A STUDY OF NETWORK FOR MULTI HOUR TRAFFIC UNDER SPLITTABLE AND NON SPLITTABLE FLOW CONDITIONS

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University of Missouri-Kansas City, 2012

ABSTRACT

In a multi hour networking environment, there are different traffic matrices operating during different time periods of the day depending on the demand. It is applicable to traffic networks such as IP networks or optical networks. In this work, we attempt to analyze how the different networking objectives relate to or differ from each other under multi hour traffic conditions. In particular, we have used three different objectives for our study, namely minimum cost routing, load balancing optimization (minimize link utilization) and minimize average delay. We consider them under both Splittable and non-Splittable flows.

Our study focuses on considering three demand conditions: low load, moderate load, and high load. Besides the objectives as specified, we also consider the following indicators: (1) Number of non-zero paths to observe how the model behaves in terms of satisfying the demand, (2) Number of demands taking more than one path to observe how likely a model tries to spread the demand to more than one paths, (3) A distance measure, which represents the difference in path allotment between Splittable and non-Splittable cases to observe how closely related the models are with respect to satisfying a demand.

Through an extensive study with four different network topologies and demand scenarios, we observe that the minimum cost routing model tries to use a single path to satisfy a demand as much as possible, while the minimizing average delay model shows the

most tendency to split the demand and use more than one paths. For the load balancing model, once it chooses a set of paths, it retains those paths unless the problem becomes infeasible.

APPROVAL PAGE

The faculty listed below, appointed by the Dean of the school of Computing and Engineering, have examined a thesis titled, “A Study of Network For Multi Hour Traffic under Splittable and Non Splittable Flow Conditions,” presented by Sudhir Mohanraj, candidate for Master of Science degree, and certify that in their opinion it is worthy of acceptance.

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LIST OF ABBREVIATIONS

UNS	Non Splittable flow Condition
T1	Time Period 1
T2	Time Period 2
T3	Time Period 3
HD	Distance Measure
H	HD Divided by Total Paths

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CHAPTER 1

INTRODUCTION

Multi hour network is having different traffic matrices in different time periods of a day or a time period (may be weeks or months). The aim here is to obtain optimal capacity such that it is less than what would be needed if we just had a single maximum traffic load for the entire day [2]. This concept is very helpful when we have different time zones and having different time periods of peak traffic. For example, consider telephone traffic in cities of east coast such as New York, Boston might have very high demand at say 8 am but during the same time period the traffic in west coast cities such as Seattle, Los Angeles might be very less [2]. The idea we are trying to implement can also be used for multi service network [5]. This model can also be used in case of link failures to provide backup.

1.1 Motivation

In thesis work [4] multi hour network design problem is tried to solve using a decomposition algorithm. In this work; the objective of minimum cost routing was compared by having 3 different sets of number of paths (3, 5 and 7 namely); the paths remained the same for all time periods. The results were reported to have an improvement of 0.29% to 3.71% from 3 to 7 paths and 0.03% to 0.06% from 5 to 7%. We wanted to extend this idea to load balancing and average delay routing cases respectively and see how the results would turn out to be.

1.2 Contribution

We try to verify what factors are contributing to better capacity utilization in multi hour case, in which cases different routing objectives will perform better, how different is the path selection for routing objectives in Splittable and Unsplittable flow conditions.

We have introduced a new metric called as “Distance”, which gives us the information as to how the models tend to choose the paths. It also gives us the information as to which time period is contributing to the objective the most. Our study also gives us the information as to the model that can be used best in different circumstances. We have extensively studied the behavior of models in networks under multi hour condition when link failures occur.

CHAPTER 2

MATERIALS AND METHODS

In this study, we consider a number of factors. They are illustrated through Figure 1. For each splittable and non-splittable case, we consider four scenarios based on routing objectives, topologies, demands, and number of paths. In addition, we consider capacity adjustment to understand solution variation.

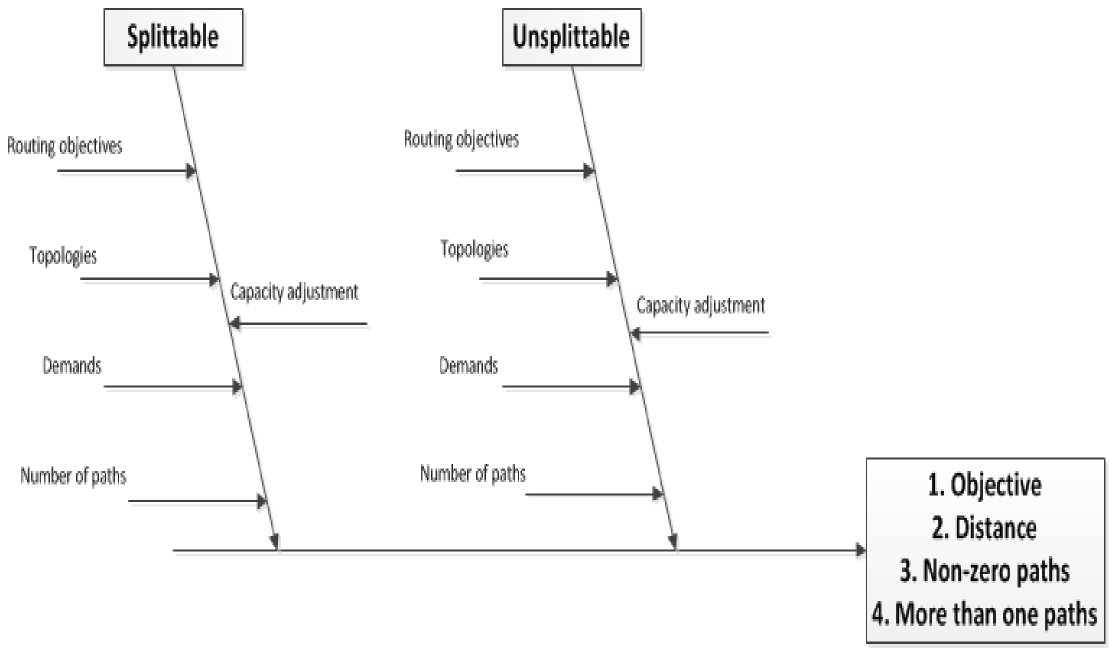

Figure 1: Diagrammatic view of our work

2.1 Routing objectives

1. Minimum cost: In this model we are trying to minimize the cost needed route a demand from a path to a destination. In my case in all the topologies I have given a cost of one for each hop.

   Case 1: Splittable flow: In this case more than one path can satisfy the demand between a path and destination.

Case 2: Non Splittable flow: In this case only one path can satisfy the demand between a path and destination.

2. Minimize Link utilization (Load balancing): Here we are trying to minimize the utilization of link which is defined as r (ratio of (capacity used/capacity available)).

Case 1: Splittable flow: In this case a single path is supposed to satisfy the demand between a path and destination.

Case 2: Non Splittable flow: In this case only one path can satisfy the demand between a path and destination.

3. Minimize Average delay: In this case we are trying to minimize the average delay of a path from source to destination.

Case 1: Splittable flow: More than one path can be used to satisfy the demand from source to destination.

Case 2: Non Splittable flow: only one path should satisfy the demand between source and destination.

2.2 Topologies

Four topologies have been used for our study,

1. Seven node topology with 7 nodes and 14 links.

2. NSF topology with 14 nodes and 21 links.

3. Twelve node topology with 12 nodes and 25 links.

4. Wrap Grid topology with 16 nodes and 32 links.

2.3 Metrics used

- Objective of each routing model: this gives us the information about the total capacity used. This information is used to see whether providing with more number of paths, how reducing the capacity of most utilized links affect the total capacity needed for the three models both in Splittable and Unsplittable condition.
- Number of non-zero paths: Gives us the information about how each of the models behaves in terms of satisfying the demand, i.e. whether they try to spread the demand or carry it on a single path.
- Number of demands using more than one path: this acts as supplement information to the above metric. In case the models are trying to spread the demand into multiple paths; in how many demand cases are they to do it.
- Distance: It is essentially the difference in path allotment between any two Models. This gives us the information about how closely related or differ the models are with respect to choosing a path to satisfy a demand.
- Distance graph: Gives us an overview of how similar or different path allocation is between routing objectives.
- Reducing the capacity of 3 most utilized links and verifying its effect on objectives and path allocations.
- Verify which time period is contributing the maximum to the total objective in multi hour case by calculating objectives in each individual time period.

We have defined three demands namely Low, medium and high which corresponds to link utilization value of ~0.45, ~ 0.6 and ~0.75 respectively.

2.4 Distance

It is essentially the difference in path allocation between two routing mechanisms i.e. if a path is chosen in minimum cost routing and a different path is chosen in average delay routing to satisfy the same demand than distance is 1. Likewise the distance is calculated for all the demands. It is calculated for each time period.

HD is distance divided by the total number of demands. (Number of paths differed/total number of demand pairs)

H is HD divided by the total number of paths. (HD/ total number of paths)

Four cases we have defined:

1. Normal case: All the links have a certain units of capacity ‘x’.
2. One link Capacity adjustment case: The most utilized link is reduced by 10%.
3. Two link Capacity adjustment case: In addition to the above case the next most utilized link is reduced by 10%.
4. Three link Capacity adjustment case: In addition to the above case the next most utilized link is reduced by 10%.

The traffic demand data that we have used for our topologies is from [7].

CHAPTER 3

OPTIMIZATION MODELS

3.1 Minimum cost routing Splittable condition [2]

Indices

d=1, 2…D	demands
p=1, 2…Pd	candidate paths for flows realizing demand d
e=1, 2….E	links
t=1, 2…T	number of time periods

Constants

δ_{edpt} =1, if link e belongs to path p realizing demand d in time period t;

0, otherwise

h_{dt} = volume of demand d during time period t

$£_{dpt}$ = unit cost of link e for the time period t

y_e= capacity of link e

Variables

x_{dpt}= flow allocated to path p of demand d during time period t

Objective

Minimize $\sum_d \sum_p \sum_t £_{dpt}\, x_{dpt}$

Constraints

$\sum_p x_{dpt}=h_{dt}$

$\sum_d \sum_p \delta_{edpt} x_{dpt} <= y_e$

An illustration is given in appendix A

3.2 Minimum cost routing in Non Splittable condition [2]

Indices

d=1, 2…D	demands
p=1, 2…Pd	candidate paths for flows realizing demand d
e=1, 2….E	links
t=1, 2…T	number of time periods

Constants

$\delta_{edpt} = 1$, if link e belongs to path p realizing demand d in time period t;

0, otherwise

h_{dt} = volume of demand d during time period t

$£_{dpt}$ = unit cost of link e for the time period t

y_e= capacity of link e

Variables

u_{dpt}= binary variable associated with the flow for demand d, path p and time period t

Objective

Minimize $\sum_d \sum_p \sum_t £_{dpt}\, h_{dpt}\, u_{dpt}$

Constraints

$\sum_p u_{dpt} = 1$

$\sum_d \sum_p \delta_{edpt} u_{dpt}\, h_{dpt} <= y_e$

An illustration is given in appendix A

3.3 Load balancing in Splittable condition [2]

Indices

d=1, 2…D	demands
p=1, 2…Pd	candidate paths for flows realizing demand d
e=1, 2….E	links
t=1, 2…T	number of time periods

Constants

δ_{edpt} =1, if link e belongs to path p realizing demand d in time period t;

0, otherwise

h_{dt} = volume of demand d during time period t

y_e= capacity of link e

Variables

x_{dpt}= flow allocated to path p of demand d during time period t

r is a variable less than 1.

Objective

Minimize r

$\sum_p x_{dpt}=h_{dt}$

$\sum_d \sum_p \delta_{edpt}\, x_{dpt} <= y_e r$

An illustration of the model is given in appendix A

3.4 Load balancing in Non Splittable condition [2]

Indices

d=1, 2…D	demands
p=1, 2…Pd	candidate paths for flows realizing demand d
e=1, 2….E	links
t=1, 2…T	number of time periods

Constants

δ_{edpt} =1, if link e belongs to path p realizing demand d in time period t;

0, otherwise

h_{dt} = volume of demand d during time period t

y_e= capacity of link e

Variables

u_{dpt}= binary variable associated with the flow for demand d, path p and time period t

r is a variable less than 1

Objective

Minimize r

$\sum_p u_{dpt} = 1$

$\sum_d \sum_p \delta_{edpt} u_{dpt} h_{dpt} <= y_e r$

An illustration of the model is given in appendix A

3.5 Piecewise linear approximation of average delay in Splittable condition[2]

Indices

d=1, 2…D	demands
p=1, 2…Pd	candidate paths for flows realizing demand d
e=1, 2….E	links
t=1, 2…T	number of time periods
AD=1,2 ….aver_del	average delay parameter
i=1,2…..iter	number of iterations

Constants

h_{dt} = volume of demand d during time period t

δ_{edpt} =1, if link e belongs to path p realizing demand d in time period t;

0, otherwise

C_e= upper bound for the capacity

j_val_n= slope for the piecewise linear graph along x axis

i_val_n= slope of the piecewise linear graph along y axis

Variable

x_{dpt}= flow allocated to path p of demand d during time period t

y_e= capacity of link e

Objective

Minimize $\sum_e z_e / y_e$

$\sum_d \sum_p \delta_{edpt} x_{dpt} <= y_e$

$z_e - j\_val_n y_e >= -(k\_val_n C_e)$

$\sum_p x_{dpt} = h_{dt}$

An illustration of the model is given in appendix A

3.6 Piecewise linear approximation of average delay in Non Splittable condition[2]

Indices

d=1, 2…D	demands
p=1, 2…Pd	candidate paths for flows realizing demand d
e=1, 2….E	links
t=1, 2…T	number of time periods
AD=1,2 ….aver_del	average delay parameter
i=1,2…..iter	number of iterations

Constants

h_{dt} = volume of demand d during time period t

δ_{edpt} =1, if link e belongs to path p realizing demand d in time period t;

0, otherwise

C_e= upper bound for the capacity

j_val_n= slope for the piecewise linear graph along x axis

i_val_n= slope of the piecewise linear graph along y axis

Variable

y_e= capacity of link e

u_{dpt}= binary variable associated with the flow for demand d, path p and time period t objective

Minimize $\sum_e z_e / y_e$

$\sum_d \sum_p \delta_{edpt} u_{dpt} h_{dt} <= y_e$

$\sum_p u_{dpt} = 1$

$z_e - j\_val_n y_e >= -(k\_val_n C_e)$

An illustration of the model is given in appendix A

CHAPTER 4

RESULTS

4.1 3 Node Topology

We start off with the computational results from 3 node topology which is completely connected.

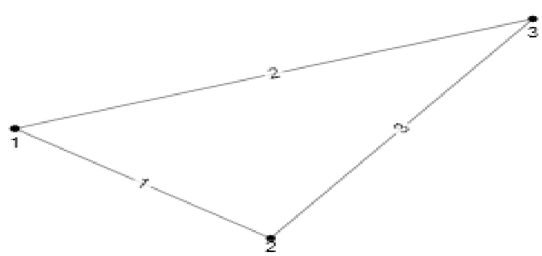

Figure 2: 3 node topology diagram

As we can see it is fully connected network with each demand pair having 2 paths each.

Table 1: Results for the 3 node topology with 2 paths for each time period under normal case

Demand		Minimum cost				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	71			71	0.43			0.5	1.5999			1.5999
	Non zero path	3	3	3		3	3	4		3	3	3	
	More than one path	0	0	0		0	0	1		0	0	0	
Med	Objective	85.2			85.2	0.52			0.6	2.31999			2.31999
	Non zero path	3	3	3		3	3	4		3	3	3	
	More than one path	0	0	0		0	0	1		0	0	0	
High	Objective	102.95			102.95	0.6283			0.725	3.3949			3.62833
	Non zero path	3	3	3		3	3	4		3	3	4	
	More than one path	0	0	0		0	0	1		0	0	1	

Table 2: Results for 3 node topology for only one time period corresponding to the highest demand

T	Minimum cost		Load balancing		Average delay	
	Splittable	Unsplittable	Splittable	Unsplittable	Splittable	Unsplittable
1	14.5	14.5	0.1933	0.2416	0.4833	0.48333
2	36.25	36.25	0.435	0.4833	1.6249	1.6249
3	52.2	52.2	0.6283	0.7250	3.3949	3.6283

4.2 4 Node Topology

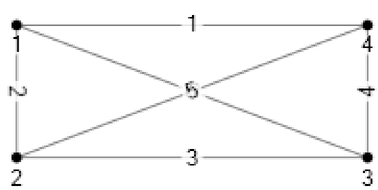

Figure 3: 4 node topology

Table 3: Results for 4 node topology with 3 paths for each time period under normal case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	143.2			143.2	0.4207			0.48	3.1330			3.1733
	Non zero path	6	6	6		6	6	10		6	6	7	
	More than one path	0	0	0		0	0	4		0	0	1	
Med	Objective	179			179	0.525925			0.60	4.63			4.6999
	Non zero path	6	6	6		6	6	10		6	6	6	
	More than one path	0	0	0		0	0	4		0	0	0	
High	Objective	214.8			214.8	0.6311			0.72	6.6399			6.9066
	Non zero path	6	6	6		6	6	10		6	6	8	
	More than one path	0	0	0		0	0	4		0	0	2	

Table 4: Results for 4 node topology for only one time period corresponding to the highest demand

Time	Minimum cost routing		Load balancing		Average delay	
	Splittable	Unsplittable	Splittable	Unsplittable	Splittable	Unsplittable
1	37.2	37.2	0.24	0.32	1.24	1.24
2	73.2	73.2	0.44	0.52	3.4399	3.4533
3	104.4	104.4	0.6311	0.72	6.6399	6.9066

Observations for 3 and 4 node topologies:

1. Mincost routing: Sum of objectives in individual time period is equal to the objective in the multi time period environment.
2. In case of Load balancing and average delay the maximum objective of the individual time periods (it corresponds to the time period which has the highest demand) becomes the objective solution in the multi hour case.
3. Minimum cost routing shows the least tendency to use more number of paths when compared to link utilization and average delay objective. To have a protection for a network from failure of links using link utilization or average delay will be a good option than minimum cost routing.
4. As we expected the highest time period corresponds to the highest objective value for average delay and load balancing models.

4.3 7 Node Topology

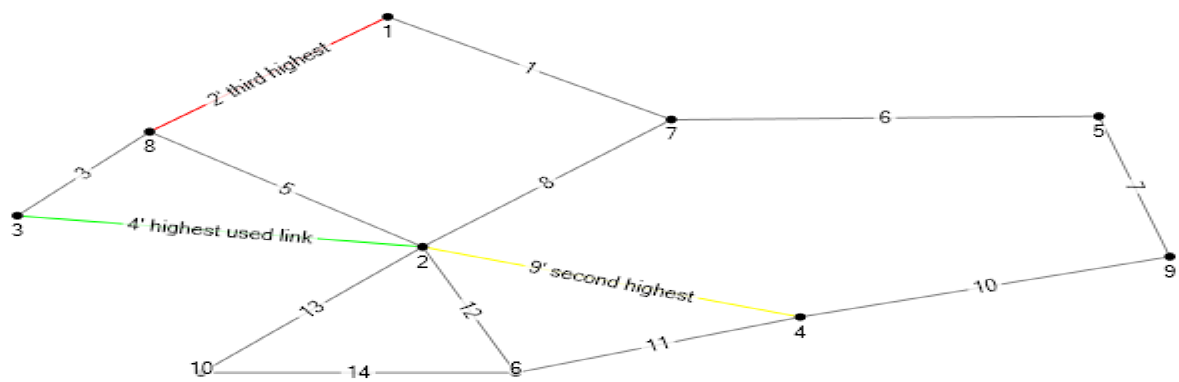

Figure 4:7 Node Topology

Amongst the paths we have chosen link 4 is the most utilized link for the topology followed by link 9 and link 2.

All the links have a maximum capacity of 3000.

Table 5: Results for 7 node topology with 3 paths for each time period under normal case

Demand		Minimum cost				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	21324			21324	0.44083			0.4636	4.7639			5.0419
	Non zero path	21	21	21		21	26	24		21	26	27	
	More than one path	0	0	0		0	5	3		0	5	6	
Med	Objective	28787.4			29157.3	0.5952			0.6259	8.703962			9.0763
	Non zero path	21	22	21		21	24	23		21	26	25	
	More than one path	0	1	0		0	3	2		0	5	4	
High	Objective	37166.4			380154.4	0.74941			0.7882	14.6488			16.0439
	Non zero path	21	23	22		21	24	23		21	25	27	
	More than one path	0	2	1		0	3	2		0	4	6	

Table 6: Results of distance for 7 node topology with 3 paths under low demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	10	10	12	32	4	15	12	31
HD	0.4762	0.4762	0.5714		0.1905	0.7143	0.5714	
H	0.1587	0.1587	0.1905		0.0635	0.2381	0.1905	
Minimum cost vs. average delay	12	10	10	32	6	13	22	41
HD	0.5714	0.4762	0.4762		0.2857	0.619	1.0476	
H	0.1905	0.1587	0.1587		0.0952	0.2063	0.3492	
Average delay vs. link utilization	12	12	12	36	10	10	16	36
HD	0.5714	0.5714	0.5714		0.4762	0.4762	0.7619	
H	0.1905	0.1905	0.1905		0.1587	0.1587	0.254	

Table 7: Results of distance for 7 node topology with 3 paths under medium demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	6	20	12	38	4	16	12	32
HD	0.2857	0.9524	0.5714		0.1905	0.7619	0.5714	
H	0.0952	0.3175	0.1905		0.0635	0.254	0.1905	
Minimum cost vs. average delay	22	8	6	36	2	10	12	24
HD	1.0476	0.381	0.2857		0.0952	0.4762	0.5714	
H	0.3492	0.127	0.0952		0.0317	0.1587	0.1905	
Average delay vs. link utilization	22	22	10	54	6	10	16	32
HD	1.0476	1.0476	0.4762		0.2857	0.4762	0.7619	
H	0.3492	0.3492	0.1587		0.0952	0.1587	0.254	

Table 8: Results of distance for 7 node topology with 3 paths under high demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	6	22	8	36	4	13	11	28
HD	0.2857	1.0476	0.381		0.1905	0.619	0.5238	
H	0.0952	0.3492	0.127		0.0635	0.2063	0.1746	
Minimum cost vs. average delay	14	10	8	32	2	8	9	19
HD	0.6667	0.4762	0.381		0.0952	0.381	0.4286	
H	0.2222	0.1587	0.127		0.0317	0.127	0.1429	
Average delay vs. link utilization	14	20	12	46	6	7	12	25
HD	0.6667	0.9524	0.5714		0.2857	0.3333	0.5714	
H	0.2222	0.3175	0.1905		0.0952	0.1111	0.1905	

Table 9: Results for 7 node topology with 6 paths for each time period under normal case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	21324			21324	0.44083			0.4636	4.76397			5.0419
	Non zero path	21	21	21		21	25	23		21	26	24	
	More than one path	0	0	0		0	4	2		0	5	3	
Med	Objective	28787.4			29157.3	0.5952			0.62595	8.703962			9.0763
	Non zero path	21	22	21		21	25	23		21	26	24	
	More than one path	0	1	0		0	4	2		0	5	3	
High	Objective	37166.4			380154.4	0.74941			0.78823	14.648			16.0439
	Non zero path	21	22	22		21	25	23		21	25	25	
	More than one path	0	1	1		0	4	2		0	4	4	

Table 10: Results for distance for 7 node topology with 6 paths under low demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	6	16	14	36	12	18	12	42
HD	0.2857	0.7619	0.6667		0.5714	0.8571	0.5714	
H	0.0476	0.127	0.1111		0.0952	0.1429	0.0952	
Minimum cost vs. average delay	26	10	10	46	6	9	11	26
HD	1.2381	0.4762	0.4762		0.2857	0.4286	0.5238	
H	0.2063	0.0794	0.0794		0.0476	0.0714	0.0873	
Average delay vs. link utilization	26	18	16	60	12	13	15	40
HD	1.2381	0.8571	0.7619		0.5714	0.619	0.7143	
H	0.2063	0.1429	0.127		0.0952	0.1032	0.119	

Table 11: Results for distance for 7 node topology with 6 paths under medium demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	6	24	14	44	12	17	12	41
HD	0.2857	1.1429	0.6667		0.5714	0.8095	0.5714	
H	0.0476	0.1905	0.1111		0.0952	0.1349	0.0952	
Minimum cost vs. average delay	8	12	12	32	6	10	7	23
HD	0.381	0.5714	0.5714		0.2857	0.4762	0.3333	
H	0.0635	0.0952	0.0952		0.0476	0.0794	0.0556	
Average delay vs. link utilization	8	20	16	44	12	11	15	38
HD	0.381	0.9524	0.7619		0.5714	0.5238	0.7143	
H	0.0635	0.1587	0.127		0.0952	0.0873	0.119	

Table 12: Results for distance for 7 node topology with 6 paths under high demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	6	22	12	40	12	15	12	39
HD	0.2857	1.0476	0.5714		0.5714	0.7143	0.5714	
H	0.0476	0.1746	0.0952		0.0952	0.119	0.0952	
Minimum cost vs. average delay	8	8	10	26	6	7	9	22
HD	0.381	0.381	0.4762		0.2857	0.3333	0.4286	
H	0.0635	0.0635	0.0794		0.0476	0.0556	0.0714	
Average delay vs. link utilization	8	22	18	48	12	20	20	52
HD	0.381	1.0476	0.8571		0.5714	0.9524	0.9524	
H	0.0635	0.1746	0.1429		0.0952	0.1587	0.1587	

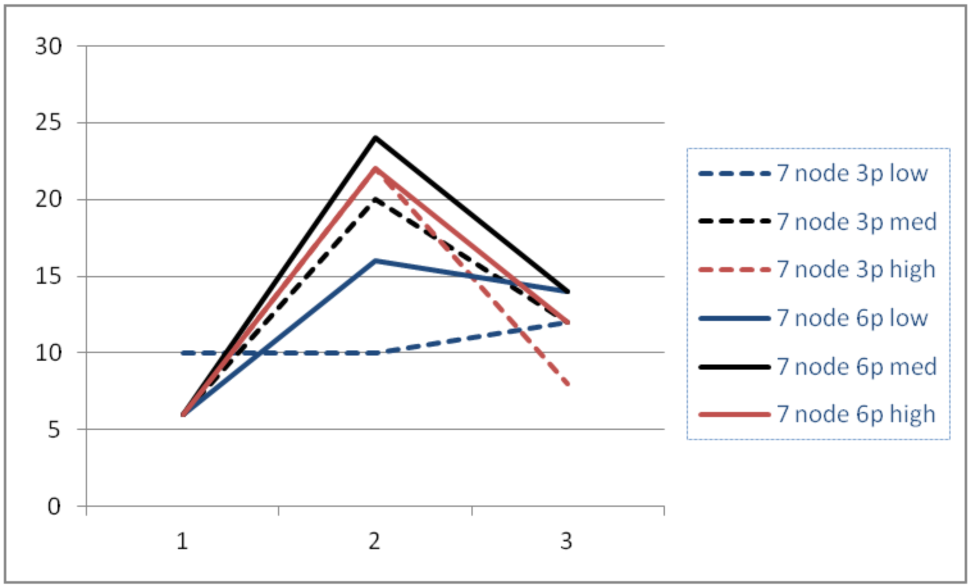

Figure 5: Distance graph for minimum cost vs. link utilization under Unsplittable condition 7 Node Topology

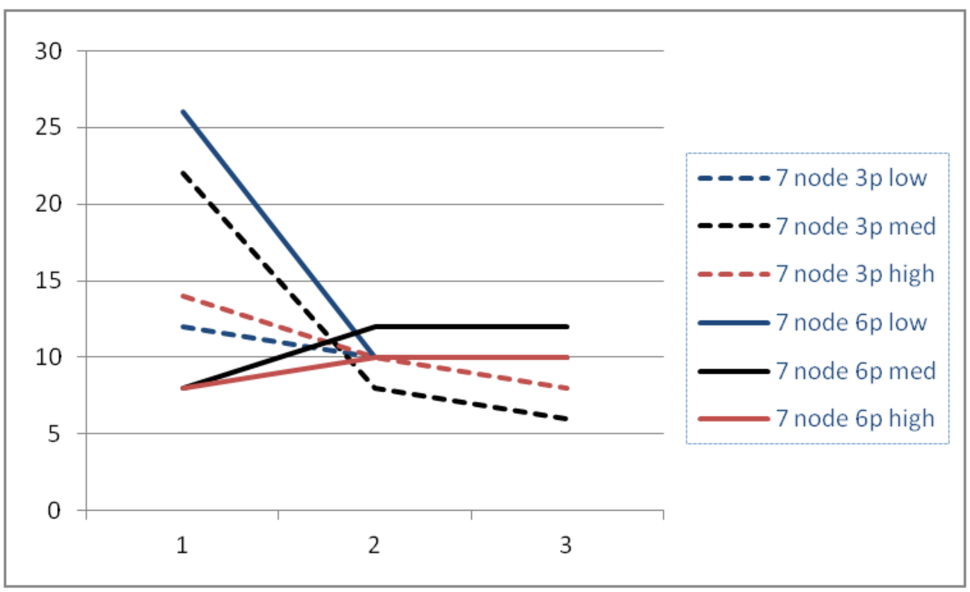

Figure 6: Distance graph for minimum cost vs. average delay under Unsplittable condition for 7 Node Topology

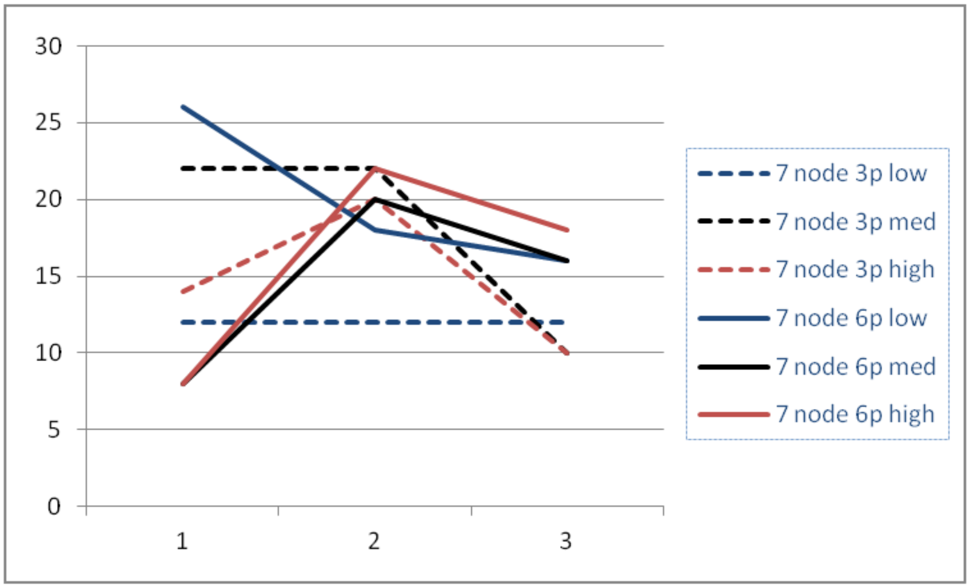

Figure 7: Distance graph for average delay vs. link utilization under Unsplittable condition 7 Node Topology

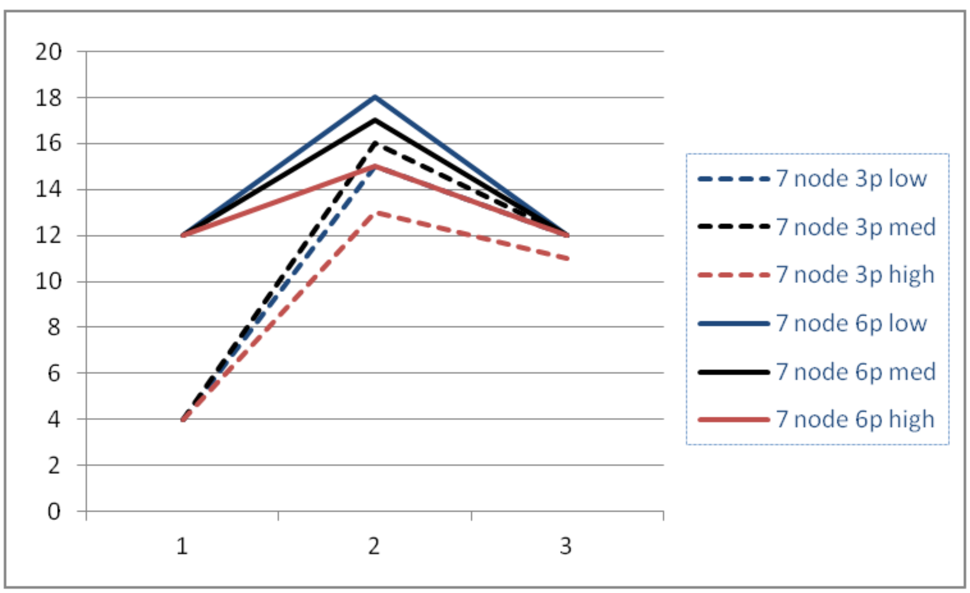

Figure 8: Distance graph for minimum cost vs. link utilization under Splittable condition 7 Node Topology

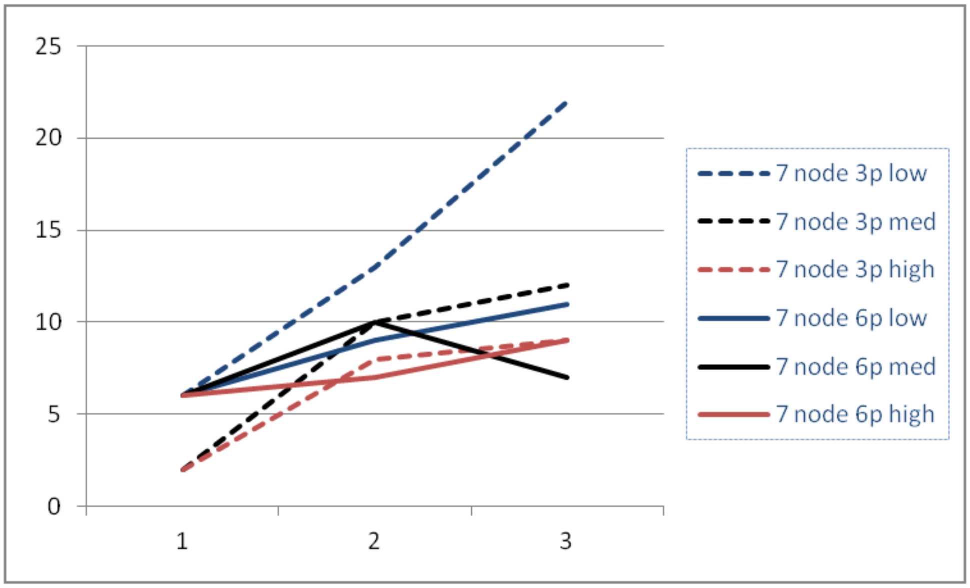

Figure 9: Distance graph for minimum cost vs. average delay under Splittable condition 7 Node Topology

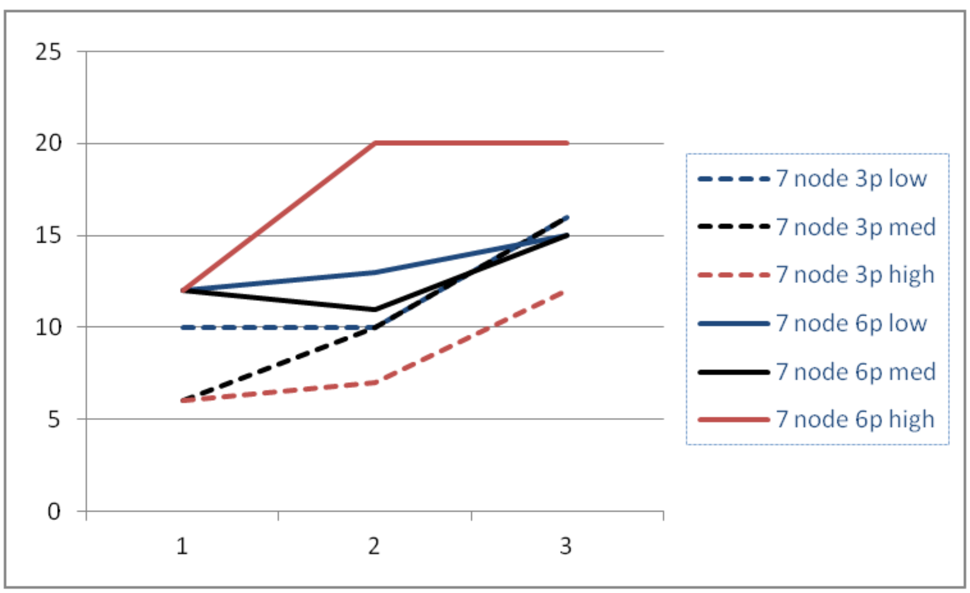

Figure 10: Distance graph for average delay vs. link utilization under Splittable condition 7 Node Topology

Heaviest used link for the network in lowest demand case is link 4, so reduce the capacity of link 4 by 10%.

Table 13: Results for 7 node topology with 3 paths for each time period under one link capacity adjustment case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	21598			21598	0.4694710685			0.4760820046	5.154225615			5.3481883
	Non zero path	21	21	21		21	25	22		21	26	27	
	More than one path	0	0	0		0	4	1		0	5	6	
Med	Objective	29216.55			29439.4	0.6337859425			0.6427107062	9.029305726			9.60074651
	Non zero path	21	23	21		21	25	22		21	26	24	
	More than one path	0	2	0		0	4	1		0	5	3	
High	Objective	37774.3			38098.7	0.7981008165			0.8093394077	15.82032881			16.7930784
	Non zero path	21	23	23		21	25	22		21	25	27	
	More than one path	0	2	2		0	4	1		0	4	6	

Each time period with **three** paths for load balancing and average delay to see which time period is having an effect on the final objective.

Table 14: Results for 7 node topology with 3 paths for all the three time periods for all the three demand cases

Demand		Load balancing		Average delay	
		Splittable	Unsplittable	Splittable	Unsplittable
Low	Time period 1	0.04419595314	0.04433333333	0.2983925588	0.2983925588
	Time period 2	0.4694710685	0.4760820046	5.154225615	5.348188345
	Time period 3	0.4045083422	0.4123006834	4.131702456	4.260727616
Medium	Time period 1	0.05966453674	0.05985	0.4028299544	0.4028299544
	Time period 2	0.6337859425	0.6427107062	9.029305726	9.600746515
	Time period 3	0.546086262	0.5566059226	7.074055726	7.454389043
High	Time period 1	0.07513312034	0.07536666667	0.50726735	0.50726735
	Time period 2	0.7981008165	0.8093394077	15.82032881	16.7930784
	Time period 3	0.6876641818	0.7009666667	11.31201848	12.01127495

Table 15: Results for 7 node topology with 6 paths for each time period under one link capacity adjustment case

Demand		Minimum cost				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	objective	21598			21598	0.4694710685			0.4760820046	5.154225615			5.316671557
	Non zero path	21	21	21		21	23	23		21	26	25	
	More than one path	0	0	0		0	2	2		0	5	4	
Med	Objective	29038.35			29439.45	0.6337859425			0.6427107062	8.79237216			9.199194601
	Non zero path	21	23	21		21	23	23		21	27	24	
	More than one path	0	2	0		0	2	2		0	6	3	
High	Objective	37774.3			38098.7	0.7981008165			0.8093394077	15.82032881			16.12665993
	Non zero path	21	23	23		21	23	23		21	25	25	
	More than one path	0	2	2		0	2	2		0	4	4	

Each time period with **six** paths for load balancing and average delay to see which time period is having an effect on the final objective.

Table 16: Results for 7 node topology with 6 paths for all the three time periods for all three demand cases

Demand		Load balancing		Average delay	
		Splittable	Unsplittable	Splittable	Unsplittable
Low	Time period 1	0.04411764706	0.04433333333	0.2981427131	0.2981427131
	Time period 2	0.4694710685	0.4760820046	5.154225615	5.316671557
	Time period 3	0.4037916371	0.4107413011	4.126192948	4.218973641
Medium	Time period 1	0.05955882353	0.05985	0.4024926626	0.4024926626
	Time period 2	0.6337859425	0.6427107062	8.79237216	9.199194601
	Time period 3	0.5451187101	0.5545007564	7.035438278	7.305814297
High	Time period 1	0.075	0.07536666667	0.5068426122	0.5068426122
	Time period 2	0.7981008165	0.8093394077	15.82032881	16.12665993
	Time period 3	0.6864457831	0.6982602118	11.18912962	11.65179134

Observations on number of “non negative paths” and” more than one path”

- Load balancing once it has selected a solution under the low demand case it takes the same paths until its condition becomes infeasible i.e. r<=1, hence we see that the number of non-zero paths and more than one paths has the same value for both cases. In this topology moving from 3 path solutions to 6 path solutions a better path is not there; even if there exists a better path a change in solution most probably will not occur since even in that case these values would remain the same.

- Average delay shows the tendency to have the most with demands being satisfied by more than one path; this is explainable since it is trying to use a link with less inherent delay it tries to find another path almost always.

- In case of minimum cost routing number of non- zero paths remain the same for both 3 path and 6 path solutions; one of the reasons is that even in the 6 path case mostly the first 3 paths are being used. This can be explained by the fact that the additional 3 paths introduced do not have a better cost than the 3 paths. To illustrate this let’s look at the path costs in 3 and 6 paths.

param	cost:=			
	[*,*,1]:	1	2	3 :=
	1	2	2	3
	2	2	3	3
	3	4	3	4
	4	2	4	5
	5	5	3	3
	6	1	3	4
	7	1	2	4
	8	1	2	3
	9	2	3	4
	10	1	2	2
	11	1	4	3
	12	2	3	4
	13	4	3	4
	14	2	3	7
	15	3	2	5
	16	2	3	5
	17	1	2	3
	18	3	2	3
	19	3	3	4
	20	1	4	5
	21	2	4	3

param	cost:=						
	[*,*,1]:	1	2	3	4	5	6:=
	1	2	2	3	5	6	7
	2	2	3	3	6	7	4
	3	4	3	4	5	3	4
	4	2	4	5	5	7	6
	5	5	3	3	4	4	4
	6	1	3	4	6	7	7
	7	1	2	4	7	8	9
	8	1	2	3	6	3	7
	9	2	3	4	4	5	5
	10	1	2	2	5	7	8
	11	1	4	3	4	5	6
	12	2	3	4	6	5	5
	13	4	3	4	5	6	5
	14	2	3	7	3	6	6
	15	3	2	5	6	3	8
	16	2	3	5	6	4	5
	17	1	2	3	8	5	6
	18	3	2	3	4	4	5
	19	3	3	4	5	5	6
	20	1	4	5	6	7	6
	21	2	4	3	3	4	5

We can see that the column highlighted in yellow does not give a better option than already given by 3 path scenario hence the non-zero paths are not affected. But we can see that in medium case it has increased by a negligible amount 0.006% and in high case there is no increase from 3 to 6 path solutions.

- We can also see that the highest demand time period corresponds to the maximum average delay and load balancing.

In addition to above the second most utilized link is reduced by 10 percent.

Table 17: Results for 7 node topology with 3 paths for each time period under two link capacity adjustments case

Demand		Minimum cost				Load balancing				Average delay			
		Splittable											
Low	objective	21598			21598	0.4694			0.4760	5.4622			5.6697
	Non zero path	21	21	21		21	25	23		21	26	25	
	More than one path	0	0	0		0	4	2		0	5	4	
Med	Objective	29263.8			29664.9	0.63378594249			0.6427	9.6241			10.1931
	Non zero path	21	24	21		21	24	23		21	25	25	
	More than one path	0	3	0		0	3	2		0	4	4	
High	Objective	38692.2			39224.1	0.7981008165			0.80933	16.846666310			17.888775141
	Non zero path	21	24	24		21	24	23		21	27	27	
	More than one path	0	3	3		0	3	2		0	6	6	

Table 18: Results for 7 node topology with 6 paths for each time period under two link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	objective	21598			21598	0.46947106851			0.47608200456	5.4622178318			5.6382724472
	Non zero path	21	21	21		21	24	24		21	25	23	
	More than one path	0	0	0		0	3	3		0	4	2	
Med	Objective	29263.8			29664.9	0.63378594249			0.64271070515	9.4617870924			9.7248482577
	Non zero path	21	24	21		21	24	24		21	25	26	
	More than one path	0	3	0		0	3	3		0	4	5	
High	Objective	38692.2			39224.1	0.79810081647			0.80933940774	16.846666310			17.165985419
	Non zero path	21	24	24		21	24	24		21	28	25	
	More than one path	0	3	3		0	3	3		0	*6	4	

When I reduced the first 3 most utilized links' capacity by 10% the solution became infeasible. Hence I increased the demand of the other 11 links by 10%.

Table 19: Results for 7 node topology with 3 paths for each time period under three link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	objective	21831			21907	0.55081917476			0.56327160494	6.805459044			6.9226120653
	Non zero path	21	22	21		21	24	23		21	24	25	
	More than one path	0	1	0		0	3	2		0	3	4	
Med	Objective	30761.1			31151.25	0.74360588592			0.76041666667	13.134009443			13.794060714
	Non zero path	21	23	22		21	24	23		21	25	24	
	More than one path	0	2	1		0	3	2		0	4	3	
High	Objective	41436.3			44407.4	0.9363925971			0.9575617274	33.108277290			37.224231584
	Non zero path	21	24	24		21	24	23		21	26	25	
	More than one path	0	3	3		0	3	2		0	5	4	

Table 20: Results for 7 node topology with 6 paths for each time period under 3 link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	objective	21664			21740	0.55081917476			0.5541543026 7	6.8054590440			6.905509728
	Non zero path	21	22	21		21	24	24		21	24	25	
	More than one path	0	1	0		0	3	3		0	3	4	
Med	Objective	30521.3			30956.85	0.7436058859			0.7481083086	13.13402461			13.64637640
	Non zero path	21	24	22		21	24	24		21	25	24	
	More than one path	0	3	3		0	3	3		0	4	3	
High	Objective	41436.3			43278.6	0.9363925971			0.9420623145	33.108277290			34.04015820
	Non zero path	21	24	24		21	24	23		21	26	25	
	More than one path	0	3	3		0	3	2		0	5	4	

Observations

- Even though there are no better paths available from 3 paths to 6 paths wrt cost (no of hops) the objective reduces from 3 to 6 paths negligible (~~0.006%); but even when the critical links availability is reduced the difference remains almost the same.

- With the failure each of link the objective increases accordingly but minimum cost remains the one that is less affected compared to load balancing and average delay.

4.4 NSF Topology

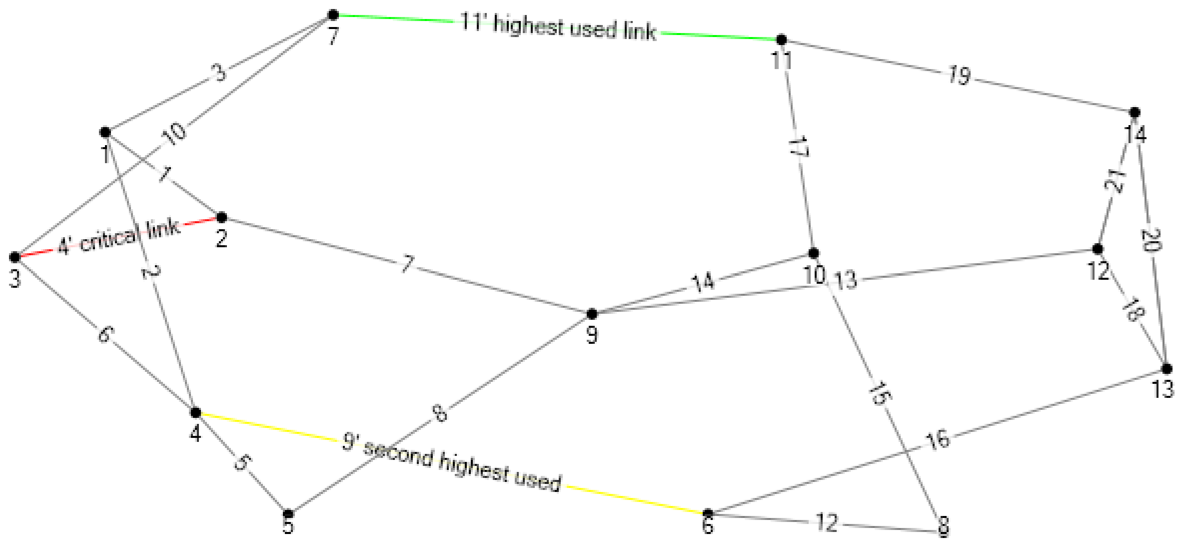

Figure 11: NSF topology diagram

In this topology the most utilized link is 11 followed by 9 and 4.

All the links have a maximum capacity of 55000.

Table 21: Results for NSF topology with 5 paths for each time period under normal case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	762849.9252			762849.9252	0.452940301			0.453223521	8.649002333			8.65353542
	Non zero path	91	91	91		91	96	93		91	98	99	
	More than one path	0	0	0		0	5	2		0	7	8	
Med	Objective	978037.0873			978037.0873	0.5807039018			0.581254438	14.08704588			14.1101720
	Non zero path	91	91	91		91	96	93		91	96	104	
	More than one path	0	0	0		0	5	2		0	5	13	
High	Objective	1222515.906			1222515.906	0.7258658669			0.726582613	24.07894537			24.1413291
	Non zero path	91	91	91		91	96	93		91	96	106	
	More than one path	0	0	0		0	5	2		0	5	** 13	

Table 22: Results of distance for NSF topology with 5 paths under low demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	94	66	22	182	24	42	19	85
HD	1.033	0.7253	0.2418		0.2637	0.4615	0.2088	
H	0.2066	0.1451	0.0484		0.0527	0.0923	0.0418	
Minimum cost vs. average delay	88	28	22	138	28	26	37	91
HD	0.967	0.3077	0.2418		0.3077	0.2857	0.4066	
H	0.1934	0.0615	0.0484		0.0615	0.0571	0.0813	
Average delay vs. link utilization	48	70	36	154	12	34	40	86
HD	0.5275	0.7692	0.3956		0.1319	0.3736	0.4396	
H	0.1055	0.1538	0.0791		0.0264	0.0747	0.0879	

Table 23: Results of distance for NSF topology with 5 paths under medium demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	94	74	30	198	24	42	19	85
HD	1.033	0.8132	0.3297		0.2637	0.4615	0.2088	
H	0.2066	0.1626	0.0659		0.0527	0.0923	0.0418	
Minimum cost vs. average delay	106	26	36	168	28	29	51	108
HD	1.1648	0.2857	0.3956		0.3077	0.3187	0.5604	
H	0.233	0.0571	0.0791		0.0615	0.0637	0.1121	
Average delay vs. link utilization	58	76	54	188	12	55	58	125
HD	0.6374	0.8352	0.5934		0.1319	0.6044	0.6374	
H	0.1275	0.167	0.1187		0.0264	0.1209	0.1275	

Table 24: Results of distance for NSF topology with 5 paths under high demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	94	89	14	197	24	42	19	85
HD	1.033	0.978	0.1538		0.2637	0.4615	0.2088	
H	0.2066	0.1956	0.0308		0.0527	0.0923	0.0418	
Minimum cost vs. average delay	88	32	36	156	28	21	19	68
HD	0.967	0.3516	0.3956		0.3077	0.2308	0.2088	
H	0.1934	0.0703	0.0791		0.0615	0.0462	0.0418	
Average delay vs. link utilization	48	84	42	174	12	29	28	69
HD	0.5275	0.9231	0.4615		0.1319	0.3187	0.3077	
H	0.1055	0.1846	0.0923		0.0264	0.0637	0.0615	

Table 25: Results for NSF topology with 10 paths for each time period under normal case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	objective	591195.9403			591195.9403	0.3290045536			0.3322500333	5.674953867			5.690440129
	Non zero path	91	91	91		91	100	95		91	96	94	
	More than one path	0	0	0		0	#7	4		0	5	3	
Med	Objective	757961.7852			757961.7852	0.4218131634			0.4255282315	8.201910806			8.278294729
	Non zero path	91	91	91		91	100	95		91	102	98	
	More than one path	0	0	0		0	#7	4		0	11	7	
High	Objective	947429.3915			947429.3915	0.5272508872			0.5315974842	12.84646357			12.86382149
	Non zero path	91	91	91		91	100	95		91	98	99	
	More than one path	0	0	0		0	#7	4		0	7	8	

Observations:

1. **'#'** More than 2 paths are used to satisfy the demand.

For the 10 path case in Load balancing:

x_3_1_2 990.994444…….4 13 11 17

x_3_3_2 249.843333……..4 1 3 11 19 20 16 12 15

x_3_10_2 1203.922222……4 7 8 5 2 3 11 17

Demand is divided into 3 paths.

Before the splitting of the paths into 3; only the path x_3_1_2 is used in 5 path scenario.

For example in the high demand case in 5 path;

x_3_1_2 1955.810000…….4 13 11 17

To understand this a bit better;

bandwidth information for 5 paths high case

link number	Bandwidth	value of r
link_capacity_1	38090.97	0.692563
link_capacity_2	28396.16	0.516294
link_capacity_3	29294.24	0.532623
link_capacity_4	42107.31	0.765587
link_capacity_5	36804.34	0.66917
link_capacity_6	26847.1967	0.488131
link_capacity_7	36666.6	0.666665
link_capacity_8	36666.6	0.666665
link_capacity_9	37610.4133	0.683826
link_capacity_10	22077.8967	0.401416
link_capacity_11	45000.0667	0.818183
link_capacity_12	21148.6008	0.38452
link_capacity_13	29732.45	0.54059

bandwidth information for the 10 paths high case

link number	bandwidth	value of r
link_capacity_1	29979.2825	0.545078
link_capacity_2	20520.33	0.373097
link_capacity_3	21468.1225	0.39033
link_capacity_4	30098.4475	0.547245
link_capacity_5	20079.68	0.365085
link_capacity_6	22504	0.409164
link_capacity_7	26899.225	0.489077
link_capacity_8	17210.02	0.312909
link_capacity_9	36666.6	0.666665
link_capacity_10	19523.2225	0.354968
link_capacity_11	32918.695	0.598522
link_capacity_12	24454.605	0.444629
link_capacity_13	31798.33	0.578151

link_capacity_14	33811.72	0.614759
link_capacity_15	22114.1008	0.402075
link_capacity_16	28548.4875	0.519063
link_capacity_17	25953.3158	0.471878
link_capacity_18	18333.4	0.333335
link_capacity_19	28188.9725	0.512527
link_capacity_20	29123.8025	0.529524
link_capacity_21	11558.11	0.210147

link_capacity_14	18333.4	0.333335
link_capacity_15	18333.4	0.333335
link_capacity_16	31145.68	0.566285
link_capacity_17	18333.4	0.333335
link_capacity_18	11845.59	0.215374
link_capacity_19	18333.4	0.333335
link_capacity_20	18333.4	0.333335
link_capacity_21	18333.4	0.333335

We can see that from the 5 path case links 4,13,15,17 to 10 path case the bandwidth used (or ‘r’) value has gone down due to the splitting of demands into more than one path. We can especially see that the link 11 whose value of ‘r’ is 0.818183 is reduced to 0.598522 which is by a considerable amount. Value of ‘r’ being very high is the primary reason in the demand being split into more than one path in the 10 path scenario.

2. 3^{rd} demand uses 3 paths in average delay high demand case; to analyze this we have considered the demand used in 5 path medium case and high case.

Path selection in case of high demand:

11^{th} demand

x_11_2_3 425.845833……3 2 5 8 13 18 20

x_11_3_3 392.755000…….3 1 7 14 17 19

X_11_4_3 1277.269167…….3 1 7 13 21

19^{th} demand

x_19_2_3 254.941667……1 2 6 10 11 17 15

x_19_4_3 1264.054167…..1 3 10 6 9 12

X_19_5_3 495.634167……1 3 11 17 15

Path selection in case of medium demand:

11^{th} demand

x_11_4_3 1854.860000……3 2 5 8 13 18 20

19^{th} demand

x_19_3_3 1611.710000……1 2 9 12

We have observed for the 11^{th} demand case; we see that the yellow colored in medium demand case are the links used by them. And the same links in high case have a higher bandwidth usage; we know that average delay parameter is inversely proportional to the total bandwidth used; hence the paths chosen by 5 path high case allows the solution to reduce the average delay parameter.

3. Having more number of paths is most beneficial in case of load balancing and average delay models.

4. We can see that there is no increase in objective for minimum cost routing we can attribute it to the fact that the numbers of paths chosen remain the same for both the cases; i.e. in Splittable and Unsplittable the same paths are being chosen.

5. Even in this case we see that the average delay objective has the highest number of demands having more than one path.

Table 26: Results of distance for NSF topology with 10 paths under low demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	22	54	38	114	30	52	39	121
HD	0.2418	0.5934	0.4176		0.3297	0.5714	0.4286	
H	0.0242	0.0593	0.0418		0.033	0.0571	0.0429	
Minimum cost vs. average delay	84	0	0	84	167	27	65	259
HD	0.9231	0	0		1.8352	0.2967	0.7143	
H	0.0923	0	0		0.1835	0.0297	0.0714	
Average delay vs. link utilization	82	54	38	174	155	53	64	272
HD	0.9011	0.5934	0.4176		1.7033	0.5824	0.7033	
H	0.0901	0.0593	0.0418		0.1703	0.0582	0.0703	

Table 27: Results of distance for NSF topology with 10 paths under medium demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	22	70	36	128	30	52	39	121
HD	0.2418	0.7692	0.3956		0.3297	0.5714	0.4286	
H	0.0242	0.0769	0.0396		0.033	0.0571	0.0429	
Minimum cost vs. average delay	98	29	32	159	167	27	65	259
HD	1.0769	0.3187	0.3516		1.8352	0.2967	0.7143	
H	0.1077	0.0319	0.0352		0.1835	0.0297	0.0714	
Average delay vs. link utilization	100	73	44	217	155	53	64	272
HD	1.0989	0.8022	0.4835		1.7033	0.5824	0.7033	
H	0.1099	0.0802	0.0484		0.1703	0.0582	0.0703	

Table 28: Results of distance for NSF topology with 10 paths under high demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	20	74	34	128	30	52	39	121
HD	0.2198	0.8132	0.3736		0.3297	0.5714	0.4286	
H	0.022	0.0813	0.0374		0.033	0.0571	0.0429	
Minimum cost vs. average delay	88	32	30	150	167	27	65	259
HD	0.967	0.3516	0.3297		1.8352	0.2967	0.7143	
H	0.0967	0.0352	0.033		0.1835	0.0297	0.0714	
Average delay vs. link utilization	90	64	56	210	155	53	64	272
HD	0.989	0.7033	0.6154		1.7033	0.5824	0.7033	
H	0.0989	0.0703	0.0615		0.1703	0.0582	0.0703	

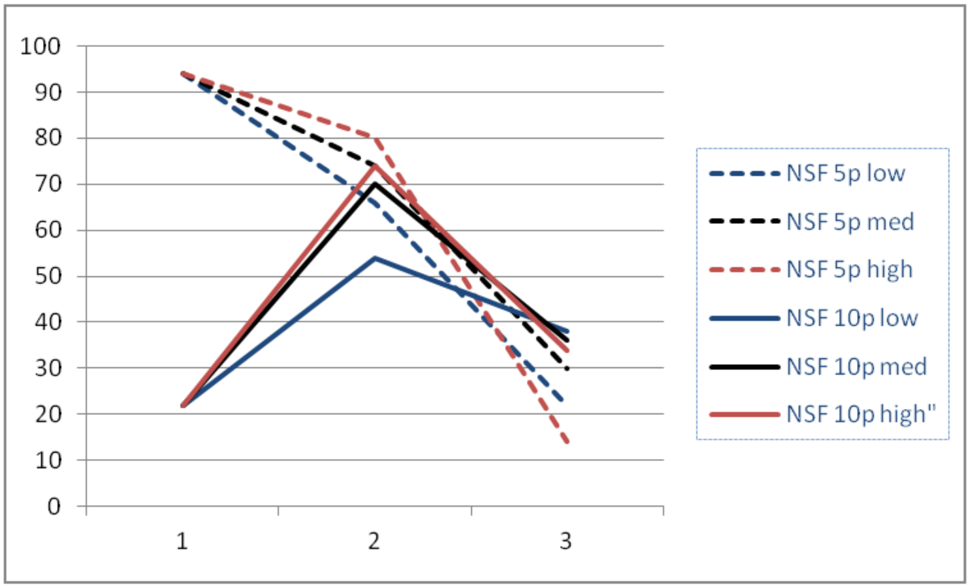

Figure 12: Distance graph for minimum cost vs. link utilization under Non Splittable condition NSF node topology

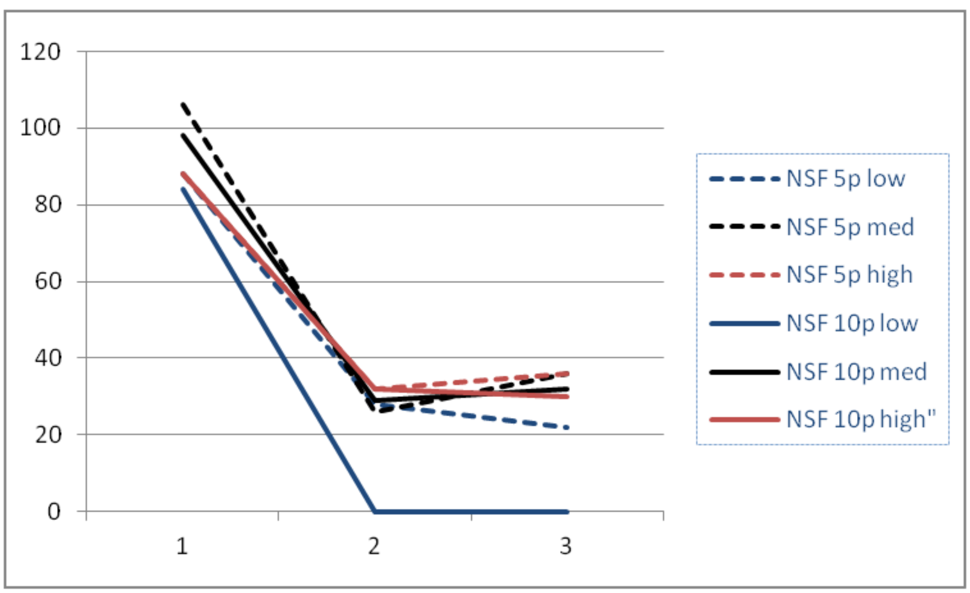

Figure 13: Distance graph for minimum cost vs. average delay under Non Splittable condition NSF node topology

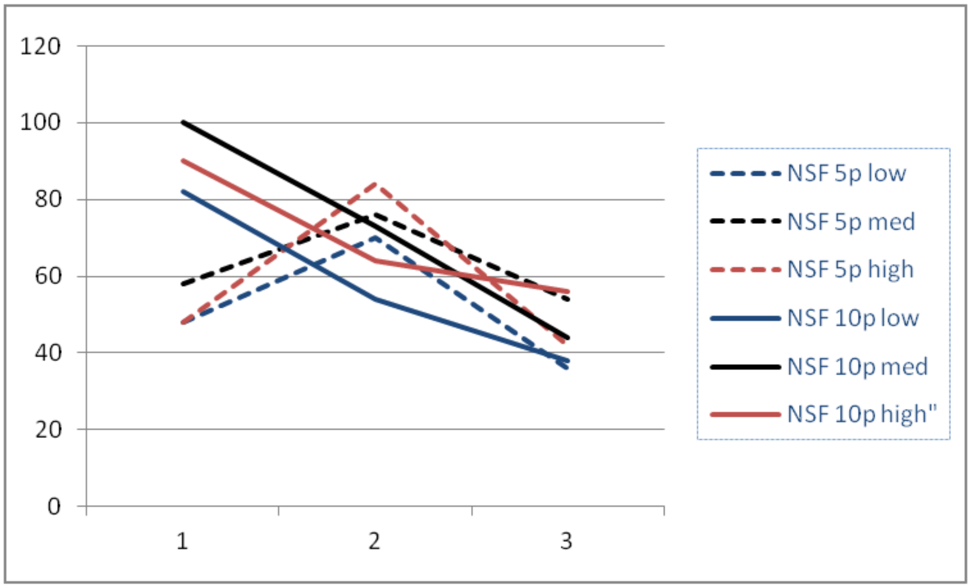

Figure 14: Distance graph for average delay vs. link utilization under Non Splittable condition NSF node topology

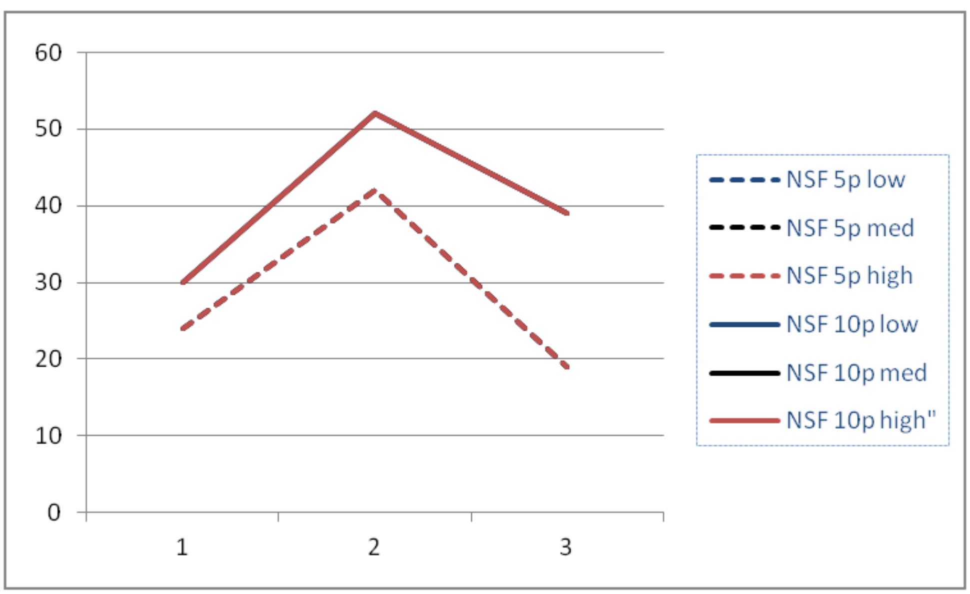

Figure 15: Distance graph for minimum cost vs. link utilization under Splittable condition NSF node topology

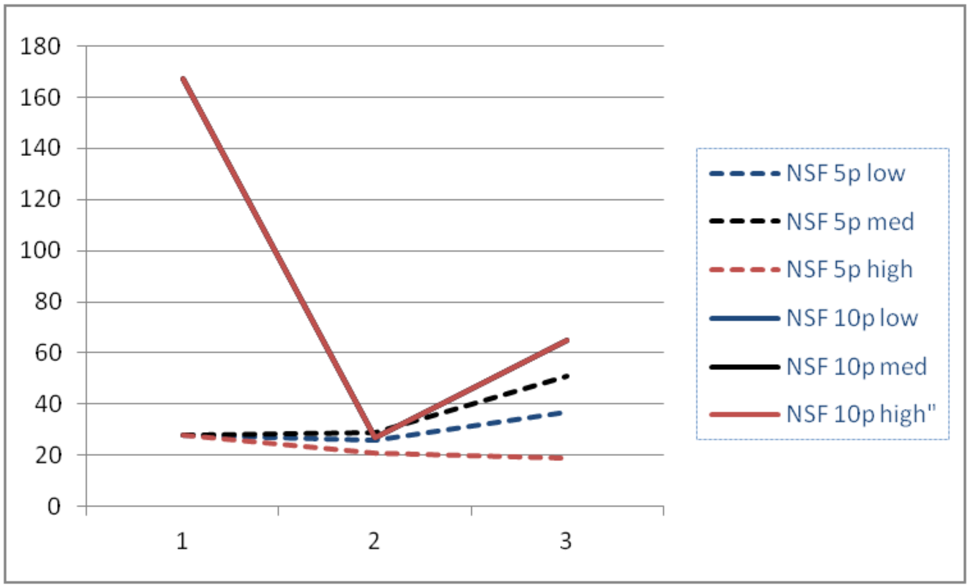

Figure 16: Distance graph for minimum cost vs. average delay under Splittable condition NSF node topology

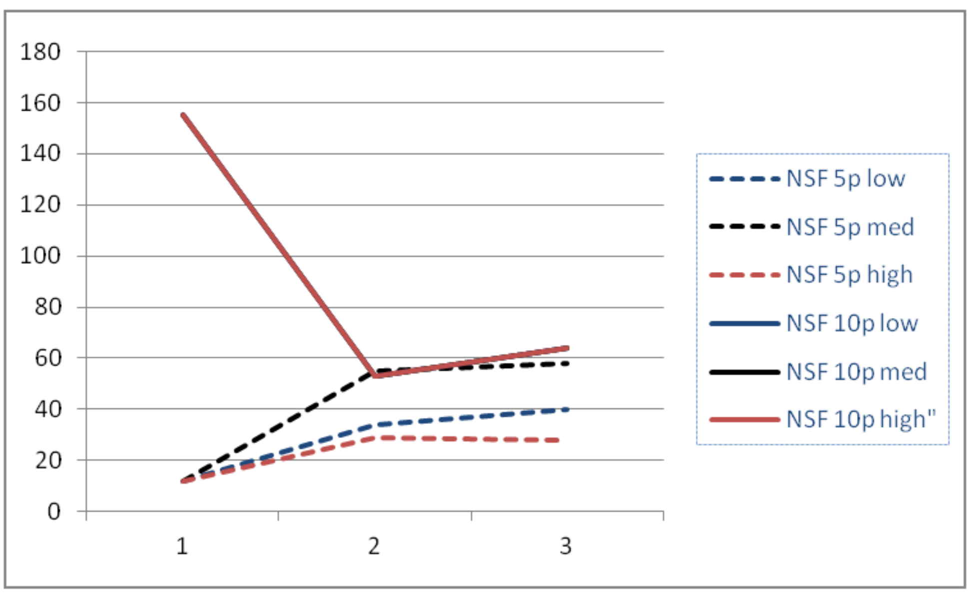

Figure 17: Distance graph for link utilization vs. average delay under Splittable condition NSF node topology

Heaviest used link for the network in lowest demand case is link 11, so reduce the capacity of link 11 by 10% (i.e. it used 34094 I reduced it to 29094).

Table 29: Results for NSF topology with 5 paths for each time period under one link capacity adjustment case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	766328.6007			766950.892	0.501791974			0.5020271884	9.794660913			9.827842295
	Non zero path	91	92	92		91	95	93		91	101	103	
	More than one path	0	1	1		0	4	2		0	10	12	
Med	Objective	998908.9276			998976.0336	0.6433389239			0.6436373624	15.83778067			15.92572093
	Non zero path	91	92	92		91	95	93		91	96	96	
	More than one path	0	1	1		0	4	2		0	5	5	
High	Objective	1263152.706			1264350.557	0.8041538045			0.8042830409	28.68748992			28.69815229
	Non zero path	91	93	92		91	95	93		91	95	94	
	More than one path	0	2	1		0	4	2		0	4	3	

Each time period with five paths for load balancing and average delay

Table 30: Results for NSF topology with 5 paths for only one time period for all three demand cases

Demand		Load balancing		Average delay	
		Splittable	Unsplittable	Splittable	Unsplittable
Low	Time period 1	0.0364622455	0.03647175575	0.5437416817	0.5437416817
	Time period 2	0.505850994	0.5058794944	9.829212581	9.870814918
	Time period 3	0.4572641674	0.4572758463	8.296633914	8.301103267
Medium	Time period 1	0.04674761326	0.04675094214	0.6971208781	0.6971208781
	Time period 2	0.6485427956	0.6486544522	15.98150568	16.07012624
	Time period 3	0.586250467	0.5863343456	13.04357202	13.10081494
High	Time period 1	0.0584496669	0.05843308573	0.8713809001	0.8713809001
	Time period 2	0.8106586442	0.8110053322	28.93999999	29.0374876
	Time period 3	0.5864603034	0.586250467	13.04357202	13.10081494

Table 31: Results for NSF topology with 5 paths for each time period under one link capacity adjustment case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	objective	591195.9403			591195.9403	0.369731256			0.3701324054	6.128820849			6.134312287
	Non zero path	91	91	91		91	96	95		91	98	96	
	More than one path	0	0	0		0	5	4		0	7	5	
Med	Objective	757961.7852			757961.7852	0.4740303111			0.4741344105	9.345203116			9.378380394
	Non zero path	91	91	91		91	96	95		91	99	95	
	More than one path	0	0	0		0	5	4		0	8	4	
High	Objective	947429.3915			947429.3915	0.5925180385			0.5957384976	14.28652125			14.32569792
	Non zero path	91	92	92		91	96	95		91	97	96	
	More than one path	0	1	1		0	5	4		0	6	5	

Each time period with ten paths for load balancing and average delay to see which time period is having an effect on the final objective.

Table 32: Results for NSF topology with 10 paths for only one time period for all three demand cases

Demand		Load balancing		Average delay	
		Splittable	Unsplittable	Splittable	Unsplittable
Low	Time period 1	0.02694317688	0.02695238752	0.4092783877	0.4092783877
	Time period 2	0.373790276	0.3740260557	6.16651549	6.174338876
	Time period 3	0.3378878392	0.3380919047	5.410428432	5.415797606
Medium	Time period 1	0.03454367914	0.03456567596	0.5247280002	0.5247280002
	Time period 2	0.4798084606	0.4792341829	9.389178254	9.402868297
	Time period 3	0.4332038925	0.4338438895	7.853862294	7.837699494
High	Time period 1	0.04317816808	0.0431907469	0.655894852	0.655894852
	Time period 2	0.5990228782	0.5991869839	14.44720948	14.4844351
	Time period 3	0.5414869218	0.5416767723	11.76571899	11.84680328

Observations:

- As we expected the highest demand corresponds to the maximum value of load balancing and average delay in both 5 and 10 path case.
- As we had seen with 7 node network the minimum cost routing uses the least number of paths and same principle explains the number of paths used. But an interesting observation is the number of non zero paths decreased from 5 path scenario to 10 path scenario the number of non zero paths has decreased this is understandable because of the presence of 5 more new paths and it has chosen a better path.
- In case of link utilization also the same principle follows. We can also see that value of link utilization has reduced from 5 paths to 10 paths due to the availability of more number of paths.
- One thing we can observe with average delay in the high demand case is the number of non zero paths is the least. We can explain this as the demand increases the path options reduces due to capacity constraint(solution space reduces).and we can also observe that in this case the average delay reduces by half (i.e. from 28(5 path) to 14(10 path) units.

Reduce 10% of capacity the first (i.e. it used 34094 I reduced it to 29094) and second highest link ‘9’ (reduced from 25437 to 19937).

Table 33: Results for NSF topology with 5 paths for each time period under two link capacity adjustment case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	777121.5195			780363.2778	0.6124265798			0.6127477695	12.95797458			13.01643703
	Non zero path	91	93	93		91	94	93		91	98	99	
	More than one path	0	2	2		0	3	2		0	7	8	
Med	Objective	1049478.431			1050929.426	0.7851816633			0.7856223366	24.11740626			24.20594542
	Non zero paths	91	93	93		91	94	93		91	98	98	
	More than one path	0	2	2		0	3	2		0	7	7	
High	Objective	1378713.992			1386931.93	0.9814528521			0.9822068648	66.12892177			67.04560267
	Non zero path	91	94	93		91	94	93		91	97	98	
	More than one path	0	3	2		0	3	2		0	6	7	

Table 34: Results for NSF topology with 10 paths for each time period under two link capacity adjustment case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	596128.8104			596896.3713	0.4512492433			0.452645407	8.108501031			8.120196258
	Non zero path	91	92	92		91	96	95		91	95	108	
	More than one path	0	1	1		0	5	4		0	4	12	
Med	Objective	776293.2689			777592.9512	0.5785440525			0.5794722123	12.40359096			12.5213391
	Non zero path	91	93	93		91	96	95		91	96	97	
	More than one path	0	2	2		0	5	4		0	5	5	
High	Objective	999800.6138			1004281.535	0.7231558386			0.7245629051	20.5786418			20.7976061
	Non zero path	91	93	93		91	96	95		91	99	95	
	More than one path	0	2	2		0	5	4		0	8	4	

Observations

1. In the average delay case there are instances where more than 2 paths are used only for the low demand case for instance demand 31 is satisfied by these 3 paths. This also suggests to us that average delay tries to use more number of paths than minimum cost and load balancing.

   x_31_2_3 317.608214…..links used 6, 10, 11

   x_31_5_3 22.670393……links used 9,16,20,19

   x_31_9_3 838.961393….links used 5, 8, 14, 17

But in the medium and high case it does not do so. This supports the fact that as the demand increases the path options reduces for average delay.

For instance in the medium case to satisfy demand 31

x_31_2_3 1511.840000…..links used 6, 10, 11

In the high case

x_31_2_3 1889.800000….links used 6, 10, 11

When I reduced the first 3 most utilized link's capacity by 10% the solution became infeasible. Hence I increased the demand of the other 18 links by 10% by 1200% and the solution was still infeasible. This explains that apart from the solutions chosen for the 5 path case there wasn't a solution without using links 11,4 and 9 to satisfy the demands.

2. With reduction of capacity of each critical link

- The number of paths used increases.
- The objective also increases.
- The difference of objective between Splittable and Unsplittable condition also increases by a considerable amount.
- Minimum cost routing is the one that changes to the smallest extent in terms of the number of non-zero paths, followed by load balancing (more non zero paths) and average delay (maximum non-zero paths.
- One interesting observation that we can see is in average delay in the high demand case there is a decrease of 50% or more from 5 paths to 10 path cases. From this we can explain that as the having more paths is most beneficial when the demand is more.

4.5 12 Node Topology

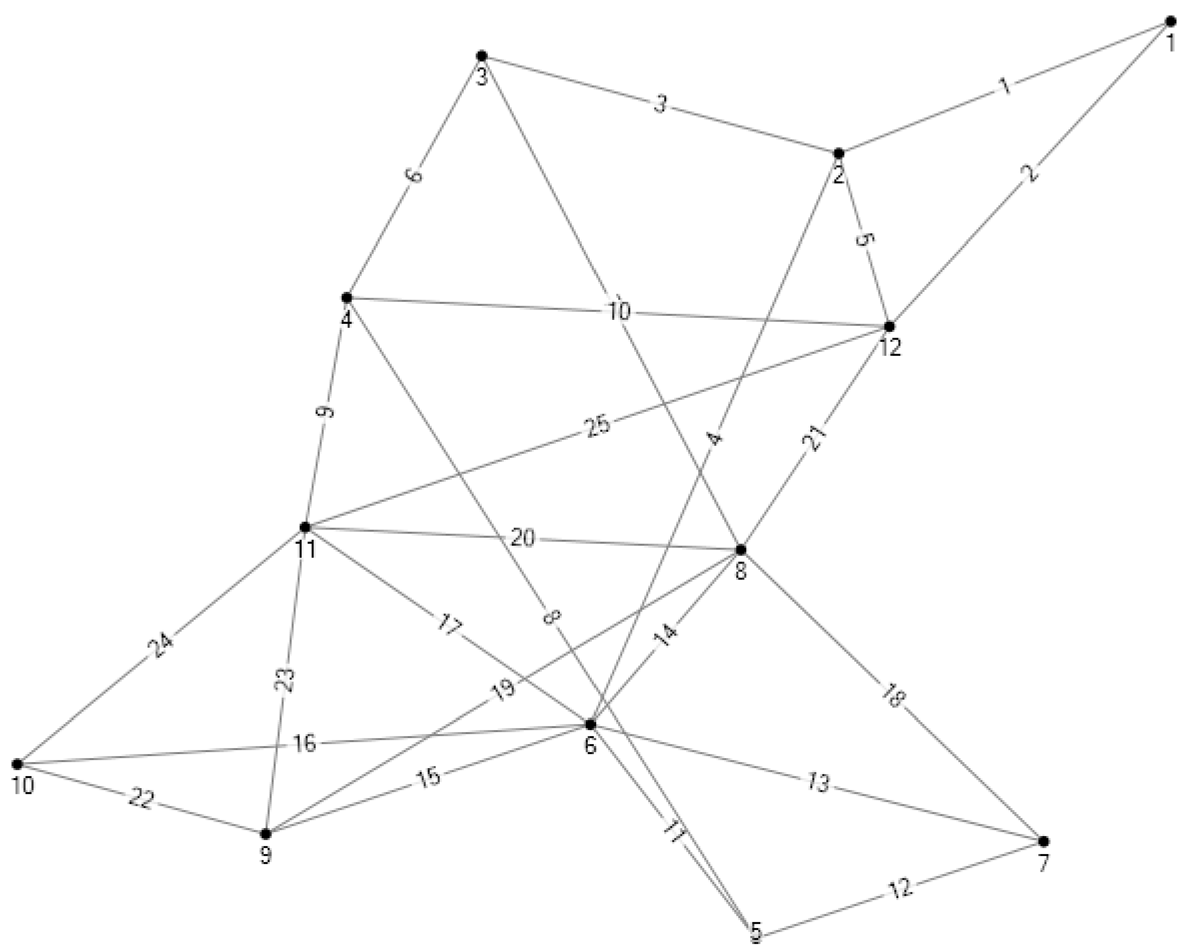

Figure 18: 12 Node topology diagram

All the links have a maximum capacity of 1000

Table 35: Results for 12 node topology with 5 paths for each time period under normal case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	21362.6			21362.6	0.36848			0.392	9.018744539			9.40239
	Non zero path	66	66	66		76	84	81		73	80	84	
	More than one path	0	0	0		10	**16	***12		*6	*13	*17	
Med	Objective	30518			30518	0.5264			0.558	16.9758234			18.1769
	Non zero path	66	67	67		76	84	81		73	79	84	
	More than one path	0	1	1		10	**16	***12		7	13	*17	
High	Objective	39786.55			39876.2	0.68432			0.715	31.18181472			33.8393
	Non zero path	68	70	69		76	84	81		72	81	83	
	More than one path	2	4	3		10	**16	***12		6	*14	17	

Table 36: Results of distance for 12 node topology with 5 paths under low demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	42	64	56	162	46	66	63	175
HD	0.6364	0.9697	0.8485		0.697	1	0.9545	
H	0.1273	0.1939	0.1697		0.1394	0.2	0.1909	
Minimum cost vs. average delay	83	91	82	256	37	42	58	137
HD	1.2576	1.3788	1.2424		0.5606	0.6364	0.8788	
H	0.2515	0.2758	0.2485		0.1121	0.1273	0.1758	
Average delay vs. link utilization	77	85	86	248	31	38	45	114
HD	1.1667	1.2879	1.303		0.4697	0.5758	0.6818	
H	0.2333	0.2576	0.2606		0.0939	0.1152	0.1364	

Table 37: Results of distance for 12 node topology with 5 paths under medium demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	34	64	62	160	46	63	62	171
HD	0.5152	0.9697	0.9394		0.697	0.9545	0.9394	
H	0.103	0.1939	0.1879		0.1394	0.1909	0.1879	
Minimum cost vs. average delay	79	90	82	251	27	32	45	104
HD	1.197	1.3636	1.2424		0.4091	0.4848	0.6818	
H	0.2394	0.2727	0.2485		0.0818	0.097	0.1364	
Average delay vs. link utilization	75	90	86	251	39	49	39	127
HD	1.1364	1.3636	1.303		0.5909	0.7424	0.5909	
H	0.2273	0.2727	0.2606		0.1182	0.1485	0.1182	

Table 38: Results of distance for 12 node topology with 5 paths under high demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	38	60	72	170	42	64	62	168
HD	0.5758	0.9091	1.0909		0.6364	0.9697	0.9394	
H	0.1152	0.1818	0.2182		0.1273	0.1939	0.1879	
Minimum cost vs. average delay	70	93	81	244	38	41	48	127
HD	1.0606	1.4091	1.2273		0.5758	0.6212	0.7273	
H	0.2121	0.2818	0.2455		0.1152	0.1242	0.1455	
Average delay vs. link utilization	72	91	85	248	34	33	36	103
HD	1.0909	1.3788	1.2879		0.5152	0.5	0.5455	
H	0.2182	0.2758	0.2576		0.103	0.1	0.1091	

Table 39: Results for 12 node topology with 10 paths for each time period under normal case

Demand		Minimum cost				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	17010.7			17010.7	0.2980551724			0.322	6.38249			6.74875
	Non zero path	66	66	66		71	82	74		87	75	88	
	More than one path	0	0	0		5	*15	8		#***16	9	*****17	
Med	Objective	24301			24301	0.4257931034			0.468	11.89445333			12.69694
	Non zero path	66	66	66		71	82	74		74	77	81	
	More than one path	0	0	0		5	*15	8		8	*10	*14	
High	Objective	31645.6			31647.2	0.5535310345			0.6045	19.07811857			20.61384
	Non zero paths	67	67	67		71	82	74		74	78	78	
	More than one paths	1	1	1		5	*15	8		8	12	*11	

	1	2	3	4	5	6	7	8	9	10
1	1	2	4	6	8	7	7	8	9	8
2	5	4	7	8	9	8	3	8	9	10
3	3	6	7	8	9	7	8	8	9	7
4	4	6	7	8	9	10	7	8	7	8
5	5	6	7	8	9	10	9	10	8	9
6	6	7	8	10	11	9	10	9	10	11
7	7	6	7	9	10	8	9	8	9	10
8	8	9	10	10	11	7	8	9	9	10
9	9	10	10	9	11	10	8	9	9	8
10	10	9	8	9	9	8	7	8	8	9
11	11	10	9	8	10	9	8	7	9	8
12	4	5	5	4	3	5	3	3	9	10
13	2	3	4	4	3	5	2	7	6	8
14	4	3	4	6	6	5	2	3	3	4
15	6	4	4	3	4	1	4	5	3	3
16	8	3	4	5	4	3	2	3	4	10
17	3	2	5	5	6	8	5	4	2	9
18	5	5	3	5	7	4	4	3	2	11
19	5	5	7	5	8	4	4	3	3	4
20	5	3	6	4	4	4	3	10	3	3
21	2	6	4	1	3	9	4	3	3	8
22	4	4	3	3	8	9	4	4	4	1

23	3	2	3	9	10	11	8	4	4	4
24	2	3	2	3	10	11	3	3	4	5
25	7	4	3	4	4	10	11	2	3	5
26	4	4	3	1	9	9	10	3	5	3
27	6	3	5	4	3	2	4	5	4	9
28	10	4	4	5	3	4	3	9	4	8
29	3	2	6	3	9	2	4	3	4	10
30	3	5	4	9	4	2	2	5	2	10
31	7	4	3	4	5	9	4	1	4	11
32	3	2	2	3	8	3	8	3	7	8
33	8	3	6	4	3	5	2	3	10	3
34	5	3	2	3	8	5	6	2	5	3
35	7	4	5	4	3	9	2	4	10	11
36	4	4	5	3	3	2	4	9	11	10
37	4	4	3	5	7	3	1	4	2	9
38	4	6	4	5	1	9	8	4	6	3
39	4	3	1	5	4	2	10	8	9	3
40	5	5	3	5	11	10	1	2	10	11
41	6	4	3	3	3	7	10	4	2	3
42	3	9	3	6	4	9	2	3	10	11
43	3	10	8	3	2	4	4	5	11	10
44	2	3	10	3	5	3	2	11	10	9
45	5	4	3	3	4	7	8	2	6	8

46	1	2	7	3	9	2	5	10	9	8
47	3	3	6	7	8	4	7	3	5	6
48	4	8	3	8	4	2	7	1	2	8
49	4	4	9	3	9	10	3	9	8	4
50	4	3	2	4	3	8	5	7	8	7
51	3	2	3	3	7	6	2	3	5	4
52	7	3	10	11	2	10	9	3	2	11
53	9	3	3	4	11	8	10	2	9	11
54	10	4	3	11	10	10	11	2	11	10
55	4	5	11	3	9	3	10	11	2	9
56	6	3	10	8	4	9	8	10	2	9
57	3	9	4	4	1	9	10	11	3	11
58	3	10	2	4	9	4	11	9	3	10
59	2	2	3	3	4	11	10	1	10	9
60	4	7	4	5	4	8	1	6	6	3
61	2	5	7	9	5	7	4	9	10	6
62	8	2	3	8	9	4	5	8	9	5
63	4	4	8	3	4	5	9	8	7	3
64	4	8	10	4	6	3	9	7	2	5
65	4	3	3	5	7	8	4	5	9	8
66	5	1	4	6	6	8	4	3	6	9

27 paths have better costs when moving from 5 paths to 10 path solutions. In this case the decrease in the cost has reduced by 20% low, medium and high demand case.

Table 40: Results of distance for 12 node topology with 10 paths under low demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	48	74	44	166	43	56	40	139
HD	0.7273	1.1212	0.6667		0.6515	0.8485	0.6061	
H	0.0727	0.1121	0.0667		0.0652	0.0848	0.0606	
Minimum cost vs. average delay	80	93	92	265	65	37	54	156
HD	1.2121	1.4091	1.3939		0.9848	0.5606	0.8182	
H	0.1212	0.1409	0.1394		0.0985	0.0561	0.0818	
Average delay vs. link utilization	90	95	92	277	62	35	54	151
HD	1.3636	1.4394	1.3939		0.9394	0.5303	0.8182	
H	0.1364	0.1439	0.1394		0.0939	0.053	0.0818	

Table 41: Results of distance for 12 node topology with 10 paths under medium demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	sum
Minimum cost vs. link utilization	60	62	34	156	43	56	40	139
HD	0.9091	0.9394	0.5152		0.6515	0.8485	0.6061	
H	0.0909	0.0939	0.0515		0.0652	0.0848	0.0606	
Minimum cost vs. average delay	76	94	94	264	38	29	37	104
HD	1.1515	1.4242	1.4242		0.5758	0.4394	0.5606	
H	0.1152	0.1424	0.1424		0.0576	0.0439	0.0561	
Average delay vs. link utilization	80	96	94	270	31	37	47	115
HD	1.2121	1.4545	1.4242		0.4697	0.5606	0.7121	
H	0.1212	0.1455	0.1424		0.047	0.0561	0.0712	

Table 42: Results of distance for 12 node topology with 10 paths under high demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Routing models	42	50	32	124	40	55	39	134
	0.6364	0.7576	0.4848		0.6061	0.8333	0.5909	
Minimum cost vs. link utilization	0.0636	0.0758	0.0485		0.0606	0.0833	0.0591	
HD	70	92	93	255	47	29	33	109
H	1.0606	1.3939	1.4091		0.7121	0.4394	0.5	
Minimum cost vs. average delay	0.1061	0.1394	0.1409		0.0712	0.0439	0.05	
HD	84	96	93	273	35	40	46	121
H	1.2727	1.4545	1.4091		0.5303	0.6061	0.697	
Average delay vs. link utilization	0.1273	0.1455	0.1409		0.053	0.0606	0.0697	

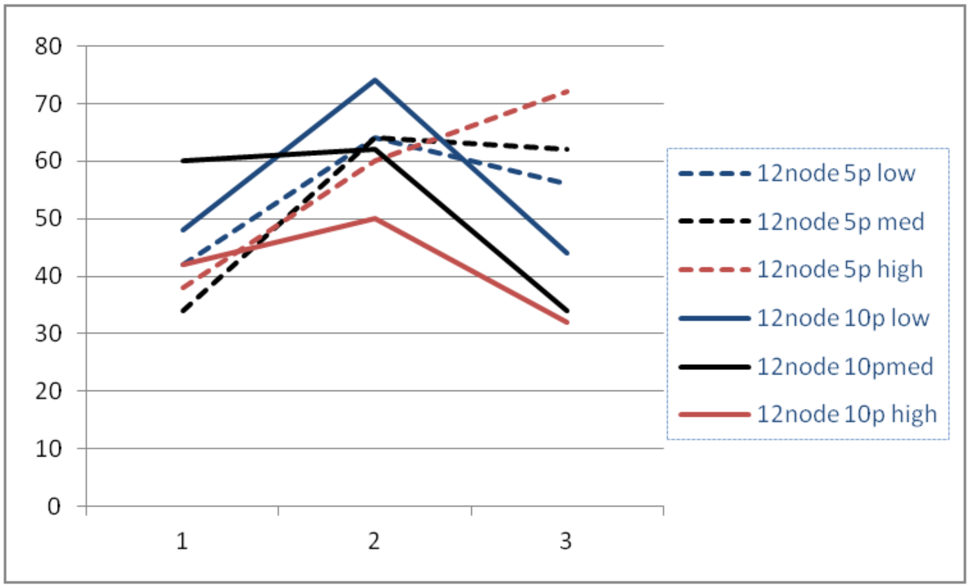

Figure 19: Distance graph for minimum cost vs. link utilization under Non Splittable condition 12 node topology

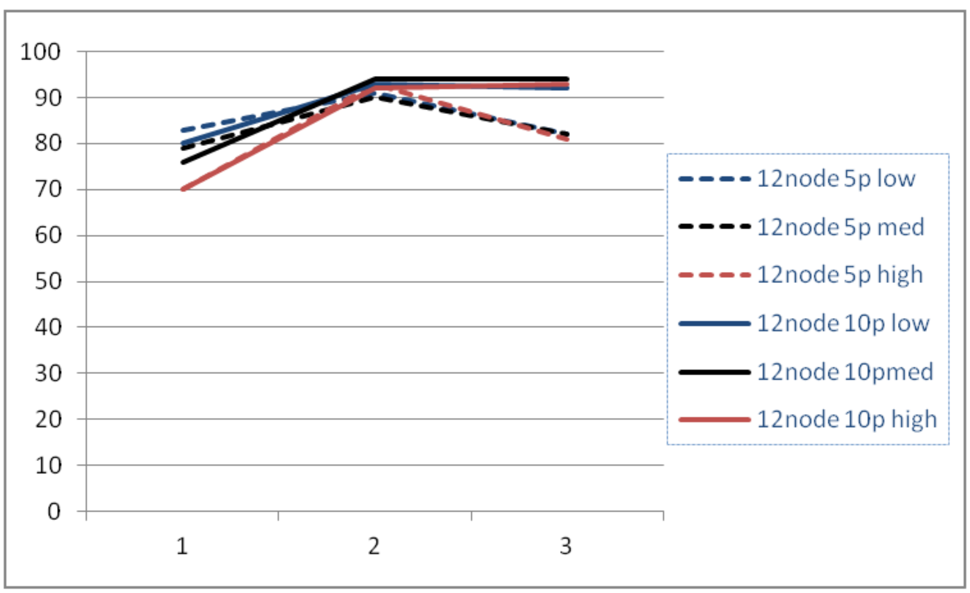

Figure 20: Distance graph for minimum cost vs. average delay under Unsplittable condition 12 node topology

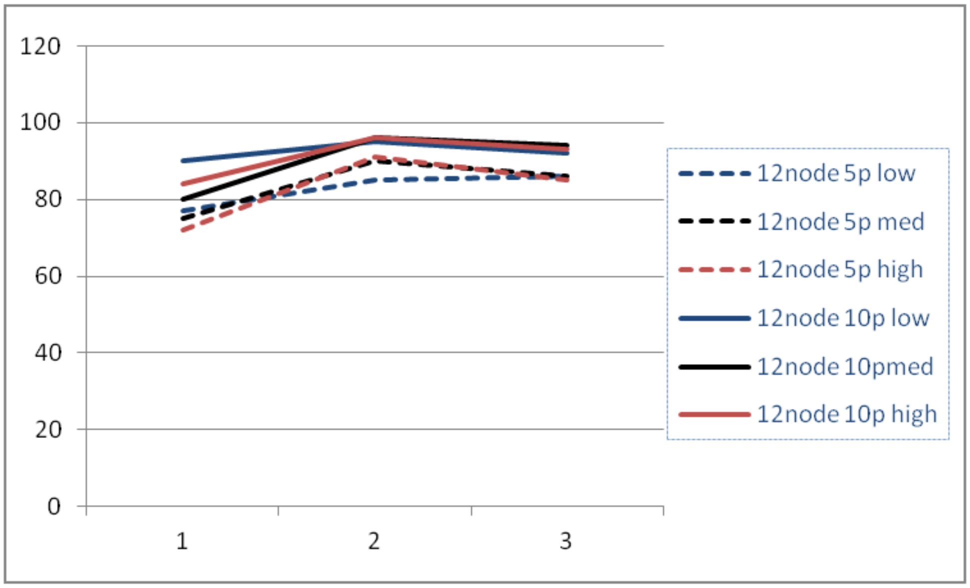

Figure 21: Distance graph for link utilization vs. average delay under Unsplittable condition 12 node topology

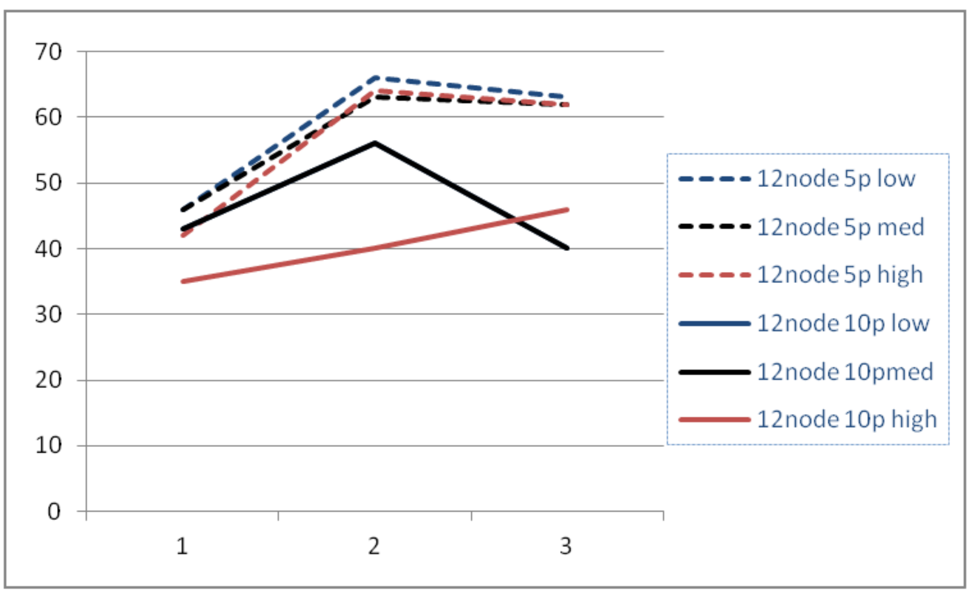

Figure 22: Distance graph for minimum cost vs. link utilization under Splittable condition 12 node topology

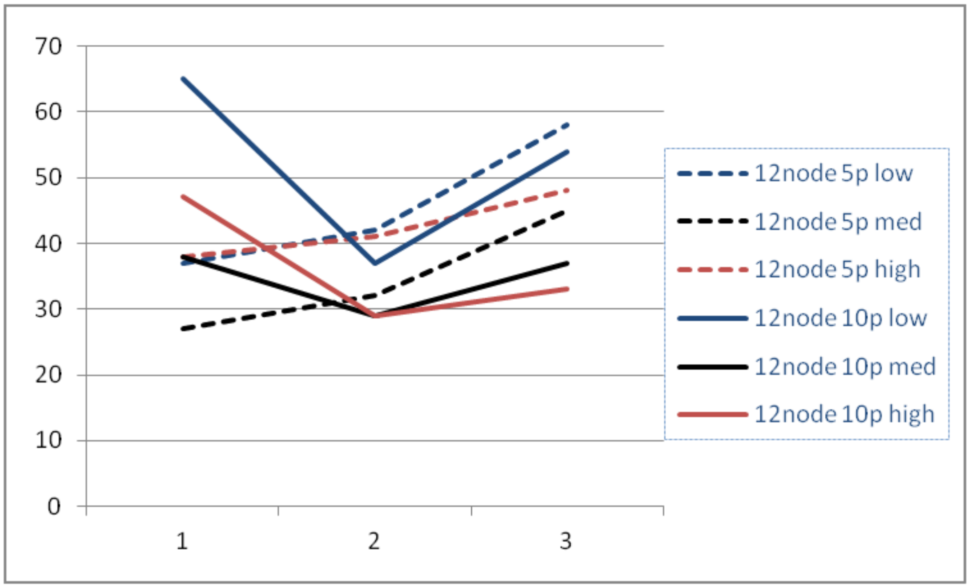

Figure 23: Distance graph for minimum cost vs. average delay under Splittable condition 12 node topology

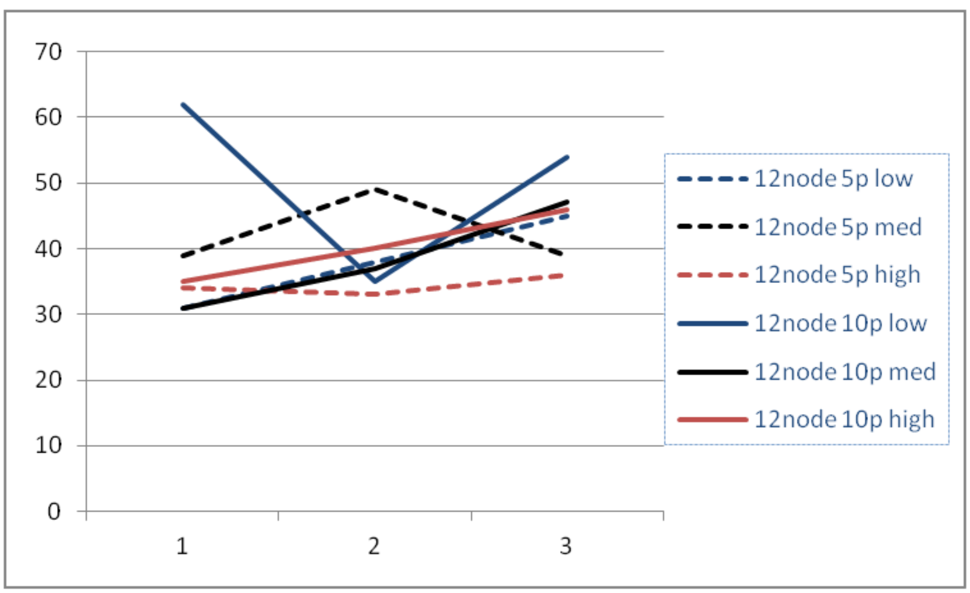

Figure 24: Distance graph for average delay vs. link utilization under Splittable condition 12 node topology

The most used link is 13; so I reduced the capacity of the link by 10% i.e. to 400.

Table 43: Results for 12 node topology with 5 paths for each time period under one link capacity adjustment case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	21731.3			21734.3	0.3991212121			0.427	10.14068917			10.65703
	Non zero path	67	67	67		72	82	79		70	77	82	
	More than one path	1	1	1		6	*15	13		4	*10	**14	
Med	Objective	31559			31562	0.5701731602			0.619	19.57317399			21.57512
	Non zero path	68	68	69		72	82	79		73	79	86	
	More than one path	2	2	3		6	*15	13		7	*12	20	
High	Objective	41474.6			41507.7	0.7412251082			0.7878	38.01289997			41.14285
	Non zero path	70	70	70		72	82	79		75	79	82	
	More than one path	4	4	4		6	*15	13		*8	**11	*15	

Table 44: Results for 12 node topology with 10 paths for each time period under one link capacity adjustment case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	17010.7			17010.7	0.3109208633			0.3395	6.734297			7.21842
	Non zero path	66	66	66		74	83	74		72	79	76	
	More than one path	0	0	0		8	**15	8		6	*13	*9	
Med	Objective	24301			24301	0.4441726619			0.49	12.62385769			13.20712
	Non zero path	66	66	66		74	83	74		72	76	81	
	More than one path	0	0	0		8	**15	8		6	*9	**13	
High	Objective	31735.7			31739.5	0.5774244604			0.637	20.67071411			21.8907
	Non zero path	67	68	67		74	83	74		74	77	83	
	More than one path	1	2	1		*7	**15	8		*7	*10	*16	

In this case one more link is reduced by 10% along with the first one. The 2nd most used link is 9; so I reduced the capacity of the link by 10% i.e. to 353.

Table 45: Results for 12 node topology with 5 paths for each time period under two link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	21731.3			21734.3	0.4431453349			0.469	11.34297824			12.0142
	Non zero path	67	68	67		74	80	80		73	80	83	
	More than one path	1	2	1		8	14	14		7	*13	**15	
Med	Objective	31559			31594	0.6330647641			0.681	23.07326911			24.5161
	Non zero path	69	69	70		74	80	80		72	78	83	
	More than one path	3	3	4		8	14	14		6	*11	**15	
High	Objective	42737.95			42944.2	0.8229841934			0.858	47.58277424			56.4138
	Non zero path	71	72	71		74	80	80		73	80	83	
	More than one path	5	6	5		8	14	14		*6	*13	**15	

Table 46: Results for 12 node topology with 10 paths for each time period under two link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	17010.7			17010.7	0.3345774139			0.3633	7.435957725			7.68989
	Non zero path	66	66	66		75	81	78		86	75	89	
	More than one path	0	0	0		9	*14	**10		#*16	9	**#19	
Med	Objective	24301			24301	0.4779677342			0.52	14.90256062			14.1075
	Non zero path	66	66	66		75	81	78		90	76	89	
	More than one path	0	0	0		9	*14	**10		*#17	10	**#19	
High	Objective	31786.7			31817.5	0.6213580544			0.6776203	23.90439125			26.1665
	Non zero path	67	69	68		75	81	78		89	78	89	
	More than one path	1	3	2		9	*14	**10		*** *19	**10	**** # 15	

In this case one more link is reduced by 10% along with the first and second one. The 3rd most used link is 14; so I reduced the capacity of the link by 10% i.e. to 316.

Table 47: Results for 12 node topology with 5 paths for each time period under 3 link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	21971.2			21981.4	0.4844250221			0.50925	13.46615929			13.965092
	Non zero path	68	69	69		75	78	79		85	79	81	
	More than one path	2	3	3		*8	*11	*12		***16	*12	*14	
Med	Objective	32858			32983	0.6920357459			0.724	28.71733195			30.605163
	Non zero path	70	71	70		75	78	79		69	81	76	
	More than one path	4	5	4		*8	*11	*12		3	15	*9	
High	Objective	45315.375			45836.7	0.8996464696			0.949	67.69522184			82.601979
	Non zero path	71	73	73		75	78	79		68	80	78	
	More than one path	*4	7	7		*8	*11	*12		2	*13	*11	

Table 48: Results for 12 node topology with 10 paths for each time period under 3 link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	17159.2			17159.8	0.3683936862			0.3829	8.434033195			8.79473
	Non zero path	67	67	67		75	76	75		90	77	90	
	More than one path	1	1	1		**7	10	*8		**#17	11	*#16	
Med	Objective	24912			24932	0.5262766945			0.543	16.60256315			17.1117
	Non zero path	67	69	68		75	76	75		90	74	86	
	More than one path	1	3	2		**7	10	*8		****20	8	*17	
High	Objective	32908.8			32987.5	0.6841597029			0.7098	28.68981472			30.3282
	Non zero path	68	71	69		75	76	75		91	78	89	
	More than one path	2	5	3		**7	10	*8		***#17	12	***23	

Decrease of objective from 5 to 10 paths is 21, 24 and 28 in low medium and high demands respectively.

4.6 Wrap Grid Topology

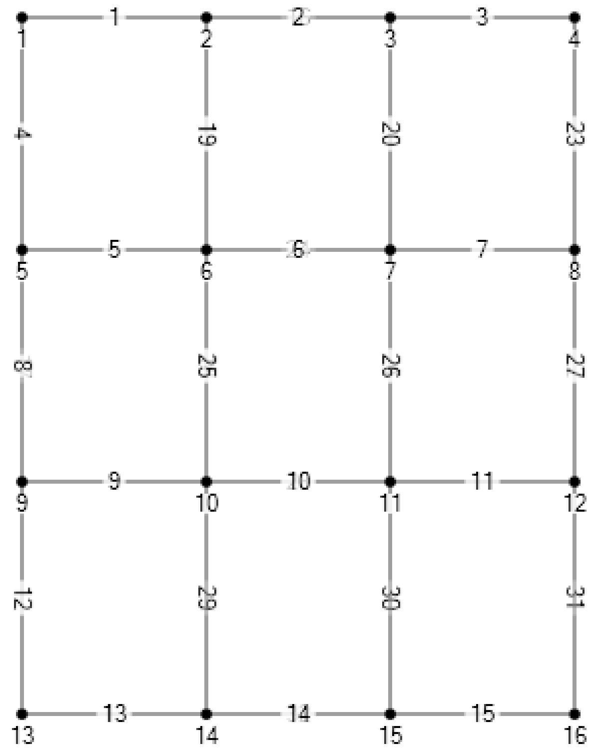

Figure 25: Wrap grid topology diagram

All the links have a maximum capacity of 4000.

Table 49: Results for wrap grid topology with 5 paths for each time period under normal case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	110443.9			110443.9	0.4176433333			0.42087	12.48518141			12.55465
	Non zero path	120	120	120		127	131	132		133	142	131	
	More than one path	0	0	0		7	*10	**10		13	*21	11	
Med	Objective	163027			163027	0.5995			0.59663	24.95949034			25.30467
	Non zero path	121	121	121		127	131	132		133	138	131	
	More than one path	1	1	1		7	*10	**10		13	*17	11	
High	Objective	219711.2			219945.7	0.7756233333			0.77935	46.53047728			47.12674
	Non zero path	122	123	123		127	131	132		130	138	134	
	More than one path	2	3	3		7	*10	**10		10	*17	14	

Table 50: Results of distance for wrap grid topology with 5 paths under low demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	110	182	152	444	145	169	162	476
HD	0.9167	1.5167	1.2667		1.2083	1.4083	1.35	
H	0.1833	0.3033	0.2533		0.2417	0.2817	0.27	
Minimum cost vs. average delay	195	158	151	504	133	120	107	360
HD	1.625	1.3167	1.2583		1.1083	1	0.8917	
H	0.325	0.2633	0.2517		0.2217	0.2	0.1783	
Average delay vs. link utilization	175	162	153	490	114	95	103	312
HD	1.4583	1.35	1.275		0.95	0.7917	0.8512	
H	0.2917	0.27	0.255		0.19	0.1583	0.1702	

Table 51: Results of distance for wrap grid topology with 5 paths under medium demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	102	146	110	358	150	142	131	423
HD	0.85	1.2167	0.9167		1.25	1.1833	1.0917	
H	0.17	0.2433	0.1833		0.25	0.2367	0.2183	
Minimum cost vs. average delay	154	159	149	462	114	85	80	279
HD	1.2833	1.325	1.2417		0.95	0.7083	0.6667	
H	0.2567	0.265	0.2483		0.19	0.1417	0.1333	
Average delay vs. link utilization	156	161	153	470	124	79	91	294
HD	1.3	1.3417	1.275		1.0333	0.6583	0.7583	
H	0.26	0.2683	0.255		0.2067	0.1317	0.1517	

Table 52: Results of distance for wrap grid topology with 5 paths under high demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	92	96	80	268	121	116	117	354
HD	0.7667	0.8	0.6667		1.0083	0.9667	0.975	
H	0.1533	0.16	0.1333		0.2017	0.1933	0.195	
Minimum cost vs. average delay	157	163	150	470	118	75	63	256
HD	1.3083	1.3583	1.25		0.9833	0.625	0.525	
H	0.2617	0.2717	0.25		0.1967	0.125	0.105	
Average delay vs. link utilization	153	161	154	468	111	69	82	262
HD	1.275	1.3417	1.2833		0.925	0.575	0.6833	
H	0.255	0.2683	0.2567		0.185	0.115	0.1367	

Table 53: Results for wrap grid topology with 10 paths for each time period under normal case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	103239.5			103239.5	0.404425			0.40083	12.02980826			12.1600
	Non zero path	120	120	120		128	134	135		149	144	131	
	More than one path	0	0	0		8	***11	** 12		** 27	24	11	
Med	Objective	156439			156441	0.5726198071			0.58	23.52221098			23.90301
	Non zero path	121	121	121		128	134	135		148	140	137	
	More than one path	1	1	1		8	***11	** 12		***25	*19	17	
High	Objective	212083.5			212189.9	0.7444057493			0.75172	43.61307054			44.595035
	Non zero paths	122	123	123		128	134	135		148	143	135	
	More than one path	2	3	3		8	***11	** 12		****24	*22	15	

Table 54: Results of distance for wrap grid topology with 10 paths under low demand condition

Routing models	Unsplittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	152	188	184	524	174	188	185	547
HD	1.2667	1.5667	1.5333		1.45	1.5667	1.5417	
H	0.1267	0.1567	0.1533		0.145	0.1567	0.1542	
Minimum cost vs. average delay	212	171	166	549	205	140	125	470
HD	1.7667	1.425	1.3833		1.7083	1.1667	1.0417	
H	0.1767	0.1425	0.1383		0.1708	0.1167	0.1042	
Average delay vs. link utilization	170	173	166	509	147	100	106	353
HD	1.4167	1.4417	1.3833		1.225	0.8333	0.8833	
H	0.1417	0.1442	0.1383		0.1225	0.0833	0.0883	

Table 55: Results of distance for wrap grid topology with 10 paths under medium demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	100	150	144	394	133	155	136	424
HD	0.8333	1.25	1.2		1.1083	1.2917	1.1333	
H	0.0833	0.125	0.12		0.1108	0.1292	0.1133	
Minimum cost vs. average delay	192	168	165	525	166	109	80	355
HD	1.6	1.4	1.375		1.3833	0.9083	0.6667	
H	0.16	0.14	0.1375		0.1383	0.0908	0.0667	
Average delay vs. link utilization	174	168	165	507	159	96	100	355
HD	1.45	1.4	1.375		1.325	0.8	0.8333	
H	0.145	0.14	0.1375		0.1325	0.08	0.0833	

Table 56: Results of distance for wrap grid topology with 10 paths under high demand condition

Routing models	Non Splittable				Splittable			
	T1	T2	T3	Sum	T1	T2	T3	Sum
Minimum cost vs. link utilization	112	114	122	348	112	109	114	335
HD	0.9333	0.95	1.0167		0.9333	0.9083	0.95	
H	0.0933	0.095	0.1017		0.0933	0.0908	0.095	
Minimum cost vs. Average delay	198	169	164	531	152	70	60	282
HD	1.65	1.4083	1.3667		1.2667	0.5833	0.5	
H	0.165	0.1408	0.1367		0.1267	0.0583	0.05	
Average delay vs. Link utilization	212	171	164	547	162	83	90	335
HD	1.7667	1.425	1.3667		1.35	0.6917	0.75	
H	0.1767	0.1425	0.1367		0.135	0.0692	0.075	

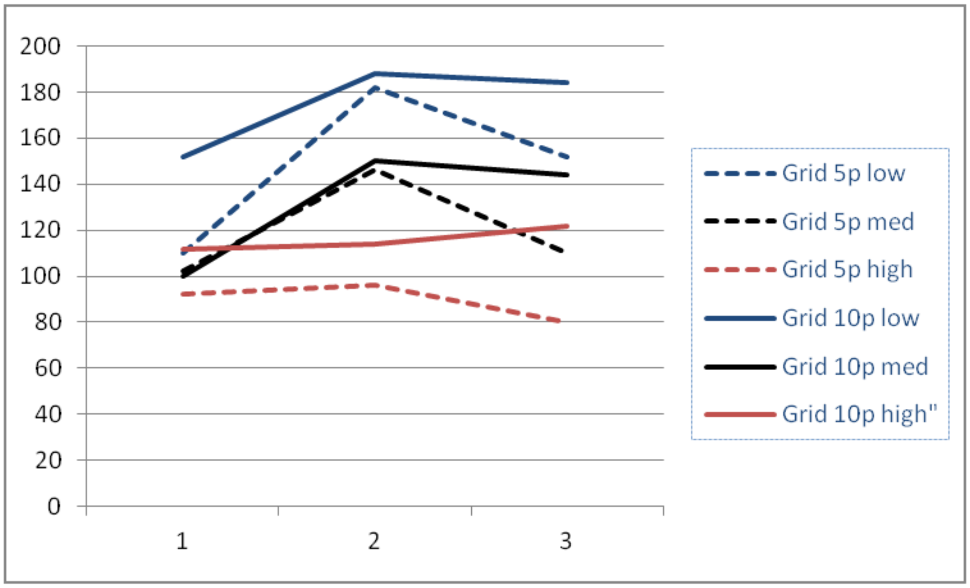

Figure 26: Distance graph for minimum cost vs. link utilization under Non Splittable condition wrap grid topology

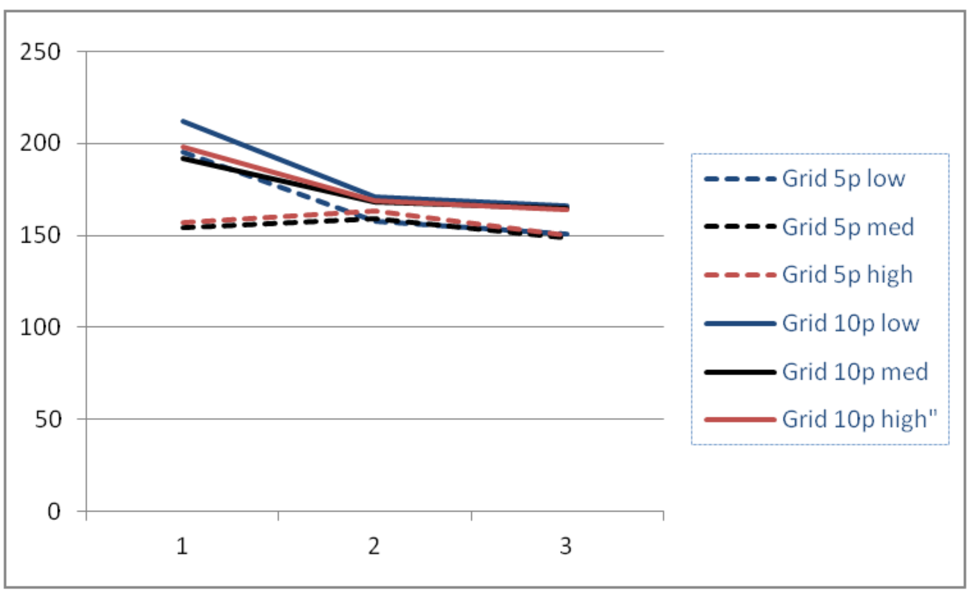

Figure 27: Distance graph for minimum cost vs. average delay under Non Splittable condition wrap grid node topology

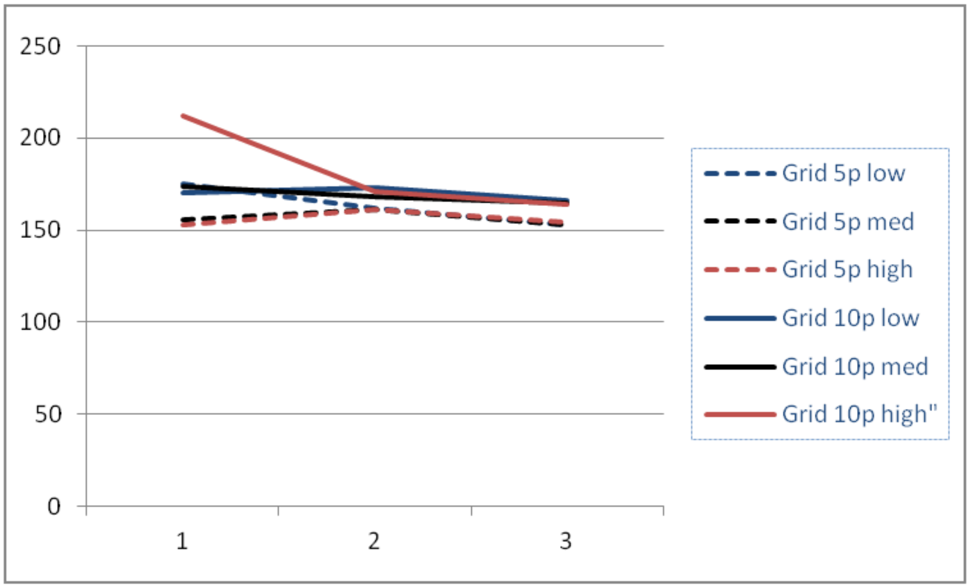

Figure 28: Distance graph for average delay vs. link utilization under Non Splittable condition wrap grid node topology

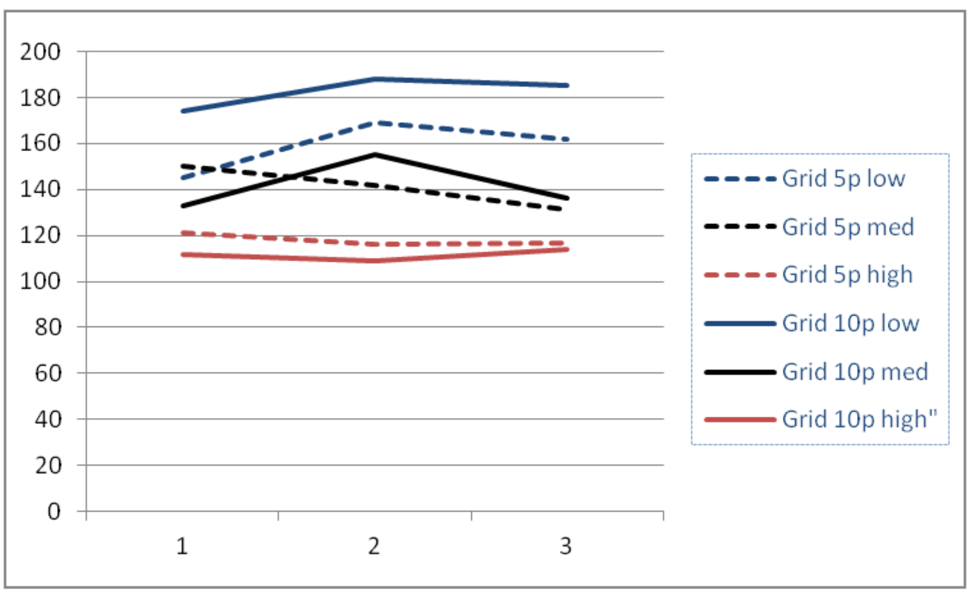

Figure 29: Distance graph for minimum cost vs. link utilization under Splittable condition wrap grid node topology

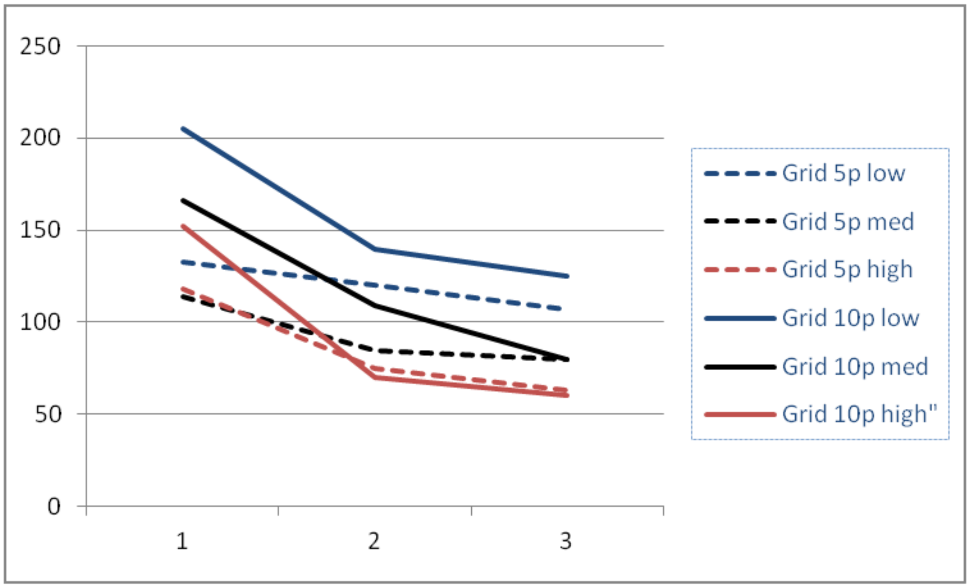

Figure 30: Distance graph for minimum cost vs. average delay under Splittable condition wrap grid node topology

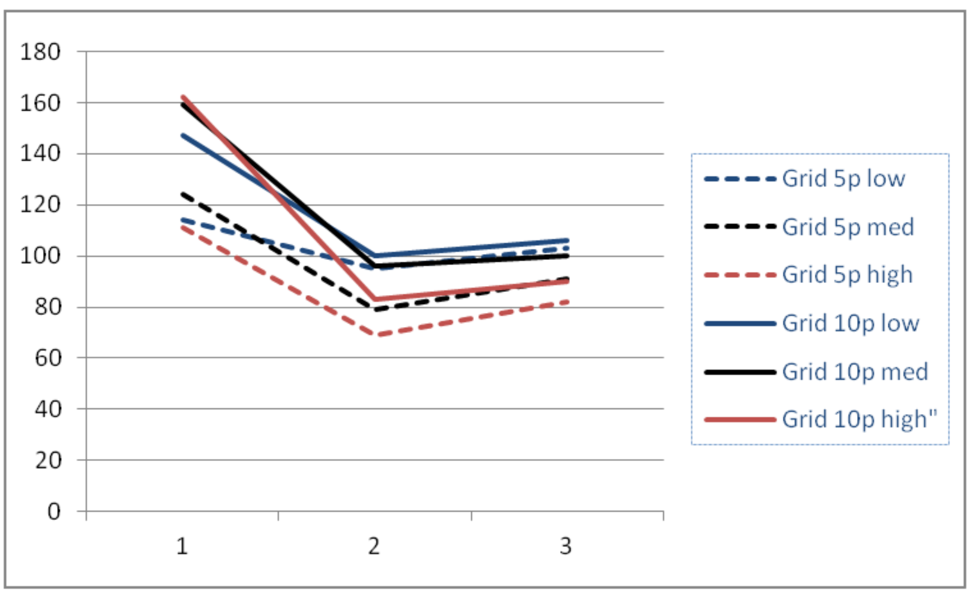

Figure 31: Distance graph for average delay vs. link utilization under Splittable condition wrap grid node topology

Most utilized link is 5; the capacity is reduced by 10%.

Table 57: Results for wrap grid topology with 5 paths for each time period under one link capacity adjustment case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	0.4395662241			0.442225	110443.9			110443.9	12.88201435			13.006945
	Non zero path	120	120	120		126	130	129		135	142	130	
	More than one path	0	0	0		6	*9	9		***12	*21	10	
Med	Objective	163027			163027	0.6279517487			0.63175	26.4562291			26.668395
	Non zero path	121	121	121		126	130	129		134	135	135	
	More than one path	1	1	1		6	*9	9		14	*15	15	
High	Objective	221633.9			221633.9	0.8163372733			0.82225	50.7843057			51.5915225
	Non zero path	123	123	123		126	130	129		134	138	134	
	More than one path	3	3	3		6	*9	9		14	*17	14	

Table 58: Results for wrap grid topology with 5 paths for each time period under one link capacity adjustment case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	103239.5			103239.5	0.4213021108			0.424025	12.40610677			12.5633562 5
	Non zero path	120	120	120		125	132	132		149	142	131	
	More than one path	0	0	0		5	12	12		#***2 5	***1 9	11	
Med	Objective	156439			156439	0.6018601583			0.6065	24.60437509			24.926555
	Non zero path	121	122	123		125	132	132		149	139	135	
	More than one path	1	2	3		5	12	12		****2 5	*18	15	
High	Objective	214055.4			214186.7	0.7824182058			0.78845	46.6443328			48.27727
	Non zero path	123	123	124		125	132	132		148	143	137	
	More than one path	3	3	4		5	12	12		****2 4	#**1 9	17	

Second most utilized link is 22; the capacity is reduced by 10% along with link 5.

Table 59: Results for wrap grid topology with 5 paths for each time period under 2 link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	110443.9			110443.9	0.4694375			0.4697	13.29592903			13.3778725
	Non zero path	120	120	120		124	128	128		132	137	134	
	More than one path	0	0	0		4	8	*7		12	17	14	
Med	Objective	163381			163391	0.670625			0.6746875	28.25218192			28.463135
	Non zero paths	122	123	123		124	128	128		135	136	130	
	More than one paths	2	3	3		4	8	*7		15	16	10	
High	Objective	228109.3			228182.5	0.8718125			0.872625	59.35047041			60.594895
	Non zero paths	123	123	123		124	128	128		129	129	135	
	More than one paths	3	3	3		4	8	*7		9	9	**13	

Table 60: Results for wrap grid topology with 10 paths for each time period under 2 link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	103239.5			103239.5	0.4276887715			0.431812	12.78044293			12.8661287
	Non zero path	120	120	120		125	132	133		151	137	137	
	More than one path	0	0	0		5	12	*12		#** 27	17	17	
Med	Objective	156439			156439	0.6109839593			0.6175	26.19112549			26.52317
	Non zero path	121	122	121		125	133	132		148	138	134	
	More than one path	1	2	1		5	13	12		** 26s	*17	14	
High	Objective	214055.4			214184.1	0.7942791471			0.79885	51.26746064			52.37165
	Non zero path	123	123	124		125	133	132		146	138	135	
	More than one path	3	3	4		5	13	12		26	*17	15	

Third most utilized link is 15; the capacity is reduced by 10% along with link 5 and 22.

Table 61: Results for wrap grid topology with 5 paths for each time period under three link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	110443.9			110443.9	0.4960125013			0.497	13.76849397			13.8903
	Non zero path	120	120	120		124	126	127		135	135	136	
	More than one path	0	0	0		4	6	7		15	15	*15	
Med	Objective	163381			163385	0.7085892876			0.7096	29.70195429			29.9302
	Non zero path	122	123	124		124	126	127		132	132	131	
	More than one paths	2	3	4		4	6	7		12	12	11	
High	Objective	229793.5			230337.9	0.9211660739			0.9213	68.41859974			69.7655
	Non zero path	123	124	124		124	126	127		132	126	134	
	More than one paths	3	4	4		4	6	7		12	6	14	

Table 62: Results for wrap grid topology with 10 paths for each time period under three link capacity adjustments case

Demand		Minimum cost routing				Load balancing				Average delay			
		Splittable			UNS	Splittable			UNS	Splittable			UNS
Low	Objective	103239.5			103239.5	0.4423563351			0.4481	13.22415769			13.4164
	Non Zero Path	120	120	120		128	131	131		145	136	135	
	MTOP	0	0	0		8	11	11		*24	16	15	
Med	Objective	156439			156439	0.6319376215			0.6346875	27.34010436			27.5510
	Non Zero Path	121	121	122		128	131	131		147	141	132	
	More than one path	1	1	2		8	11	11		*** 24	#* 18	12	
High	Objective	214844.4			214966.7	0.821518908			0.824525	54.94128737			55.7087
	Non Zero Path	124	124	124		128	131	131		145	137	135	
	More than one path	4	4	4		8	11	11		** 23	*16	15	

Observations for distance

1. In 3 out of the 4 topologies we have tested; Unsplittable cases have more paths being differed than the Splittable condition.
2. More number of paths we have more the difference between the routing objectives both in Splittable and Unsplittable condition.
3. The difference is usually the highest during time periods 1 and 2. We can infer from this that the models tend to use more paths either in time period 1 or time period 2.
4. The most common pattern we have observed is

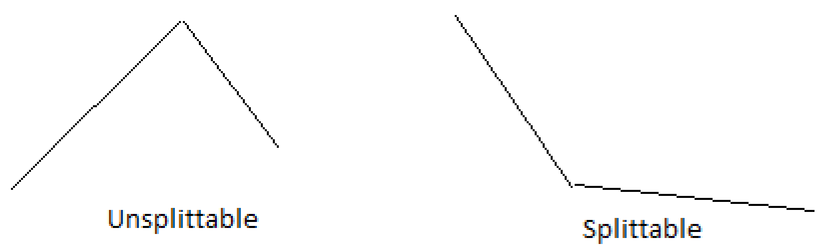

Table 63: Results of comparison of objective with less and more paths with minimum cost routing model

Topology	Number of paths				
	Less paths		More paths		Decrease in percent
7 node	Low	21324	Low	21324	0
	Med	28787.4	Med	28787.4	0
	High	37166.4	High	37166.4	0
NSF	low	762849.92	Low	591195.94	22
	Med	978037.08	Med	757961.78	22
	High	1222515.9	High	947429.39	22
12 node	Low	21362.6	Low	17010.7	20
	Med	30518	Med	24301	20
	High	39786.55	High	31645.6	20
Wrap Grid	Low	110443.9	Low	103239.5	6.5
	Med	163027	Med	156439	4
	High	219711.2	High	212083.5	3.47

Decrease in objective with increased path options for minimum cost.

Table 64: Results of comparison of objective with less and more paths with Average delay routing model

Topology	Number of paths				
	Less paths		More paths		Decrease in percent
7 node	Low	5.04	Low	5.04	0
	Med	9.07	Med	9.07	0
	High	16.04	High	16.04	0
NSF	Low	8.65	Low	5.67	34
	Med	14.08	Med	8.20	41
	High	24.07	High	12.84	46
12 node	Low	9.01	Low	6.38	29
	Med	16.97	Med	11.89	29
	High	31.18	High	19.07	38.83
Wrap Grid	Low	12.48	Low	12.08	3.2
	Med	24.95	Med	23.52	5.3
	High	46.53	High	43.61	6.27

CHAPTER 5

PRE ANALYSIS

5.1 Common links

To understand the results in more detail we verify how many links are common in each set of paths that we have chosen for our cases.

Table 65: Number of occurrences of each link in the paths we have chosen for 7 node topology

	3 Paths		6 Paths	
Link number	Occurrence(O)	O/Paths	Occurrence	O/Paths
1	15	0.238095238	49	0.388889
2	16	0.253968254	48	0.380952
3	9	0.142857143	30	0.238095
4	15	0.238095238	38	0.301587
5	8	0.126984127	23	0.18254
6	18	0.285714286	52	0.412698
7	16	0.253968254	53	0.420635
8	23	0.365079365	40	0.31746
9	17	0.26984127	34	0.269841
10	16	0.253968254	53	0.420635
11	15	0.238095238	43	0.34127
12	13	0.206349206	29	0.230159
13	3	0.047619048	21	0.166667
14	3	0.047619048	21	0.166667

Heaviest utilized link does not correspond to the maximum number of link occurrences. First three most utilized links are 4,2 and 9 respectively.

Table 66: Number of occurrences of each link in the paths we have chosen for NSF topology

	5 paths		6 Paths	
link number	occurrence(O)	O/Paths	Occurrence(O)	O/Paths
1	125	0.274725	251	0.275824
2	117	0.257143	239	0.262637
3	121	0.265934	272	0.298901
4	133	0.292308	264	0.29011
5	125	0.274725	255	0.28022
6	136	0.298901	300	0.32967
7	142	0.312088	308	0.338462
8	120	0.263736	241	0.264835
9	161	0.353846	346	0.38022
10	106	0.232967	244	0.268132
11	137	0.301099	291	0.31978
12	104	0.228571	210	0.230769
13	138	0.303297	286	0.314286
14	106	0.232967	254	0.279121
15	105	0.230769	208	0.228571
16	124	0.272527	274	0.301099
17	102	0.224176	218	0.23956
18	103	0.226374	202	0.221978
19	108	0.237363	219	0.240659
20	95	0.208791	192	0.210989
21	87	0.191209	187	0.205495

Heaviest utilized link does not correspond to the maximum number of link occurrences. First three most utilized links are 11, 9 and 4 respectively.

Table 67: Number of occurrences of each link in the paths we have chosen for 12 node topology

	5 Paths		10 Paths	
Link number	Occurrence(O)	O/Paths	Occurrence(O)	O/Paths
1	121	0.366667	268	0.406061
2	85	0.257576	192	0.290909
3	116	0.351515	282	0.427273
4	78	0.236364	167	0.25303
5	35	0.106061	52	0.078788
6	117	0.354545	264	0.4
7	60	0.181818	128	0.193939
8	127	0.384848	269	0.407576
9	75	0.227273	139	0.210606
10	62	0.187879	135	0.204545
11	108	0.327273	193	0.292424
12	61	0.184848	144	0.218182
13	52	0.157576	102	0.154545
14	84	0.254545	145	0.219697
15	59	0.178788	131	0.198485
16	38	0.115152	95	0.143939
17	57	0.172727	107	0.162121
18	68	0.206061	140	0.212121
19	63	0.190909	143	0.216667
20	34	0.10303	105	0.159091
21	54	0.163636	143	0.216667
22	53	0.160606	111	0.168182
23	42	0.127273	122	0.184848
24	42	0.127273	104	0.157576
25	43	0.130303	100	0.151515

Heaviest utilized link does not correspond to the maximum number of link occurrences. First three most utilized links are 13, 9 and 14 respectively.

Table 68: Number of occurrences of each link in the paths we have chosen for wrap grid topology

	5 Paths		10 Paths	
Link number	Occurrence(O)	O/Paths	Occurrence(O)	O/Paths
1	477	0.795	1072	0.893333
2	420	0.7	915	0.7625
3	381	0.635	839	0.699167
4	421	0.701667	936	0.78
5	342	0.57	688	0.573333
6	341	0.568333	699	0.5825
7	301	0.501667	652	0.543333
8	215	0.358333	402	0.335
9	268	0.446667	564	0.47
10	299	0.498333	569	0.474167
11	271	0.451667	533	0.444167
12	277	0.461667	521	0.434167
13	279	0.465	525	0.4375
14	326	0.543333	584	0.486667
15	342	0.57	628	0.523333
16	67	0.111667	77	0.064167
17	54	0.09	59	0.049167
18	63	0.105	92	0.076667
19	48	0.08	54	0.045
20	78	0.13	124	0.103333
21	92	0.153333	178	0.148333
22	375	0.625	732	0.61
23	186	0.31	370	0.308333
24	65	0.108333	98	0.081667
25	121	0.201667	218	0.181667
26	103	0.171667	227	0.189167
27	172	0.286667	395	0.329167
28	136	0.226667	318	0.265
29	164	0.273333	372	0.31
30	161	0.268333	384	0.32
31	136	0.226667	366	0.305
32	100	0.166667	230	0.191667

Heaviest utilized link does not correspond to the maximum number of link occurrences. First three most utilized links are 5, 22 and 15 respectively. So from these results we can conclude that number of occurrences of a link is not an indication of its usage by the paths.

5.2 Variance of the average delay

We have shown the variance of the average delay of the 7 node topology and NSF topology. The other two topologies have similar results.

Variance for 7 node topology

Table 69: Results of variance for 7 node topology under normal case in low and Splittable condition

	3 Paths		6 Paths	
Links	value of z	average delay	value of z	average delay
z_1	1000.005	0.333335	922.505	0.307501667
z_2	1648.005	0.549335	2210.5	0.736833333
z_3	1000.005	0.333335	1000	0.333333333
z_4	2934.975	0.978325	2934.97	0.978323333
z_5	326	0.108666667	403.5	0.1345
z_6	1000.005	0.333335	1000	0.333333333
z_7	1000.005	0.333335	1000	0.333333333
z_8	2244.96	0.74832	1682.46	0.56082
z_9	1000.005	0.333335	1000	0.333333333
z_10	1000.005	0.333335	1000	0.333333333
z_11	867.99	0.28933	867.99	0.28933
z_12	269.99	0.089996667	754.99	0.251663333
z_13	485	0.161666667	0	0
z_14	485	0.161666667	0	0
	Variance	0.0609853	Variance	0.068985735

Table 70: Results of variance for 7 node topology under normal case in medium low and non Splittable condition

	3 Paths		6 Paths	
Links	value of z	average delay	value of z	average delay
z_1	914	0.304666667	914	0.304666667
z_2	1389.99	0.46333	1389.99	0.46333
z_3	859	0.286333333	859	0.286333333
z_4	3357.99	1.11933	3357.99	1.11933
z_5	271	0.090333333	271	0.090333333
z_6	913	0.304333333	913	0.304333333
z_7	917	0.305666667	917	0.305666667
z_8	2751.99	0.91733	2751.99	0.91733
z_9	962	0.320666667	962	0.320666667
z_10	917	0.305666667	917	0.305666667
z_11	993	0.331	993	0.331
z_12	880	0.293333333	880	0.293333333
z_13	0	0	0	0
z_14	0	0	0	0
	Variance	0.096470754	Variance	0.096470754

Table 71: Results of variance for 7 node topology under normal case in medium and Splittable condition

	3 Paths		6 Paths	
Links	value of z	average delay	value of z	average delay
z_1	2830.7293	0.943576429	1478.03	0.492676667
z_2	3705.5293	1.235176429	3705.53	1.235176667
z_3	2712.2443	0.904081429	2712.25	0.904083333
z_4	3999.9857	1.333328571	3999.98	1.333326667
z_5	400.605	0.133535	851.505	0.283835
z_6	2211.99	0.73733	2325.39	0.77513
z_7	1000.005	0.333335	1000	0.333333333
z_8	3999.9857	1.333328571	3999.98	1.333326667
z_9	2403.375	0.801125	2289.98	0.763326667
z_10	1000.005	0.333335	1000	0.333333333
z_11	1000.005	0.333335	1000	0.333333333
z_12	847.455	0.282485	847.455	0.282485
z_13	0	0	0	0
z_14	0	0	0	0
	Variance	0.230303684	Variance	0.213417315

Table 72: Results of variance for 7 node topology under normal case in medium and non Splittable condition

	3 Paths		6 Paths	
Links	value of z	average delay	value of z	average delay
z_1	3366.24	1.12208	3665.94	1.22198
z_2	4803.5	1.601166667	3941.34	1.31378
z_3	3143.49	1.04783	3143.49	1.04783
z_4	3568.74	1.18958	3568.74	1.18958
z_5	365.85	0.12195	265.95	0.08865
z_6	1053.69	0.35123	1053.69	0.35123
z_7	1005.09	0.33503	1005.09	0.33503
z_8	3459.39	1.15313	3759.09	1.25303
z_9	3414.84	1.13828	3414.84	1.13828
z_10	1005.09	0.33503	1005.09	0.33503
z_11	1146.84	0.38228	1146.84	0.38228
z_12	896.4	0.2988	896.4	0.2988
z_13	0	0	0	0
z_14	0	0	0	0
	Variance	0.28269524	Variance	0.266196643

Table 73: Results of variance for 7 node topology under normal case in high and Splittable condition

	3 Paths		6 Paths	
Links	Value of z	Average delay	value of z	average delay
z_1	2898.3857	0.966129	1485.36	0.49512
z_2	3999.9857	1.333329	3999.98	1.333326667
z_3	3999.9857	1.333329	3999.98	1.333326667
z_4	8965.0143	2.988338	8965.04	2.988346667
z_5	921.4	0.307133	2177.21	0.725736667
z_6	3979.5857	1.326529	3979.58	1.326526667
z_7	3999.9857	1.333329	3999.98	1.333326667
z_8	5727.0286	1.90901	5727.07	1.909023333
z_9	3647.1793	1.215726	3647.18	1.215726667
z_10	3999.9857	1.333329	3999.98	1.333326667
z_11	1000.005	0.333335	1000	0.333333333
z_12	807.905	0.269302	807.905	0.269301667
z_13		0	0	0
z_14		0	0	0
	Variance	0.677452	Variance	0.666193109

Table 74: Results of variance for 7 node topology with 3 paths for each time period under normal case in high and non Splittable condition

	3 Paths		6 Paths	
Links	value of z	average delay	value of z	average delay
z_1	1595.49	0.53183	1105.89	0.36863
z_2	2697.09	0.89903	3910.89	1.30363
z_3	7647	2.549	5590.01	1.863336667
z_4	5318	1.772666667	7375.01	2.458336667
z_5	3161.19	1.05373	3757.89	1.25263
z_6	3834.39	1.27813	3834.39	1.27813
z_7	3854.79	1.28493	3854.79	1.28493
z_8	10554	3.518	6508.01	2.169336667
z_9	3747.69	1.24923	3492.69	1.16423
z_10	3854.79	1.28493	3854.79	1.28493
z_11	1044.69	0.34823	1299.69	0.43323
z_12	822.8	0.274266667	907.8	0.3026
z_13		0	0	0
z_14		0	0	0
	Variance	0.963378667	Variance	0.597816036

Variance for NSF topology

Table 75: Results of variance for NSF node topology under normal case in low and Splittable condition

	5 Paths		10 Paths	
Links	value of z	average delay	value of z	average delay
z_1	32158.951	0.5847082	18333.4	0.333334545
z_2	18333.4	0.333334545	12173.241	0.221331655
z_3	18333.4	0.333334545	12888.563	0.234337509
z_4	33174.796	0.603178109	18333.4	0.333334545
z_5	18591.286	0.338023382	12685.1785	0.230639609
z_6	16210.903	0.294743704	17126.462	0.311390218
z_7	20509.276	0.372895927	18043.085	0.328056091
z_8	18333.4	0.333334545	10894.4945	0.198081718
z_9	46417.137	0.843947948	34356.3955	0.624661736
z_10	16110.388	0.292916151	13070.831	0.237651473
z_11	66666.142	1.212111688	18333.4	0.333334545
z_12	16572.896	0.301325382	14841.5405	0.269846191
z_13	18131.396	0.329661758	18333.4	0.333334545
z_14	17210.205	0.312912831	14367.6685	0.261230336
z_15	17175.359	0.312279255	8941.0445	0.162564445
z_16	18333.4	0.333334545	17002.475	0.309135909
z_17	18333.4	0.333334545	11836.494	0.215208982
z_18	11018.384	0.200334268	4004.454	0.072808255
z_19	17227.251	0.313222756	13136.48	0.238845091
z_20	18333.4	0.333334545	10102.089	0.183674345
z_21	7212.262	0.131132036	13318.468	0.242153964
	Variance	0.056768057	Variance	0.011127828

Table 76: Results of variance for NSF node topology under normal case in low and non Splittable condition

	5 Paths		10 Paths	
Links	value of z	average delay	value of z	average delay
z_1	28884.706	0.525176473	18427.435	0.335044273
z_2	17540.922	0.318925855	13182.711	0.239685655
z_3	18218.663	0.331248418	12832.813	0.233323873
z_4	36172.741	0.6576862	18326.223	0.333204055
z_5	18867.586	0.343047018	15054.059	0.273710164
z_6	17252.656	0.313684655	11941.141	0.217111655
z_7	20232.976	0.367872291	18067.253	0.328495509
z_8	18609.7	0.338358182	13263.375	0.241152273
z_9	46888.66	0.852521091	27099.679	0.492721436
z_10	16067.951	0.292144564	13293.538	0.241700691
z_11	66194.62	1.203538545	18410.971	0.334744927
z_12	17559.185	0.319257909	16747.74	0.304504364
z_13	17535.156	0.318821018	18957.265	0.344677545
z_14	18298.267	0.332695764	17150.252	0.311822764
z_15	18161.648	0.330211782	11482.455	0.208771909
z_16	16592.842	0.301688036	16665.86	0.303015636
z_17	18305.907	0.332834673	16102.717	0.292776673
z_18	10422.144	0.189493527	4004.454	0.072808255
z_19	17517.193	0.318494418	10660.82	0.193833091
z_20	18100.526	0.329100473	7197.714	0.130867527
z_21	7212.262	0.131132036	13526.423	0.245934964
	Variance	0.056791439	Variance	0.007532223

Table 77: Results of variance for NSF node topology under normal case in medium and Splittable condition

	5 Paths		10 Paths	
Links	value of z	average delay	value of z	average delay
z_1	60649.24	1.102713455	22628.965	0.411435727
z_2	23700.777	0.430923221	18333.4	0.333334545
z_3	27334.283	0.496986961	18333.4	0.333334545
z_4	72665.41	1.321189273	26839.15	0.487984545
z_5	45154.197	0.820985403	18333.4	0.333334545
z_6	22313.384	0.405697896	18333.4	0.333334545
z_7	57165.73	1.039376909	18333.4	0.333334545
z_8	44823.627	0.814975039	16037.67	0.291594
z_9	65617.78	1.193050545	54904.645	0.998266273
z_10	18333.4	0.333334545	18333.4	0.333334545
z_11	66666.143	1.212111688	45352.75	0.824595455
z_12	18333.4	0.333334545	18333.4	0.333334545
z_13	39574.03	0.719527818	37776.985	0.686854273
z_14	50206.433	0.912844234	18333.4	0.333334545
z_15	20650.6	0.375465455	11117.705	0.202140091
z_16	27449.65	0.499084545	18333.4	0.333334545
z_17	28120.253	0.511277325	18333.4	0.333334545
z_18	18333.4	0.333334545	4978.4975	0.090518136
z_19	21419.14	0.389438909	18333.4	0.333334545
z_20	18333.4	0.333334545	11574.8625	0.210452045
z_21	14715.29	0.267550727	18226.6925	0.331394409
	Variance	0.120510716	Variance	0.043134725

Table 78: Results of variance for NSF node topology under normal case in medium and non Splittable condition

	5 Paths		10 Paths	
Links	value of z	average delay	value of z	average delay
z_1	60481.24	1.099658909	28060.33	0.510187818
z_2	31359.67	0.570175818	19325.35	0.35137
z_3	19507.39	0.354679818	18608.86	0.338342909
z_4	72833.41	1.324243818	24901.6	0.452756364
z_5	45254.5	0.822809091	20925.55	0.380464545
z_6	23666.14	0.430293455	18274.55	0.332264545
z_7	57165.73	1.039376909	18221.95	0.331308182
z_8	44923.93	0.816798727	16901.72	0.307304
z_9	65612.98	1.192963273	57803.68	1.050976
z_10	30661.27	0.557477636	18727.93	0.340507818
z_11	66570.64	1.210375273	36367.75	0.661231818
z_12	18350.44	0.333644364	16945.66	0.308102909
z_13	42947.65	0.780866364	41366.65	0.752120909
z_14	43455.43	0.790098727	18063.24	0.328422545
z_15	20667.64	0.375775273	11839.72	0.215267636
z_16	27427.81	0.498687455	19066.39	0.346661636
z_17	20588.35	0.374333636	19308.82	0.351069455
z_18	18255.32	0.331914909	3923.62	0.071338545
z_19	18820.6	0.342192727	17558.24	0.319240727
z_20	18545.8	0.337196364	10941.38	0.198934182
z_21	15761.75	0.286577273	18438.31	0.335242
	Variance	0.117990149	Variance	0.043517835

Table 79: Results of variance for NSF node topology under normal case in high and Splittable condition

Links	value of z	average delay	value of z	average delay
z_1	87576.7	1.592303636	88219.5	1.603990909
z_2	48521.68	0.882212364	52787.02	0.959764
z_3	51215.92	0.931198545	46739.92	0.849816727
z_4	127740.1	2.322547273	127771.4	2.323116364
z_5	74710.4	1.358370909	74381.1	1.352383636
z_6	43874.79	0.797723455	34470.61	0.626738364
z_7	73333	1.333327273	74007.1	1.345583636
z_8	73333	1.333327273	73234.21	1.331531091
z_9	109438.13	1.989784242	119747.3	2.177223636
z_10	29566.89	0.537579818	41228.08	0.749601455
z_11	183334.67	3.333357576	172680.7	3.139649091
z_12	26779.003	0.486890955	31601.17	0.574566727
z_13	52530.55	0.955100909	57409.96	1.043817455
z_14	64768.36	1.177606545	69751.21	1.268203818
z_15	29675.503	0.539554591	34497.67	0.627230364
z_16	48978.663	0.890521136	46620.46	0.847644727
z_17	41193.148	0.748966318	41661.25	0.757477273
z_18	18333.4	0.333334545	23212.81	0.422051091
z_19	47900.118	0.870911227	32857.21	0.597403818
z_20	50704.608	0.921901955	44095.78	0.801741455
z_21	11558.11	0.210147455	11558.11	0.210147455
	Variance	0.525713913	Variance	0.498495847

We see that from the above results we can conclude that the variance increases as the demand increases.

CHAPTER 6

CONCLUSIONS

- There is a decrease in objective even when there are no better paths; when more paths are given (in our case 5 and 10 respectively).
- If there needs to be stability in network when there is some failure in links than load balancing or average delay model would be a better option than minimum cost model.
- It is the most beneficial to have more paths; when average delay routing is used. We see in our case that the decrease in average delay from 5 to 10 path case is 30-40% on average.
- From the distance graph we can see that the paths being chosen by link utilization and minimum cost routing are similar.
- Average delay is the one that varies the most with respect to paths chosen when compared with link utilization and minimum cost routing.
- Minimum cost routing shows the least tendency to split its demand into more paths to satisfy the demand, while load balancing once it has decided upon a set of paths does not change it until the condition becomes infeasible, average delay model behaves in a different manner when the demand is low it tends to spread the demand into more paths and as the demand increases it tends to spread the demands into less paths.

Appendix

A. Model files for the different objectives

Minimum cost routing model in Splittable case

```
 param D >0 integer;
param E>0 integer;
param N>0 integer;
param Pd>0 integer;
param T>0 integer;

set Nodes:= 1..N;
set link_nos:= 1..E;
set demand_nos:= 1..D;
set route_nos:=  1..Pd;
set hours_nos:=1..T;

#generation of links
param link_src{link_nos} within Nodes;
param link_dest{link_nos} within Nodes;
param link_capacity{link_nos} >=0 integer;
param cost{demand_nos,route_nos,hours_nos}>=0 ;

#generation of demands
param demand_src{demand_nos} within Nodes;
param demand_dest{demand_nos} within Nodes;
param time_demand{hours_nos} within hours_nos;
```

```
#generation of routes

set Routes{demand_nos,route_nos,hours_nos} within link_nos;

#generation of variables

var x{d in demand_nos,p in route_nos,t in hours_nos}>=0;

param h{demand_nos,hours_nos}>=0 ;


#generation of variables required for optimization

param delta{e in link_nos,d in demand_nos,p in route_nos,t in hours_nos} = if e in
routes[d,p,t] then 1 else 0;

minimize capacity: sum{d in demand_nos}sum{p in route_nos}sum{t in
hours_nos}((cost[d,p,t]*x[d,p,t]));

subject to all_demands{d in demand_nos,t in hours_nos}:sum{p in
route_nos}(x[d,p,t])=h[d,t];

subject to capacity_constraints{e in link_nos,t in hours_nos}:sum{d in      demand_nos}sum{p
in route_nos}(delta[e,d,p,t]*x[d,p,t])-link_capacity[e]<=0;
```

Minimum cost routing models under Unsplittable condition

```
param D >0 integer;
param E>0 integer;
param N>0 integer;
param Pd>0 integer;
param T>0 integer;

set Nodes:= 1..N;
set link_nos:= 1..E;
set demand_nos:= 1..D;
set route_nos:=  1..Pd;
set hours_nos:=1..T;

#generation of links
param link_src{link_nos} within Nodes;
param link_dest{link_nos} within Nodes;
param link_capacity{link_nos} >=0 integer;
param cost{demand_nos,route_nos,hours_nos}>=0 ;

#generation of demands
param demand_src{demand_nos} within Nodes;
param demand_dest{demand_nos} within Nodes;
param time_demand{hours_nos} within hours_nos;

#generation of routes
set Routes{demand_nos,route_nos,hours_nos} within link_nos;


param h{demand_nos,hours_nos}>=0 ;

#generation of variables
var u{d in demand_nos,P in route_nos,t in hours_nos} binary;

#generation of variables required for optimization
param delta{e in link_nos,d in demand_nos,p in route_nos,t in hours_nos} = if e in
Routes[d,p,t] then 1 else 0;



minimize capacity: sum{d in demand_nos}sum{p in route_nos}sum{t in
hours_nos}((cost[d,p,t]*h[d,t]*u[d,p,t]));
```

```
# subject to all_demands{d in demand_nos,t in hours_nos}:sum{p in
route_nos}((x[d,p,t])*u[d,p,t])=h[d,t];
subject to path_constraint{d in demand_nos,t in hours_nos}:sum{p in
route_nos}(u[d,p,t])=1;

subject to capacity_constraints{e in link_nos,t in hours_nos}:sum{d in demand_nos}sum{p in
route_nos}(delta[e,d,p,t]*h[d,t]*u[d,p,t])
-link_capacity[e]<=0;
```

Load balancing model under Splittable condition

```
param D >0 integer;
param E>0 integer;
param N>0 integer;
param Pd>0 integer;
param T>0 integer;

set Nodes:= 1..N;
set link_nos:= 1..E;
set demand_nos:= 1..D;
set route_nos:=  1..Pd;
set hours_nos:=1..T;

#generation of links
param link_src{link_nos} within Nodes;
param link_dest{link_nos} within Nodes;
param link_capacity{link_nos} >=0 integer;

#generation of demands
param demand_src{demand_nos} within Nodes;
param demand_dest{demand_nos} within Nodes;
param time_demand{hours_nos} within hours_nos;

#generation of routes
set Routes{demand_nos,route_nos,hours_nos} within link_nos;
param h{demand_nos,hours_nos}>=0 ;

#generation of variables
var x{d in demand_nos,p in route_nos,t in hours_nos}>=0;
var r;

#generation of variables required for optimization
param delta{e in link_nos,d in demand_nos,p in route_nos,t in hours_nos} = if e in
Routes[d,p,t] then 1 else 0;

minimize link_utilization: r;

subject to all_demands{d in demand_nos,t in hours_nos}:sum{p in
route_nos}(x[d,p,t])=h[d,t];
```

```
#formulation for minimizing link_utilization
subject to capacity_constraints{e in link_nos,t in hours_nos}:sum{d in demand_nos}sum{p in
route_nos}(delta[e,d,p,t]*x[d,p,t])-link_capacity[e]*r<=0;
```

Load balancing model under Unsplittable condition

```
param D >0 integer;
param E>0 integer;
param N>0 integer;
param Pd>0 integer;
param T>0 integer;

set Nodes:= 1..N;
set link_nos:= 1..E;
set demand_nos:= 1..D;
set route_nos:=  1..Pd;
set hours_nos:=1..T;

#generation of links
param link_src{link_nos} within Nodes;
param link_dest{link_nos} within Nodes;
param link_capacity{link_nos} >=0 integer;

#generation of demands
param demand_src{demand_nos} within Nodes;
param demand_dest{demand_nos} within Nodes;
param time_demand{hours_nos} within hours_nos;

#generation of routes
set Routes{demand_nos,route_nos,hours_nos} within link_nos;


param h{demand_nos,hours_nos}>=0 ;

#generation of variables
#var x{d in demand_nos,p in route_nos,t in hours_nos}>=0;
var u{d in demand_nos,p in route_nos,t in hours_nos} binary;
var r<=1;

#generation of variables required for optimization
param delta{e in link_nos,d in demand_nos,p in route_nos,t in hours_nos} = if e in
Routes[d,p,t] then 1 else 0;


minimize link_utilization: r;
```

```
#subject to all_demands{d in demand_nos,t in hours_nos}:sum{p in
route_nos}(x[d,p,t])=h[d,t];
subject to path_constraint{d in demand_nos,t in hours_nos}:sum{p in
route_nos}(u[d,p,t])=1;

#formulation for minimizing link_utilization

subject to capacity_constraints{e in link_nos,t in hours_nos}:sum{d in demand_nos}sum{p in
route_nos}(delta[e,d,p,t]*u[d,p,t]*h[d,t])
-link_capacity[e]*r<=0;
```

Piecewise linear approximation of average delay under Splittable condition

```
param D >0 integer;
param E>0 integer;
param N>0 integer;
param Pd>0 integer;
param T>0 integer;
#average delay changes
param AD>0 integer;
param i>0 integer;

set Nodes:= 1..N;
set link_nos:= 1..E;
set demand_nos:= 1..D;
set route_nos:=  1..Pd;
set hours_nos:=1..T;
set aver_delay:=1..AD;# average delay parameter#
set iter:=1..i;

#generation of links
param link_src{link_nos} within Nodes;
param link_dest{link_nos} within Nodes;
#param capacity{link_nos} >=0 integer;
param j_val{iter} >=0;
param k_val{iter} >= 0;

#generation of demands
param demand_src{demand_nos} within Nodes;
param demand_dest{demand_nos} within Nodes;
param time_demand{hours_nos} within hours_nos;
param capacity{link_nos} >=0 integer;
#generation of routes
set Routes{demand_nos,route_nos,hours_nos} within link_nos;


param h{demand_nos,hours_nos}>=0 ;

#generation of variables
var x{d in demand_nos,p in route_nos,t in hours_nos}>=0;
var link_capacity{link_nos} >=0;
var z{link_nos} >= 0;
#var z{g in aver_delay} >=0;
#generation of variables required for optimization
```

```
param delta{e in link_nos,d in demand_nos,p in route_nos,t in hours_nos} = if e in
Routes[d,p,t] then 1 else 0;

minimize average_delay:sum{e in link_nos}(z[e]/capacity[e]);
subject to all_demands{d in demand_nos,t in hours_nos}:sum{p in
route_nos}(x[d,p,t])=h[d,t];

subject to capacity_constraints{e in link_nos,t in hours_nos}:sum{d in demand_nos}sum{p in
route_nos}(delta[e,d,p,t]*x[d,p,t])<=link_capacity[e];

subject to delay{e in link_nos,n in iter}:(z[e])-(j_val[n]*link_capacity[e])>=-
(k_val[n]*capacity[e]);
```

Piecewise linear approximation of average delay under Unsplittable condition

```
param D >0 integer;
param E>0 integer;
param N>0 integer;
param Pd>0 integer;
param T>0 integer;
#average delay changes
param AD>0 integer;
 param i>0 integer;

set Nodes:= 1..N;
set link_nos:= 1..E;
set demand_nos:= 1..D;
set route_nos:=  1..Pd;
set hours_nos:=1..T;
set aver_delay:=1..AD;# average delay parameter#
set iter:=1..i;

#generation of links
param link_src{link_nos} within Nodes;
param link_dest{link_nos} within Nodes;
#param capacity{link_nos} >=0 integer;
param j_val{iter} >=0;
param k_val{iter} >= 0;

#generation of demands
param demand_src{demand_nos} within Nodes;
param demand_dest{demand_nos} within Nodes;
param time_demand{hours_nos} within hours_nos;
param capacity{link_nos} >=0 integer;
#generation of routes
set Routes{demand_nos,route_nos,hours_nos} within link_nos;


param h{demand_nos,hours_nos}>=0 ;

#generation of variables
var u{d in demand_nos,p in route_nos,t in hours_nos} binary;
var link_capacity{link_nos} >=0;
var z{link_nos} >= 0;
#var z{g in aver_delay} >=0;
#generation of variables required for optimization
```

```
param delta{e in link_nos,d in demand_nos,p in route_nos,t in hours_nos} = if e in
Routes[d,p,t] then 1 else 0;

minimize average_delay:sum{e in link_nos}(z[e]/capacity[e]);
subject to path_constraint{d in demand_nos,t in hours_nos}:sum{p in
route_nos}(u[d,p,t])=1;
subject to capacity_constraints{e in link_nos,t in hours_nos}:sum{d in demand_nos}sum{p in
route_nos}(delta[e,d,p,t]*u[d,p,t]*h[d,t])<=link_capacity[e];

subject to delay{e in link_nos,n in iter}:(z[e])-(j_val[n]*link_capacity[e])>=-
(k_val[n]*capacity[e]);
```

REFERENCE LIST

1. Fortz, B. and M. Thorup (2002). "Optimizing OSPF/IS-IS weights in a changing world." Selected Areas in Communications, IEEE Journal on 20(4): 756-767

2. Pióro, M., D. Medhi, et al. (2004). Routing, flow, and capacity design in communication and computer networks, Elsevier/Morgan Kaufmann.

3. Srivastava, S., G. Agrawal, et al. (2005). "Determining link weight system under various objectives for OSPF networks using a Lagrangian relaxation-based approach." Network and Service Management, IEEE Transactions on **2**(1): 9-18.

4. Zhou,"Experience with a decomposition algorithm for solving a multi-hour computer communications network design problem with multi-path routing". MS Thesis. University of Missouri-Kansas City, Kansas City,1995. Print

5. Gavish, B. and I. Neuman (1992). Routing in a network with unreliable components. Communications, IEEE Transactions on **40**(7): 1248-1258.

6. B. Fortz and M. Thorup, "Internet traffic engineering by optimizing OSPF weights", Proceedings of INFOCOM'2000, Tel-Aviv, Israel, Mar. 2000.

7. Medhi, D. (1995). "Multi-hour, multi-traffic class network design for virtual path-based dynamically reconfigurable wide-area ATM networks." Networking, IEEE/ACM Transactions on **3**(6): 809-818.

VITA

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