This thesis is subject to
library loan regulations

UM Libraries Depository
103334706018

LIBRARY OF THE
UNIVERSITY OF MISSOURI

This Thesis Has Been

MICROFILMED

Negative No. T- 1341

1952

Form 26

THE ABSOLUTE MEASUREMENT OF ELECTRICAL CAPACITIES

by

Benjamin Estill Shackelford, A. B.

UNIVERSITY
LIBRARY
OF MISSOURI

SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

in the

GRADUATE SCHOOL

of the

UNIVERSITY OF MISSOURI

1913

378.7M71
XSh1

May 16, 1913.

Dr. L. M. Defoe,

Engineering Building.

Dear Dr. Defoe:

It is customary for the Graduate Committee to refer dissertations, submitted by candidates for the degree of Master of Arts, to some member of the Group who is not connected with the Department in which the candidate's work has been done. I am sending you herewith a dissertation which has been submitted by Benj. Shackleford.
I shall be greatly obliged if you will kindly examine the same at your earliest convenience and report to us for the Graduate Committee whether in your opinion the dissertation meets the general standard which has been established in this University for the Master's dissertation.

Very truly yours,

Walter Miller

Chairman, Graduate Committee.

117860 59

THE ABSOLUTE MEASUREMENT OF ELECTRICAL CAPACITIES.

Of the various methods in use at the present time for the absolute measurement of electrical capacities in electromagnetic units, none give the satisfaction desired. Only two of these methods are in common use and they depend on the alternate charging and discharging of a condenser. If the galvanometer used be of a fairly long period, this intermittent current may be considered as a constant current of a definite intensity.

In the first and simpler method of the two, a galvanometer (G), a cell (B), and a capacity (C) are connected as indicated in Fig.1, the alternate charging and discharging of the condenser being effected by a vibrating spring (I), alternately making contact at I' and I". When this tongue is connected to I' the circuit is closed and a charge flows through the galvanometer in charging the condenser. Then when it touches I", the condenser discharges. When this process is repeated at regular intervals by means of the vibrating spring, the galvanometer has a steady deflection. If q is the charge on the condenser when charged by the difference of potential E at the terminals, then $q = EC$, or if n is the frequency of the vibrator,

$$nq = nEC. \qquad (1)$$

This is the current flowing around the circuit. A high resistance R' is substituted for the condenser, and the galvanometer is shunted by a resistance s, so the capacity may be calculated as follows. Let d' be the constant de-

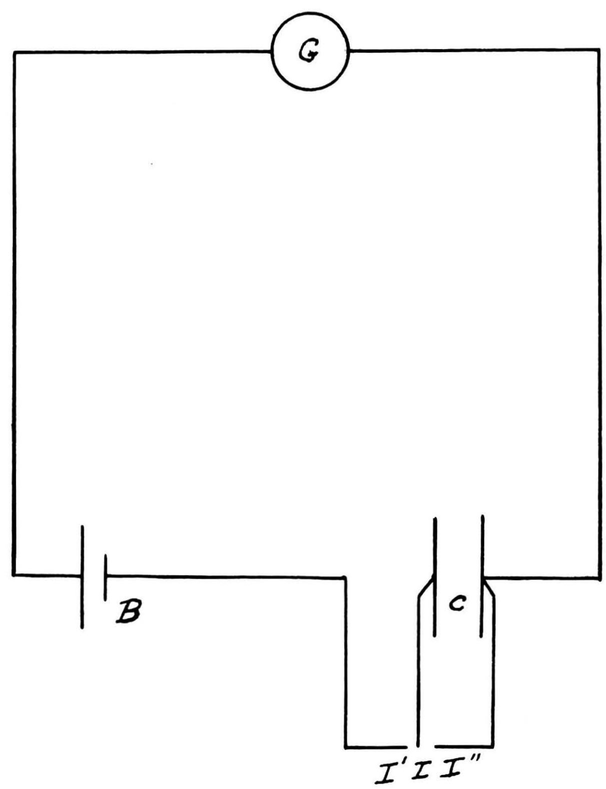

Fig. 1.

flection in the first part of the experiment. Then $md' = nEC$, where m is the current through the galvanometer necessary to produce unit deflection. Let d" be the deflection after the capacity has been replaced by the resistance. Then

$$md'' = \frac{E}{R' + \frac{sg}{s+g}} \frac{s}{s+g}, \tag{2}$$

where g is the resistance of the galvanometer. Dividing (1) by (2),

$$\frac{d'}{d''} = nRC \frac{s+g}{s}, \tag{3}$$

when R is the total resistance of the circuit. Combining (1) and (3),

$$C = \frac{d'}{d''} \cdot \frac{1}{nR} \cdot \frac{s}{s+g}. \tag{4}$$

All of the terms on the right hand side of the equation are known and therefore C can be calculated.*

There are several objections to this method. If the frequency of the spring and the electromotive force of the cell are not constant, an error is introduced. One of the greatest objections is that it is a deflection method, so that with all other defects left out of consideration, an accuracy of one-half of one per cent is about the limit.

In the second method, first proposed by Maxwell for determining the ratio between the electromagnetic and electrostatic units of capacity, the condenser and vibrator are placed in one arm of a Wheatstone bridge and balance obtained as in a resistance measurement. This

---

* Carhart and Patterson, "Electrical Measurements". p.229.

latter method has the advantage over the former in that it is a zero method, and almost absolutely independent of the constancy of the cell, and also requires only one setting. It is therefore much more accurate. Both methods however are subject to the difficulty of keeping the frequency of the vibrator constant. When the charging and discharging is done by means of contacts on a rotating armature, as is often the case, it is very difficult to maintain this at a constant speed. Consequently a perfect balance is out of the question. A tuning fork which charges and discharges the condenser by means of contacts on it dipping into mercury cups is used very much in this kind of a determination. The trouble, however, has been that the contact would occasionally fail. That is, if the amplitude was small and a platinum point was used, the mercury in the cups would become oxidised and electrical contact would not be made between the platinum and the mercury. Also the mercury would stream up after the point and the contact, once made, would in some cases not be broken when the point was raised from the liquid. Either of these happening even occasionally would prevent accurate balancing. The galvanometer would suddenly deflect a centimeter or more in one direction and continue swinging for some time. Then as it was just about at rest again, it would take another jump and the whole process be repeated. Thus it is a very annoying, and at times, almost impossible measurement.

The present experiment was undertaken to learn

the exact conditions necessary for best results, the second method being used with the tuning fork vibrator. A diagram of the apparatus and connections is given in Fig. 2, and the theory is as follows.

Let A, B, and D be the resistances of the parts of the bridge as indicated. Let g be the resistance of the galvanometer, and b that of the battery. C is the capacity under investigation. The point 1 is connected, by means of the vibrations of the tuning fork, first to n and then to m. In this way the condenser is alternately charged and discharged. If the period of the galvanometer is long compared with that of the fork, this alternate charge and discharge is equivalent to a steady current. The current through the galvanometer may be divided into two parts. First there is that due to the steady current flowing in the arms A, B, and D of the bridge when the condenser is not being charged. Let us call this dy/dt. The current through the galvanometer is decreased while the condenser is being charged. This then is the equivalent of an intermittent current flowing in a direction opposite to that of dy/dt, and we will denote it by the sum of the charges Y, which flow at each charging. When taken over some length of time the condition for balance is that, $dy/dt = nY$. Y is the quantity of electricity that would flow through the galvanometer if the cell were removed and the condenser permitted to dicharge through the bridge. Let dZ/dt be the current from the condenser during this discharge, dY/dt the current through the

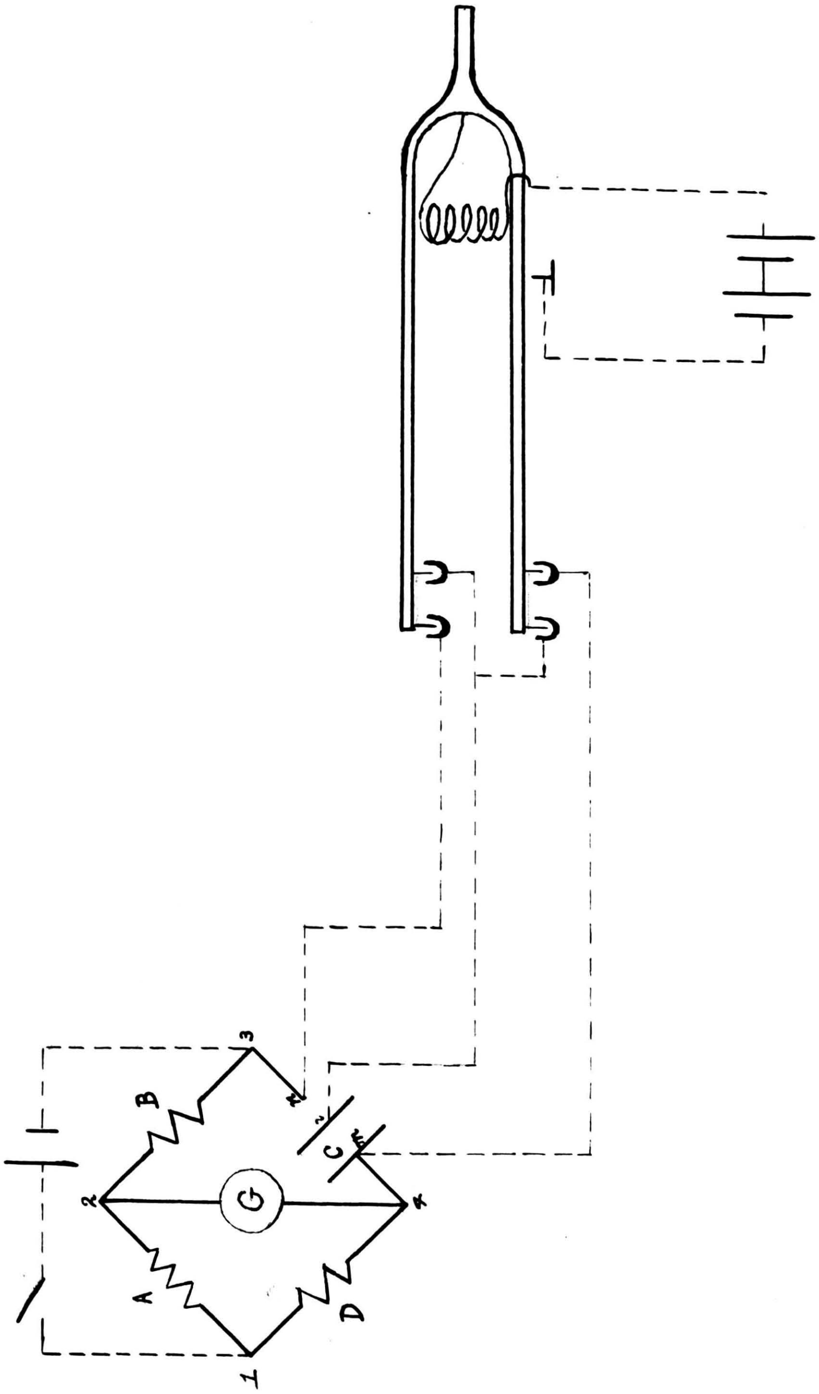

Fig 2.

galvanometer, and dW/dt the current through A. Let the coefficients of self induction of A, B, D, g, and b, be L_a, L_b, L_d, L_g, and L_c respectively. Then from the circuit (1241), by applying Kirchoff's laws,

$$L_g \frac{d^2Y}{dt^2} + L_d \frac{d^2(Y-Z)}{dt^2} - L_a \frac{d^2W}{dt^2} + g \frac{dY}{dt} + D\left[\frac{dY}{dt} - \frac{dZ}{dt}\right] - A \frac{dW}{dt} = 0$$

Integrating over the time of discharge,

$$GY + D(Y-Z) - AW = 0 \qquad (5)$$

since dY/dt, dZ/dt, and dW/dt vanish at the limits, and Y, Z, and W are the respective quantities. In the same way, from 1 to 3 through the battery,

$$(A+B+b)W + (B+b)Y - bZ = 0. \qquad (6)$$

Combining these last two equations,

$$Y = \frac{D(A+B+b) + bA}{A(B+b) + (G+D)(A+B+b)} Z. \qquad (7)$$

If V is the difference of potential of the plates of the condenser when fully charged,

$$Z = CV. \qquad (8)$$

But $V_{2,4} = G\ dy/dt$, and

$$V_{2,3} = B \frac{A+G+D}{A} \frac{dy}{dt}.$$

Therefore,

$$V = G + B + \frac{B}{A}(G+D) \frac{dy}{dt} \qquad (9)$$

Then from (7), (8), and (9), since $nY = dy/dt$,

$$\frac{1}{n\ C} = \frac{D(A+B+b) + bA}{A(B+b) + (G+D)(A+B+b)} \left[G + B + \frac{B}{A}(G+D)\right]$$

Since b is very small in comparison with the other resistances,

$$C = \frac{A}{nDB} \frac{1 - \frac{A}{(G+D+A)(A+B)}}{1 + \frac{GA}{B(G+D+A)}}. \tag{10}$$

This formula was used in all the calculations. However the formula,

$$C = \frac{A}{nDB} \tag{11}$$

is a fairly close approximation since the numerators of the complex fraction are small in comparison with the denominators. Equation (11) was used in the preliminary calculations, and as an aid in getting the best adjustment of the fork and the resistances.*

In choosing a fork, one of the things most desired was a fairly large amplitude, as this naturally aided in overcoming both of the difficulties mentioned above, namely, that of occasional failure of contact, and the streaming up of the mercury far enough to interfere. A fork made of one-eighth inch steel rod was first used. With a current up to one and one-half amperes this had an amplitude which could be adjusted from one to three-quarters of an inch. The driving current was led into the fork by means of a mercury cup near the heel of the fork, the mercury being covered with glycerine to reduce arcing. Contacts for the charge and discharge were made by fine copper wires hardened and pointed, which were attached, insulated, to the ends of the prongs of the fork. At first this arrangement worked very well, the usual difficulties not being experienced; but two new

* Thompson, J. J. "Elements of Elec. and Mag.", Par. 263.

troubles arose. The fork seemed a little light, not being very regular at all times, and the natural planes of linear vibration of the prongs not only were not in the plane of the fork as a whole, but were not in ~~the same~~ planes with each other.

This fork also gave a very pretty illustration of the phenomena of forced vibrations with a period which differs from the free period of the body set in vibration. When started, the prong controlled directly by the magnet would vibrate regularly, as would the other for a very short while. Then the whole being rigidly fastened to a table, the amplitude of the second prong would die down, showing the transfer of energy. This would continue back and forth for a short while, gradually dying out until finally the second prong was vibrating with the period of the first, or what is the same thing, the period of the impressed force.

At first these were not serious difficulties and could be controlled by the magnet, but later, for some unknown reason they increased until they became such a serious factor that another fork had to be substituted. This second fork was much heavier and of smaller though sufficient amplitude. The cross-section of the prongs of this fork was rectangular and the vibrations took place in the plane of the fork as a whole. Due to these facts this instrument eliminated both of the difficulties experienced with the former one. Copper wires attached to metal strips were fasten-

ed to the prongs, small pieces of mica being used as insulating washers. These wires were stiffened by filing and were pointed to prevent splashing. The fine copper wires were used instead of platinum because they were easily amalgamated, thus ensuring better contacts. Contrary to what might have been expected this did not cause an appreciable streaming up of the mercury when the point lifted from the liquid. Referring to Fig. (2), it will be seen that when the fork is in that part of the vibration such that the two prongs are pulled toward each other, contact is made through the upper cups and the circuit closed, thus charging the condenser. Half a vibration later when the prongs are apart, this contact is broken and a new one made through the lower set of cups. In this case the condenser terminals are connected and the capacity discharged. The large amplitude also insured that the mercury was stirred so that a clean surface of the unoxidised metal was always presented to the descending wire, thus making the contacts always for the same length of time. With this arrangement, using a little over two amperes, no difficulty whatever was experienced in keeping the galvanometer absolutely steady.

The next problem was that of rating the fork. The first method tried was that of a synchronous motor operated by the current used in running the fork. A wooden wheel with eight soft iron bars in it symmetrically located near the edge, was mounted on adjustable bearings so that the bars came in turn between the poles of an

electromagnet excited by the tuning fork current. This wheel was speeded up until it came in step with the current and then would continue running as long as the fork vibrated. If the nearest iron bar passed the magnet a little ahead of the making of the circuit, it would be pulled back thus slowing up the wheel. If it were a little slow it would be speeded up by the momentary current, and so kept in synchronism with the vibrations of the fork. This at first seemed to afford an easy method of getting the frequency of the fork as it made eight complete vibrations for every revolution of the wheel. However, after running a few times nothing could be done with it, probably because of a very slight residual magnetism in the soft iron bars.

The final arrangement for rating the fork was one in which a combination of Lissajous' figures and Michelson's stroboscopic method were used. A very light piece of paper was attached to one prong of a fork of 320 vibrations per second, and a small mirror was attached to the fork used to charge and discharge the condenser. This mirror of course remained on the fork throughout all the experiments. The second fork vibrated vertically and the first one horizontally. When a light was placed behind the hole in the paper on the comparison fork, and the two set vibrating, the figure compounded of the two motions was observed in a telescope. This figure corresponded to the ratio 1:4, and returned to the same phase approximately two and one-half times per second. The

integer nearest to twice the frequency was determined from these values as follows. Let N be the number of vibrations the slow fork makes during the time necessary for a return of the same phase of the figure, and N' the number of vibrations of the fast one in the same time. Then $N' = 4N \pm 1$. This occurs approximately 2.5 times per second, or

$2.5\ N = n$, the frequency of the slow fork, and

$2.5\ N' = n'$, the frequency of the fast fork.

Then $n' = 4n \pm 2.5$, or

$$n = \frac{n'}{4} \pm 0.6$$

The fork was then loaded with a small piece of wax, and it was found that the figure returned to the same phase less frequently, indicating the positive sign as the correct one. For when slowed down it differed less from 80, one-fourth of the frequency of the comparison fork. This latter fork was compared with a Koenig fork and found to differ from frequency stamped on it, by less than one-tenth of a vibration per second, which does not effect the value of n as determined here. Hence

$n = 80.6$ vibrations per second, approximately,

or the nearest integer to twice the frequency is 161.

The final determination of the frequency was made by a stroboscopic method, as shown in Fig. (3). A telescope was placed in front of the fork and focused, normally, on the vibrating edge. A small Geissler tube was placed end on behind this edge, and was flashed every

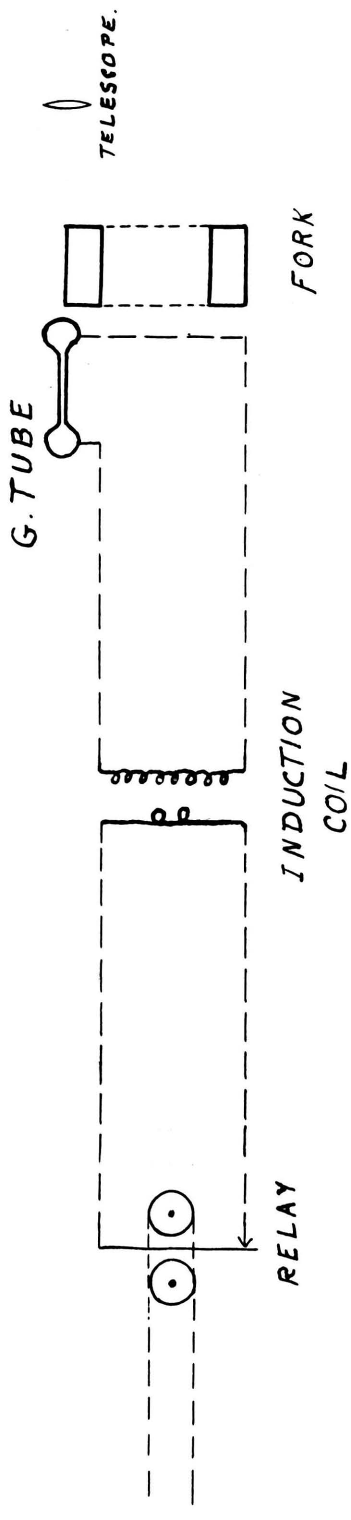

Fig. 3.

two seconds by means of a small induction coil. A chronometer was used to operate a relay, which made and broke the circuit through the primary of the induction coil. When the fork was vibrating, the entire flash would at times be seen on looking through the telescope. It would have been seen to the same extent every time, if twice the frequency had been an integer, as the tube was flashed every two seconds. However at some times no flash would be observed and again a fraction of it might be seen. It was found that a little more than two flashes, or four seconds, were required for a complete return, or, more correctly, it made 10 cycles in the time of 21 flashes, giving 2.1 flashes per cycle. Other trials gave values of 2.11 and 2.17, or an average of 2.13 flashes per cycle.

The nearest integer corresponding to twice the frequency of the fork used in charging is 161. Now if twice the true frequency is, say 161.1, there would be 10 flashes during the time required for a complete cycle. The same time would be required for the return if the frequency were 159.9. Evidently there must be subtracted from or added to twice the approximate frequency, a number which is the reciprocal of the number of flashes per cycle. Thus if n' is the frequency of the fork, and 2N is the nearest integer to 2n',

$$2n' = 2N \pm \frac{1}{a}, \qquad (12)$$

where a is the number of flashes per cycle. It was found that _a_ decreased when the fork was very slightly

(slightly) loaded. Then the negative sign in equation (12) is the proper one, since n' decreases on loading, and 1/a is found to increase. Substituting in (12), n' was found to be 80.266 vibrations per second where the unit of time is that of the chronometer, siderial seconds. Reducing to mean solar time we have,

$$n = 80.491 \text{ *}$$

vibrations per second.

Wolff resistance boxes were used in the measurements and all of the coils were checked with a new Wolff Wheatstone bridge. A Leeds and Northrup one micro-farad condenser which had been in service several years was the capacity measured. The following results were obtained; equation (10) being used in the calculations.

B	A	D	Cap.in M.F.	Diff.from mean.
5000	200	513.0	.9544	.0004
5000	100	257.9	.9540	.0000
5000	50	129.5	.9541	.0001
2000	50	323.0	.9536	-.0004
5000	50	129.5	.9541	.0001
5000	75	194.0	.9541	.0001
5000	100	258.0	.9537	-.0003
3000	100	429.5	.9538	-.0002
3000	75	322.5	.9544	.0004
3000	50	215.5	.9540	.0000
Gal. Res = 120 Ω		Mean	.9540	.0002
				or .02 of 1 %.

* Michelson, A.A. Phil. Mag. Vol.15, page 84.

It was found after a number of trials that the above proportions of the resistances were the best for the experiment, and that balance could be determined most accurately with this arrangement. The accuracy of the results is about that to which the resistances are known, though errors of the same sign tend to offset each other, as is seen from equation (11).

Aside from the ordinary requirements of a carefull Wheatstone bridge measurement, there are several special things to be taken care of. The most of these have to do with the fork which charges and discharges the condenser. The planes of vibration of both prongs of the fork must very nearly coincide with the plane of the fork as a whole; the fork must have a comparatively large amplitude, and not be too light. Copper wires seem to be the best for contact with the mercury cups, and must be stiff, as well as small and pointed. When these things are properly provided for there should be no trouble in obtaining the desired accuracy.

BIBLIOGRAPHY.

Carhart and Patterson, "Electrical Measurements".
Paragraphs 107-109.

Rentschler, H. C., "Laboratory Eperiments in Electrical Measurements". #23.

Thompson, J. J., Electricity and Magnetism".
Paragraph 263.

Watson, W., "Practical Physics".
Paragraph 227.

Grover, F. W., "Measurement of Capacities", Bulletin Bureau of Standards, Vol.3. #3, Reprint 64.

Michelson, A. A., "Rating Tuning-forks". Phil. Mag.
Volume 15. p.84.

Reed, J. O.," Rating Tuning-forks". Physical Review.
Volume 12. p.279.

Rayleigh, Lord, "Phonic Wheel". Phil. Trans.
1883.

Starling, S. G., "Electricity and Magnetism".
Page 393.

Maxwell, Clerk, "Electricity and Magnetism". Volume 2.
Paragraphs 755-776.

378.7M71
XSh1

University of Missouri - Columbia
010-100944141

378.7M71
XSh1