Characterization of
a piezoelectric transformer
plasma source

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by

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The undersigned, appointed by the Dean of the Graduate School, have examined the dissertation entitled:

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**a piezoelectric transformer**

**plasma source**

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Chapter 1

Introduction

The piezoelectric transformer plasma source (PTPS) is a compact radio frequency (RF) driven plasma source. The PTPS uses an integrated piezoelectric transformer (PT) to aid in plasma production. Research on the PTPS has focused on optimizing the integrated PT within the PTPS and characterizing the plasma source operation at background gas pressures of 100-2000 mTorr.

Research on the PTPS started in support of a ferroelectric plasma thruster (FEPT) [1, 2]. The FEPT is a compact micropropulsion device that relies on ion mass transfer to produce thrust by accelerating ions out of a discharge plasma. Early in the development of the FEPT the importance of the piezoelectric effect with respect to device operation was noted. It was observed that the applied voltage necessary to produce plasma in the aperture of the FEPT was significantly reduced at certain frequencies corresponding to the piezoelectric resonance [3]. The PTPS and FEPT are identical in design and operation. The difference in nomenclature was established to highlight the importance of the piezoelectric effect in the operation of the device.

The PTPS is similar in design to many ferroelectric plasma sources (FPS). Both devices take advantage of the nonlinear characteristics of ferroelectric/piezoelectric materials to aid in plasma production. The main difference between a FPS and the PTPS is the applied signal to the device. A FPS typically uses a high-voltage
pulsed driving signal to create a large electric field at the triple-point at the edge of an electrode on the surface of the ferroelectric material [4]. Pulsed voltages of 5-40 kV are typical to initiate plasma formation in a FPS [5, 6]. The PTPS uses a comparatively low voltage applied RF signal of 100-500 V and takes advantage of the piezoelectric effect of the material. The integrated piezoelectric transformer steps up the input voltage to generate the necessary electric fields to create plasma.

1.1 Piezoelectric Effect

The piezoelectric effect is the linear interaction between electrical and mechanical energy. Piezoelectricity was first observed in quartz and Rochelle salt by Jacques and Pierre Curie [7]. While much research had been done on naturally occurring piezoelectric materials, transducer applications of piezoelectric materials were not widespread until barium titanate and other piezoelectric compounds were discovered. When poled by exposure to a high electric field these compounds exhibit a much stronger piezoelectric effect compared to natural piezoelectric crystals allowing for increased sensitivity and control of transducers [7].

The piezoelectric effect can be broken into two parts, the direct effect and the converse effect. The direct piezoelectric effect refers to a materials ability to couple mechanical strain to electrical displacement. Piezoelectric sensors take advantage of the direct piezoelectric effect to translate an applied force into a detectable electrical signal. Piezoelectric sensors are used in accelerometers [8] and pressure sensors [9, 10]. The converse piezoelectric effect produces a mechanical stress due to an applied electric field. Piezoelectric actuators use the converse piezoelectric effect to induce displacement with an applied electric field. Common uses of piezoelectric actuators include fluid flow control in ink jet printers [11] and micro-positioning for applications such as machining, Fabry-Perot laser cavities, and hard drives [12, 13].
A piezoelectric transformer takes advantage of both the direct and converse piezoelectric effects in a single device [14, 15]. A piezoelectric transformer uses the converse piezoelectric effect to convert a voltage applied at the transformer input into a mechanical displacement within the piezoelectric material. The displacement is then converted back into an output voltage at the transformer secondary through the direct piezoelectric effect. Similar to magnetic transformers, they can be used to step-up or step-down voltages. Piezoelectric transformers offer some advantages to traditional magnetic transformers including low noise, low weight, high voltage isolation, and high efficiency [16]. Piezoelectric transformers have been integrated into power converter circuits [17, 18] and fluorescent lamp ballasts [19, 20]. Similar to the PTPS, other plasma sources have integrated piezoelectric transformers into their design to generate the electric fields necessary to ionize gas. Plasma sources with integrated PTs have been investigated for driving dielectric barrier discharges [21], atmospheric pressure plasma jets [22, 23], atmospheric plasma sources [24].

The constitutive relationships for piezoelectricity are given in equations (1.1) and (1.2), where $T$ is the stress, $S$ is the strain, $E$ is the electric field, and $D$ is the electric displacement. The parameters $c$, $e$, and $\epsilon$ represent the elastic, piezoelectric, and dielectric constants for the material. The subscripts $i$, $j$, $k$, and $l$ represent the indices of the tensors for the material constants [25].

\[ T_{ij} = c_{ijkl}^E S_{kl} - e_{kij} E_k \]  
\[ D_i = e_{ikl} S_{kl} + \epsilon_{ij}^S E_k \]

Equation (1.1) is the relationship for the converse piezoelectric effect. If the piezoelectric term is removed, the relationship reduces to the constitutive relationship for a linear elastic material [26]. Equation (1.2) describes the direct piezoelectric effect. If the piezoelectric term is removed the relationship reduces to the typical
constitutive relationship for a dielectric. The addition of the piezoelectric term in each equation couples the two equations, such that the stress and electric displacement depend on both the electric field and strain.

1.2 Piezoelectric Transformer Plasma Source

The PTPS consists of a cylindrical piezoelectric disk driven by an RF voltage source. The voltage is applied through the thickness of the disk with a driven electrode covering one side of the disk and an annular electrode on the opposite side. Figure 1.1 shows a cross-section diagram of a simple electrode geometry for the PTPS. The piezoelectric disk is 10 mm in diameter and 2 mm in thickness. The piezoelectric material used in the PTPS is lithium niobate (LiNbO$_3$). Lithium niobate was chosen because of its high dielectric breakdown strength, strong electromechanical coupling, and low losses [27].

As the driving frequency approaches the resonance frequency of the disk, the electromechanical coupling and voltage transformation in the piezoelectric material increases. The voltage output of the PT is generated within the aperture of the annular electrode. The electrode geometry of the PTPS creates a step-up transformer with a generated output voltage that is larger than the applied voltage. The output voltage profile within the aperture is shown in Figure 1.2 for an ideal, radially

Figure 1.1: Cross-section diagram of the PTPS.
Figure 1.2: Electric potential generated within aperture of an ideal, radially symmetric, PTPS.

symmetric, resonance mode. The large induced RF voltage in the aperture produces the electric fields necessary to ionize background gas to produce plasma. The resulting electric field provides the electric force to accelerate charged particles from the PTPS. The electric field within the aperture are shown in a two-dimensional electrostatic field simulation of the PTPS in Figure 1.3. Equipotential lines show the rapidly decreasing electric field near the PT surface and the electric field direction is indicated by arrows.

The PTPS can be operated with a single aperture electrode as seen in Figure 1.1 or a stacked aperture electrode arrangement as shown in Figure 1.3. The fundamental operation in either case is the same. The PT effect occurs as a result of the applied voltage between the driven and aperture electrodes. Plasma formation is initiated
Figure 1.3: Electrostatic simulation of the PTPS with stacked apertures electrode during the ion emission half-cycle. Electric field direction is shown with arrows along with equipotential lines.
at the piezoelectric/electrode/background gas triple point. Additional aperture electrodes were only used when a gas flow was used to create a high pressure volume local to the piezoelectric surface. The electrodes may also serve to shape the electric field within the aperture volume to affect the charged particle beam shape.

1.3 Dissertation Overview

Chapter 2 covers the basic setup for the finite element simulations of the PTPS. Chapter 3 covers experiments and simulations completed to optimize the piezoelectric transformer effect used in the operation of the PTPS. Chapter 4 details the operation of the PTPS, the diagnostics used on the PTPS, and preliminary experiments evaluating the plasma source dependence on background pressure. Chapter 5 covers the plasma source characterization at background gas pressures of 100-2000 mTorr. Chapter 6 concludes the thesis and suggests potential future work.
Chapter 2

Simulation Setup

Finite element code was used to simulate the piezoelectric transformer (PT) properties of the plasma source. COMSOL Multiphysics [28] was chosen to model the PT because it easily allows different physics modules to be coupled together in a simulation. In the case of a piezoelectric device, this includes both solid mechanics and electrostatics. The piezoelectric devices interface, part of the structural mechanics or MEMS module in COMSOL, was used for most of the simulations presented. However, some simulations also included the electric circuit interface, a part of the AC/DC module, to determine the how the piezoelectric properties of the source varied with an additional electrical load at the transformer output.

2.1 Simulation Geometry

Nearly all simulations were completed using a three-dimensional simulation space, due to the rotated polarization axis for optimal electromechanical coupling in lithium niobate, described in detail in Chapter 3. The rotated polarization removed any cylindrical symmetry from the problem, preventing a two-dimensional axisymmetric simulation or the use of symmetry planes in a three-dimensional simulation. Three typical simulation geometries are shown in Figure 2.1. All geometries were drawn in the simulation global coordinate system with the z-axis normal to the disk surface.
The basic properties of a piezoelectric disk were modeled using a simulation space consisting only of the lithium niobate disk, as shown in Figure 2.1a. Both domains were modeled using the piezoelectric material model. As a result, electrostatic and structural boundary conditions needed to be applied to each boundary surface. The highlighted boundary in Figure 2.1a represented the aperture electrode and had a ground boundary condition. The entire opposite side of the disk represented the driven electrode and had an electric potential boundary condition applied. For eigenfrequency and frequency response simulations, the amplitude of the driving voltage was specified. In time-dependent simulations a sinusoidal function with the time, $t$, as a variable was specified. All other boundaries had a zero charge boundary
condition. Structurally, the free boundary condition was applied all boundaries. The simple geometry was used only to investigate the resonance modes and basic piezoelectric transformer characteristics of the PTPS.

The simulation geometry used to investigate the electric potentials and fields generated within the aperture of the PTPS is shown in Figure 2.1b. This geometry includes the aperture electrodes that shape the electric field within the aperture and an additional domain (not shown) encompassing the volume within the aperture and above the aperture electrodes. These additional domains were modeled only using electrostatic physics. As a result, structural effects of the electrodes were not modeled using solid mechanics. The mechanical effects of the electrodes were instead accounted for through the use of a mechanical loss factor in the lithium niobate domains. The mechanical loss factor is further explained in section 2.2.2.

Finally, the geometry shown in Figure 2.1c was used to verify the PT characteristics with an attached electrical load. The highlighted domain modeled an output voltage electrode placed in the center of the aperture of the PTPS. Simulations using this geometry also included the electric circuit interface in COMSOL. The highlighted surfaces had a terminal boundary condition applied. When a terminal is specified as an electric circuit terminal the electrostatic conditions at the terminal are coupled into a SPICE model of an electric circuit. All simulations performed used the voltage generated at the terminal to act as a voltage source for a SPICE model. The resulting current draw due to the circuit described by the SPICE model is coupled back to the finite-element model of the piezoelectric transformer. The electric circuit model allowed an electrical load to be modeled at the output of the transformer.
2.2 Simulation Parameters

Several parameters in COMSOL were important to implement in order to achieve realistic simulation results. These parameters include loss factors and the definition of a local coordinate system for the piezoelectric material.

2.2.1 Dielectric Loss Factor

Piezoelectric materials commonly list a loss tangent, tan $\delta$, for the material. In COMSOL, this factor is implemented as a loss factor for $\epsilon^S$ (see equation (1.2)) and is referred to as the dielectric loss factor, $\eta_{\epsilon S}$. The dielectric loss factor modifies the dielectric constant, $\epsilon^S$, according to equation (2.1).

$$
\epsilon^{(i,j)}_S \rightarrow (1 - \eta^{(i,j)}_{\epsilon S})\epsilon^{(i,j)}_S
$$

The dielectric loss factor used in the simulations was 0.0013. This value was reported by Ohmachi and is assumed to be constant for frequencies up to 9 Ghz [29].

2.2.2 Mechanical Loss Factor

Mechanical properties of a piezoelectric material are often characterized by a mechanical quality factor, $Q_m$. In general, a quality factor, $Q$, for a parameter is related to the loss factor, $\eta$, by an inverse relationship, as in equation (2.2).

$$
\eta = \frac{1}{Q}
$$

Therefore, the mechanical loss factor in COMSOL, $\eta_{cE}$, is the inverse of the the mechanical quality factor. The mechanical loss factor modifies the elastic constant, $c^E$ (see equation (1.1)) according to equation (2.3).

$$
c^{(i,j)}_E \rightarrow (1 + \eta^{(i,j)}_{cE})c^{(i,j)}_E
$$
The mechanical quality factor was used to account for any mechanical energy losses in the PTPS. This includes losses inherent to the material itself as well as additional losses due to the construction of the PTPS, including the external electrodes affixed to the lithium niobate. The mechanical quality factor had a large dependence on the materials used and could vary significantly from one device to the next due to compression of the lithium niobate disk between the driven electrode and aperture electrode, as will be shown in Chapter 3. Due to the variability of the mechanical quality factor, a typical value of 75 was used for the majority of simulations in this research.

2.2.3 Local Coordinate System

Piezoelectric material constants are defined for a coordinate system in which the polarization of the material is in the z-direction. In order to simulate a rotated polarization, either the simulation geometry needs to be rotated or a local coordinate system for the piezoelectric domains needs to be defined. Rotating the simulation geometry could cause simulation problems as the finite-element mesh would need to be regenerated each time the geometry was redrawn. This issue can be avoided if a local coordinate system is defined.

A local coordinate system for the piezoelectric domains was defined as a rotated system. The rotated system is defined by three consecutive rotation angles, \( \alpha \), \( \beta \), and \( \gamma \), as shown in Figure 2.2. A rotated system is especially important for modeling a lithium niobate piezoelectric transformer because the maximum electromechanical coupling occurs for polarization directions that are not orthogonal to the disk surface. The rotated coordinate system was only applied to domains modeling the piezoelectric transformer. For example, the electrodes in Figure 2.1b were modeled using the global coordinate system.
2.3 Simulation Quality

Generally, the simulation goals were to get trends in the piezoelectric transformer performance and not to exactly predict PTPS operation for a specific set point. With this in mind, the use of a typical values for $Q_m$ and polarization rotation angles were sufficient.

However, in some cases a simulation was used to verify experimental data and good agreement between simulation and experiment was necessary. In these cases the mechanical quality factor and the polarization rotation angles would be varied until a good fit was achieved. These parameters were chosen due to the variability of $Q_m$ each time a PTPS was assembled and the variability of the polarization direction between lithium niobate samples. The variability in polarization direction was a result of the $+/- 15^\circ$ rotation angle tolerance as specified by the manufacturer.

As an example, the results of a measured and simulated transformer ratio ($V_{out}/V_{in}$) are shown in Figure 2.3. The lithium niobate disk was specified with $\beta = 45^\circ$ and $\gamma = 0^\circ$ rotation angles and the initial estimate of the mechanical quality factor from experimental measurements was 63. The simulation was done using the geometry in Figure 2.1c, as the experiment had a high voltage probe attached to an
output voltage electrode to measure the transformer ratio. After least squares fit was performed on a parameter sweep of $Q_m$ and the polarization rotation angles, $\beta$ and $\gamma$, it can be seen that the simulation results are in good agreement with experimental results. The best fit was achieved with a simulated $Q_m$ of 70 and rotation angles of $\beta = 45^\circ$, $\gamma = 4^\circ$. 

Figure 2.3: Comparison of measured and simulated transformer ratio for a radial mode PT.
Chapter 3

Piezoelectric Effect Utilization

The PTPS utilizes the piezoelectric transformer effect to aid in plasma production. The applied RF signal induces a vibration in the crystal through the converse piezoelectric effect. The strain induced in the crystal then induces a voltage in the aperture of the PTPS. With an electrode pattern of a large annular electrode with a small aperture in the center the induced voltage is larger than the applied voltage. Near the resonant frequency, the voltage step-up ratio can be very large. The large induced RF voltage in the aperture results in ionization of gas to produce plasma due to the high electric field inside the aperture. The plasma is also accelerated from the PTPS by the generated voltage.

This chapter presents some studies completed to more effectively utilize the piezoelectric resonance in the PTPS. Experiments and simulations were performed to determine the optimal crystal orientations, verify resonance frequencies, investigate methods to increase the mechanical quality factor of the piezoelectric transformer, and determine the optimal electrode arrangement for the PTPS.

3.1 Crystal Rotation

The choice of lithium niobate as the dielectric in the PTPS has several advantages over common piezoelectric materials as mentioned in the introduction. However,
one disadvantage is that the optimal piezoelectric coupling is achieved for crystal orientations where the polarization direction is not orthogonal to the disk surface. Due to the rotated polarization, it becomes difficult to determine analytical expressions for a cylindrical disk because the problem no longer has radial symmetry. Using the local coordinate system described in section 2.2.3 and a parameter sweep of the rotation angles, the optimal crystal orientations were determined for a cylindrical lithium niobate disk in COMSOL. The rotation angles specified, with respect to the crystal geometry are shown in Figure 3.1. Only the $\beta$ and $\gamma$ rotations were necessary with our geometry. The first rotation, $\beta$, rotates about the X1 axis, resulting in the polarization axis, X3, rotating away from the disk surface normal. The second rotation, $\gamma$, rotates about the polarization axis. It is important to note that lithium niobate exhibits trigonal crystal structure and has three symmetry planes, separated by 120°, about the spontaneous polarization axis [30]. Due to this symmetry, it was only necessary to simulate rotation about the polarization axis, $\gamma$, for angles between 0° and 120°.

Eigenfrequency simulations were completed first to determine the resonance frequencies for each set of rotation angles. Then with known resonance frequencies
for each set of rotations, a frequency response analysis was performed. The frequency
response simulations were used to determine the peak output voltage generated at
the output of the piezoelectric transformer as well as the peak von Mises stress within
the crystal. The von Mises stress, $\sigma_e$, is a parameter calculated from the principal
stresses, $\sigma_1$, $\sigma_2$, and $\sigma_3$, in the material, as shown in equation (3.1).

$$\sigma_e = \sqrt{\frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]}$$ (3.1)

This parameter was used because von Mises yield stress for lithium niobate is known to
be between 30-120 MPa [30]. The optimal rotation angles were determined by taking
the ratio of the peak voltage induced in the aperture ($V_{\text{peak}}$) to the maximum von
Mises stress generated in the crystal ($T_{\text{vonMises}}$). The ratio $V_{\text{peak}}/T_{\text{vonMises}}$ determines
the maximum generated voltage with the least chance of exceeding the yield stress of
the material.

The eigenfrequency simulations yielded two dominant resonance modes for the
cylindrical disks. The two modes are shown in Figure 3.2. The first resonance
mode resulted in a radial displacement and occurred at frequencies around 385 kHz,
dependent on rotation angle. The second resonance mode resulted in an elliptical
displacement where the disk elongates then compresses in the direction of the rotated
polarization and occurred at frequencies near 280 kHz.

Both resonance modes produced a high output voltage at the crystal surface within
the aperture of the PTPS, however the optimal rotations for each mode were different.
The results of the rotation angle parameter sweep simulations for each mode are
shown in Figure 3.3. The radial mode resonance has optimal rotation angles of
$\beta = 45^\circ, \gamma = 0^\circ$ or $\beta = 135^\circ, \gamma = 60^\circ$. Also, as seen at $\beta = 45^\circ, \gamma = 120^\circ$, $\gamma$ can
be incremented by $120^\circ$ and achieve the same results by the trigonal symmetry of
lithium niobate. The elliptical mode has optimal rotation angles of $\beta = 45^\circ, \gamma = 60^\circ$
or $\beta = 135^\circ, \gamma = 0^\circ$. For a given resonance mode, all of the possible combinations
of optimal rotation angles can be represented with a single disk. The only difference between a disk with $\beta = 45^\circ$ or $\beta = 135^\circ$ is that the driven electrode and aperture electrode are on opposite sides.

The simulations yielded peak output voltage to von Mises stress ratios of 59 V/MPa for the radial mode and 50 V/MPa for the elliptical mode. The maximum von Mises yield stress for LiNbO$_3$ has been reported to be between 30-120 MPa [30]. As a result the maximum output voltage of the radial mode can be expected to be 1.7-7 kV and the maximum output voltage of the elliptical mode can be expected to be 1.5-6 kV. These ranges agree with charged particle emission measurements, detailed in chapter 5, that indicate peak voltages of approximately 4 kV.

These simulation results were used to order samples optimized for each resonance mode. These samples were then tested to verify the resonance frequencies predicted from simulation. Figure 3.4 shows a typical measured transformer ratio for disks optimized for the elliptical mode and radial mode. The transformation ratios were measured on samples with an output electrode on the disk surface at the center of the aperture. The input and output voltage for the samples were measured with a
Figure 3.3: Ratio of PT output voltage to von Mises stress for (a) radial and (b) elliptical mode piezoelectric transformers.
Figure 3.4: Measured transformer ratio for LiNbO$_3$ disk optimized for the elliptical ($\beta = 45^\circ, \gamma = 60^\circ$) and radial ($\beta = 45^\circ, \gamma = 0^\circ$) mode.

Tektronix p5100a high voltage probe with a 40 MΩ, 1.5 pF impedance. Both samples performed as expected from the simulation results, the elliptical mode resonance occurred near 280 kHz while the radial mode resonance occurred near 385 kHz. Each sample demonstrated a large transformer ratio near the resonance frequency. Separate simulations investigated the effect of the probe impedance on the output voltage of the piezoelectric transformer. A result from those simulations is shown in Figure 3.5 and indicates that the actual transformer ratio for these samples may be almost twice the measured transformer ratio due to the presence of the high voltage probe.

### 3.2 Mechanical Quality Factor

The mechanical quality factor of a piezoelectric resonator can be estimated from input impedance measurements. Using an equivalent circuit analysis, Ikeda derived a method of estimating the mechanical quality factor from input impedance measurements near the resonant frequency [7]. Equation (3.2) shows the relationship,
Figure 3.5: Simulated transformer ratio for a radial mode LiNbO$_3$ disk with and without a Tektronix p5100a high voltage probe load modeled at the transformer output.

where $Q_m$ is the mechanical quality factor, $\omega_r$ and $\omega_a$ are the resonance and anti-resonance frequencies, and $Z_{max}$ and $Z_{min}$ are the maximum and minimum input impedances of the resonator [7].

$$Q_m \approx \frac{\omega_r}{2(\omega_a - \omega_r)} \left( \frac{1 + |Z_{max}|}{|Z_{min}|} \right) \sqrt{\frac{|Z_{max}|}{|Z_{min}|}}$$  \hspace{1cm} (3.2)

Equation (3.2) is an estimate of the mechanical quality factor that is more accurate for large values of the ratio of $Z_{max}$ to $Z_{min}$ or resonators with large mechanical quality factors. Ikeda also proposed a correction to equation (3.2) to improve the estimate for lower quality factors. For the measurements presented, both methods gave nearly identical results.

Input impedance measurements and estimation of the mechanical quality was used to investigate how the mechanical restriction resulting from assembling the PTPS affects the mechanical quality factor. Two main factors that affect the mechanical
quality are compression of the disk between the driven and aperture electrodes and the conductive material used to maintain good electrical contact between the PT and the electrodes.

The PT needs to be held in place between the two electrodes and was accomplished by compressing the disk through the application of torque to bolts that secure the PTPS together. The mechanical quality factor was estimated for three different amounts of compression. Figure 3.6 shows the measured input impedance curves. The calculated mechanical quality factors for the low, medium, and high compression were 152, 70, and 30, respectively. COMSOL simulation results show the transformer ratio dependence on the mechanical quality factor in Figure 3.7. While the applied compression force was not measured and can only be described qualitatively, the results show the sensitivity of the mechanical quality and transformer ratio for different amounts of compression. The high, medium, and low compression were varied only through 1/2 turns of a 0-80 bolt. In practice, the medium compression force was a good balance between minimizing the mechanical losses while holding the lithium niobate in place. At the low compression force the lithium niobate disk would occasionally vibrate out of position.

The electrical contact between the electrodes painted onto the PT and the solid electrodes was initially made by using an additional small amount of silver paint. Using this method the plasma formation was observed to be unreliable, requiring higher voltages or no longer occurring, after several hours. The unreliable plasma formation was attributed to a reduced piezoelectric transformer effect from the mechanical clamping of the drying and hardening silver paint [2]. The clamping effect was verified by measuring the input impedance of the crystal one hour after affixing the disk to the solid electrodes and remeasuring after the paint was allowed to dry for twelve hours. The experiment was performed for both a radial mode and
Figure 3.6: PTPS input impedance near resonance for varying amounts of disk compression. Mechanical quality decreases as compression increases. Low, medium, and high clamping refer to a mechanical quality factor of 152, 70, and 30, respectively.

Figure 3.7: PTPS transformer ratio near resonance for varying amounts of disk compression. Low, medium, and high clamping refer to a mechanical quality factor of 152, 70, and 30, respectively.
an elliptical mode PT.

The results for a radial mode PT shown in Figure 3.8a indicate a weaker resonance after the paint was allowed to dry overnight. Using equation (3.2), the mechanical quality factor reduced from 97 after one hour to 62 after drying twelve hours.

Results for the elliptical mode PT, shown in Figure 3.9a, show a much stronger clamping effect. The mechanical quality factor was reduced from 58 after one hour to 18 after twelve hours. The much larger change is attributed to the fact that an elliptical mode PT has two null displacement points, compared to the radial mode which has a single null displacement point.

In an effort to reduce mechanical clamping, silver grease was used to make electrical contact between the crystal and solid electrodes. Similar experiments with both the radial and elliptical mode transformers revealed that the input impedance was nearly unchanged between one and twelve hours after construction. Input impedance measurements for the radial mode and elliptical mode transformers are shown in Figure 3.8b and Figure 3.9b, respectively. The mechanical quality factor for the radial mode transformer remained nearly unchanged, 75 after one hour and 77 after twelve hours. The mechanical quality factor for the elliptical mode transformer was calculated to be 47 after one hour and 53 after twelve hours.

Due to the large effect that the compression has on the mechanical quality factor it is difficult to compare the mechanical quality factor if the PTPS is disassembled and reassembled. To eliminate this effect on the mechanical quality factor, the PTPS was not disassembled between the measurement at one hour and at twelve hours after construction. Obviously, the PTPS had to be reassembled when using a different disk (radial or elliptical) or material (paint or grease). Therefore only the relative change in the mechanical quality factor should be compared from this experiment. Attempting to compare the absolute value of the mechanical quality factor between
Figure 3.8: PTPS input impedance near resonance when (a) silver paint and (b) silver grease was used to affix a radial mode ($\beta = 45^\circ$, $\gamma = 0^\circ$) LiNbO$_3$ disk to metal electrodes.
Figure 3.9: PTPS input impedance near resonance when (a) silver paint and (b) silver grease was used to affix an elliptical mode ($\beta = 45^\circ$, $\gamma = 45^\circ$) LiNbO$_3$ disk to metal electrodes.
the elliptical and radial mode, for example, is likely influenced by the assembling of
the PTPS.

3.3 Aperture Offset

Another consequence of the off-axis polarization required in lithium niobate for
efficient piezoelectric transformer operation is that the cylindrical disk does not
deform in a uniform radial motion. Figure 3.10a shows a cross section of the
displacement magnitude for a 45° rotated Z-cut (radial mode) crystal. It can be
seen that the displacement is not uniform in the radial dimension, and that the null
displacement line through the crystal is rotated in the direction of the polarization
(z) axis. The non-radial displacement leads to an electric potential profile as shown
in Figure 3.10b.

The lack of symmetry in the aperture of the PTPS alters the PT behavior in
two ways. First, the transformer ratio of the piezoelectric transformer is reduced.
If the aperture is offset in the y-z plane of the crystal, the transformer ratio can be
improved as shown in Figure 3.11. Second, as the aperture is offset the electric
potential profile in the aperture becomes symmetric as shown in photographs of
the discharge at atmospheric pressure in Figure 3.12. The PT was driven with an
applied voltage amplitude of approximately 200 V near the resonance frequency of
385 kHz. At atmospheric pressure the discharge remains near the triple point of
the electrode/dielectric/air boundary and near the surface of the dielectric. The
photographs show a non-uniform discharge, originating only from one side of the
aperture when the aperture was centered on the disk. As the aperture was offset away
from the polarization axis, the discharges become uniformly distributed around the
aperture. Discharges occurring uniformly around the aperture may help to increase
the plasma source lifetime by reducing degradation of the dielectric surface.
Figure 3.10: Simulated (a) displacement in the y-z plane and (b) electric potential at the aperture surface of a 45°rotated Z-cut crystal.
Figure 3.11: PT transformation ratio versus aperture location. The aperture is offset in the y-z plane of the crystal as shown in Figure 3.10a.

Figure 3.12: Open-shutter pictures of discharge formation at atmospheric pressure for three aperture locations.
The high voltage developed in the aperture both creates plasma through ionization of the background gas and accelerates charged particles away from the surface. The aperture offset may also affect the trajectory of the charged particles as they are accelerated from the device. Figure 3.13 presents the results of a particle trajectory calculation for ions near the surface of the crystal. The particle trajectory plot in COMSOL determines the trajectory of a test particle with a specified mass and charge that is placed within the simulation space, according to equation (3.3), where $m$ is the particle mass, $\ddot{x}$ is the acceleration, and $F$ is the net force applied to the particle.

\[
m\ddot{x} = F(t, x, \dot{x}) \tag{3.3}
\]

In the case of an electrostatic simulation, the force is the product of electric field and charge, $F = qE$. The particle trajectory plots shown in Figure 3.13 were calculated for a static electric field as calculated at a given phase of the simulation solution. In Figure 3.13a, the aperture is centered on the piezoelectric disk and the non-uniform electric potential results in a particle flux from the surface with a net component tangent to the surface of the disk. The aperture in Figure 3.13b is offset towards the edge of the disk resulting in a more uniform electric potential in the aperture. The particle flux from the surface here is more uniform and directed along the surface normal.
Figure 3.13: Particle trace simulations of ions accelerated from disk surface for an aperture offset of (a) 0 mm and (b) 2 mm.
Chapter 4

PTPS Operation

The PTPS is capable of operating at a wide range of pressures from atmospheric pressure [24] to high vacuum [2]. This chapter details the equipment and diagnostics used to operate and characterize the PTPS. It will also cover preliminary experiments testing the PTPS over a range of background pressures.

4.1 Experiment Setup

4.1.1 Driving the PTPS

The basic setup used to operate the PTPS is shown in Figure 4.1. The plasma source was driven with an RF voltage, near the resonant frequency of piezoelectric disk. The RF voltage was applied in a burst mode with a low duty cycle. Typical

Figure 4.1: Diagram of basic experiment setup for a PTPS.
bursts include 100-3000 cycles with a period of 0.1-1 second, corresponding to duty cycles of under 10%. Continuous, or near continuous, driving of the PT resulted in significant heating of the piezoelectric disk, eventually leading to the disk breaking. The RF driving signal was generated by an Agilent 33210A waveform generator. The output of the waveform generator was typically 200-400 mV peak-to-peak. The waveform generator output signal was input into an Amplifier Research 25A250A RF amplifier with variable gain to boost the power of the driving signal. The output of the RF amplifier was fed into the primary of a toroidal, iron powder core, 1:6 step-up transformer used to improve the impedance match between the 50 Ω output impedance of the amplifier and the larger input impedance of the plasma source. The input voltage and current waveforms of the plasma source were measured on the secondary side of the matching transformer with a Tektronix P5100A (100X) high-voltage probe and a Pearson 2877 (1 V/A) current monitor.

Most experiments required a more complicated experiment setup that included operating in a vacuum chamber, floating the plasma source, or operating the plasma source with a gas feed. A diagram of an experiment setup for operating a floating plasma source inside the vacuum chamber is shown in Figure 4.2. The equipment used was the same as was listed for the basic setup. Additional complication comes from electrically isolating the plasma source from the vacuum chamber by removing the ground on the secondary side of the impedance matching transformer. The electrical isolation allowed the plasma source to float to a potential based on the charge transfer from the source. Experiments with a floating plasma source used a second Tektronix P5100A high-voltage probe to monitor the reference voltage. When the plasma source was floating, the input voltage to the PTPS was either calculated as the difference of the driving voltage and reference voltage or measured directly with a Tektronix P5200A (50X/500X) differential probe.
Operating the plasma source with a gas flow required a stacked electrode arrangement to create a local high pressure volume near the surface of the piezoelectric disk. The electrode arrangement is shown in Figure 4.3. The stacked electrodes had an 8 mm aperture diameter. The electrodes were separated with two annular dielectric layers. The layers had a hole bored in one side to allow a gas flow into the aperture and served to prevent gas pressure from flowing out the edges of the PTPS. The aperture diameter of the outermost aperture electrode was varied in diameter from <1 mm to 8 mm. A thin mesh grid with 30% open area fraction was also placed at the aperture to reduce the penetration of the electric field lines outside the PTPS. The gas flow was controlled by a Celerity 7300 mass flow controller with a maximum $N_2$ flow rate of $0.05 \text{ Pa} \cdot \text{m}^3 \cdot \text{s}^{-1}$ (30 sccm) and a maximum Ar flow rate of $0.071 \text{ Pa} \cdot \text{m}^3 \cdot \text{s}^{-1}$ (42 sccm). Gas pressure within the aperture volume was controlled based on the gas
Figure 4.3: Stacked aperture electrode configuration for PTPS operation with a gas flow rate and the output aperture diameter.

4.1.2 Diagnostics

Diagnostics used with the PTPS included imaging with a photomultiplier tube (PMT), gated images with an intensified CCD camera, optical spectroscopy and charged particle emission measurements using a Faraday cup and a retarding potential analyzer.

The voltage and current diagnostics already mentioned are summarized in Figure 4.4. This figure details the nomenclature for any measured potentials on the PTPS. $V_{\text{applied}}$ is the output of the impedance matching transformer. When the plasma source was grounded $V_{\text{applied}}$ was equal to $V_{\text{drv}}$. If the plasma source was floating, $V_{\text{applied}}$ was measured as the potential difference of $V_{\text{drv}}$ and $V_{\text{ref}}$, and $V_{\text{ref}}$ was the floating potential of the source. $V_{\text{pt}}$ was only measured in a few experiments and is the output voltage of the PT measured with respect to ground. The PT output voltage was measured by placing an electrode near the center of the aperture on the surface of the disk and measuring the voltage with a high-voltage probe.
Optical diagnostics were used to observe light formation and image the discharge. Time-resolved light emission was measured with a Hamamatsu 1P28A photomultiplier tube (PMT). The PMT has a detection range of 185-700 nm and a rise time of 2.2 ns. Open-shutter and gated images were captured with a PI-MAX ICCD camera. Optical spectroscopy was done with an Acton SpectraPro 2300i 0.3 m spectrograph. The PI-MAX ICCD camera was used to image the output slit of the spectrograph.

Charged particle emission from the FEPT was detected with a Faraday cup and retarding potential analyzer. The diagram in Figure 4.5 shows the Faraday cup and four grids (G1, G2, G3, and G4) that formed the retarding potential analyzer. Current emission measurements were recorded on an oscilloscope using a 1000 Ω current viewing resistor in the path to ground from the inner cup. The retarding potential analyzer grids were configured as follows; Grid G1 was grounded, G2 had a large negative bias voltage applied to screen out electron charge with energies less than the bias voltage, G3 was only occasionally used and had a positive bias voltage to screen out ion charge, and G4 had a small negative bias voltage to suppress secondary

\[
V_{\text{applied}} = V_{\text{drv}} - V_{\text{ref}}
\]

Figure 4.4: Nomenclature for measured potentials on the PTPS.
electron emission from the Faraday cup.

When using the Faraday cup to measure emission current, two correction factors need to be considered. One correction factor accounts for emission current collected on the copper grids placed in front of the Faraday cup. The open area fraction of a single grid was 0.68. The total open area fraction for three grids was $0.68^3 = 0.314$ and for four grids was $0.68^4 = 0.214$. The second correction factor accounts for divergence in the charged particle beam that prevents the Faraday cup from collecting all the emitted current. The percent of the beam collected is given by the solid angle, $\Omega$,

$$\Omega = \frac{\int_0^{2\pi} \int_0^{\theta_c} F(\theta) \sin \theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi/2} F(\theta) \sin \theta \, d\theta \, d\phi}$$  \hspace{1cm}(4.1)$$

where $\theta$ is the inclination angle and $\phi$ is the azimuth angle in spherical coordinates, $F(\theta)$ is a distribution function describing the beam profile, and $\theta_c$ is the collection cone angle between the source and Faraday cup. The collection cone angle, is defined by the radius of the Faraday cup, $r_{cup}$, and the distance between the source and Faraday cup, $z$.

$$\theta_c = \tan^{-1}\left(\frac{r_{cup}}{z}\right)$$  \hspace{1cm}(4.2)
Table 4.1: Summary of solid angle correction factor data.

<table>
<thead>
<tr>
<th>Distance</th>
<th>$\cos^2 \theta$</th>
<th>$I_{raw}$ (mA)</th>
<th>$I_{cor}$ (mA)</th>
<th>$I_{raw}$ (mA)</th>
<th>$I_{cor}$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 mm</td>
<td>0.555</td>
<td>0.0422±0.0062</td>
<td>0.076±0.0137</td>
<td>0.1336±0.0098</td>
<td>0.2408±0.018</td>
</tr>
<tr>
<td>47 mm</td>
<td>0.319</td>
<td>0.0243±0.0039</td>
<td>0.076±0.0238</td>
<td>0.0870±0.0110</td>
<td>0.2729±0.034</td>
</tr>
</tbody>
</table>

The results from an experiment detecting electron and ion current at various distances was used to determine a suitable distribution function to correct for divergence in the beam. At each distance the source was operating under similar conditions and it was assumed that the true emitted current at both distances was equal. Corrections factors for several distributions, isotropic, $\cos \theta$, and $\cos^2 \theta$ were applied to the data. A $\cos^2 \theta$ distribution was found to be the best fit for the data, resulting in corrected ion and electron currents that were nearly equal. The data is summarized in Table 4.1.

4.2 Plasma Formation Dependence on Background Pressure

Plasma formation with the PTPS varied as the background gas pressure was varied. The FEPT was originally characterized at a high vacuum pressures of $10^{-6}$ Torr. Operation at these pressures would make the FEPT ideal for deep space propulsion, especially if an additional propellant was not needed. However it was often difficult to produce reliable discharges at these pressures because the plasma is formed by a surface discharge. Imaging showed that the discharges appeared local to the surface. Optical and mass spectroscopy revealed that the ion beam was mostly composed of silver, lithium, and niobium ions, materials at the surface of the PT [2].

At higher pressures, approximately 100-2000 mTorr, the discharge transitioned to a glow discharge and appeared to occupy a volume above the surface of the PT. At
Figure 4.6: Minimum applied voltage necessary to generate plasma versus background pressure for three PTPS samples.

These pressures the plasma is formed though ionization of the background gas and the plasma formation is very reliable, occurring during every applied RF burst. These levels of background pressure correspond to altitudes of 130,000-200,000 ft meaning the PTPS could have near-space propulsion applications without an additional gas propellant [31]. Propulsion at higher altitudes would require a gas flow propellant to maintain appropriate gas pressure.

Several preliminary experiments were completed to evaluate the plasma formation for a range of pressures applicable to near space propulsion. The PMT was used to determine the applied voltage necessary for plasma creation over a range of pressures. Three PTPS samples were tested and a similar response was seen in all three samples. The results shown in Figure 4.6 reveal a Paschen law relation in all three tested configurations. As the background pressure dropped below 500 mTorr, larger applied voltages (and corresponding larger PT output voltages) were needed for plasma formation.
In addition to the PMT diagnostic, the ICCD diagnostic was used to image the plasma creation. The ICCD camera was used to observe the plasma creation throughout an applied RF cycle. As shown in Figure 4.7, the applied RF cycle was broken into ten frames. Each frame was an accumulation of ten images taken during sequential RF bursts.

The images in Figures 4.7a and 4.7b reveal that plasma emission occurs twice during each RF cycle, once during the positive half cycle and once during the negative half cycle. It can be seen in both cases that the plasma emission tends to occur after the maximum and minimum points of the input voltage waveform. The plasma emission timing indicates the importance of the piezoelectric transformer effect because the PT output voltage lags the PT input voltage.

As pressure was reduced a transition in the plasma formation takes place. At 360 mTorr a large, diffuse glow discharge was formed. As the pressure was reduced, a transition to an intense, localized vacuum arc discharge took place. The plasma formation pressure dependence can be explained by evaluating the particle interactions within the volume of the discharge. As an example, the electron-neutral collisions for $N_2$ were examined. Two particle collision processes important for plasma formation are ionization and momentum transfer. These interactions can be described by two parameters, the effective cross-section, $\sigma$, and mean free path, $\lambda$. These parameters are related by the gas density, $n$, as shown in equation (4.3).

$$\lambda = \frac{1}{n\sigma} \tag{4.3}$$

Qualitatively, when the ionization cross-section is large, collisions between the electrons and $N_2$ are likely to produce $N_2$ ions. When the momentum transfer cross section is large, electrons are likely to lose a significant amount of energy each collision. When ionization and momentum transfer are both likely to occur, electrons collide, lose energy, then begin to gain energy from an applied electric field to collide again. As
Figure 4.7: Gated images of plasma formation at background pressures of (a) 360 mTorr and (b) 85 mTorr during a single RF cycle. CCD gating is shown in (c).
this process is repeated, the background gas rapidly ionizes, resulting in the formation of plasma.

The ionization and momentum transfer cross sections for electron-N\textsubscript{2} collisions, tabulated in [32], are shown in Figure 4.8a. The corresponding calculated mean free paths for four pressures are shown in Figure 4.8b. In order for a collision to take place in the aperture of the PTPS, the mean free path should be less than approximately 1 cm, the scale length of the device. According to Figure 4.8b, this occurs at background pressures of approximately 100 mTorr or greater, in agreement with the observations from discharge imaging.
Figure 4.8: Ionization and momentum transfer collision (a) cross-section and (b) mean free path for electron-N$_2$ interactions. Ionization is shown with a solid line and momentum transfer with a dashed line.
Chapter 5

Plasma Source Characterization

Further experiments on the PTPS focused on operation at gas pressures between 100-2000 mTorr. Operation at these pressures produced very reliable discharges though the ionization of the background gas. Experiments were performed using a gas flow to create a local high pressure volume near the surface of the PT, while a base pressure of $10^{-5}$-$10^{-6}$ Torr was maintained in the remainder of the vacuum chamber. The low base pressure prevented excess ionization from occurring outside of the plasma source, in the volume between the PTPS and the Faraday cup, and resulted in more accurate measurements of emission current from the source.

5.1 Charged Particle Emission

Charged particle emission current measurements with the Faraday cup and retarding potential analyzer demonstrated both electron and ion emission from the PTPS during a single RF cycle. A sample measurement is shown in Figure 5.1. Electron emission was measured as negative emission current and the ion emission was measured as positive emission current. To clearly observe both ion and electron emission, a negative bias voltage was applied to grid G2 of the retarding potential analyzer. Without the bias voltage, the relatively slow ion current was still being collected as electron current emitted during the the subsequent half-cycle was reaching
Figure 5.1: Electron and ion emission from a PTPS measured with a Faraday Cup. A 
−2 kV bias voltage was used on grid G2 of the retarding potential analyzer to screen 
out a portion of the electron current.

the Faraday cup. The ion current could not be resolved if the electron current was 
unattenuated.

5.1.1 Electron Energy

The retarding potential analyzer was used to determine the energy distributions 
of the emitted electron and ion currents. The energy distributions were determined 
using a PTPS with a stacked electrode configuration. The output aperture was 2 mm 
in diameter covered with a 30% open area grid. A nitrogen gas flow at 0.05 Pa·m$^3$·s$^{-1}$ 
(30 sccm) was used and the PTPS was driven with a 430 V amplitude driving signal.

For electron energy measurements, the emission current was measured while a 
negative bias voltage, $V_{G2bias}$, was applied to grid G2 of the retarding potential
Figure 5.2: Measured emission current for (a) $-260$ V, (b) $-1.4$ kV, (c) $-2.2$ kV, (d) $-3$ kV, and (e) $-3.4$ kV, applied to grid G2 of the retarding potential analyzer. Additional bias voltages used are omitted from this figure for clarity. Applied voltage and input current are scaled to the primary y-axis and the five emission current traces are scaled to the secondary y-axis.

Analyzer. The negative bias voltage prevented electrons emitted with energies (in eV) less than $V_{G2\text{bias}}$ from being collected at the Faraday cup. A sample of measured electron currents from a PTPS with a 430 V amplitude driving signal is shown in Figure 5.2. The electron energy distribution was calculated by integrating the measured currents with respect to time to obtain the total charge emitted with energies greater than or equal to the bias voltage, then subtracting to determine the charge per energy bin. For example, the area between lines (c) and (d) in Figure 5.2 represents the electron charge emitted with energy between 2.2-3 keV.

The resulting electron energy distribution is shown in Figure 5.3. It shows a wide distribution of energies, with a significant peak between 2.6 and 3 keV. The peak electron energies, shown in Figure 5.4, were measured to be near 4 keV and were dependent on the gas flow rate. The electron energy measurements demonstrate the importance of the piezoelectric transformer effect in the PTPS operation. The
The majority of emitted electrons had an energy greater than the applied voltage and the peak electron energies were up to eight times larger than the applied voltage. The electron energy dependence on flow rate could be explained by a loading effect on the output of the piezoelectric transformer. The plasma formed at higher background gas pressures presents a greater load to the output of the transformer reducing the voltage gain and accelerating potential of the PTPS.

5.1.2 Ion Energy

The ion energy was measured using two different methods. The first method used the retarding potential analyzer, similar to the electron energy measurement, to screen out ions with energy less than an applied potential. The second method measured the ion velocity to determine the kinetic energy of the ions. The kinetic energy was calculated from

$$ E = \frac{1}{2} mv^2 $$

where $E$ is the ion energy, $m$ is the ion mass, and $v$ is the ion velocity. In order to determine the kinetic energy the ion mass needs to be known. Using either method,
the charge state of the ion needs to be known to estimate the accelerating potential. The accelerating potential of the ion, $V$, is given by,

$$V = \frac{U}{q} \quad (5.2)$$

where $U$ is the energy of the ion and $q$ is the ion charge.

Both experiments used an $N_2$ gas flow and it was assumed the beam consisted only of $N_2^+$ ions. This assumption was made based on the ionization cross section data for electron-$N_2$ collisions shown in Figure 5.5. The ionization cross-sections for $N_2^+$ and $N^+$ are larger than $N_2^{++}$ by two orders of magnitude so it was assumed that all ions were singly ionized. The cross-section for $N_2^+$ is 3-5 times larger than $N^+$. While it is likely that $N^+$ ions were present in small quantities, the calculated kinetic energy would not change significantly when assuming an ion mass of 28 amu for an $N_2^+$ ion beam versus using a weighted average of the two ion species masses.

The ion energy distribution was measured using the retarding potential analyzer with a fourth grid. In order to resolve the ion current, all electron current was first
Figure 5.5: $N_2^+$, $N^+$, and $N_2^{++}$ ionization cross-sections for electron-$N_2$ collisions.

screened out using a large negative potential on grid G2, then the ion current was filtered by applying a positive potential to grid G3. A sample of measured ion currents is shown in Figure 5.6. The resulting ion energy distribution is shown in Figure 5.7. The measured ion energies were unexpectedly low, with over 30% of the ion current less than 400 eV. The peak ion energy was near 650 eV.

The second ion energy calculation required measuring the ion transit time. The ion transit time was defined as the time between a “blip” seen on the input current signal, indicating current emission from the plasma source, and detection of the positive emission current at the Faraday cup. The ion transit time is shown in Figure 5.8a. A separate signal from a photomultiplier tube (PMT), shown in Figure 5.8b, was used to confirm that the plasma discharge began near the “blip” seen in the ion current. The PMT signal indicates ion emission as a shorter, more intense discharge and electron emission as a longer, more diffuse discharge.

Ion transit times were measured with varying distances between the PTPS and the Faraday cup. The results are shown in Figure 5.9. The mean ion transit time for each
Figure 5.6: Measured emission current for (a) 180 V, (b) 290 V, and (c) 330 V, applied to grid G3 of the retarding potential analyzer. Additional bias voltages used are omitted from this figure for clarity. Applied voltage and input current are scaled to the primary y-axis and the three emission current traces are scaled to the secondary y-axis.

Figure 5.7: Ion energy distribution for PTPS with 430 V applied voltage.
Figure 5.8: Measurement of the ion transit time.
distance is represented by a point and the error bars indicate the range of measured times. The fit line is a linear regression with the y-intercept set equal to zero. The inverse of the slope of the fit line represents the ion velocity. Using equation (5.1) and a mass of 28 amu for a $N_2^+$ ion, the peak ion energy was determined to be 734 eV. If it was assumed the beam was also composed of $N^+$ ions, the peak energy would be slightly reduced.

The ion energies determined from the Faraday cup and retarding potential analyzer were in good agreement with the peak ion energy determined from the time of flight measurement. The ion energies were lower than expected, but peak ion energies were still greater than the applied voltage to the PT.
5.2 Charged Particle Accelerating Voltage

The output voltage of the piezoelectric transformer ($V_{pt}$) provides the accelerating potential for charged particle emission. To investigate the difference in the measured electron and ion energies, the output voltage of the piezoelectric transformer was directly measured by placing an electrode on the surface of the disk and using a Tektronix P5100A high voltage probe with a 40 MΩ, 1.5 pF impedance. Although the addition of the electrode and voltage probe reduced the gain of the transformer due to additional loading, plasma formation and charged particle acceleration still occurred allowing direct measurement of the PT output voltage during plasma emission.

A typical result of the piezoelectric transformer output voltage diagnostic is shown in Figure 5.10. Plasma formation in the aperture of the PTPS occurs approximately 125 μs into the applied RF burst. Prior to plasma formation, $V_{pt}$ increases as the lithium niobate disk begins to resonate. When the output voltage is high enough, plasma formation occurs and $V_{pt}$ biases negatively. The piezoelectric transformer output voltage remains biased negatively until the end of the applied RF burst. While the AC magnitude of $V_{pt}$ remains much greater than $V_{applied}$, the negative bias of $V_{pt}$ reduces the maximum ion accelerating potential and increases the electron accelerating potential. The negative bias of $V_{pt}$ will have a large impact on the thrust capabilities of the PTPS when operating with a gas flow due to the reduced ion energies.

5.2.1 PT Output Voltage Bias Theory

The bias voltage at the PT output can be explained by examining RF-driven capacitively coupled discharges. When an RF-driven capacitive discharge is generated between two electrodes with unequal electrode areas, a DC self-bias voltage develops at the driven electrode [33, 34]. These types of discharges are often used in sputtering
Figure 5.10: Negative bias of piezoelectric transformer output voltage during plasma formation.

applications where one of the discharge electrodes contains a sputtering target. When the plasma is formed, ions from the plasma are accelerated through the plasma sheath and bombard the target. The energy of the ion at impact is dependent on the difference between the plasma potential and the potential at the target electrode. In order to increase the ion energy to the target and decrease ion energy to the walls of the system, electrodes with unequal areas are used to develop a bias voltage [35].

The self-bias of the driven electrode can be qualitatively explained by analyzing the current from the plasma to the driven electrode, similar to a floating plasma probe. A simple circuit model for an asymmetric discharge is shown in Figure 5.11 and a sample voltage waveform for the biased electrode, $V_b$, is shown in Figure 5.12.

The capacitance, $C$, can be a simple blocking capacitor in the driving circuit or the capacitance of an insulator covering one of the electrodes. The capacitor prevents a net current from flowing in the circuit. If the net current in the circuit is zero, then the total current to the biased electrode in one period also has to be zero. During time $t_1$ the driven electrode potential is positive and electron current flows from the
Figure 5.11: Circuit schematic of an asymmetric capacitively coupled discharge.

Figure 5.12: Sample voltage waveform for a negatively biased electrode in an asymmetric capacitively coupled discharge.
plasma to the electrode. During time $t_2$ ion current flows to the electrode. Since the ions are heavy and move relatively slow compared to the electrons, $t_2$ must be greater than $t_1$ to result in zero net current to the electrode.

To more accurately model the behavior of the RF discharge, the characteristics of the plasma sheath need to be considered. The plasma sheath behavior depends largely on the operating frequency, at high frequencies the sheath looks capacitive and at low frequencies the sheath looks resistive. High and low frequencies are defined by the plasma ion frequency, $\omega_{pi}$. If the driving frequency is less than the plasma ion frequency ($\omega < \omega_{pi}$) then the discharge is low frequency. If the driving frequency is greater than the plasma ion frequency ($\omega > \omega_{pi}$) then the discharge is high frequency. Many equivalent circuit models have been developed to account for the varying behavior based on frequency. The plasma sheath has been modeled as a parallel diode/capacitor [36], a parallel diode/capacitor/resistor [37], a parallel capacitor/resistor [38, 39], or only a capacitor or resistor [39, 40]. The most developed models are based on capacitive plasma sheaths because many plasma processing applications occur at 13.56 MHz, where a capacitive model or combination model is most applicable.

To adapt the analysis to the PTPS, the appropriate model has to be determined. The plasma ion frequency, $\omega_{pi}$, for the PTPS can be calculated from,

$$\omega_{pi} = \left(\frac{ne^2}{m_i \epsilon_0}\right)^{\frac{1}{2}}$$

(5.3)

where $n$ is the plasma density, $e$ is the charge of an electron, and $m_i$ is the ion mass. A lower bound for the plasma density in the PTPS is estimated to be on the order of $10^{15}$ m$^{-3}$. The calculated lower bound for the plasma ion frequency is 1.2 MHz for nitrogen and 1 MHz for argon. A resistive sheath model is applicable to the PTPS since the 400 kHz driving frequency is less than the plasma ion frequency.
Several approaches have been used to determine the self-bias voltage. In [40], the bias voltage, \( V_{DC} \), as a function of the applied voltage, \( V_{RF} \), and electrode areas was obtained by equating the ion current to the driven electrode during time \( t_2 \) (from Figure 5.12) to the ion current to the grounded electrode during time \( t_1 \). Assuming the ion current does not have a voltage dependence, i.e. the ion current is equal to the Bohm current, the result is shown in equation (5.4). The area of the electrodes are defined as \( A_t \) for the driven electrode and \( A_w \) for the grounded electrode.

\[
-V_{DC} = V_{RF} \cos \left( \pi \frac{A_t}{A_w + A_t} \right)
\]

This analysis is simple, in that the bias voltage is dependent only on the electrode areas and applied voltage. However it is not straightforward to apply to the PTPS because the electrode areas for the PTPS are not clearly defined. The analysis assumes a planar discharge where the plasma density, and therefore ion current density, is likely to be uniform across both electrodes. The PTPS has a radial discharge and it is not likely that the plasma density is uniform within the aperture volume. Measurements on a Grimm-type glow discharge plasma source, which has a similar radial geometry, show that the plasma density can decrease several orders of magnitude as the distance increases from the driven electrode [41].

A second analysis equates the electron current and ion current in a single RF cycle to the driven electrode. The electron current density, \( j_e \), to the driven electrode can be described using Boltzmann’s law,

\[
j_e = \frac{en\bar{v}_e}{4} \exp \left( \frac{-e(V_p - V_b)}{kT_e} \right)
\]

where \( \bar{v}_e \) is the average electron velocity given by,

\[
\bar{v}_e = \left( \frac{8kT_e}{\pi m} \right)^{1/2}
\]

In equations (5.5) and (5.6), \( e \) is the electron charge, \( n \) is the plasma density, \( V_p \) is the plasma potential, \( V_b \) is the potential of the driven electrode, \( k \) is Boltzmann’s
Figure 5.13: Plasma potential, $V_p(t)$, and voltage on the driven electrode, $V_b(t)$, in a low frequency RF discharge.

constant, $T_e$ is the electron temperature, and $m$ is the mass of an electron. The ion current density, $j_i$, is given by the Bohm current,

$$j_i = 0.61en \left( \frac{kT_e}{M} \right)^{1/2}$$

where $M$ is the ion mass.

By limiting the current balance to a single electrode, the potential issue of non-uniform plasma densities is removed. However, this approach is difficult analytically for a low frequency RF discharge because the plasma potential is non-sinusoidal as shown in Figure 5.13. In the case of a low frequency discharge the plasma potential is assumed to be constant with respect to the higher potential electrode, because the plasma ion frequency is greater than the driving frequency and the plasma ions can effectively respond to the applied RF field instantaneously. The resulting potential difference from the plasma to the driven electrode ($V_p(t) - V_b(t)$) is shown in Figure 5.14. The potential difference shown in Figure 5.14 controls the electron current to the
Figure 5.14: Retarding potential, $V_p(t)-V_b(t)$, for electron current from the plasma to the driven electrode.

driven electrode, as described in equation (5.5). During time $t_1$, the electron current to the electrode is limited only by the plasma potential. During time $t_2$ the electron current is effectively cut off due to the large potential difference between the plasma and a driven electrode. The ion current has no dependence on the plasma potential and is assumed to be constant throughout the RF cycle.

A simplified analysis can be performed using the assumption that the electron current is zero during time $t_2$ to determine the bias voltage in a low frequency RF discharge. The total electron charge, $Q_e$, in one RF cycle is given by,

$$Q_e = A \int_{t_1} j_e \, dt = j_e A t_1$$

where $A$ is the electrode area. The ion charge $Q_i$ is given by,

$$Q_i = A \int_{0}^{2\pi} j_i \, dt = j_i A (2\pi)$$

Equating the electron and ion charge and substituting in equations (5.5), (5.6), and (5.7),

$$\frac{1}{4} en \left( \frac{8kT_e}{\pi m} \right)^{1/2} \exp \left( -\frac{-eV_b}{kT_e} \right) A t_1 = 0.61 en \left( \frac{kT_e}{M} \right)^{1/2} A (2\pi)$$
Simplifying and solving for $t_1$,

$$t_1 = 4.88\pi \left( \frac{\pi m}{8M} \right)^{1/2} \exp \left( \frac{-eV_p}{kT_e} \right)$$  \hspace{1cm} (5.11)

The value of $t_1$ can be related to the ratio of $V_{DC}/V_{RF}$.

$$t_1 = 2 \cos^{-1} \left( \frac{V_{DC}}{V_{RF}} \right)$$ \hspace{1cm} (5.12)

Finally solving for $V_{DC}/V_{RF}$.

$$\frac{V_{DC}}{V_{RF}} = \cos \left[ 2.44\pi \left( \frac{\pi m}{8M} \right)^{1/2} \exp \left( \frac{-eV_p}{kT_e} \right) \right]$$ \hspace{1cm} (5.13)

The ratio of $V_{DC}/V_{RF}$ is highly dependent on the ratio of $V_p/T_e$, with a slight dependence on the ion mass. The results of this analysis for argon and nitrogen are shown in Figure 5.15. The results suggest that $V_p/T_e$ for the PT output voltage measurement in Figure 5.10 is around four. Since this analysis ignores electron current during time $t_2$, which may be significant near the voltage zero crossings, the ratio is likely a littler larger.
5.2.2 PT Output Current Measurement

An analysis of the PT output current shows that during the first few RF cycles after plasma formation there was a large electron current to the output electrode. In this analysis, the output current is the current measured through the PT output load resistor with a Pearson 2877 current monitor as shown in Figure 5.16.

The analysis is explained in Figure 5.17. The output voltage and current waveforms and an ideal sinusoidal fit to the output current are shown in Figure 5.17a. The markers a, b, and c, indicate when excess electron current was being collected at the PT output electrode resulting in an increased output current magnitude that can be easily seen when compared to the ideal sinusoidal output current. The difference between the measured and ideal current is shown red in Figure 5.17b. The current difference was then integrated to obtain the total excess charge collected at the output electrode. The charge at the output electrode accumulated quickly during the first several RF cycles after plasma formation. Figure 5.18 shows this analysis extended for several more RF cycles to show that the charge transfer from the plasma to the electrode eventually reached a steady state along with the output voltage bias.
Figure 5.17: Analysis of excess electron current to the PT output electrode during plasma formation.
5.2.3 PT Output Voltage Bias Simulation

The charge transfer from the plasma to the piezoelectric transformer was also modeled in COMSOL. The simulation geometry used is shown in Figure 5.19. The simulations used a surface charge accumulation boundary condition at the output electrode of the piezoelectric transformer. The electron and ion current densities to the boundary were specified according to equations (5.5) and (5.7). The simulation used N₂ for the ion mass and the electron temperature and plasma potential were varied.

A time-dependent simulation was needed to allow the charge to accumulate at the output electrode over several RF periods. To reduce simulation time, a frequency response simulation was first completed, without surface charge accumulation, to set initial conditions for the time dependent simulation. Using the results of the frequency response solution effectively allowed the time-dependent simulation to begin with a steady-state resonance established. More details on the simulation setup are included in Appendix A.

Figure 5.18: Excess electron charge collected at PT output electrode.
In the first simulations, the bias voltage would develop over many RF cycles, and in most cases a current balance would not be reached in a reasonable amount of time. Experiment results however showed that the bias reached a steady state after less than 20 RF cycles. To reduce simulation time a large electron flux to the PT surface, similar to Figure 5.17, was simulated at the time of plasma formation. It is not unreasonable that the electron current to the PT output electrode could be greater than given by equation (5.5) during this time because the plasma potential that retards the electron current will not yet be established. Several simulation results are shown in Figure 5.20. The simulation results agree with the previous analysis that the bias voltage is heavily dependent on the plasma potential. The bias voltage ranged from 30% to nearly 100% of the PT output voltage amplitude for a plasma potential difference of only 1.1 V.

5.3 Control of emission current

Various aperture diameters, flow rates, and applied voltages were studied to determine the best method to control charged particle emission from the PTPS. The
Figure 5.20: COMSOL simulation results of PT output voltage and surface charge for $T_e = 3$ V, and a) $V_p = 12.8$ V, b) $V_p = 12.5$ V, c) $V_p = 12$ V, and d) $V_p = 11.7$ V.
output aperture electrode was tested with 2, 4.2, and 8 mm aperture diameters. A thin mesh grid with 30% open area was also placed at the aperture to reduce the penetration of the electric field lines into the gap between the PTPS and the Faraday cup. The maximum gas flow rate was $0.05 \text{ Pa} \cdot \text{m}^3 \cdot \text{s}^{-1}$ and the minimum flow rate was dependent on the aperture size used. The larger apertures required higher flow rates to reliably form plasma. Applied voltages to the PTPS ranged between 100 and 450 V with input powers between 2 and 20 W.

The total emission charge and peak energies were measured with the Faraday cup. The emission charge was obtained by integrating the current over a single RF period. The results are shown in Figure 5.21. The emitted charge, both ion and electron, is mainly dependent on input power and aperture size. The peak electron and ion energy is shown in Figure 5.22. As the input power to the PTPS increases, the peak ion energy decreases while the peak electron energy increases. For propulsion applications, this may mean that higher input power may not lead to greater thrust. The opposite trends for the ion and electron energies may indicate a larger PT output voltage bias at higher input power. If the peak ion energy and peak electron energies are assumed to be the maximum and minimum PT output voltages, then the PT output voltage bias and the PT output voltage amplitude can be determined. The ratio of the two, $V_{DC}/V_{out}$, is shown in Figure 5.23. The increased bias voltage ratio is similar to results reported by Roosmalen and Pointu for a RF glow discharge source [38, 42]. The PT output voltage amplitude was also used to determine the transformer ratio, shown in Figure 5.24.

The plasma density can be estimated from ion emission measurements. The emitted ion current, $I_i$, through the aperture is given by,

$$I_i = en \nu_i A$$

(5.14)

where $e$ is the ion electron charge (assuming singly ionized), $n$ is the plasma density,
Figure 5.21: Charged particle emission versus input power for a 2 mm, 4.2 mm, and 8 mm aperture.
Figure 5.22: Charged particle energy versus input power for a 2 mm, 4.2 mm, and 8 mm aperture.
Figure 5.23: PT output voltage bias versus input power for a 2 mm, 4.2 mm, and 8 mm aperture.

Figure 5.24: Transformer ratio versus input power for a 2 mm, 4.2 mm, and 8 mm aperture.
Figure 5.25: Estimated plasma density versus input power for a 2 mm, 4.2 mm, and 8 mm aperture.

\( \nu_i \) is the ion velocity determined from a time of flight measurement, and A is the aperture area. The resulting plasma densities are shown in Figure 5.25.

The estimated density ranges between \( 10^{15} - 10^{16} \text{ m}^{-3} \). This value may be a lower bound of the actual plasma density because the peak ion velocity was used in equation (5.14). The ion energy distribution shows that a significant portion of the ions have a lower energy, which would result in an increase in plasma density.

5.4 Estimate of Thruster Parameters

The thruster parameters for the PTPS were estimated using ion emission measurements from the previous section. These parameters include the thrust, specific impulse, mass flow utilization, and efficiency.

The thrust, \( T \), is given by,

\[
|T| = m \cdot |a| = \dot{m} \cdot |v_{eff}|
\]  

(5.15)

where \( \dot{m} \) is the mass flow rate and \( v_{eff} \) is the effective velocity of particles expelled
from the thruster. If it is assumed that the only ions present are the ionized background gas, a portion of the expelled particles will be ionized gas with a high effective velocity and the rest will be neutral gas with a low effective velocity. The thrust produced by the ions can be estimated by using the ion mass flow rate and the ion velocity. The ion mass flow rate, \( \dot{m}_{\text{ion}} \), is given by,

\[
\dot{m}_{\text{ion}} = \frac{I_{\text{avg}} \cdot m_{\text{ion}}}{q}
\]

(5.16)

where \( I_{\text{avg}} \) is the average ion current measured with the Faraday cup, \( m_{\text{ion}} \) is the ion mass, and \( q \) is the charge of an electron. The ion velocity is determined from the ion energy, as in equation (5.1). Combining equations (5.15), (5.16), and (5.1) gives an expression for the ion thrust,

\[
|T_{\text{ion}}| = I_{\text{avg}} \sqrt{\frac{2m_{\text{ion}}E_{\text{ion}}}{q}}
\]

(5.17)

The ion thrust versus applied power for each aperture diameter is shown in Figure 5.26. The thrust did not increase significantly as the applied power increased because of the DC bias voltage of the PT output. As applied power increased, the DC bias increased and the ion energy decreased. The reduced ion energy nearly cancels out the increased ion charge at higher applied power.

The thrust from the background gas neutrals is determined using the neutral mass flow rate and average velocity of the neutral particles. The neutral mass flow rate is given by,

\[
\dot{m}_{\text{neutral}} = \dot{m}_{\text{total}} - \dot{m}_{\text{ion}}
\]

(5.18)

where \( \dot{m}_{\text{total}} \) is the total mass flow rate measured by the flow controller. The mass flow controller measures throughput, \( Q \), in SI units of Pa·m³·s⁻¹. The total mass flow rate (\( \dot{m}_{\text{total}} \)) in kg/s can be given in terms of the throughput. The relationship is derived from the ideal gas law and is given by,

\[
\dot{m}_{\text{total}} = \frac{m_{\text{neutral}} Q}{kT}
\]

(5.19)
Figure 5.26: Calculated ion thrust versus applied power for a 2 mm, 4.2 mm, and 8 mm aperture.

where $k$ is Boltzmann’s constant and $T$ is the gas temperature in Kelvin. The average neutral velocity, $v_{\text{neutral}}$ is given by,

$$v_{\text{neutral}} = \left( \frac{8kT}{\pi m_{\text{neutral}}} \right)^{1/2} \tag{5.20}$$

where $m_{\text{neutral}}$ is the mass of the neutral particle. Combining equations (5.15), (5.18), and (5.20) gives the thrust due to neutrals as

$$|T_{\text{neutral}}| = \dot{m}_{\text{neutral}} \left( \frac{8kT}{\pi m_{\text{neutral}}} \right)^{1/2} \tag{5.21}$$

Due to the large gas flow rate needed to maintain a background gas pressure of 100-2000 mTorr, the total thrust ($T_{\text{neutral}} + T_{\text{ion}}$) from the PTPS was dominated by neutral gas flow as shown in Figure 5.27. The total thrust was effectively dependent only on neutral flow rate because of comparatively low ion mass flow rates. The mass flow utilization, calculated as the ratio of $\dot{m}_{\text{ion}}/\dot{m}_{\text{total}}$ is shown in Figure 5.28.

The specific impulse of the thruster is given by,

$$I_{sp} = \frac{v_{\text{eff}}}{g} = \frac{T}{\dot{m} g} \tag{5.22}$$
Figure 5.27: Total thrust and ion thrust versus flow rate for a 2 mm, 4.2 mm, and 8 mm aperture. The total thrust is shown with solid data markers and ion thrust is shown with open data markers.

Figure 5.28: Mass flow utilization versus flow rate for a 2 mm, 4.2 mm, and 8 mm aperture.
where $g$ is the gravitational constant. The specific impulse for the thruster, including neutral gas flow, was approximately 48 s. As with total thrust, the specific impulse was dominated by the neutral gas flow.

The efficiency, $\eta$, of the thruster was calculated as the ion thrust power divided by the input power,

$$\eta = \frac{\frac{1}{2} T_{ion} v_{ion}}{P_{in}}$$

and is shown in Figure 5.29.

The PTPS is not suitable for propulsion applications when it is operated with such large flow rates. The amount of ionized particles expelled from the thruster is insignificant when compared to the neutral particles. As a result the specific impulse and efficiency due to the ion thrust are extremely low.
5.5 Floating Potential

When the PTPS is operated isolated from the vacuum chamber ground, it will bias itself to a floating potential based on the emitted charge. The results in Figure 5.21 show that the emitted electron charge is an order of magnitude greater than the ion charge. As a result the plasma source biases positively to attempt to balance the emitted charge. Figure 5.30 shows how the electron current, ion current, and floating potential vary during an applied burst of 250 RF cycles. Initially, the emitted electron current was greater than the emitted ion current and as a result the began to float to a positive potential. As the floating potential increased the emitted electron current decreased while the ion current increased. The emitted electron and ion currents should eventually equilibrate. However, in our experiments the electron current remained approximately five times greater than the ion current as the PTPS approached a steady state. The steady state difference between ion and electron emission current was likely due to insufficient isolation from the chamber ground due to the impedance matching transformer having a high, but significant, impedance to ground.

5.6 Optical Spectroscopy

Optical spectroscopy was performed using an argon gas flow to investigate the ion species in the beam. The results are shown in Figure 5.31. Emission lines were identified based on the tables from the Handbook of Basic Atomic Spectroscopic Data published by NIST [43]. Many emission lines from Ar$^+$ and Ar$^{++}$ were present within the beam due to ionization of the background gas. Strong emission lines were also observed for Li$^+$ and Nb$^+$, indicating that sputtering of the piezoelectric surface was occurring. Some lines were left unlabeled due to uncertainty of their source.
Figure 5.30: Charged particle, (a) electron and (b) ion, emission and (c) floating potential for an applied burst of 250 cycles. The data presented is the average value per RF cycle. Electron and ion emission measurements are recorded for two different RF bursts.
For example, NIST data indicated that Ar\(^+\) and Nb\(^+\) both had emission lines near 420 nm, but neither was listed as a primary emission wavelength. The line at 420 nm in Figure 5.31a may have occurred from a combination of both.

Optical spectroscopy using a nitrogen gas flow was done to determine if the plasma heavy particle temperature of the plasma was significantly different between the ion emission and electron emission half-cycles. The ICCD camera was gated to split the applied RF cycle into two parts. The ion emission spectra was recorded only during the 1 \(\mu s\) after the “blip” in the input current (see Figure 5.8) and the electron emission spectra was recorded only during the 1.5 \(\mu s\) prior to the “blip” in the input current. The 337.13 nm N\(_2\) emission line was observed to model the heavy particle temperature of the plasma. The unnormalized spectra in Figure 5.32a shows that the plasma is much more intense during ion emission than electron emission. The intensity difference is consistent with the photomultiplier tube diagnostic in Figure 5.8b. When the intensity is normalized to the maximum intensity, as shown in Figure 5.32b, it is clear that the shape of the emission line is similar during ion and electron emission. The similar shape of the emission lines indicate that the ion temperature in the plasma is the same throughout the RF cycle and likely does not dissipate and reform every half-cycle.
Figure 5.31: Optical emission spectroscopy results for PTPS with an argon gas flow.
Figure 5.32: Optical emission spectroscopy results for PTPS with a nitrogen gas flow.
Chapter 6

Conclusions and Future Work

6.1 Conclusions

Research on the PTPS focused on two aspects. The first was to improve the utilization of the piezoelectric resonance in the device. The second was to characterize the plasma formation at gas pressures between 100-2000 mTorr, corresponding to near-space altitudes of 130,000-200,000 ft.

COMSOL simulations were used to determine the optimal poling direction to maximize the piezoelectric transformer effect in the PTPS. The simulation results yielded two resonance modes, radial and elliptical, that were capable of generating a large output voltage within the aperture. The PT performance was evaluated by maximizing the ratio of the peak output voltage generated by the PT to the peak von Mises stress within the crystal. The goal was to find crystal orientations that would produce the maximum voltage without exceeding the yield stress of the crystal. The optimal radial mode polarization was rotated $\beta = 45^\circ, \gamma = 0^\circ$ from a $z$-cut disk. The optimal elliptical mode polarization was rotated $\beta = 45^\circ, \gamma = 60^\circ$ from a $z$-cut disk. The results of the simulations were used to order two sets of lithium niobate samples, one optimized for each mode. These samples were used in all subsequent testing.

The assembly of a PTPS was evaluated by determining the mechanical quality
factor for several PTPS samples. The piezoelectric transformer output was affected by mechanical energy losses from the construction of the source. It was found that the mechanical quality of the PTPS was greatly reduced when the lithium niobate disk was excessively compressed between the aperture and driven electrodes. The PTPS operated most effectively when it was assembled with just enough compression to hold the piezoelectric disk in place and maintain good electrical contact between the electrodes and the disk. Evaluating the mechanical quality factor also lead to using silver grease, rather than silver paint, to maintain electrical contact between the electrodes and the disk. When silver paint was used the mechanical quality factor decreased over time as the paint dried and hardened. Silver grease eliminated this problem and the mechanical quality factor was not significantly affected over time.

The output voltage of the piezoelectric transformer was improved by offsetting the aperture electrode from the center of the lithium niobate disk. The need to offset the aperture electrode is a side effect of the rotated polarization required for efficient electromechanical coupling. The rotated polarization prevents a radially symmetric displacement within the lithium niobate disk, resulting in a generated voltage within aperture that is offset towards the edge of the aperture electrode. When the aperture was offset towards the edge of the disk, the gain of the PT increased. Operation at atmospheric pressure also revealed that the discharges became more uniformly distributed around the aperture electrode when the aperture was offset. The uniform discharge distribution may help increase the lifetime of the PTPS by reducing degradation of the piezoelectric surface.

A stacked electrode configuration was used to test the plasma emission characteristics for background pressures of 100-2000 mTorr. A gas flow was used to create a local high pressure volume at the disk surface while maintaining a high vacuum in the rest of the test chamber. Charged particle emission measurements with a Faraday cup
and retarding potential analyzer confirmed ion and electron emission from the source during opposite half-cycles of the applied signal. Energy measurements revealed a significant fraction of electron charge near 3 keV and peak electron energies of up to 4 keV when the PTPS was driven with a 430 V applied RF voltage. The large peak electron energies indicated that the voltage generated at the output of the piezoelectric transformer was responsible for accelerating charged particles from the source. Ion energy measurements were unexpectedly low, with peak energies of 650-700 eV. The peak ion energy was still greater than the applied voltage but was 4-5 times less than the peak electron energy.

To investigate the significant energy difference between the ions and electrons, the output voltage of the piezoelectric transformer was directly measured with a high-voltage probe during plasma formation. The output voltage amplitude steadily increased while the disk began to resonate. At the time of plasma formation, the output voltage of the transformer remained large in amplitude but began to offset with a negative DC bias. The negative bias reached a steady state and remained for the duration of the applied burst. The negative bias was attributed to self-biasing of the PT output in order to equalize electron and ion current from the plasma to the PT. The same effect is seen in RF glow discharges with unequal electrode sizes. The electrode biases itself negatively to reduce the electron current flux from the plasma. The negative bias ensures that the net current to the electrode in a single RF cycle is zero, a requirement for a capacitively coupled discharge. The low ion current flux from the plasma throughout the entire RF cycle is balanced by only a short electron current pulse while the PT output voltage is positive. Simulations and analysis of the PT output voltage and current waveforms confirmed that excess electron current to the PT output electrode resulted in the negative bias of the output voltage.

The PTPS was configured various aperture diameters and operated with varying
flow rates and applied voltages to determine the best method to control charged particle emission. The aperture size and applied voltage both had a large effect on the emission current. The flow rate did not have a significant influence. Larger applied voltages resulted in a greater charged particle flux from the plasma source. However, the PT output voltage bias also appeared to have a dependence on the applied voltage which affected the charged particle energies. The ion energy decreased and the electron energy increased for increasing applied voltage. While the output voltage amplitude of the PT increased with larger applied voltages, as expected, the gain of the PT decreased for larger applied voltages indicating additional energy losses or an increased electrical load from the plasma.

Thruster parameters were determined for the PTPS when operating with a 2, 4.2, and 8 mm aperture. It was determined that at the large mass flow rates lead to the neutral gas flow dominating the thruster parameters. As a result the specific impulse and efficiency due to the ion thrust were very low. In order to increase efficiency the mass flow rate needs to be greatly reduced. Operation at lower flow rates may be accomplished though further reduction in the aperture diameter or an improved gas flow design for the PTPS.

Finally, the PTPS was isolated from the vacuum chamber ground to investigate plasma source self-neutralization. It was observed that the plasma source as a whole biased positively because the electron charge emission was greater than the ion charge emission. Near the beginning of the RF burst, electron current was twenty times greater than the ion current. Throughout the burst the floating potential increased, resulting in a decreased electron current and increased ion current. When the floating potential reached approximately 80 V the ratio of electron current to ion current had decreased to a factor of five. While it was not observed in the laboratory, it is believed that the floating potential will continue to rise until the electron and ion currents are
equal. If the emitted current equilibrates at an acceptable floating potential, that could be effectively isolated from other systems in the spacecraft, the PTPS would be have an advantage over other ion propulsion technologies since a separate neutralizing system would not be needed.

6.2 Future Work

In order to further improve the utilization of the PT effect, alternate electrode configurations or assembly methods could be investigated. A method using a painted or deposited electrode, with an appropriate offset from center, that was connected to a larger aperture electrode through a radial array of wires would likely be less mechanically restrictive than compressing the disk with the aperture electrode. A wire array would be difficult to do by hand and would require a standardized assembly process. If the compression method is used a standardized method of assembling the PTPS with a known compressive force would help to reduce variation in the characteristics of the PTPS from experiment to experiment.

Lithium niobate is a pyroelectric material and its properties can vary based on temperature. The emission characteristics may change if the PTPS is operated for a larger number of RF cycles per burst or at higher duty cycles. Duty cycles were intentionally kept low to avoid problems with the disk breaking when operated in a near continuous mode. It may be possible that as the disk heats up the input signal could be reduced or altered in frequency to avoid exceeding the yield stress of the disk. Operation for a longer number of RF bursts would also reduce the dead time when an RF signal is applied to the PT, but the plasma has not yet formed.

A measurement of the beam divergence during both ion and electron emission half-cycles will help result in more accurate current measurements with the Faraday cup. The true beam divergence can be determined by measuring the emitted current
at varying angles, rather than directly in front of the PTPS. Additionally, it would be beneficial to see if the beam profile can be shaped using different configurations of the stacked aperture electrodes. The number of electrodes, the aperture diameter, and the electrode spacing could all be varied.

Further characterization of the PTPS could be done versus background pressure. The pressure in the aperture was never directly measured, making it difficult to determine the effect of background pressure on the emission characteristics of the source. Background pressures were only estimated from the flow rate and aperture diameter. The estimated background pressures prevented comparison of emission characteristics versus background pressure across the different sized aperture electrodes.

Finally, additional experiments need to be done to characterize the PTPS for near-space propulsion. The thrust due to ion emission could be measured on the thrust stand. The thrust measurement could confirm that the thrust should not be greatly affected by the applied voltage due to the DC bias voltage. Methods to reduce the gas flow rate while still reliably generating plasma need to be investigated. If the gas flow rate can be reduced, a redesign of the PTPS should be considered to reduce gas leaks from the aperture volume. As the gas flow rate is decreased any unwanted gas leaks become more significant.
Appendix A

COMSOL Simulation Guidelines

This appendix will explain some of the common simulation setup for PTPS simulations on COMSOL. Specific examples are used from the surface charge accumulation simulations presented in section 5.2.3.

New Simulation

When starting a new simulation you to choose three items, the geometry space, physics to model, and a study type. The common choices I used are listed below.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physics</td>
<td>Structural Mechanics → Piezoelectric Devices</td>
</tr>
<tr>
<td></td>
<td>AC\DC → Electrical Circuit</td>
</tr>
<tr>
<td>Study Type</td>
<td>Eigenfrequency</td>
</tr>
<tr>
<td></td>
<td>Frequency Response</td>
</tr>
<tr>
<td></td>
<td>Time Dependent</td>
</tr>
</tbody>
</table>
Global Definitions

I define any input parameter into the simulation as a parameter under the global definitions. A sample parameter table from the surface charge accumulation simulation is shown in Figure A.1.

<table>
<thead>
<tr>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
</tr>
<tr>
<td>tanDelta</td>
</tr>
<tr>
<td>Qm</td>
</tr>
<tr>
<td>appliedV</td>
</tr>
<tr>
<td>fres</td>
</tr>
<tr>
<td>appliedVt</td>
</tr>
<tr>
<td>echg</td>
</tr>
<tr>
<td>np</td>
</tr>
<tr>
<td>Te</td>
</tr>
<tr>
<td>mi</td>
</tr>
<tr>
<td>me</td>
</tr>
<tr>
<td>ub</td>
</tr>
<tr>
<td>ve</td>
</tr>
<tr>
<td>Vp</td>
</tr>
<tr>
<td>ji</td>
</tr>
<tr>
<td>jcl</td>
</tr>
</tbody>
</table>

Figure A.1: COMSOL global definitions.
Model Definitions

Probes

A probe is used to monitor a parameter in the simulation. Any data from a probe can be obtained later through post processing, but I find it easier to set up a probe before running the simulation. I typically use a boundary probe to monitor the input voltage, input current, output voltage, and surface charge.

For example the surface charge boundary is an integral probe and integrates the surface charge density (sigs) on the output electrode boundary to obtain the total surface charge on the boundary.

Figure A.2: COMSOL probe definition.
Functions

Functions can be defined and operate on any variable. I used a step function to begin the surface charge accumulation after 2 RF cycles in a time-dependent simulation. When the function is used, you specify the variable it acts on.

![Step Function](image)

Figure A.3: COMSOL function definition.

For example on a surface charge accumulation boundary condition, I specified the ion current density as:

\[ j_i \times \text{step1}(t \text{ [1/s]}) \]

\( j_i \) was defined as the Bohm current (see Global Definitions) and \text{step1}() was defined to be equal to 0 when the parameter \( t \) was less than 2 RF periods and 1 when \( t > 2 \) RF periods. Note: COMSOL expects the input to a function to be unitless. Since \( t \) has units of seconds, a unit of 1/s was specified in the argument of the function.
Model Couplings

Model couplings are used to couple a variable that is being solved for in the simulation as an input to another parameter. For example, when I had electron charge accumulating on the PT output I used an average operator to determine if the PT output voltage was positive or negative.

![COMSOL Operator Definition](image)

Figure A.4: COMSOL operator definition.

When I specified the electron current density accumulating on the surface I used the average operator to specify one electron current when the output voltage was positive and another electron current when the output voltage was negative.

```plaintext
if(aveop1(V[1/V])>=0,
   je1*step1(t [1/s]),
   je1*exp(aveop1(V[1/V])/Te)*step1(t [1/s]))
```
Local Coordinate System

Piezoelectric material constants are defined for a coordinate system in which the polarization of the material is in the z-direction. A rotated local coordinate system for the piezoelectric domains was defined. The rotated system is defined by three consecutive rotation angles, $\alpha$, $\beta$, and $\gamma$. For a z-cut disk only the angles $\beta$ and $\gamma$ are needed. It is helpful to define the angles as global variables and reference the variable in the definition. The default unit is radians, so degrees needs to be specified if they are used.

![Rotated System](image)

Figure A.5: COMSOL local coordinate system definition.
Model→Geometry

When drawing the simulation geometry I found it easiest to draw in the global coordinate system, such that the z-axis is perpendicular to the disk surface. When creating the geometry, the geometry domains can overlap. To model the basic PT, you can simply draw two concentric cylinders. When the geometry is finished, COMSOL splits the overlapping geometries into two domains.

Model→Materials

All the required materials are in the material library.

Model→Piezoelectric Devices

Here you define any boundary conditions. The input voltage is modeled with an Electric Potential boundary. In an eigenfrequency or frequency response simulation only the amplitude is specified. In a time-dependent simulation a function needs to be input, i.e.

\[ \text{appliedV*sin}(2\pi\text{fres}\*t) \]

Any losses in the simulation can be specified under Piezoelectric Material Model. A good starting estimate for mechanical losses is \(1/Qm\) for the loss factor for \(cE\) and the loss tangent, loss factor for \(cS\), for lithium niobate is 0.0013.
Figure A.6: COMSOL damping and loss definition.
Model → Electric Circuit

When attaching an electric circuit (spice circuit) to a boundary in the simulation the *Electric Circuit* interface needs to be loaded in the simulation. The boundary that the circuit is attached to (for example an output voltage electrode) is specified with a *Terminal* boundary condition under the *Piezoelectric Devices* interface. This terminal type needs to be specified as a *Circuit* as shown in Figure A.7.

![Figure A.7: Piezoelectric Devices boundary for a COMSOL circuit simulation.](image)

The spice circuit is then defined under the *Electric Circuit* interface. I used an *External I-Terminal* to interface the *Terminal* boundary condition with the spice circuit. For this to work correctly the *External Terminal* for the *External I-Terminal* in the *Electric Circuit* interface needs to be specified as the *Terminal* defined in the
Piezoelectric Devices interface. This is shown in Figure A.8. When this is done, the voltage at the boundary in the finite element model is used as a voltage source in the spice circuit. The resulting current draw from the spice circuit is automatically coupled back into the finite-element model. The rest of the circuit is specified just like a spice netlist.

Figure A.8: Electric Circuit ‘boundary’ for a COMSOL circuit simulation.
Appendix B

Lithium Niobate Order Specifications

The lithium niobate disks used in this research were ordered from Foreal Spectrum. It is important to realize that when specifying a crystal orientation from Foreal that the rotation angles are defined in the opposite direction when compared to COMSOL. Images of the crystal specifications from Foreal for the radial and elliptical mode disks are included in Figures B.1 and B.2.
Orientations for 10mm dia. X 2mm LiNbO$_3$ disk  
(Updated with only one rotation)

X, Y, Z: global axis  
X1, X2, X3: crystal axis  
X3: polarization direction

Fig. 1. Start Position (start with Z-cut).  
Fig. 2. Rotation (rotate $-45^\circ$/+$5^\circ$ around global X axis)

Figure B.1: Specification for radial mode LiNbO$_3$ crystal.
Orientations for 10mm dia. X 2mm LiNbO3 disk_revised

X, Y, Z: global axis
X1, X2, X3: crystal axis
X3: polarization direction

Fig. 1. Start Position (start with Z-cut).

Fig. 2. First Rotation (rotate -45°/+5° around global X axis)

Fig. 3. Second Rotation (rotate -60°/+5° around X3 axis).

Fig. 4. Final Orientation

Figure B.2: Specification for elliptical mode LiNbO3 crystal.
Appendix C

Matlab Tektronix Load Script

This script will load files from the small Tektronix oscilloscopes much quicker than the Tekload.m script found on the internet. I have cleverly named it Tekload2.m.

function data = TekLoad2(filename,numRows)
% TEKLOAD Helper function for loading saved waveform data
%
rawData=dlmread(filename,',',0,3,[0 3 numRows-1 4]);
data = struct('Waveform', [], 'Time', []);
data.Waveform=rawData(:,2);
data.Time=rawData(:,1);

To use, just supply the filename of the .csv file saved by the oscilloscope and the number of data points, typically 2500. For example running:

data=TekLoad2(strcat('F0000CH1.CSV'),2500);

would produce a structure called data with the oscilloscope information. The time would be referenced as data.Time and the waveform would be data.Waveform.
Bibliography


Vita

Brian Hutse was born in St. Louis, Missouri on April 7, 1984. He received the B.S. in Electrical Engineering and B.S. in Computer Engineering in May 2007 from the University of Missouri. He received his M.S. in Electrical Engineering from the University of Missouri in December 2008. The title of his Master’s Thesis was “Runtime and jitter of a laser triggered gas switch” and was supported by Sandia National Laboratories. He will earn the Ph.D. in May of 2012. For this work, he was supported by the Air Force Office of Scientific Research.

Brian has presented his research at several international conferences and has x conference papers and presentations. As of this publication, he has two peer-reviewed journal papers with two more journal papers in press.

After graduation, Brian will be employed by Sandia National Laboratories in Albuquerque, New Mexico. In July 2011 he married Krystle Azerolo.