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UNIVERSITY OF MISSOURI COLLEGE OF AGRICULTURE
AGRICULTURAL EXPERIMENT STATION

F. B. MUMFORD, *Director*

GROWTH AND DEVELOPMENT

With Special Reference to Domestic Animals

XLI. Relation Between Live Weight and Chest Girth in Dairy Cattle of Unknown Age.

(Includes the Missouri-Nebraska Standard for Estimating Live Weight
From Chest Girth)

S. BRODY, H. P. DAVIS, AND A. C. RAGSDALE

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FOREWORD

The special investigation on growth and development is a cooperative enterprise in which the departments of Animal Husbandry, Dairy Husbandry, Agricultural Chemistry, and Poultry Husbandry have each contributed a substantial part. The parts for the investigation in the beginning were inaugurated by a committee including A. C. Ragsdale, E. A. Trowbridge, H. L. Kempster, A. G. Hogan, F. B. Mumford. Samuel Brody served as Chairman of this committee and has been chiefly responsible for the execution of the plans, interpretation of results and the preparation of the publications resulting from this enterprise.

The investigation has been made possible through a grant by the Herman Frasch Foundation, now represented by Dr. F. J. Sievers.

F. B. MUMFORD

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Acknowledgment is made to Miss Margaret Sappington for computations; to Cleo Mull, John Campbell, and Hudson Kibler for computations and preparations of charts.

ABSTRACT

The relation between body weight and chest girth of dairy cattle of mixed ages may be formally represented by the equation $Y=aX^n$ in which Y is body weight, X chest girth. The value of the exponent n is of the order of 2.82 (that is the *percentage* increase in body weight tends to be about 2.82 times as great as the *percentage* increase in chest girth), but in the 6 groups of cattle investigated the values of n ranged from 2.72 to 2.88. There are also variations with group and with size in the value of the coefficient a . It thus appears that the relation between weight and chest girth is not altogether independent of nutritive level. However, the relation between weight and chest girth is close enough for practical estimation of weight from chest girth and for the theoretical generalization that within certain limits weight is a power function of chest girth as indicated by the equation $Y=aX^n$. This equation $Y=aX^n$ was fitted by the method of least squares to the following weight-chest pairs of measurements: 2454 Missouri Holsteins; 2235 Missouri Jerseys; 4689 combined Missouri pairs; 5866 Nebraska Holsteins; 2758 Nebraska Jerseys; 840 Nebraska Guernseys; 1457 Nebraska Ayrshires; 10,921 combined Nebraska pairs; 15,610, grand total of all the pairs. The fitted equations and their statistical constants are shown in Fig. 3. Tables and charts are presented for estimating live weight from chest girth of cattle of mixed ages. A later bulletin will present similar analyses for given age intervals.

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XLI. RELATION BETWEEN LIVE WEIGHT AND CHEST GIRTH IN DAIRY CATTLE OF UNKNOWN AGE*

(Includes the Missouri-Nebraska Standard for Estimating Live Weight From Chest Girth)

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I. INTRODUCTION

Growth in size is associated with change in form, which is due to unequal growth of the constituent parts of the body.¹ Figs. 1a and 1b illustrate change in form and unequal growth in dairy cattle. The problem of estimating live weight from chest girth is one aspect of the larger problem of defining change in form with increasing size, and of the relation of part to whole.

We shall estimate live weight from chest girth with the aid of the power equation

$$Y=aX^n \quad (1a)$$

or what is the *same*, the logarithmic equation

$$\log Y=\log a+n \log X \quad (1b)$$

in which Y is body weight and X chest girth. In view of the novel terminology proposed by Huxley and Tessier,² it may be noted that this power or logarithmic equation (1) was long known as a parabola when the exponent n is positive, and as a hyperbola when n is negative. This power equation is as well known and as widely used in the physical sciences and engineering as the simpler linear equation. Indeed, the power equation $Y=aX^n$ or $Y=aX^n+b$ include the linear equations $Y=aX$ or $Y=aX+b$ (when $n=1$). The power equation thus embraces also a great linear territory. Moreover, there is no reason for thinking that a logarithmic, or constant-percentage, relation is more remarkable than a linear, or constant-absolute, relation.

We would like to note that since 1916 equation (1) has been used at the Missouri Station for representing various interrelationships between part and whole, including relations between surface area and body weight³⁻⁵ of dairy cattle, beef cattle, horses, swine, rats, monkeys,

*This bulletin is based on data collected by the Missouri and Nebraska Stations. The Missouri data were collected under the supervision of A. C. Ragsdale; the Nebraska data under H. P. Davis. S. Brody is chiefly responsible for the synthesis of the data and preparation of the MS.

1. Cf. Thompson, D'Arcy W., *On Growth and Form*, Cambridge (England), 1917.
2. Huxley, J. S., & Tessier, G., *C. R. Soc. Biol.* 121, 1936; *Nature*, 137, 780, 1936.
3. Moulton, C. R., *J. Biol. Chem.*, 24, 249, 1916.
4. Brody, S., & Elting, E. C., *Mo. Agric. Exp. Sta. Res. Bul.* 89, 1926.
5. Brody, S., Comfort, J. E., and Mathews, J. S., *Id.*, *Res. Bul.* 115, 1928.

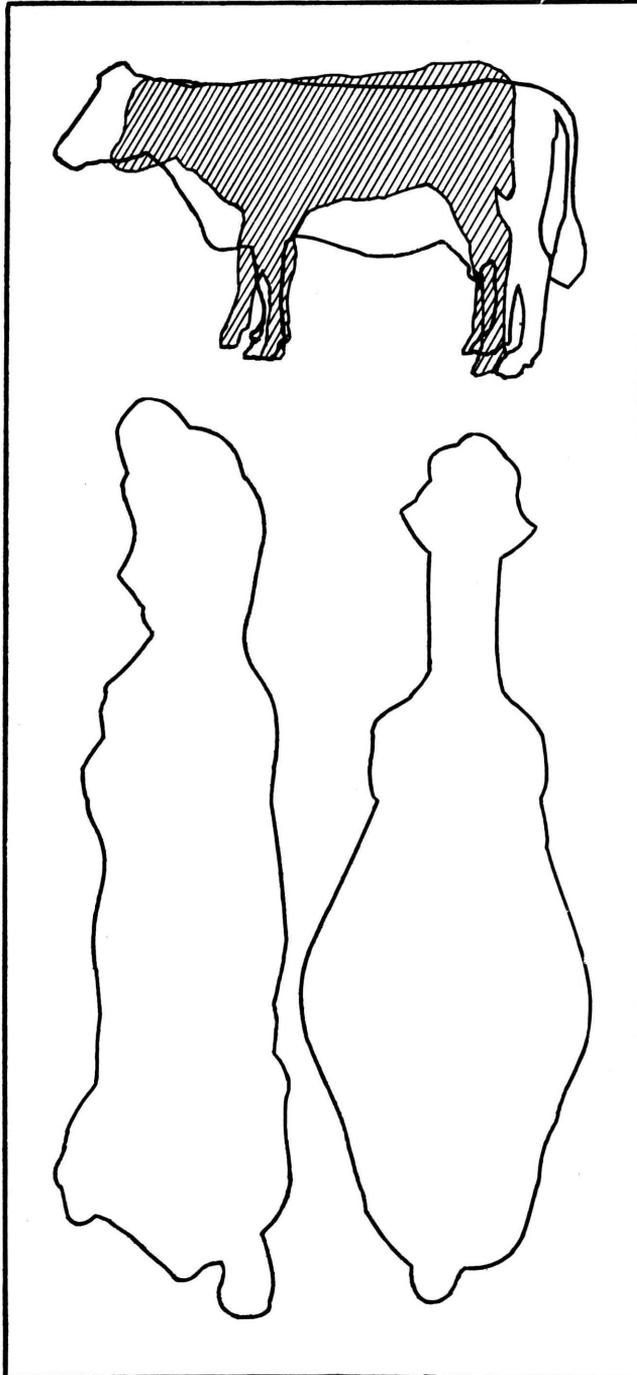


Fig. 1a.—Change in form with increasing age in dairy cattle.

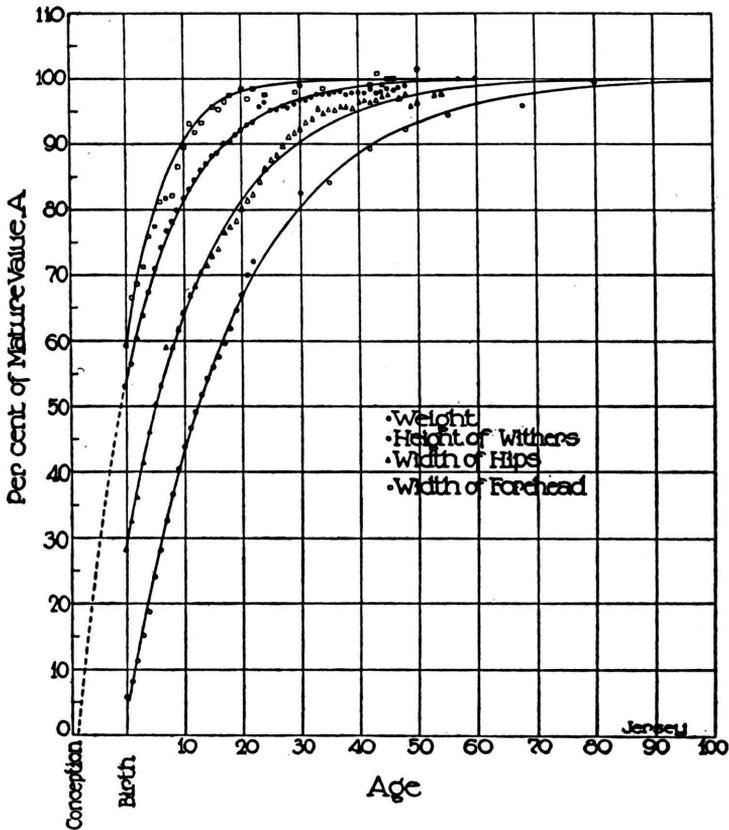


Fig. 1b.—Age curves of growth in weight and in several skeletal structures plotted in terms of percentage of maximum size. While all these age curves can be formally represented by the same equation $Y=A-Be^{-kt}$, (in which Y is size at age t , and A is maximum size) yet the rate, k , of approach to maturity, A , is different for each of these structures. It is these differences in the rate of approach to maturity which are responsible for change in form with increasing age. While each structure approaches maturity at a rate characteristic of the given structure, and different from the rates of growth of other structures, yet the relation between the rate of growth of two structures tend to remain constant during the entire growth period, as indicated by the equation $Y=aX^n$ in which Y is weight or size, of any structure, and X size of any other structure.

humans; the weight relations⁵ between kidneys, lung, liver, stomach, intestine, blood and body; the relation^{6, 7} between body weight and aortic cross-section area, vital capacity, food consumption, heat production, endogenous nitrogen excretion, neutral urinary sulphur excretion; the relation between skeletal growth as represented by some 21 different linear measurements and growth in body weight.⁸

This equation has been used by others in biology at least since 1898 when it was used by Lapicque, although we did not know this until

6. Brody, S., and Procter, R. C., Id., Res. Bul. 166, p. 89-97, 1932.

7. Brody, S., Procter, R. C., Ashworth, U. S., Id., Res. Bul. 220, 1934.

8. Brody, S., Id., Res. Bul. 103, 1927.

we read Teissier's⁹ historic analyses of growth equations. Teissier notes the analyses of the growth equation problem, including the use of equation (1), that were made at about the same time by Schmalhausen in Russia, Teissier in France, and Brody in the United States. Teissier, Needham,¹⁰ Robb,¹¹ and Huxley,¹² not only made important original contributions but did extremely effective educational work in popularizing the use of equation (1) in zoology and biochemistry. Sinnott¹³ popularized this equation in botany.

As previously noted, there is nothing surprising about the applicability of equation (1) to the representation of interrelationships between parts, or parts and whole. If two parts, Y and X, grow exponentially with age,¹⁴ that is if Y changes in a geometric or logarithmic progression with age, and if X likewise changes in a geometric or logarithmic progression with age, then obviously the corresponding relation between Y and X, is geometric or logarithmic. This is indicated in the following derivation.

Equation (1) is merely an algebraic statement of the fact that the *percentage* change in size Y of one part (or of the whole) tends to increase n times the *percentage* change in size X of the related part. In other words, the relation between *percentage* changes in Y and X tends to be constant, namely n. A more technical way of saying the same thing is the *logarithms* of Y tend to vary linearly with the *logarithms* of X, as is indicated by equation 1b.

Derivation of equation (1): Preceding the point of inflection on the S-shaped curve of growth (self-accelerated phase), the course of growth may be represented by the equations:

$$Y = b_1 e^{k_1 t} \quad (a)$$

$$X = b_2 e^{k_2 t} \quad (b)$$

where Y is size of one structure; X, size of another structure; b_1 and b_2 constants; k_1 and k_2 , growth rates; t, time or age. Taking natural logarithms of (a) and (b),

$$\ln Y = \ln b_1 + k_1 t$$

$$\ln X = \ln b_2 + k_2 t$$

Differentiating

$$\frac{dy}{y} = k_1 dt \quad (c)$$

$$\frac{dx}{x} = k_2 dt \quad (d)$$

9. Teissier, G., *Recherches morphologiques et physiologiques sur la croissance des insectes. Travaux de la Station Biologique de Roscoff*, 9, pp. 29-239, 1931 (Les Presses Universitaires de France, Paris, 1931); *Ann. Physiol. et Phys.-Chim. biol.*, 12, 527, 1936.

10. Needham, Joseph, *Bull. Societe Philomatique de Paris*, 115, 11, 1932; *Biol. Rev.* 9, 79, 1934.

11. Robb, R. C., *British J. Exper. Biol.*, 6, 311, 1928-9; *Proc. Inter. Cong. Genetics*, 2, 166, 1932.

12. Huxley, J., *Problems of Relative Growth*, New York, 1932.

13. Sinnott, E. W., *Science*, 85, 61, 1937; *Genetics*, 20, 12, 1935; *Bull. Torrey Bot. Club*, 23, 418 and 602, 1936; *Am. Naturalist*, LXX, 245, 1936.

14. Cf. Brody, S., Ragsdale, A. C., and Turner, C. W., Jr., *J. Gen. Physiol.*, 5, 445, 1923; 6, 21, 1923, 5, 441, 1923; 5, 777, 1923; Brody, S., *Univ. Mo., Agric. Expt. Sta. Res. Bul.* 97, 1926.

Dividing (c) by (d)

$$\frac{dy}{y} \div \frac{dx}{x} = \frac{k_1}{k_2}$$

or

$$\frac{dy}{y} = n \frac{dx}{x}$$

where $\frac{k_1}{k_2} = n$ (a constant).

Integrating

$$\log Y = n \log X + \log a$$

and

$$Y = aX^n$$

which is equation (1).

It is rather more confusing to derive equation (1) from the equation for the phase of growth following the inflection (self-inhibited growth phase). For the latter,

$$Y = A_1 - B_1 e^{-k_1 t} \tag{e}$$

$$X = A_2 - B_2 e^{-k_2 t} \tag{f}$$

Y and X have the same significance as in equation (a) and (b); A_1 and A_2 represent mature values of Y and X; k_1 and k_2 are growth rates with respect to growth yet to be made [(i. e., with respect to $(A_1 - Y)$, or $(A_2 - X)$]. Transposing, taking natural logarithms, and differentiating,

$$\frac{dy}{dt} = k_1; \quad \frac{dx}{dt} = k_2.$$

$$\frac{\frac{k_1}{A_1 - Y} dy}{k_2} = \frac{\frac{dx}{A_2 - X}}{k_2} \div \frac{dx}{A_2 - X}$$

Transposing

$$\frac{dy}{A_1 - Y} = \frac{k_1}{k_2} \frac{dx}{A_2 - X}$$

Integrating,

$$1n(A_1 - Y) = \frac{k_1}{k_2} 1n(A_2 - X) + 1n C \tag{g}$$

$$A_1 - Y = C(A_2 - X)^{\frac{k_1}{k_2}} \tag{h}$$

or

$$Y = A_1 - C(A_2 - X)^{\frac{k_1}{k_2}}$$

$$Y = A_1 - C(A_2 - X)^n \tag{i}$$

in which $n = \frac{k_1}{k_2}$.

Equation (g) indicates that if values of $(A_1 - Y)$ are plotted against corresponding values of $(A_2 - X)$ on logarithmic paper, the result will be a straight line of slope n,

or $\frac{k_1}{k_2}$. Equation (1) can be derived from (i) by transformation of axes.

It is not desirable to place much emphasis on the derivation of equation (1), $Y = aX^n$ from the *time* equations (a), (b), (e), and (f), because the equation $Y = aX^n$ was fitted to data of animals of mixed ages. Time did not enter in fitting it

to the data, so a demonstration of its derivation from the *time* equations is perhaps without significance. Besides equation (1) can be derived from many types of equations other than (a) and (b). Moreover, equation (i) is not in satisfactory form, nor is it in satisfactory form after transformation of axes. In view of the breaks, and major inflection, in the age curve, a single equation can not represent the relations between two parts during the entire growth period. It is therefore perhaps as well to consider equation (1) as a purely empirical representation of the fact that the *percentage* change in size *Y* of one part *tends* to increase *n* times the *percentage* change in size *X* another part. This fact is practically very useful regardless of biologic significance. The power equations $Y = aX^n$ and $Y = aX^n + b$ of course include the linear equations $Y = aX$ and $Y = aX + b$ ($n = 1$). The power equation thus takes in a tremendous territory of possibilities, and there is no reason for surprise that it represents many relations between structures, or functions, regardless of rationalizations.

Addendum to proof; Since the above was written, two important papers have appeared on the significance of the so-called allometric or (what we call the parabolic or power) equation $Y = aX^n$. (L. Lapique, sur l'emploi de la formule dite allométrique, Bull. de la Soc. de Chimie Biol., 19, 434, 1937; Hyman Lumer, The Consequences of Sigmoid Growth for Relative Growth Functions. GROWTH, 1, 140, 1937). Lapique, the distinguished Parisian physiologist who used this equation 30 years ago for representing the relation between brain weight and body weight in different species, gives a charming historic and philosophic discussion of the problem, chiding enthusiastic biochemists (such as Meunier) who see significance in the applicability of this very flexible equation to biochemical relations. This equation may be used to represent the relation between brain weight and body weight and also the physical phenomenon of adsorption. Shall one then say that the growth of the brain occurs according to the law of adsorption? "La croissance de l'encephale se fait suivant la loi d'adsorption." It is equally illogical to say that chemical relations are allometric. The term "allometry" is but, we might say, a trade name especially coined by Huxley and Teissier for equation (1), generally known as the parabola or power equation, when used to represent particular anatomic relations which they observed. If biochemists wish to use this power equation for representing relations between chemical constituents, they may do so—as has been done in the past (e.g., for solubility, adsorption, etc.)—but they should not use the morphologists' special term "allometry" for this relation. Lapique notes that the relation between brain and body sizes long occupied the attention of naturalists and philosophers. Eugene Dubois in 1895 demonstrated that the brain weight, *E*, is related to body weight, *P*, by the power equation $E = K P^{0.66}$, and it is this equation for which Huxley and Teissier coined the term allometry. "Vous reconnaissez ce qu'on appelle aujourd'hui la formule allométrique." Lapique notes that the wide applicability of this equation to growth data is due to its flexibility—it can represent linear, and also (within limits of experimental error) exponential equations. What is the significance of the wide applicability of this equation? "Que signifie-t-elle? Il y a une infinité de cas scientifiques ou on a affaire a deux grandeurs mesurables fonction l'une de l'autre sans etre proportionnelles, et il est assez general qu'on puisse traduire, avec une approximation convenable, leur relation par cette expression." Lumer's paper demonstrates algebraically that power equation (1) in only a first approximation of the relation between two parts of an organism; that the approximation is not valid over the entire growth period; that the deviation may be explained by the determinate and the sigmoid nature of the time course of growth. Neither Lapique nor Lumer attaches ulterior biologic significance to the numerical values of the constants, nor apparently to the general applicability of this equation to data.

The basic "law" of interrelation between parts, or parts and whole, during growth and development is that the relation between the *logarithms*—or what is the same, *percentages*—of parts and whole tends to remain constant. The well-known and widely used equation (1) is an algebraic statement of this fact, which does not call for new

terminology, such as the term "dysharmony" proposed by Champy¹⁵; "heterogony" proposed by Pezzard¹⁶; "allometry" proposed by Huxley and Teissier². These technical terms only becloud a perfectly understandable phenomenon, that of a tendency for the relations between *percentage* changes of various body parts, or parts and whole, to remain constant.

Incidentally, the above discussion clarifies our reasons for preferring to plot the relation between parts, or parts and whole, on logarithmically divided paper (rather than on arithmetically divided paper). The relation between the *percentage*—or *logarithmic*—changes of the parts tends to be constant, or linear; therefore when the data are plotted on logarithmically divided paper, the distribution of data will tend to be linear, that is along an approximately *straight line*. But if the data are plotted on arithmetically divided paper, the distribution is along a logarithmic *curve*. The eye is very much more sensitive to deviations from a straight line than from a curve; so that plotting the data on log-log paper, which results in approximately linear distribution of the data, enables the reader to judge more effectively deviations from the average straight line.

Another reason for plotting data on log-log paper is that deviations of individual points from the average are shown in proportion to percentage. A 10-pound deviation on a 100-pound calf is shown on this paper by the same distance as a 100-pound deviation on a 1000-pound cow, because in both cases the *percentage* deviation is the same. The practical dairyman is more interested in having a table of a constant *percentage* than of a constant absolute accuracy for predicting weight from chest girth. Thus a 10-pound error is insignificant in estimating the weight of a 1000-pound cow, but quite significant in estimating the weight of a 50-pound calf. While arithmetic paper emphasizes *absolute* deviations, log-log paper emphasizes the more important *percentage* deviations.

Before leaving this introduction, we like to point out that about two years ago the Missouri Station published a table for estimating live weight from chest girth of dairy cattle of known and unknown age.¹⁷ These prediction tables were derived with the aid of equation (1). At roughly the same time, Kendrick and Parker¹⁸ derived independently a prediction table by a different procedure (tracing

15. Champy, Chr., *Sexualite et hormones*, Paris, 1924; also in *Ann. Sc. Nat.* 12, 193, 1929.

16. Pezzard, A., *Bul. Biol. Fr. et Belg.*, 52, 1, 1918.

17. Ragsdale, A. C., and Brody, S., Estimating live weights of dairy cattle, *Univ. Missouri Agric. Expt. Sta. Bul.* 354, 1935; see also Brody, S., and Ragsdale, A. C., *Estimating Conditions in dairy cattle*, Id., *Bul.* 355, 1935.

18. Kendrick, J. F., and Parker, J. B., Estimating the weights of dairy cows from heart-girth measurements, *U. S. Dept. Agric. Bureau of Dairy Industry, B. D. I.*, 695, 1936.

free-hand a curve through weight-chest-girth data). Kendrick and Parker's curve, shown in Fig. 7, is based on 1721 pairs of measurements. The Missouri-Nebraska Standard, also shown in Fig. 7, is based on 15,610 pairs of measurements of which 10,921 pairs were secured at the Nebraska Station under the direction of H. P. Davis, and 4,689 were secured at the Missouri Station under the direction of A. C. Ragsdale.

II. RESULTS

We desired very much to be able to present our 15,610 data points on one page. We cannot say that Fig. 2, which represents the effort to satisfy this desire, pleases us, with regard to the representation of each separate data point. Fig. 2 is not, however, devoid of interest.

In Fig. 2 the continuous line represents the least-squares fit of equation (1) to the data. Incidentally, but little need be said about the method of fitting equation (1) to data by the method of least squares. Equation (1b) has the same form as a linear equation, except that instead of numbers we have logarithms of the numbers. Equation (1b) was therefore fitted to the data by the method of least squares exactly as a linear equation is fitted, except that before fitting the equation we had to get the logarithms of the numbers.

The central continuous lines in Fig. 2 then represent equation (1) fitted by the method of least squares separately to each of the 6 groups of data. The age factor was ignored in fitting the equation to the data. The broken curves represent the standard errors of estimate, $+S_R$ and $-S_R$, which include $\frac{2}{3}$ of the data. The numerical values of the equations, and of S_R , are given for the Missouri data on the left side of Fig. 2, and for the Nebraska data on the right side. The letter P on Fig. 2 represents the "correlation coefficient" for logarithmic relations in the same sense that the letter r represents the correlation coefficient for linear relations.

Fig. 2 makes it clear that the distribution of the data on the log-log grid is approximately linear. There are deviations, some very great ones, but in some cases they appear to be fortuitous due to compensatory changes in nutritive condition rather than seriously systematic. The detailed causes for these variations invite investigation. This after all is the main purpose of Fig. 2: to bring out the character of the variation of the individual data points.

It seemed best to group the data according to chest girth levels. Since we felt that chest-girth intervals of one inch would meet practical

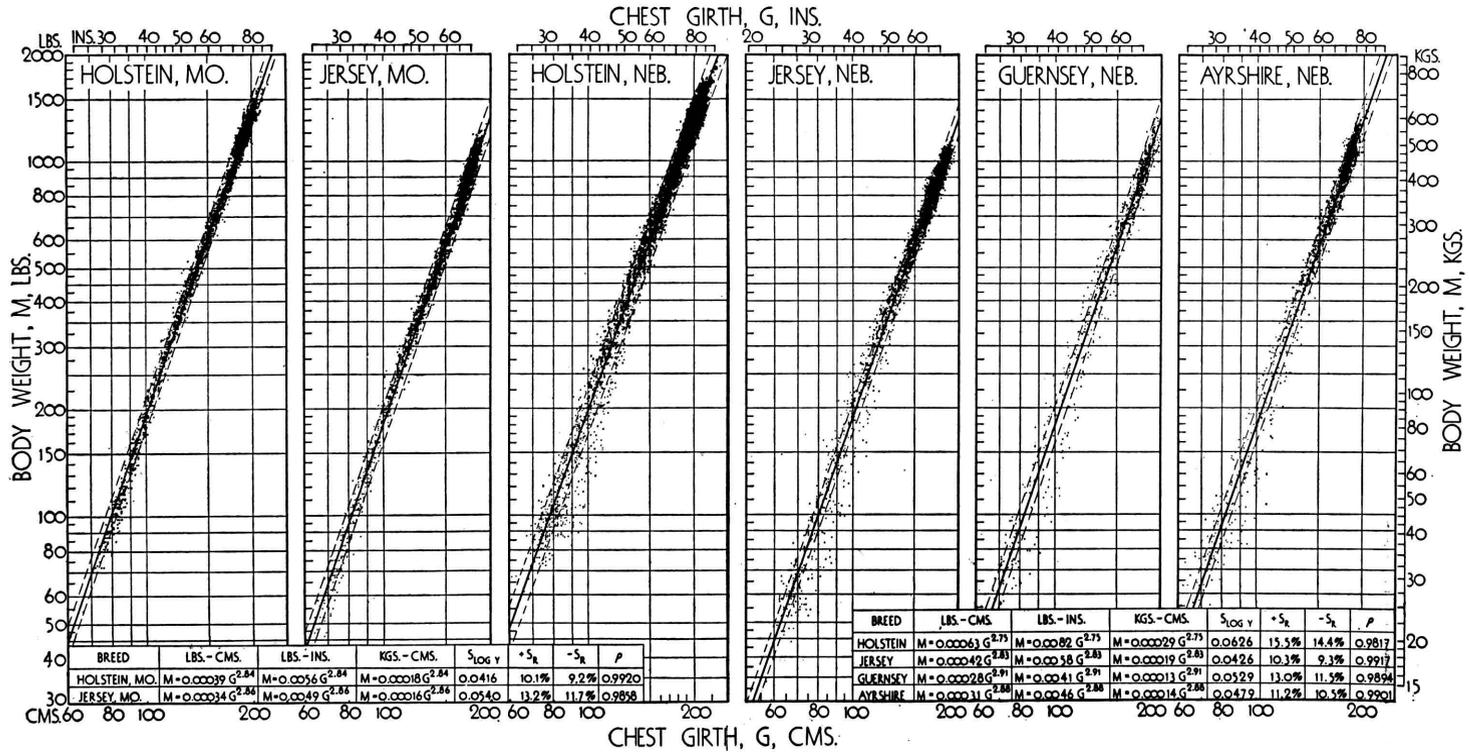


Fig. 2.—Each of the 15,610 weights was plotted against the corresponding chest girths on this logarithmically divided paper. The heavy lines represent the equation $Y=aX^b$ fitted to the data (equations for the Missouri data are on the left, of Nebraska data on the right). The broken curves represent the standard errors of estimate, $+S_r$, and $-S_r$, between which are included 2/3 of all the data. The axes, and the equations are represented in terms of the metric as well as the English scales.

TABLE 1. AVERAGE BODY WEIGHTS FOR GIVEN CHEST GIRTHS*

Weight in Pounds

Chest Girth, Inches	Nebraska										Missouri						Grand Av.	
	Holstein		Jersey		Guernsey		Ayrshire		Average		Holstein		Jersey		Average			
	Obs.1	Com.2	Obs.	Com.	Obs.	Com.	Obs.	Com.	Obs.	Com.	Obs.	Com.	Obs.	Com.	Obs.	Com.		
22.5	--	--	41	38	--	--	--	--	41	39	--	--	41	41	41	38	41	39
23.5	53	48	42	43	--	--	--	--	47	44	--	--	41	41	41	43	45	44
24.5	67	54	54	49	49	45	63	46	55	50	--	--	47	47	47	48	55	50
25.5	--	60	53	55	60	50	65	51	55	56	--	--	51	52	51	54	54	56
26.5	70	66	59	61	63	56	66	57	61	62	54	62	55	58	54	60	60	62
27.5	82	74	63	68	69	63	69	64	68	69	64	69	62	65	62	67	67	69
28.5	89	81	72	75	72	70	77	71	78	76	73	76	67	72	68	74	75	76
29.5	90	89	75	83	78	77	79	78	83	84	83	84	71	79	75	81	81	84
30.5	93	99	84	91	81	85	84	86	88	92	87	92	83	87	85	90	87	92
31.5	102	107	92	99	90	93	86	94	96	101	96	101	91	96	94	98	95	101
32.5	110	116	106	109	103	102	100	103	107	110	103	111	101	105	102	107	105	110
33.5	114	127	115	118	95	111	110	113	112	120	111	120	112	114	111	117	112	120
34.5	125	137	125	129	127	121	114	123	124	130	118	131	122	124	120	127	123	130
35.5	129	148	141	139	127	132	115	133	129	141	147	142	136	135	141	138	134	141
36.5	153	160	144	151	147	143	136	144	147	153	149	154	141	146	146	150	147	153
37.5	160	172	167	163	139	155	146	156	157	165	159	166	160	167	159	162	158	165
38.5	166	186	184	175	166	167	142	168	168	178	176	179	172	170	174	174	170	178
39.5	192	199	189	188	184	190	182	181	191	191	204	192	196	183	201	187	195	191
40.5	207	213	192	202	174	194	193	195	199	205	219	207	203	196	212	201	204	205
41.5	225	228	247	217	201	208	214	209	225	219	225	221	218	210	222	216	224	219
42.5	238	243	234	232	228	223	208	224	233	234	244	237	233	225	240	231	235	235
43.5	250	259	246	248	234	238	231	239	242	250	263	253	250	241	256	247	246	250
44.5	277	276	287	264	249	254	265	256	276	267	278	270	269	257	274	264	275	267
45.5	295	293	297	281	268	271	294	272	292	284	296	288	288	274	293	281	292	284
46.5	314	312	311	299	293	289	269	290	306	302	308	306	300	291	305	299	305	302
47.5	326	330	328	318	318	308	313	308	323	320	338	325	321	310	329	317	326	321
48.5	352	350	350	337	340	327	333	327	348	339	356	345	336	329	348	337	348	340
49.5	367	370	371	357	359	347	347	347	363	359	381	366	346	348	370	357	362	361
50.5	389	391	390	377	373	367	390	368	388	380	405	387	372	369	391	378	389	382
51.5	428	412	411	399	419	389	395	389	419	402	423	409	395	390	408	400	415	403

52.5	445	435	439	421	427	411	413	411	434	424	448	432	413	412	430	422	433	426
53.5	480	458	456	444	450	435	446	434	463	447	467	456	451	435	450	446	458	449
54.5	502	482	472	468	464	458	485	458	486	471	497	481	457	459	482	470	485	473
55.5	525	506	499	493	497	483	493	483	508	496	512	506	481	484	496	495	503	498
56.5	546	532	536	518	518	509	519	508	535	521	546	532	498	509	525	521	532	523
57.5	581	558	546	545	532	536	549	535	557	548	566	560	498	535	533	548	548	550
58.5	591	585	576	572	551	563	556	562	575	575	590	588	535	562	559	575	570	578
59.5	629	613	598	600	587	592	606	590	611	603	618	617	576	590	599	604	607	606
60.5	655	642	630	629	641	621	618	619	639	632	648	647	609	619	630	633	636	635
61.5	675	671	659	659	664	651	647	649	664	661	659	677	637	649	647	664	659	665
62.5	713	702	689	689	698	683	682	680	797	692	698	709	668	679	684	695	693	696
63.5	744	733	731	711	698	715	714	711	731	723	734	742	720	711	726	727	730	728
64.5	772	765	772	754	695	748	752	744	765	756	781	776	765	743	771	760	766	761
65.5	816	798	799	787	762	782	770	777	798	789	804	810	794	776	797	794	798	794
66.5	846	832	820	821	835	817	812	812	829	823	853	846	817	811	826	829	828	829
67.5	884	867	844	857	836	854	840	848	853	859	884	883	870	846	873	866	860	865
68.5	914	902	882	893	903	891	893	885	897	897	908	920	898	883	900	903	898	901
69.5	945	939	913	930	922	929	946	923	931	932	962	959	930	920	939	941	933	939
70.5	983	977	948	969	984	968	977	961	970	970	1017	999	964	959	982	980	974	977
71.5	1036	1015	982	1008	976	1009	1006	1001	1009	1009	1061	1039	1016	998	1037	1020	1018	1017
72.5	1066	1054	1007	1048	1032	1050	1038	1042	1044	1049	1097	1081	1023	1038	1065	1061	1052	1057
73.5	1109	1095	1038	1090	1079	1093	1072	1084	1091	1091	1128	1124	1061	1080	1108	1104	1096	1100
74.5	1155	1137	1053	1133	1150	1137	1100	1127	1138	1133	1169	1169	1115	1123	1161	1147	1146	1142
75.5	1194	1178	1073	1175	1175	1181	1139	1171	1182	1176	1199	1213	1145	1166	1198	1191	1186	1185
76.5	1234	1222	1025	1221	1188	1228	1194	1216	1227	1220	1223	1260	1132	1211	1220	1238	1225	1231
77.5	1277	1266	-----	-----	1171	1275	1208	1262	1271	1266	1245	1307	1196	1257	1244	1284	1266	1276
78.5	1320	1312	-----	-----	1160	1324	1197	1310	1318	1312	1311	1356	-----	-----	1311	1332	1317	1324
79.5	1374	1359	-----	-----	-----	-----	-----	1278	1359	1373	1360	1362	1406	-----	1362	1381	1372	1372
80.5	1391	1406	-----	-----	-----	-----	-----	1386	1408	1391	1408	1365	1456	-----	1365	1431	1389	1421
81.5	1426	1454	-----	-----	-----	-----	-----	1377	1460	1426	1458	1467	1509	-----	1407	1483	1425	1472
82.5	1472	1504	-----	-----	-----	-----	1356	1512	1471	1509	1358	1562	-----	-----	1358	1535	1470	1523
83.5	1484	1554	-----	-----	-----	-----	-----	-----	1484	1560	1535	1616	-----	-----	1535	1589	1485	1575
84.5	1542	1606	-----	-----	-----	-----	-----	-----	1542	1614	1577	1672	-----	-----	1577	1644	1543	1630
85.5	1598	1659	-----	-----	-----	-----	-----	-----	1598	1668	-----	-----	-----	-----	-----	-----	1598	1684
86.5	1676	1712	-----	-----	-----	-----	-----	-----	1676	1723	-----	-----	-----	-----	-----	-----	1676	1740
87.5	1676	1767	-----	-----	-----	-----	-----	-----	1676	1779	-----	-----	-----	-----	-----	-----	1676	1797
88.5	1697	1823	-----	-----	-----	-----	-----	-----	1697	1837	-----	-----	-----	-----	-----	-----	1697	1856
89.5	1776	1880	-----	-----	-----	-----	-----	-----	1776	1895	-----	-----	-----	-----	-----	-----	1776	1916
90.5	1861	1938	-----	-----	-----	-----	-----	-----	1861	1955	-----	-----	-----	-----	-----	-----	1861	1976
91.5	1998	-----	-----	-----	-----	-----	-----	-----	-----	2017	-----	-----	-----	-----	-----	-----	-----	2038
92.5	1865	2058	-----	-----	-----	-----	-----	-----	1865	2079	-----	-----	-----	-----	-----	-----	1865	2102

¹Observed geometric averages.

²Computed from equation $Y = aX^b$. For numerical values of formula constants see Figs. 2 and 3.

purposes, we grouped the data by one-inch chest-girth intervals *disregarding age*. The weights were then averaged for these groups. Incidentally, while from a practical point of view it is immaterial for such narrow groups whether the averages are arithmetic or geometric, it is necessary to point out for theoretical reasons that the weight averages listed in Table 1 are not arithmetic but geometric. Our reason for preferring geometric to arithmetic averages is that our basic assumption is that weight and chest girth are related not arithmetically but geometrically (logarithmically), and it seems to us more consistent to deal with geometric means. We also computed arithmetic means for the given chest-girth intervals, but the differences were practically negligible. There therefore seemed no need for including both, and since the geometrical means seemed theoretically more appropriate, we present the geometric means only.

Table 1 shows the observed average weights for the corresponding chest-girth intervals and the computed weights from the corresponding equations. Table 2 shows the number of measurements, or of animals, in each group.

Fig. 3 represents the lines of equation (1) fitted to each of the six groups of cattle separately and also to all the combined data. In all cases the equation was fitted to the data before they were subdivided by chest-girth intervals. It is only the group averages of the observed data that were obtained after grouping the data by one-inch chest-girth intervals.

Fig. 3 is the final graphic result of our work. *The heavy continuous line represents the proposed Missouri-Nebraska Standard for estimating weight from chest girth.* The other six numbered lines represent the averages of the six constituent sub-groups. The two outside broken lines represent the standard errors of estimate of the combined data. The equations and statistical constants are also given on the chart.

Fig. 3 invites attention to several peculiarities of the data, the most important of which is that during the earlier periods the Nebraska animals, particularly the Nebraska Holsteins, are considerably heavier for a given chest girth than the Missouri animals. This is illustrated to better advantage in Fig. 4. But the Missouri and Nebraska curves merge and, in the case of the Holsteins, tend to cross over with increasing size. Fig. 4 shows the Missouri-Nebraska comparison on *logarithmic* coordinate paper; Fig. 5, on *arithmetic* coordinate paper.

TABLE 2. NUMBER OF ANIMALS INCLUDED IN TABLE 1.

Chest Girth, Inches	Nebraska				Total	Missouri			Grand Total
	Hol- stein	Jer- sey	Guern- sey	Ayr- shire		Hol- stein	Jer- sey	Total	
22.5	1	2	--	--	2	--	--	2	
23.5	--	1	--	--	1	--	--	1	
24.5	2	12	4	3	21	--	--	21	
25.5	--	14	--	--	14	--	--	14	
26.5	3	41	5	2	51	1	12	67	
27.5	16	39	11	13	79	1	19	99	
28.5	14	15	9	9	47	4	15	66	
29.5	47	22	15	24	108	16	26	150	
30.5	50	18	12	12	92	20	17	129	
31.5	76	28	10	21	135	30	23	188	
32.5	46	12	5	8	71	23	16	110	
33.5	52	28	8	13	101	41	17	159	
34.5	36	13	10	7	66	15	14	95	
35.5	41	27	7	14	89	29	26	144	
36.5	34	15	5	11	65	26	11	102	
37.5	46	26	13	13	98	37	15	150	
38.5	34	23	9	10	76	29	24	129	
39.5	33	23	5	15	76	26	15	117	
40.5	54	18	8	12	92	29	22	143	
41.5	32	15	9	9	65	26	18	109	
42.5	41	31	12	8	92	38	16	146	
43.5	32	25	7	23	87	16	19	122	
44.5	54	30	9	9	102	38	26	166	
45.5	33	13	8	12	66	25	19	110	
46.5	44	36	13	10	103	27	23	153	
47.5	39	26	11	17	93	26	28	147	
48.5	54	37	13	15	119	39	28	186	
49.5	42	17	5	15	79	26	39	144	
50.5	46	42	11	19	118	52	36	206	
51.5	85	49	20	16	170	41	44	255	
52.5	35	21	9	19	84	33	32	149	
53.5	63	46	20	22	151	50	45	246	
54.5	42	30	11	15	98	37	21	156	
55.5	69	61	16	27	173	55	57	285	
56.5	47	50	13	17	127	36	26	189	
57.5	64	61	19	28	172	58	52	282	
58.5	55	50	19	20	144	34	41	219	
59.5	96	72	24	37	229	66	55	350	
60.5	59	51	11	21	142	38	31	211	
61.5	105	93	24	41	263	52	64	379	
62.5	55	67	18	26	166	40	34	240	
63.5	118	149	26	34	327	63	74	464	
64.5	111	201	22	51	385	59	100	544	
65.5	81	124	18	31	254	30	74	358	
66.5	133	186	34	55	408	61	174	643	
67.5	76	129	25	53	283	36	108	427	
68.5	145	175	56	94	470	51	183	704	
69.5	100	121	26	70	317	40	99	456	
70.5	166	154	48	80	448	85	155	688	
71.5	122	74	29	54	279	59	66	404	
72.5	187	80	39	85	391	134	98	623	
73.5	168	34	9	53	264	81	34	379	
74.5	273	22	36	80	411	172	28	611	
75.5	198	5	13	36	252	67	1	320	
76.5	316	4	8	29	357	128	4	489	
77.5	398	--	10	23	431	89	2	522	
78.5	272	--	1	3	276	49	--	325	
79.5	374	--	--	3	377	38	--	415	
80.5	252	--	--	3	255	17	--	272	
81.5	264	--	--	1	265	11	--	276	
82.5	130	--	--	1	131	1	--	132	
83.5	95	--	--	--	95	2	--	97	
84.5	55	--	--	--	55	1	--	56	
85.5	22	--	--	--	22	--	--	22	
86.5	5	--	--	--	5	--	--	5	
87.5	13	--	--	--	13	--	--	13	
88.5	6	--	--	--	6	--	--	6	
89.5	4	--	--	--	4	--	--	4	
90.5	4	--	--	--	4	--	--	4	
91.5	--	--	--	--	--	--	--	--	
92.5	1	--	--	--	1	--	--	1	
Total	5866	2758	840	1457	10921	2454	2235	4689	15610

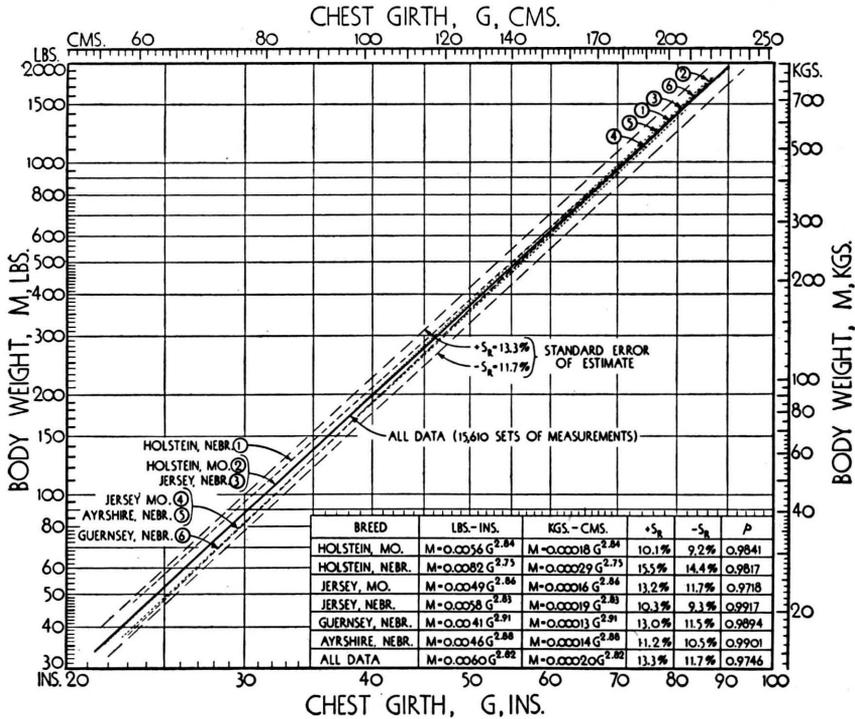


Fig. 3.—The central heavy line represents the equation $Y=aX^n$ fitted to all the 15,610 pairs of measurements. The outside broken line represents the standard errors of estimate, $+S_e$, and $-S_e$, between the limits of which are included 2/3 of these 15,610 pairs of measurements. The six numbered lines represent the six indicated subgroups of data. The equations of these six subgroups, as well as of the combined data, are given on the chart. As in Fig. 2, the data are plotted on logarithmically divided paper, the metric as well as in English units.

Another way of comparing the Missouri and Nebraska data is indicated in Fig. 6. The ratios of the average weight of the *Missouri* cattle of given chest girth to the average weight of *all* cattle of the same chest girth are plotted against chest girth. On the same scale are plotted similar ratios of the average of the *Nebraska* cattle of given chest girth to the average weight of *all* cattle of the same chest girth. In the case of the Holstein, the Nebraska and Missouri curves practically coincide following chest girth 34 inches; preceding 34 inches chest girth, the Nebraska curve is considerably above the Missouri curve. In the case of the Jersey cattle, the Nebraska and Missouri curves are quite close for all sizes. In other words, at the earlier ages there is a decided tendency for the Nebraska Holsteins to be heavier than for the Missouri Holsteins of the same chest girth but not for the Jerseys. Fig. 6 shows several other group comparisons.

One naturally wonders why the Nebraska Holsteins are heavier than the Missouri Holsteins of the same chest girth in the early

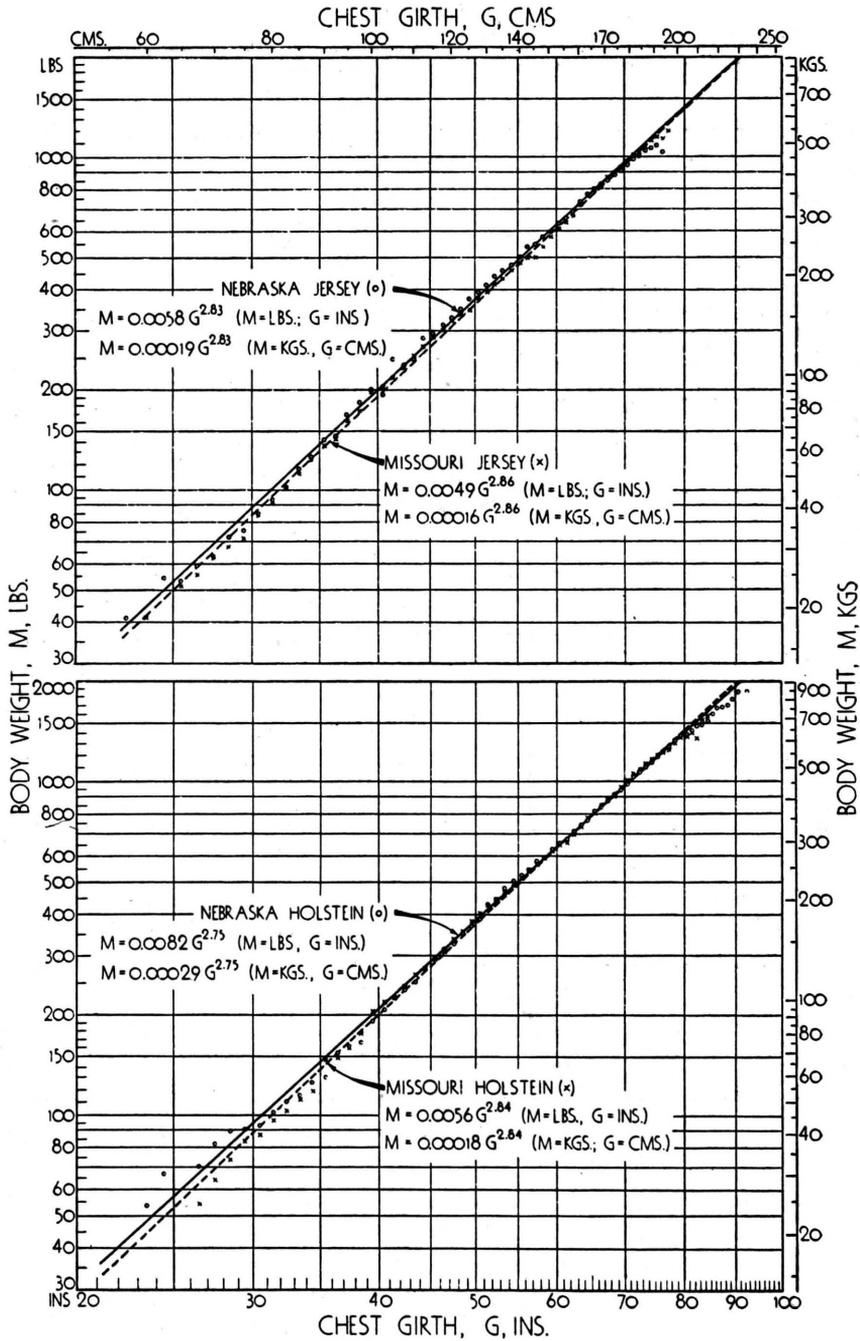


Fig. 4.—Comparison of the Missouri and Nebraska data on log-log paper. The averages of the grouped data as well as the fitted lines are shown. Note that for the lower girth regions, the Nebraska animals, particularly the Holsteins, tend to be heavier than the Missouri animals of the same chest girth.

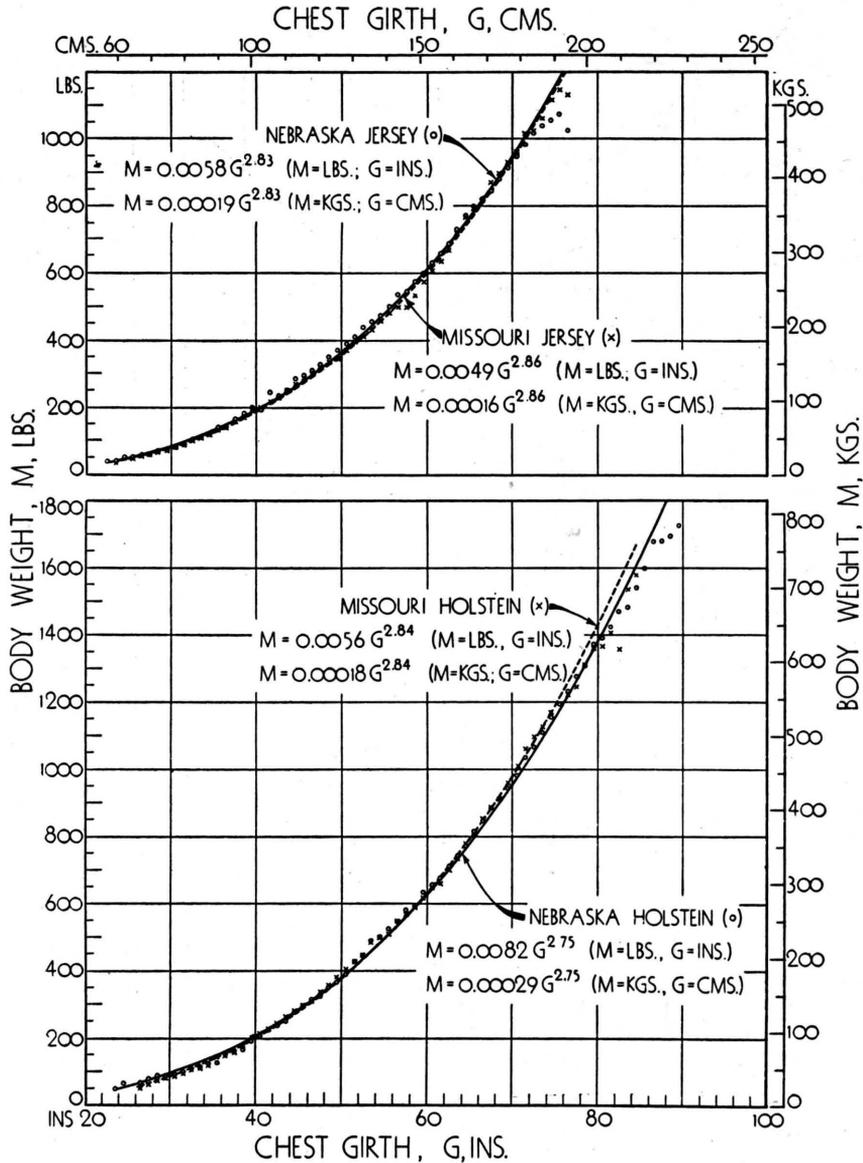


Fig. 5.—Comparison of the Missouri and Nebraska data on ordinary squared paper: The same data and lines as in Fig. 4 but plotted on arithmetic instead of logarithmic paper.

brackets, and light in the highest brackets. The tentative conclusion must be that a high nutritive condition in early life tends to be associated with an excess weight which does not proportionately affect the chest girth. This excess weight in comparison to chest girth in early life apparently tends to be compensated by a relatively lower

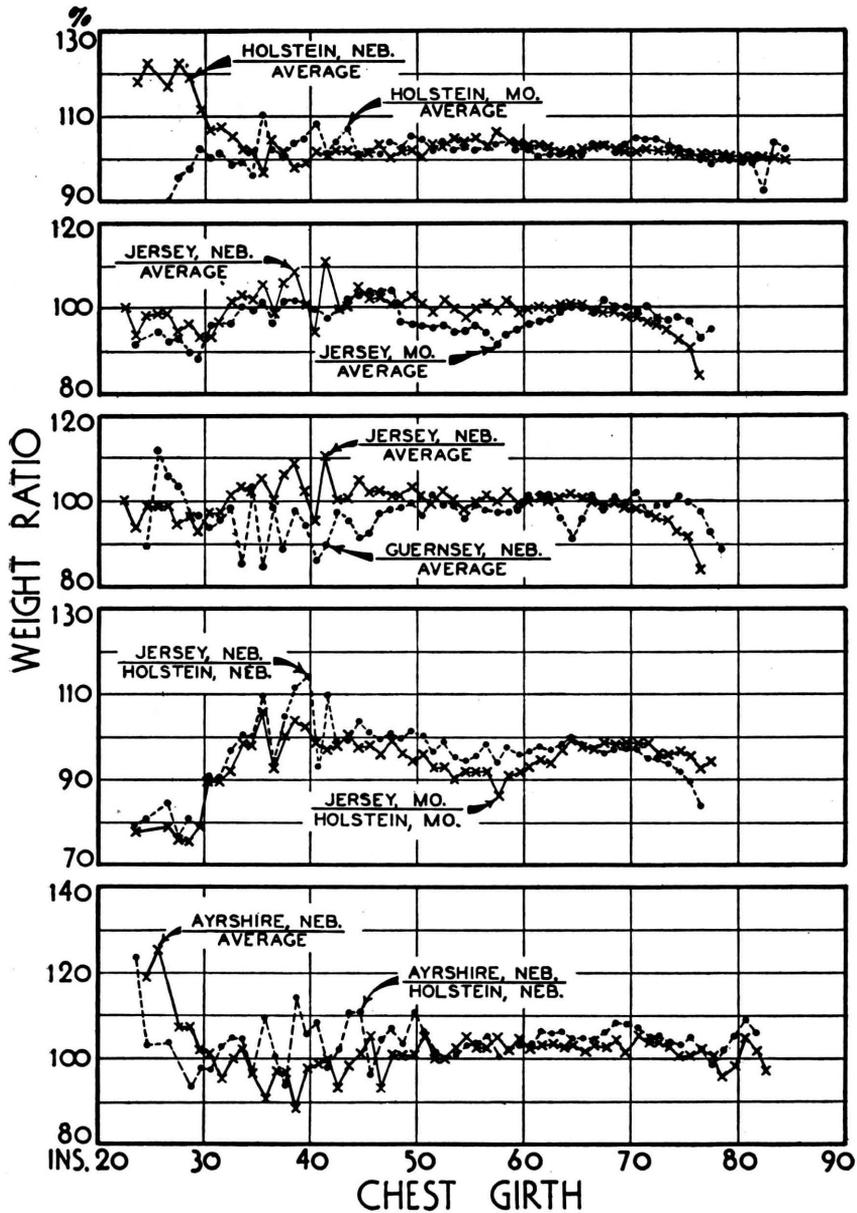


Fig. 6.—In this chart are plotted against chest girth several interesting ratios. In the upper quadrant one curve represents the ratios of the average body weight of *Nebraska* Holsteins to the average body weight of *all* the cattle at successive chest girth levels. A second curve in the upper quadrant represents similar ratios but of the *Missouri* Holsteins to *all* the cattle, plotted against chest girth. Note that preceding chest girth 34 inches the *Nebraska* curve is very much above the *Missouri* curve; following 34 inches, the two curves nearly coincide. If equation (1) represented perfectly each set of data, then the ratio of the average weight of one group of cattle to another group of the same chest girth would always be unity or 100%. This chart is designed to emphasize the deviation from the theoretical 100% level, and to question for the possible causes of the deviations.

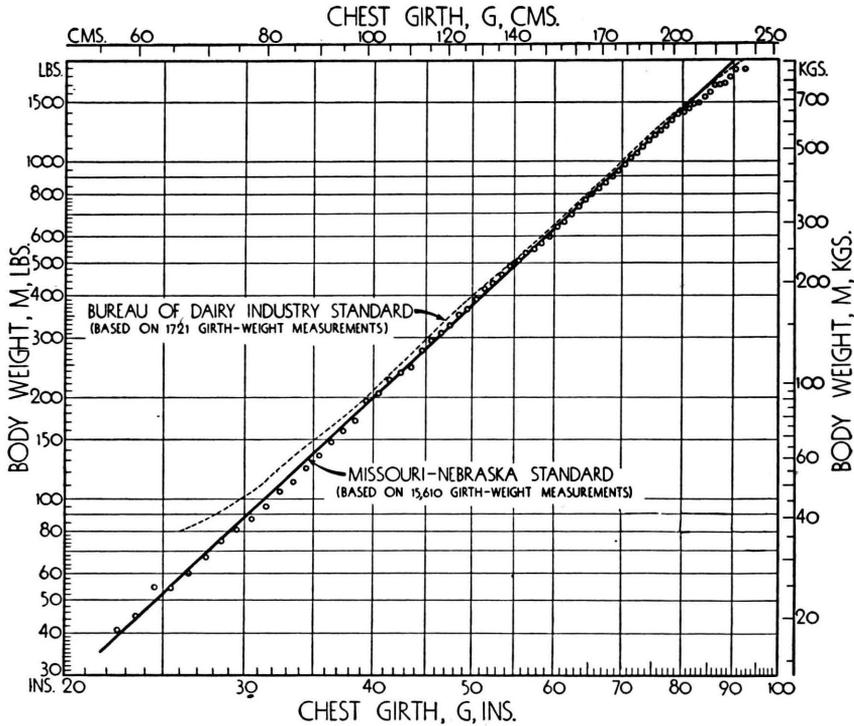


Fig. 7.—Comparison between the proposed Missouri-Nebraska Standard (circles and continuous curve), and the Kendrick-Parker prediction (broken curve). The differences for prediction are not important for heavier animals, but they are quite serious for small animals. There is also the difference of principle: the Kendrick-Parker prediction is based on free-hand judgment, while the Missouri-Nebraska prediction is based on a supposed "law" of interrelation fitted to the data by the method of least squares.

weight for a given chest girth in later life. This statement is obviously not an explanation but a calling of attention to a problem needing explanation.

We may point out that according to the usual statistical tests for significance of differences, the differences between the equation constants of the Missouri and Nebraska data for the same breeds are definitely significant. That is to say, the numerical differences between the statistical constants are real. But the fact that differences are real does not mean that they are *practically significant*. The practical significance of differences depends on the use of the data. From the point of view of practical estimation of weight from chest girth, the weight differences between the Missouri and Nebraska animals at given chest girth do not appear to be significant.

The following table indicates the degree of statistical significance that can be attached to the differences between the equation constants of the indicated groups of cattle. In interpreting the table, differences between the values of constants in

equation (1) are significant if the ratio of $\frac{x}{\sigma}$ is 2 or over; or if the values in the col-

umn labelled chances per 100 are 5 or less. 19.7 chances per 100 means that there are 19.7 chances in 100 that if additional samples of the same kind were secured the differences between the values of n would be as large as they are in this one. These chances were obtained from Pearson's *Tables for Statisticians and Biometricians* (pp. 2-8). The table shows that the difference between the Missouri Jerseys and Nebraska Ayrshires; or Nebraska Guernseys and Nebraska Ayrshires; or Missouri Holsteins and Nebraska Jerseys are not significant. The differences between the other indicated pairs are significant.

Breeds Compared	$\frac{x}{\sigma}$	Chances per 100	$\frac{x}{\sigma}$	Chances per 100
	for n		for log A	
Mo. Jersey and Neb. Ayrshire.....	1.29	19.70	1.29	19.70
Neb. Guernsey and Neb. Ayrshire....	1.38	16.76	1.36	17.38
Mo. Holstein and Mo. Jersey.....	1.46	14.40	2.14	3.24
Mo. Holstein and Neb. Jersey.....	1.67	9.50	1.13	25.64
Mo. Jersey and Neb. Guernsey.....	2.46	1.38	-----	-----
Mo. Jersey and Neb. Jersey.....	2.83	.46	-----	-----
Mo. Holstein and Neb. Ayrshire.....	2.95	.32	-----	-----
Mo. Holstein and Neb. Guernsey....	3.85	.012	-----	-----
Neb. Jersey and Neb. Ayrshire.....	4.27	.0018	-----	-----
Neb. Jersey and Neb. Guernsey....	6.24	*	-----	-----
Neb. Jersey and Neb. Holstein.....	8.06	*	-----	-----
Mo. Jersey and Neb. Holstein.....	9.23	*	-----	-----
Mo. Holstein and Neb. Holstein....	9.56	*	-----	-----
Neb. Holstein and Neb. Guernsey ..	9.75	*	-----	-----
Neb. Holstein and Neb. Ayrshire...-	10.50	*	-----	-----

*Less than 2 chances in a million.

To carry out the comparisons in the table, it was necessary to have the standard errors of the values to be tested and also of the differences to be tested. For this purpose Doolittle's table (Mill's Statistical Methods, p. 578) and the table in Snedecar and Wallace's Correlation and Machine Calculation p. 46 were used. The formulas for the standard error of the differences of n and log A are (Croxtton and Cowden's Practical Business Statistics, pp. 226-7).

$$\sigma_{n_1 - n_2} = \sqrt{\sigma_{n_1}^2 + \sigma_{n_2}^2} \text{ and } \sigma_{\log A_1 - \log A_2} = \sqrt{\sigma_{\log A_1}^2 + \sigma_{\log A_2}^2}$$

The final ratios that we need are:

$$\frac{n_1 - n_2}{\sigma_{n_1 - n_2}} = \frac{x}{\sigma} \text{ and } \frac{\log A_1 - \log A_2}{\sigma_{\log A_1 - \log A_2}} = \frac{x}{\sigma}$$

R. A. Fischer's Statistical Methods for Research Workers explains the philosophy of the problem.

Too much importance must not be placed on the statistical significance of differences as it is based on too many confusing factors such as nature of material, condition of observer, measuring instruments and methods, etc. A statistically significant difference may have no practical significance. Two shipments of fence posts may differ on the average by 1/16 inch in length, and this difference is established by analysis to be statistically significant. In other words there is a real difference of 1/16 inch. Does this mean that this 1/16 inch in length of the posts is practically significant? In the above comparison of constants of the logarithmic equation (1) the comparison is rather complex involving a comparison of slope n as well as initial level A, thus adding to the interpretative difficulties. In ordinary comparison of two means, the difference is said to be significant if the difference between the means is three or more times the probable error.

While we are on the problem of making comparison, it seems desirable to compare the proposed *Missouri-Nebraska Standard* with the Kendrick-Parker prediction. This comparison is indicated in Fig. 7. The circles represent the average weights of our data grouped by one-inch chest-girth intervals; the smooth line represents equation (1) fitted to all of the Missouri-Nebraska data; the broken curve represents the Kendrick-Parker free-hand curve.

III. SUMMARY AND CONCLUSIONS

The major contribution of this bulletin is of course the welding together of a vast body of data on body weight and chest girth with the aid of equation (1). The practical contribution of this work is the formulation of a thoroughly representative and reliable table for estimating weight from chest girth of *dairy cattle of unknown, or of mixed, ages*. The standard errors $+S_R$ and $-S_R$ in Fig. 3 indicate the degree of reliance that may be placed on the prediction of weight from chest girth of cattle of mixed ages: $\frac{2}{3}$ of the predictions over the whole range of weights may be expected to fall within $\pm 13\%$ of the average curve. The predictions can be greatly improved by separating the animals by age groups. Such a prediction in which age is considered will be reported in a later bulletin. The theoretic contribution of this bulletin is the demonstration that the relation between weight and chest girth can be formally represented by equation (1). This means that the *percentage increase* in body weight tends to keep a constant ratio to the *percentage increase* in chest girth. This ratio is of the order of 2.82: the percentage increase in body weight tends to be about 2.82 times the percentage increase in chest girth. In other words, the power equation (1a) may be written $Y=aX^{2.82}$. But there are some variations in the value of the exponent n from group to group as well as in the coefficient a . The highest value for the exponent n was found to be 2.88 for the Nebraska Ayrshires, and the lowest 2.75 for the Nebraska Holsteins. The differences are significant statistically, but probably not practically so.

The value of a in the above equation should be constant, namely Y/X^n , independent of size of animal. Actually, as shown in Fig. 8, when the ratio is computed independently for each one-inch chest girth interval, there are considerable trends. However, the fact that the nature of the trends varies from group to group indicates that the trends are probably caused by management (feeding) conditions. Thus the Nebraska curves show a high initial level for the ratios while the Missouri curves show a low initial level. The group differences in weight at given chest girth levels indicate that the relation between

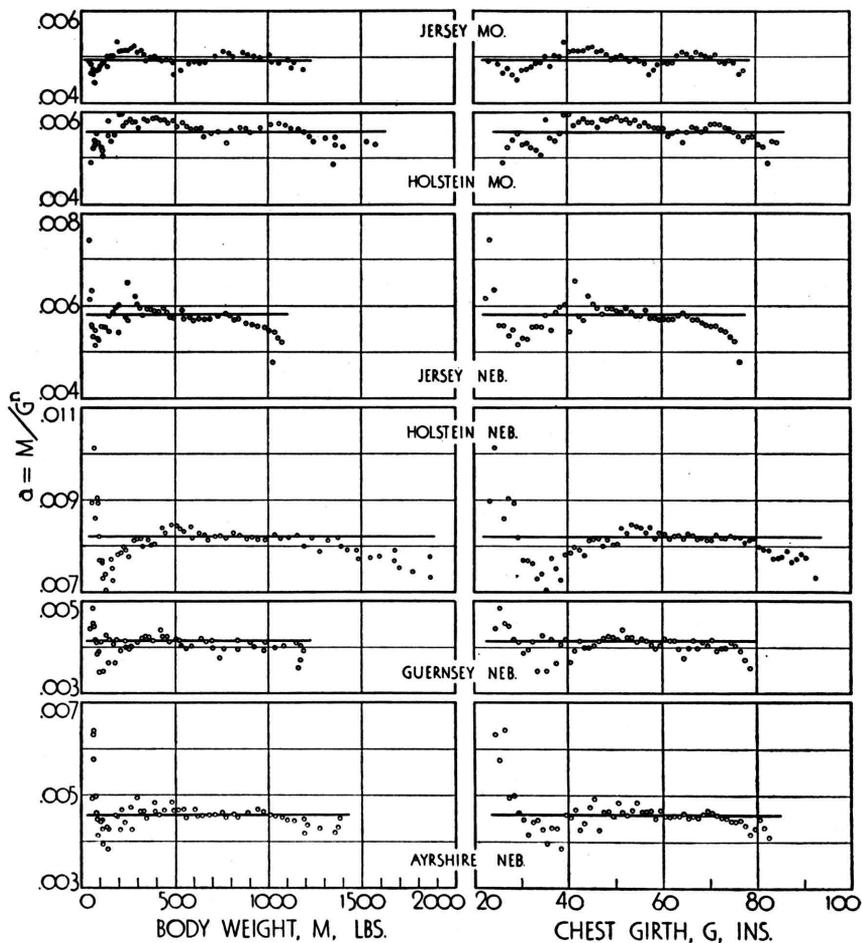


Fig. 8.—If the equation $Y=aX^n$ represented the data perfectly, then $a=Y/X^n$ should be constant regardless of size of animal. This chart, in which a was computed separately for each one-inch chest girth interval and plotted against the size of the animal, shows that the value of a fluctuates considerably. However, the fluctuations are not systematic. Thus the initial values of a are relatively high for the Nebraska data and low for the Missouri data, indicating that environmental (e. g., feeding) rather than genetic factors are responsible for the fluctuations.

weight and chest girth is in part dependent on nutritive condition: a relatively high nutritive level at the early ages causes a relatively (with regard to the equation level) greater weight increase than chest-girth increase, presumably due to relatively greater weight deposition in other regions than around the chest.

Historically, this bulletin is an extension of Missouri Research Bulletin 103 (1927). In bulletin 103, 21 different linear measurements on a *few* animals were related to body weight; in this bulletin a vast body of data on *one* measurement is related to weight, using the same equation and method, but extending the result to the practical issue of formulating a table for predicting weight from chest girth.