

**EXTENSION OF EOQ MODEL
WITH EMERGENCY ORDERS AND
EXPLICIT ENERGY COST CONSIDERATIONS**

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The undersigned, appointed by the dean of the Graduate School,
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EXPLICIT ENERGY COST CONSIDERATIONS**

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And hereby certify that, in their opinion, it is worthy of acceptance.

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ABSTRACT

As customer expectations continue to rise, so too do the costs of producing and distributing globally competitive products and services that are in line with such demanding expectations. This trend includes not only the cost of materials and labor, but also the cost of energy to procure, produce, and deliver such products and services across the global market. In fact, the price of gas has nearly quadrupled in the last two decades. Even so, the demand for such non-renewable energy as well as the fear of its limited availability continues to rise and thus threaten its price more.

Given the trend in energy costs, this research investigates the effect of energy on logistics decisions by analyzing the effect of energy on an inventory ordering policy. The inventory model developed and analyzed in this paper is based on the actual environment at a leading aircraft manufacturer. In particular, the proposed model is applicable for production systems with constant production rates but small, underlying possibilities for undesirable circumstances to threaten the intricately planned production schedule. Rather than ignoring the possibility for undesirable circumstances and subsequently fulfilling any emergency demand with a more expensive and energy cost sensitive emergency order, the proposed model provides multiple scenarios to fulfill the emergency demand more cost effectively compared to the traditional EOQ model. These options include fulfilling the emergency demand from safety stock alone, a combination of safety stock and an emergency order, and lastly an emergency order alone if the regularly scheduled order is already in route to the production facility.

Thus, the objective of the inventory model developed in this paper is to structure an inventory policy under explicit energy cost considerations with optimal sizes for a scheduled order quantity, safety stock, and inventory cycle length that minimizes the total expected cost per unit time for a system with a constant production rate but a small, underlying possibility for undesirable circumstances to threaten the production schedule. Whilst varying most of the model parameters, this model is comparatively analyzed to the traditional EOQ model which satisfies the regular demand generated by the production system with a regularly scheduled order and ignores the possibility of an undesirable circumstance threatening the intricately planned production schedule.

By varying most of model parameters, the analysis reveals key production environments in which inventory policies are most significantly affected by changes to energy cost as well as the environments in which the proposed inventory model is most cost effective compared to the traditional EOQ model. These environments, as illustrated and discussed in analysis, consist of high levels of at least one of the following key parameters: the weight of the product, the regular demand of the product, or the emergency demand of the product. As any one of the three key factors increase, the change in many of the inventory decisions or related logistics costs become more significant as energy cost changes. Moreover, the cost effectiveness of implementing the proposed inventory model in place of the traditional EOQ model becomes more significant as any one of the three key factors increase as energy cost rises. Therefore, production environments with relatively high levels of at least one of the three key factors are particularly receptive to the proposed inventory model and its cost savings.

CHAPTER 1

INTRODUCTION

1.1 Background & Motivation

Industry changes over the past several decades have been driven by several factors including technology advancement and globalization. As a result, market expectations around the world have increased significantly. Customers expect not only an even greater amount of quality and customization, but also a faster delivery of many if not all products and services regardless of the place of origin. Furthermore, there is a persistent expectation for these products and services to be available at a lower price.

Nevertheless, the costs of producing and distributing globally competitive products and services that are in line with such expectations are also increasing. This trend includes not only the cost of materials and labor, but also the cost of energy to procure, produce, and deliver such products and services. In fact, the price of gasoline has nearly quadrupled in the last two decades. As a result of these increasing trends in costs and market expectations, businesses are compelled to manage their resources and facilities more efficiently and effectively in order to minimize costs and maintain competitiveness across the global market.

Even so, increased global awareness of the environmental costs associated with consuming non-renewable energy such as gasoline is forcing businesses to think again. Rather than minimizing monetary expenses alone, businesses are researching for

alternative options in order to increase energy efficiency, reduce energy consumption, and improve their environmental image.

One of the most promising areas to employ alternative energy options for an actual monetary return is transportation. According to Leinbach and Capineri (2007), energy consumption within the transportation sector has increased nearly 47 percent in the past two decades compared to 4 percent in the other industrial sectors. Given this increased consumption within transportation activities and the increased cost of energy sources like gasoline, businesses are compelled to consider alternative options with regard to energy and transportation in order to manage their resources more efficiently and effectively and thus compete in this environmentally conscious market.

For these reasons, businesses should explicitly consider energy cost as it relates to transportation decisions as well as many other decisions contingent upon transportation. That is, businesses should explicitly consider the costs of energy as it relates to all decisions essential for moving products from the suppliers to the customers. These decisions which are integrated for effective supply chain management include but are not limited to those associated with location, production, inventory, and transportation.

Though successful coordination of all the strategic, tactical, and operational decisions are important for effective supply chain management, the focus area for this research is inventory management. More specifically, this research explicitly considers energy cost as it relates to inventory decisions including order quantity, safety stock, inventory cycle length, and transportation.

1.2 Problem Description

The situation under scrutiny in this study consists of a single part in a single stage of a business supply chain. This stage consists of a production system which has a number of characteristics; most importantly, the production rate of the system is mostly constant from year to year regardless of any fluctuations to the actual market demand of the product. This constant production rate may be due to any number of reasons, but one genuine possibility is that the production capacity is limited. So, customer orders which are processed very early and delivered to customers at a much later date can be scheduled for production far in advance.

Given such a system with a constant production rate and a strict production schedule, the demand for a part required by the system may be effortlessly assumed. That is, since the production rate is constant it may be assumed that the demand for a part required by the system is also constant. Hence, the inventory decisions for such a system can follow a simple economic order policy with a regularly delivered order quantity and zero safety stock.

However, within almost any industry there are undesirable circumstances that arise and threaten to affect the production schedule. These circumstances include failures to meet quality standards, requests for repairs, requests for maintenance, and urgent changes to the production schedule. Regardless of the exact situation, if there is zero safety stock, a notably expensive emergency order from a supplier or distributor to the production system is required to satisfy the unexpected emergency demand and to maintain the highly intricate production schedule.

In the situation under investigation, there is a small probability of an undesirable circumstance to arise and generate unexpected emergency demand. Nevertheless, there are multiple options available to satisfy the emergency demand without backlogging or stopping the production schedule. These options include carrying safety stock, increasing the size of the regularly scheduled order from the supplier, and placing a more expensive emergency order from the supplier or other comparable distributor.

The high cost of such an emergency order is primarily associated with higher handling costs, higher fuel costs, and greater energy consumption for faster delivery via faster, less energy efficient transportation modes such as airplanes. Given the aforementioned rise in energy costs, the costs of these emergency orders are expected to increase significantly. Therefore, it is necessary to explicitly consider the cost of energy as it affects transportation costs as well as other costs including those associated with procurement and production when establishing inventory replenishment policies.

Given the detailed problem situation above, the objective of this research is to determine the optimal sizes for the scheduled order quantity and the safety stock that minimizes the total cost of the system with respect to procurement, transportation, inventory, and particularly energy costs of a single part in this production system. In other words, this study explicitly considers energy cost to determine the optimal inventory policy that minimizes all associated costs for a single part in a production system with a constant production rate but small probability for an undesirable circumstance to arise and generate unexpected emergency demand.

1.3 Problem Motivation: Boeing Corporation

The problem situation, inventory model and numerical analysis in the subsequent sections are motivated by actual challenges faced by the Boeing Corporation. With operations across 49 states and in 70 countries as well as customers in more than 90 countries, Boeing is challenged to operate effectively and efficiently throughout its entire supply chain in order to minimize its costs and improve its competitiveness. This challenge may become even more difficult as the cost of energy increases.

Given the rising costs of gas, especially in 2008 when gas prices exceeded four dollars a gallon, managers at Boeing questioned the effect of increasing energy costs on many of its logistics decisions. With such a large scale logistics network, it was questioned whether or not such decisions like inventory ordering policies are in fact significantly impacted by changes to energy cost, even if such changes were small. As a result of this luring challenge, a group of individuals at Boeing Corporation approached the Center for Engineering Logistics and Distribution at the University of Missouri with the task of investigating the effect of energy cost on many logistics decisions, but more specifically on the inventory ordering policy at Boeing Corporation.

Although the actual inventory ordering policy currently employed at Boeing is not clear, the inventory model developed throughout this research is based at least on the production environment at Boeing as well as other aircraft manufacturers. The production environment, like that in the problem description, is differentiated with fairly constant production rate – a characteristic that is quite common in the aircraft industry. In fact, the aircraft industry is prone to a very limited manufacturing capacity. Orders for aircraft placed by customers of companies like Boeing are processed very early and

delivered at a much later date. Consequently, the production schedules are planned far in advance. So whether or not the actual customer demand fluctuates from year to year, the production rate is fairly constant.

Given such a system, the demand for a part required for production may be frequently assumed as constant. However, undesirable circumstances do arise and threaten to impact the production schedule. These circumstances include failures to meet quality standards, requests for repairs, requests for maintenance, and urgent changes to the production schedule. Regardless of the exact situation, if there is no safety stock for the part in trouble, a notably expensive emergency shipment of parts from the supplier or a comparable distributor is required to satisfy the unexpected emergency demand for that part and to maintain the highly intricate production schedule.

In the situation under investigation at Boeing, there is a small probability of an undesirable circumstance to arise and generate unexpected emergency demand. So, for many parts, there may be virtually no safety stock. When such a circumstance arises, an emergency shipment of the part in trouble is required for a notably higher cost than that of a regularly scheduled shipment of the part. Since this higher cost is primarily associated with higher handling costs, higher fuel costs, and greater energy consumption for a faster deliver by way of a faster, less energy efficient transportation mode such as an airplane, these costs are expected to increase significantly as energy costs rise.

This study compares the expected cost of a traditional economic ordering policy without safety stock with an inventory model designed to determine optimal inventory decisions with respect to procurement, transportation, inventory, and particularly energy costs of a single part in a similar production environment as described prior.

1.4 Effect of Energy on Logistics

Though there is little research that explicitly considers or mentions energy cost as it relates to inventory decisions, there is evidence that energy cost as well as energy consumption is related to many transportation decisions relevant to supply chains. These decisions include but are not limited to modes of transportation, types of carriers, degree of consolidation, vehicle routes, and vehicle load plans. The diagram in Figure 1.1 shows some of interrelationships between the aforementioned transportation decisions.

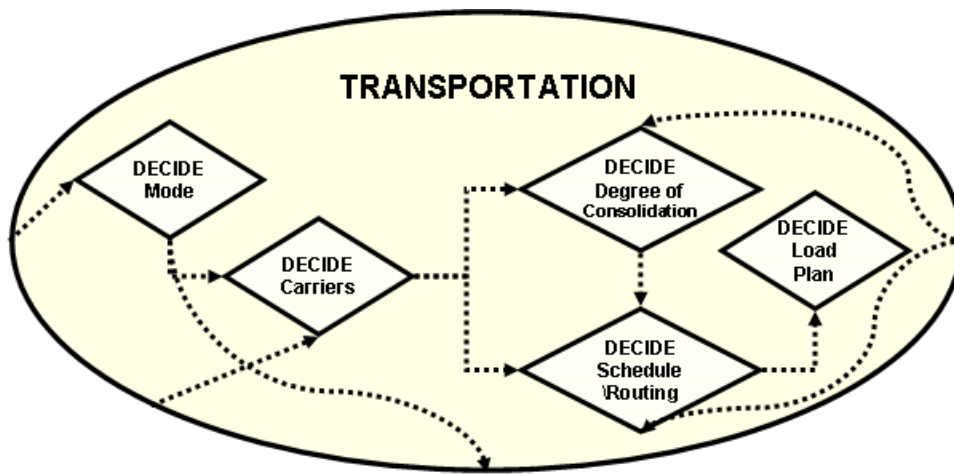


Figure 1.1: Interrelationships of Transportation Decisions in Supply Chains

The costs associated to any of these transportation decisions illustrated in the diagram above include the costs to operate a vehicle or fleet and the costs to handle individual packages that are transported from an origin to a destination. One of the primary operating costs associated to many of these transportation decisions is the cost of fuel consumption. As fuel consumption increases, so does the cost of transportation.

Fuel consumption can be reduced in multiple ways, but one frequently researched approach to reducing fuel consumption associated with transportation is shifting the

mode of transportation. Each mode of transportation such as water, rail, road, and air vary by energy efficiency which is directly related to the cost-effectiveness of a mode. A barge for instance, is one of the most energy efficient and thus cost effective modes of transportation. With a fraction of the fuel consumed by other modes, a single barge can transport the same quantity of materials as 15 railcars or 80 trucks (Murphy 2009). Furthermore, McKinnon cites that the energy consumption of road transportation is 4.3 times higher than that of rail and 6.8 times higher than that of water (1999). So, the energy efficiency and the cost effectiveness of a transportation mode are strongly related to optimizing the transportation mode decision.

Similar to the energy efficiency of a transportation mode, the transportation distance is directly related to fuel consumption. As transportation distance increases, fuel consumption and thus transportation cost increases; and different transportation modes can become more cost effective. So, transportation decisions related to distance including vehicle routing or even package consolidation can significantly affect the resultant transportation cost. Conversely, changes in energy cost can affect the degree of consolidation or other transportation decisions that aim to minimize cost.

Though energy cost and energy consumption may not directly and explicitly affect every transportation decision, since the decisions are interrelated as illustrated in Figure 1.1, all the transportation decisions in some way are affected by energy cost and energy consumption. Likewise, since the essential functions in the logistics system are interrelated with each other as well as transportation, all the decisions prevalent in the logistics system are affected explicitly or implicitly by transportation decisions. The interrelationships between these essential functions are illustrated in Figure 1.2.

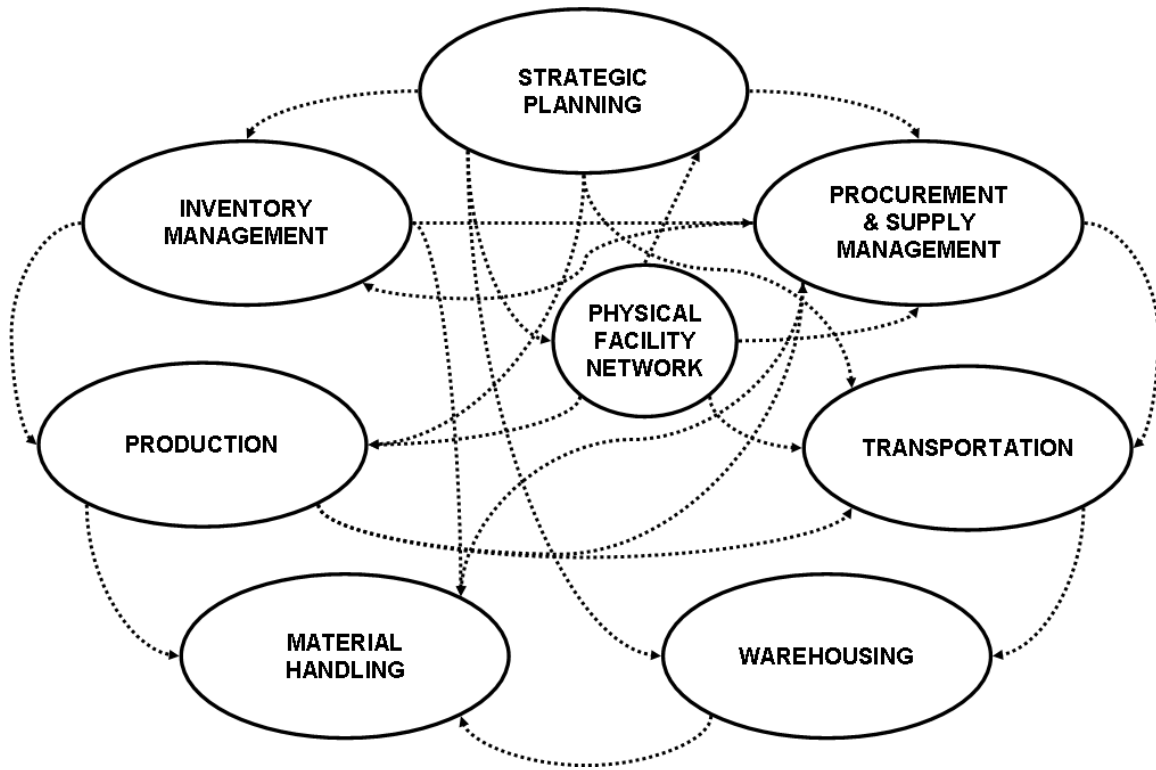


Figure 1.2: Interrelationships of Essential Decisions in Supply Chains

The relationship diagram in Figure 1.2 illustrates the interrelationships between all the essential decisions and functions in the logistics system. As illustrated by the diagram, transportation decisions are explicitly related to the decisions prevalent in strategic planning, physical network organization, procurement and supply management, production, and warehousing. Given that these decision areas are explicitly related to transportation decisions, and the cost of energy affects all transportation decisions in some way, it can be concluded that these five decision areas are also affected to some degree by the cost of energy.

Although the remaining two decision areas – inventory management and material handling – are not explicitly related to transportation decisions, it can be reasoned that

these decisions, and thus all decisions prevalent to the logistics system, are also affected to some degree by the cost of energy. Such reasoning is possible given the complex system of interrelationships prevalent in the logistics system and illustrated in Figure 1.2. So, even though transportation is not directly linked to inventory management or material handling in the diagram, these decision areas are implicitly related to transportation and thus subsequently affected by the cost of energy to some degree through other decisions and interrelationships in the system.

These relationships can be indirectly connected through multiple decisions, but it is easy to illustrate and understand the implicit relationship given only one degree of separation between the transportation decisions and the remaining two decision areas. For instance, transportation decisions can be linked to inventory management decisions through any of three decision areas – strategic planning, procurement and supply management, and production. Likewise, transportation decisions can be linked to material handling decisions through the decision areas of procurement and supply management, production, or warehousing.

Given that the remaining two decision areas – inventory management and material handling – are implicitly related to transportation decisions, and the cost of energy affects all transportation decisions in some way, it can be concluded that the decisions associated to inventory management and material handling are also affected to some degree by the cost of energy. Furthermore, since every decision prevalent in the logistics system is either explicitly or implicitly related to transportation decisions, it can be concluded that every decision essential in the logistic system are affected to some degree by the cost of energy.

A more detailed view of the interrelationships between the decisions within each of the essential functional areas of a logistics system is illustrated in Figure 1.3. This detailed relationship diagram illustrates more specifically the relationships between all the decisions prevalent within each functional area as well as the relationships between all the decisions in the whole logistics system.

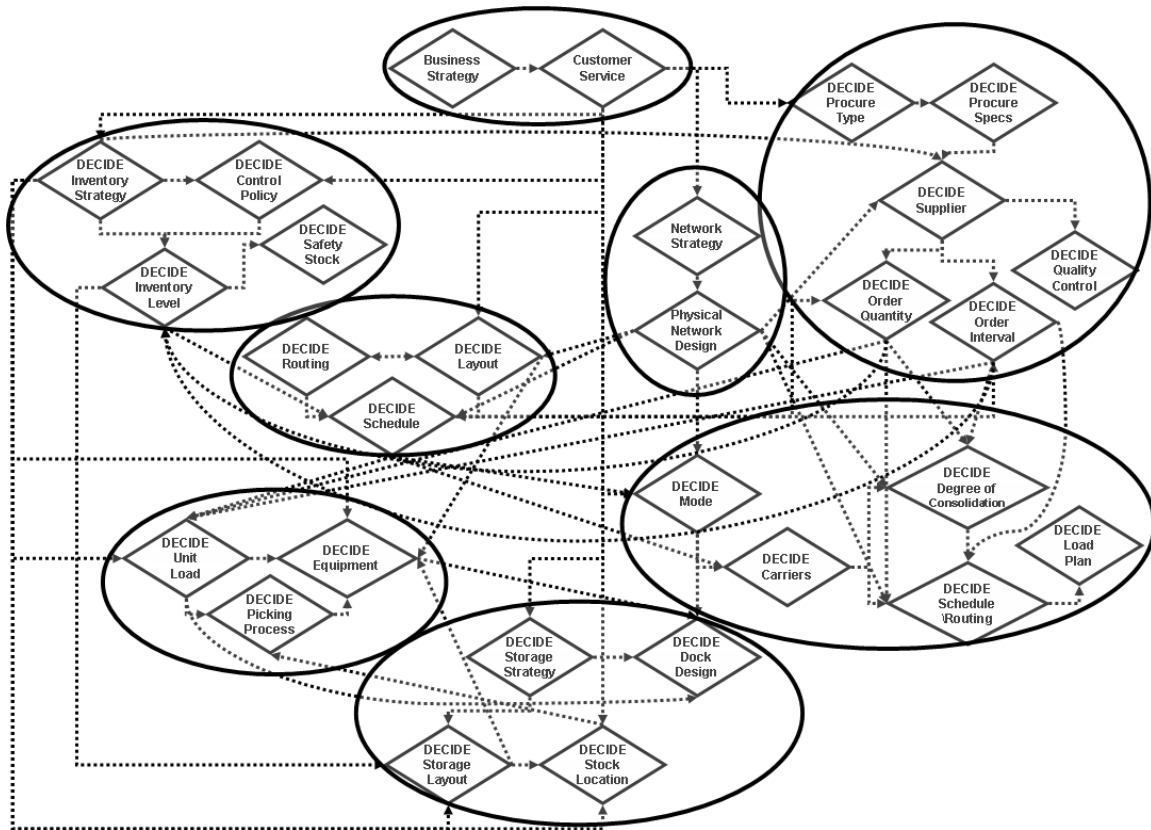


Figure 1.3: Detailed Interrelationships of Essential Decisions in Supply Chains

Following a similar reasoning for the effect of energy cost on the essential decision areas illustrated in Figure 1.2, it is reasoned that all the decisions within each functional area of the logistics system illustrated in Figure 1.3 are affected to some degree by the cost of energy. Rather than explicitly showing the effect of energy cost on

each specific decision prevalent in the logistics system illustrated in Figure 1.3, the purpose of this research is to investigate this effect through an example logistics problem situation. That is, the objective of this research is to investigate the effect of energy cost on logistics by investigating the effect of energy cost on a common logistics problem – the inventory order policy – and the overall cost associated to decisions essential in the policy. The decisions essential to the inventory order policy as well as the direct relationships between these decisions and other decisions in the logistics system are emphasized in the relationship diagram illustrated in Figure 1.4.

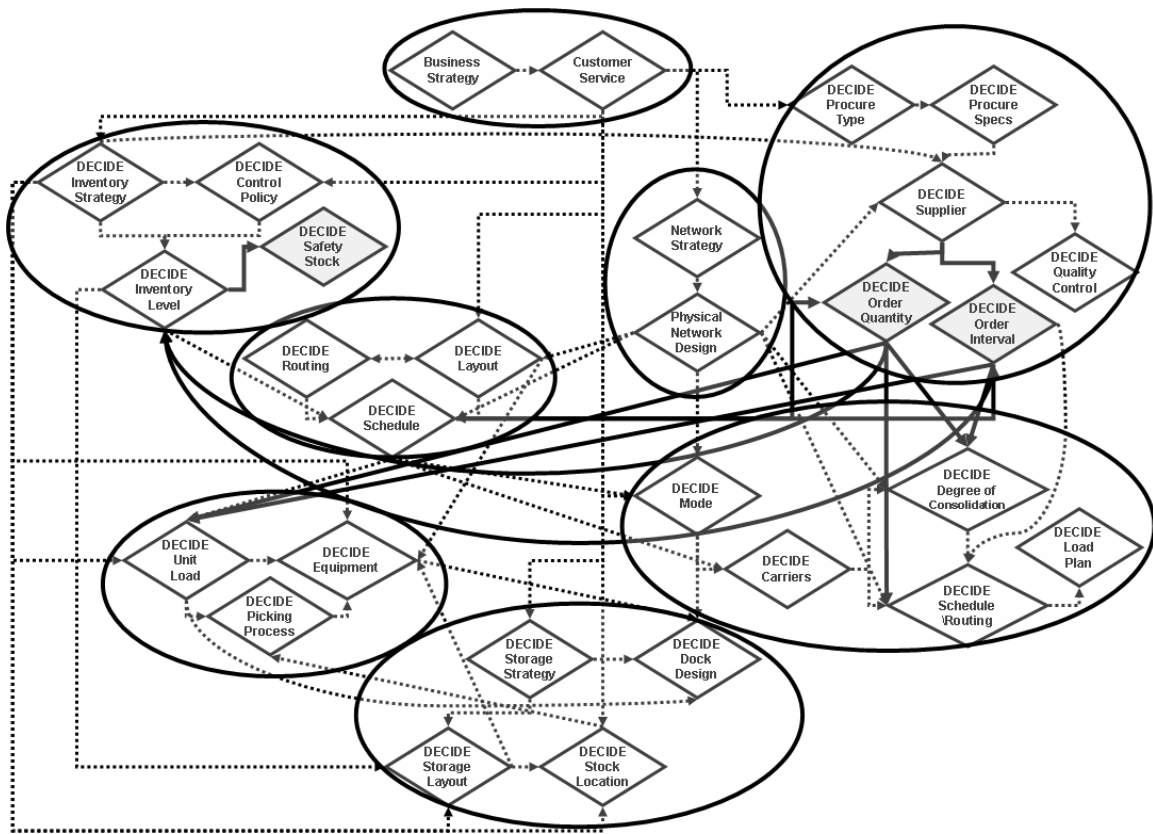


Figure 1.4: Detailed Interrelationships of Essential Decisions in Supply Chains with Emphasis on Inventory Policy Decisions

1.5 Value & Contribution

The purpose of this research is to investigate the effect of energy cost on logistics decisions by more specifically investigating the effect of changes to energy cost on the decisions and costs related to the inventory ordering policy. Though there is little research that explicitly considers or mentions energy cost as it relates to inventory decisions, there is evidence that energy cost as well as energy consumption is directly related to many transportation decisions relevant to supply chains. For instance, the energy cost of a transportation mode increases as the energy efficiency of the mode decreases. Similarly, the energy consumption and thus the energy proportion of the transportation cost increases as the transportation distance increases.

Since the inventory ordering policy is related to procurement, inventory, and transportation costs, it is theorized that inventory decisions are affected by changes to energy cost and consumption. Therefore, the inventory model developed and analyzed in this paper explicitly considers the cost of energy in the formulation of the inventory model. Such a formulation of an inventory model is contrary to many if not all the current research on inventory ordering policies. Though some inventory models consider different transportation modes or emergency shipments, there is little to no research that explicitly considers the cost of energy in transportation or other logistics costs.

Also, unlike much of the current research, the inventory model developed and analyzed in this paper is based on a simple economic ordering policy even though the demand by the production system is not entirely constant. That is, while the production system may have a fairly constant production rate, there is a small, underlying possibility for undesirable circumstances to threaten the rigidly set production schedule. Rather than

ignoring the possibility for undesirable circumstances and subsequently fulfilling any emergency demand with a more expensive and energy cost sensitive emergency order from a supplier, the proposed model provides multiple scenarios to fulfill the emergency demand more cost effectively. These options include fulfilling the emergency demand from safety stock alone, a combination of safety stock and an emergency order, and lastly an emergency order alone if the scheduled order is already in route to the plant location. Given these multiple options to replenish the emergency demand more cost effectively, the inventory model developed later in this paper determines an optimal inventory ordering policy similarly to an economic inventory ordering policy but with safety stock in addition to a scheduled order quantity.

Besides developing a unique inventory policy that explicitly considers energy cost and optimizes the inventory decisions for a system with a constant production rate but a small, underlying possibility for emergency demand, the purpose of this research is to discover and understand the production environments in which inventory policies are most significantly affected by changes to energy cost as well as the environments in which the proposed inventory model is most cost effective. Provided a set of factors characteristic of these production environments, guidelines can be developed to direct businesses to manage their inventory as well as other resources more efficiently and effectively as energy cost and consumption rise. Such guidelines are expected to be especially beneficial for businesses with extensive energy usage or logistics systems.

As is presented later in this paper, the research analysis reveals three factors which are significant to the effect of energy cost on inventory decisions and related logistics costs. As any of the three factors increase, the change in many of the inventory

decisions and related logistics cost becomes more significant with respect to changes in energy cost. Moreover, the cost effectiveness of implementing the proposed inventory policy in place of simple economic ordering policy becomes more significant as any one of the three key factors increase with respect to energy cost. Therefore, production environments with relatively high levels of at least one of the three key factors are particularly receptive to the proposed inventory model and its cost savings.

Before the results are presented, a review of the current and related research on inventory models is discussed in Chapter 2. Then, the proposed model is described, defined, and formulated in Chapter 3. Provided the solution conditions also formulated in Chapter 3, procedures for numerically solving the complex inventory model are developed in Chapter 4. Subsequently, the model is analyzed and compared to the traditional economic order quantity model with respect to changes in several model parameters in Chapter 4 in order to develop guidelines for businesses at the end of Chapter 4 and in Chapter 5. Lastly, the conclusions of the research and the analysis are presented in Chapter 5 along with future extensions to this research.

CHAPTER 2

LITERATURE REVIEW

Many researchers study inventory systems in which there is more than one supply option. The objective of most if not all the past research in this area is to minimize the total expected cost per unit time, per inventory cycle, per year, or for the finite horizon. The primary differences between the models are the replenishment policies and the assumptions.

Much of the past inventory literature is based on the classic economic order quantity model introduced by F.W. Harris (Ghiani et. al, 2004). The classic EOQ model assumes instantaneous supply and deterministic demand. Given that demand varies and supply is not always instantaneous, the EOQ has been extended to the common (Q, R) order policy which determines the economic order quantity and reorder point. Such an order policy is common in inventory systems with continuous review policies.

There have been several models developed within the class of continuous review inventory policies that allow for emergency orders. White and White (1992) compare an extension of the (r,Q) model that allows emergency ordering and safety stock in addition to the original (r,Q) model with no emergency ordering.

Moinzadeh and Nahmias (1988) analyze an extension of the (r,Q) policy in which two supply modes with continuous lead times are available. Instead of determining only one set of ordering policy parameters, the model develops two sets of ordering policies parameters. That is, the suggested ordering policy is of the form (r1, r2, Q1, Q2) in

which (r_1, Q_1) represent the optimal order quantity and reorder point for a regular order and (r_2, Q_2) represent an optimal order quantity and reorder point for an emergency order. The optimal parameters are determined by minimizing the expected cost of procurement, holding, and shortages. An order of each type has an associated fixed ordering cost and unit procurement cost.

Johansen and Thortstenson (1998) analyze a similar model in which the lead time of the emergency order is assumed to be much smaller than the lead time of the regular order. Between replenishments, the state of the system is reviewed at certain time points rather than continuously. The order policy is thus driven by the state and the time of the system. In order to minimize the inventory cost rate with state-dependent emergency orders, a tailor made policy-iteration algorithm is designed based on the Markov decision process. Results show that an emergency order option may have considerable impact on the average costs for a single-item in the system. However, this may not be the case if shortage costs are high compared to the emergency order costs.

Many more research has been done within the class of periodic review inventory policies than continuous review inventory policies that allow for emergency orders. In periodic review inventory policies, the inventory level is checked at regular periods and orders are made to raise the inventory a specified threshold.

Fukuda (1964) presents a dynamic inventory problem in which stock is delivered by either a regular less expensive mode one period later or an emergency more expensive mode instantaneously. In every even period, an optimal ordering policy is determined based on the current inventory state and the regular and emergency order-up-to levels. That is, if the inventory level is below the emergency order-up-to level, an emergency

order is made in order to return to the emergency order-up-to level, and a regular order is made in order to return to the cumulative order-up-to level. If the inventory level, on the other hand, is between the emergency order-up-to level and the cumulative order-up-to level, then the difference between the current inventory level and the cumulative order-up-to level is ordered. Wright (1969) studies an extension to this model for both a single-product and a multi-product inventory system in which there exists a capacity on the size of the emergency order.

The modeled developed by Vlachos and Tagaras (2001) also include a capacity on the size of the emergency order in addition to analyzing the option of placing an emergency order early or late in the period. Unlike some models, both the models presented by Vlachos and Tagaras (2001) assume the fixed order cost for both order types are negligible and the variable cost of the regular order is negligible. This assumption is true if the regular order is part of a larger order with more products that are delivered at every period regardless of a different ordering policy. These assumptions can also be made on standing orders which are delivered with the same quantity in every period.

A special case of the multiply-supply mode inventory system is one in which a standing order is deliver in every period. Rosenshine and Obee (1976) investigate such a system under a periodic review inventory system. At each review period, if the inventory falls below a known emergency stock level, an emergency order is place and delivered instantaneously to increase the inventory level to the optimal threshold. The dynamic inventory model determines the optimal emergency order quantity and standing order quantity to minimize the expected total cost of the system. Unlike past research, if the total inventory exceeds the maximum inventory level, the excess inventory is sold.

In general, analysis by Rosenshine and Obee (1976) show that the unit penalty cost on the purchasing price of an emergency order decreases, so does the lead-time at which the standing order system becomes more economical. This model was extended by Chiang (2007) who does not assume a minimum and maximum inventory level is known. Instead, the dynamic inventory model derived by Chiang (2007) determines the dispose-down-to level and order-up-to level. If inventory in a review period is lower than the order-up-to level, an emergency order is placed to raise the inventory to this level instantaneously; and if the inventory at a review period is higher than the dispose-down-to level, inventory is sold down to this level. The model is solved for either the average-cost or discounted-cost criterion as well as the backlogged or lost-sales problems.

Rather than having an option for emergency orders, some research considers the option of expediting an outstanding an order. Chiang (2002) proposes two continuous-review single-item order policies in which expediting is allowed either by a certain threshold time point or within a the lead time of an outstanding ordering.

The first policy extends another model similar to the (r,Q) which added a third optimal ordering parameter – order expediting level – two the original two optimal parameters for the regular order – order quantity and reorder point. The first policy extends this model by ensuring that the expedited order does not arrive after the regular order by assuming that the last time point at which an expediting decision can be made is the elapsed time after a regular order is less than (not equal to) the difference between the two order lead times. The model of the first policy thus determines the optimal regular order quantity, reorder point, expediting ordering point, and threshold time point.

The second policy proposed by Chiang (2002) assumes the lead time of the order consists of two components: a manufacturing lead time and a delivery lead time. While the manufacturing lead time is assumed to be deterministic, the delivery period can be a variable interval. The model determines the optimal order quantity, reorder point, and expediting level by minimizing the total cost per unit time with respect to a service level constraint.

Duran et. al (2004) extends the second policy proposed by Chiang (2002) which assumes the lead time of an order consists of two components. In this model, if the inventory is below the expediting level at the end of the manufacturing lead time, the order is expedited; otherwise, the order is not expedited.

CHAPTER 3

INVENTORY MODEL

3.1 Model Description

The inventory model developed in this chapter is motivated by the actual environment at Boeing Corporation. Like many other businesses in the aircraft industry, Boeing has a limited manufacturing capacity. Orders from customers are processed very early and delivered at a much later date. As a result, the production schedules are planned far in advance, and the production rate is fairly constant from year to year regardless of whether or not actual customer demand fluctuates.

Given an environment in which the production rate is fairly constant, it may be frequently assumed that the demand for a part required for production is also constant. Thus, the inventory policy for parts required by the production system can follow a traditional economic order policy with a regularly delivered order from the supplier or distributor and zero safety stock.

However, in this production environment, there is a possibility for undesirable circumstances to arise and threaten the intricately planned production schedule. These circumstances include failures to meet quality standards, requests for repairs, requests for maintenance, and urgent changes to the production schedule. Regardless of the exact situation, if there is no safety stock for the part, a notably expensive emergency order of the part from a supplier or distributor is required to satisfy the unexpected emergency demand for that part and to maintain the highly intricate production schedule.

The high cost of such an emergency order is primarily associated with higher handling and fuel costs as well as greater energy consumption for a faster delivery via a faster, less energy efficient transportation mode such as an airplane. Given the rise in energy costs, the costs of these emergency orders from suppliers or distributors are expected to increase significantly. Therefore, the inventory control policies in environments similar to the one described become much more critical. So, it is necessary to explicitly consider the cost of energy as it affects the procurement cost, transportation cost, and inventory cost to determine a cost effective inventory policy.

Based on the described production environment, the following model explicitly considers the cost of energy to determine an optimal inventory policy of a single item with a possibility of emergency demand in addition to a fairly constant regular demand. Given that the regular demand is constant, a scheduled order of size Q that satisfies the regular demand during the inventory cycle is delivered from a supplier after a deterministic lead time τ at the beginning of each inventory cycle. It is assumed that the length between consecutive scheduled orders is identical and equal to T time units, where the inventory cycle length T is always greater than the deterministic lead time τ ($\tau < T$).

In addition to constant regular demand, there exists a small probability that an undesirable circumstance such as a failure to meet quality standards or a request for part repair will arise and threaten to change the intricately planned production schedule. Such a circumstance creates an unexpected emergency demand for the part required by the production system. In order to satisfy this emergency demand as well as to maintain the highly intricate production schedule with minimum cost, there exists some additional inventory quantity s for safety stock.

Without safety stock, a more expensive emergency order from a supplier is required to maintain the production schedule and meet customer demands with zero stock-outs. Conversely, too much safety stock can lead to unnecessary inventory holding costs. In order to balance this tradeoff, it is assumed that in addition to the safety stock, an emergency replenishment option is available for certain demand scenarios.

Regardless of the demand scenario, it is assumed that each cycle begins at an identical inventory level equal to $Q + s$ in order to satisfy the expected demand within each inventory cycle. Thus, it is assumed that the safety stock level is full or replenished at the beginning of each inventory cycle. The safety stock can be replenished by either adjusting the regularly scheduled order or placing an emergency order from the supplier.

Since the regularly scheduled order requires a deterministic lead time τ to be shipped from the supplier, the size of the regularly scheduled order can only be adjusted within the first $T - \tau$ time of the inventory cycle before it leaves the supplier. Therefore, any emergency demand that occurs during the scheduled order lead time τ – after the scheduled order leaves the supplier – requires an emergency order from the supplier or a comparable distributor to replenish the safety stock and prevent an imminent stock-out.

An emergency order of a part may also be placed before the scheduled order leaves the supplier if the emergency demand exceeds the safety stock level. In such a scenario, an emergency order is necessary to prevent a stock-out before the beginning of the next inventory cycle. In any case, such an emergency order requires negligible lead time and thus is delivered from a supplier immediately after emergency demand occurs.

The purpose of this model is to determine optimal sizes for the scheduled order quantity Q , the safety stock level s , and the inventory cycle length T that minimizes the

expected total cost per unit time of a product with a probability of stochastic emergency demand between regularly scheduled orders. In the model, it is assumed that there is proportional probability equal to pt of one occurrence of stochastic emergency demand arising within a given time period of length t . Hence, there is a proportional probability equal to $(1 - pt)$ of no emergency demand arising within a given time period of length t .

The possibility of more than one occurrence of emergency demand during any time period is not considered in this model. Though it may be possible for multiple instances of emergency demand within a given time period, the probability per unit time of a single instance of stochastic emergency demand is assumed to be very small so that the probability of more than one instance of emergency demand within a given time period would be negligible.

If there is no emergency demand during an inventory cycle, a regular inventory replenishment scenario occurs. Within a regular inventory replenishment scenario, the regularly scheduled order quantity Q will be received by way of an economical ground transportation after a deterministic lead time τ at the beginning of the next inventory cycle. If, on the other hand, an instance of emergency demand does occur during an inventory cycle, one of three irregular replenishment scenarios will arise depending on the time point and the quantity of the emergency demand. The three irregular replenishment scenarios are depicted in the following figure alongside the regular replenishment scenario.

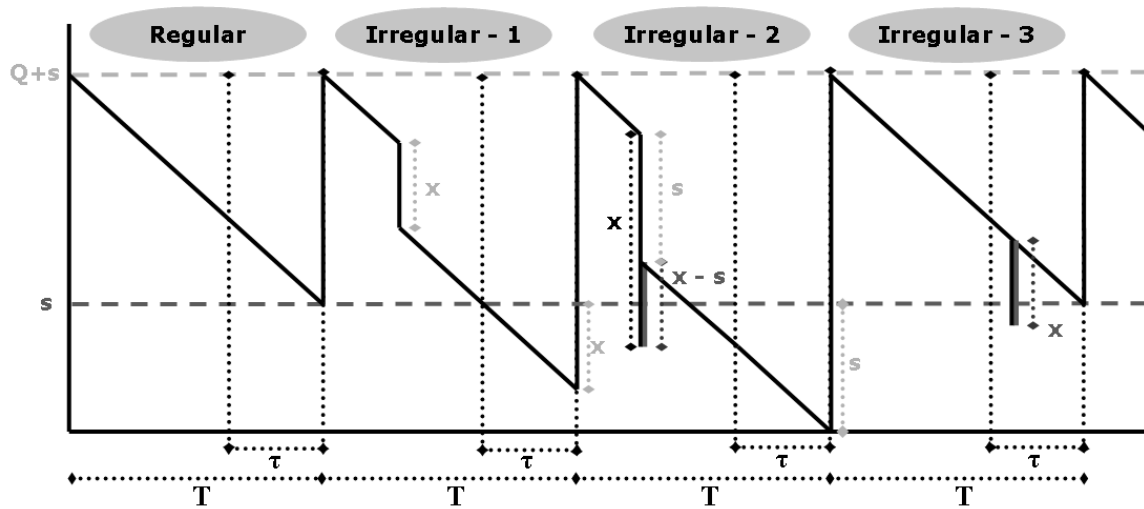


Figure 3.1: Replenishment Scenarios for the Proposed Inventory Policy

The first irregular replenishment scenario depicted in the figure arises when the emergency demand x occurs before the next scheduled order leaves the supplier and is less than or equal to the safety stock level s . If the emergency demand is less than the safety stock level, no inventory stock-outs will occur before the next regularly scheduled order is received. Nevertheless, the depleted safety stock must be replenished by the beginning of the next inventory cycle in order to have an adequate amount of inventory to satisfy the expected demand in the next inventory cycle. Since the emergency demand occurs before the regularly scheduled order leaves the supplier in this scenario, the size of this order from the supplier is increased by an amount equal to the emergency demand x . That is, at the beginning of the next inventory cycle, a scheduled order of size $(Q + x)$ will be delivered from the supplier via ground transportation.

In the second irregular replenishment scenario, a large instance of emergency demand occurs before the next scheduled order ships out from the supplier. In such a scenario, the inventory level will decrease to zero before the beginning of the next

inventory cycle because the emergency demand x is greater than the safety stock level s . So, an emergency order from the supplier or a comparable distributor is required. Yet since it is assumed that no other instance of emergency demand will occur in the current inventory cycle, there is no pressing need to replenish the completely depleted safety stock with the more expensive emergency order. Instead, the completely depleted safety stock is replenished by increasing the size of the next scheduled order to $(Q + s)$; and the emergency demand that exceeds the safety stock level is replenished by an emergency order of size $(x - s)$ which is triggered and delivered immediately via air transportation.

The third and final irregular replenishment scenario depicted in the figure arises when the emergency demand occurs after the next scheduled order leaves the supplier. Since it is assumed that each inventory cycle starts with the inventory level $(Q + s)$ in order to satisfy the expected demand within any inventory cycle, an emergency order from a supplier is necessary to replenish the depleted safety stock before the beginning of the next inventory cycle. Thus, an emergency order equal to the size of the emergency demand x is triggered and received immediately after the emergency demand occurs via air transportation that is assumed to be more expensive than ground transportation and have a negligible lead time.

3.2 Model Definitions

The purpose of the proposed model is to develop an inventory model under explicit energy cost considerations. Specifically, it is an inventory model with optimal sizes for the scheduled order quantity Q , the safety stock level s , and the inventory cycle length T that minimizes the expected total cost per unit time with respect to procurement, transportation, inventory, and energy costs of a part in the production stage of a supply chain. Though the fairly constant demand of the part is satisfied by regularly scheduled orders, there is a small probability p of stochastic emergency demand x between orders.

It is assumed that a regularly scheduled order is delivered at the beginning of each inventory cycle and each inventory cycle has an identical length of time T . Given the inventory cycle length T and a constant regular demand D , the scheduled order quantity Q can be derived by the product of the inventory cycle length T and the constant regular demand D . This order quantity, which is one of the inventory model decisions, only satisfies the constant regular demand within each inventory cycle. Another inventory model decision, the level of safety stock s , is designed to satisfy the expected emergency demand at a minimal cost.

Emergency demand arises when an undesirable circumstance such as a failure to meet quality standards or a request for part repair threatens to change the intricately planned production schedule. It is assumed that there exists a small probability p per unit time of a stochastic emergency demand such that the proportional probability of a single occurrence of stochastic emergency demand within a given inventory cycle of time period of length T is pT . Hence, the proportional probability of no emergency demand

within a given inventory cycle is $(1 - pT)$. This is equivalent to the probability for a regular replenishment scenario in which no emergency demand occurs.

It is assumed that at most one occurrence of emergency demand can arise within an inventory cycle since the probability of multiple occurrences of emergency demand is very small. In the proposed model, emergency demand may occur before or after the regularly scheduled order leaves the supplier. So the probability that the emergency demand occurs after the regularly scheduled order leaves the supplier within the last τ time units of the inventory cycle, as it does in the third irregular replenishment scenario, is $p\tau$. Conversely, the probability that the emergency demand occurs before the regularly scheduled order leaves the supplier within the first $(T - \tau)$ time units of the inventory cycle, as it does in either the first or second irregular replenishment scenario, is $p(T - \tau)$.

Since the occurrence of either the first or the second irregular replenishment scenario depends on the size of the emergency demand, the probability of one of the two scenarios arising is the product the emergency demand probability $p(T - \tau)$ and the probability that the emergency demand x is less than or equal to the safety stock level s (for the first irregular replenishment scenario) or the emergency demand x is greater than the safety stock level s (for the second irregular replenishment scenario). The total costs of an inventory cycle depend on the associated replenishment scenario.

The total cost for each replenishment scenario consists of procurement costs, transportation costs, inventory costs, and explicit energy related costs. The procurement and transportation costs consist of a fixed cost component k_m , a variable cost component e_m associated to energy cost, and another variable cost component c_m associated to everything but energy cost. The index $m \in \{o, g, a\}$ of each cost component represents

the origin of the cost. Any production or procurement cost is represented by the index $m = o$. So the fixed contracting or ordering cost to procure the product from supplier is represented by k_o ; and the per unit non-energy-related production cost to procure a product from the supplier is represented by c_o ; and per unit energy-related production cost to procure a product from the supplier is represented e_o . Alternatively, any shipping activity cost is represented by the index $m \in (g, a)$.

The transportation and shipping costs depend on the transportation mode and thus the replenishment scenario. The transportation mode for any regularly scheduled order is assumed to be an economical ground transportation denoted by the index $m = g$ that takes a deterministic time of length τ to deliver. An emergency order, on the other hand, is assumed to be a more expensive and faster air transportation denoted by the index $m = a$ that takes a theoretically negligible time to deliver.

For either transportation mode $m \in (g, a)$, there is a fixed cost component and two variable cost components that compose the total cost of transportation and shipping. In particular, there is a fixed cost k_m to ship an order such that the fixed cost for ground transportation is less than the fixed cost for air transportation ($k_g < k_a$). Additionally, there is a per unit cost c_m to ship one item via a specified transportation mode $m \in (g, a)$ such that the unit shipping cost via ground transportation is again less than the unit shipping cost via air transportation ($c_g < c_a$). Lastly, there is an energy-related cost component e_m such as fuel cost to ship one unit of product via a specified transportation mode $m \in (g, a)$. Similar to the prior transportation cost components, the energy-related shipping cost per unit of product is less for ground transportation than for air ($e_g < e_a$).

The final component of the total cost in each inventory cycle is the inventory holding cost. The total inventory holding cost is a function of the demand curve and the holding cost h per unit product and per unit time. Though the holding cost h is identical for any inventory cycle, the total inventory cost is dependent on the replenishment scenario.

Indices

m cost origin $m \in \{o, g, a\}$

Parameters

D Regular demand of product per cycle time

x Emergency demand of product per cycle time

p Probability of emergency demand in each unit time

k_m Fixed cost of an order via cost origin m

c_m Non-energy related cost of one unit of product via cost origin m

e_m Energy cost of one unit of product via cost origin m

h Holding cost per unit product and per unit time

τ Lead time of scheduled order

Integer Variables

Q Order quantity of the scheduled order

s Safety stock level of product

Noninteger Variables

T Length between consecutive schedule orders or length of the inventory cycle

3.3 Model Formulation

The purpose of the proposed model is to determine optimal sizes for the scheduled order quantity Q , the safety stock level s , and the inventory cycle length T that minimizes the expected total cost under explicit energy cost considerations of a part with a probability p of stochastic emergency demand x between regularly scheduled orders. In order to determine the expected total cost per unit time of the inventory model, the total cost and probability of each replenishment scenario must be determined. Given the total costs and the probability of each replenishment scenario, the objective function to minimize the total expected cost can be formulated.

3.3.1 Total Costs for each Replenishment Scenario

The occurrence of a replenishment scenario depends on whether emergency demand occurs or not. If there is no emergency demand during an inventory cycle, a regular inventory replenishment scenario occurs. The probability of this regular inventory replenishment scenario in which no emergency demand occurs during the inventory cycle is $(1 - pT)$.

Within a regular inventory replenishment scenario, the regularly scheduled order quantity Q will be received by an economical ground transportation after a deterministic lead time τ at the beginning of the next inventory cycle. The costs of these orders are depicted in equations (1-2). While equation (1) depicts the cost to procure the regularly scheduled order, equation (2) depicts the cost to ship the regularly scheduled order. Equation (3), on the other hand, depicts the inventory holding cost of this scenario in which the inventory level decreases at a constant demand rate and the safety stock level

remains constant throughout the inventory cycle. Given these three individually depicted cost components in equations (1-3), the total cost of the regular replenishment scenario can be illustrated in equation (4).

$$k_o + Q(c_o + e_o) \quad (1)$$

$$k_g + Q(c_g + e_g) \quad (2)$$

$$h\left(\frac{QT}{2} + sT\right) \quad (3)$$

$$k_o + k_g + Q(c_o + e_o + c_g + e_g) + \frac{hT}{2}(Q + 2s) \quad (4)$$

Whereas the occurrence of the regular replenishment scenario is contingent upon no emergency demand arising during an inventory cycle, the occurrence of one of the other three irregular replenishment scenarios is dependent upon when the emergency demand occurs within the inventory cycle and how much is demanded at that time. If the emergency demand occurs within the first $(T - \tau)$ time units of the inventory cycle – before the regularly scheduled order leaves the supplier, either the first or the second irregular replenishment scenario will arise with a probability of $p(T - \tau)$. If, on the other hand, the emergency demand occurs within the last τ time units of the inventory cycle – after the regularly scheduled order leaves the supplier, the third irregular replenishment scenario will arise with a probability of $p\tau$. Since the replenishments vary in each of the three irregular replenishment scenarios, each irregular replenishment scenario incurs a different amount of cost.

The total cost of the first irregular replenishment scenario in which the emergency demand x occurs before the next scheduled order leaves the supplier and is less than or

equal to the safety stock level s is illustrated in equation (8). In this scenario, no inventory stock-outs occur before the next order is received since the emergency demand is less than the level of safety stock. However, a portion of the safety stock is depleted and thus requires replenishment by the beginning of the next inventory cycle in order to satisfy the next cycle's demand. Given that this occurs before the scheduled order leaves the supplier, the scheduled order quantity is increased by an amount equal to the emergency demand x .

In terms of cost, the change in the scheduled order quantity for the first irregular replenishment scenario translates to an increase in the cost of the scheduled order by an amount proportional to the emergency demand. This increase is incorporated in equations (5-6) which illustrate the costs of procurement and transportation for this scenario. In particular, the cost to procure the regularly scheduled order is increased by the unit cost to procure the emergency demand x as shown in equation (5); and the cost to ship the regularly scheduled order is increased by the unit cost to ship the emergency demand x as shown in equation (6).

Contrary to the increase in procurement and transportation costs for this irregular replenishment scenario, the inventory holding cost shown in equation (7) decreases. In this scenario as well as the second irregular replenishment scenario, it is assumed that the emergency demand can occur at any time between the beginning of an inventory cycle and the time at which the regularly scheduled order is shipped. Yet on average, the emergency demand occurs half way between these two time points. So similarly, the time point at which the inventory level is reduced by the emergency demand is on average, half way between the time period $(T - \tau)$. Given that the inventory level is

reduced by an amount equal to the emergency demand for the remaining time period, the inventory holding cost is reduced by an amount proportional to the emergency demand x and the remaining time period which is equal to half the time period $(T + \tau)$.

Given the procurement, transportation, and inventory costs individually depicted in equations (5-7), the total cost of the first irregular replenishment scenario can be illustrated in equation (8). The probability of incurring the total cost illustrated in equation (8) is equal to the product of the following two probabilities: the first being the probability $p(T - \tau)$ that an emergency demand occurs before the scheduled order leaves the supplier and the second being the probability $f(x \leq s)$ that an emergency demand is less than or equal to the safety stock.

$$k_o + (Q + x)(c_o + e_o) \quad (5)$$

$$k_g + (Q + x)(c_g + e_g) \quad (6)$$

$$h \left(\frac{QT}{2} + sT - \frac{x(T + \tau)}{2} \right) \quad (7)$$

$$k_o + k_g + (Q + x)(c_o + e_o + c_g + e_g) + \frac{hT}{2} \left(Q + 2s - x \left(1 + \frac{\tau}{T} \right) \right) \quad (8)$$

Similarly, in the second irregular replenishment scenario, the probability of incurring the total cost characteristic of this scenario in equation (12) is equal to the product of the next two probabilities: the probability $p(T - \tau)$ that an emergency demand occurs before the scheduled order leaves the supplier and the probability $f(x > s)$ that an emergency demand is greater than the safety stock level. In such a scenario, the inventory level will decrease to zero before the beginning of the next inventory cycle

because the emergency demand x is greater than the safety stock level s . So, an emergency order is required to maintain the intricate production.

Nevertheless, since it is assumed that no other instance of emergency demand will occur in the current inventory cycle, there is no pressing need to replenish all the stock, specifically the completely depleted safety stock, with the more expensive emergency order before the beginning of the next inventory cycle. Instead, the emergency order from the supplier should only satisfy the emergency demand which exceeds the level of safety stock and thus would not be satisfied otherwise; and the completely depleted safety stock should be replenished by adjusting the next scheduled order. .

So, in the second irregular replenishment scenario, two separate inventory replenishments must occur. First, an emergency order of size $(x - s)$ will be triggered and delivered immediately from the supplier via air transportation that is assumed to be more expensive than ground transportation and have a negligible lead time. Second, the next scheduled order that is delivered from the supplier via ground transportation will be increased to the size $(Q + s)$ so as to replenish the entirely depleted safety stock. The procurement and transportation costs associated to these inventory replenishments are represented in equations (9-10), respectively; and the resultant inventory cost of this scenario is shown in equation (11).

The procurement costs in equation (9) consist of two separately incurred costs. For one, it includes the cost to procure the regularly scheduled order which is increased to the size $(Q + s)$ so as to replenish the depleted safety stock and start the next inventory cycle with an identical inventory level of $(Q + s)$. Secondly, it includes the cost to procure the emergency order of size $(x - s)$ so as to satisfy the emergency demand that

exceeds the safety stock level. As compared to the procurement costs in equation (1) of the regular replenishment scenario, these procurement costs are increased by a fixed cost to procure the emergency order as well as the unit costs to procure an amount of parts equal to the emergency demand between the two replenishment methods.

Like the procurement costs in equation (9), the transportation costs in equation (10) consist of two separately incurred costs. That is, it includes the cost to ship the regularly schedule order via ground transportation as well as the cost to ship the emergency order via air transportation. As compared to the transportation costs in equation (2) of the regular replenishment scenario, these transportation costs are increased by three factors: the fixed cost to ship the emergency order via the more expensive air transportation; the unit costs to ship the larger regularly scheduled order which replenishes the completely depleted safety stock with ground transportation; and lastly, the unit costs to ship the emergency order of size $(x - s)$ with air transportation.

Whereas both the procurement and transportation costs are higher in this scenario as compared to the regular replenishment scenario, the resultant inventory cost shown in equation (11) is lower in this scenario as compared to the regular replenishment scenario. This is true for both the first and second irregular replenishment scenarios. In fact, the resultant inventory cost for the two scenarios are almost identical.

Like in the first irregular replenishment scenario, it is assumed that the emergency demand can occur at any time between the beginning of an inventory cycle and the time at which the regularly scheduled order is shipped from the supplier. Yet on average, the emergency demand occurs half way between these two time points – at the point in time equal to half the time period $(T - \tau)$. So, like the first irregular replenishment scenario,

the time point at which the inventory level is reduced by the emergency demand is, on average, half way between the two points in time. Given that the inventory level is reduced by an amount equal to the depleted safety stock for the remaining time period, the inventory holding cost is reduced by an amount proportional to the depleted safety stock s and the remaining time period which is equal to half the time period $(T + \tau)$. The only difference in inventory cost between the two scenarios is that the reduction in inventory cost of the first is proportional to the emergency demand x whereas the reduction in inventory cost of the second is proportional to the depleted safety stock s .

Given the inventory holding costs outlined in equation (11) as well as the procurement and transportation costs shown separately in equations (9-10), the total costs of the second replenishment scenario can be illustrated in equation (12).

$$2k_o + (Q + x)(c_o + e_o) \quad (9)$$

$$k_g + k_a + (Q + s)(c_g + e_g) + (x - s)(c_a + e_a) \quad (10)$$

$$h \left(\frac{QT}{2} + sT - \frac{s(T + \tau)}{2} \right) \quad (11)$$

$$2k_o + k_g + k_a + (Q + x)(c_o + e_o) + (Q + s)(c_g + e_g) + (x - s)(c_a + e_a) + \frac{hT}{2} \left(Q + s \left(1 - \frac{\tau}{T} \right) \right) \quad (12)$$

The final cost scenario for the third irregular replenishment scenario incurs the total costs illustrated separately in equations (13-15). The probability for these costs to incur is equal to $p\tau$, which is the probability for an emergency demand to occur after the scheduled order leaves the supplier. Since the emergency demand occurs during the scheduled order lead time, an emergency order equal to the size of the emergency demand is necessary to begin the next inventory cycle at an identical inventory level.

Thus, as compared to the costs of the regular replenishment scenario shown in equations (1-3), the costs of this scenario shown in equation (13-15) are only increased by the procurement and transportation costs associated with replenishing the inventory that was reduced by the emergency demand x . That is, as compared to the procurement costs of the regular replenishment scenario shown in equation (1), the procurement costs of this scenario shown in equation (13) are increased by only the additional fixed and unit costs to procure the emergency order of size x from the supplier. Similarly, as compared to the transportation costs of the regular replenishment scenario shown in equation (2), the transportation costs of this scenario shown in equation (14) are increased by only the additional fixed and unit costs to ship the emergency order of size x via the more expensive air transportation method.

Lastly, the inventory holding cost shown in equation (15) is identical to that for the regular replenishment scenario shown in equation (3). In both scenarios, the inventory level decreases at a constant demand rate and the final inventory level is equal to the safety stock level. So the inventory holding cost is proportional to the inventory cycle length, the constant demand rate, and the constant safety stock level. Combined with the prior costs, the total cost of this third scenario can be illustrated in equation (16).

$$2k_o + (Q + x)(c_o + e_o) \quad (13)$$

$$k_g + k_a + Q(c_g + e_g) + x(c_a + e_a) \quad (14)$$

$$h\left(\frac{QT}{2} + sT\right) \quad (15)$$

$$2k_o + k_g + k_a + (Q + x)(c_o + e_o) + Q(c_g + e_g) + x(c_a + e_a) + \frac{hT}{2}(Q + 2s) \quad (16)$$

3.3.2 Objective Function

The objective of the proposed model is to determine the optimal scheduled order quantity Q and the safety stock level s that minimize the expected total cost per unit time with respect to procurement, transportation, inventory, and energy costs of a part required by a production system that has a probability p of stochastic emergency demand x , which follows $f(x)$, between regularly scheduled orders from suppliers and distributors. The expected total cost per unit time of the inventory model shown in equation (17) is the aggregated products of the total costs and corresponding probability of each replenishment scenario divided by the inventory cycle length T .

$$TC(s, Q) = (1 - pT) \frac{1}{T} \left[k_o + k_g + Q(c_o + e_o + c_g + e_g) + \frac{hT}{2}(Q + 2s) \right] \quad (17a)$$

$$+ p(T - \tau) \frac{1}{T} \int_0^s \left[k_o + k_g + (Q + x)(c_o + e_o + c_g + e_g) + \frac{hT}{2} \left(Q + 2s - x \left(1 + \frac{\tau}{T} \right) \right) \right] f(x) dx \quad (17b)$$

$$+ p(T - \tau) \frac{1}{T} \int_s^\infty \left[\begin{aligned} &2k_o + k_g + k_a + (Q + x)(c_o + e_o) \\ &+ (Q + s)(c_g + e_g) + (x - s)(c_a + e_a) \\ &+ \frac{hT}{2} \left(Q + 2s - s \left(1 + \frac{\tau}{T} \right) \right) \end{aligned} \right] f(x) dx \quad (17c)$$

$$+ p\tau \frac{1}{T} \int_0^\infty \left[\begin{aligned} &2k_o + k_g + k_a + (Q + x)(c_o + e_o) \\ &+ Q(c_g + e_g) + x(c_a + e_a) \\ &+ \frac{hT}{2}(Q + 2s) \end{aligned} \right] f(x) dx \quad (17d)$$

Based on the probability of each of the four replenishment scenarios, the objective function shown in equation (17) is separated into each of the four scenarios (a-d). The total cost per unit time of the scenario illustrated in (17a) is associated with the regular replenishment scenario in which no emergency demand occurs during the inventory cycle. The probability of such a scenario with no emergency demand within an inventory cycle is $(1 - pT)$. Conversely, the probability for a single occurrence of emergency demand during the entire inventory cycle is pT .

Given an occurrence of emergency demand during an inventory cycle, one of three irregular replenishment scenarios will arise depending on the time point and the quantity of the emergency demand during the inventory cycle. If the emergency demand occurs before the regularly scheduled order leaves the supplier within the first $(T - \tau)$ time units of the inventory cycle, either the first or the second irregular replenishment scenario will arise with a probability of $p(T - \tau)$.

The total cost per unit time of the first irregular replenishment scenario is depicted in equation (17b). The probability of this scenario is equal to the product of the following two probabilities: the first being the probability $p(T - \tau)$ that an emergency demand occurs before the scheduled order leaves the supplier and the second being the probability $f(x \leq s)$ that an emergency demand is less than or equal to the safety stock. If the emergency demand is greater than the safety stock as it is for second irregular replenishment scenario, the probability of the scenario equals the product of the next two probabilities: the probability $p(T - \tau)$ that an emergency demand occurs before the scheduled order leaves the supplier and the probability $f(x > s)$ that an emergency demand

is greater than the safety stock level. The total cost per unit time associated to this irregular replenishment scenario is illustrated in equation (17c).

The final piece of the objective function (17d) is the total cost per unit time of the third and last irregular replenishment scenario. In this final scenario, the emergency demand occurs after the regularly scheduled order leaves the supplier or distributor within the last τ time units of the inventory cycle; thus, the probability that the third irregular replenishment scenario will arise is $p\tau$. Together, the equations illustrate the expected total costs per unit time of the inventory model of a product with a probability p of stochastic emergency demand x between regularly scheduled orders.

Given that in each scenario a regularly scheduled order quantity Q is delivered from the supplier at the beginning of each inventory cycle, the objection function in (17) can be rewritten as following:

$$TC(s, Q) = \frac{1}{T} \left[k_o + k_g + Q(c_o + e_o + c_g + e_g) + \frac{hT}{2}(Q + 2s) \right] \quad (18a)$$

$$+ p \left(1 - \frac{\tau}{T}\right) \int_0^s \left[x(c_o + e_o + c_g + e_g) - \frac{hx}{2}(T + \tau) \right] f(x) dx \quad (18b)$$

$$+ p \left(1 - \frac{\tau}{T}\right) \int_s^\infty \left[\begin{array}{l} k_o + k_a + x(c_o + e_o) \\ + s(c_g + e_g) + (x - s)(c_a + e_a) \\ - \frac{hs}{2}(T + \tau) \end{array} \right] f(x) dx \quad (18c)$$

$$+ p \frac{\tau}{T} \int_0^\infty [k_o + k_a + x(c_o + e_o) + x(c_a + e_a)] f(x) dx \quad (18d)$$

In equation (18), the total cost per unit time of the regularly scheduled order quantity Q is disconnected from each of the replenishment scenarios since it is incurred

regardless of the scenario. So, equation (18a) depicts the procurement, transportation, and inventory cost per unit time of the regularly scheduled order quantity Q which is incurred during the regular replenishment cycle as well as each of the irregular replenishment cycles. Equations (18b-d), on the other hand, depict the additional expected cost per unit time of each of the irregular replenishment scenarios.

The additional expected cost per unit time of the first irregular replenishment scenario is depicted in equation (18b). In this scenario, the emergency demand that occurs before the scheduled order leaves the supplier is less than the safety stock level. So, even though no inventory stock-outs will occur before the next scheduled order is delivered from the supplier, a portion of the safety stock is depleted and requires replenishment by the beginning of the next inventory cycle. Thus, the scheduled order quantity is increased by an amount equal to the emergency demand x in order to replenish the inventory level to the target $(Q + s)$ by the beginning of the next inventory cycle.

The additional expected cost per unit time depicted in (18b) for this scenario consists of an increase to the procurement and transportation costs but a decrease to the inventory cost. In particular, the procurement and transportation costs are increased by the unit costs to procure and ship the emergency demand x by way of ground transportation. The inventory cost, on the other hand, decreases by an amount proportional to the emergency demand x and the length of time – equal to half the time period $(T + \tau)$ – remaining after the emergency demand reduces the inventory level. Since the decrease in inventory cost will always be less than the increase in the procurement and transportation costs, the expected cost per unit time depicted in (18b) for this scenario will always be positive – an increase to the total cost.

The second irregular replenishment scenario, which is similar to the first irregular replenishment scenario, incurs the additional expected costs per unit time that are depicted in equation (18c). In this scenario, the emergency demand that occurs before the scheduled order leaves the supplier is greater than the safety stock level. So, without an emergency order, the inventory level will decrease to zero before the beginning of the next inventory cycle. Thus, an emergency order of size $(x - s)$ which satisfies only the emergency demand not satisfied by the safety stock in the current inventory cycle is triggered immediately; and the next scheduled order is increased by a quantity equal to the completed depleted safety stock level s .

Overall, the additional expected cost per unit time illustrated in (18c) for this scenario is greater than that of any other scenario. For one, procurement costs are increased by a fixed cost to procure the emergency order as well as the unit costs to procure an amount of parts equal to the emergency demand between the two replenishment methods. Secondly, the transportation costs are increased by three factors: the fixed cost to ship the emergency order via the more expensive air transportation; the unit costs to ship the larger regularly scheduled order which replenishes the completely depleted safety stock with ground transportation; and lastly, the unit costs to ship the emergency order of size $(x - s)$ with air transportation. Even with the decrease in inventory cost – which like the first irregular replenishment scenario is decreased by an amount proportional to the depleted safety stock and the length of time remaining after the emergency demand depletes the safety stock – the additional expected cost per unit time for this scenario will always be positive and greater than that of any other scenario.

The third and final irregular replenishment scenario incurs the additional expected total costs per unit time depicted in equation (18d). In this scenario, the emergency demand occurs after the scheduled order leaves the supplier so that without an emergency order, the inventory level at the beginning of the next inventory cycle will be less than the targeted $(Q + s)$ level. The resultant cost of this necessary emergency order is additional fixed and variable costs to procure and transport an order size equal to the size of the emergency demand x via the more expensive but faster mode of air transportation.

Both the previous formulations of the objective function in equations (17) and (18) are structured in a way that can be logically understood with respect to each replenishment scenario. However, these formulations can and should be simplified. Yet before doing so, the function $x(c_g + e_g)$ is added and subtracted to parts (18c-d) so as to not actually change the result of the objective function but aid in later simplification.

$$TC(s, Q) = \frac{1}{T} [k_o + k_g + Q(c_o + e_o + c_g + e_g)] + \frac{h}{2}(Q + 2s) \quad (19a)$$

$$+ p(c_o + e_o + c_g + e_g)E[x] \quad (19b)$$

$$+ p\left(1 - \frac{\tau}{T}\right) \int_s^\infty [k_o + k_a + (x - s)(c_a + e_a - c_g - e_g)] f(x) dx \quad (19c)$$

$$+ p\left(\frac{\tau}{T}\right) [(k_o + k_a) + (c_a + e_a - c_g - e_g)E[x]] \quad (19d)$$

$$- p\left(1 - \frac{\tau}{T}\right) \left(\frac{h}{2}\right) (T + \tau) \left[\int_0^s xf(x) dx + \int_s^\infty sf(x) dx \right] \quad (19e)$$

Given the revised objective function in equation (19), a few assumptions can be introduced to assist in simplifying the objective function. For one, the inventory cycle length T is a function of the regularly scheduled order quantity Q and the constant

demand D . Specifically, the inventory cycle length T is equivalent to the quotient of the scheduled order quantity Q and the constant demand D . So, from this point on, every instance of T in the objective function is replaced with the quotient of Q and D .

Another approach to simplify the readability of the objective function is to replace parameters that are summed multiple times with representative symbols. For instance, since the sum $(c_a + e_a - c_g - e_g)$ appears multiple times in the objective function, it is hereby replaced by the symbol δ . Thus, every time the symbol δ appears in the objective function, it will represent the difference between the unit costs of air transportation and the unit costs of ground transportation $(c_a + e_a - c_g - e_g)$. Since it is already assumed that all the transportation costs via air transportation are greater than those via ground transportation $(c_a + e_a > c_g + e_g)$, it can also be assumed that δ , which represents the positive difference between the two costs, is greater than zero ($\delta > 0$). Likewise, the sum $(k_o + k_g)$ is hereby replaced by the symbol γ ; the sum $(k_o + k_a)$ is hereby replaced by the symbol α ; and lastly, the sum $(c_o + e_o + c_g + e_g)$ is hereby replaced by the symbol β . Given these replacements, the objective function can be rewritten as the following:

$$TC(s, Q) = \frac{D}{Q} \gamma + D\beta + \frac{h}{2}(Q + 2s) \quad (20a)$$

$$+ p\beta E[x] \quad (20b)$$

$$+ p \left(1 - \frac{\tau D}{Q}\right) \int_s^\infty [\alpha + \delta(x - s)] f(x) dx \quad (20c)$$

$$+ p \left(\frac{\tau D}{Q}\right) [\alpha + \delta E[x]] \quad (20d)$$

$$- p \left(1 - \frac{\tau D}{Q}\right) \left(\frac{h}{2}\right) \left(\frac{Q}{D} + \tau\right) \left[\int_0^s x f(x) dx + \int_s^\infty s f(x) dx \right] \quad (20e)$$

Equation (20) can then be simplified into the final form of the objective function which is shown in equation (21):

$$TC(s, Q) = \frac{D}{Q} [\gamma + p\tau(\alpha + \delta E[x])] + \beta(D + pE[x]) + \frac{h}{2}(Q + 2s) \quad (21a)$$

$$+ p \left(1 - \frac{\tau D}{Q} \right) \int_s^{\infty} [\alpha + \delta(x - s)] f(x) dx \quad (21b)$$

$$- p \frac{h}{2} \left(\frac{Q}{D} - \frac{\tau^2 D}{Q} \right) \left[\int_0^s x f(x) dx + \int_s^{\infty} s f(x) dx \right] \quad (21c)$$

3.4 Solution Procedure

The economically optimal solution to operating an inventory system under constant demand and a probability of stochastic emergency demand is defined by the inventory policy that minimizes the total cost per unit time. Since the inventory policy is determined by two decision variables – safety stock s and order quantity Q – the optimal solution set contains all values of s and Q such that the derivative of the total cost per unit time is equal to zero. Therefore, the optimal decisions s and Q must satisfy

$$\frac{d}{ds} TC(s, Q) = 0 \quad \text{and} \quad \frac{d}{dQ} TC(s, Q) = 0. \quad (22-23)$$

The optimal policy must also satisfy the conditions for which the Hessian matrix below is positive-definite. This additional condition guarantees that the solution is the optimal minimum cost and not a maximum cost or some other local optimal cost.

$$\nabla^2 TC(s^*, Q^*) = \begin{bmatrix} \frac{d}{ds} TC(s, Q) & \frac{d}{dsdQ} TC(s, Q) \\ \frac{d}{dsdQ} TC(s, Q) & \frac{d}{d^2Q} TC(s, Q) \end{bmatrix} \quad (24)$$

3.4.1 First-Order Derivative Condition

The first set of optimal conditions are derived by setting the first order derivative of the total cost per unit with respect to the safety stock level s and the scheduled order quantity Q equal to zero. The derivative of the total cost per unit time with respect to the safety stock level s is shown in equation (25):

$$\frac{d}{ds} \text{TC}(s, Q) = h \quad (25a)$$

$$+ p \left(1 - \frac{\tau D}{Q} \right) \left[-\alpha f(s) - \delta s f(s) - \delta (F(\infty) - F(s) - s f(s)) \right] \quad (25b)$$

$$- p \left(\frac{h}{2} \right) \left(\frac{Q}{D} - \frac{\tau^2 D}{Q} \right) \left[s f(s) + F(\infty) - F(s) - s f(s) \right] \quad (25c)$$

The derivative in (25) can be rewritten in a simpler form as shown in equation (26).

$$\frac{d}{ds} \text{TC}(s, Q) = h \quad (26a)$$

$$+ p \left(1 - \frac{\tau D}{Q} \right) \left[-\alpha f(s) - \delta (1 - F(s)) \right] \quad (26b)$$

$$- p \left(\frac{h}{2} \right) \left(\frac{Q}{D} - \frac{\tau^2 D}{Q} \right) \left[1 - F(s) \right] \quad (26c)$$

Thus, one condition the optimal solution must satisfy is that the first order partial derivative of the total cost per unit value with respect to the safety stock quantity s be equal to zero as shown in equation (27).

$$h - p \left(1 - \frac{\tau D}{Q} \right) \left[\alpha f(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) (1 - F(s)) \right] = 0 \quad (27)$$

The second optimal conditions is derived by setting the first order derivative of the total cost per unit with respect to the scheduled order quantity Q equal to zero. This first order derivative with respect to the scheduled order quantity Q is shown in (28):

$$TC(s, Q) = -\frac{D}{Q^2} [\gamma + p\tau(\delta E[x] + \alpha)] + \frac{h}{2} \quad (28a)$$

$$+ p\left(\frac{\tau D}{Q^2}\right) \int_s^\infty [\alpha + \delta(x-s)] f(x) dx \quad (28b)$$

$$- p\left(\frac{h}{2}\right) \left(\frac{1}{D} + \frac{\tau^2 D}{Q^2}\right) \left[\int_0^s xf(x) dx + \int_s^\infty sf(x) dx \right] \quad (28c)$$

After integrating equation (28), the derivative can be written as the following:

$$TC(s, Q) = -\frac{D}{Q^2} [\gamma + p\tau(\delta E[x] + \alpha)] + \frac{h}{2} \quad (29a)$$

$$+ p\left(\frac{\tau D}{Q^2}\right) \left[\delta \int_s^\infty xf(x) dx + (\alpha - \delta s)(F(\infty) - F(s)) \right] \quad (29b)$$

$$- p\left(\frac{h}{2}\right) \left(\frac{1}{D} + \frac{\tau^2 D}{Q^2}\right) \left[\int_0^s xf(x) dx + s(F(\infty) - F(s)) \right] \quad (29c)$$

For easier simplification later, the derivative can be the rewritten as the following:

$$\frac{d}{dQ} TC(s, Q) = -\frac{D}{Q^2} (\gamma + p\tau\alpha - p\tau\alpha(1 - F(s))) + \frac{h}{2} \quad (30a)$$

$$- p\left(\frac{\tau D}{Q^2}\right) \left[\delta \left(E[x] - \int_s^\infty xf(x) dx \right) - \delta s(1 - F(s)) \right] \quad (30b)$$

$$- p\left(\frac{h}{2}\right) \left(\frac{1}{D} + \frac{\tau^2 D}{Q^2}\right) \left[\int_0^s xf(x) dx + s(F(\infty) - F(s)) \right] \quad (30c)$$

The derivative can then be simplified to the following:

$$\frac{d}{dQ} \text{TC}(s, Q) = -\frac{D}{Q^2}(\gamma + p\tau\alpha F(s)) + \frac{h}{2} \quad (31a)$$

$$- p\delta \left(\frac{\tau D}{Q^2} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] \quad (31b)$$

$$- p \left(\frac{h}{2} \right) \left(\frac{1}{D} + \frac{\tau^2 D}{Q^2} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] \quad (31c)$$

Thus, the second condition that the optimal solution set must satisfy in which the first order partial derivative of the total cost per unit time with respect to the scheduled order quantity Q is equal to zero is shown in equation (32):

$$\frac{h}{2} - \frac{D}{Q^2} [\gamma + p\tau\alpha F(s)] - p \left(\delta \frac{\tau D}{Q^2} + \frac{h}{2D} + \frac{h\tau^2 D}{2Q^2} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] = 0 \quad (32)$$

So, the first set of optimal conditions that are derived by setting the first order derivative of the total cost per unit time with respect to the safety stock level s and the scheduled order quantity Q equal to zero are shown again in equations (33) and (34):

$$h - p \left(1 - \frac{\tau D}{Q} \right) \left[\alpha f(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) (1 - F(s)) \right] = 0 \quad (33)$$

$$\frac{h}{2} - \frac{D}{Q^2} [\gamma + p\tau\alpha F(s)] - p \left(\delta \frac{\tau D}{Q^2} + \frac{h}{2D} + \frac{h\tau^2 D}{2Q^2} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] = 0 \quad (34)$$

3.4.2 Second-Order Derivative Condition

The second set of optimal conditions satisfies the conditions for which the Hessian matrix (35) is positive-definite. This condition guarantees that the solution is the global optimal minimum cost and not a maximum cost or some other local optimal cost.

There are at least two conditions that the Hessian matrix must satisfy for it to be defined as positive definite. For one, the matrix must be symmetric. That is, the components in the diagonals of the matrix must share the same sign. For a matrix to be positive-definite, the components of the primary diagonal must be positive. Secondly, the determinant of the matrix must be positive. The determinant of the Hessian matrix in (35) is illustrated in (36).

$$\nabla^2 \text{TC}(s^*, Q^*) = \begin{bmatrix} \frac{d}{d^2 s} \text{TC}(s, Q) & \frac{d}{dsdQ} \text{TC}(s, Q) \\ \frac{d}{dsdQ} \text{TC}(s, Q) & \frac{d}{d^2 Q} \text{TC}(s, Q) \end{bmatrix} \quad (35)$$

$$\det(\nabla^2 \text{TC}(s^*, Q^*)) = \left(\frac{d}{d^2 s} \text{TC}(s, Q) \right) \left(\frac{d}{d^2 Q} \text{TC}(s, Q) \right) - \left(\frac{d}{dsdQ} \text{TC}(s, Q) \right)^2 \quad (36)$$

In order to determine whether the Hessian matrix is positive-definite, the second order partial derivatives of the total cost per unit time with respect to the safety stock level s and the scheduled order quantity Q must be determined. The second order partial derivative of the total cost per unit time with respect to the safety stock level s alone is derived from the equation (33) and shown in equation (37):

$$\frac{d}{d^2 s} \text{TC}(s, Q) = p \left(1 - \frac{\tau D}{Q} \right) \left[-\alpha f'(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) f(s) \right] \quad (37)$$

The second order partial derivative of the total cost per unit time with respect to the scheduled order quantity Q alone is derived from the equation (34) and shown in equation (38). Equation (39) is the simplified form of the second order partial derivative with respect to Q .

$$\frac{d}{d^2 Q} \text{TC}(s, Q) = \frac{D}{Q^3} [\gamma + p\tau\alpha F(s)] - p \left(-\delta \frac{\tau D}{Q^3} - \frac{h\tau^2 D}{2Q^3} \right) \left[\int_0^s x f(x) dx + s(1 - F(s)) \right] \quad (38)$$

$$\frac{d}{d^2Q} \text{TC}(s, Q) = \frac{D}{Q^3} \left[\gamma + p\tau\alpha F(s) + p\tau \left(\delta + \frac{h\tau}{2} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] \right] \quad (39)$$

The second order partial derivative of the total cost per unit time with respect to first the safety stock level s and then the scheduled order quantity Q is derived from the equation (33) and shown in equation (40). This second order partial derivative can be simplified to the form in equation (41).

$$\frac{d}{dsdQ} \text{TC}(s, Q) = -p\alpha \left(\frac{\tau D}{Q^2} \right) f(s) - p \left(\delta \frac{\tau D}{Q^2} + \frac{h\tau^2 D}{2Q^2} + \frac{h}{2D} \right) (1 - F(s)) \quad (40)$$

$$\frac{d}{dsdQ} \text{TC}(s, Q) = -p \frac{\tau D}{Q^2} [\alpha f(s) + \delta(1 - F(s))] - p \left(\frac{h\tau^2 D}{2Q^2} + \frac{h}{2D} \right) (1 - F(s)) \quad (41)$$

To verify the equations, the second order partial derivative of the total cost per unit time with respect to first the scheduled order quantity Q and then the safety stock level s is derived from the equation (34) and shown in equation (43). This should be equivalent to the second order partial derivative derived in equation (40) and simplified in equation (41).

$$\frac{d}{dQds} \text{TC}(s, Q) = -\frac{D}{Q^2} [p\tau\alpha f(s)] - p \left(\delta \frac{\tau D}{Q^2} + \frac{h\tau^2 D}{2Q^2} + \frac{h}{2D} \right) [sf(s) + 1 - F(s) - sf(s)] \quad (42)$$

$$\frac{d}{dQds} \text{TC}(s, Q) = -p \left(\frac{\tau D}{Q^2} \right) \left[\alpha f(s) + \left(\delta + \frac{h\tau}{2} \right) (1 - F(s)) \right] - p \left(\frac{h}{2D} \right) (1 - F(s)) \quad (43)$$

As expected, second order partial derivative of the total cost per unit time with respect to the safety stock level s and to the scheduled order quantity Q in equations (41) and (43) are equivalent even though each was derived from a different first order partial derivative. Thus, the derivations thus far are at least mathematically accurate.

Now that the second order partial derivatives of the total cost per unit time are derived, the Hessian matrix can be determined and thus the second set of solution conditions. The Hessian matrix for the current model is shown in equation (44). The determinant of this Hessian is shown in equation (45).

$$\nabla^2 \text{TC}(s^*, Q^*) = \begin{bmatrix} p \left(1 - \frac{\tau D}{Q}\right) \left[-\alpha f'(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau D}{2D} \right) f(s) \right] & -p \left[\alpha \frac{\tau D}{Q^2} f(s) + \left(\delta \frac{\tau D}{Q^2} + \frac{h}{2D} + \frac{h\tau^2 D}{2Q^2} \right) (1 - F(s)) \right] \\ -p \left[\alpha \frac{\tau D}{Q^2} f(s) + \left(\delta \frac{\tau D}{Q^2} + \frac{h}{2D} + \frac{h\tau^2 D}{2Q^2} \right) (1 - F(s)) \right] & \frac{D}{Q^3} \left[\gamma + p\tau\alpha F(s) + p\tau \left(\delta + \frac{h\tau}{2} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] \right] \end{bmatrix} \quad (44)$$

$$\det(\nabla^2 \text{TC}(s^*, Q^*)) = p \left(1 - \frac{\tau D}{Q}\right) \left[-\alpha f'(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) f(s) \right] \quad (45a)$$

$$\times \frac{D}{Q^3} \left[\gamma + p\tau\alpha F(s) + p\tau \left(\delta + \frac{h\tau}{2} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] \right] \quad (45b)$$

$$- p^2 \left[\alpha \frac{\tau D}{Q^2} f(s) + \left(\delta \frac{\tau D}{Q^2} + \frac{h}{2D} + \frac{h\tau^2 D}{2Q^2} \right) (1 - F(s)) \right]^2 \quad (45c)$$

To satisfy the condition for which the Hessian matrix in (44) is positive definite, the matrix must first be symmetric. That is, the components in the diagonals of the matrix must share the same sign. Since the components in the bottom-left to top-right diagonal are the same second-order derivative of the total cost, it can be inferred that the components in this diagonal share the same sign. Though the sign of this diagonal is inconsequential, the sign of the components in the top-left to bottom-right diagonal must be positive in order for the matrix to be positive definite. Thus, it is necessary to prove that these components are positive.

The component in the top-left of the matrix is the second order partial derivative of the total cost per unit time with respect to the safety stock level s alone as shown in

equation (37). In order to logically determine the sign of this diagonal component, the equation in (37) is separated into several segments shown in equations (46-48).

The first segment of the equation in the top-left position of the matrix shown in (46) consists of the ratio of constant demand to order quantity. This ratio is equivalent to the inverse of the inventory cycle length. Given the fact that the scheduled order lead time is always less than the length of the inventory cycle, the product the prior ratio – the inverse of the inventory cycle length – and the scheduled order lead time will always be less than one. So the first segment of the top-left matrix component is always positive.

$$p\left(1 - \frac{\tau D}{Q}\right) \quad (46)$$

$$- \alpha f'(s) \quad (47)$$

$$\left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2}\right) f(s) \quad (48)$$

The other segments of the top-left matrix component shown in equations (47-48) are multiplied by the first segment illustrated in equation (46) to obtain equation (37). For the entire component to be positive, the sum of the segments in equations (47-48) must also be positive. It is simple to observe that the segment illustrated in equation (48) is always positive, because all the terms and the signs in the equation are positive.

However, the segment illustrated in equation (47) cannot be easily assumed as positive or negative since it comprises the derivative of a probability distribution function. Depending on the distribution for the emergency demand, equation (47) may be positive or negative. If the derivative of the probability distribution function is less than or equal to zero, then equation (47) is positive and thus the whole equation is guaranteed to be positive. However, if the derivative of the probability distribution

function is greater than zero, then equation (47) is negative and the sum of the equation (47) and (48) may either be positive or negative.

In order for the top-left component of the Hessian matrix to be positive and partially satisfy the conditions for which the Hessian matrix is positive definite, the optimal solution must satisfy the necessary condition in which sum of equations (47) and (48) are greater than zero and thus positive as shown in (49):

$$-\alpha f'(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) f(s) > 0 \quad (49)$$

In addition to the requirement for the top-left component of the Hessian matrix, there is a requirement for the bottom-right component of the Hessian matrix in order for the matrix to partially satisfy the conditions for which the Hessian matrix is positive definite. The component in the bottom-right of the matrix is the second order partial derivative of the total cost per unit time with respect to the scheduled order quantity Q alone as shown in equation (39). In order to logically determine the sign of this diagonal component, the equation in (39) is separated into two segments shown in (50-51).

$$\frac{D}{Q^3} [\gamma + p\tau\alpha F(s)] \quad (50)$$

$$\frac{D}{Q^3} \left[p\tau \left(\delta + \frac{h\tau}{2} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] \right] \quad (51)$$

The first segment of the equation as shown in (50) consists of all positive terms and signs; whereas the second segment of the equation in (51) consists of all positive terms but a negative sign. Even with the negative sign, the segment illustrated in equation (51) is always positive because the term being subtracted – the cumulative probability of a continuously distribution – is always less than or equal to one. Thus, the

segment in equation (51) is always positive; and the whole portion of the bottom-right component of the Hessian matrix is always positive.

Consequently, there are then only two conditions necessary for Hessian Matrix to be positive definite. For one, the Hessian matrix in (44) must satisfy the condition in (49) such that the diagonals of the matrix share the same sign and the primary diagonal from top-left to bottom-right is positive. Secondly, the solution set must satisfy the condition in (52) in which the determinant of the Hessian matrix is greater than zero.

$$0 < p \left(1 - \frac{\tau D}{Q} \right) \left[-\alpha f'(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) f(s) \right] \quad (52a)$$

$$\times \frac{D}{Q^3} \left[\gamma + p\tau\alpha F(s) + p\tau \left(\delta + \frac{h\tau}{2} \right) \left[\int_0^s xf(x)dx + s(1-F(s)) \right] \right] \quad (52b)$$

$$- p^2 \left[\alpha \frac{\tau D}{Q^2} f(s) + \left(\delta \frac{\tau D}{Q^2} + \frac{h\tau^2 D}{2Q^2} + \frac{h}{2D} \right) (1-F(s)) \right]^2 \quad (52c)$$

3.4.3 Summary of Solution Conditions

The economically optimal solution to operating an inventory system under constant demand and a probability of stochastic emergency demand is defined by the inventory policy that minimizes the total cost per unit time. Since the inventory policy is determined by two decision variables – safety stock s and order quantity Q – the optimal solution set contains all values of s and Q such that the first-order partial derivatives of the total cost per unit time are equal to zero and the Hessian matrix of the second-order partial derivatives is positive definite. Therefore, the optimal decisions s and Q must satisfy two sets of optimal conditions.

The first set of optimal conditions are derived by setting the first order derivative of the total cost per unit with respect to the safety stock level s and the scheduled order quantity Q equal to zero. These conditions which are reiterated in (53) and (54) guarantee that the total cost per unit time is either minimized or maximized.

$$h - p \left(1 - \frac{\tau D}{Q} \right) \left[\alpha f(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) (1 - F(s)) \right] = 0 \quad (53)$$

$$\frac{h}{2} - \frac{D}{Q^2} [\gamma + p\tau\alpha F(s)] - p \left(\delta \frac{\tau D}{Q^2} + \frac{h\tau^2 D}{2Q^2} + \frac{h}{2D} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] = 0 \quad (54)$$

The second set of conditions guarantees that the total cost per unit time is a global minimum and not a local minimum or even a global maximum. These conditions which are reiterated in (55) and (56) satisfy the necessary requirements for the Hessian matrix of the second-order partial derivatives of the total cost per unit time to be positive-definite. That is, conditions for which the Hessian matrix is symmetric with positive diagonals and the determinant of the Hessian matrix to be greater than zero.

$$-\alpha f'(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) f(s) > 0 \quad (55)$$

$$0 < p \left(1 - \frac{\tau D}{Q} \right) \left[-\alpha f'(s) + \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) f(s) \right] \quad (56a)$$

$$\times \frac{D}{Q^3} \left[\gamma + p\tau\alpha F(s) + p\tau \left(\delta + \frac{h\tau}{2} \right) \left[\int_0^s xf(x)dx + s(1 - F(s)) \right] \right] \quad (56b)$$

$$- p^2 \left[\alpha \frac{\tau D}{Q^2} f(s) + \left(\delta \frac{\tau D}{Q^2} + \frac{h\tau^2 D}{2Q^2} + \frac{h}{2D} \right) (1 - F(s)) \right]^2 \quad (56c)$$

CHAPTER 4

NUMERICAL ANALYSIS & RESULTS

The purpose of the inventory model developed in the previous sections is to determine an optimal inventory policy under explicit energy cost considerations. Specifically, the objective of the model is to find an inventory policy with optimal sizes for a scheduled order quantity Q , a safety stock level s , and an inventory cycle length T that minimizes the expected total cost per unit time of a part with a fairly regular demand but a small probability of emergency demand. Such an inventory policy is expected to be applicable for production systems with constant production rates but small, underlying possibilities for undesirable circumstances to threaten the planned production schedules.

In order to illustrate the effect of energy on inventory policies, the inventory model developed in the previous sections is numerically analyzed with respect to changes in energy cost as well as numerous other model parameters that are reasonable to similar production environments. The resultant inventory policy decisions and respective logistics costs for the various model parameters are analyzed and compared to the traditional EOQ model in order to further validate the inventory model and illustrate the cases in which it is most effective.

4.1 Numerical Analysis Parameters

In the numerical analysis to follow, most of the model parameters are initially varied between only two levels in order to identify the key parameters that affect inventory policy decisions and the resultant logistics costs. These parameters are organized into the following three logistics functions: supply procurement, production, and transportation. After key parameters are identified, the levels at which the key parameters vary need not be limited to the two initial levels in the subsequent analysis. Nevertheless, the purpose of varying most of the model parameters by only two levels in the analysis is to discover and understand the environments in which the inventory policies are most significantly affected by changes to energy cost as well as the environments in which the proposed inventory model is most cost effective.

4.1.1 Supply Procurement Parameters

The first set of model parameters are the purchasing costs associated with the procurement of raw materials or unfinished products from a supplier or distributor required by the production system. These procurement costs include a fixed purchasing cost k_o to order any number of products from the supplier, a variable purchasing cost e_o associated to the energy consumed in order to supply a single unit of the product, and a variable purchasing cost c_o associated to everything but the energy cost to supply a single unit of the product. As displayed in Table 4.1, the fixed purchasing cost and the total variable purchasing costs ($c_o + e_o$) are varied between two levels that are reasonable to the procurement activities at similar production environments.

Set	Description	Low Level	High Level
Supply	Fixed Purchasing Cost to Order k_o	\$25	\$100
	Total Unit Purchasing Cost to Procure $(c_o + e_o)$	\$100	\$500

Table 4.1: Supply Procurement Parameters

4.1.2 Production Parameters

The second set of model parameters are the factors associated with the demand of the production system. Again, the production system is characterized with a limited manufacturing capacity. As a result, customer orders can be processed very early and delivered at a much later date. So, production schedules are planned far in advance, and the production rate is fairly constant from regardless of the actual customer demand.

Even though the production rate and thus the demand of raw materials and unfinished products by the production system are fairly constant, there is a possibility for undesirable circumstances to arise and threaten the intricately planned production schedule. The inventory model developed in the previous sections is expected to be applicable for production systems with constant production rates but small, underlying possibilities for undesirable circumstances to threaten the production schedule.

Though the possibility of more than one undesirable circumstance occurring within any inventory cycle is so small it is presumed negligible in the proposed model, the volume of the emergency demand generated by the undesirable circumstance can be any number. Furthermore, the probability and the total cost for any of the irregular replenishment scenarios depend on the random size of the emergency demand. So, the optimal inventory policy is contingent upon the probability distribution of the emergency demand volume generated by the undesirable circumstance.

The inventory model developed in the previous sections is formulated in such a way that any probability distribution can be selected to represent that of the stochastic emergency demand. In the subsequent analysis of the inventory model, two different distributions – the Uniform distribution and the Exponential distribution – are selected to portray the behavior of the emergency demand in similar manufacturing systems.

While the objective function and solution approaches are rewritten with respect to either the Uniform distribution or the Exponential distribution in the subsequent sections, the model parameters for the two distributions used in the numerical analysis are shown in Table 4.2. For both distributions, the mean emergency demand varies between two levels – low and high – which depend on the size of the regular demand. Hence, there are essentially four levels at which the mean emergency demand varies in the subsequent numerical analysis. This assumption is reasonable given that the volume of the emergency demand depends partially on the regular demand from the production system.

Set	Description	Low Level High Level			
Production	Regular Demand per unit time D	3		30	
	Mean Emergency Demand	Low	High	Low	High
	Uniform Distribution $x \sim U(a,b)$	1, 3	2, 4	2, 4	10, 20
	Exponential Distribution $x \sim E(u)$	2	3	3	15
	Probability of Emergency Demand p	0.01 == 1.0%			

Table 4.2: Production Parameters

In addition to the varying levels of regular and emergency demand, Table 4.2 shows the probability of the emergency demand held constant throughout all the numerical analysis to follow. The value is assumed to be 0.01 in order to represent the very small probability of an undesirable circumstance occurring during an inventory

cycle and the negligible possibility of more than one undesirable circumstance occurring during an inventory cycle.

4.1.3 Transportation Parameters

The third set of model parameters are factors associated to transportation activities. These parameters include the transportation lead time of the regularly scheduled order as well as the transportation costs. Like many of the aforementioned parameters, the transportation lead time τ varies between two levels – short and long – as shown in Table 4.3. Ground shipment often requires a three to five day lead time, but sometimes requires an even longer lead time for various reasons including longer shipping distance. Furthermore, the total lead time which includes the supplier or manufacturing lead time may be even longer.

Set	Description		Low Level		High Level		
Transportation	Lead Time	τ	3 days		<i>10 days</i>		
	Distance		Near		<i>Far</i>		
	Fixed Cost to Ship via Ground Transport	k_g	\$5		<i>\$20</i>		
	Fixed Cost to Ship via Air Transport	k_a	\$30		<i>\$60</i>		
	Weight		Light		<i>Heavy</i>	Light	<i>Heavy</i>
	Unit Cost to Ship via Ground Transport	c_g	\$3	<i>\$10</i>	\$5	<i>\$40</i>	
Unit Cost to Ship via Air Transport	c_a	\$20	<i>\$85</i>	\$30	<i>\$105</i>		

Table 4.3: Transportation Parameters

In addition to lead time, Table 4.3 shows the values at which transportation cost vary in the subsequent analysis. Since transportation costs are dependent on several factors including shipping distance, package weight, and fuel cost, the transportation costs in the subsequent analysis vary based on changes to these factors. That is, each

component of transportation cost for either transportation mode $m \in (g, a)$ varies between two levels for each factor that affects the respective cost component.

The first transportation cost factor shown in Table 4.3 – distance – is related to all the transportation cost components and varies between two levels – near and far. Since the fixed transportation cost k_m to ship any number of parts via either transportation mode $m \in (g, a)$ is dependent only on the distance of the shipment, these fixed transportation costs vary between only two levels that are dependent upon the two levels at which the shipping distance varies.

Alternatively, the variable transportation cost c_m associated to everything but the energy cost to ship a single unit of the product via either transportation mode $m \in (g, a)$, is dependent on both the shipping distance and the package weight. Similarly to the shipping distance, the package weight varies between two levels – light and heavy. So, the unit cost c_m to ship a product for either transportation mode $m \in (g, a)$ varies between four levels – two levels of weight for each level of the two levels of shipping distance.

The final transportation cost e_m associated to the energy consumed in order to ship a single unit of the product via either transportation mode $m \in (g, a)$ is dependent on shipping distance, package weight, and energy cost. Though this cost component is incurred due to transportation activities, it is strongly related to energy cost and thus presented in the following section with energy parameters.

4.1.4 Energy Parameters

The set of model parameters associated to energy cost and consumption is directly related to the variable energy cost to procure or ship a single product. More specifically,

the energy parameters shown in Table 4.4 affect the unit purchasing cost e_o associated to the energy consumed to procure a single unit of product as well as the unit transportation cost e_m associated to the energy consumed to ship a single unit of product via either transportation mode $m \in (g, a)$.

Set	Description	Low Level	High Level
Energy	Energy Proportion of Total Unit Purchasing Cost to Procure	$e_o / (c_o + e_o)$	5% 20%
	Energy cost - gas	ε	\$2 \$9
	Fuel Surcharge for Mode m	$r_m \geq s_m * \varepsilon + b_m$	
	Unit Energy Cost to Ship	$e_m = (c_m \times r_m)$	

Table 4.4: Energy Parameters

The first energy parameter shown in Table 4.4 affects the unit energy purchasing cost e_o to procure a single product. This cost is related to the energy consumed by any activity not including transportation before the product ships to the production system. Throughout the subsequent analysis, the unit energy purchasing cost e_o is modeled as a proportion of the total unit purchasing cost ($c_o + e_o$) to procure a single unit of product, rather than a function of the energy source, energy cost, and energy consumption. However, since the total unit purchasing cost already varies between two levels as shown in Table 4.1 with the supply procurement parameters, the energy proportion of the total unit purchasing cost only affects the individual unit costs to procure a single product and not the total unit cost to procure a product.

The second energy parameter shown in Table 4.4 affects the unit transportation cost e_m associated to the energy consumed to ship a single unit of product via either transportation mode $m \in (g, a)$. As noted previously, the unit energy cost to ship is dependent upon shipping distance, package weight, and energy cost. Generally,

transportation and shipping businesses base the energy cost portion of the total unit shipping cost on a fuel surcharge rate, which is a function of fuel cost and shipping mode. An example of such a fuel surcharge rate utilized by a major logistics company in 2010 for two possible transportation modes is shown in Figure 4.1.

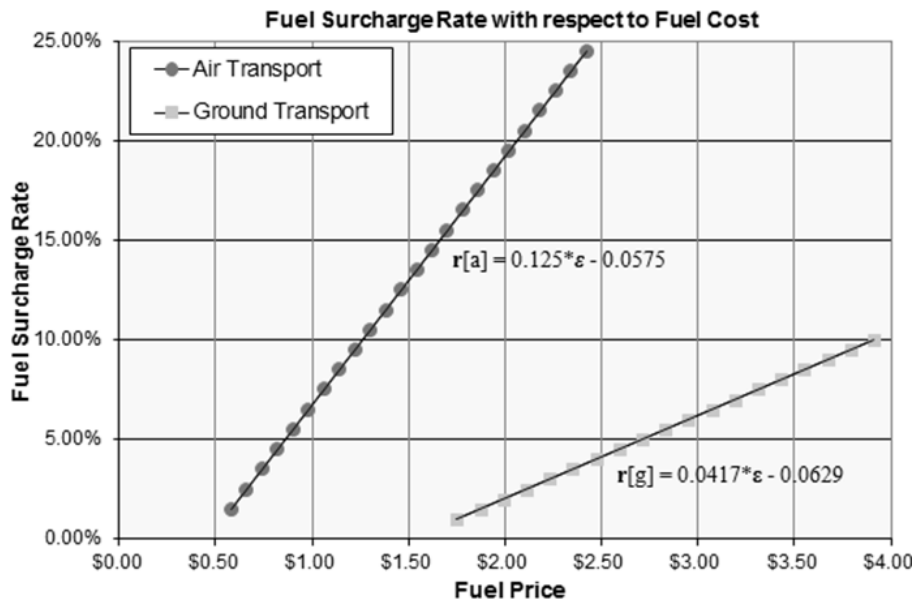


Figure 4.1: Fuel Surcharge Rates with respect to Fuel Cost

In addition to a fuel surcharge rate, transportation and shipping businesses often base the unit energy cost e_m to ship a product on a function of shipping distance and package weight. Since the unit shipping cost c_m not associated to energy is already a function of shipping distance and package weight, transportation and shipping businesses frequently use the product this function and the function of fuel surcharge rate to price the unit energy cost to ship via a specified transportation mode.

Accordingly, the unit energy cost e_m to ship a single unit of product via either transportation mode $m \in \{g, a\}$ is modeled in the subsequent analysis as the product of the unit shipping cost c_m and the fuel surcharge rate r_m . The model parameters for these

energy factors in the subsequent numerical analysis are shown in Tables 4.3 and 4.4 as well as Figures 4.1. As shown in Table 4.3, the unit shipping cost c_m varies between two values for package weight for each of the two values at which the shipping distance varies. The fuel cost varies between the two levels shown in Table 4.4; and the parameters for the fuel surcharge rate r_m excluding the fuel cost for either transportation mode $m \in (g, a)$ are held constant and shown in Figure 4.1.

4.1.5 Miscellaneous Parameters

The final set of model parameters are those that are specified for the subsequent numerical analysis but not included in the prior sets of parameters. These include the time unit of the model and the inventory cost parameters. As shown in Table 4.5, the time unit of the inventory model is held constant to one day throughout all the subsequent analysis. So, the resultant logistics costs of the inventory decisions are presented in terms of costs per day; and, the resultant inventory cycle length is presented in terms of days.

Set	Description	Low Level	High Level
Miscellaneous	Time Unit	One Day	
	Inventory Holding Rate per Year r_h	30%	
	Holding Cost per Year $h = r_h * (c_o + e_o)$		

Table 4.5: Miscellaneous Parameters

In addition to the model time unit, Table 4.5 shows the inventory cost parameters that are held constant throughout the subsequent analysis. Similarly to most textbooks and research, the unit inventory holding cost per time period is modeled as a function of the total unit procurement costs ($c_o + e_o$) and the inventory holding cost rate r_h per time

period. Because the annual inventory holding cost rate varies between 25 and 50 percent in most textbooks and research, the annual inventory holding cost rate in the subsequent analysis is held constant at 30 percent. This rate and thus the unit inventory holding cost is translated into days, however, for the subsequent analysis.

4.1.6 Summary of Model Parameters

Given the five sets of parameters described above, a table summarizing all the parameters for the following numerical analysis is shown in Table 4.6.

Set	Description	Low Level High Level			
Supply	Fixed Purchasing Cost to Order k_o	\$25		\$100	
	Total Unit Purchasing Cost to Procure $(c_o + e_o)$	\$100		\$500	
Production	Regular Demand per unit time D	3		30	
	Mean Emergency Demand	Low	High	Low	High
	Uniform Distribution $x \sim U(a,b)$	1, 3	2, 4	2, 4	10, 20
	Exponential Distribution $x \sim E(u)$	2	3	3	15
	Probability of Emergency Demand p	0.01 == 1.0%			
Transportation	Lead Time τ	3 days		10 days	
	Distance	Near		Far	
	Fixed Cost to Ship via Ground Transport k_g	\$5		\$20	
	Fixed Cost to Ship via Air Transport k_a	\$30		\$60	
	Weight	Light	Heavy	Light	Heavy
	Unit Cost to Ship via Ground Transport c_g	\$3	\$10	\$5	\$40
Unit Cost to Ship via Air Transport c_a	\$20	\$85	\$30	\$105	
Energy	Energy Proportion of Total Unit Purchasing Cost to Procure $e_o / (c_o + e_o)$	Any %			
	Energy cost - gas ε	\$2		\$9	
	Fuel Surcharge for Mode m $r_m \geq s_m * \varepsilon + b_m$ Unit Energy Cost to Ship $e_m = (c_m * r_m)$				
Miscellaneous	Time Unit	One Day			
	Inventory Holding Rate per Year r_h Holding Cost per Year $h = r_h * (c_o + e_o)$	30%			

Table 4.6: Model Parameters for Numerical

4.2 Numerical Solution Approach given Uniform Distribution

Assuming that the emergency demand is distributed uniformly between a minimum point a and a maximum point b , the inventory model formulated in Chapter 3 can be modified to the following objective function and solution conditions. Given the modifications, the model is numerically analyzed over a variety of parameters. Finally, the solution to the inventory model is compared to a traditional economic ordering policy assuming again that the emergency demand is distributed uniformly.

4.2.1 Objective Function

With the assumption that the emergency demand x follows a uniform distribution from minimum a to a maximum b , the total cost per unit time can be written as follows:

$$TC(s, Q) = \frac{D}{Q} \left[\gamma + p\tau \left(\alpha + \delta \frac{(a+b)}{2} \right) \right] + \beta \left(D + p \frac{(a+b)}{2} \right) + h \left(\frac{Q}{2} + s \right) \quad (57a)$$

$$+ p \left(1 - \frac{\tau D}{Q} \right) \int_s^b \left[\alpha + \delta(x-s) \right] \left(\frac{1}{b-a} \right) dx \quad (57b)$$

$$- p \frac{h}{2} \left(\frac{Q}{D} - \frac{\tau^2 D}{Q} \right) \left[\int_a^s \frac{x}{b-a} dx + \int_s^b \frac{s}{b-a} dx \right] \quad (57c)$$

After integrating the distribution, equation (57) transforms to equation (58):

$$TC(s, Q) = \frac{D}{Q} \left[\gamma + p\tau \left(\frac{\delta}{2} (a+b) + \alpha \right) \right] + \left(D + p \frac{1}{2} (a+b) \right) \beta + h \left(\frac{Q}{2} + s \right) \quad (58a)$$

$$+ \left(\frac{p}{b-a} \right) \left(1 - \frac{\tau D}{Q} \right) \left[\alpha(b-s) + \delta \left(\frac{1}{2} (b^2 - s^2) - s(b-s) \right) \right] \quad (58c)$$

$$- \frac{h}{2} \left(\frac{p}{b-a} \right) \left(\frac{Q}{D} - \frac{\tau^2 D}{Q} \right) \left[\frac{1}{2} (s^2 - a^2) + s(b-s) \right] \quad (58e)$$

The final objective function with the assumption that the emergency demand follows a continuous uniform distribution is shown in equation (59):

$$TC(s, Q) = \frac{D}{Q} \left[\gamma + p\tau \left(\frac{\delta}{2}(a+b) + \alpha \right) \right] + \left(D + p\frac{1}{2}(a+b) \right) \beta + h \left(\frac{Q}{2} + s \right) \quad (59a)$$

$$+ \left(\frac{p}{b-a} \right) \left(1 - \frac{\tau D}{Q} \right) \left[\alpha(b-s) + \delta \left(\frac{1}{2}b^2 - bs + \frac{1}{2}s^2 \right) \right] \quad (59c)$$

$$- \frac{h}{2} \left(\frac{p}{b-a} \right) \left(\frac{Q}{D} - \frac{\tau^2 D}{Q} \right) \left[-\frac{1}{2}a^2 + bs - \frac{1}{2}s^2 \right] \quad (59e)$$

4.2.2 Optimal Solution Conditions

For the solution of operating an inventory system under constant demand and a probability of uniformly distributed emergency demand to be economically optimal, it must satisfy two sets of conditions. The first set of conditions states that in order to minimize or maximize the total cost per unit time the optimal solution must satisfy the condition in which the first-order derivatives of the total cost per unit time are equal to zero. The second set of conditions states that in order to minimize the total cost per unit time, the optimal solution must satisfy the conditions in which the Hessian matrix of the second-order derivatives of the total cost per unit time is positive-definite.

The first set of optimal conditions are derived by setting the first order derivatives of the total cost per unit with respect to the safety stock level s and the scheduled order quantity Q equal to zero. The condition in which the derivative of the total cost per unit time with respect to the safety stock level s is equal to zero shown in equation (60); and the condition in which the derivative of the total cost per unit time with respect to the scheduled order quantity Q is equal to zero is shown in equation (61).

$$h - \left(\frac{p}{b-a} \right) \left(1 - \frac{\tau D}{Q} \right) \left[\alpha + \left(\delta + \frac{hQ}{2D} + \frac{h\tau D}{2D} \right) (b-s) \right] = 0 \quad (60)$$

$$\frac{h}{2} - \frac{D}{Q^2} \left[\gamma + \left(\frac{p}{b-a} \right) \tau \alpha (s-a) \right] - \left(\frac{p}{b-a} \right) \left[\delta \frac{\tau D}{Q^2} + \frac{h}{2D} + \frac{h\tau^2 D}{2Q^2} \right] \left(-\frac{1}{2} a^2 + bs - \frac{1}{2} s^2 \right) = 0 \quad (61)$$

The second set of optimal conditions in which the Hessian matrix (35) is positive-definite requires the second-order partial derivatives of the total cost per unit time with respect to the safety stock level s and the scheduled order quantity Q be determined first. For the assumption that the emergency demand is uniformly distributed, the second order partial derivative of the total cost per unit time with respect to the safety stock level s alone is shown in equation (62). The second order partial derivative of the total cost per unit time with respect to the scheduled order quantity Q alone is shown in equation (63). Finally, the second order partial derivative of the total cost per unit time with respect to the safety stock level s and the scheduled order quantity Q is shown in equation (64).

$$\frac{d}{ds} \text{TC}(s, Q) = \left(\frac{p}{b-a} \right) \left(1 - \frac{\tau D}{Q} \right) \left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right) \quad (62)$$

$$\frac{d}{dQ} \text{TC}(s, Q) = \frac{D}{Q^3} \left[\gamma + \left(\frac{p\tau}{b-a} \right) \left(\alpha(s-a) + \left(\delta + \frac{h\tau}{2} \right) \left(-\frac{1}{2} a^2 + bs - \frac{1}{2} s^2 \right) \right) \right] \quad (63)$$

$$\frac{d}{dsdQ} \text{TC}(s, Q) = - \left(\frac{p}{b-a} \right) \left[\alpha \frac{\tau D}{Q^2} + \left(\delta \frac{\tau D}{Q^2} + \frac{h\tau^2 D}{2Q^2} + \frac{h}{2D} \right) (b-s) \right] \quad (64)$$

Given the second order partial derivatives of the total cost per unit time, the Hessian matrix for the inventory problem with a probability of uniformly distributed emergency demand between regularly scheduled orders can be written as the following:

$$\nabla^2 \text{TC}(s^*, Q^*) = \begin{bmatrix} \left(\frac{p}{b-a} \right) \left(1 - \frac{\tau D}{Q} \right) \left(\delta + \frac{h}{2D} (Q + \tau D) \right) & - \left(\frac{p}{b-a} \right) \left[\alpha \frac{\tau D}{Q^2} + \left(\delta \frac{\tau D}{Q^2} + \frac{h\tau^2 D}{2Q^2} + \frac{h}{2D} \right) (b-s) \right] \\ - \left(\frac{p}{b-a} \right) \left[\alpha \frac{\tau D}{Q^2} + \left(\delta \frac{\tau D}{Q^2} + \frac{h\tau^2 D}{2Q^2} + \frac{h}{2D} \right) (b-s) \right] & \frac{D}{Q^3} \left[\gamma + \left(\frac{p\tau}{b-a} \right) \left(\alpha(s-a) + \left(\delta + \frac{h\tau}{2} \right) \left(-\frac{1}{2} a^2 + bs - \frac{1}{2} s^2 \right) \right) \right] \end{bmatrix} \quad (65)$$

The second set of optimal conditions in which the solution set satisfies the conditions for which the Hessian matrix (65) is positive-definite guarantees that the solution is at the global optimal minimum. For a positive definite matrix, two conditions must be satisfied. For one, the matrix must be symmetric. That is, the components in the diagonals of the matrix must share the same sign. Secondly, the determinant of the matrix must be positive. Given the emergency demand distribution, the previously derived conditions shown in (55) and (56) can be redeveloped as (66) and (67).

$$\left(\frac{1}{b-a}\right)\left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2}\right) > 0 \quad (66)$$

$$0 < \left(\frac{p}{b-a}\right)\left(1 - \frac{\tau D}{Q}\right)\left(\delta + \frac{hQ}{2D} + \frac{h\tau}{2}\right) \quad (67a)$$

$$\times \frac{D}{Q^3} \left[\gamma + \left(\frac{p\tau}{b-a}\right)\left(\alpha(s-a) + \left(\delta + \frac{h\tau}{2}\right)\left(-\frac{1}{2}a^2 + bs - \frac{1}{2}s^2\right)\right) \right] \quad (67b)$$

$$- \left(\frac{p}{b-a}\right)^2 \left[\alpha \frac{\tau D}{Q^2} + \left(\delta \frac{\tau D}{Q^2} + \frac{h\tau^2 D}{2Q^2} + \frac{h}{2D}\right)(b-s) \right]^2 \quad (67c)$$

4.2.3 Numerical Solution Procedure

Before a numerical solution can be generated for the inventory model given the uniformly distributed volume of emergency demand, the safety stock level s and the scheduled order quantity Q must be derived. The solution for safety stock level s and scheduled order quantity Q are derived from the first-order optimal solution conditions shown in (60) and (61), respectively, as follows:

$$s = b - \frac{2hDQ - 2\alpha D \left(\frac{P}{b-a} \right) (Q - \tau D)}{\left(\frac{P}{b-a} \right) (Q - \tau D) (hQ + D(2\delta + h\tau))} \quad (68)$$

$$Q^2 = \frac{4D^2\gamma + \tau D^2 \left(\frac{P}{b-a} \right) [4\alpha(s-a) + (2\delta + h\tau)(-a^2 + 2bs - s^2)]}{h \left[2D - \left(\frac{P}{b-a} \right) (-a^2 + 2bs - s^2) \right]} \quad (69)$$

The equations for safety stock level s and scheduled order quantity Q shown in (68) and (69) are nonlinear. In fact, if the two equations in (68) and (69) are combined to form a function of the scheduled order quantity Q , the equation would be a polynomial to the sixth degree. Given this complexity, a numerical solution must be determined through an iterative process. The iterative solution process for the inventory model with uniformly distributed emergency demand size is as follows:

- STEP 0: Estimate an initial value of the safety stock level starting at the minimum level of the emergency demand a . Label the value s_o .
- STEP 1: Calculate the scheduled order quantity from equation (69) using the estimated initial value of safety stock s_o . Only calculate the positive root to the quadratic function in (69) because the scheduled order quantity must be greater than 0 and thus not negative. Label the positive value of the scheduled order quantity as Q_I .
- STEP 2: Calculate the safety stock level from equation (68) using the value of the scheduled order quantity Q_I derived in the prior step. Label the value of the safety stock s_I .

- STEP 3: If $s_I = s_o$, solve the inequalities in equations (66) and (67) given the scheduled order quantity Q_I derived in STEP 1 and the safety stock level s_I derived in STEP 2 to test the second-order solution conditions. Otherwise, if $s_I \neq s_o$, increment the initial value of safety stock s_o by 1% of the difference between the minimum and the maximum emergency demand and go to STEP 1 to repeat the process.
- STEP 4: If the second-order solution conditions are TRUE, the numerical solution for the inventory policy consists of the scheduled order quantity Q_I derived in STEP 1 and the safety stock level s_I derived in STEP 2. Solve for other results including cost and cycle length. Otherwise, if the second-order solution conditions are FALSE, increment the initial value of safety stock s_o by 1% of the difference between the minimum and the maximum emergency demand and go to STEP 1 to repeat the process.
- STOP: If no solution is found, the inventory policy should be identical to a traditional EOQ model with zero safety stock.

4.2.4 Comparative Solution with Traditional EOQ Model

The numerical solution derived in the previous section for the inventory model given a uniformly distributed volume of emergency demand is compared to a traditional EOQ model. In the traditional EOQ model, the regular demand generated by the production schedule is satisfied by the constant order quantity; and any emergency demand generated by undesirable circumstances is satisfied by a more expensive

emergency order from a supplier or distributor via the more expensive mode of transportation – air. Thus, there is zero safety stock. So, given a traditional EOQ model, the total cost per unit time is shown in equation (70); and the optimal scheduled order quantity derived by taking the first derivative of total cost function with respect to the scheduled order quantity in equation (71) is shown in equation (72).

$$TC(Q) = \frac{D}{Q}\gamma + \beta D + \frac{h}{2}Q + p \left[\alpha + (\beta + \delta) \frac{(a+b)}{2} \right] \quad (70)$$

$$\frac{d}{dQ}TC(s, Q) = -\frac{D}{Q^2}\gamma + \frac{h}{2} \quad (71)$$

$$Q = \sqrt{\frac{2D\gamma}{h}} \quad (72)$$

4.3 Numerical Solution Approach given Exponential Distribution

The numerical solution to the inventory model in this section corresponds to the assumption that the emergency demand volume follows an Exponential distribution with mean μ^{-1} . Given the assumption on the probability distribution of emergency demand size, the inventory model formulated in Chapter 3 can be modified to develop the following objective function and solution conditions. Given these modifications, the model is numerically analyzed with a variety of changes to the parameters.

4.3.1 Objective Function

With the assumption that the emergency demand quantity x follows an exponential distribution with a mean μ^{-1} , the total cost per unit time is as follows:

$$TC(s, Q) = \frac{D}{Q} \left[\gamma + p\tau \left(\alpha + \delta \frac{1}{\mu} \right) \right] + \beta \left(D + p \frac{1}{\mu} \right) + \frac{h}{2} (Q + 2s) \quad (73a)$$

$$+ p \left(1 - \frac{\tau D}{Q} \right) \int_s^{\infty} [\alpha + \delta(x-s)] \mu e^{-\mu x} dx \quad (73b)$$

$$- p \frac{h}{2} \left(\frac{Q}{D} - \frac{\tau^2 D}{Q} \right) \left[\int_0^s \mu x e^{-\mu x} dx + \int_s^{\infty} \mu s e^{-\mu x} dx \right] \quad (73c)$$

After integrating the distribution, equation (73) transforms to equation (74):

$$TC(s, Q) = \frac{D}{Q} \left[\gamma + p\tau \left(\alpha + \delta \frac{1}{\mu} \right) \right] + \beta \left(D + p \frac{1}{\mu} \right) + \frac{h}{2} (Q + 2s) \quad (74a)$$

$$+ p \left(1 - \frac{\tau D}{Q} \right) \left[(\alpha - s\delta) (-e^{-\infty} + e^{-\mu s}) + \delta \left(-\infty e^{-\infty} - \frac{1}{\mu} e^{-\infty} + s e^{-\mu s} + \frac{1}{\mu} e^{-\mu s} \right) \right] \quad (74b)$$

$$- p \frac{h}{2} \left(\frac{Q}{D} - \frac{\tau^2 D}{Q} \right) \left[\left(-s e^{-\mu s} - \frac{1}{\mu} e^{-\mu s} + 0 e^{-0} + \frac{1}{\mu} e^{-0} \right) + s (-e^{-\infty} + e^{-\mu s}) \right] \quad (74c)$$

The final objective function with the assumption that the emergency demand follows a continuous Exponential distribution is shown in equation (75):

$$TC(s, Q) = \frac{D}{Q} \left[\gamma + p\tau \left(\alpha + \delta \frac{1}{\mu} \right) \right] + \beta \left(D + p \frac{1}{\mu} \right) + \frac{h}{2} (Q + 2s) \quad (75a)$$

$$+ p \left(1 - \frac{\tau D}{Q} \right) \left(\alpha + \delta \frac{1}{\mu} \right) e^{-\mu s} - p \left(\frac{hQ}{2D} - \frac{h\tau^2 D}{2Q} \right) \frac{1}{\mu} (1 - e^{-\mu s}) \quad (75b)$$

4.3.2 Optimal Solution Conditions

For the solution of operating an inventory system under constant demand and a probability of exponentially distributed emergency demand to be economically optimal, it must satisfy two sets of conditions. The first set of conditions states that in order to minimize or maximize the total cost per unit time the optimal solution must satisfy the condition in which the first-order derivatives of the total cost per unit time are equal to zero. The second set of conditions states that in order to minimize the total cost per unit time, the optimal solution must satisfy the conditions in which the Hessian matrix of the second-order derivatives of the total cost per unit time is positive-definite.

The first set of optimal conditions are derived by setting the first order derivatives of the total cost per unit with respect to the safety stock level s and the scheduled order quantity Q equal to zero. The condition in which the derivative of the total cost per unit time with respect to the safety stock level s is equal to zero shown in equation (76); and the condition in which the derivative of the total cost per unit time with respect to the scheduled order quantity Q is equal to zero is shown in equation (77).

$$h - pe^{-\mu s} \left(1 - \frac{\tau D}{Q} \right) \left[\alpha \mu + \delta + \frac{hQ}{2D} + \frac{h\tau}{2} \right] = 0 \quad (76)$$

$$\frac{h}{2} - \gamma \frac{D}{Q^2} - \frac{p}{\mu} \left(\frac{\tau D}{2Q^2} (2\alpha\mu + 2\delta + h\tau) + \frac{h}{2D} \right) (1 - e^{-\mu s}) = 0 \quad (77)$$

The second set of optimal conditions in which the Hessian matrix (35) is positive-definite requires the second-order partial derivatives of the total cost per unit time with respect to the safety stock level s and the scheduled order quantity Q be determined first. For the assumption that the emergency demand is exponentially distributed, the second order partial derivative of the total cost per unit time with respect to the safety stock level s alone is shown in equation (78). The second order partial derivative of the total cost per unit time with respect to the scheduled order quantity Q alone is shown in equation (79). Finally, the second order partial derivative of the total cost per unit time with respect to the safety stock level s and the scheduled order quantity Q is shown in equation (80).

$$\frac{d}{d^2s} \text{TC}(s, Q) = p\mu e^{-\mu s} \left(1 - \frac{\tau D}{Q} \right) \left[\alpha\mu + \left(\delta + \frac{h}{2D} (Q + \tau D) \right) \right] \quad (78)$$

$$\frac{d}{d^2Q} \text{TC}(s, Q) = \frac{D}{Q^3} \left[\gamma + p\tau \left(\alpha + \frac{1}{\mu} \left(\delta + \frac{h\tau}{2} \right) \right) (1 - e^{-\mu s}) \right] \quad (79)$$

$$\frac{d}{dQds} \text{TC}(s, Q) = -pe^{-\mu s} \left[\frac{\tau D}{Q^2} \left[\alpha\mu + \left(\delta + \frac{h\tau}{2} \right) \right] + \frac{h}{2D} \right] \quad (80)$$

Given the second order partial derivatives of the total cost per unit time, the Hessian matrix for the inventory problem with a probability of exponentially distributed emergency demand between regularly scheduled orders can be written as the following:

$$\nabla^2 \text{TC}(s^*, Q^*) = \begin{bmatrix} p\mu e^{-\mu s} \left(1 - \frac{\tau D}{Q} \right) \left[\alpha\mu + \left(\delta + \frac{h}{2D} (Q + \tau D) \right) \right] & -pe^{-\mu s} \left[\frac{\tau D}{Q^2} \left[\alpha\mu + \left(\delta + \frac{h\tau}{2} \right) \right] + \frac{h}{2D} \right] \\ -pe^{-\mu s} \left[\frac{\tau D}{Q^2} \left[\alpha\mu + \left(\delta + \frac{h\tau}{2} \right) \right] + \frac{h}{2D} \right] & \frac{D}{Q^3} \left[\gamma + p\tau \left(\alpha + \frac{1}{\mu} \left(\delta + \frac{h\tau}{2} \right) \right) (1 - e^{-\mu s}) \right] \end{bmatrix} \quad (81)$$

The second set of optimal conditions in which the solution set satisfies the conditions for which the Hessian matrix (81) is positive-definite guarantees that the solution is at the global optimal minimum. For a positive definite matrix, two conditions must be satisfied. For one, the matrix must be symmetric. That is, the components in the diagonals of the matrix must share the same sign. Secondly, the determinant of the matrix must be positive. Given the emergency demand distribution, the previously derived conditions shown in (55) and (56) can be redeveloped as (82) and (83).

$$\mu e^{-\mu s} \left[\alpha \mu + \left(\delta + \frac{h}{2} \left(\frac{Q}{D} + \tau \right) \right) \right] > 0 \quad (82)$$

$$0 < p \mu e^{-\mu s} \left(1 - \frac{\tau D}{Q} \right) \left[\alpha \mu + \left(\delta + \frac{h}{2D} (Q + \tau D) \right) \right] \quad (83a)$$

$$\times \frac{D}{Q^3} \left[\gamma + p \tau \left(\alpha + \frac{1}{\mu} \left(\delta + \frac{h \tau}{2} \right) \right) (1 - e^{-\mu s}) \right] \quad (83b)$$

$$- p^2 e^{-2\mu s} \left[\frac{\tau D}{Q^2} \left[\alpha \mu + \left(\delta + \frac{h \tau}{2} \right) \right] + \frac{h}{2D} \right]^2 \quad (83c)$$

4.3.3 Numerical Solution Procedure

Before a numerical solution can be generated for the inventory model given the exponentially distributed volume of emergency demand, the safety stock level s and the scheduled order quantity Q must be derived. The solution for safety stock level s (as a function of the exponential value e) and scheduled order quantity Q are derived from the first-order optimal solution conditions shown in (76) and (77), respectively, as follows:

$$e^{-\mu s} = \frac{2hDQ}{p(Q - \tau D)[hQ + D(2\alpha\mu + 2\delta + \tau h)]} \quad (84)$$

$$Q^2 = \frac{2\gamma\mu D^2 + pD^2\tau(2\alpha\mu + 2\delta + h\tau)(1 - e^{-\mu s})}{h(\mu D - p(1 - e^{-\mu s}))} \quad (85)$$

The equations for safety stock level s and scheduled order quantity Q shown in (84) and (85) are nonlinear. Nevertheless, if the two equations in (84) and (85) are combined to form a function of the scheduled order quantity Q and not the safety stock level s , the equation would be a polynomial to the fourth degree as shown in (86). Such a polynomial is solvable provided a rough numerical process.

$$AQ^4 + BQ^3 + CQ^2 + EQ + F = 0 \quad (86a)$$

$$\text{where } A = h^2(\mu D - p) \quad (86b)$$

$$B = hD[(\mu D - p)(K - h\tau) - 2h] \quad (86c)$$

$$C = -h\mu D^2(\tau DK + 2\gamma) \quad (86d)$$

$$D = -D^3[(2\gamma\mu + p\tau K)(K - h\tau) + 2h\tau K] \quad (86e)$$

$$E = \tau D^4 K(2\gamma\mu + p\tau K) \quad (86f)$$

$$\text{and } K = (2\alpha\mu + 2\delta + \tau h) \quad (86g)$$

Given that there are four numerical solutions to the scheduled order quantity Q derived by solving for the four roots to the fourth degree polynomial equation (86a-g), a short iterative process is required to determine the final solutions for the inventory policy. The short iterative solution process for the inventory model with exponentially distributed emergency demand size is as follows:

STEP 0: Derive the four roots to the fourth degree polynomial equation (86a-g) for the scheduled order quantity. Label the values of the scheduled order quantity Q_1 , Q_2 , Q_3 , and Q_4 .

- STEP 1: Calculate the safety stock level from equation (84) for each value of the scheduled order quantity $Q_1, Q_2, Q_3,$ and Q_4 derived in STEP 0. Label the value of the safety stock $s_1, s_2, s_3,$ and s_4
- STEP 2: Check whether each respective pair of values for the scheduled order quantity Q and the safety stock level s found in STEP 0 and STEP 1, respectively, satisfy the inequalities: $Q \geq 0$ and $s \geq 0$. Go to STEP 3 with any of the respective pairs that satisfy the above inequalities. Otherwise, if no pairs satisfy the above inequalities, go to STEP 4.
- STEP 3: Test the second-order solution conditions in equations (82-83) given the solution pairs from STEP 2. If the conditions are TRUE, a solution to the inventory model given the current parameters has been found. Otherwise, if the conditions are FALSE, go to STEP 4.
- STEP 4: No solution is found for the inventory policy given the current model parameters. Thus, the inventory policy should be identical to a traditional economic ordering policy with zero safety stock.

4.3.4 Comparative Solution for Traditional EOQ Model

The numerical solution derived in the previous section for the inventory model given an exponentially distributed volume of emergency demand is compared to a traditional EOQ model. With a traditional economic order policy, the regular demand generated by the production schedule is satisfied by a regular order quantity; and any emergency demand generated by undesirable circumstances is satisfied by a more expensive emergency order from a supplier or distributor via the more expensive mode of

transportation – air. Thus, there is zero safety stock. So, given a traditional economic ordering policy, the total cost per unit time is shown in equation (87); and the optimal scheduled order quantity derived by taking the first derivative of total cost function with respect to the scheduled order quantity in equation (88) is shown in equation (89).

$$TC(s, Q) = \frac{D}{Q}\gamma + \beta\left(D + p\frac{1}{\mu}\right) + \frac{h}{2}Q + p\left[\alpha + \delta\frac{1}{\mu}\right] \quad (87)$$

$$\frac{d}{dQ}TC(s, Q) = -\frac{D}{Q^2}\gamma + \frac{h}{2} \quad (88)$$

$$Q = \sqrt{\frac{2D\gamma}{h}} \quad (89)$$

Note, the traditional economic ordering policy does not change with respect to the distribution of the emergency demand. In fact, the traditional model is not related to the emergency demand at all. Rather, the emergency demand only affects the total procurement and transportation cost and thus the total cost of the policy.

4.4 Numerical Analysis & Results

In order to illustrate the effect of energy on inventory policy decisions and corresponding logistics costs, the inventory model developed in the previous sections is numerically analyzed with respect to changes in energy cost as well as numerous other model parameters. The resultant inventory policy decisions and respective logistics costs for the various model parameters are compared to those of a traditional economic ordering policy to further validate the inventory model and illustrate the cases in which the proposed model is most cost effective. That is, the purpose of varying most of the model parameters between two levels for this analysis is to discover and understand the situations in which the inventory policies are most significantly affected by changes to energy cost as well as the situations in which the proposed model is most effective.

4.4.1 Effect of Model Parameters with respect to Energy Cost

In the analysis, the model parameters described in Section 4.1 are each varied between two levels in order to analyze the effect of each model parameter on various results including inventory policy decisions and logistics costs. Since the effect of energy cost is one of the primary focuses in this research, the results analyzed in the analysis are in terms of changes with respect to energy cost. That is, each result represents the difference between the result given high energy cost and the result given low energy cost. So, in the analysis, each model parameter excluding energy cost is varied between two levels in order to analyze the effect of each model parameter on changes to various results with respect to energy cost.

The results collected from the comparative analysis are analyzed statistically via the analysis of variance. For each response, which represents the change in an inventory policy decision or logistics cost with respect to energy cost, the analysis of variance determines a degree of significance for each model parameter's effect on the response. As the degree of significance decreases (or increases), the effect becomes more important (or less important) to the response. Only if the degree of significance is less than or equal to 0.050 is the effect particularly significant to the response.

In the subsequent presentation of the results, the significance of each effect on each response is presented as one of five levels of significance. Since there are two numerical analyses conducted for each of the two distributions assumed to represent the volume of the emergency demand, the specific degree of significance for each model parameter on each response may vary between the two analyses. However, the general conclusions of the two analyses are reasonably identical. So, rather than displaying the degree of significance for each model parameter on each response as an average between the two analyses, the degree of significance is marked as one of five levels.

The five levels at which the degree of significance is presented in the subsequent presentation of the results for the analysis of variance include < 0.001 , < 0.050 , < 0.100 , < 0.50 , and > 0.75 . If the degree of significance is marked as either < 0.001 or < 0.050 , the effect is significant to the response; otherwise, the effect is not significant to the response. Nevertheless, a degree of significance marked as < 0.100 is still considered noteworthy in the analysis of variance results of this comparative analysis given the slight variances between the results of the two numerical analyses conducted for each of the two distributions assumed to represent the volume of the emergency demand.

In addition to presenting the degree of significance of each model parameter's effect on changes in various results with respect to energy cost, the subsequent results for the analysis of variance display the relationship of each model parameter on changes in various results with respect to energy cost. Such a relationship may either be positive, negative, or negligible. If the relationship is positive, the change in the result with respect to energy cost increases as the model parameter increases; whereas, if the relationship is negative, the change in the result with respect to energy cost decreases as the model parameter increases.

The results of the analysis of variance which include both the degree of significance and the relationship of each model parameter on changes to inventory policy decisions and logistics costs with respect to energy cost are shown in Tables 4.7-4.12. While Tables 4.8-4.12 present the effect of each model parameter on the change in logistics costs with respect to energy cost, Table 4.7 presents the effect of each model parameter on the change in inventory policy decisions with respect to energy cost. Since the traditional inventory policy decisions do not change as energy cost changes, only the results concerning the changes in the proposed inventory policy decisions with respect to energy cost are shown in Table 4.7.

The responses presented in Table 4.7 include the changes in the inventory cycle length, the scheduled order quantity, the safety stock, and the probability of an emergency order with respect to energy cost for the proposed inventory policy. Whereas the change in each inventory decision with respect to energy cost – otherwise known as the response – is presented in a set of two columns, each model parameter is presented in a single row. So, each row displays the effect of the specified model parameter on the

response(s); and each column of displays the effects of all the model parameters on the response. In particular, for each response, the left column displays the degree of significance a model parameter affects the response, and the right column displays the relationship a model parameter has on the response.

Description	Change in Proposed Policy Decisions wrt Energy						Probability of an Emergency Order	
	Cycle Length		Order Quantity		Safety Stock			
Fixed Purchasing Cost to Order	< 0.001	Negative	<< 0.50	Negative	<< 0.50	Negative	< 0.050	Negative
Total Unit Purchasing Cost to Procure	< 0.050	Negative	<< 0.50	Negative	<< 0.50	Positive	< 0.001	Negative
Regular Demand per unit time	>> 0.75	Negligible	< 0.001	Positive	< 0.001	Positive	< 0.001	Positive
Mean Emergency Demand	< 0.001	Positive	< 0.001	Positive	< 0.050	Positive	< 0.050	Negative
Lead Time	< 0.001	Positive	<< 0.50	Positive	>> 0.75	Positive	<< 0.50	Negative
Distance	>> 0.75	Negligible	>> 0.75	Negligible	<< 0.75	Negligible	<< 0.50	Negative
Weight	< 0.001	Positive	< 0.050	Positive	< 0.100	Positive	< 0.050	Positive

Table 4.7: Effect of Model Parameters on Changes to Inventory Decisions with respect to Energy Cost for the Proposed Inventory Policy

For each response shown in Table 4.7, both the product weight and the average emergency demand are found to be significant to the change in the inventory decision with respect to energy cost. In particular, these two parameters, unlike the other parameters, directly affect the transportation cost required to fulfill an emergency demand. For example, as energy cost increases, the difference between the unit energy cost to ship a product via a faster, less energy efficient mode such as air and a slower, more energy efficient mode such as ground increases significantly. Similarly, as either one of these two parameters increase, the difference between the total energy cost to ship the emergency demand via an emergency order and a regularly scheduled order increases. Thus, the inventory decisions in the proposed inventory model change significantly as either one of these two parameters change with respect to energy cost in order to reduce

the possibility of an expensive emergency order and likewise reduce the expected total cost of fulfilling an emergency demand.

In the proposed inventory model, unlike the traditional economic ordering policy, this possibility of an emergency order may be decreased by adding or increasing safety stock. However, safety stock affects not only the possibility of an emergency order, but also the inventory cycle length which subsequently affects the scheduled order quantity and both the probability of an emergency demand and an emergency order. If the regular demand of the production system remains constant, a significant increase in safety stock causes a significant increase to the inventory cycle length and proportionally, a significant increase to the scheduled order quantity. Yet, if the regular demand of the production system changes with respect to energy cost, a significant increase in the safety stock does not significantly affect the inventory cycle length because the scheduled order quantity, which is related to the inventory cycle length, is significantly affected by the change in the regular demand.

So, as the regular demand changes with respect to energy cost, the level of safety stock becomes more significant to reducing the possibility of an expensive emergency order and likewise the expected total cost of fulfilling an emergency demand. In fact, the degree of significance for the regular demand on the change in safety stock with respect to energy cost is the lowest compared to all the other parameters. Likewise, the degree of significance for the regular demand on the change in either the scheduled order quantity or the probability of an emergency order is one of the lowest compared to all other parameters. The only response regular demand does not significantly affect is the change in the inventory cycle length with respect to energy cost.

Therefore, the three model parameters determined to be most significant to changes in inventory decisions with respect to energy cost for the proposed inventory policy are the regular demand, the average emergency demand, and the product weight. The remaining parameters are not considered noteworthy to the proposed inventory policy because neither one is significant to more than two of the responses shown in Table 4.7. Transportation distance, for instance, is insignificant to each and every inventory decision, even as energy cost changes. The primary reason that these model parameters are less significant than the three factors discussed prior to changes in inventory decisions with respect to energy cost is that these parameters do not significantly affect the size or the cost to replenish an emergency demand as energy cost changes. Thus, each of the remaining factors is insignificant to the change in both the scheduled order quantity and the safety stock; and the other inventory decisions shown in Table 4.7 are less significantly affected by the remaining factors as energy cost changes.

Since neither energy cost nor emergency demand is explicitly considered in the traditional EOQ model, any change in energy cost is negligible to the inventory policy decisions. Consequently, any change in energy cost is also insignificant to both the total procurement cost and the total inventory cost for the traditional policy. The only logistics activity affected by energy cost in the traditional policy is transportation.

Conversely, all the logistics activities including procurement, transportation, and inventory are affected by energy cost in the proposed inventory policy because both the energy cost and the emergency demand are explicitly considered in the proposed model. So, accordingly, the effect of each model parameter on the change in the total cost of each activity for the proposed inventory policy is shown in Table 4.8.

Description	Change in Total Cost per Activity wrt Energy					
	Procurement		Transportation		Inventory	
Fixed Purchasing Cost to Order	<< 0.50	Negative	>> 0.75	Negligible	< 0.050	Negative
Total Unit Purchasing Cost to Procure	< 0.050	Negative	<< 0.75	Negative	< 0.050	Positive
Regular Demand per unit time	< 0.001	Negative	< 0.001	Positive	< 0.001	Positive
Mean Emergency Demand	< 0.050	Negative	< 0.001	Positive	< 0.001	Positive
Lead Time	< 0.100	Negative	<< 0.75	Negative	<< 0.50	Positive
Distance	<< 0.50	Positive	<< 0.50	Positive	>> 0.75	Negligible
Weight	< 0.001	Negative	< 0.001	Positive	< 0.001	Positive

Table 4.8: Effect of Model Parameters on Changes to Total Costs of Each Logistics

Activity with respect to Energy Cost for the Proposed Policy

Similarly to the results shown in Table 4.7, the regular demand, the average emergency demand, and the product weight are determined to be the most significant factors to the change in the total cost of each logistics activity with respect to energy cost. In fact, these three parameters are significant to all of the logistics activities shown in Table 4.8. All of the other parameters, on the other hand, are significant to at most two logistics activities in the proposed inventory policy; and the transportation distance is insignificant to all of the logistics activities as energy cost changes.

While not all the individual activity costs are affected by changes to energy cost for both the proposed inventory policy and the traditional inventory policy, the total transportation cost is affected by energy cost for both policies. Likewise, the total cost of all the logistics activities for both policies is affected by energy cost. So the effect of each model parameter on the change in total transportation cost with respect to energy cost for either inventory policy is shown in Table 4.9; and the effect of each model parameter on the change in the total cost of all activities with respect to energy cost for either inventory policy is shown in Table 4.10.

Set	Description	Change in Shipping Cost wrt Energy			
		Traditional		Proposed	
Supply	Fixed Purchasing Cost to Order	>> 0.75	Negligible	>> 0.75	Negligible
	Total Unit Purchasing Cost to Procure	<< 0.50	Negative	<< 0.50	Negative
Production	Regular Demand per unit time	< 0.001	Positive	< 0.001	Positive
	Mean Emergency Demand	>> 0.75	Negligible	>> 0.75	Negligible
Transportation	Lead Time	>> 0.75	Negligible	>> 0.75	Negligible
	Distance	< 0.050	Positive	< 0.050	Positive
	Weight	< 0.001	Positive	< 0.001	Positive

Table 4.9: Effect of Model Parameters on Changes to Total Transportation Cost with respect to Energy Cost for each Inventory Policy

Set	Description	Change in Total Cost wrt Energy			
		Traditional		Proposed	
Supply	Fixed Purchasing Cost to Order	>> 0.75	Negligible	>> 0.75	Negligible
	Total Unit Purchasing Cost to Procure	<< 0.50	Negative	<< 0.50	Negative
Production	Regular Demand per unit time	< 0.001	Positive	< 0.001	Positive
	Mean Emergency Demand	>> 0.75	Negligible	>> 0.75	Negligible
Transportation	Lead Time	>> 0.75	Negligible	>> 0.75	Negligible
	Distance	< 0.050	Positive	< 0.050	Positive
	Weight	< 0.001	Positive	< 0.001	Positive

Table 4.10: Effect of Model Parameters on Changes to Total Cost of all Logistics Activities with respect to Energy Cost for either Inventory Policy

According to the results shown in Tables 4.9-4.10, the regular demand size, the transportation distance, and the product weight are significant to both the change in the total transportation cost and the change in the total cost of all the activities with respect to energy cost for either inventory policy. All of the other parameters, on the other hand, are not significant to either the total transportation cost or the total cost of all activities. The primary reason for these results is that the three key model parameters are significant to the size or the cost of the regularly scheduled order as energy cost changes whereas the other model parameters are not, even if there is no instance of an emergency demand.

For instance, the regular demand directly affects the scheduled order quantity and thus indirectly affects the total cost of any logistics activity, regardless of any change in

energy cost. As the regular demand increases, the scheduled order quantity increases. Thus, the total cost of any logistics activity to procure, transport, or store the regularly scheduled order quantity throughout an inventory cycle increases. Yet, as the energy cost increases along with the regular demand, the proportional increase to the scheduled order quantity more significantly affects the total energy cost to ship the regularly scheduled order quantity via ground transportation.

Similarly, both the transportation distance and the product weight affect the unit costs to ship a product via a specified transportation mode. As either the transportation distance or the product weight increases, so too does the unit transportation cost associated to a specified mode. This is true for both the unit transportation cost related to energy and not related to energy. In fact, the unit transportation cost related to energy is proportional to the unit transportation cost not related to energy. So, as the energy cost increases along with either the transportation distance or the product weight, the unit energy cost to ship a single product via a specified transportation mode such as ground transportation increases more significantly. As a result, the change in the total transportation and likewise the total cost of all the logistics activities with respect to energy cost is significant to transportation distance, product weight, and regular demand.

Since at least one logistics activity (transportation) is affected by changes to energy cost in either inventory policy, the proportion of the total cost allocated between procurement, transportation, and inventory activities is also affected by changes to energy cost. The effects of each model parameter on these responses are shown in Table 4.11. According to the results, the total unit purchasing cost, the transportation distance, and the product weight are the only factors significant to the change in the proportion of total

cost allocated to either procurement or transportation activities in both inventory policies as energy cost changes. Conversely, the change in the proportion of total cost allocated to the inventory activity is significantly affected by a majority – at least five – of the model parameters in both inventory policies. However, the insignificant parameters to the inventory activity are not the same for both the inventory policies.

Change in Allocation of Total Cost Between Logistics Activities wrt Energy for Traditional Policy						
Description	Procurement		Transportation		Inventory	
Fixed Purchasing Cost to Order	<< 0.75	<i>Negative</i>	<< 0.75	Positive	< 0.001	<i>Negative</i>
Total Unit Purchasing Cost to Procure	< 0.001	Positive	< 0.001	<i>Negative</i>	< 0.001	Positive
Regular Demand per unit time	<< 0.75	Positive	<< 0.50	<i>Negative</i>	< 0.001	Positive
Mean Emergency Demand	>> 0.75	<i>Negligible</i>	>> 0.75	<i>Negligible</i>	>> 0.75	<i>Negligible</i>
Lead Time	>> 0.75	Positive	>> 0.75	<i>Negligible</i>	>> 0.75	<i>Negligible</i>
Distance	< 0.001	<i>Negative</i>	< 0.001	Positive	< 0.001	<i>Negative</i>
Weight	< 0.001	<i>Negative</i>	< 0.001	Positive	< 0.001	<i>Negative</i>

Change in Allocation of Total Cost Between Logistics Activities wrt Energy for Proposed Policy						
Description	Procurement		Transportation		Inventory	
Fixed Purchasing Cost to Order	>> 0.75	<i>Negative</i>	<< 0.75	Positive	< 0.001	<i>Negative</i>
Total Unit Purchasing Cost to Procure	< 0.001	Positive	< 0.001	<i>Negative</i>	< 0.001	Positive
Regular Demand per unit time	>> 0.75	<i>Negligible</i>	>> 0.75	<i>Negligible</i>	< 0.100	Positive
Mean Emergency Demand	>> 0.75	Positive	<< 0.75	<i>Negative</i>	< 0.001	Positive
Lead Time	>> 0.75	<i>Negligible</i>	>> 0.75	<i>Negligible</i>	< 0.001	Positive
Distance	< 0.001	<i>Negative</i>	< 0.001	Positive	< 0.001	<i>Negative</i>
Weight	< 0.001	<i>Negative</i>	< 0.001	Positive	<< 0.50	Positive

Table 4.11: Effect of Model Parameters on Changes to the Proportion of Total Cost Allocated to each Logistics Activities with respect to Energy Cost

These results, at least with regard to the change in the proportion of total cost allocated to either procurement or transportation activities with respect to energy cost, are similar to the results shown in Tables 4.9-4.10. For instance, since both the transportation distance and the product weight significantly affect the total transportation cost as energy cost changes, the proportion of total cost allocated between procurement and transportation activities with respect to energy cost changes significantly but

proportionally. This proportional relationship of the total cost allocation between procurement and transportation transpires because the two activities comprise over 98 percent of the total cost; yet, only the total transportation cost is significantly affected by changes to energy cost. So, as either the transportation distance or the product weight increases, for example, the total transportation cost increases, and thus the proportion of the total cost allocated to transportation increases while the proportion of the total cost allocated to procurement decreases.

The effect of the total unit purchasing cost on the proportion of the total cost between procurement and transportation activities is very similar to that of the transportation distance and the product weight. However, where as the transportation distance and the product weight significantly affect the transportation cost positively, the total unit purchasing cost significantly affects the transportation cost negatively. That is, as the total unit purchasing cost increases, the significance of the total unit purchasing cost overshadows the significance of the unit transportation cost. So, as the total unit purchasing cost increases with respect to energy cost, the total transportation cost decreases, and thus the proportion of the total cost allocated to transportation decreases while the proportion of the total cost allocated to procurement increases.

The change in the proportion of the total cost allocated to inventory with respect to energy cost, on the other hand, is affected by a majority of the model parameters. The only parameters not significant to the change in the proportion of total cost allocated to inventory with respect to energy cost for the traditional inventory policy include the average emergency demand size and the transportation lead time. Conversely, the only parameter not significant to the change in the proportion of total cost allocated to

inventory with respect to energy cost for the proposed inventory policy is the product weight. Even though the majority of the parameters are significant to this response, the response and likewise the reasons for the response are not significant to the analysis. That is, since the proportion of total cost allocated to inventory in either policy provided any change in model parameters or energy cost is less than 2 percent, the effects on this response are not significant to the analysis.

The final results of the analysis of variance concern the effectiveness of the proposed inventory policy if implemented in place of the traditional inventory policy. Table 4.12 presents the effect of each model parameter on the change in the difference between the total transportation cost of the proposed inventory policy and the traditional inventory policy with respect to energy cost as well as the change in the difference between the total cost of the proposed policy and the traditional policy with respect to energy cost. These changes represent the cost effectiveness of the proposed inventory policy on reducing the total transportation cost or similarly the total cost of all activities if implemented in place of the traditional policy, especially if energy cost changes.

According to the results shown in Table 4.12, the regular demand size, the average emergency demand, and the product weight are the only factors significant to the cost effectiveness of the proposed inventory policy at reducing both the total transportation cost and the total cost of all the logistics activities with respect to energy cost if implemented in place of the traditional inventory policy. By no coincidence, these key parameters are identical to the key parameters determined to be significant to changes in inventory decisions with respect to energy cost for the proposed inventory policy, as shown in Table 4.7. Because each of these key parameters significantly affects

the probability of fulfilling an emergency demand with an emergency order in the proposed inventory policy, the expected total transportation cost of fulfilling an emergency demand via any of the irregular replenishment scenarios is also significantly affected with respect to energy cost. That is, as the probability of an emergency order decreases, the expected total transportation cost to fulfill an emergency demand via the irregular replenishment scenarios decreases.

Set	Description	Change in the Difference between the two Inventory Policies			
		Transportation Cost		Total Cost	
Supply	Fixed Purchasing Cost to Order	>> 0.75	Negligible	<< 0.50	Positive
	Total Unit Purchasing Cost to Procure	<< 0.75	<i>Negative</i>	<< 0.50	<i>Negative</i>
Production	Regular Demand per unit time	< 0.001	Positive	< 0.050	Positive
	Mean Emergency Demand	< 0.001	Positive	< 0.001	Positive
Transportation	Lead Time	<< 0.75	<i>Negative</i>	<< 0.50	<i>Negative</i>
	Distance	<< 0.50	Positive	<< 0.50	Positive
	Weight	< 0.001	Positive	< 0.001	Positive

Table 4.12: Effect of Model Parameters on Changes to the Cost Effectiveness of the Proposed Policy in place of the Traditional Policy with respect to Energy Cost

The effects on the probability of an emergency order and other inventory decisions, however, only transpire in the proposed inventory policy as energy cost changes. Since neither the energy cost nor the emergency demand is explicitly considered in traditional EOQ model, the traditional policy does not change as energy cost changes. Therefore, the cost savings with respect to fulfilling the emergency demand via the scenarios in the proposed inventory policy as opposed to the traditional inventory policy which only fulfills the emergency demand by the more expensive emergency order increases as either one of the three key parameters increase. This result is more significant as either one of the key parameters increase as energy cost changes.

4.4.2 Key Parameters with respect to Energy Cost

Given the various relationships and significances of model parameters on changes to inventory policy decisions and logistics costs shown in Tables 4.7-4.12 with respect to energy, certain model parameters can be combined to illustrate situations in which inventory policy decisions and their respective costs are most significantly affected by changes to energy cost. Furthermore, similar combinations of such key factors can represent situations in which the proposed inventory model is most effective at reducing cost if implemented in place of the traditional EOQ inventory model. These key factors, according to the initial comparative analysis, include the product weight, the size of the regular demand per unit time, and last but not least, the size of the emergency demand per unit time. All other parameters including the fixed purchasing cost, the unit purchasing cost, the transportation lead time, and the transportation distance are not as significant to changes in inventory policies and costs with respect to energy cost.

Set	Description	1	2	3
Supply	Fixed Purchasing Cost to Order	<i>High</i>		
	Total Unit Purchasing Cost to Procure	Low		
Production	Regular Demand per unit time	<i>High</i>	<i>Vary</i>	<i>High</i>
	Mean Emergency Demand	<i>High</i>		<i>Vary</i>
Transportation	Lead Time	Short		
	Distance	<i>Far</i>		
	Weight	<i>Vary</i>	<i>Heavy</i>	

Table 4.13: Parameter Levels to Validate Significant Effects of Key Parameters

Consequently, in the analysis conducted to illustrate the degree of significance for each of the key parameters, the parameters which are least affected by changes to energy cost are held constant at an arbitrary level shown in Table 4.13; and the parameters which are most significantly affected by changes to energy cost are each independently varied

between two levels. That is, only one significant factor is varied at any time. All other significant factors are held at the level which results in the most significant change with respect to energy cost as shown in Table 4.13.

4.4.3 Effects of Product Weight with respect to Energy Cost

According to the comparative analysis, one of the most significant factors to changes in inventory policy decisions and the logistics costs with respect to energy cost is the product weight. As noted in the introduction of the model parameters, the product weight affects both the non-energy related and the energy related unit cost to ship a single product via a specified transportation mode. More specifically, as the product weight increases, so too does the unit cost to ship via a specified mode.

Similarly, as the energy cost increases along with the product weight, the unit transportation cost associated to the energy required to ship a single product via a specified transportation mode increases more significantly. As a result, the expected total transportation cost per day for both policies increases more significantly as the weight of the product increases with respect to energy cost. Figure 4.2 illustrates the effect of product weight, in particular a light weighted product versus a heavy weight product, on change the in transportation cost with respect to energy cost. As the graph on the left illustrates, the change in total transportation cost per day with respect to energy cost is more significant for products with heavier weight than products with lighter weight. Yet regardless of the weight, the expected transportation cost per day for the proposed policy is less than that of the traditional policy, as is illustrated in the graphs on the right in 4.2.

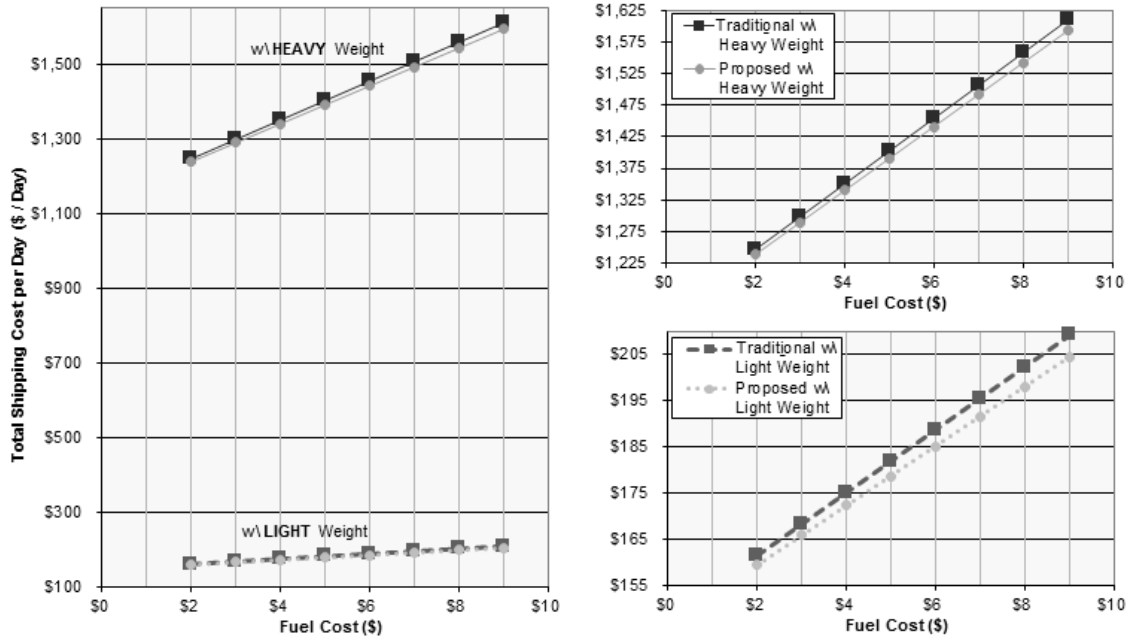


Figure 4.2: Change in Expected Total Transportation Cost per Day with respect to Energy Cost and Product Weight for the Exponential Distribution

The affect of product weight and energy cost on the daily total transportation cost shown in Figure 4.2 corresponds to variations in the proportion of total cost allocated to procurement, transportation, and inventory activities. For either inventory policy, over 98 percent of the total cost is allocated to procurement and transportation activities. However, since the procurement cost in both inventory policies is not significantly affected by energy cost alone, the change in the proportions of total cost allocated to procurement or to transportation corresponds proportionally with the change in the expected total transportation cost per day with respect to energy cost and product weight.

In other words, as the change in the expected total transportation cost with respect to energy increases, so too do the proportion of total cost allocated to transportation activities. Conversely, as the proportion of the total cost allocated to transportation

activities increases, the proportion of the total cost allocated to procurement decreases. As the graph on the left in Figure 4.3 illustrates, the change in the proportion of the total cost allocated to transportation with respect to energy cost increases more significantly provided a heavier product than a lighter product. Similarly, the proportion of the total cost allocated to procurement with respect to energy decreases more significantly provided a heavier product than a lighter product.

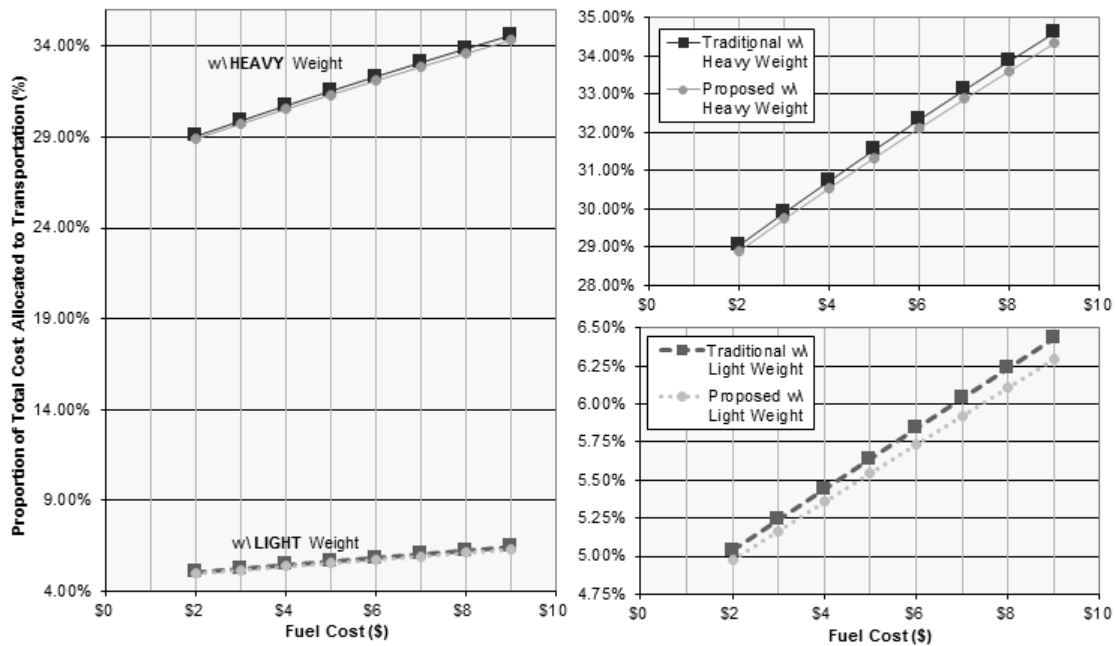


Figure 4.3: Change in the Proportion of Total Cost Allocated to Transportation with respect to Energy Cost and Product Weight for the Exponential Distribution

As the two graphs on the right in Figure 4.3 illustrate, the proportion of total cost allocated to transportation activities is always less for the proposed inventory policy than the traditional inventory policy. However, the proportion of total cost allocated to procurement activities for the proposed inventory policy is not always more or less than that of the traditional inventory policy. The reason for these results is that the proposed

inventory policy, unlike the traditional inventory policy, explicitly considers energy cost and emergency demand in the model. Also, the expected total procurement cost per day is not significantly affected by changes to energy cost for both policies.

Consequently, like the change in the expected transportation cost per day with respect to energy cost, the change in the expected total cost per day with respect to energy cost is significantly affected by changes to product weight. In particular, the change in the expected total cost per day with respect to energy increases more significantly provided a heavier product than a lighter product. Additionally, the expected total cost per day for the proposed inventory policy is always less than that of the traditional inventory policy. This result is proportionally identical to the result illustrated in the graphs in Figure 4.2 for the change in the expected total transportation cost per day.

Product weight affects not only the change in the unit energy cost to ship with a specified transportation mode with respect to changes in energy cost, but also the change in the difference in the unit energy cost to ship between specified transportation modes with respect to changes in energy cost. As the energy efficiency of the transportation mode decreases, the rate at which the fuel surcharge changes with respect to energy cost increases, and the rate at which the total unit transportation cost changes with respect to weight increases. So, the difference between the unit energy cost to ship a product via a faster, less energy efficient mode such as air and a slower, more energy efficient mode such as ground increases significantly with respect to weight and/or energy.

Consequently, the need to reduce the probability of fulfilling emergency demand with the more expensive and energy sensitive emergency order which utilizes air transportation becomes more significant to minimize cost as weight increases with

respect to energy cost. With regards to the proposed policy, the probability of fulfilling an emergency demand with the more expensive and energy sensitive emergency order can be decreased by adding or increasing safety stock. So, in order to minimize this probability and thus minimize cost, the safety stock must increase more significantly as product weight increases with respect to changes in energy cost. Figure 4.4 illustrates that the change in safety stock with respect to energy cost does in fact increase more significant provided a heavier product than a lighter product.

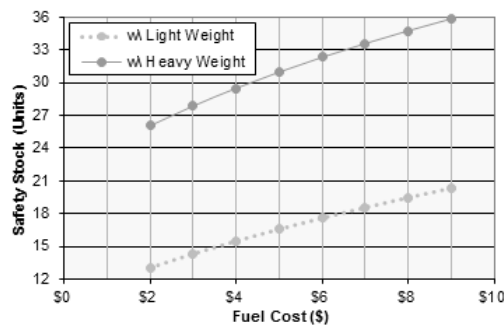


Figure 4.4: Change in Safety Stock with respect to Energy Cost and Product Weight for the Exponential Distribution

However, safety stock affects not only the probability of an emergency order, but also the inventory cycle length which subsequently affects the scheduled order quantity, the probability of an emergency demand, and the probability of fulfilling the emergency demand with an emergency order. As safety stock increases, the inventory cycle length and thus the scheduled order quantity increases proportionally since there is no change in the regular demand. Therefore, similarly to safety stock, both the inventory cycle length and the scheduled order quantity increase more significantly as weight increases with respect to energy cost. This result is illustrated in two separate graphs in Figure 4.5.

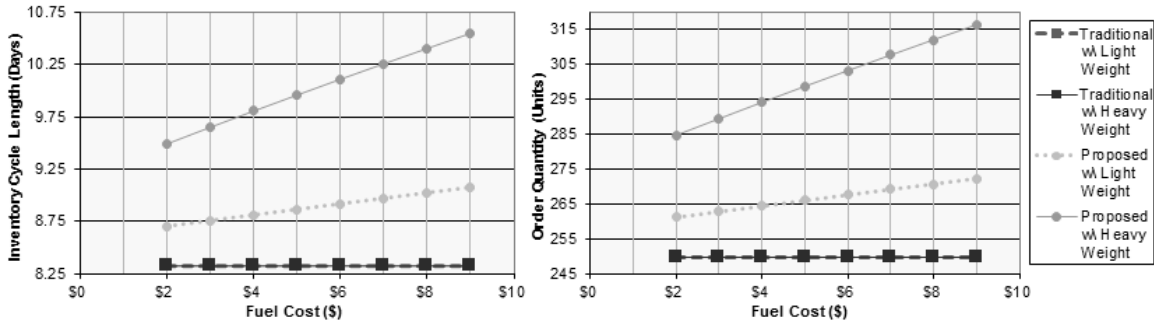


Figure 4.5: Change in the Inventory Cycle Length and Change in the Scheduled Order Quantity with respect to Energy Cost and Product Weight for the Exponential Distribution

As both graphs in Figure 4.5 illustrate, the change in the inventory cycle length with respect to energy cost (on the left) is proportional to the change in the scheduled order quantity with respect to energy cost (on the right) when the regular demand size is held constant. Furthermore, the change in either the inventory cycle length or the scheduled order quantity with respect to energy cost for the proposed inventory policy is more significant for a heavier product than a lighter product but negligible for the traditional inventory policy. In fact, because traditional inventory policy, unlike the proposed inventory policy, does not explicitly consider energy cost or emergency demand, the traditional inventory policy decisions do not change as product weight changes with respect to energy cost.

Therefore, given that the inventory cycle length directly affects the probability of an emergency demand, the change in the probability of an emergency demand with respect to energy cost is negligible for the traditional inventory policy but significant for the proposed inventory policy. Moreover, the effect of energy cost and product weight

on this probability is similar if not identical to that of both the inventory cycle length and the scheduled order quantity. In particular, as the graph on the left in Figure 4.6 illustrates, the change in the probability of an emergency demand with respect to energy cost is more significant for heavier products than lighter products.

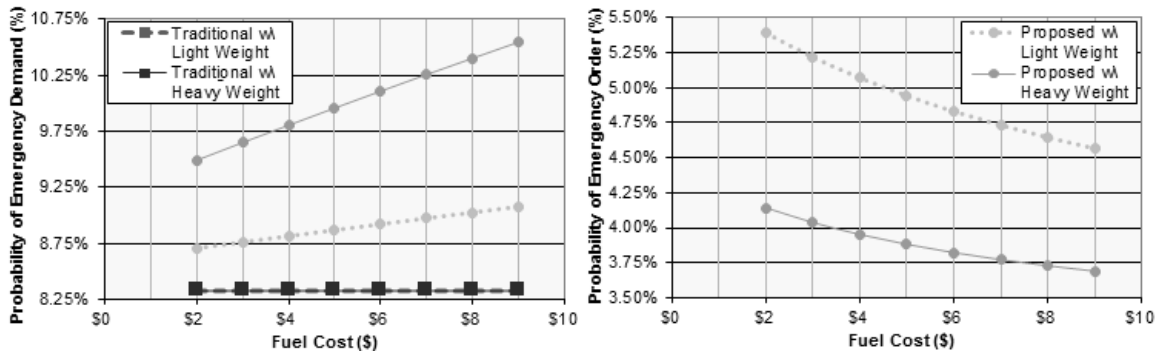


Figure 4.6: Change in the Probability of an Emergency Demand and Change in the Probability of an Emergency Order

with respect to Energy Cost and Product Weight for the Exponential Distribution

Similarly to the probability of an emergency demand, the probability of fulfilling the emergency demand with an emergency order increases as the inventory cycle length increases. However, if safety stock also increases, the probability of fulfilling the emergency demand with an emergency order decreases. So, the change in the probability of an emergency order actually decreases as energy cost increases. Yet, as the graph on the right in Figure 4.6 illustrates, the change in the probability of an emergency order with respect to energy cost is negatively related to the product weight. That is, the probability of an emergency order decreases more significantly with a light weighted product than a heavy weighted product. Nevertheless, as the graph of the change in the

probability of an emergency demand with respect to energy cost in Figure 4.6 illustrates, the probability still decreases as the product weight increases.

Since the probability of an emergency order decreases as either the product weight or the energy cost increase, the expected total transportation cost of fulfilling an emergency demand via any of the irregular replenishment scenarios also decreases as either the product weight or the energy cost increase. Therefore, the cost savings with respect to fulfilling the emergency demand via the scenarios in the proposed inventory policy as opposed to the traditional inventory policy increases as either the energy cost or the product weight increases. This result is realized more significantly as the product weight increases with respect to energy cost.

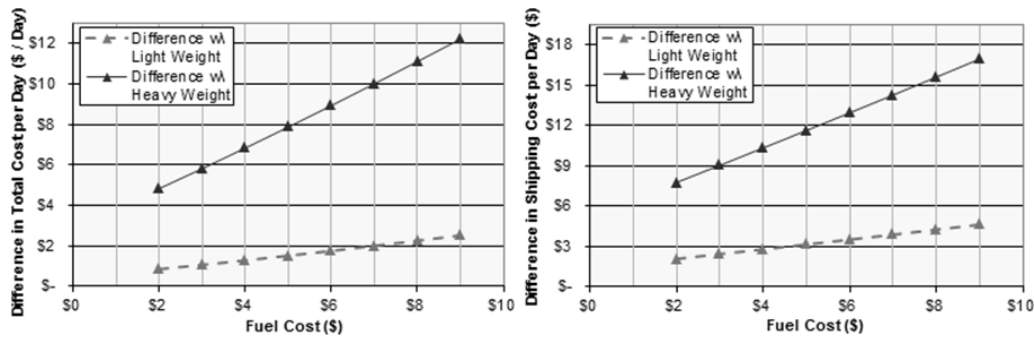


Figure 4.7: Change in the Cost Effectiveness of the Proposed Inventory Policy at Reducing the Total Transportation Cost per Day and the Total Cost of all Logistics Activities per Day with respect to Energy Cost and Product Weight for the Exponential Distribution

As a result, the total transportation cost savings per day of implementing the proposed inventory policy in place of the traditional inventory policy increases more significantly as the product weight increases with respect to energy cost. That is, the total

transportation cost savings per day changes with respect to energy cost more significantly for a heavier weighted product than a lighter weighted product, as illustrated in the graph on the right in Figure 4.7. Likewise, the total cost savings per day for all logistics activities of implementing the proposed inventory policy in place of the traditional inventory policy increases more significantly as the product weight increases with respect to energy cost. This result, which is illustrated in the graph on the left in Figure 4.7, occurs because transportation is the only activity that comprises a large proportion of the total cost and is significantly affected by energy cost alone.

4.4.4 Effects of Regular Demand with respect to Energy Cost

Another very significant factor to changes in inventory policy decisions and the logistics costs with respect to energy cost, according to the comparative analysis, is the size of the regular demand. Without any change to energy cost, the size of the regular demand directly affects the scheduled order quantity and thus indirectly affects the total procurement, transportation, and inventory cost for the regularly scheduled order. Yet, as the energy cost increases along with the size of the regular demand, the proportional increase to the scheduled order quantity more significantly affects the total energy cost to ship the scheduled order quantity via a specified transportation mode. As a result, change in the expected total transportation cost per day with respect to energy cost for both policies increases more significantly as the regular demand increases.

The effect of the regular demand size on the change in the total transportation cost per day with respect to energy cost is illustrated in Figure 4.8. In particular, the change in total transportation cost per day with respect to energy cost is more significant for

products with higher regular demand than products with lower regular demand, as illustrated in the graph on the left in Figure 4.8. Yet, regardless of the size of the regular demand, the expected transportation cost per day for the proposed policy is less than that of the traditional policy, as is illustrated in the two graphs on the right in Figure 4.8.

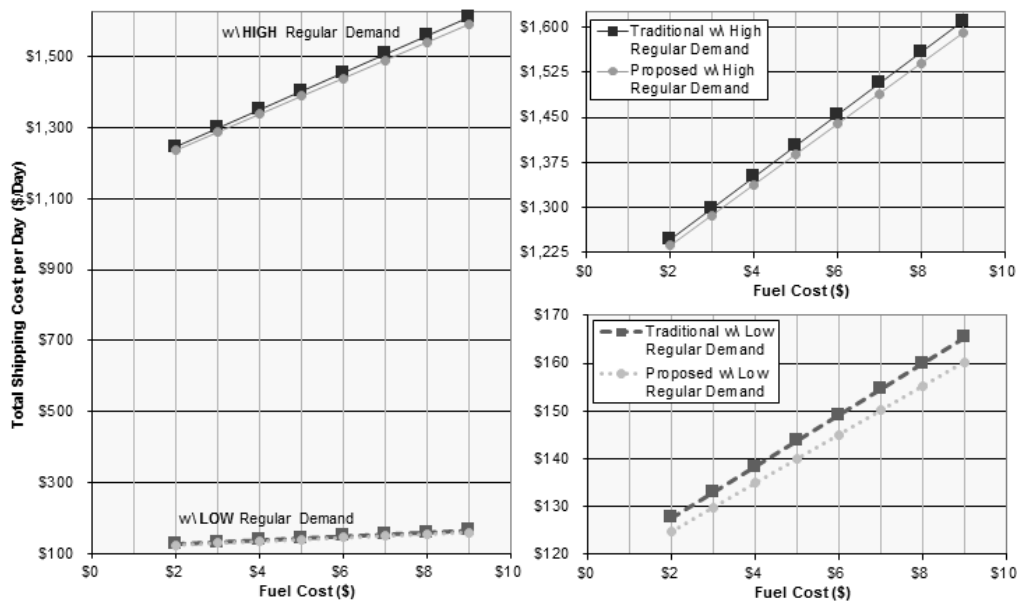


Figure 4.8: Change in Expected Total Transportation Cost per Day with respect to Energy Cost and Regular Demand for the Uniform Distribution

The effects on the total transportation cost per day illustrated in Figure 4.8 are proportionally identical to the effects the same model parameters on the total cost per day of all logistics activities related to the inventory policy. This response transpires because transportation is the only activity that comprises a large proportion of the total cost and is significantly affected by energy cost alone. Procurement, on the other hand, comprises the remaining major proportion of the total cost but is not significantly affected by energy cost alone. So, similarly to the effects illustrated in Figure 4.8, the change in the expected total cost per day of all the logistics activities for either policy with respect to

energy cost increases more significantly for products with higher volumes of regular demand than products with lower volumes of regular demand.

The change in regular demand and energy cost affects not only the regularly scheduled order, but also the irregular replenishment scenarios. In particular, the change in energy cost, regardless of a change in regular demand, affects the difference in the unit energy cost to ship an order via different modes of transportation. More specifically, as the energy efficiency of the transportation mode decreases, the rate at which the fuel surcharge changes with respect to energy cost increases. So, the difference between the unit energy cost to ship a product via a faster, less energy efficient mode such as air and a slower, more energy efficient mode such as ground increases with respect to energy cost.

Consequently, the necessity to reduce the probability of fulfilling the emergency demand with the more expensive and energy sensitive emergency order which utilizes air transportation continues to become more significant to the objective of minimizing cost as energy cost increases, regardless of any change regular demand size. This probability can be decreased by adding or increasing safety stock in the proposed policy. However, contrary to the prior logic, the change in the safety stock is significantly affected by not only energy cost, but also the change in regular demand size. That is, the change in safety stock with respect to energy cost is significantly affected by changes in the regular demand size, because the probability of fulfilling the emergency demand with an emergency order is affected by more than safety stock alone.

As previously noted, safety stock affects not only the probability of an emergency order, but also the inventory cycle length which subsequently affects the scheduled order quantity, the probability of an emergency demand, and the probability of fulfilling an

emergency demand with an emergency order. Without any change in the size of the regular demand, an increase in safety stock causes an increase to the inventory cycle length and subsequently, a proportional increase to the scheduled order quantity. Yet, if regular demand changes along with safety stock, the change in the scheduled order quantity becomes more significant than the change in the inventory cycle length which is not affected by changes to the regular demand alone. In fact, the change in scheduled order quantity with respect to energy cost is significantly affected by the regular demand size whereas the change in the inventory cycle length with respect to energy cost is not significantly affected by regular demand size.

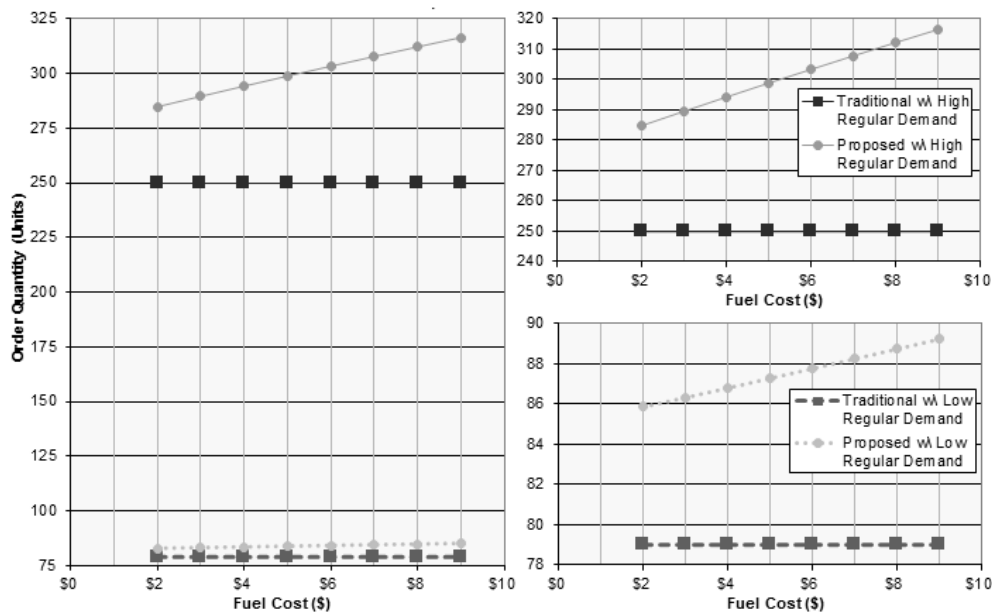


Figure 4.9: Change in the Scheduled Order Quantity with respect to Energy Cost and Regular Demand for the Exponential Distribution

Figure 4.9 illustrates the significant effect of the regular demand size on the change in the scheduled order quantity with respect to energy cost. As the graph on the

left in Figure 4.9 shows, the change in the scheduled order quantity with respect to energy cost increases more significantly for products with higher regular demand volumes than products with lower regular demand volumes. Regardless of the regular demand size, the scheduled order quantity is always greater in the proposed inventory policy than the traditional inventory policy, as is illustrated in the two graphs on the right in Figure 4.9. Moreover, the scheduled order quantity is always greater with higher regular demand sizes than lower regular demand sizes, regardless of the inventory policy.

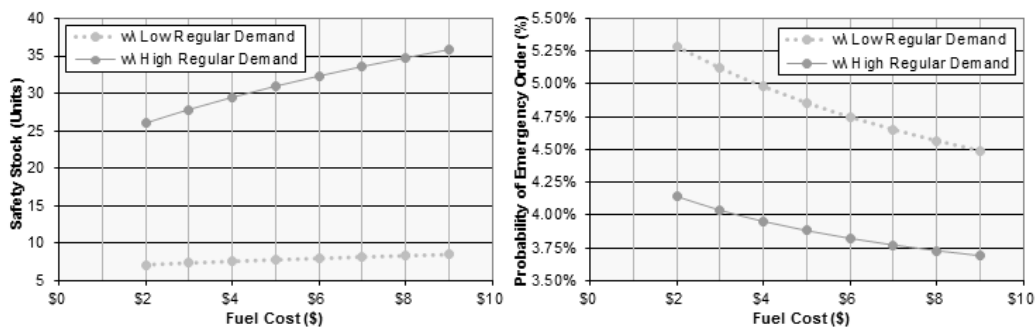


Figure 4.10: Change in Safety Stock

and Change in the Probability of an Emergency Order

with respect to Energy Cost and Regular Demand for the Exponential Distribution

Although the inventory cycle length and likewise the probability of an emergency demand are not significantly affected by changes in the regular demand size with respect to energy cost, the two results are still significantly affected by changes to energy cost alone. Consequently, as energy cost increases, the inventory cycle length as well as the safety stock increases, and thus the probability of fulfilling an emergency order decreases with respect to energy cost. So as the graph on the left in Figure 4.10 illustrates, the change in safety stock with respect to energy cost increases more significantly for higher

regular demand volumes than lower regular demand volumes. Yet the probability of an emergency order decreases more significantly for lower regular demand volumes than higher regular demand volumes. Nevertheless, as the graph of the change in the probability of an emergency demand with respect to energy cost in Figure 4.10 illustrates, the probability of an emergency order still decreases as the regular demand size increases.

Since the probability of an emergency order decreases as either the regular demand or the energy cost increase, the expected total transportation cost of fulfilling an emergency demand via any of the irregular replenishment scenarios also decreases as either the regular demand size or the energy cost increase. Therefore, the cost savings with respect to fulfilling the emergency demand via the scenarios in the proposed inventory policy as opposed to the traditional current inventory policy increases as either the energy cost or the regular demand size increase. That is, the cost effectiveness of implementing the proposed policy in place of the traditional inventory policy becomes more significant as either the energy cost or the regular demand increase; however, the cost effectiveness becomes more evident as the regular demand size increases.

As a result, the total transportation cost savings per day of implementing the proposed inventory policy in place of the traditional inventory policy increases more significantly as the regular demand size increases with respect to energy cost. That is, the total transportation cost savings per day changes with respect to energy cost more significantly for higher regular demand volumes than lower regular demand volumes, as illustrated in the graph on the right in Figure 4.11. Likewise, the total cost savings per day for all logistics activities of implementing the proposed inventory policy in place of the traditional inventory policy increases more significantly as the regular demand size

increases with respect to energy cost. This result, which is illustrated in the graph on the left in Figure 4.11, occurs because transportation is the only activity that comprises a large proportion of the total cost and is significantly affected by energy cost alone.

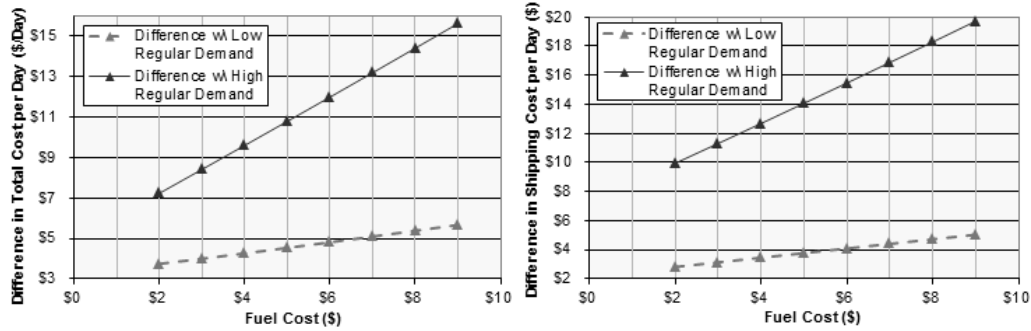


Figure 4.11: Change in Total Cost per Day and Change in Shipping Cost per Day with respect to Energy Cost and Regular Demand for the Uniform Distribution

4.4.5 Effects of Emergency Demand with respect to Energy Cost

The final factor significant to the cost effectiveness of the proposed inventory policy over the traditional inventory policy with respect to energy cost, according to the comparative analysis, is the expected size of the emergency demand. Unlike changes to the regular demand, changes to the expected size of the emergency demand only affect the irregular replenishment scenarios. Therefore, only the inventory decisions and respective costs in the proposed inventory policy are affected by changes to the expected size of the emergency demand, because the traditional inventory policy does not consider emergency demand in its inventory policy decisions. Instead, changes to the size of the emergency demand only affect the total procurement and the total transportation cost of the traditional inventory policy.

Though the change in the total transportation cost with respect to energy cost for either of the two inventory policies is not significantly affected by the change in the average emergency demand size, the change in the total transportation cost associated only to the irregular replenishment scenarios with respect to energy cost is significantly affected by the average emergency demand. In particular, the change in the average emergency demand with respect to energy cost affects the difference in the total energy cost to fulfill the emergency demand via different modes of transportation. As the energy efficiency of the transportation mode decreases, the rate at which the fuel surcharge changes with respect to energy cost increases. So, the difference between the total energy cost to fulfill the emergency demand via the faster, less energy efficient air transportation and the slower, more energy efficient ground transportation increases more significantly as the size of the emergency demand increases with respect to energy cost.

Consequently, the necessity to reduce the probability of fulfilling emergency demand with the more expensive and energy sensitive emergency order which utilizes air transportation continues to become more significant to the objective of minimizing cost as the size of the emergency demand increases with respect to energy cost. So, like the response of the proposed model to changes in product weight or regular demand with respect to energy cost, the response of the proposed model to increases in the emergency demand volume with respect to energy cost is to add or increase safety stock in order to reduce the probability of fulfilling the emergency demand with an emergency order. In fact, as Figure 4.12 shows, the change in the safety stock with respect to energy cost increases more significantly for higher average emergency demand volumes than lower average emergency demand volumes.

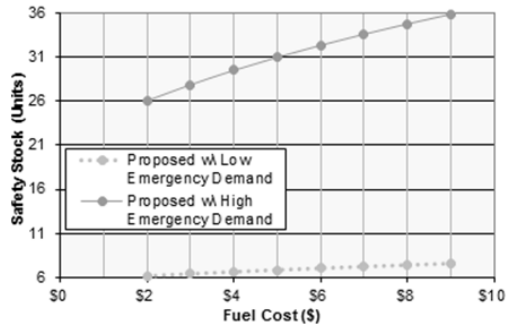


Figure 4.12: Change in Safety Stock with respect to Energy Cost and Average Emergency Demand for the Exponential Distribution

However, as previously noted, the probability of fulfilling an emergency demand with an emergency order is affected by more than safety stock alone. For instance, the inventory cycle length, which is also affected by safety stock, affects the probability of an emergency order as well as the probability of an emergency demand. As the inventory cycle length increases, both the probability of an emergency demand and the probability of fulfilling the emergency demand with an emergency order increase, provided no other changes. Yet, as both the safety stock and inventory cycle length increase, the probability of an emergency order decreases more significantly as the inventory cycle length increases with respect to the safety stock level.

Since the safety stock level also affects the inventory cycle length, the inventory cycle length is similarly affected by changes to the average size of the emergency demand with respect to energy cost. That is, as either the safety stock level or the average size of the emergency demand increases, the inventory cycle length increases. Likewise, the scheduled order quantity and the probability of an emergency demand, which are proportional to the inventory cycle length provided that there is no change in

the size of the regular demand, are increase significantly with respect co changes in the energy cost and the average size of the emergency demand. These results are illustrated in two separate graphs in Figure 4.13.

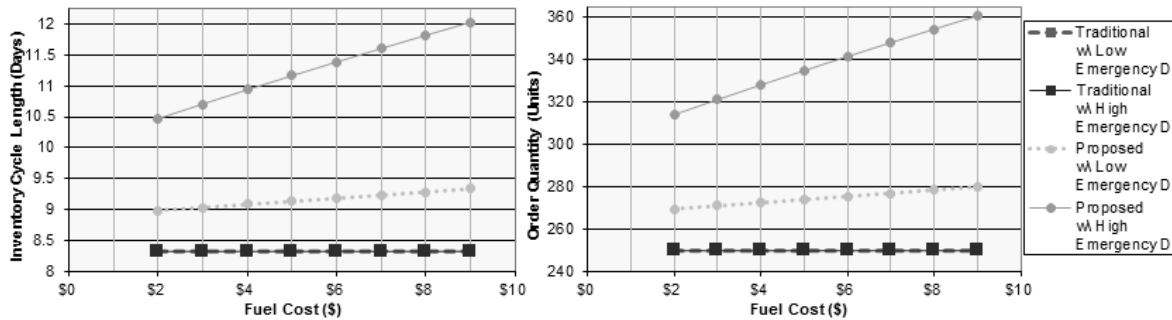


Figure 4.13: Change in the Inventory Cycle Length and Change in the Scheduled Order Quantity with respect to Energy Cost and Average Emergency Demand for the Uniform Distribution

As both graphs in Figure 4.13 illustrate, the change in the inventory cycle length with respect to energy cost (on the left) is proportional to the change in the scheduled order quantity with respect to energy cost (on the right) when the regular demand size is held constant. Furthermore, the change in either the inventory cycle length or the scheduled order quantity with respect to energy cost for the proposed inventory policy is more significant for a higher average emergency demand than a lower average emergency demand. However, this result is only true for the proposed policy because the traditional policy does not explicitly consider energy cost or emergency demand.

Since the inventory cycle length proportionally affects the probability of an emergency demand, the change in the probability of an emergency demand with respect to energy cost is significantly affected by the average emergency demand volume in the

proposed inventory policy. In particular, the effect of energy cost and the average emergency demand size on this probability is similar if not identical to that of both the inventory cycle length and the scheduled order quantity. That is, the change in the probability of an emergency demand with respect to energy cost is more significant for a higher average emergency demand size than a lower emergency demand size, as the graph on the left in Figure 4.14 illustrates.

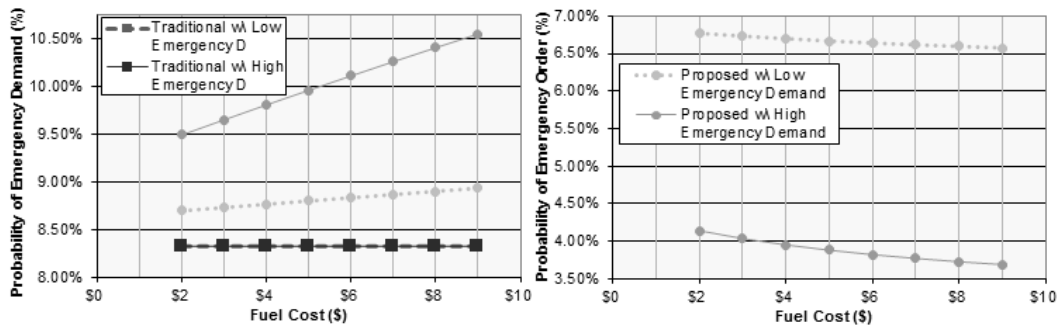


Figure 4.14: Change in the Probability of Emergency Demand and Change in the Probability of an Emergency Order with respect to Energy Cost and Average Emergency Demand for the Exponential Distribution

Similarly to the probability of an emergency demand, the probability of fulfilling the emergency demand with an emergency order increases as the inventory cycle length increases. However, because safety stock increases in addition to the inventory cycle length as the energy cost increases, the probability of fulfilling the emergency demand with an emergency order decreases. Yet, as the graph on the right in Figure 4.14 illustrates, the change in the probability of an emergency order with respect to energy cost is positively related to the product weight. That is, the probability of an emergency order decreases more significantly with a higher average emergency demand than a lower

average emergency demand. Also, as the graph on the right in Figure 4.14 shows, the probability of an emergency demand decreases as either the energy cost or the average emergency demand size increase.

Since the probability of an emergency order decreases as either the average size of the emergency demand or the energy cost increase, the expected total transportation cost of fulfilling an emergency demand via any of the irregular replenishment scenarios also decreases as either the average size of the emergency demand or the energy cost increase. Moreover, since the change in the probability of an emergency order with respect to energy cost decreases more significantly as the average size of the emergency demand increases, the change in the total transportation cost of fulfilling an emergency demand via any of the irregular replenishment scenarios with respect to energy cost decreases more significantly as the average size of the emergency demand increases. Therefore, the cost savings with respect to fulfilling the emergency demand via the scenarios in the proposed policy as opposed to the traditional model increases more significantly as the average emergency demand increases with respect to energy cost.

As a result, the total transportation cost savings per day of implementing the proposed inventory policy in place of the traditional inventory policy increases more significantly as the average emergency demand size increases with respect to energy cost. That is, the total transportation cost savings per day changes with respect to energy cost more significantly for a higher average emergency demand size than a lower average emergency demand size, as illustrated in the graph on the right in Figure 4.15. Likewise, the total cost savings per day for all logistics activities of implementing the proposed inventory policy in place of the traditional inventory policy increases more significantly

as the average size of the emergency demand increases with respect to energy cost. This result, which is illustrated in the graph on the left in Figure 4.15, occurs because transportation is the only activity that comprises a large proportion of the total cost and is significantly affected by energy cost alone.

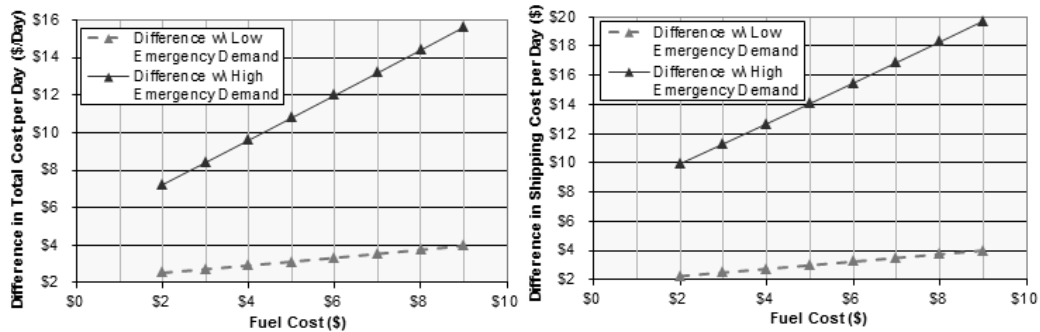


Figure 4.15: Change in Total Cost per Day and Change in Shipping Cost per Day with respect to Energy Cost and Emergency Demand for the Uniform Distribution

4.4.6 Summary of Numerical Analysis & Results

The purpose of analyzing the proposed inventory policy comparatively to the traditional economic ordering policy with respect to changes in energy cost as well as numerous other model parameters is to discover and understand the environments in which the inventory policies are most significantly affected by changes to energy cost as well as the environments in which the proposed inventory model is most cost effective. As the analysis reveals, the three key model parameters – product weight, regular demand, and emergency demand – affect inventory decisions and related logistics costs more significantly than all other parameters as energy cost changes. In particular, as any of the three key model parameters increase, the change in many of the inventory decisions or related logistics costs become more significant as energy cost changes. Furthermore, the cost effectiveness and thus savings of implementing the proposed

inventory policy in place of the traditional policy becomes more significant as product weight, regular demand, or emergency demand increase with respect to energy cost.

Therefore, the production environments with relatively heavy product weights, high regular demand, and high emergency demand are most significantly affected by changes to energy cost. Such environments are most receptive to the proposed inventory model rather than a simple economic order policy which ignores the possibility for emergency demand. Nonetheless, the production environments with high levels of at least one of the three factors are also receptive to the proposed inventory policy since each factor alone significantly affects many inventory decisions and related logistics costs with respect to energy cost.

Provided these results, guidelines can be developed to assist businesses in managing inventory or other logistics functions more efficiently and effectively as energy cost and consumption continue to rise. For instance, with regard to inventory management, businesses with similar production environments should implement the proposed inventory ordering policy if the weight of the product, the regular demand of the product, or the emergency demand of the product is relatively high. In particular, when more than one of the three factors – product weight, regular demand, and emergency demand – is relatively high, businesses should employ the proposed inventory policy rather than a simple economic ordering policy in order to effectively and efficiently compete globally as energy cost rises.

CHAPTER 5

CONCLUSION & EXTENSIONS

5.1 Conclusions

The inventory model developed and analyzed in this paper is based on the actual environment at an aircraft manufacturer. In particular, the proposed inventory model is applicable for production systems with constant production rates but small, underlying possibilities for undesirable circumstances to threaten the intricately planned production schedule. Rather than ignoring the possibility for undesirable circumstances and subsequently fulfilling any emergency demand with a more expensive and energy cost sensitive emergency order from a supplier, the proposed model provides multiple scenarios to fulfill the emergency demand more cost effectively. These options include fulfilling the emergency demand from safety stock alone, a combination of safety stock and an emergency order, and lastly an emergency order alone if the regularly scheduled order is already in route to the production facility.

So, the objective of the inventory model developed in this paper was to structure an inventory policy under explicit energy cost considerations with optimal sizes for the scheduled order quantity, safety stock, and inventory cycle length that minimizes the total expected cost per unit time for a system with a constant production rate but a small, underlying possibility for undesirable circumstances to threaten the intricately planned production schedule. After solution conditions and procedures were developed to ensure a globally optimal solution to the given problem situation, the model was comparatively

analyzed to a traditional economic ordering policy which satisfies the regular demand generated by the production system and ignores the possibility of an undesirable circumstance threatening the intricately planned production schedule.

Throughout the numerical analysis, the model parameters including the energy cost are varied in order to identify the key parameters that affect the inventory policy decisions and the resultant costs. By varying most of the model parameters, the analysis unveils the environments in which inventory policies are most significantly affected by changes to energy cost as well as the environments in which the proposed inventory policy is most cost effective. As the analysis reveals, the three key model parameters – product weight, regular demand, and emergency demand – affect inventory decisions and related logistics cost more significantly than all other parameters as energy cost changes. In particular, as any of these three key model parameters increase, the change in inventory decisions or related logistics cost becomes more significant as energy cost changes. Moreover, the cost effectiveness of implementing the proposed inventory policy in place of the traditional inventory policy also becomes more significant as any of these key model parameters increase with respect to energy cost.

Therefore, the production environments with relatively heavy product weights, high regular demand, and high emergency demand are most significantly affected by changes to energy cost. Such environments are also particularly receptive to the proposed inventory model and the cost savings it offers in comparison to the simple economic order policy which ignores the possibility for emergency demand. Nonetheless, the production environments with high levels of at least one of the three factors are also

receptive to the proposed inventory policy since each factor alone significantly affects many inventory decisions and related logistics costs with respect to changes in energy cost.

Provided these results, guidelines can be developed to assist businesses in managing inventory and many other logistics functions more efficiently and effectively as energy cost and consumption continue to rise. For instance, with regard to inventory management, businesses with similar production environments should implement the proposed inventory ordering policy if the weight of the product, the regular demand of the product, or the emergency demand of the product is relatively high, but especially when all three factors are relatively high.

5.2 Extensions

There are many possible extensions to this research. Many of the following extensions are based on expanding the scope of the research. For instance, future research may expand the scope of the current research with regard to the inventory applications, the supply chain network, and the production environment.

The first possible research extension involves broadening the scope of the inventory model applications. That is, future research should develop an inventory model that still explicitly considers energy cost but is applicable to more production environments. By developing a more generalized inventory model with respect to changes in energy cost, a more generalized guideline for businesses to manage inventory more efficiently and effectively as energy cost and consumption rise can be developed.

The second possible research extension involves broadening the scope of the supply chain network. The scope of the supply chain network can be expanding

vertically, horizontally, or both. To expand the current research vertically, future research should consider more levels in the supply chain network. Instead of focusing on at the plant level, future research can on the inventory at the plant level as it relates to backwards interactions from suppliers and distributors or as it relates to forwards interactions from warehouses, distributors, and customers.

The scope of the supply chain network can also be expanded horizontally. To expand the supply chain scope of the current research, future research should consider more locations at each or any level of the supply chain. For instance, future research could develop an inventory model that explicitly considers energy cost in order to determine the optimal inventory policy for each plant location in the supply chain, provided there are multiple locations in the supply chain. If the scope of the supply chain is expanded horizontally and vertically, an optimal inventory policy for multiple locations at multiple levels could be determined.

The third possible research extension involves broadening the scope of the production environment by means of considering multiple products. Thus, an inventory ordering model that explicitly considers the cost of energy can be developed to determine the optimal inventory ordering policy for all the products in a production system.

Last but not least, the energy portion of the current research can be extended in the scope of the model formulation. In particular, the unit purchasing cost associated to energy should be reformulated in order to test the true significance of energy cost on the purchasing cost. Rather than testing the effect of energy cost on transportation alone, the effect of energy cost on both production costs and thus purchasing costs and transportation costs should be investigated in future research.

BIBLIOGRAPHY

- [1] Chiang, C. Ordering policies for inventory systems with expediting. *Journal of the Operational Research Society* 93-94 (2002) 357-373.
- [2] Chiang, C. Optimal control policy for a standing order inventory system. *European Journal of Operational Research* 182 (2007) 695-703.
- [3] Duran A., Gutierrez G., Zequeira R.I. A continuous review inventory model with order expediting. *International Journal Production Economics* 87 (2004) 157–169.
- [4] Fukuda, Y. Optimal Policies For Inventory Problem with Negotiable Leadtime. *Management Science* 10 (1964) 690-708.
- [5] Ghiani, G., Laporte, G., and Musmanno, G. *Introduction to Logistics Systems Planning and Control*. England: John Wiley & Sons, Inc., 2004.
- [6] Johansen, S.G., Thorstenson, A. An inventory model with Poisson demands and emergency orders. *International Journal of Production Economics* 56–57 (1998) 275–289.
- [7] Leinbach and Capineri. *Globalized Freight Transport: Intermodality, E-Commerce, Logistics, and Sustainability*. Massachusetts: Edward Elgar Publishing, Inc., 2007.
- [8] McKinnon, A.C. A logistical perspective on the fuel efficiency of road freight transport. Report presented to the workshop ‘Improving Fuel Efficiency in Road Freight: The Role of Information Technologies’, Paris, France: 1999.
- [9] Minner, S. Multiple-supplier inventory models in supply chain management: A review *International Journal of Production Economics* 81–82 (2003) 265–279.
- [10] Moinzadeh, K., Nahmias, S. A continuous review model for an inventory system with two supply modes. *Management Science* 34 (1988) 761–773.
- [11] Murphy, R. Strategic Trends Impacting Logistics in the United States. Presented to Logistics Seminar, Helsinki, Finland: 2009.
- [12] Rosenshine M., Obee, B. Analysis of a Standing Order Inventory System with Emergency Orders. *Operations Research* 24 (1976) 1143-1153.

- [13] Tagaras, G., Vlachos, D. A periodic review inventory system with emergency replenishments. *Management Science* 47 (2001) 415–429.
- [14] Vlachos, D., Tagaras, G. An inventory system with two supply modes and capacity constraints. *International Journal of Production Economics* 72 (2001) 41–58.
- [15] White C.R., B.R. White. Emergency Inventory Ordering assuming Poisson Demands. *Production Planning and Control* 3 (1992) 168-174.
- [16] Wright, G.P. Optimal Ordering Policies for Inventory Systems with Emergency Ordering. *Operations Research* 20 (1969) 111-123.