GEOMETRIC AND NONLINEAR LIMIT THEOREMS IN PROBABILITY THEORY

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ABSTRACT

The concentration of measure phenomenon is a nonlinear equivalent of the law of large numbers that deals with real valued Lipschitz functions and includes linear functionals such as the sample mean. In the first part of this dissertation we study functions that take values in more general metric-like spaces and have the property that they are invariant under coordinate permutations. In Chapter 1 we study functions that take values in the space of convex bodies, in Chapter 2 we study order statistics and in Chapter 3 we prove abstract concentration inequalities for functions taking values in an arbitrary metric space.

In the second part of the dissertation we study the central limit theorem. We show that if one conditions on certain tail events then convergence to the normal distribution can be achived without having to take a large number of summands. In fact 2 summands is enough.