UNIVERSITY OF MISSOURI COLLEGE OF AGRICULTURE
AGRICULTURAL EXPERIMENT STATION

Research Bulletin 222

GROWTH AND DEVELOPMENT

With Special Reference to Domestic Animals

XXXV. Energetic Efficiency of Milk Production and the Influence of Body Weight Thereon

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(Publication Authorized March 10, 1935)

COLUMBIA, MISSOURI
MARCH, 1935

FOREWORD

The special investigation on growth and development is a cooperative enterprise in which the departments of Animal Husbandry, Dairy Husbandry, Agricultural Chemistry, and Poultry Husbandry have each contributed a substantial part. The plans for the investigation in the beginning were inaugurated by a committee including A. C. Ragsdale, E. A. Trowbridge, H. L. Kempster, A. G. Hogan, F. B. Mumford. Samuel Brody served as Chairman of this committee and has been chiefly responsible for the execution of the plans, interpretation of results and the preparation of the publications resulting from this enterprise.

The investigation has been made possible through a grant by the Herman Frasch Foundation, now represented by Dr. F. J. Sievers.

F. B. MUMFORD
Director Agricultural Experiment Station

ERRATA

In Bulletin No. XXXIV of the Growth and Development Series (Missouri Agricultural Experiment Station Research Bulletin 220, issued in October, 1934) the following correction of data should be made on Page 32: The estimated basal metabolism of the 8450-pound elephants should be 30924 calories instead of 20924 calories.

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ABSTRACT

Analyses of records for feed consumption, milk production, and body weight changes of 243 mature cows (5 years or over) showed that 0.305 pound TDN (total digestible nutrients) is required to produce 1 pound FCM (4% milk); 2.1 pounds TDN are required to gain 1 pound live weight; 0.053 pound TDN is required to maintain 1 pound live weight at body weight 1 lb if it is assumed that maintenance cost increases not with simple body weight, but with body weight raised to the 0.73 power (as was found for basal metabolism and endogenous nitrogen). From these results the following conclusions were deduced: (A) The *net* digestible feed energy cost of milk production (not counting maintenance cost or live-weight gain cost) is about 1.6 times the milk energy; or the *net* (or partial) energetic efficiency (ratio of milk energy to digestible feed energy less maintenance energy) is about 60%. (B) The *gross* digestible feed energy cost of milk production (including maintenance cost) is about 3 times the milk energy; or the *gross* (or overall, or total) energetic efficiency of milk production (ratio of milk energy to total digestible feed energy) is about 30% (exact value depending on milk yield). (C) The digestible energy cost of maintenance is about 2.4 times the basal (energy) metabolism. All these conclusions are but rough approximations to the true values because the basic data are not homogeneous.

Gross efficiency of milk production in the given group of cattle declined with increasing live weight, thereby confirming Gaines' conclusion. However, while this decline in efficiency with increasing body weight is significant statistically, it may not be significant physiologically; available evidence indicates that larger cows tend to be fed somewhat more liberally than smaller cows, and that efficiency tends to decrease with increasing plane of nutrition.

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INTRODUCTION

1. **Aims.**—This is the second report on efficiency of energy transformations with special reference to the influence of live weight thereon. The first paper (Missouri Research Bulletin 209, 1934) was concerned with efficiency of work (pulling loads) in horses; this paper is concerned with efficiency of milk production in dairy cattle.

2. **Plan.**—Before attempting to compute efficiency of milk production, it is necessary to determine how the digestible feed consumed by the cow is distributed between maintenance, gain (or loss) in body weight, and milk production. The first step in this research will therefore be to fit an equation relating digestible feed consumption to maintenance, gain (or loss) in live weight, and milk production. In fitting an equation to the data we shall make the following assumptions: (1) The dietary energy need for milk production is directly proportional to the milk-energy production. (2) The digestible nutrient need for gain in weight is directy proportional to the gain in weight. Unit gain or loss in body weight has the same feed equivalent. The last assumption, while not strictly true, is perhaps justifiable because the feed-equivalent difference between body gain and loss appears to be within the limits of experimental error. (3) The digestible nutrient need for maintenance is proportional not to simple body weight, but to body weight raised to the 0.73 power. This assumption is based on a study (Missouri Research Bulletin 220) indicating that both basal metabolism and endogenous nitrogen are proportional to body weight raised to the 0.73 power. Before fitting the equation to the data the milks will be converted to Gaines' 4% milk (FCM) and to Gaines' caloric equivalent of milk. In the following computations it will be assumed that 1 pound TDN has a physiological fuel value of 1814 Calories (1 gm = 4 Cal.) and 1 pound FCM (4% milk) has a gross fuel value of 340 Calories (1 Kg = 750 Cal.)

After fitting the equation to the data and computing statistical constants, we shall compare the TDN allowances for milk, gain (or loss) in live weight, and maintenance as computed from our equation, with the corresponding allowances from Morrison's standards. The computation of the efficiency of milk secretion and the evaluation of live weight thereon must necessarily be left to the end.

This is paper 90 in the Herman Frasch Foundation Series

The review of the literature together with the quoted data are given in the Appendix (see page 29).

Three sets of data will be employed in this study: A. Data on Experimental animals in college herds for which feed, body weight, and milk production records were kept. B. Data on *Register of Merit* cattle for which milk records and body weight estimates are available, but not feed consumption records. C. We shall also mention briefly McDowell's Dairy Herd Improvement Association data, as quoted by Gaines' (see Appendix and Fig. 5).

A. EXPERIMENTAL ANIMALS

1. Equations Relating Feed Consumption to Milk Production, Maintenance, and Gain or Loss in Live Weight.—The consumed available food energy must equal the algebraic sum of the energies of milk secreted, "work" of converting milk precursors into milk, "work" of secretion, maintenance cost, gain (or loss) in live weight. This statement may be represented by the equation:

$$\text{TDN} = \text{B(FCM)} + \text{C}(\text{M}^{0.73}) + \text{D}(\Delta\text{M}) \qquad (1a)$$

in which TDN = total digestible nutrients; FCM = 4% milk as explained by Gaines; $\text{M}^{0.73}$ = live weight raised to the 0.73 power; ΔM = gain or loss in body weight. When other variables are held constant, B and D represent respectively units of TDN required to produce unit of 4% milk, FCM, and to gain unit live weight, ΔM. The TDN cost of maintenance is C times body weight raised to the 0.73 power.

The equation was fitted to the data by minimizing the squares of the residuals with respect to the three constants, B, C, D, to yearly lactation records of the 243 Holstein and Jersey (and a few Guernsey) cows 5 years of age or over listed in Table A in the appendix. The energies expended for converting milk precursors into milk and for the process of secretion, are lumped with the FCM term. Solving equation (1a) in term of pounds per day we obtain

$$\text{TDN} = 0.305\ \text{FCM} + 0.053\ \text{M}^{0.73} + 2.1\Delta\text{M} \qquad (1b)$$

The meaning of the constants in equation (1b) is as follows: if maintenance and weight-gain costs are held constant, 0.305 pounds TDN is used per pound FCM produced; if milk yield and maintenance costs are held constant, 2.1 pounds TDN is used for 1 pound gain in live weight; if milk yield and weight gain costs are held constant the TDN cost of maintenance is 0.053 times weight raised to the 0.73 power. In other words the digestible energy cost of milk production is 1.64 times the energy in the milk $(1.64 = \frac{0.305 \times 1814}{340})$. The digestible energy cost of maintenance is about 2.4 times the energy of basal metabolism ($2.4 = \frac{0.053 \times 1814}{39.5}$ where 39.5 is the constant relating basal metabolism to weight in pounds raised to the 0.73 power).

Equation (1b) was formulated on the assumption that there were no experimental errors, and that the TDN consumption was not influenced by factors other than FCM, M, and ΔM. But of course this assumption is not strictly true, and so logically there should be another constant A to take up errors and unknown influencing factors, as indicated by the equation

$$\text{TDN} = A + B(\text{FCM}) + C(M^{0.73}) + D(\Delta M) \qquad (2a)$$

Solving equation (2a), we obtain

$$\text{TDN} = 0.478 + 0.303\,(\text{FCM}) + 0.051\,M^{0.73} + 2.18\,\Delta M \qquad (2b)$$

The standard errors of regression of coefficients B, C and D are respectively 0.012, 0.0032, 0.246. Equation (2b) shows that the numerical value of A is small in comparison to the total TDN consumed by a lactating cow, and the values of B, C and D are inappreciably affected by omitting A. We shall therefore omit A, and use the simpler equation (1a) in preference to (1b).

Needless to say, the distribution of TDN between milk production, weight gain and maintenance indicated by equation (1b) is not applicable to maximum-fed cows on advanced registry test. (Animals that are heavily fed have a higher heat production due to specific dynamic effect than animals lightly fed; moreover, the average digestibilities used for computations in this bulletin are probably not applicable to maximum-fed cows, because digestibility decreases with increased food intake above a certain optimum level.)

The numerical values of the constants in equations (1b) and (2b) are but rough approximations to the true values, since the data on which they are based are not homogeneous, and not always comparable.

2. Correlations:*

(a) *Coefficient of multiple correlation*, $R_{1.234}$, measures the closeness of association between TDN, FCM, $M^{0.73}$ and ΔM. By definition, perfect correlation is represented by 1, no correlation, by 0. The adjusted multiple correlation for equation (1b) is

$$R_{1.234} = 0.936 \pm .008$$

which is very high. [For equation (2b), $R_{1.234} = 0.9403$, i.e., somewhat higher than for equation (1b)].

(b) *Standard error of estimate*, $S_{1.234}$, measures in absolute units the closeness with which TDN can be estimated from FCM, $M^{0.73}$,

*In the following discussions subscripts 1, 2, 3 and 4 represent respectively the variables TDN, FCM, maintenance, and weight-gain. Subscripts preceding decimal point represent constants for the moment. Thus $R_{1.234}$ is the correlation [in equation (1b)] between TDN on one side and FCM, $M^{0.73}$, ΔM on the other side; $\beta 12.34$ is [for equation (1b)] the beta coefficient between 1(=TDN) and 2(=FCM) while 3(=$M^{0.73}$) and 4(=ΔM) are held constant.

and ΔM. $S_{1.234}$ for equation (1b) is 1.02, which means that two-thirds of the time, equation (1b) will predict the variable 1 (i. e., TDN) to within ± 1.02 pound, from variables 2, 3 and 4. [For equation (2b), $S_{1.234} = 0.8971$ lbs.]

(c) *Beta Coefficients,** presented in table 1, indicate the relative importance of each of the independent variables while the others ar. held constant.

TABLE 1.—BETA COEFFICIENTS

	Equation (1b)		Equation (2b)	
	Absolute values	Percentage values	Absolute values	Percentage values
$\beta 12.34$	0.631	49.2	0.623	49.5
$\beta 13.24$	0.446	34.8	0.423	33.6
$\beta 14.23$	0.206	16.0	0.213	16.9
Sum	1.238	100%	1.259	100%

Table 1 indicates that in apportioning the TDN, FCM is the most important, maintenance next, weight gains least.

3. Comparison of Results of Equations with Morrison's Feeding Standards:

(a) *TDN allowance for milk:* Morrison's feeding standard (p. 746 of Henry & Morrison's "Feeds and Feeding", 1923) allows from 0.311 to 0.346 pounds TDN per pound FCM. Our equation (1b) indicates that (for the given group of animals) an average of 0.305 pounds TDN was used to produce 1 pound 4% milk.

(b) *TDN allowance for weight gains:* Our equation (1b) indicates that (for the given group of animals) 2.1 pounds TDN were consumed per pound gain in live weight in the lactating animals.

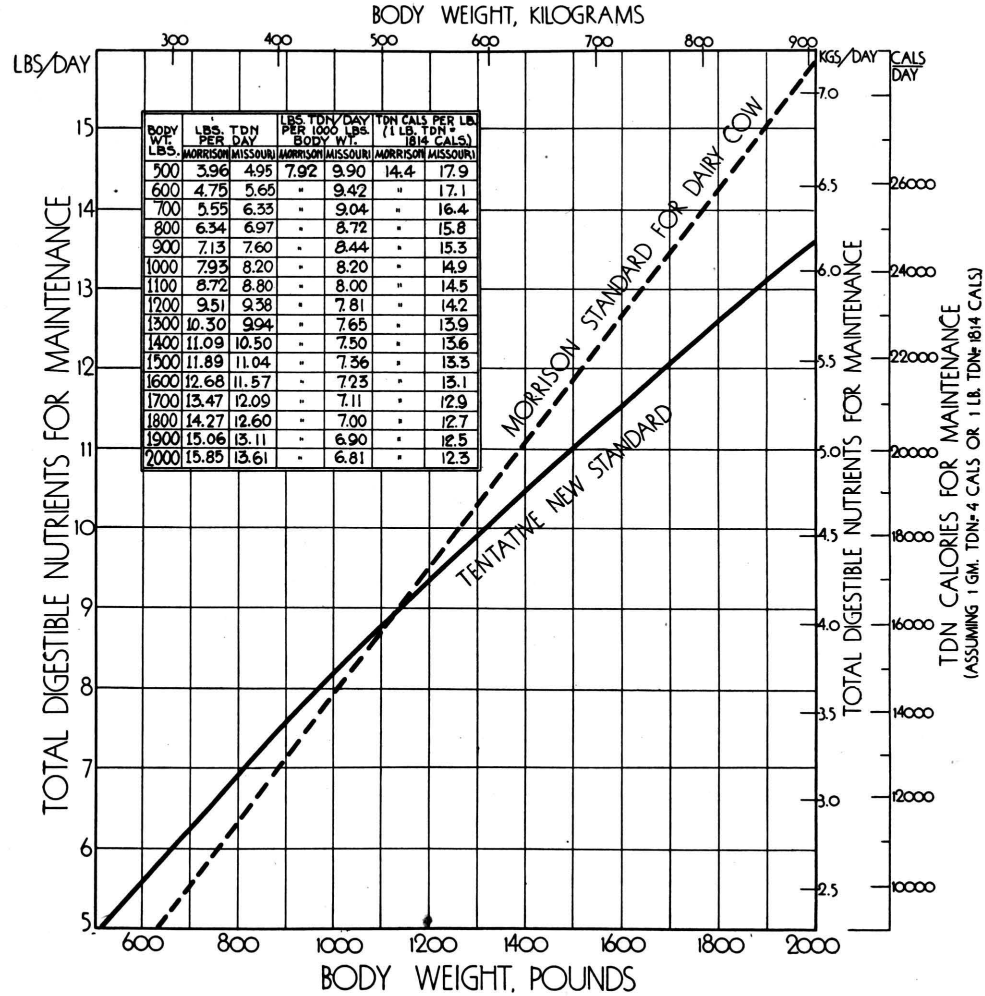
(c) *TDN allowance for maintenance:* Morrison's maintenance allowance is 7.925 pounds TDN per 1000 pounds live weight of cow regardless of its live weight. Our equation indicates a declining maintenance allowance per 1000 pounds live weight with increasing live weight. Fig. 1, with its table, carries out the comparison in graphic and tabular forms. According to Morrison's schedule, doubling live weight doubles maintenance requirements; according to our equations, doubling live weight increases maintenance cost one and two thirds. In brief, according to equation (1b) TDN is related to maintenance by the equation

$$TDN = 0.053\ M^{0.73} \qquad (3)$$

*Beta coefficients are computed from equation

$$\beta 12.34 = B\frac{\sigma_2}{\sigma_1};\ \beta 13.23 = C\frac{\sigma_3}{\sigma_1};\ \beta 14.23 = D\frac{\sigma_4}{\sigma_1}$$

in which B, C, D are the regression coefficients in equation (1), and subscripts *1, 2, 3,* and *4* represent respectively TDN, FCM, maintenance and gain in weight.



BODY WT. LBS.	LBS. TDN PER DAY		LBS. TDN/DAY PER 1000 LBS. BODY WT.		TDN CALS PER LB. (1 LB. TDN = 1814 CALS.)	
	MORRISON	MISSOURI	MORRISON	MISSOURI	MORRISON	MISSOURI
500	3.96	4.95	7.92	9.90	14.4	17.9
600	4.75	5.65	"	9.42	"	17.1
700	5.55	6.33	"	9.04	"	16.4
800	6.34	6.97	"	8.72	"	15.8
900	7.13	7.60	"	8.44	"	15.3
1000	7.93	8.20	"	8.20	"	14.9
1100	8.72	8.80	"	8.00	"	14.5
1200	9.51	9.38	"	7.81	"	14.2
1300	10.30	9.94	"	7.65	"	13.9
1400	11.09	10.50	"	7.50	"	13.6
1500	11.89	11.04	"	7.36	"	13.3
1600	12.68	11.57	"	7.23	"	13.1
1700	13.47	12.09	"	7.11	"	12.9
1800	14.27	12.60	"	7.00	"	12.7
1900	15.06	13.11	"	6.90	"	12.5
2000	15.85	13.61	"	6.81	"	12.3

Fig. 1.—Comparison of Morrison's and the proposed feeding standard. In comparison to the suggested standard, Morrison's standard tends to overfeed heavy animals and underfeed light animals.

while according to Morrison's standard TDN is related to maintenance by the equation

$$TDN = 0.007925\ M \qquad (4)$$

Equation (4) shows that at live weight 1 pound, TDN requirement for maintenance is 0.007925 pounds, at live weight 1000 pounds it is 7.925, at 2000 pounds it is 15.85 pounds. Equation (3) shows that at live weight 1 pound the TDN requirement for maintenance is 0.053 pounds, at 1000 pound it is 8.20 pounds, and at 2000 pounds it is 13.61 pounds. In other words Morrison's allowance overfeeds the heavy animals and underfeeds the light animals in comparison to our allowance.*

*The difference between Morrison's and the suggested standard is that Morrison assumed that the TDN cost of maintenance varies with 1.0 power of body weight while we assume that it varies with the 0.73 power of body weight, as does basal metabolism. Our assumption must be proved before it can be accepted as superior to Morrison's.

TABLE 2.—JERSEY AND HOLSTEIN DATA AVERAGED FROM HAECKER; ECKLES; SAVAGE; HILLS; PERKINS; HARRISON & SAVAGE (SEE APPENDIX FOR INDIVIDUAL RECORDS). COMPARISON BETWEEN MORRISON'S AND OUR "OBSERVED" AND COMPUTED VALUES FOR MAINTENANCE. AGE OF COWS 5 YEARS OR OVER

Body weight lbs.	Number of cows	TDN consumed lbs/day	Milk produced FCM lbs/day	Change in body wt. lbs/day	Gross efficiency %	Gross efficiency corrected for change in body weight %	*Comparison with Morrison's maintenance standard TDN lbs/day: Our tentative formula, observed	Our tentative formula, computed	Our tentative formula, difference	Morrison's standard, observed	Morrison's standard, computed	Morrison's standard, difference
700- 799	21	13.63	23.69	−.055	32.6	32.3	6.52	6.65	−0.13	5.98	5.94	0.04
800- 899	51	15.08	26.18	−.068	32.8	32.4	7.24	7.29	−0.05	6.63	6.74	−0.11
900- 999	28	17.00	26.90	.119	30.6	30.9	8.55	7.91	0.64	7.93	7.53	0.40
1000-1099	14	18.85	29.61	.222	29.4	30.2	9.35	8.51	0.84	8.67	8.32	0.35
1100-1199	29	19.44	29.83	.205	29.3	29.9	9.90	9.09	0.81	9.22	9.11	0.11
1200-1299	48	19.32	30.17	.201	29.3	29.9	9.69	9.66	0.03	8.99	9.91	−0.92
1300-1399	44	19.63	30.01	.252	29.8	30.4	9.94	10.22	−0.28	9.25	10.70	−1.45
1400-1499	6	19.21	30.41	.080	29.7	29.2	9.77	10.76	−0.99	9.07	11.49	−2.42

*Our computed maintenance requirements were obtained from equation (3), TDN = 0.053 $M^{0.73}$; Morrison's computed maintenance were obtained from equation (4), TDN = 0.007925M; our "observed" maintenance values were computed from equation (1), observed maintenance TDN = total TDN consumed less 0.307 FCM and less 2.13ΔM; Morrison's "observed" maintenance values were computed from equation TDN = total TDN consumed less 0.328 FCM and less 2.13ΔM.

Footnote to Table 2.—Some peculiarities about the course of change of TDN consumption FCM production and efficiency with increasing live weights may be due to the heterogeneous distribution of the data as indicated by the following table.

Body Weight pounds	Percentage distribution of data by sources: Haecker	Savage	Harrison and Savage 16 20 & 24 X protein planes: 16% protein	20% protein	24% protein	Other Sources
600- 699	100	--	--	--	--	--
700- 799	100	--	--	--	--	--
800- 899	82	8	--	--	--	10
900- 999	59	17	--	--	--	14
1000-1099	43	43	--	--	7	7
1100-1199	17	14	31	7	28	3
1200-1299	6	4	40	17	33	--
1300-1399	2	2	11	43	23	19
1400-1499	--	--	20	40	40	--
1500-1599	--	--	--	100	--	--

Moreover the frequency distribution curves of superior milkers differ from the poor milkers. Other conditions being the same superior milkers are of course more efficient than inferior.

Table 2, showing the relative losses in body weight of cows of different size during lactation, substantiates the above conclusion that small cows tend to be underfed and heavy cows tend to be overfed, when fed by present standards, and that the allowance in our equation tends to correct this feeding error.

4. **Efficiency of Milk Production.**—The generally accepted definition of efficiency is percentage ratio of output to input. In the case of milk production it is the ratio of energy in milk produced in a given time to energy in feed consumed during the same time. Thus Forbes and Voris [J. Nutrition, 5, 395 (1932)] divided the energy of milk produced by nine Holstein cows during a period of about 300 days by the total energy of the feed consumed during the same time. They found that the energy of the milk varied from 18% to 23% of the total energy of the feed. However, the efficiency computed by Forbes and Voris is not comparable to our efficiency, because they computed milk energy as percentage of the *total* feed energy, while we computed milk energy as percentage of *digestible* nutrients only.

(a) *Gross* (*or overall, or total*) *efficency of milk production*, is defined by the equation:

$$\text{gross efficiency} = \frac{100 \times \text{energy in milk}}{\text{energy in digestible nutrients consumed}} \quad (5a)$$

Since it is assumed that the energy in a pound of FCM (4% milk) is 340 Calories and the energy in a pound of TDN is 1814 Calories, therefore equation (5a) may be written

$$\text{gross efficiency} = \frac{100 \times 340 \times \text{FCM (lbs.)}}{1814 \times \text{TDN (lbs.)}} \% \quad (5b)$$

$$= \frac{100 \times 340 \times \text{FCM}}{1814\,(0.305\,\text{FCM} + 0.053\text{M}^{0.73} + 2.04\,\Delta\text{M})}$$

$$= \frac{100 \times 340\ \text{FCM}}{553\ \text{FCM} + 96.1\ \text{M}^{0.73} + 3700\ \Delta\text{M}} \%$$

$$= 0.61\, \frac{100 \times \text{FCM}}{\text{FCM} + 0.173\ \text{M}^{0.73} + 6.64\ \Delta\text{M}} \% \quad (5c)$$

Factor .61 is the net efficiency (see equation 6b). It is the ratio of calories in 1 pound FCM (340) to Calories in feed (553) required to produce 1 pound FCM, not including cost of maintenance of the body.

With the aid of equation (5b) average gross efficiency was found to be 30.4% ± 0.26% (coefficient of variation 13.8%) for 243 lactation periods (31.9% for 67 Jersey cows; 29.9% for 146 Holstein cows). In the above values, no correction was made for gain or loss in live weight during lactation; if correction is made for gain or loss, then the average gross efficiency of the population is 30.9%. Good producers showed efficiencies of 25-35%; inferior producers, 15-25%; superior producers, 35-40%.

(b) *Net (or partial) efficiency of milk production*, is defined by the equation

$$\text{net efficiency} = \frac{100 \times \text{energy in milk}}{\text{energy in digestible nutrients less digestible nutrients expended for maintenance and for gain (or loss) in weight.}} \qquad (6a)$$

Or numerically

$$\text{net efficiency} = \frac{100 \times 340 \text{ FCM}}{1814 \{\text{TDN} - (0.053 \text{ M}^{0.73} + 2.04 \Delta\text{M})\}} \% = 61\% \qquad (6b)$$

From equation (6) it is clear that *net* efficiency is the ratio (or percentage) of calories in milk produced to calories in digestible feed consumed *above maintenance*. *Gross* energetic efficiency includes maintenance cost while *net* efficiency does not. *Gross* energetic efficiency is the efficiency of the entire body as milk producer; *net* energetic efficiency is the efficiency of the mammary gland mechanism as milk producer (under the given conditions) independent of the rest of the body. The average net efficiency of milk production is, according to equation (1b), 61% $\left(= \frac{340 \times 100}{(0.305 \times 1814)}\right)$. It is interesting to note that the net efficiency of milk production is of the same order as the net efficiency of growth of chick embryos (Needham) and of chicks (Kleiber), and is about twice as great as that found for maximum work (pulling loads on a horizontal plane) of horses (Missouri Research Bulletin 209).

(c) *Influence of live weight on energetic efficiency of milk production:* Other conditions being equal, is a large cow more energetically efficient as a milk producer than a small cow? It is not possible to give a categorical reply, because other conditions cannot be kept equal. Statistical investigations of the 243 lactation records tabulated in detail in the appendix of this bulletin, and summarized in Table 2 and Fig. 2, show that the *average* gross efficiency of small cows is somewhat higher than of large cows. The differences are in most cases *statistically* significant. However, we are not convinced that they are physiologically significant because heavier cows may be fatter or they may be overfed in comparison to lighter cows. It is generally known that while excess fat in the body does not produce extra milk, it does increase maintenance cost and therefore decreases gross efficiency. It was previously noted (Fig. 1) that

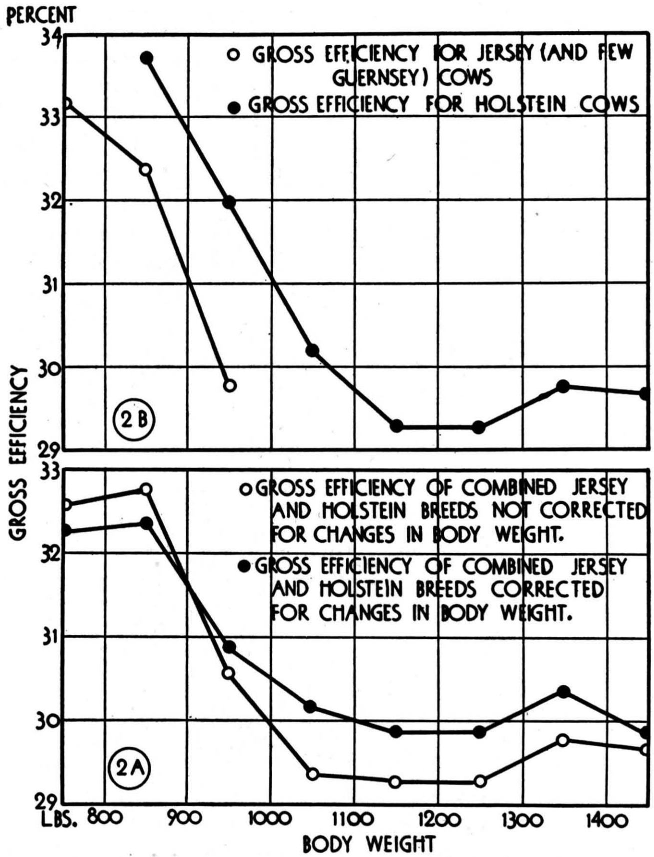


Fig. 2.—Gross efficiency as function of body weight of the experimental cows. The efficiency differences between the Holstein and Jersey curves for the same live weights are not significant statistically (Fishers tables).

current feeding standards, and customs, tend to overfeed heavier cows in comparison to lighter ones, and substantiated by the greater weight losses of the lighter cows during lactation (See Table 2).

The lower curves in Fig. 2 show that the *average* gross efficiency $(=\frac{100 \times 340 \times FCM}{1814 \times TDN})$ of the cows declines from 32.3% for 750 pound cows to 30% for 1000 pound cows; it remains constant (at about 30%) between live weights of 1000 and 1500 lbs. The upper curves in Fig. 2 show different efficiencies for Jersey and Holstein cows; but these differences for corresponding live weights are not statistically significant. The 2½% differences of 900 and 1000 pound cows of the same breed are significant statistically, but as previously noted, we are inclined to believe that if fatness, relative feed levels and other conditions were the same, gross energetic efficiency would be independent of body weight. Theoretical reasons for this belief were furnished by Kleiber as explained in the review of literature (in the appendix). We are planning to secure more comparable data for a future analysis.

In this connection, Fig. 3 may be of interest on account of its similarity to Fig. 9 in Missouri Research Bulletin 209 which showed the relation between efficiency of work in horses, body weight and rate of work. Similarly Fig. 3 shows the relation between efficiency of milk production, body weight and rate of milk production. The curves in Fig. 3 were drawn from equation (1b).

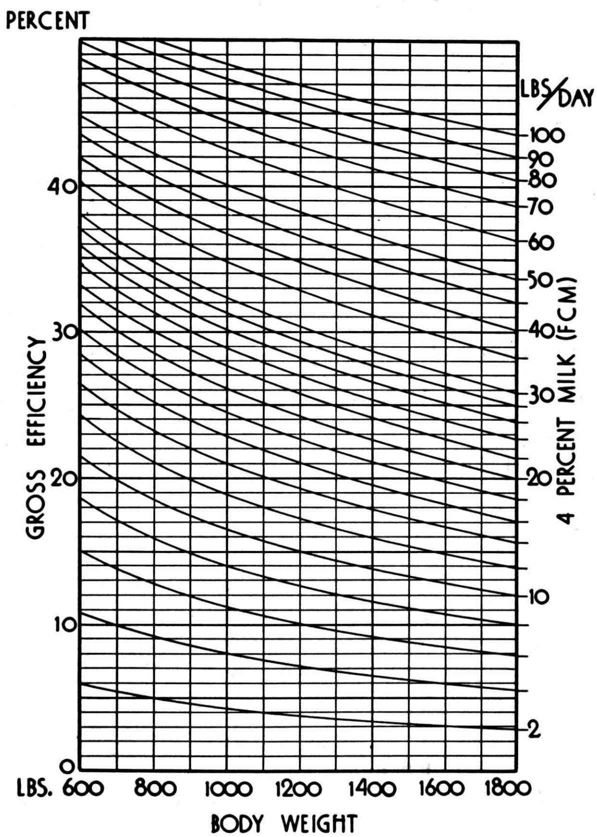


Fig. 3.—Gross efficiency of milk production of cows of different live weights producing milk at different rates. Compare to Fig. 9, page 29, Missouri Research Bulletin 209.

TABLE 3.—*Register of Merit* JERSEY DATA

Body wt. lbs.	2 yr. olds (23, 24, 25 Mos.)		3 yr. olds (35, 36, 37 Mos.)		4 yr. olds (47, 48, 49 Mos.)		5 yr. olds (59, 60, 61 Mos.)		6-8 yr. olds		Reentries 6-8 yr. olds		Reentries 8-10 yr. olds	
	No. cows	FCM lbs/yr	No. cows	FCM lbs/yr	No. cows	FCM lbs/yr	No. cows	FCM lbs/yr	No. cows	FCM lbs/yr	No. cows	FCM lbs/yr	No. cows	FCM lbs/yr
500	5	7448	--	----	--	----	--	----	--	----	--	---	---	----
550	--	----	--	----	---	----	--	----	--	----	--	----	--	----
600	26	7485	1	6250	--	----	--	----	2	10650	--	----	--	----
650	43	7448	7	8106	2	8969	--	----	1	8250	--	----	--	----
700	151	7535	26	8197	8	8085	6	9810	5	9760	--	----	1	12250
750	163	8053	38	8208	23	8888	11	9853	17	10270	14	10830	4	11870
800	228	8120	87	8673	70	9514	44	10140	81	10220	29	11560	10	12000
850	98	8254	64	9155	58	9772	42	9864	76	10790	52	11060	17	11010
900	91	8840	58	8591	82	9797	48	9976	134	10430	104	11540	32	11370
950	29	9312	26	9440	31	10040	38	10720	95	10960	91	12040	26	13000
1000	11	8928	21	9096	20	10810	37	10600	121	11250	120	12460	57	12130
1050	2	9972	6	9999	9	8938	8	10720	26	11350	36	12840	15	15010
1100	1	8525	2	12980	1	11250	5	9703	16	10690	36	13590	18	13570
1150	1	10062	--	----	--	----	4	10510	3	10560	7	17140	4	12850
1200	--	----	1	8250	--	----	1	12750	3	12030	28	13930	6	14320
1250	--	----	--	----	--	----	1	9250	--	----	1	12250	--	----
1300	--	----	--	----	--	----	--	----	1	12250	--	----	1	12250
1350	--	----	--	----	--	----	--	----	--	----	1	13750	--	----

B. REGISTER OF MERIT JERSEY CATTLE

The above discussion on efficiency was concerned with milk production of cows for which feed consumption and weight-gain data were available. There is also available a large body of milk production data on *Register of Merit* Jersey cattle but without feed consumption and weight-gain records. We shall attempt to substantiate the conclusion obtained on the experimental animals, presented in the preceding section by analyses of the *Register of Merit* data in this section. To avoid time complications (due to improvements in breeding, feeding, etc.) we confined the analyses to data in *Register of Merit* year books 1918, 1919, 1920. To avoid age complications, we confined the analyses to 3-month intervals in ages 2, 3, 4 and 5 years; and to 2-year intervals in ages 6 to 8 and 8 to 10 years as shown in Tables 3 and 4A.

TABLE 4A.—STATISTICAL CONSTANTS* FOR EQUATIONS RELATING ENERGY IN MILK TO BODY WEIGHT

Age	No. of rec-ords	Constants of equations (7) and (8)				Constants for equation Y = C+Dx	
		A	B	n	ρ	C	D
2 yrs. (23, 24, 25 Mos.)	849	390	363	.455	.268	4177	5.01
3 yrs. (35, 36, 37 Mos.)	338	717	668	.372	.227	5624	3.74
4 yrs. (47, 48, 49 Mos.)	304	809	753	.367	.193	5886	4.38
5 yrs. (59, 60, 61 Mos.)	245	2662	2480	.198	.145	8165	2.28
6-8 yrs.	581	1256	1170	.315	.175	7453	3.60
6-8 yrs. (re-entries)	449	728	678	.411	.209	3984	8.57
8-10 yrs. (re-entries)	191	294	274	.545	.252	5408	7.26

**A* is the regression constant of equation (7) when FCM is in pounds per year; *B* is the regression constant of equation (8) when FCM is Calories per day; *n* is the exponent of power equations (7) and (8); ρ is the correlation coefficient between logs of live weight and logs FCM (usually referred to as index of correlation, or coefficient of curvilinear correlation). C and D are regression coefficients in the given linear equation in which Y is pounds FCM per *year*.

1. Milk Production as Function of Body Weight at Constant Age:— Fig. 4 represents FCM (4% milk) plotted against live weight for the indicated ages. The power equation

$$\text{FCM} = \text{AM}^{\text{n}} \qquad (7)$$

was fitted to the data. FCM represents pounds 4% milk produced per year, and M represents pounds live weight. Equation (7) was fitted to the data by the method of least squares with the results shown in Fig. 4. Table 4A gives the numerical values of A and n of the power equation (7) and also of a linear equation.

Instead of pounds FCM per year, the results may be represented in terms of Calories per day. Converting pounds FCM per year to Calories per day we have

$$\text{FCM} = \frac{340}{365} \text{AM}^n = \text{BM}^n \quad (8)$$

in which $B = \frac{340}{365}$ A. The numerical values of A, B, n and ρ are given in Table 4A. Corresponding equation constants for McDowell's data are given in Table 4B.

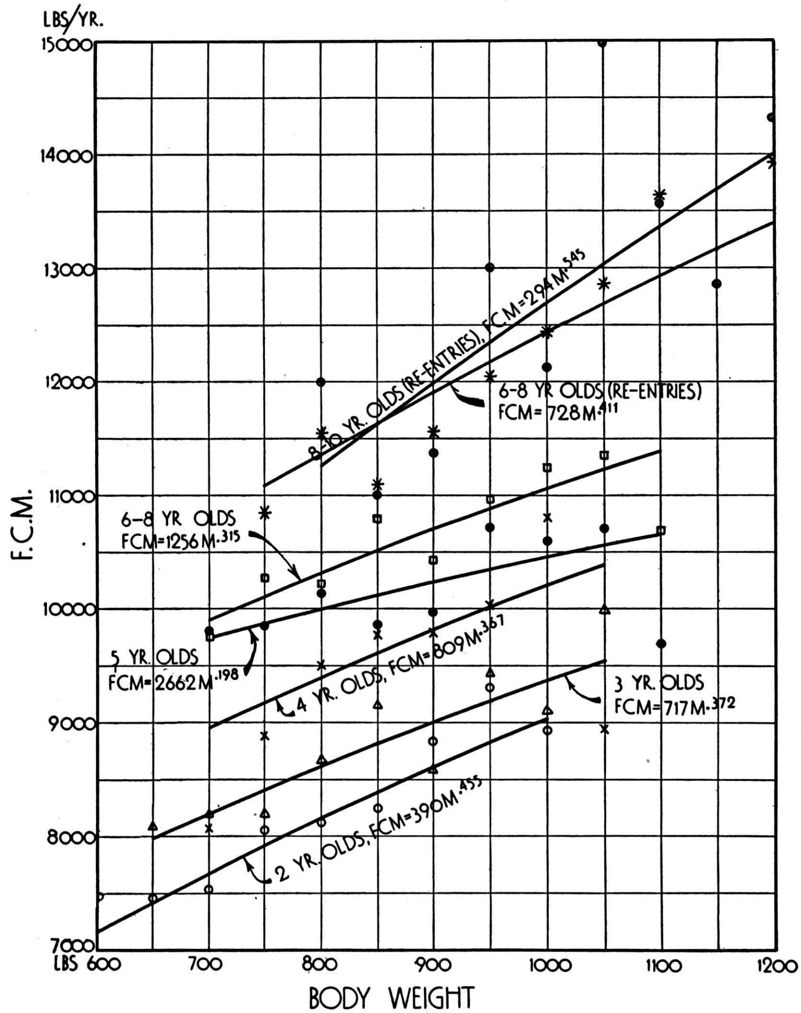


Fig. 4.—FCM (4% milk) production at given ages as function of body weight, plotted from the data in table 3.

TABLE 4B.—CONSTANTS FOR EQUATION RELATING MILK PRODUCTION (FCM) TO BODY WEIGHT OF McDOWELL'S DATA

	Log-log equation		Linear equation	
	A	n	A	B
Grade Jerseys	462	.405	4380	3.19
Register Jerseys	627	.366	4853	2.99
Grade Holstein	184	.536	3629	3.81
Register Holstein	158	.567	3832	4.13
Grade Guernsey	531	.382	4542	2.86
Register Guernsey	1059	.280	5202	2.09
Grade Ayrshire	124	.585	2794	4.26
Register Ayrshire	53.5	.704	2056	4.88
Grade Shorthorn	218	.485	3232	2.98

2. Influence of Live Weight of Cows on Gross Efficiency of Milk Production.—Since the value of the exponent, n, for milk production in Table 4A, and in Fig. 4, is between 0.20 and 0.55, while the value of the exponent in the maintenance equation (see equations 1, 2, 3) is 0.73, therefore maintenance cost increases more rapidly with live weight than milk production. This means that the larger the cow the greater the maintenance cost in comparison to milk production, and therefore the less the gross efficiency. This conclusion is substantiated by the following computations.

As defined by equation (5), gross efficiency of milk production is the ratio of energy in FCM produced to energy in TDN consumed (assuming that there is no gain or loss in live weight during lactation). We have no data on TDN consumed for these *Register of Merit* cows, but for our purpose we may assume that the TDN requirements (for maintenance and for FCM) are the same as for the experimental animals represented by equation (1b). Gross efficiency was then computed from equations (1b) and (5b)

$$\text{TDN} = 0.305\text{FCM} + 0.053\text{M}^{0.73} \qquad (1b)$$

$$\text{gross efficiency} = \frac{100 \times 340 \times \text{FCM}}{1814 \times \text{TDN}} \% \qquad (5b)$$

with the results shown in Table 5 and Fig. 5 (in which McDowell's data are also plotted).

Table 5 and Fig. 5 show that gross efficiencies tend to decline with increasing live weight (at constant age). However, as previously noted, such small declines while statistically significant, might not have been so if other conditions had been the same. It may be noted incidentally that the *Register of Merit* cows (Table 5) show a higher gross efficiency than the experimental animals (Table 2). This is no doubt due to the fact that the R. M. cows produced more milk in comparison to maintenance costs.

*TABLE 5.—INFLUENCE OF LIVE WEIGHT ON PER CENT GROSS EFFICIENCY OF MILK PRODUCTION IN REGISTER OF MERIT JERSEY DATA. TDN CONSUMPTION WAS COMPUTED FROM EQUATION (1b)

Body weight lbs.	Age, years						
						Re-entries	
	2	3	4	5	6-8	6-8	8-10
600	32.2	----	----	----	----	----	----
650	31.2	----	----	----	----	----	----
700	30.5	31.8	----	----	----	----	----
750	30.8	31.1	32.3	33.9	34.5	35.3	----
800	30.2	31.2	32.6	33.6	33.7	35.6	36.1
850	29.8	31.4	32.4	32.5	33.8	34.2	34.2
900	30.2	29.7	31.8	32.0	32.7	34.2	34.0
950	30.4	30.6	31.5	32.5	32.9	34.3	35.4
1000	29.2	29.4	32.1	31.8	32.7	34.2	33.8
1050	----	----	----	----	32.3	34.1	36.5
1100	----	----	----	----	31.0	34.7	34.7

*Each figure represents average of 10 or more observations of FCM. TDN was computed from equation (1b).

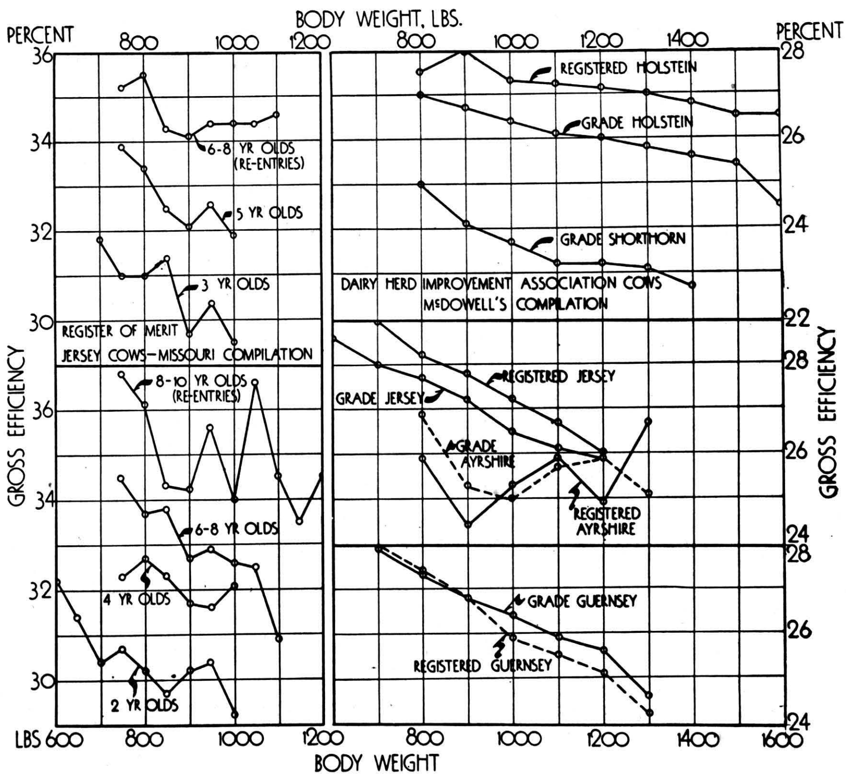


Fig. 5.—Gross efficiency as function of body weight. The left side represents data compiled by us from the Jersey *Register of Merit*; the right side represents McDowell's compilation from Dairy Herd Improvement Association data (plotted from the McDowell data as quoted by Gaines. See appendix for discussion of Mc Dowell and Gaines' results.)

The above results may be criticised on the grounds that the efficiency computations on the *Register of Merit* data were based on the TDN results of the experimental cows with all their errors of fact and treatment. However, the same results are obtained on the basis of other assumptions as follows: If live weight remains constant, then TDN is used for (a) milk production and (b) maintenance, and equation (5b) may be written

$$\text{gross efficiency} = \frac{100 \times 340 \times \text{FCM}}{\text{A}(340 \times \text{FCM}) + \text{B}(39.5\text{M}^{0.73})}\% \qquad (9)$$

in which (340 × FCM) represents caloric value of the milk produced; $39.5\text{M}^{0.73}$ represents basal metabolism Calories (as explained in Missouri Research Bulletin 220); *A* is a factor for converting energy in milk to total energy for producing milk; *B* is a factor for converting basal metabolism calories to total maintenance calories. Dividing numerator and denominator of equation (9) by 340 × FCM, we obtain

$$\text{gross efficiency} = \frac{100}{\text{A} + \dfrac{\text{B} \times 39.5\text{M}^{0.73}}{340 \times \text{FCM}}} = \frac{100}{\text{A} + \text{C}\left(\dfrac{\text{M}^{0.73}}{\text{FCM}}\right)}\% \qquad (10)$$

in which $\text{C} = \dfrac{\text{B} \times 39.5}{340}$. Assuming, as shown in Fig. 4, that FCM is a function of weight, M, raised to some power, n, we may replace FCM by DM^{n} in equation (10) and obtain

$$\text{gross efficiency} = \frac{100}{\text{A} + \text{C}\left(\dfrac{\text{M}^{0.73}}{\text{DM}^{\text{n}}}\right)} = \frac{100}{\text{A} + \text{EM}^{0.73-\text{n}}}$$

From equations (10) and (11) it is clear that if n (the exponent relating milk production with weight) is less than 0.73 (the exponent relating maintenance cost with live weight) the efficiency declines with increasing live weight. Assuming, as seems reasonable, that B = 2 (i. e., that the digestible energy cost of maintenance is double the energy of basal metabolism) and A = 2 (i. e., that the digestible energy cost of milk production is double the energy in the milk), let us compute the gross efficiencies of milk production by different body weights. The results in Table 6 show, as before, that efficiency declines somewhat with increasing live weight.

Incidentally, the efficiency values given in Tables 5 and 6 tend to increase with increasing age, because as shown in Fig. 4, milk production tends to increase with increasing age for given body weights.

There are small differences between *absolute* efficiency values in Tables 5 and 6, on account of small differences in the conversion factors.

Thus as regards digestible energy cost of milk production, in Table 6 it was assumed to be double the energy in the milk, while in table 5 it was assumed to be only 1.64 $\left(= \frac{0.305 \times 1814}{340}\right)$ times the energy in the milk. As regards digestible energy cost of maintenance, in Table 6 it was assumed to be double the basal metabolism, while in Table 5 it was assumed to be 2.43 $\left(= \frac{0.043 \times 1814}{39.5}\right)$ times the energy of basal metabolism. However, the *relative* declines in efficiency of milk production with increasing live weight are practically the same in Tables 5 and 6.

TABLE 6.—INFLUENCE OF LIVE WEIGHT ON ESTIMATED PERCENTAGE GROSS EFFICIENCY. COMPUTED FOR *Register of Merit* JERSEY COWS FROM EQUATION (11). (ASSUMING THAT IN EQUATION (10) A = 2, AND B = 2.)

Body weight lbs.	Age, years						
						Re-entries	
	2	3	4	5	6-8	6-8	8-10
500	30.9	32.2	33.5	35.0	34.9	35.1	34.7
600	30.3	31.4	32.8	34.0	34.1	34.5	34.4
700	29.8	30.8	32.2	33.1	33.5	33.9	34.1
800	29.3	30.2	31.6	32.3	32.8	33.5	33.8
900	28.9	29.7	31.1	31.5	32.3	33.0	33.6
1000	28.5	29.2	30.7	30.9	31.8	32.7	33.4
1100	28.2	28.8	30.3	30.3	31.3	32.3	33.2
1200	27.9	28.4	29.9	29.7	30.9	32.0	33.0
1300	27.7	28.1	29.6	29.2	30.5	31.7	32.8
1400	27.4	27.8	29.2	28.7	30.2	31.4	32.7

C. APPLICATIONS

It will be instructive to conclude the above theoretical discussion with one or two practical applications.

It is necessary to emphasize that this is a preliminary report and the equation constants are altogether tentative (the given "experimental" data from different sources lead to different conclusions indicating that the experimental conditions are not comparable).

From equation (1b) it is easy to construct several charts which will enable the dairyman to estimate the TDN (total digestible nutrients) requirements of cows of different live weights* and productivities. Such charts are presented, by way of illustration, in Figs. 6 and 7, and in Table 7. The legends accompanying these charts supply full directions for their use.

Before using Figs. 6 or 7, or Table 7, the given milk must first be converted to standard (4%) milk by the method of Gaines' (Univ. Ill. Agric. Expt. Sta. Bul. 308, 1928) or by modifications of this method

*We assumed that the TDN needs for maintenance increase with the 0.73 power of body weight, which assumption may be wrong.

as explained in Missouri Station Bulletin 351, or by the alignment chart shown in Fig. 8 (both based on Gaines' formula).

Equation (1b) may also be used for preparing an alignment chart for estimating efficiencies of milk production of individual cows of different body weights and productivities. Thus dividing both sides of equation (1b) by FCM, we obtain

$$\frac{TDN}{FCM} = 0.305 + \frac{0.053M^{0.73}}{FCM}$$

or

$$\frac{FCM}{TDN} = \frac{1}{0.305 + \dfrac{0.053M^{0.73}}{FCM}}$$

Since 1 pound FCM = 340 Calories, and 1 pound TDN = 1814 Calories; and $\frac{340}{1814} \times 100 = 18.74$, therefore gross efficiency per cent

$$= \frac{1}{0.305 + \dfrac{0.0535M^{0.73}}{FCM}} \times 18.74.$$

An alignment chart for estimating percentage efficiencies for different body weights and productivities may be constructed from this equation and from the fact that when FCM becomes infinitely large, *gross* efficiency approaches *net* efficiency (i.e., 61, or $\frac{18.7}{0.305}$) as limit. Such an alignment chart is presented for dairymen's use in Missouri Station bulletin 351. Bulletin 351 gives a table showing

TABLE 7.—POUNDS DIGESTIBLE NUTRIENTS REQUIRED BY COWS OF DIFFERENT LIVE WEIGHTS (UPPER HORIZONTAL ROW) PRODUCING DIFFERENT AMOUNTS OF 4% MILK (LEFT COLUMN). THIS TABLE WAS COMPUTED FROM EQUATION (1b) IN THE TEXT.

4% milk lbs. per day	Body Weight, Pounds														
	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
0	5.7	6.3	7.0	7.6	8.2	8.8	9.4	10.0	10.5	11.0	11.6	12.1	12.6	13.1	13.6
5	7.2	7.9	8.5	9.1	9.7	10.3	10.9	11.5	12.0	12.6	13.1	13.6	14.1	14.6	15.1
10	8.7	9.4	10.0	10.7	11.3	11.9	12.4	13.0	13.5	14.1	14.6	15.1	15.7	16.2	16.8
15	10.2	10.9	11.5	12.2	12.8	13.4	14.0	14.5	15.1	15.6	16.1	16.7	17.2	17.7	18.2
20	11.8	12.4	13.1	13.7	14.3	14.9	15.5	16.0	16.6	17.1	17.7	18.2	18.7	19.2	19.7
25	13.3	14.0	14.6	15.2	15.8	16.4	17.0	17.6	18.1	18.7	19.2	19.7	20.2	20.7	21.2
30	14.8	15.5	16.1	16.8	17.4	18.0	18.5	19.1	19.6	20.2	20.7	21.2	21.8	22.3	22.8
35	16.3	17.0	17.6	18.3	18.9	19.5	20.1	20.6	21.2	21.7	22.2	22.8	23.3	23.8	24.3
40	17.9	18.5	19.2	19.8	20.4	21.0	21.6	22.1	22.7	23.2	23.8	24.3	24.8	25.3	25.8
45	19.4	20.1	20.7	21.3	21.9	22.5	23.1	23.7	24.2	24.8	25.3	25.8	26.3	26.8	27.3
50	20.9	21.6	22.2	22.9	23.5	24.1	24.6	25.2	25.7	26.3	26.8	27.3	27.9	28.4	28.9
55	22.4	23.1	23.7	24.4	25.0	25.6	26.2	26.7	27.3	27.8	28.3	28.9	29.4	29.9	30.4
60	24.0	24.6	25.3	25.9	26.5	27.1	27.7	28.2	28.8	29.3	29.9	30.4	30.9	31.4	31.9
65	25.5	26.2	26.8	27.4	28.0	28.6	29.2	29.8	30.3	30.9	31.4	31.9	32.4	32.9	33.4
70	27.0	27.7	28.3	29.0	29.6	30.2	30.7	31.3	31.8	32.4	32.9	33.4	34.0	34.5	35.0
75	28.5	29.2	29.8	30.5	31.1	31.7	32.3	32.8	33.4	33.9	34.4	35.0	35.5	36.0	36.5
80	30.1	30.7	31.4	32.0	32.6	33.2	33.8	34.3	34.9	35.4	36.0	36.5	37.0	37.5	38.0
85	31.6	32.3	32.9	33.5	34.1	34.7	35.3	35.9	36.4	37.0	37.5	38.0	38.5	39.0	39.5
90	33.1	33.8	34.4	35.1	35.7	36.3	36.8	37.4	37.9	38.5	39.0	39.5	40.1	40.6	41.1
95	34.6	35.3	36.0	36.6	37.2	37.8	38.4	38.9	39.5	40.0	40.5	41.1	41.6	42.1	42.6
100	36.2	36.8	37.5	38.1	38.7	39.3	39.9	40.4	41.0	41.5	42.1	42.6	43.1	43.6	44.1

gross efficiencies of milk production corresponding to different live weights and productivities of cows. By these tables animals can be selected for breeding purposes on the basis of individual gross efficiencies of milk production.

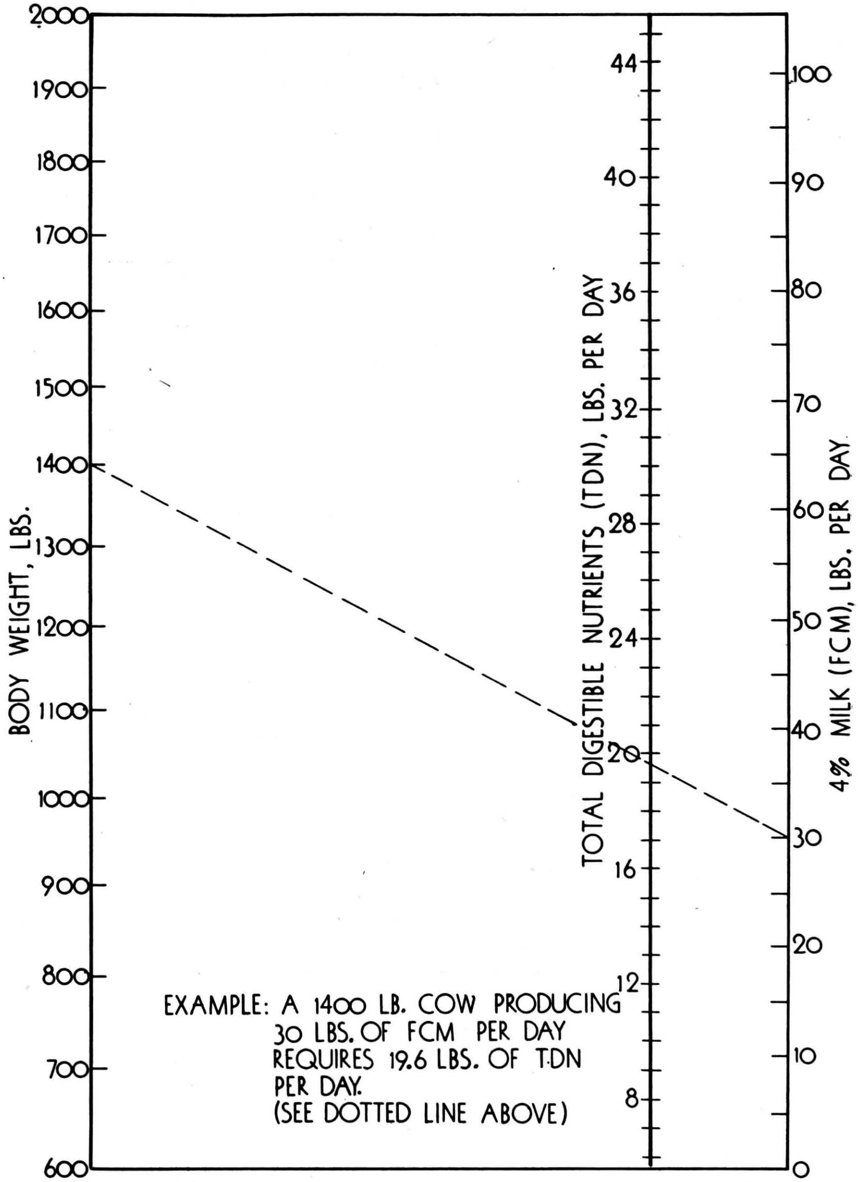


Fig. 6.—An alignment chart for estimating TDN (total digestible nutrients) required by cows of different live weights producing different amounts of standard (4%) milk. Stretch a string across the chart from a point of the left (body-weight) scale representing the cows live weight, to a point on the right (milk-production) scale representing 4 per cent milk production. The inside (TDN) scale crossed by the string gives the total digestible nutrients required by the cow. Thus a 1400 pound cow producing 30 pounds of FCM (4% milk) per day requires 19.6 pounds of TDN (total digestible nutrients) per day to just cover her needs (without gain or loss in body weight).

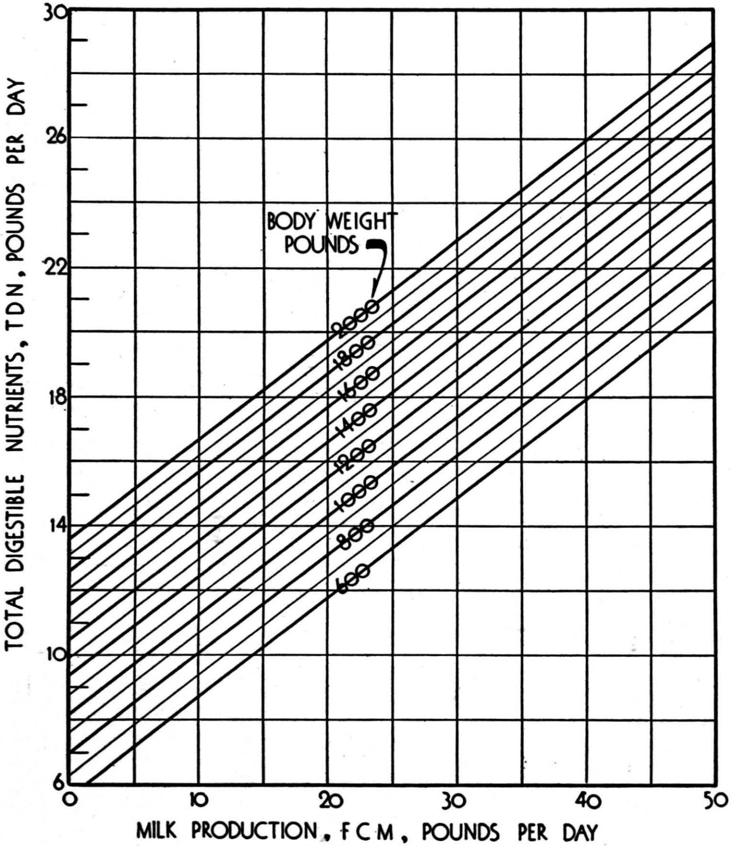


Fig. 7a.—Total digestible nutrients as function of milk production (from 0 to 50 pounds FCM) for different live weights.

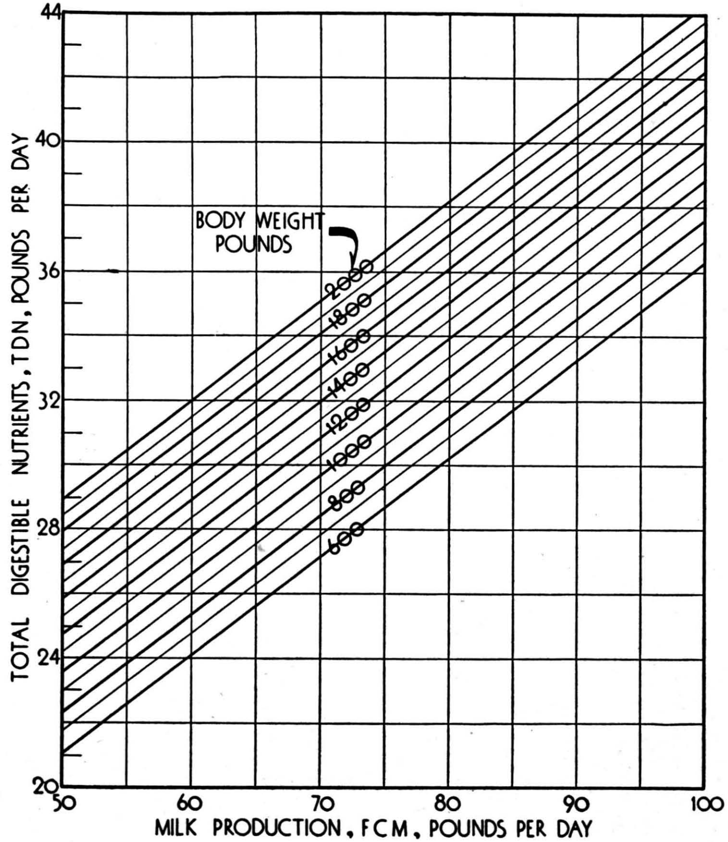


Fig. 7b.—Continuation of Fig. 7a for milk production levels 50 to 100 pounds.

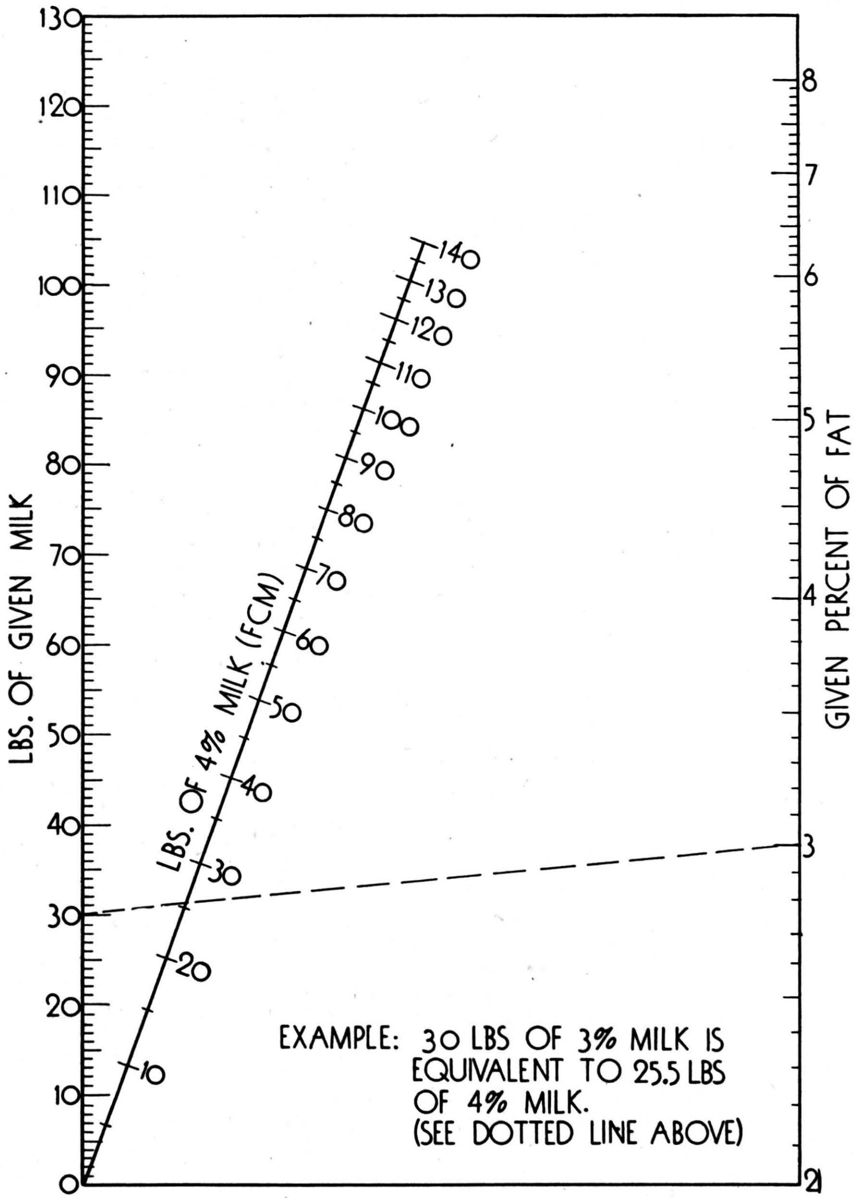


Fig. 8.—An alignment chart for converting pounds of milk containing any per cent of fat to milk containing 4 per cent of fat. Thus to convert 30 pounds 3% milk to pounds 4% milk, stretch a string between 30 on the left scale and 3 on the right scale and read the answer (25.5 pounds 4% milk) on the middle scale. This alignment chart was constructed from Gaines' well known formula, as was also the conversion table in Missouri Station Bulletin 351.

Since the *net* efficiency of milk production, or the efficiency of the mammary gland as a machine (not counting maintenance cost of the cow), is about 60%, the *gross* efficiency (including maintenance cost of cow) cannot reach 60%; but the greater the milk production, the less in proportion is the maintenance cost and the greater the gross efficiency. The gross efficiency will therefore increase at decreasing rate in its approach to the theoretical 60% maximum. This is illustrated in Fig. 9.

SUMMARY AND CONCLUSIONS

We determined by statistical methods the distribution of the dietary digestible nutrients between milk production, maintenance cost, and gain in live weight of 243 mature Holstein, Jersey and Guernsey cows. The results are given by the equation $TDN = 0.305\ FCM + 0.053M^{0.73} + 2.1\ \Delta M$, in which TDN is pounds total digestible nutrients consumed per day, FCM pounds 4% milk produced per day, M pounds live weight, ΔM pounds gain or loss in live weight. This equation shows that if other variables are held constant, 0.305 pound TDN is required to produce 1 pound FCM (= 4% milk); 2.1 pounds TDN are required to gain 1 pound live weight. This equation may be interpreted as follows: 1. The *net* digestible feed energy cost of milk production (not counting costs of maintenance or of gain in weight) is about 1.6 times the milk energy; or what is the same, the *net* (or partial) energetic efficiency of milk production (ratio of milk energy to digestible feed energy less maintenance energy) is about 60%. This is the efficiency of the mammary gland, and is the theoretical maximum energetic efficiency of milk production. 2. The *gross* digestible feed energy cost of milk production (including maintenance cost of cows) is about 3 times the milk energy; or what is the same, the *gross* (or overall, or total) energetic efficiency of milk production (ratio of milk energy to total digestible feed energy) is about 30% (15-25% in poor producers; 25-35% in average producers; 35-45% in superior producers). The computations were carried out on the basis of 12-month records. 3. The digestible energy cost of maintenance is about 2.4 times the basal energy metabolism.

In carrying out the computations it was assumed that the energy equivalence of TDN (total digestible nutrients) is 1814 Calories per pound or 4 Calories per gram, and that 1 pound FCM (4% milk) has a combustion value of 340 Calories (Gaines).

It was assumed (on the basis of results presented in Missouri Research Bulletin 220) that the maintenance cost increases not with

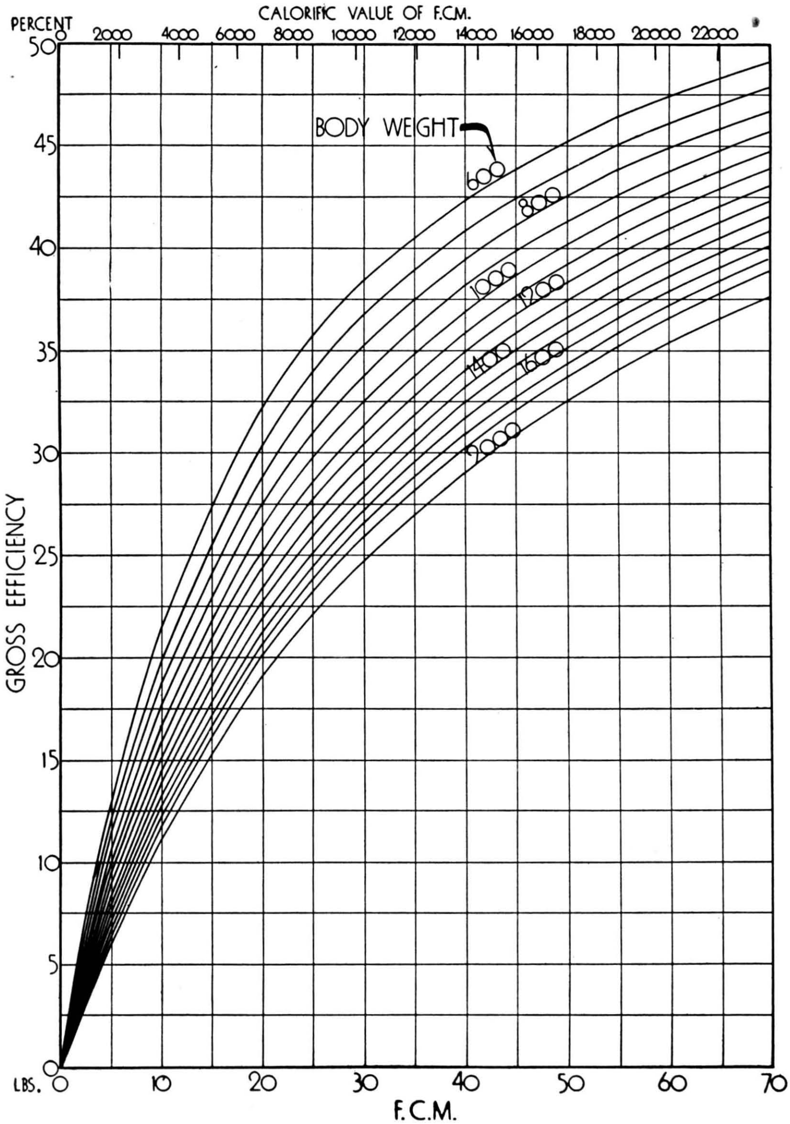


Fig. 9.—Gross efficiency as function of FCM (4% milk) production at different live weights. As milk production increases the gross efficiency likewise increases but at decreasing rates in accordance with the law of diminishing returns, approaching a theoretical maximum of about 60% efficiency (the efficiency of the mammary gland itself).

simple body weight, as is implicit in the current feeding standards, but with the 0.73 power of body weight (increasing body weight by 100% increases maintenance cost not by 100% but by about 70%).

Gross efficiency of milk production in the given group of cattle declined with increasing live weight, as shown in Table 2 and Fig. 2. Similar results were obtained on a large body of *Register of Merit* Jersey Cattle, as indicated in Tables 5 and 6 and in Figs. 4 and 5. These results confirm Gaines' conclusion derived from McDowell's Dairy Herd Improvement Association data. However, we believe that if other conditions were the same, gross energetic efficiency of milk production would probably be independent of live weight. The larger cows were probably overfed and were fatter in comparison to the smaller ones as inferred from the following facts: 1. From considerations of basal metabolism and endogenous nitrogen relationship it appears that Morrison's feeding standard allows relatively more feed for heavier than for lighter cows. 2. Lighter cows in this research tended to lose more weight during lactation than heavier cows. 3. Since larger cows tend to produce more milk than smaller there is a tendency to be more liberal in feeding large than small cows. Efficiency tends to decline with increasing nutritive level due to decreasing digestibility, increasing specific dynamic action and greater maintenance cost. We believe therefore that if hereditary capacity for milk production, nutritive level including body fatness, and activity were the same, the energetic efficiency of milk production would probably be the same regardless of body size. Under customary conditions, however, gross energetic efficiency tends to decline with increasing live weight.

APPENDIX

1. BRIEF REVIEW OF LITERATURE

Since the present investigation proposes a new maintenance standards (see also Missouri Research Bulletin 220) we should perhaps review the various feeding standards [Grouven, 1858; Wolff, 1864; Wall, 1894; Smith, 1897; Atwater & Phelps, 1894-97; Kuhn, 1897; Lehmann, 1899; Maucker, 1902; Hansson, 1902; Kellner, 1905-6; Wall & Humphrey, 1910; Haecker, 1904, 1912, 1914; Savage, 1912; Eckles, 1913; Armsby, 1916; Morrison, 1923]. Since such reviews have been given before, it does not seem appropriate to add another. But we will quote a tabular summary from Kriss (J. Nutrition, **4**, 141, 1931).

SUMMARY OF FEEDING STANDARDS IN TERMS OF POUNDS TOTAL DIGESTIBLE NUTRIENTS (TDN).

Author	Feeding standard for maintenance per 1000 lbs. live weight	Feeding standard for milk production per lb. FCM (= 4% milk) lbs. TDN
Kellner	6.67	-----
Armsby	6.46	0.285
Mollgaard	5.86	0.302
Hansson	5.64	0.302
Morrison	7.93	0.328
Forbes & Kriss	5.97	0.300
Tentative Missouri "Standard"	varies with body wt. according to formula $0.053M^{0.73}$ as shown in Fig. 1.	0.305

Our second concern is *efficiency* of milk production, and the influence of live weight of cows thereon.

We have already mentioned that Forbes & Voris (J. Nutrition, **5**, 395, 1932) found that the ratio of energy in milk produced to *total* energy of feed consumed by nine cows during a lactation period of about 300 days was 18% to 23%. However, the efficiency values of Forbes & Voris are not comparable to ours, because they computed milk energy as percentage of *total* feed energy consumed, while we computed milk energy as percentage of *digestible* nutrients. Since cattle feeds vary enormously in their digestibilities, it seemed to us illogical to use in the computations the indigestible part of the feed which animals did not use. Our computations, therefore, on the basis of *digestible* feed are nearer "true" efficiency.

McDowell, Gaines, and Kleiber studied the influence of live weight of cows on efficiency of milk production. McDowell concluded that "within the breed the big dairy cows excell" (U. S. D. A. Circ. 114, 1930). Gaines (J. Dairy Sc., *14*, 14, 1931) examined McDowell's data from the standpoint of *energetic* (rather than dollar) efficiency and concluded, on the contrary, that small cows are more efficient. Gaines found the following fallacies in treatment of the data: 1. The bookkeeping tended to overcharge the small cow for her feed and undercharge the large cow. Thus pasturage charges per cow disregard size (the same was probably true of roughage charges). 2. A large cow may be more profitable not because of size, but becau e of lower fat percentage. If the price per quart of milk is the same, the larger cows' milk is sold at a higher price per calorie. However, "this market advantage of low-energy milk is now pretty much a thing of the past." 3. Larger cows may be more advantageous commercially because fewer are needed to produce a given volume of milk. Thus the cost of milking is greater in goats (than in cows) "due to the small size and output of the individual machine". 4. Mc-

Dowell stated that the relationship "in regards to production. . . . .as existed among individuals of the same breed did not exist among individuals of different breeds". Gaines proved on the basis of McDowell's data that the effect of increase of live weight on milk yield is the same for Jersey, Guernsey, Ayrshire, and Holstein cattle. 5. McDowell's computations were based on milk yield. Since energy of milk varies with its fat content, the milk should have been converted to a uniform fat percentage (i. e., to FCM or 4% milk as proposed by Gaines) before computing efficiency.

Gaines accepted Morrison's feeding standard: 0.327 pounds TDN to 1 pound FCM (4% milk) and 2.893 pounds TDN to maintain 1 pound live weight per year. He assumed that 1 pound FCM is equivalent to 0.172 pounds digestible nutrients. It follows that the net efficiency of milk production ("the efficiency of the mammary gland as apart from the body") is $\frac{0.172}{0.327} = 52.6\%$ (as compared to 61% estimated by us). The gross efficiency is of course (see equation 5b in text for details)

$$\frac{100 \times 0.172\text{FCM}}{0.327\text{FCM} + 2.893\text{M}} = 100 \times \frac{\frac{0.172\ \text{FCM}}{0.327}}{\text{FCM} + \frac{2.893\text{M}}{0.327}} = 52.6 \times \frac{\text{FCM}}{\text{FCM} + 8.47\text{M}}$$

where FCM is 4% milk and M is live weight of cow. Gaines called the solution of this equation the "coefficient of efficiency", while we would have called it gross (or total or overall) efficiency. (Compare this equation of Gaines with our equations 5a, 5b, 5c, 6a, and 6b.) Gaines substituted McDowell's data in the above formula, and found the "coefficient of efficiency" to decline from about 28 for 600-pound cows to about 22 for 1400-1600 pound cows. (See Figs. 2 and 5 for our findings on influence of live weight on gross efficiency).

McDowell found that efficiency of milk production increases with increasing live weight. Gaines found that efficiency of milk production decreases with increasing live weight. Kleiber [Beiderman's zentrbl. Abt. B (Tierernähr.) *5*, 1933] argued that efficiency of milk secretion is independent of live weight. The essence of Kleiber's argument is simpler than its formulation. The available food is used for maintenance and production. If the ratio of production to maintenance is the same in two animals of different size, then their productive efficiencies must be the same. Actually, Kleiber argued on the basis of the ratio of maximum food intake (U) to basal metabolism (B). He found the $\frac{U}{B}$ ratio is the same (about 5) for steers, chicks, and rabbits; hence the generalization that productive efficiency is independent of size. We believe that our data on the efficiency of work of horses (Missouri Research Bulletin 209), and the data analyzed in this bulletin on efficiency of milk production, favor Kleiber's generalization rather than the particular conclusions of either McDowell or Gaines. While it is undoubtedly true that gross energetic efficiency of milk production decreased somewhat with increasing size of cow, we believe that this decrease was not due to size as such but to other factors, the most important of which was the tendency to increasing overfeeding and fatness with increasing body size.

Incidentally, the "*efficiency quotient*" of Palmer & Kennedy (Proc. Soc. Exp. Biol. and Med., *26*, 427, 1929; J. Biol. Chem. *90*, 545, 1931) and Morris, Palmer & Kennedy (Univ. Minn. Agric. Exp. Station, Tech. Bull. 92, 1933) can not be true if our assumption of the independence of efficiency from body weight is true, because Palmer's "efficiency quotient", which may be defined by the ratio $\frac{\text{food consumed}}{\text{gain in weight} \times \text{weight} \times 100}$, assumes that efficiency is an inverse function of body weight. However, this assumption has not been proved (see Brody, Annual Review of Biochemistry, Vol. IV, 1935).

Finally, our "gross efficiency" is termed by Kleiber "total" efficiency. Our "net efficiency" is termed by Kleiber "partial efficiency", since it represents a *part* of the *total* available food used for production. Kleiber used in his discussion the following definitions and equations:

$$W_p = \frac{N}{U - E} \qquad \text{(a)}$$

where U = available energy in total food; E available energy in food used for maintenance. U — E = "available energy" for production = digested energy less energy in urine and feces. N = heat of combustion of product (e. g., of milk) or "net energy" of the product. W_p is therefore net or "partial" efficiency.

Kleiber defined total (gross) efficiency, and therefore the practical usefulness of animals by the equation

$$W_t = \frac{U}{N} \qquad \text{(b)}$$

where W_t = total (gross) efficiency, N and U the same as in (a).

If N is calculated from equation (a)

$$N = W_p \left(1 - \frac{E}{U}\right)$$

and the result substituted in equation (b), we obtain

$$W_t = W_p \left(1 - \frac{E}{U}\right) \qquad \text{(c)}$$

indicating symbolically, what we previously explained verbally, that for a given partial (net) efficiency, the total (gross) efficiency, W_t, is determined by the ratio of $\frac{E}{U}$ (i. e., of ratio of available energy for maintenance to available energy in total food). If E = U (if food is only enough for maintenance), the total efficiency in (c) = O; if U is less than E (not enough food for maintenance), its efficiency is negative (it spends its reserves); the greater the ratio of $\frac{U}{E}$ (the more it eats in comparison to its maintenance needs) the greater the total efficiency. Since experimental values for maintenance, E, are scarce, Kleiber used basal metabolism values, B, and the ratio $\frac{U}{B}$ (ratio of "available" food consumed to basal metabolism). Kleiber found the $\frac{U}{B}$ ratios for a group of chicks (4.43), for a rabbit (5.03), for 2 steers (4.24 and 5.57). As the $\frac{U}{B}$ ratios between steers are greater than between steers and rabbits and chicks, he concluded that efficiency is independent of body size.

In connection with our correction for body weight changes, we should mention the method described by I. W. Rupel at the Cornell (Summer 1934) Meeting of the American Dairy Science Association. *1st*, according to Forbes, utilization of nutrients for body gain is 77% as efficient as for milk production. Hence, the quantity of food producing 336 Cal. of milk (1 pound FCM), can produce 260 Cal. body gain. *2nd*, according to Haecker, 1 pound of body increase in a mature cow is equivalent to 2700 Cal. Hence 1 pound body gain during lactation is equivalent to 10.4 pounds FCM (= 2700/260); 1 pound body loss is equivalent to 8.03 pound FCM (= 2700/336). Thus when a cow gains in weight, her milk yield is corrected to constant weight by increasing the yield by 10.4 pounds milk for each pound body gain. When weight is lost, the actual milk yield is reduced at the rate of 8.03 pounds per pound decrease in body weight. J. C. Knott, R. E. Hodgson and E. V. Ellington (Washington Sta, Bull. 295, 1934) estimated that 3.53 lbs. TDN is required to gain 1 lb. live weight, and that 2.73 TDN is equivalent to 1 lb. loss in live weight.

2. INDIVIDUAL RECORDS OF THE EXPERIMENTAL COWS

DATA ANALYZED IN TEXT. *A.* EXPERIMENTAL COWS
(Starred Rows not included in deriving equation in text).

Source of data	Table or Number	Breed	Av. TDN/day lbs. X_1	Av. FCM/day lbs. X_2	Av. body weight M lbs.	$M^{0.83}$ X_3	ΔM/day av. gain per/day lbs. X_4	Av. gross efficiency % (not corrected for gain or loss in wt)	Age years
Haecker	1-3	Jersey	13.40	23.12	858	138.5	−.500	32.3	13
"	"	Holstein	16.22	27.79	919	145.6	−.214	32.1	6
*	"	Shorthorn	13.74	17.45	997		.036	23.8	4
* "	"	Native	10.53	16.35	742		−.304	29.1	11
"	"	Jersey	14.68	25.64	867	139.6	−.214	32.7	7
"	"	Guernsey	13.98	29.68	887	141.9	−.488	39.8	6
"	"	Jersey	13.09	31.60	778	126.0	−.893	45.2	7
* "	"	Jersey	11.11	19.02	696		−.429	32.1	3
* "	"	Shorthorn	14.33	20.68	928		−.259	27.0	3
"	"	Guernsey	12.62	16.55	765	127.4	.196	24.6	7
* "	"	Holstein	13.04	19.11	802		.125	27.5	2
"	"	Holstein	19.66	36.06	1292	186.7	.117	34.4	9
"	"	Jersey	15.27	23.50	856	138.3	−.080	28.8	6
* "	"	Holstein	15.41	31.74	853		−.232	38.6	4
"	"	Jersey	13.96	28.39	836	135.9	−.143	38.1	7
"	"	Jersey	15.81	23.18	1013	160.0	−.009	27.5	9
"	"	Jersey	13.78	23.49	754	126.0	−.027	31.9	8
"	"	Jersey	14.17	30.90	809	132.7	−.414	40.9	7
* "	"	Ayrshire	14.73	25.55	821		−.018	32.5	--
"	4-6	Jersey	12.74	25.65	817	133.6	−.364	37.7	8
"	"	Jersey	12.28	25.95	859	138.6	−.311	39.6	6
"	"	Holstein	14.80	28.55	888	142.0	−.230	36.1	7
"	"	Guernsey	12.39	22.67	801	131.7	−.133	34.3	5
"	"	Jersey	13.61	23.15	845	137.0	−.010	31.9	8
"	"	Guernsey	13.51	26.95	843	136.7	−.240	37.4	7
* "	"	Shorthorn	13.26	18.65	956		.026	26.3	4
"	"	Guernsey	12.50	21.86	782	129.4	.102	32.8	5
"	"	Holstein	18.64	31.84	1315	189.2	.321	32.0	10
"	"	Jersey	13.85	24.01	876	140.6	−.082	32.5	7
"	"	Jersey	14.28	20.43	1046	160.1	.148	26.8	10

DATA ANALYZED IN TEXT. *A.* EXPERIMENTAL COWS
(Starred Rows not included in deriving equation in text).

	Source of data	Table or Number	Breed	Av. TDN/day lbs. X_1	Av. FCM/day lbs. X_2	M Av. body weight lbs.	$M^{0.73}$ X_3	ΔM/day av. gain per/day lbs. X_4	Av. gross efficiency % (not corrected for gain or loss in wt.)	Age years
	„	„	Jersey	12.51	27.48	734	123.6	−.010	41.2	9
	„	„	Jersey	12.54	25.51	796	131.1	−.092	38.1	8
*	„	„	Jersey	11.89	20.63	798		.097	32.5	3
	„	8-10	Jersey	11.79	16.34	760	126.8	.008	26.0	7
	„	„	Jersey	14.50	27.77	792	130.6	−.398	35.9	7
	„	„	Holstein	15.97	17.27	872	140.1	.165	20.3	8
	„	„	Guernsey	13.81	24.73	790	130.4	−.293	33.6	6
*	„	„	Brown Swiss	14.26	15.34	1006		.466	20.2	3
	„	„	Jersey	15.08	18.47	820	134.0	.196	23.0	9
	„	„	Guernsey	15.94	23.43	855	138.1	.008	27.6	8
	„	„	Jersey	14.66	25.48	706	120.1	−.286	32.6	9
	„	„	Jersey	11.10	18.65	683	117.3	−.053	31.5	6
*	„	„	Shorthorn	13.95	21.92	880		−.090	29.4	5
	„	„	Guernsey	13.85	22.66	766	127.6	.050	30.7	6
	„	„	Holstein	20.07	30.41	1298	187.4	.192	28.4	11
	„	„	Jersey	15.16	23.04	890	142.2	.071	28.5	8
	„	„	Holstein	16.86	35.17	880	141.1	.050	39.1	6
	„	„	Jersey	13.56	26.21	838	136.2	−.097	36.2	9
	„	„	Jersey	13.05	15.41	1071	162.8	.077	22.1	11
	„	„	Jersey	14.21	22.85	752	125.8	.006	30.1	10
	„	„	Jersey	14.27	23.63	793	130.8	.115	31.0	9
	„	„	Jersey	13.73	27.02	817	133.6	−.060	36.9	4
	„	13-15	Holstein	18.81	26.65	976	152.2	.223	36.5	5
	„	„	Holstein	15.00	23.56	910		−.008	29.4	2
	„	„	Jersey	15.44	22.05	887	141.9	−.103	26.8	9
	„	„	Jersey	12.49	19.35	783	129.6	−.080	29.0	8
*	„	„	Holstein	15.06	22.99	884		.314	28.6	3
	„	„	Holstein	14.60	29.64	844	136.8	−.429	38.0	7
	„	„	Jersey	13.75	24.06	811	132.9	−.333	32.8	10
	„	„	Jersey	12.08	22.69	604		−.066	35.2	4
	„	„	Holstein	19.88	36.99	985	153.2	−.120	34.9	9
	„	„	Jersey	15.14	27.59	758	126.5	−.114	34.1	11
	„	„	Jersey	14.50	21.09	854	138.0	.017	27.3	10
	„	„	Guernsey	16.95	30.38	864	139.2	−.109	33.6	7

	"	"	Jersey	17.63	31.33	935	147.5	−.074	33.3	10
	"	"	Guernsey	15.78	21.07	996	154.4	.103	25.0	9
	"	"	Jersey	12.57	20.79	735	123.7	−.034	31.0	7
*	"	"	Shorthorn	14.63	20.94	1035		.017	26.8	6
	"	16-18	Jersey	13.70	17.99	915	145.2	.036	24.6	5
	"	"	Holstein	20.23	24.71	1137	170.1	.226	22.9	6
*	"	"	Jersey	11.29	17.67	593		.048	29.3	2
	"	"	Holstein	18.18	30.57	1273	184.7	−.137	31.5	13
*	"	"	Holstein	18.88	31.86	1027		.089	31.6	3
	"	"	Jersey	16.85	30.14	849	137.4	−.225	33.5	10
	"	"	Jersey	16.01	29.25	855	138.1	−.025	34.2	9
*	"	"	Holstein	17.43	23.57	990		.311	25.3	4
*	"	"	Holstein	18.34	37.83	890	142.2	−.319	38.7	8
	"	"	Jersey	11.96	17.91	734		−.071	28.1	3
	"	"	Jersey	14.78	23.77	898	143.2	−.131	30.1	11
*	"	"	Holstein	14.90	21.92	861		−.220	27.6	2
	"	"	Holstein	19.15	32.29	978	152.4	−.196	31.6	10
	"	"	Holstein	15.08	22.49	781	129.3	.077	27.9	3
*	"	"	Jersey	13.25	19.41	720		.000	27.5	3
	"	"	Jersey	15.29	29.30	811	132.9	−.184	36.0	11
	"	"	Guernsey	17.97	32.79	909	144.5	−.082	34.2	8
	"	"	Jersey	18.62	28.77	936	147.6	−.113	28.9	11
	"	"	Guernsey	17.37	26.28	1019	157.0	.143	28.4	10
	"	"	Jersey	14.94	21.07	843	140.3	.143	26.4	5
	"	"	Jersey	14.10	28.56	800	131.6	−.095	38.0	8
*	"	"	Shorthorn	15.92	26.48	1112		−.125	31.2	7
	"	19-22	Holstein	18.44	29.83	1095	165.5	−.042	30.2	7
*	"	"	Holstein	18.81	29.59	1075		.136	29.5	4
	"	"	Jersey	16.60	26.67	824	134.5	−.036	30.1	11
*	"	"	Jersey	16.01	28.66	897		.052	33.6	3
	"	"	Jersey	14.85	24 34	861	138.9	.101	30.7	10
	"	"	Holstein	17.54	33.24	906	144.1	−.071	35.5	9
	"	"	Jersey	13.96	22.53	820	134.0	.113	30.3	5
*	"	"	Jersey	14.45	25.47	734		.060	33.0	4
	"	23-25	Jersey	14.45	28.48	634	111.1	−.137	36.9	6
	"	"	Holstein	17.35	29.35	877	140.7	−.089	31.7	11
*	"	"	Holstein	18.31	29.77	809		.083	30.5	4
*	"	"	Jersey	14.38	25.39	742		.006	33.1	4

DATA ANALYZED IN TEXT. *A.* EXPERIMENTAL COWS
(Starred Rows not included in deriving equation in text).

Source of data	Table or Number	Breed	Av. TDN/day lbs. X_1	Av. FCM/day lbs. X_2	M Av. body weight lbs.	$M^{0.33}$ X_3	ΔM/day av. gain per/day lbs. X_4	Av. gross efficiency % (not corrected for gain or loss in wt.)	Age years
”	”	Guernsey	17.19	29.83	840	136.4	.029	32.5	9
”	’	Guernsey	15.08	22.27	925	146.3	−.060	27.7	10
”	”	Jersey	13.45	24.09	789	130.3	.202	33.6	9
”	”	Shorthorn	14.97	20.73	1011		−.089	25.9	8
”	”	Holstein	19.74	28.25	1114	167.6	.161	26.8	8
* ”	”	Holstein	18.23	29.96	923		.042	30.8	3
”	”	Holstein	16.07	27.71	925		.030	32.3	3
”	”	Holstein	18.40	25.41	1128	169.1	.238	25.9	8
”	”	Holstein	18.65	28.30	1117	167.9	.113	28.4	5
”	”	Jersey	16.21	30.04	833	135.5	−.161	34.7	12
* ”	”	Jersey	12.61	24.03	623		−.198	35.7	3
”	”	Jersey	16.24	30.87	902	143.6	−.036	35.6	11
”	”	Holstein	16.37	23.51	975	152.0	.191	26.9	12
”	”	Holstein	13.42	30.15	887	141.9	.278	42.1	10
* ”	”	Jersey	12.78	24.33	737		.030	35.7	3
”	”	Jersey	14.96	24.65	863	139.1	.018	30.9	13
”	”	Holstein	16.47	31.43	923	146.1	−.286	35.8	12
”	”	Holstein	14.14	22.69	817	133.6	.095	30.1	5
”	”	Holstein	18.75	27.69	1086	164.5	.143	27.7	6
”	”	Jersey	12.84	21.79	752	125.8	−.060	31.8	14
”	”	Jersey	15.93	28.85	782	129.4	.137	33.9	5
”	”	Jersey	14.64	22.60	735	123.7	.238	28.9	
”	”	Guernsey	15.29	20.13	947	148.8	−.095	24.7	10
* ”	”	Brown Swiss	16.84	20.54	1243		.262	22.9	
”	”	Jersey	18.65	37.05	910	144.6	.024	37.1	13
”	”	Guernsey	20.53	37.62	998	154.7	−.191	34.3	11
”	”	Holstein	15.13	20.88	1106	166.7	.226	25.9	.
”	”	Jersey	15.91	26.06	989	153.7	.250	30.7	4
* ”	”	Shorthorn	17.61	29.03	1081		.060	30.0	8
Eckles	”	Holstein	18.13	29.89	1319	189.6	.019	30.9	8
* ”	”	Ayrshire	14.82	24.57	976		.000	31.1	5
* ”	”	Shorthorn	12.22	15.02	1144		.000	23.0	8

”	”	Jersey	15.47	24.75	807	132.4	.041	30.0	5
”	”	Jersey	14.91	22.75	824	134.6	.137	28.6	7
”	”	Jersey	15.03	21.73	952	149.4	.000	27.1	5
* ”	”	Ayrshire	14.19	27.13	1020		.092	35.8	5
”	”	Holstein	20.03	34.26	1056	161.2	.000	32.1	7
”	”	Jersey	9.22	10.46	902	143.6	.049	21.3	5
”	”	Jersey	16.42	28.65	899	143.3	.044	32.7	5
Savage	”	Jersey	17.05	26.59	860	138.7	.095	29.2	
”	”	Jersey	16.64	24.71	925	146.3	.571	27.8	
”	”	Holstein	21.20	35.81	985	153.2	.695	31.7	
”	”	Holstein	21.38	32.36	1175	174.2	.581	28.4	
”	”	Holstein	21.69	38.51	1150	171.5	.638	33.3	
”	”	Holstein	22.71	37.69	1090	164.9	.657	31.1	
”	”	Jersey	18.81	34.67	812	133.0	.009	34.5	
* ”	”	Shorthorn	18.53	25.79	1040		.924	26.1	
”	”	Holstein	20.94	29.71	1035	158.8	.838	26.6	
”	”	Holstein	20.15	29.90	1054	160.9	.029	27.8	
”	”	Jersey	17.57	23.42	909	144.5	.505	25.0	
”	”	Jersey	19.73	32.31	931	147.0	.752	30.7	
”	”	Guernsey	19.27	32.74	1072	162.9	.514	31.8	
”	”	Jersey	17.86	34.44	865	139.3	.248	36.1	
”	”	Holstein	24.36	40.55	1184	175.2	.209	31.2	
”	”	Holstein	22.13	30.12	1341	191.9	.438	25.5	
”	”	Holstein	33.83	40.15	1239	181.1	.419	31.6	
”	”	Holstein	22.86	42.53	1053	160.8	.295	34.9	
”	”	Holstein	21.21	36.87	990	153.7	.086	32.6	
”	”	Holstein	20.84	30.39	1073	163.1	.162	27.3	
”	”	Holstein	22.70	37.48	1179	174.7	.019	30.9	
”	”	Guernsey	16.05	24.42	846	137.1	.200	28.5	
”	”	Holstein	23.36	38.24	1253	182.6	.752	30.7	
Harrison	”	Holstein	17.45	22.70	1172	173.9	.432	24.4	4
&	”	Holstein	16.25	22.23	1120	168.2	.192	25.6	4
Savage	”	Holstein	21.72	39.39	1203	177.3	−.030	34.0	4
”	”	Holstein	20.42	33.19	1232	180.4	.087	30.5	4
”	”	Holstein	19.91	30.70	1170	173.7	.711	28.9	4
”	”	Holstein	18.94	28.13	1163	172.9	.214	27.8	4
”	”	Holstein	16.50	24.36	1159	172.5	.425	27.7	4-7
”	”	Holstein	18.14	23.31	1203	177.3	.530	24.1	4-7
”	”	Holstein	17.94	20.25	1332	190.9	.688	21.2	4-7
”	”	Holstein	20.73	32.63	1297	187.3	.470	29.5	4-7

Data Analyzed in Text. *A.* Experimental Cows
(Starred Rows not included in deriving equation in text).

Source of data	Table or Number	Breed	Av. TDN/day lbs. X_1	Av. FCM/day lbs. X_2	M Av. body weight lbs.	$M^{0.73}$ X_3	ΔM/day av. gain per/day lbs. X_4	Av. gross efficiency % (not corrected for gain or loss in wt.)	Age years
,,	,,	Holstein	18.41	25.93	1187	175.5	.244	26.4	4-7
,,	,,	Holstein	18.60	29.31	1192	176.1	.004	29.5	4-7
,,	,,	Holstein	18.40	26.62	1364	194.3	.421	27.1	4-7
,,	,,	Holstein	18.67	27.25	1271	184.5	.587	27.4	4-7
,,	,,	Holstein	20.00	33.45	1341	191.9	.192	31.4	4-7
,,	,,	Holstein	19.67	31.60	1250	182.3	.188	30.1	4-7
,,	,,	Holstein	21.96	41.04	1195	176.4	.120	35.0	4-7
,,	,,	Holstein	20.13	29.22	1275	184.9	.470	27.2	4-7
,,	,,	Holstein	18.28	26.35	1238	181.0	−.278	27.0	4-7
,,	,,	Holstein	21.53	39.18	1301	187.7	.090	34.1	4-7
,,	,,	Holstein	19.55	31.23	1267	184.1	.455	29.9	4-7
,,	,,	Holstein	19.30	28.38	1217	178.8	.384	27.6	4-7
,,	,,	Holstein	18.09	21.97	1344	192.2	.853	22.8	4-7
,,	,,	Holstein	17.39	20.55	1272	184.6	.327	22.2	4-7
,,	,,	Holstein	19.11	28.00	1224	179.5	1.05	27.5	4-7
,,	,,	Holstein	19.59	32.82	1134	169.8	.075	31.4	4-7
,,	,,	Holstein	23.07	43.19	1170	173.7	.169	35.1	4-7
,,	,,	Holstein	18.50	27.76	1306	188.2	.511	28.1	4-7
,,	,,	Holstein	19.95	32.81	1154	172.0	−.150	30.8	4-7
,,	,,	Holstein	17.34	19.84	1332	190.9	−.011	21.4	4-7
,,	,,	Holstein	19.49	27.62	1366	194.5	.518	26.6	4-7
,,	,,	Holstein	18.18	27.19	1210	178.0	.150	28.0	4-7
,,	,,	Holstein	21.48	35.65	1333	191.0	.261	31.1	4-7
,,	,,	Holstein	18.45	27.41	1453	203.5	.494	27.9	4-7
,,	,,	Holstein	19.08	30.56	1379	195.8	.147	30.0	4-7
,,	,,	Holstein	21.71	39.84	1331	190.8	−.237	34.3	4-7
,,	,,	Holstein	18.24	29.49	1255	182.8	−.049	30.4	4-7
,,	,,	Holstein	18.02	32.76	134—	191.8	.327	34.1	4-7
,,	,,	Holstein	19.35	33.53	1337	191.5	.351	32.5	4-7
,,	,,	Holstein	16.76	26.85	1223	179.4	.188	30.0	4-7
,,	,,	Holstein	21.31	41.03	1386	196.6	.016	36.1	4-7
,,	,,	Holstein	21.47	38.71	1494	207.6	−.327	33.8	4-7
,,	,,	Holstein	19.61	33.87	1379	195.8	−.131	32.4	4-7
,,	,,	Holstein	19.70	33.91	1323	190.0	.176	32.3	4-7

”	”	Holstein	17.74	23.72	1266	184.0	.282	25.1	4-7
”	”	Holstein	20.68	35.09	1292	186.7	.045	31.8	4-7
”	”	Holstein	20.76	32.38	1305	188.1	.384	29.2	4-7
”	”	Holstein	20.80	35.29	1338	191.6	.004	31.8	4-7
”	”	Holstein	18.86	31.31	1138	170.2	.367	31.1	4-7
”	”	Holstein	19.22	28.55	1436	201.7	.082	27.8	4-7
”	”	Holstein	19.38	34.49	1081	163.9	.155	33.4	4-7
”	”	Holstein	15.83	22.86	1404	198.4	.637	27.1	4-7
”	”	Holstein	16.16	25.11	1234	180.6	−.200	29.1	4-7
”	”	Holstein	18.57	30.46	1272	184.6	.098	30.7	4-7
”	”	Holstein	22.20	41.90	1184	175.2	.171	35.4	4-7
”	”	Holstein	16.14	23.41	1220	179.1	.004	27.2	4-7
”	”	Holstein	16.81	23.36	1230	180.1	.249	26.1	4-7
”	”	Holstein	19.65	30.79	1320	189.7	.131	29.4	4-7
”	”	Holstein	20.05	38.16	1239	181.1	.074	35.7	4-7
”	”	Holstein	18.80	29.80	1182	175.0	−.078	29.7	4-7
”	”	Holstein	21.00	32.88	1346	192.4	.343	29.4	4-7
”	”	Holstein	20.98	36.36	1298	187.4	.065	32.5	4-7
”	”	Holstein	19.53	31.55	1254	182.7	−.188	30.3	4-7
”	”	Holstein	21.31	40.14	1394	197.4	−.690	35.3	4-7
”	”	Holstein	21.82	38.77	1388	196.8	.265	33.3	4-7
”	”	Holstein	20.83	34.91	1272	184.6	.216	31.4	4-7
”	”	Holstein	20.86	36.03	1261	183.5	.155	32.4	4-7
”	”	Holstein	21.48	35.65	1333	191.0	.261	31.1	4-7
”	”	Holstein	18.45	27.41	1453	203.5	.494	27.9	4-7
”	”	Holstein	19.08	30.56	1379	195.8	−.147	30.0	4-7
”	”	Holstein	21.71	39.74	1331	190.8	−.237	34.3	4-7
”	”	Holstein	18.24	29.59	1255	182.8	−.049	30.4	4-7
”	”	Holstein	18.02	32.76	1340	191.8	.327	34.1	4-7
”	”	Holstein	19.35	33.53	1337	191.5	.351	32.5	4-7
”	”	Holstein	16.76	26.75	1223	179.4	.188	30.0	4-7
”	”	Holstein	21.31	41.03	1386	196.6	.016	36.1	4-7
”	”	Holstein	21.47	38.71	1494	207.6	−.327	33.8	4-7
”	”	Holstein	19.61	33.87	1379	195.8	−.131	32.4	4-7
”	”	Holstein	19.70	33.91	1323	190.0	.176	32.3	4-7
”	”	Holstein	17.74	23.72	1266	184.0	.282	25.1	4-7
”	”	Holstein	20.68	35.09	1292	186.7	.045	31.8	4-7
”	”	Holstein	20.76	32.38	1305	188.1	.384	29.2	4-7
”	”	Holstein	20.80	35.29	1338	191.6	.004	31.8	4-7
”	”	Holstein	18.86	31.31	1138	170.2	.367	31.1	4-7
”	”	Holstein	19.22	28.55	1436	201.7	.081	27.8	4-7
”	”	Holstein	19.38	34.49	1081	163.9	.155	33.4	4-7

DATA ANALYZED IN TEXT. *A*. EXPERIMENTAL COWS
(Starred Rows not included in deriving equation in text).

Source of data	Table or Number	Breed	Av. TDN/day lbs. X_1	Av. FCM/day lbs. X_2	M Av. body weight lbs.	$M^{0.73}$ X_3	ΔM/day av. gain per/day lbs. X_4	Av. gross efficiency % (not corrected for gain or loss in wt.)	Age years
"	"	Holstein	15.83	22.86	1404	198.4	.637	27.1	4-7
"	"	Holstein	16.16	25.11	1234	180.6	—.200	29.1	4-7
"	"	Holstein	18.57	30.46	1272	184.6	.098	30.7	4-7
"	"	Holstein	22.20	41.90	1184	175.2	.171	35.4	4-7
"	"	Holstein	16.14	23.41	1220	179.1	.004	27.2	4-7
Hills	"	Holstein	18.40	20.49	1390	197.0	.307	20.9	10
"	"	Holstein	17.20	20.70	1317	189.4	.752	22.5	10
"	"	Holstein	11.25	9.40	1184	175.2	.487	15.7	11
"	"	Holstein	11.20	11.41	1341	191.9	.893	13.2	12
"	"	Jersey	13.70	13.60	978	152.4	.458	18.6	8
"	"	Jersey	14.20	19.43	899	143.3	.548	25.6	9
"	"	Jersey	15.20	17.16	920	145.7	.613	21.1	8
"	"	Jersey	16.55	19.45	929	146.8	.410	22.0	8
"	"	Jersey	13.95	17.89	885	141.7	.627	24.0	9
Perkins	"	Holstein	18.33	24.76	1360	193.9	.432	25.3	
"	"	Holstein	21.36	27.83	1393	197.3	.371	24.4	
"	"	Holstein	21.96	28.58	1288	196.8	.336	24.4	
"	"	Holstein	22.03	28.23	1330	190.7	.781	24.0	

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