} \left( 1 + \frac{PV_{\text{in}} \xi_{\text{in}}}{2T_{\text{w}} \eta_{\text{in}} \eta_{\text{r}}} \hat{\alpha}_{\text{m}} \right),
\]

where $\eta_{\text{in}}$ and $\xi_{\text{in}}$ are the effective speed and friction efficiencies and the product of which form the drivetrain’s effective efficiency. Note that the speed ratio in Equation (2.37) was linearized for a small $\hat{\alpha}_{\text{m}}$. Furthermore, a design constraint is placed upon the speed ratios along the mechanical path such that

\[
\Xi_{1} \Xi_{2} \Xi_{3} \Xi_{4} \equiv 1.
\]

This constraint ensures that when the swashplate angle of the hydraulic motor is zero, the vehicle operates the same as a conventional vehicle, whose drivetrain consists of only a mechanical transmission and differential. To simplify notation, a new variable, $\psi$, is introduced such that

\[
\frac{PV_{\text{in}} \xi_{\text{in}}}{2T_{\text{w}} \eta_{\text{in}}} \equiv \psi.
\]
The simplified relations from Equation (2.37) written in matrix form are given as

\[
\begin{bmatrix}
\frac{\omega_c}{\omega_w} \\
\frac{T_c}{2T_w} \\
\psi
\end{bmatrix} = 
\begin{bmatrix}
-\Xi & \psi \\
\eta_s & \eta_p \\
\Xi & \psi
\end{bmatrix} \mathbf{X} + 
\begin{bmatrix}
\Xi \\
1
\end{bmatrix},
\]  
(2.40)

These ratios will be used to describe the overall power conveyance through the drivetrain as they contain information about the transmission efficiency and effective speed ratio.

2.4 Engine Model

In modeling an engine, the input power and output power are of interest. The input power, \( \Pi_f \), is provided by the combustion of fuel, forcing the engine’s pistons to spin the crankshaft; the output power, \( \Pi_e \), is related to the input power by

\[ \Pi_e = \eta_e \Pi_f, \]  
(2.41)

where \( \eta_e \) is the engine efficiency. The power generated by the engine that is transmitted to the driveshaft is called the brake power, or the power a brake attached to the crankshaft must overcome to stop the vehicle. In order to produce a certain desired vehicle velocity, power is demanded by the vehicle from the engine through the transmission. Thus, the brake power can be represented in terms of the power demanded at the wheel, \( \Pi_w \), as

\[ \Pi_e = \frac{2\Pi_w}{\eta_{eff}}, \]  
(2.42)

where \( \eta_{eff} \) is the effective efficiency of the transmission. The power demanded by the wheel is dependent on the wheel torque and speed requirement such that

\[ \Pi_w = T_w \omega_w. \]  
(2.43)
Recall $\omega_w$ and $T_w$ are expressed in Equations (2.6) and (2.7) in Section 2.2, respectively. A complete expression for the brake power, or power output, demanded is

$$\Pi_e = \frac{v_x}{\eta_{eff}} \left[ \left(m + 2J \right) \dot{v}_x + \frac{1}{2} \rho C_d A_y v_x^2 + mg \left(C_r + \sin(\beta)\right) \right];$$

(2.44)

in other words, to maintain a vehicle velocity of $v_x$, the wheels must receive $\Pi_e$ units of power from the engine.

A limit exists on the amount of brake power that is available at any given engine speed; this maximum available power is not constant. To determine the maximum available power as a function of engine speed, this function, $\Pi'_e(\omega_e)$, and the engine speed can be nondimensionalized using the absolute maximum power capability, $\Pi_{e \text{ max}}$, of the engine and the engine speed at this maximum power, $\omega_{e \text{ max}}$, such that

$$\hat{\omega}_e = \frac{\omega_e}{\omega_{e \text{ max}}},$$

(2.45)

where $\hat{\omega}_e$ is the nondimensionalized engine speed, and

$$\hat{\Pi}_e (\hat{\omega}_e) = \frac{\Pi'_e(\omega_e)}{\Pi_{e \text{ max}}},$$

(2.46)

where $\hat{\Pi}_e (\hat{\omega}_e)$ is the nondimensionalized maximum power as a function of the nondimensionalized engine speed; this method is used in [14] to model a generic engine. The nondimensionalized maximum power can be approximated with a third-order polynomial,

$$\hat{\Pi}_e (\hat{\omega}_e) = a_1 \hat{\omega}_e + a_2 \hat{\omega}_e^2 - a_3 \hat{\omega}_e^3,$$

(2.47)

where $a_1$, $a_2$, and $a_3$ are coefficients chosen to satisfy the following conditions:
1) \[ \hat{\Pi}_e(1) = a_1 + a_2 - a_3 = 1, \]

2) \[ \left( \frac{d\hat{\Pi}_e}{d\hat{\omega}_e} \right)_{\hat{\omega}_e = 1} = a_1 + 2a_2 - 3a_3 = 0. \]

Condition 1 ensures the absolute maximum power output occurs at the proper engine speed, i.e. \( \omega_e \), and condition 2 ensures the maximum power output is indeed situated at the maximum of the polynomial. Choosing \( a_1 = 1 \) also allows \( a_2 = a_3 = 1 \). Working back to Equation (2.42) with knowledge of Equations (2.45), (2.46), and (2.47), the maximum brake power, \( \Pi_e'(\omega_e) \), is shown as

\[ \Pi_e'(\omega_e) = \frac{\Pi_{\text{e max}}}{\omega_{e_0}} \omega_e + \frac{\Pi_{\text{e max}}}{\omega_{e_0}^2} \omega_e^2 - \frac{\Pi_{\text{e max}}}{\omega_{e_0}^3} \omega_e^3. \] (2.48)

Thus, the maximum power function is dependent on the engine’s absolute maximum power capability, the engine speed at this power level, and the current engine speed. Dividing Equation (2.48) by the engine speed will result in the maximum available engine torque, \( T_e'(\omega_e) \), such that

\[ T_e'(\omega_e) = \frac{\Pi_{\text{e max}}}{\omega_{e_0}} + \frac{\Pi_{\text{e max}}}{\omega_{e_0}^2} \omega_e - \frac{\Pi_{\text{e max}}}{\omega_{e_0}^3} \omega_e^2. \] (2.49)

The maximum available engine torque function in Equation (2.49) is called the wide-open-throttle (WOT) line as the maximum available engine torque occurs, just as the maximum brake power, when at full throttle; this is shown in Figure 2-7. Note that Equation (2.49) is an approximation of an actual WOT line just as Equation (2.48) is an approximate of maximum available brake power; true models must be done experimentally for individual engines. Furthermore, limits exist on the engine speed, as shown in Figure 2-7, such that
where \( \omega_{e_{\text{min}}} \) and \( \omega_{e_{\text{max}}} \) are the minimum and maximum rated engine speeds, respectively. These values vary among specific engine types.

![Figure 2-7. Engine WOT line featuring important engine parameters.](image)

The power input to the engine is primarily based on the fuel being used. The power input to the engine, \( \Pi_f \), is defined in [15] as

\[
\Pi_f = \dot{m}_f \cdot \Delta H_c^0,
\]

where \( \dot{m}_f \) is the mass flow rate of fuel into the engine and \( \Delta H_c^0 \) is the fuel’s heat of combustion and in this case represents its Lower Heating Value (LHV). A substance’s heat of combustion is a measure of the energy released as heat when the substance undergoes complete combustion. The LHV of the heat of combustion disregards the
energy taken to vaporize water contained in the fuel and describes the energy released solely by fuel. Combining Equation (2.6) from Section 2.2 and Equations (2.41), (2.42), (2.43), and (2.51) along with the following definitions from calculus:

\[
v_x = \frac{dx}{dt},
\]

\[
\dot{m}_f = \rho_f \frac{dV}{dt},
\]

where \(x\) is distance traveled, \(\rho_f\) is fuel density, and \(V\) is fuel volume, an expression for distance traveled per change in fuel volume is obtained:

\[
\frac{dx}{dV} = \frac{\eta_e \eta_{eff} r \rho_f \cdot \Delta H_c^0}{2T_w},
\]

with proper unit conversions, Equation (2.53) will result in the instantaneous miles per gallon rating of the vehicle; this equation concurs with the mile-per-gallon equation presented in [11] with the added benefit of including component efficiencies. In this equation, the wheel torque, \(T_w\), contains the dynamics of the vehicle as shown in Equation (2.7) and relates vehicle parameters, such as its weight and aerodynamic qualities, to the vehicle fuel economy. Furthermore, maximizing the engine efficiency, \(\eta_e\), and transmission efficiency, \(\eta_{eff}\), increases the fuel economy of the vehicle.

2.5 Power Efficiency

The power efficiency of a vehicle is the percentage of power contained in the fuel that is passed through the engine and transmission to the wheels. Furthermore, the overall power efficiency is the product of the engine efficiency and transmission efficiency. These relationships are described as
\[ H = \eta_e \eta_{\text{eff}} = \frac{2\Pi_w}{\Pi_f}; \]  

where \( H \) is the overall vehicle efficiency. As shown in Equation (2.54), to understand the overall vehicle efficiency, engine efficiency and transmission efficiency must be understood.

Engine efficiency is the ratio of brake power to fuel power; recall Equation (2.41) in Section 2.4. Using the definition of fuel power, engine efficiency is described as

\[ \eta_e = \frac{\Pi_e}{m_f \cdot \Delta H_e^0}. \]  

The ratio of fuel mass flow rate, \( \dot{m}_f \), and brake power, \( \Pi_e \), is called the Brake Specific Fuel Consumption (BSFC) such that

\[ g_e = \frac{\dot{m}_f}{\Pi_e}, \]  

where \( g_e \) is the engine’s BSFC. The BSFC is a measure of fuel economy and is determined experimentally. Using Equation (2.56), engine efficiency is redefined as

\[ \eta_e = \frac{1}{g_e \cdot \Delta H_e^0}, \]  

and since \( \Delta H_e^0 \) is constant for a given fuel type, an expression for \( g_e \) is needed. In [16], Golverk models the BSFC map using a second order polynomial:

\[ g_e (\omega_e, T_e) = K_1 + K_2 \omega_e + K_3 T_e + K_4 \omega_e^2 + K_5 \omega_e T_e + K_6 T_e^2, \]  

where \( K_i \) (\( i = 1, \ldots, 6 \)) are experimentally determined coefficients; these coefficients vary with engine type and model. It will be useful to alter Equation (2.58) to be in terms of the CVT control, \( \hat{\alpha}_m \), and this can be done using the ratios in Equation (2.40).
The second component of the vehicle efficiency is the drivetrain’s effective efficiency. As shown in Equation (2.37) in Section 2.3, $\eta_{\text{eff}}$ is the inverse of the product of the ratios in Equation (2.40) such that

$$\eta_{\text{eff}} = \frac{\eta_s \eta_f}{\eta_s \eta_f + (1 - \eta_s \eta_f) \psi \hat{a}_m}. \quad (2.59)$$

Recall the definitions of $\eta_s$ and $\eta_f$ in Equation (2.25). Expressions for $\eta_p$, $\eta_v$, $\eta_m$, and $\eta_{\text{sw}}$ are defined as the following [17]:

$$\eta_p = 1 - A \exp \left( -B \frac{\mu \omega_h}{P \alpha_{\text{max}} \hat{a}_p} \right) - C \sqrt{\frac{\mu \omega_h}{P \alpha_{\text{max}} \hat{a}_p}} - D \frac{1}{P \alpha_{\text{max}} \hat{a}_p},$$

$$\eta_v = 1 - \frac{P}{K_3} - K_1 \frac{P}{\mu \alpha_{\text{max}} \hat{a}_p \omega_h} - K_2 \sqrt{\frac{P}{\alpha_{\text{max}} \hat{a}_p \omega_h}}, \quad (2.60)$$

$$\eta_m = 1 - A \exp \left( -B \frac{\mu \omega_2}{P \alpha_{\text{max}} \hat{a}_m} \right) - C \sqrt{\frac{\mu \omega_2}{P \alpha_{\text{max}} \hat{a}_m}} - D \frac{1}{P \alpha_{\text{max}} \hat{a}_m},$$

$$\eta_{\text{sw}} = 1 - K_1 \frac{P}{\mu \alpha_{\text{max}} \hat{a}_m \omega_2} - K_2 \frac{\sqrt{P}}{\alpha_{\text{max}} \hat{a}_m \omega_2},$$

where the coefficients $A, B, C, D, K_1, K_2,$ and $K_3$ are listed in Table 2-1 and $\mu$ is the viscosity of the hydraulic fluid used. Note that the efficiencies in Equation (2.60) go to negative infinity as $\hat{a}_m$ and $\hat{a}_p$ approach zero. To avoid negative efficiency values, an assumption is made that these efficiencies have a minimum value of 65%. The efficiencies in Equation (2.60) can be rewritten in terms of $\hat{a}_m$, $\omega_w$, and $T_w$ with the following equations:

$$\omega_2 = \frac{\xi_z}{\xi_d} \xi_{\omega_h} \omega_h,$$

$$\omega_h = \frac{\xi_d \omega_h \psi}{\xi_z \xi_h} \hat{a}_m,$$

$$\hat{a}_p = 2 \frac{\xi_z \xi_{T_w}}{V \xi_d P \psi} (\psi \hat{a}_m + 1). \quad (2.61)$$
Table 2-1. Efficiency coefficients and parameters [17].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pump</th>
<th>Motor</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.17</td>
<td>0.17</td>
<td>no units</td>
</tr>
<tr>
<td>$B$</td>
<td>$9.5511 \times 10^8$</td>
<td>$4.7755 \times 10^8$</td>
<td>deg</td>
</tr>
<tr>
<td>$C$</td>
<td>30.9049</td>
<td>154.5243</td>
<td>$\sqrt{\text{deg}}$</td>
</tr>
<tr>
<td>$D$</td>
<td>$1.25 \times 10^7$</td>
<td>$1.25 \times 10^7$</td>
<td>Pa deg</td>
</tr>
<tr>
<td>$K_1$</td>
<td>$1.047 \times 10^{-7}$</td>
<td>$1.047 \times 10^{-7}$</td>
<td>deg</td>
</tr>
<tr>
<td>$K_2$</td>
<td>0.0033</td>
<td>0.0066</td>
<td>$\frac{\text{deg}}{\sqrt{\text{Pa s}}}$</td>
</tr>
<tr>
<td>$K_3$</td>
<td>$10^9$</td>
<td>not applicable</td>
<td>Pa</td>
</tr>
</tbody>
</table>

In summary, the efficiency of the vehicle is the product of Equations (2.57) and (2.59). Both of these efficiency equations are dependent on the torque and volumetric efficiencies of both the pump and motor in the CVT laid out in Equation (2.60). Using the chain of equations describing the transmission of power through the drivetrain, the efficiency equations can be expressed in terms of the motor control input, and wheel torque and speed; the relevance of this will become more apparent in the next chapter.

2.6 Summary

Models for the various drivetrain components have been presented to describe the complete power conveyance from fuel intake to the road. Furthermore, Equation (2.53) shows that maximizing the overall vehicle efficiency improves gas mileage, an important
fact when setting up an optimization problem. The models established within this chapter will be used either directly or indirectly in writing and solving the optimization problem discussed in the next chapter.
CHAPTER 3. NUMERICAL SIMULATION

3.1 Introduction

This chapter will test the models in the previous chapter to determine the differences in fuel economy between the hydraulic hybrid vehicle equipped with a CVT with a conventional vehicle. In the next section, values will be given to all the variables discussed thus far. Upon assigning values to the variables, the simulation will run using two driving schedules: one driving schedule represents urban driving and the other highway driving. Finally, an alternate simulation will be introduced to simulate a conventional vehicle, i.e. one without a CVT, in which to compare with the simulation results using the hybrid vehicle.

3.2 Simulation Parameters

The vehicle under consideration for the purposes of the simulations discussed in this chapter is a 1997 Ford Ranger, which was chosen as further physical testing and prototyping, beyond this thesis, will occur on this vehicle model. Table 3-1 details the parameters of the Ranger’s body. In addition to the physical parameters of the vehicle body, the values for air density, \( \rho \), and gravitational acceleration, \( g \), are needed for Equation (2.7); the density of air is 1.2041 kg m\(^{-3}\), and the gravitational acceleration is 9.81 m s\(^{-2}\). Furthermore, for the purposes of these simulations, the driving surface include angle, \( \beta \), will be held a constant zero such that the only effect gravity has on the vehicle is in the rolling resistance force.
3.2.1 Vehicle Parameters and Driving Conditions

Additional information needed to calculate wheel torque and speed, which will ultimately be used for calculations in the optimization algorithm, are the vehicle velocity and acceleration. As mentioned, two driving schedules will be used to conduct simulations [18, 19]; both driving schedules were constructed by the Environmental Protection Agency (EPA). The first driving schedule, the New York City Cycle (NYCC), features low speed stop-and-go traffic conditions, simulating city driving. Lasting 10 minutes, this driving schedule also exhibits relatively aggressive acceleration and deceleration profiles; Figure 3-1 shows the NYCC profile.

Table 3-1. Overall vehicle parameters for a 1997 Ford Ranger.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$m$</td>
<td>1347.1</td>
<td>kg</td>
</tr>
<tr>
<td>Wheel Radius</td>
<td>$r$</td>
<td>0.305</td>
<td>m</td>
</tr>
<tr>
<td>Wheel Moment of Inertia</td>
<td>$J$</td>
<td>13</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>Drag Coefficient</td>
<td>$C_d$</td>
<td>0.4</td>
<td>no units</td>
</tr>
<tr>
<td>Frontal Area</td>
<td>$A_d$</td>
<td>2</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Rolling Resistance Coefficient</td>
<td>$C_r$</td>
<td>0.01</td>
<td>no units</td>
</tr>
</tbody>
</table>

The second driving schedule is the Highway Fuel Economy Driving Schedule (HWFET) and mimics highway driving under 60 mph. Differing from the NYCC, this driving schedule has very little velocity change and lasts 13 minutes; Figure 3-2 shows the
HWFET profile. Using the MATLAB `diff()` command on the data comprising the two driving schedules, the vehicle accelerations can be computed; these are shown in Figures 3-3 and 3-4. This native MATLAB function simply uses a finite difference method to compute the acceleration. Now, using vehicle velocity and acceleration data and parameters from Table 3-1, the wheel torque and speed can be calculated, which will act as inputs into the drivetrain that contains the control input.

![Figure 3-1. NYCC vehicle velocity profile.](image)

![Figure 3-2. HWFET vehicle velocity profile.](image)
3.2.2 Engine Parameters

The test vehicle uses a 4.0 liter Over Head Valve (OHV) V-6 internal combustion engine. The engine performance specifications, listed in Table 3-2, are crucial for implementing the constraints laid out in the previous chapter. Refer to Figure 2-7, the WOT line, for a graphical representation of the parameters listed in Table 3-2.
Table 3-2. Engine performance specifications [20].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Engine Power</td>
<td>$\Pi_{e_{\text{max}}}$</td>
<td>160</td>
<td>hp</td>
</tr>
<tr>
<td>Speed at Maximum Power</td>
<td>$\omega_{\omega_{e}}$</td>
<td>4200</td>
<td>rpm</td>
</tr>
<tr>
<td>Minimum Engine Speed</td>
<td>$\omega_{e_{\text{min}}}$</td>
<td>1000</td>
<td>rpm</td>
</tr>
<tr>
<td>Maximum Engine Speed</td>
<td>$\omega_{e_{\text{max}}}$</td>
<td>5000</td>
<td>rpm</td>
</tr>
</tbody>
</table>

The coefficients for the BSFC map in Equation (2.58) are to be determined experimentally using a procedure prescribed in [16]. However, to avoid stripping the engine from the test vehicle for experimentation, example values from [16] are used and are listed in Table 3-3; however, when used, these values produce a BSFC map in the

Table 3-3. BSFC map coefficients [16].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1$</td>
<td>437.0</td>
<td>$\frac{g}{J}$</td>
</tr>
<tr>
<td>$K_2$</td>
<td>-6.2646</td>
<td>$\frac{g}{s J}$</td>
</tr>
<tr>
<td>$K_3$</td>
<td>-0.46711</td>
<td>$\frac{g}{J^2}$</td>
</tr>
<tr>
<td>$K_4$</td>
<td>0.15043</td>
<td>$\frac{g}{s J}$</td>
</tr>
<tr>
<td>$K_5$</td>
<td>-0.0028646</td>
<td>$\frac{g}{s J^2}$</td>
</tr>
<tr>
<td>$K_6$</td>
<td>0.00053406</td>
<td>$\frac{g}{J^3}$</td>
</tr>
</tbody>
</table>
form shown in Figure 3-5. Figure 3-5 shows that the BSFC is outside the engine speed range defined in Table 3-2, and the WOT line does not match the WOT line for the test vehicle’s engine. To move the map to the operating range of the engine, the inputs, i.e. engine speed and torque, are scaled in the BSFC equation such that

$$g_e (\omega_e, T_e) = K_1 + K_2 \omega_e^* + K_3 T_e^* + K_4 (\omega_e^*)^2 + K_5 \omega_e^* T_e^* + K_6 (T_e^*)^2,$$

(3.1)

where

$$\omega_e^* = \frac{\omega_e}{10},$$

(3.2)

$$T_e^* = 3(T_e - 100).$$

Note that this scaling procedure is done by observation to result in a BSFC map where the maximum is inside the engine speed and torque constraints. The scaled BSFC map with the vehicle’s WOT line is shown in Figure 3-6. Lastly, the vehicle will consume generic, unleaded gasoline. The parameters needed to describe the fuel are its LHV and density; these are listed in Table 3-4.

Figure 3-5. BSFC map using coefficient values listed in [16].
Table 3-4. Fuel specifications [21].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel LHV</td>
<td>Δ$H_c^0$</td>
<td>0.0122</td>
<td>kW h g⁻¹</td>
</tr>
<tr>
<td>Fuel Density</td>
<td>$\rho_f$</td>
<td>718.95</td>
<td>kg m⁻³</td>
</tr>
</tbody>
</table>

3.2.3 Drivetrain Parameters

The prescribed parameters concerning the drivetrain are the gear ratios shown in Figure 2-2, as well as the CVT properties such as operating pressure, $P$, pump and motor maximum volumetric displacement, $V_{p_{\text{max}}}$ and $V_{m_{\text{max}}}$, and pump and motor maximum swashplate angles, $\alpha_{p_{\text{max}}}$ and $\alpha_{m_{\text{max}}}$. The pump and motor parameters chosen are typical for the size of pump and motor that would be used for this CVT and are not based on a specific pump or motor on the market. The pump and motor parameters are listed in Table 3-5. Next, the operating pressure and hydraulic fluid properties must be defined;
Table 3-5. Hydraulic pump and motor specifications [13].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Pump</th>
<th>Motor</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Volumetric Displacement</td>
<td>$V_{\text{max}}$</td>
<td>20</td>
<td>20</td>
<td>cm$^3$ rev</td>
</tr>
<tr>
<td>Max. Swashplate Angle</td>
<td>$\alpha_{\text{max}}$</td>
<td>18</td>
<td>18</td>
<td>deg</td>
</tr>
</tbody>
</table>

Table 3-6. Hydraulic fluid specifications [22].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Pressure</td>
<td>$P$</td>
<td>3000</td>
<td>psi</td>
</tr>
<tr>
<td>Hydraulic Fluid Viscosity</td>
<td>$\mu$</td>
<td>0.0062</td>
<td>Pa s</td>
</tr>
</tbody>
</table>

these are listed in Table 3-6. Not all gear ratios present in the drive train need to be defined since constraints where placed on these gear ratios in Chapter 2, see Equation (2.38). From the test vehicle, the differential gear ratio, $\xi_d$, is 3.55. Furthermore, the test vehicle has a 5-speed transmission with gear ratios listed in Table 3-7. Table 3-7 also provides a shifting schedule based on vehicle velocity, which is in units of meters per second. From the second constraint placed upon the drive train stated in Chapter 2, $\xi_2$ can be computed from the following:

$$\xi_2 = \frac{2T_{\text{max}} \psi_{\text{max}}}{\xi_d PV_{\text{max}}}$$

(3.3)

here, the maximum constraint variable, $\psi_{\text{max}}$, is set to 0.5. The maximum wheel torque can be calculated from the maximum engine torque using the wheel-engine relation in Equation (2.40). The result of Equation (3.3) sets $\xi_2$ equal to 2.31. The final gear ratios required to be defined are $\xi_r$ and $\xi_h$. These can be calculated from the pump control input
Table 3-7. Test vehicle manual transmission gear ratios and shift schedule [23, 24].

<table>
<thead>
<tr>
<th>Gear</th>
<th>Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>2.47</td>
<td>$0 \leq v_x \leq 4.47$</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>1.87</td>
<td>$4.47 &lt; v_x \leq 8.49$</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>1.47</td>
<td>$8.49 &lt; v_x \leq 12.5$</td>
</tr>
<tr>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>1.00</td>
<td>$12.5 &lt; v_x \leq 17.5$</td>
</tr>
<tr>
<td>5&lt;sup&gt;th&lt;/sup&gt;</td>
<td>0.75</td>
<td>$17.5 &lt; v_x$</td>
</tr>
</tbody>
</table>

equation. Suppose when the engine torque is at its maximum, the pump control input is also to be one such that

$$\hat{\alpha}_p = \frac{\xi_r \xi_h T_{max}}{PV_{max}} \equiv 1;$$  \hspace{1cm} (3.4)

solving for the product of $\xi_r$ and $\xi_h$, this is set to 0.18; thusly, these ratios are chosen such that their product equals 0.18 because they only occur in this form, never alone. A summary of the gear ratios used in the drive trains is listed in Table 3-8.

Table 3-8. Transmission gear ratios.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_r$</td>
<td>1.50</td>
</tr>
<tr>
<td>$\xi_h$</td>
<td>0.12</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>2.31</td>
</tr>
<tr>
<td>$\xi_d$</td>
<td>3.55</td>
</tr>
</tbody>
</table>
In summary, all physical parameters for simulation of the vehicle have been set. Within the simulation, much care is given to ensure all units correspond such that the simulation will result with proper numbers in their proper units.

3.3 Simulation Algorithms

The testing of the hydraulic hybrid transmission takes place over four distinct simulations. Two vehicle configurations are tested against the two driving schedules introduced in the last section. The first vehicle configuration is that of a conventional vehicle not equipped with a CVT, and the second is the vehicle with the CVT. Although the models derived in Chapter 2 are specifically for a CVT equipped vehicle, the vehicle has been designed in such a way that if the control input, \( \dot{\alpha}_m \), remains a constant zero, the vehicle operates just as a conventional, non-hybrid vehicle would. All simulations are done in the MATLAB environment.

3.3.1 Conventional Vehicle Simulation

The first algorithm to be described is for the conventional vehicle, i.e. the control input remains a constant zero. This algorithm is made up of two functions. The first and most important function is named \texttt{objFun} and is presented in Appendix A. In this function, the parameters specified in the previous section are defined. The inputs for this function are the vehicle’s velocity and acceleration as well as the control input; the function outputs the instantaneous overall vehicle efficiency, \( H \), the instantaneous fuel economy, \( \frac{dx}{dV} \), a row vector of inequality constraint values, \([g_1, g_2, g_3, g_4, g_5]\), and
the value of the equality constraint, $h_1$. The final two outputs are not important for the simulation of the conventional vehicle as they are constraints on the control input, which
is a constant zero. The second function, \texttt{convTrans}, is presented in Appendix B and manages the simulation of the conventional vehicle. The input to the function is a flag to instruct the function to use the NYCC or HWFET driving schedule. The integration of the two functions is shown in Figure 3-7.

3.3.2 Hybrid Vehicle Simulation

The second algorithm simulates the hybrid vehicle, where the control input is allowed to change in order to produce a higher vehicle efficiency. The function, \texttt{HHV}, manages the simulation’s iterative process between driving conditions, similar to \texttt{convTrans}. This function is presented in Appendix C and its process is shown in Figure 3-8.

As shown in Equation (2.53), maximizing the overall efficiency of the vehicle maximizes its fuel economy. As such, and in keeping with the general practice of referring to minimization, the negative of the overall vehicle efficiency, $H$, will act as the objective function such that

$$\min_{\hat{\alpha}_m} -H(\hat{\alpha}_m),$$

where the control input, $\hat{\alpha}_m$, acts as the design variable; the objective function is written this way as to keep with convention. However, physical constraints and limits exist on the system, such as the WOT line is a constraint on engine torque. Constraints occur in two forms, inequality constraints and equality constraints, written as

$$g_i(\hat{\alpha}_m) \leq 0, \quad i = 1, \ldots, k,$$

$$h_j(\hat{\alpha}_m) = 0, \quad j = 1, \ldots, y,$$
Figure 3-8. Hydraulic hybrid vehicle simulation.
where \( k \) and \( y \) are the numbers of inequality and equality constraints, respectively. The WOT line is a major inequality constraint. Equation (2.49) can be rewritten in the form of an inequality constraint with the engine torque, known from the wheel torque through the ratio in Equation (2.40), as in the following:

\[
g_1(\hat{\alpha}_m) = T_e(\hat{\alpha}_m) - T'(\hat{\alpha}_m) \leq 0. \tag{3.7}
\]

This constraint forces the engine torque to remain under the WOT line. The range of viable engine speeds presented in Equation (2.50) can be split into two inequality constraints as in the following:

\[
g_2(\hat{\alpha}_m) = \omega_{e_{\text{min}}} - \omega_c(\hat{\alpha}_m) \leq 0, \\
g_3(\hat{\alpha}_m) = \omega_c(\hat{\alpha}_m) - \omega_{e_{\text{max}}} \leq 0. \tag{3.8}
\]

Likewise, the control input must remain between 0 and 1, adding two more inequality constraints written as

\[
g_4(\hat{\alpha}_m) = -\hat{\alpha}_m \leq 0, \\
g_5(\hat{\alpha}_m) = \hat{\alpha}_m - 1 \leq 0. \tag{3.9}
\]

In addition to the five inequality constraints listed, one equality constraint is placed on the system. This constraint will ensure the engine produces the same amount of power after the optimization process as it produced before. This constraint is important as a certain amount of power is requested from the engine by the wheels in order to maintain a certain velocity, and this amount of power cannot change or else the vehicle will not have the desired velocity. The equality constraint is written as

\[
h_i(\hat{\alpha}_m) = \Pi_e(\hat{\alpha}_m^*) - \Pi_e(\hat{\alpha}_m) = 0, \tag{3.10}
\]

where \( \hat{\alpha}_m^* \) is some non-optimal control input at which the motor is set before the optimization process and thus remains constants throughout this process. The expression
for engine power is presented in Equation (2.44), and within this expression, $\eta_{eff}$ is a function of the control input.

Since all feasible control inputs must be roots of equality constraint in Equation (3.10), the roots of this constraint are found using \texttt{fzero}, a native MATLAB function, as shown in Figure 3-8. The roots are then compared to the constraints in Equations (3.7) through (3.9), and if these constraints are satisfied, the surviving roots are compared to each other on the vehicle efficiency they produce. The surviving control input is the root of Equation (3.10) that satisfies the constraints in Equations (3.7) through (3.9) and produces the highest vehicle efficiency.

Within \texttt{objFun}, the torque on the wheel turns negative when the vehicle decelerates. When this occurs, the power demand to the engine is forced to zero so the engine is not asked to produce a negative power. Furthermore, whenever the vehicle is stationary, i.e. its velocity is zero, or when the power demand to the engine is zero, i.e. the vehicle’s acceleration is negative, the control input is forced to zero. Since the engine always has an associated, non-zero speed, setting the control input to zero when no rotational speed is needed by the wheel prevents the transmission of speed from the engine to the wheel. Also, when the wheel torque is negative, having the control input at zero allows the CVT to completely absorb this negative torque to keep the engine torque from going negative.

3.4 Summary

Two simulation algorithms are constructed to retrieve efficiency and fuel economy values as the vehicle completes two driving schedules using the parameters laid
out in Section 3.2. The first algorithm simulates a conventional vehicle and requires no iterative process as the control input is fixed at zero. The second algorithm utilizes an optimization method to find an optimal engine operating point, and from this point, the necessary control input is determined. The results of the two simulations are compared and discussed in the next chapter.
CHAPTER 4. RESULTS AND DISCUSSION

4.1 Introduction

Equations, algorithms, and parameters allowing the simulation of the conventional and hybrid vehicles are presented in Chapters 2 and 3. The results of these simulations are provided within this chapter. Section 4.2 is split into two parts: one reporting the results of the NYCC simulation, and the other reporting the results of the HWFET simulation. In each subsection compares the fuel economy and vehicle efficiency of the conventional vehicle to those of the hybrid vehicle. In addition to comparisons between the vehicle configurations, the impact of the CVT on the vehicle is presented. In Section 4.3, a discussion of the results is provided to explain trends found in the operation of the CVT.

4.2 Results

A summary of the results is given for each driving schedule along with a comparison between the conventional vehicle and hybrid vehicle. Since the efficiency, H, is the only variable to change in Equation (2.53) between the simulations for the two vehicle configurations, the improvements listed are the same for both efficiency and fuel economy. Profiles of the vehicle efficiencies are presented to show the differences between the vehicle configurations and how the CVT affects them. A profile of how the control variable input changes for the hybrid vehicle simulation is also presented for both the NYCC and HWFET driving schedules. Where ever the control input is non-zero, the efficiency of the hybrid vehicle will be different than that of the conventional vehicle.
4.2.1 New York City Cycle Results

The results of the simulations using the NYCC driving schedule are shown in Table 4-1. Vehicle efficiency and fuel economy in the hybrid vehicle increased over those of the conventional vehicle, however, not by a large margin. A comparison between the efficiency profiles of the conventional and hybrid vehicles is shown in Figure 4-1. The profiles in Figure 4-1 are very similar, but it can be noticed that at several points along the efficiency profile for the hybrid vehicle, the efficiency is higher than at the corresponding point on the conventional vehicle efficiency profile. These points are when the CVT is active.

The control input of the CVT across the entire simulation is shown in Figure 4-2. This shows the CVT is only activated a few times, but this still causes a 1.28% increase in fuel economy. Most of the driving schedule has conditions where the vehicle is not moving or is decelerating; by design, the control input is zero under these conditions.

Table 4-1. Simulation results for NYCC driving schedule for both conventional and hybrid vehicles.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conventional</th>
<th>Hybrid</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Efficiency</td>
<td>15.33</td>
<td>15.53</td>
<td>%</td>
</tr>
<tr>
<td>Average Fuel Economy</td>
<td>24.33</td>
<td>24.64</td>
<td>miles/gallon</td>
</tr>
<tr>
<td>Improvement</td>
<td>-</td>
<td>1.28</td>
<td>%</td>
</tr>
</tbody>
</table>
Figure 4-1. NYCC Simulation: A) Vehicle efficiency profile of the conventional vehicle. B) Vehicle efficiency profile of the hybrid vehicle.

As mentioned in the previous chapter, Figure 4-3 shows that all the inequality constraints are satisfied for each velocity and acceleration input. The most active constraints are $g_2$ and $g_4$, where $g_2$ is the constraint ensuring engine speed is above its minimum operating point and $g_4$ is the constraint ensuring the control input is equal to or greater than zero. Figure 4-4 shows the equality constraint across the NYCC simulation;
notice that the magnitude of the errors is very small, so the equality constraint can be considered satisfied for each velocity and acceleration input. The small errors seen are attributed to the \texttt{fzero} function.

Figure 4-3. Inequality constraint values over the course of the NYCC simulation.
4.2.2 Highway Fuel Economy Driving Schedule Results

The results of the simulations using the HWFET driving schedule are shown in Table 4-2. Vehicle efficiency and fuel economy in the hybrid vehicle increased over those of the conventional vehicle and at a higher margin than in the NYCC simulation. A comparison between the efficiency profiles of the conventional and hybrid vehicles is shown in Figure 4-3. As shown in this figure, the efficiency of the hybrid vehicle has

Table 4-2. Simulation results for HWFET driving schedule for both conventional and hybrid vehicles.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conventional</th>
<th>Hybrid</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Efficiency</td>
<td>17.59</td>
<td>21.56</td>
<td>%</td>
</tr>
<tr>
<td>Average Fuel Economy</td>
<td>29.24</td>
<td>35.85</td>
<td>miles/gallon</td>
</tr>
<tr>
<td>Improvement</td>
<td>-</td>
<td>22.62</td>
<td>%</td>
</tr>
</tbody>
</table>
less drastic fluctuations and is generally much higher at a given time than that of the conventional vehicle at the same time. The control input of the CVT across the entire simulation is shown in Figure 4-3. Compared to the NYCC, the CVT is more active.

Figure 4-5. HWFET Simulation: A) Vehicle efficiency profile of the conventional vehicle. B) Vehicle efficiency profile of the hybrid vehicle.

Figure 4-6. CVT control input over the course of the HWFET simulation.
Just as with the NYCC simulation, the inequality constraints are satisfied across the HWFET simulation as shown in Figure 4-7. Here, the constraint on the lower bound of the control input, $g_4$, is much less active. Furthermore, just as with the NYCC simulation, the errors seen in the equality constraint, shown in Figure 4-8, are small enough to be considered zero.

Figure 4-7. Inequality constraint values over the course of the HWFET simulation.
4.3 Interpretations

The use of a CVT improves fuel economy in both city and highway driving. However, the CVT plays a larger role in highway driving than city driving. This is due to the fact that the CVT becomes advantageous when the vehicle is operating at high speeds and low accelerations. This conclusion confirms those reached in [12]. The NYCC driving schedule has fewer instances of high speeds with low accelerations than the HWFET schedule, so this is why the CVT is more active with highway driving. The results found in [11], which also uses a hydraulic CVT within the drivetrain, are the opposite of those found in this thesis. In that research, greater fuel economy improvement is found in city driving than in highway driving. However, [11] does not include efficiency information of the hydraulic components, and only considers WOT conditions in the engine, whereas this thesis includes component efficiencies and allows the engine to operate anywhere below the WOT line.

The profile of the vehicle efficiency over all control inputs is split into two regions; Figure 4-9 displays these regions. The first region centers has a local maximum
Figure 4-9. Vehicle efficiency profile illustrating the two distinct regions when
\[ v_x = 10 \, \text{ms}^{-1} \] and \[ \dot{v}_x = 0 \, \text{ms}^{-2} \].

when the control input is zero. As the control input increases in the second region, the
engine operates at a more efficient level, and the individual component efficiencies are at
their highest and by correlation, the drivetrain efficiency is high. As such, the local
maximum in the second region is always where the control input equals one; however,
constraints may be unsatisfied when this occurs. This is why control inputs other than
zero or one occur during the HWFET simulation of the hybrid vehicle, see Figure 4-6.
The location of the threshold separating the two regions seems dependent on the vehicle
velocity, \( v_x \); as the velocity increases, the threshold is reached with a smaller control input
Figure 4-10. Comparison of threshold locations. A) Efficiency profile when $v_x = 2 \text{ ms}^{-1}$ and $\dot{v}_x = 0 \text{ ms}^{-2}$ B) Efficiency profile when $v_x = 30 \text{ ms}^{-1}$ and $\dot{v}_x = 0 \text{ ms}^{-2}$.

Figure 4-11. Vehicle efficiency profile illustrating the two distinct regions when $v_x = 10 \text{ ms}^{-1}$ and $\dot{v}_x = 1 \text{ ms}^{-2}$.
value. This is shown in Figure 4-10. Furthermore, the “sharpness”, or rate of change of
the slope, appears greater when the vehicle is operating at lower accelerations. At higher
accelerations, the slope around the threshold has a more gradual change; compare Figures
4-9 and 4-11. Notice in Figure 4-11 that the threshold is less distinct than that shown in
Figure 4-9. The discontinuous nature of these efficiency profiles can be explained
through the assumption made on the efficiencies of the pump and motor; recall that a
minimum efficiency of 0.65 is assumed for $\eta_{p}$, $\eta_{m}$, $\eta_{w}$, and $\eta_{n}$. Once these
efficiencies become greater than 0.65, the threshold has been reached and the vehicle
efficiency begins to increase.

4.4 Summary

The results of all simulations are presented and compared to show that the hybrid
vehicle increases fuel economy over the conventional vehicle. Furthermore, the CVT is
shown to be more effective and active at high vehicle speeds paired with low vehicle
accelerations. The results attained in this thesis are also compared to results found in [11]
since this thesis expands upon that work and show that the efficiency of the drivetrain
plays an important role when optimizing the entire vehicle’s efficiency.
CHAPTER 5. CONCLUSION

5.1 Introduction

In this chapter, conclusions are made based and supported by the results discussed in the previous chapter. Section 5.2 lists these conclusions as well as summarizes this entire document. Finally, the last section, Section 5.3, recommends future endeavors to be taken to further the understanding and development of the hybrid vehicle analyzed in this thesis.

5.2 Conclusions

Chapter 1 describes the need for more fuel efficient vehicles due to volatile fuel costs and the effect of combustion by-products on the environment. Several alternatives exist, in which hybrid vehicles are included. Comparing electric and hydraulic hybrid vehicles, hydraulic hybrids are shown to have a lower associated cost. Chapter 2 models the vehicle under study and provides expressions for the efficiencies of the major components as functions of the control input, $\hat{\alpha}_m$, which is a nondimensionalization of the swashplate angle of the hydraulic motor. Because the component efficiencies are dependent on the control input, the overall vehicle efficiency is dependent on the control input. As such, the vehicle efficiency is optimized by altering the control input.

After running the simulations laid out in Chapter 3, results are obtained and described in Chapter 4, where the use of a CVT is shown to increase vehicle efficiency and fuel economy over those of a conventional vehicle. Other major conclusions reached in Chapter 4 are the following:
1) The CVT becomes advantageous to use when the vehicle is operating at high speeds with low acceleration,

2) A threshold exists in the vehicle efficiency profile with respect to the control input that creates two distinct regions: one where the efficiency is dominated by the drivetrain efficiency and the other where the efficiency is dominated by the engine efficiency,

3) The location of the threshold is dependent on the vehicle velocity, and the rate of change of the efficiency slope with respect to the control input decreases around the threshold when the vehicle is operating at higher accelerations,

4) The CVT tends to operate at the minimum and maximum control input when not violating any constraints since the vehicle efficiency profile has two local maxima where the control input equals zero and one.

Recall from Chapter 1 that hybrid vehicles also include power storage devices. The configuration of the vehicle under study does not include a power storage device so all efficiency gains are made only by changing the CVT gear ratio. Introducing power storage in the form of a hydraulic accumulator will supplement power from the engine to increase fuel economy even further.

5.3 Scope of Future Work

The analysis done during the course of this thesis provides important insight on the operation of a hydraulic hybrid vehicle taking component efficiencies in to account. However, there is room to improve and build upon the work in this thesis. First of all, this thesis is contained in a virtual environment; real-world testing is necessary to
reaffirm the conclusions attained in this document. Furthermore, only the steady state of the control input was of concern; a control system may be needed to affect the dynamics of the swashplate angle as it changes from one control input to another.

Aside from improving and confirming the results of the thesis, alternate transmission configurations can and should be analyzed as well. As stated above, a major component of most hybrid vehicles, the power storage device, is omitted in this thesis. Adding a hydraulic accumulator to the CVT for energy storage will provide supplementary power to increase the overall fuel economy of the vehicle. Integration of an accumulator is crucial to further development of this hybrid vehicle. With the integration of an accumulator, the control input can go negative and charge the accumulator as the vehicle goes through deceleration. Recall that when the vehicle was decelerating in the simulations of Chapter 4, the control input is forced to zero.
REFERENCES


APPENDIX A. MATLAB FUNCTION \texttt{objFun}

function \([\text{Eta}, \text{dmpg}, g, h] = \text{objFun}(v_x, v_{dot\_x}, \alpha_{\hat{\text{m}}})\)

% Definitions of Parameters

% Pump and Motor Efficiency Parameters (Table 2-1)
\[
\begin{align*}
A &= [0.17 \quad 0.17]; \\
B &= [9.5511e8 \quad 4.7755e8]; \% [\text{deg}] \\
C &= [30.9049 \quad 154.5243]; \% [\text{deg}^{1/2}] \\
D &= [1.27e7 \quad 1.27e7]; \% [\text{Pa*deg}] \\
Kappa_1 &= [1.047e-7 \quad 1.047e-7]; \% [\text{deg}] \\
Kappa_2 &= [0.0033 \quad 0.0066]; \% [\text{deg}/(\text{Pa}^{1/2}\text{s})] \\
Kappa_3 &= 10^9; \% [\text{Pa}] \\
\end{align*}
\]

% Vehicle Parameters (Table 4-1)
\[
\begin{align*}
m &= 1347.1; \% [\text{kg}] \\
r &= 0.305; \% [\text{m}] \\
J &= 13; \% [\text{kg*m}^2] \\
C_d &= 0.4; \\
A_d &= 2; \% [\text{m}^2] \\
C_r &= 0.01; \\
\end{align*}
\]

% Environmental Parameters
\[
\begin{align*}
\beta &= 0; \% [\text{rad}] \\
gerav &= 9.81; \% [\text{m/s}] \\
\rho &= 1.2041; \% [\text{kg/m}^3] \\
\end{align*}
\]

% Engine Performance Parameters (Table 4-2)
\[
\begin{align*}
\Pi_e_{\text{max}} &= 160*745.7; \% [\text{N*}m/s] \\
\omega_e_{\text{o}} &= 4200*0.1047; \% [\text{rad/s}] \\
\end{align*}
\]
\(\omega_e_{\text{min}} = 1000 \times 0.1047\); \% [rad/s]
\(\omega_e_{\text{max}} = 5000 \times 0.1047\); \% [rad/s]

% Fuel Parameters (Table 4-4)

\(\delta H = 0.0122\); \% [kW*h/g]
\(\rho_f = 718.95\); \% [kg/m^3]

% BFSC Map Coefficients (Table 4-3)

\(K_1 = 437.0 - 300 \times -0.46711 + 9 \times 0.00053406 \times 10000;\)
\(K_2 = -6.2646/10 - 30 \times -0.0028646;\)
\(K_3 = 3 \times -0.46711 - 9 \times 200 \times 0.00053406;\)
\(K_4 = 0.15043/100;\)
\(K_5 = -0.0028646 \times 3/10;\)
\(K_6 = 0.00053406 \times 9;\)

% CVT Parameters (Tables 4-5 and 4-6)

\(V_{p_{\text{max}}} = 20 \times 1.5915 \times 10^{-007};\) \% [m^3/rad]
\(V_{m_{\text{max}}} = 20 \times 1.5915 \times 10^{-007};\) \% [m^3/rad]
\(\alpha_{p_{\text{max}}} = 18;\) \% [deg]
\(\alpha_{m_{\text{max}}} = 18;\) \% [deg]
\(P = 3000 \times 6895;\) \% [N/m^2]
\(\mu = 0.0062;\) \% [Pa*s]

% Gear Ratios (Table 4-7)

\(\xi_r = 1.50;\)
\(\xi_h = 0.12;\)
\(\xi_2 = 2.31;\)
\(\xi_d = 3.55;\)

if \(v_x \leq 4.47\)

\(\Xi_t = 2.47;\)

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elseif (v_x > 4.47) && (v_x <= 8.49)
    Xi_t = 1.87;
elseif (v_x > 8.49) && (v_x <= 12.5)
    Xi_t = 1.47;
elseif (v_x > 12.5) && (v_x <= 17.5)
    Xi_t = 1.00;
else
    Xi_t = 0.75;
end

% Equations

% Wheel Speed and Acceleration (Equation 2.6)

omega_w = v_x/r;
omega_dot_w = v_dot_x/r;

% Wheel Torque (Equation 2.7)

T_w = (0.5*m*r^2 + J)*omega_dot_w + 0.25*rho*C_d*A_d*r^3*omega_w^2 + 0.5*m*grav*r*(C_r + sin(beta));

% Wheel Power Demand

Pi_w = omega_w*T_w;

% Psi

psi = xi_2*xi_d*P*V_m_max/(2*T_w);

% Transmission Relations (Equation 6.1)

omega_2 = xi_2*xi_d*omega_w;
omega_h = alpha_hat_m*(xi_d*psi*omega_w)/(xi_r*xi_h);

alpha_hat_p = (psi*alpha_hat_m + 1)*(2*abs(T_w)*xi_h*xi_r)/(V_p_max*P*xi_d);

% Pump and Motor Torque Efficiencies (Equation 2.59/60)

if (omega_w == 0) || (alpha_hat_m == 0)
\[ \eta_t_p = 1; \]
\[ \eta_t_m = 1; \]

else

\[ x_p = \mu \omega_h / (P \alpha_p_{\text{max}} \alpha_{\hat{p}}); \]
\[ \eta_t_p = 1 - A(1) \exp(-B(1) x_p) - C(1) \sqrt{x_p} - D(1)/(P \alpha_p_{\text{max}} \alpha_{\hat{p}}); \]
\[ x_m = \mu \omega_m / (P \alpha_m_{\text{max}} \alpha_{\hat{m}}); \]
\[ \eta_t_m = 1 - A(2) \exp(-B(2) x_m) - C(2) \sqrt{x_m} - D(2)/(P \alpha_m_{\text{max}} \alpha_{\hat{m}}); \]

if (\eta_t_p < 0.65)

\[ \eta_t_p = 0.65; \]

end

if (\eta_t_m < 0.65)

\[ \eta_t_m = 0.65; \]

end

end

\% Pump and Motor Volumetric Efficiencies (Equation 2.59/61)

if (\omega_w == 0) || (\alpha_{\hat{m}} == 0)

\[ \eta_v_p = 1; \]
\[ \eta_v_m = 1; \]

else

\[ \eta_v_p = 1 - P/Kappa_3 - Kappa_1(1) \mu/(\alpha_p_{\text{max}} \alpha_{\hat{p}} \omega_h) - Kappa_2(1)/\sqrt{\alpha_p_{\text{max}} \alpha_{\hat{p}} \omega_h}; \]
\[ \eta_v_m = 1 - Kappa_1(2) \mu/(\alpha_m_{\text{max}} \alpha_{\hat{m}} \omega_2) - Kappa_2(2)/\sqrt{\alpha_m_{\text{max}} \alpha_{\hat{m}} \omega_2}; \]

if (\eta_v_p < 0.65)

\[ \eta_v_p = 0.65; \]

end
if (eta_v_m < 0.65)
    eta_v_m = 0.65;
end

% Speed and Friction Efficiencies (Equation 2.25)
eta_s = eta_v_p*eta_v_m;
eta_f = eta_t_p*eta_t_m;

% Drive Train Effective Efficiency (Equation 2.57)
eta_eff = eta_s*eta_f/((1 - eta_s*eta_f)*(psi*alpha_hat_m) + eta_s*eta_f);

% Engine Speed, Torque, and Power (Equation 2.38)
Pi_e = 2*Pi_w/eta_eff;
if Pi_e < 0
    Pi_e = 0;
end
T_e = 2*T_w/(Xi_t*xi_d)*(1 + psi*eta_t_m*alpha_hat_m);
if T_e < 0
    T_e = 0;
end
omega_e = Pi_e/T_e;
if (omega_e < omega_e_min) || (T_e == 0)
    omega_e = omega_e_min;
end

% Brake Specific Fuel Consumption (Equation 4.1)
g_e = K_1 + K_2*omega_e + K_3*T_e + K_4*omega_e^2 + K_5*omega_e*T_e + K_6*T_e^2;
% Engine Efficiency (Equation 2.55)
eta_e = 1/(g_e*deltaH);

% Overall Efficiency (Equation 2.52)
Eta = eta_e*eta_eff;

% Instantaneous Fuel Economy [mpg] (Equation 2.51)
dmpg = 8.4677e+003*Eta*r*rho_f*deltaH/(2*T_w);

% Inequality Constraint 1
\[ g(1) = T_e - Pi_e_max/omega_e_o - Pi_e_max*omega_e/omega_e_o^2 + Pi_e_max*omega_e.^2/omega_e_o^3; \]

% Inequality Constraint 2
\[ g(2) = omega_e_min - omega_e; \]

% Inequality Constraint 3
\[ g(3) = omega_e - omega_e_max; \]

% Inequality Constraint 4
\[ g(4) = - alpha_hat_m; \]

% Inequality Constraint 5
\[ g(5) = alpha_hat_m - 1; \]

% Equality Constraint 1
\[ h = -(psi)^2*(alpha_hat_m - (1 - eta_s*eta_f))^2 + (psi)*(alpha_hat_m - (1 - eta_s*eta_f)) + (psi)^2(1 - eta_s*eta_f)^2 + (psi)*(1 - eta_s*eta_f); \]

end
APPENDIX B. MATLAB FUNCTION convTrans

function [hist,avg] = convTrans(flag)

% Driving Schedule Data
% Import Data Based on Flag
  if flag == 1
    M = dlmread('nycccol.txt');
  else
    M = dlmread('hwycol.txt');
  end

% Convert to [m/s]
  M(:,2) = M(:,2)*0.45;

% Compute Acceleration
  M(:,3) = [0; diff(M(:,2))];

% Initialize Parameters
  alpha_hat_m = 0;
  Pi_e        = 0;

% Simulation Loop
% Initialize Matrix
  hist = zeros(length(M),14);

for i = 1:length(M)
  % Initialize Parameters
    Pi_e_star = Pi_e;
    t = M(i,1);
    v_x = M(i,2);
    v_dot_x = M(i,3);

% Compute Instantaneous Efficiency and Fuel Economy
[\text{Eta}, \text{dmpg}, g, h] = \text{objFun}(v_x, v_\text{dot}_x, \alpha_\text{hat}_m);

% History Array Update

\text{hist}(i,:) = [t \ v_x \ v_\text{dot}_x \ \text{Eta} \ \text{dmpg} \ g(1) \ g(2) \ g(3) \ g(4) \ g(5) \ \Pi_e \ e(1) \ e(2)];

end

% Average Efficiency and Fuel Economy

avg(1) = \text{mean}(\text{hist}(:,4));

avg(2) = \text{mean}(\text{hist}( :, 5));

end
function [hist, avg] = HHV(flag)

% Driving Schedule Data
% Import Data Based on Flag
if flag == 1
    M = dlmread('nycccol.txt');
else
    M = dlmread('hwycol.txt');
end

% Convert to [m/s]
M(:,2) = M(:,2)*0.45;

% Compute Acceleration
M(:,3) = [0; diff(M(:,2))];

% Simulation Loop
% Initializations
q = length(M);
hist = zeros(q,10);
for i = 1:q

% Initialize Parameters
alpha_hat_m = 0;
t = M(i,1);
v_x = M(i,2);
v_dot_x = M(i,3);

% Minimization
if (v_x > 0) && (v_dot_x >= 0)
    for k = 0:0.1:1
\[
\text{Eta}_o = \text{objFun}(v_x,v_{dot_x},\alpha_{hat_m});
\]

\[
x = \text{fzero}(\theta(y) \text{myFunc}(v_x,v_{dot_x},y),k);
\]

\[
[\text{Eta},\sim,\sim,\sim] = \text{objFun}(v_x,v_{dot_x},x);
\]

\[
\text{if } (g(1) \leq 0) \land (g(2) \leq 0) \land (g(3) \leq 0) \land (x \\leq 1) \land (x \geq 0) \land (\text{Eta} > \text{Eta}_o)
\]

\[
\alpha_{hat_m} = x;
\]

\end

end

end

% History Array Update

\[
[\text{Eta},dmpg,g,h] = \text{objFun}(v_x,v_{dot_x},\alpha_{hat_m});
\]

\[
\text{hist}(i,:) = [t \text{ Et}\text{a} \text{ dmpg} g(1) g(2) g(3) g(4) g(5) h \alpha_{hat_m}];
\]

end

% Average Efficiency and Fuel Economy

\[
\text{avg}(1) = 100*\text{mean(hist}(:,2));
\]

\[
\text{avg}(2) = \text{mean(hist}( :, 3));
\]

end

function h = myFunc(v_x,v_{dot_x},\alpha_{hat_m})

\[
[\sim,\sim,\sim,h] = \text{objFun}(v_x,v_{dot_x},\alpha_{hat_m});
\]

end