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The theoretical foundation of statistics includes three fundamental theorems from probability theory: the law of large numbers, the Glivenko-Cantelli theorem and the central limit theorem. The first two provide justification of the idea that a large random sample taken from a population contains meaningful information about the original population. The third explains the ubiquity of the normal distribution (the bell curve).

In this dissertation we study various abstractions and generalizations of these theorems. The first main result deals with large random samples of points in Euclidean space of any dimension. We show that, under appropriate assumptions, the shape of the cloud is essentially pre-determined, even though the experiment is random. The second main result concerns the one dimensional case. We show, again under appropriate conditions, that the entire sample can be accurately predicted, except for the order in which the points are chosen. The final result is a modification of the central limit theorem that applies not only to large samples, but to any sample with at least two points.

The primary contribution of this work is to advance academic scholarship in the fields of probability theory and convex geometry. This is done by gaining a deeper understanding of random processes, as well as the geometry of basic mathematical objects such as sets and functions with convexity properties.