PRACTICAL PROBLEMS DEPENDING UPON PRINCIPLES OF GEOMETRY

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Preface.

A personal interest in the problem of increasing the efficiency of geometry teaching in the secondary schools led to a study of Real Problems Depending upon Principles of Geometry. The paper is more of a resume of a plan that was tried in the Mound City, Missouri High School, than any attempt to lay down a plan for others. If the following plan is taken up by other teachers it should be done only with the idea of using the plan in a suggestive way. Problem material and class possibilities will vary according to locality and time. The teacher must depend upon himself for interpretation of local conditions and source of problems, and not be content with the type problems given in the paper.

I am indebted to Dr. L. D. Ames, Assistant Professor of Mathematics, University of Missouri for his courtesy in reviewing the manuscript, and to Dr. J. L. Meriam, Professor of School Supervision, University of Missouri, who has been a source of inspiration and help to me in the preparation of the paper.

Practical Problems Depending upon Principles of Geometry.

Introduction.

One of the recent questions in education deals with the efficiency of our secondary schools. The question is of vital importance to all classes of educators, but to no class is it more pertinent than to the High School teacher of Mathematics. Mathematics has held a decided place in the courses of study in the past because of the supposed value it gave in formal discipline. A change in our conception of formal discipline, together with other causes, has brought about the violent agitation of the question of how to present High School mathematics, the arrangement, the amount and the kind for each grade. Although the decade just drawing to a close has been marked by the publication of improved texts, by the introduction of improved methods, by the interest that has been manifested by some of our best thinkers, and by the propagation of organizations for the study and discussion of all High School mathematics, the fact still remains that High School teaching of Mathematics has not reached satisfactory efficiency.

Geometry, probably better than any other subject in Mathematics, will help in reasoning, induction, correlation, space intuition, precise thought, accurate expression, observation, neatness and exactness.[1]

For two thousand years Geometry has assumed an almost crystallized form due partly to the belief that Mathematics is a perfect science and partly to the careful and thorough study of Euclid. Although Geometry in its early growth was developed from

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1. Young, The Teaching of Mathematics. Page 259.

a practical problem, arising out of the needs of the early Egyptians for a plan of measuring land, it was taken up by the Greeks, who loved to theorize and speculate. It was a fruitful field to them - one based entirely upon assumption with the liberty to discard any and all things that did not agree with the primary assumptions. They worked out a beautiful theory which was systematized by Euclid. With a very little deviation Euclid is the text used in English schools to-day, and with only a trifle greater deviation in American schools.

The basis of any subject should be understood. In any study in Mathematics there is a need for exactness but probably no more so than in Geometry. Geometry is founded upon a series of assumptions, called definitions, axioms and postulates. These assumptions mean whatever the users decide upon. It is easy to see that these assumptions might change from year to year.[1] In the last few years many teachers and authors have tried to get away from the pure Euclidean idea and back to the fundamental beginnings of Geometry.

Any change is attended by much waste of time and varied discussion. While several texts have intended to make a change in their subject matter and method of presentation custom has held them closely to the old. Some of the thinkers along the lines of Mathematics have attempted correlation with physics,[2] astronomy and other subjects. Others have tried the heuristic[3] plan, and others

1. Smith, The Teaching of Elementary Mathematics. Page 257.

2. Iles, My Class in Geometry,

3. Hopkins, Plane Geometry.

the plan of induction,[1] while others have advocated a Geometry that was more concrete, practical,[2] inventional[3] and empirical.[4]

The teachers of Mathematics, and those who have studied Mathematics and child psychology are convinced that a change is necessary and we find the more progressive teachers devoting a part of their time to trying to formulate a plan that will enable us to present Geometry in an empirical way at first and as the child grows more mature the logical reasoning.

In order to do this it is necessary to understand the child, the subject matter and modern methods of instruction.

The first part of this thesis will be a discussion of the present day tendencies as shown by current texts and literature; the second part will be an outline of work done by pupils in the tenth grade in Geometry in the Mound City, Missouri, High School, which will be the main part of the thesis; while the third part will be a discussion and criticism of the second part.

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1. Sanders, Plane Geometry. Shutts, Plane Geometry.
2. Harrison, Practical Plane and Solid Geometry.
3. Spencer, Inventional Geometry.
4. Warren, Experimental and Theoretical Geometry.

Chapter I.

A brief exposition and criticism of present day thought and tendencies relative to Geometry as seen in texts and current literature.

Exposition.

ORIGIN AND DEFINITION.

Geometry was born of a practical need. The Egyptians were a practical people. They were not interested in Geometry for its own sake as the Greeks later were but because it would be an aid to a practical need. They needed Geometry to measure their land, plan their pyramids and other buildings. The word Geometry is taken from γη, the earth, and μετρειν, to measure, and meant to the Egyptian what our word surveying means to us to-day. The early beginnings of Geometry then grew out of the concrete. If we believe in the culture epoch theory it is worth while to remember the origin of geometrical science. As development of the individual omits the non-essentials that the race experienced it is well to carry a certain amount of the theoretical along with the practical, and to co-ordinate some of the practical with the theoretical.

Most of our experiences are concrete arising out of the practical life of the individual. And since Mathematics is supposed to be an exact science, clear and lucid in its subject matter, it is necessary for us to define Geometry in specific, definite terms. We understand definitions and terms as they fit into our experiences. A definition should be a working tool; built up out of possible experiences or easily attainable experiences of the student. In order to get a definition that is plausible or at least one that is generally accepted, the following are taken from current texts:

1. " Geometry is the science of space." [1]

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1. Phillips and Fisher, Elements of Plane and Solid Geometry. Page 1

2. "Geometry is a vector subject. Vector is the general term used for all directed quantities like displacements, velocities, forces, and similar things, which are compounded by or subjected to the law of parallelograms.[1]"

3. "Geometry is the science which treats of position, form and magnitude.[2]"

4. "Geometry is the science which treats of the properties of continuous magnitudes and space.[3]"

5. "Geometry is the science which treats of the properties, the construction, and the measurement of geometrical figures.[4]"

6. "Geometry treats of the properties, construction and measurement of geometrical figures.[5]"

7. "Plane Geometry is that branch of Mathematics in which are considered the properties of magnitudes lying on the same plane.[6]"

8. "Geometry is the science which treats of the properties of geometrical magnitudes.[7]"

9. "In Geometry he is to study form.[8]"

10. "Geometry is the science which treats of the properties of space.[9]"

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1. Harrison, Practical Plane and Solid Geometry. Pages viii and 124.

2. Wentworth, Plane and Solid Geometry, Page 3.

3. Durell, Plane and Solid Geometry. Page 11.

4. Sanders, Elements of Plane and Solid Geometry. Page 9.

5. Wells, Plane and Solid Geometry. Page 7.

6. Bush and Clarke, Elements of Geometry. Page 12.

7. Schultze and Sevenoak, Plane and Solid Geometry. Page 2.

8. Beman and Smith, Plane and Solid Geometry. Page 1.

9. Halstead, Elements of Geometry. Page 8.

PURPOSE. There are really two reasons for the study of Geometry. The first or practical as readily seen in physics or architecture, and the second the cultural.[1]

Observational Geometry should deal with material that will easily find a place in common things and yet include accurate geometric concepts. The constructing of neat and accurate figures has its value in forming correct habits of conduct. The exercise in logic as a means of mental training, as a discipline in the habits of neatness, order, diligence, and above all honesty, since Mathematics must be right or wrong is worth while.

Pedagogy[2] must play its part in Geometry teaching. The student as well as the teacher must bear in mind the specific end and the means of reaching that end. The student should always understand why he selects a certain process. In material construction the boy is taught why he uses the try-square at one time and the T-square at another time. In constructing a house the student sees how the building goes up, and in the same manner he should see how he build up his Geometry. If he knows the purpose, he stands a better chance to be a discoverer. "The pupil must know from the beginning what is aimed at, if he is to display and employ his whole strength in the effort of learning; and he will employ it provided he knows definitely what is to be reached. To lead him up unconsciously by question and task whose purpose he does not clearly see, has the disadvantage that neither a free rising mental activity nor a clear internally connected insight takes place.[2]"

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1. Smith, The Teaching of Elementary Mathematics. Page 237.

2. A. L. Baker, Purposive Geometry, Sch. Sci. and Math. 1906, P.511

This means that the student should see some reason for the assigned task.

VALUE. A great deal has been said about the practical side of Mathematics but it has a cultural value as well. Lately there appeared a series of articles in School Science and Mathematics entitled "A Cultural Course in Mathematics"[1] in which graphic plotting of school attendance, sensible representation of various activities, and various economic problems growing out of statistics were emphasized. This cultural training reaches its highest efficiency through the practical.

Of the various branches of Mathematics, probably Geometry more than any other may be better used to develop the powers of observation and retention. All construction work requires Geometry. This early work in drawing demands geometry, and children naturally like to make things. G. Stanley Hall has collected evidence to show that boys like to do constructive work and girls have the same tendency as is shown in their making of doll dresses. If this characteristic is present it is not only economic to make use of it but also to develop it, in our school work. Although secondary education should stress the cultural side, this cultural development should bear a close relation to the material. Although Mathematics has been considered a classic in cultural development probably its greatest fitness in doing so is shown in making it of practical value. Geometry teaching[2] should be freed from the restriction of college entrance requirement and put on a basis of utility. The existing deductive and demonstrational methods should be replaced

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1. T. M. Blakslee, Culture Course. Sch. Sci. and Math. 1906 P.133.
2. Pearson, Practical Application of Geometry, Nature. 1891. P. 273.

by the observational and inductive. The latter brings out the individual initiative in the discovery of new truths something that each pupil may work at, while the first is pure logic and beyond many children. The one offers the incentive to usefulness and the other dulls because of its abstruseness.

"The modern development of graphical and geometrical methods has placed a powerful instrument of calculation and investigation in the hands of those who have neither the time nor the opportunity to handle the abstruse tools of analytical mathematics.[1]"

Practical problems are continually coming up and have for a basis the principles of elementary geometry. The engineer has to deal with the questions of force and motion, strength of materials, the architecture of houses and bridges; he has to figure on excavations, gradation and other engineering problems. The farmer, the carpenter, the mason and the other artisans are confronted with problems each in his own particular field.

At the present there is a decided tendency to emphasize the practical and the concrete; leading up from the known to the unknown. Geometry instead of being a form of pure logical reasoning is a psychological arrangement of facts that have or may have presented themselves to the child. The arithmetical computations are not to be pretty puzzles but emphatic in their practicality. The graphical idea, or mechanical drawing is becoming more and more pronounced. Only a few years since Euclid was taught in

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1. Pearson, Nature, 1891, Page 273.

English schools without ruler, compass or protractor - to-day we never think of trying to teach geometry without ruler and compass, and by some the protractor is early used and squared paper by many.[1]

In order to easily get the pupil to grasp the graphical idea, we are falling away from the old idea of teaching mathematics in "water tight compartments," as brought about by the narrowness and limited knowledge of the teacher, and instead find the elementary principles[2] as they may be presented in any mathematics. Myers in his little book, "First Year Mathematics", has attempted this, and Bailey and Wood's book, "Course in Mathematics", does the same for algebraic equations, analytics, conic sections, differential and integral calculus, and differential equations. This movement has in mind the presenting of material at the stage at which it meets the child's development. Subjects[3] in business and social life have a relation. School work in English is the same from one years work to another except as to gradation. In a large measure this is true of other High School subjects; why should there not be a relation between the mathematical subjects? In this arrangement the strictly philosophical could be left to more mature years.

1.E. H. Moore, Foundation of Mathematics, Sch. Rev., 1903. P.521.

2.Klein, "In all domains of mathematics those parts are to be called elementary which can be understood by a pupil of average ability without long continued special study." Quoted by E. H. Moore, Foundations of Mathematics, Sch. Rev. 1903. Page 521.

3. Compare reference 2.

Although[1] for many years mathematicians have denounced our methods of teaching, it is only lately that the real practical problem has been given a place at all. A late definition for education is to make one useful and happy. Following this idea usefulness should be a factor in determining what should be taught in geometry. People outside of a subject are liable to consider some of the things set forth in the subject as being foolish or unnecessary - or the results are not understood or useful to them. Work in pure mathematics should offer something useful to the physicist, astronomer, or other profession, or to the artisan, other than mere principles from which to make deductions. This does not mean that a pursuit of pure knowledge may not have an educational value of its own, but in addition to the mental and emotional powers produced other things should follow. The principles of geometry supply the physicist and engineer with their fundamentals. The study began because it was useful and continues because it is useful, and is valuable to the world because of the usefulness of its results. The pure mathematics must allow one to think along his own line. In this way one will get a mental training as well as mathematical knowledge.

The following is an outline by John Perry[2] of England, as to what the course of study in mathematics should do:

1. Produce the higher emotions and mental pleasure, hitherto neglected in teaching almost all boys,

a. In brain development.

b, In mere mathematical study.

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1. John Perry, Teaching of Mathematics, Educa. Rev. 1902. Page 158.

2. Compare reference 1.

2. Aid in giving mathematical weapons in the study in physical science, hitherto neglected in teaching almost all boys.

3.Aid in passing examinations. The only form not really neglected. The only form really recognized by teachers.

4. Help in giving men mental tools as easy to use as their legs or arms; enabling them to go on with their education throughout their lives, utilizing for this purpose all of their experiences. This is exactly analogous with the powers to educate one's self for the fondness for reading.

5. To teach the importance of thinking things out for himself and so delivering him from the yoke of authority and convincing him whether he obeys or commands he is one of the highest beings. This is usually left to other than mathematical study.

6. Make men in any profession of applied science feel that they know the principles on which it is founded and according to which it is being developed.

7. Aid in giving to acute philosophical minds a logical counsel of perfection altogether charming and satisfying, and so preventing their attempting to develop any philosophical subject from the purely abstract point of view because of the absurdity of such an attempthhas beenme obvious.

The value[1] lies in doing and not in knowing. "A student's ability to prove a proposition is no assurance that he knows it. The test as to whether he knows it is whether he can do it.[2]"

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1. Clara Hart, Teaching of Geometry, Sch. Sci. and Math. 1905. P 649
   E. S. Loomis, Teaching of Mathematics in High School School Science and Mathematics. 1900, Page 102.

2. Osgood, quoted by Clara Hart in Cf. Reference 1, Pages 649, 717.

A subject that is purely theoretical does not give the student much chance for doing. If the teaching is thoroughly objective, the student's activity is called into action and developed. One of the difficulties in mathematical teaching is the lack of the knowledge of English. Lack in thorough training in English is the reason that originals are so difficult. And ability to do originals is the true test of a student's knowledge of geometrical fundamentals. As[1] many originals should be done as time will permit. The originals to be chosen so that they have idea and common sense, and covering the field of the student's experience. Mathematics has and ought to have humanity in it. If a student plots graphically, or constructs out of material the figures there is no doubt but that he and the teacher both know when the student has a correct idea of the proposition.

There also seems to be a tendency[2] at the present day to quit teaching geometry as though every student was to become a teacher of geometry. The method of instruction is worth attention.

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1. B. F. Brown, Mathematics in Secondary Schools. Sch. Rev. 1903. Page 292.

A definition that in a measure covers the above paragraph is: "Geometry is the science of measure for the determination of relations of magnitudes either ideal or practical, and is therefore one of the best out of the many instruments that man has for overcoming his environment and becoming master of it; through which process he declares his individual personality, the ultimate object of human effort."[a]

a. E.S.Loomis,Teaching of Math. in H. S. Sch.Sc.and Math.'00.P.102.

2.John Perry, Teaching of Math. Educa. Rev. 1902. Page 158.

The man who teaches geometry must keep in mind hundreds of propositions, corollaries, and problems of which only a few are used in practical life. The man who does not teach needs to know a few propositions and to know them so well that he can apply them at any time that the problem requiring these principles is presented. Therefore the student who expects to teach should do a greater amount of work, but not necessarily different work, than the average student of geometry.

METHOD. According to magazine articles and discussions at mathematical organizations one is led to believe that there is a growing tendency to study the child. We wish to know more about his social experiences and his psychological nature. Due to this fact and to the desire to co-ordinate the work with the student's present equipment, we have found men who were willing to spend much time in studying the child - in order to get his view-point.

Put[1] the student to work as soon as possible with the ruler and dividers. Let him prove in the ealy part of the work by cutting out paper and applying the superposition idea. In this way he cannot help but get the principle and in a little while use his imagination in the application. In the study of the area of the triangle use squared paper, also in trapezoids it is easy to see that the area is equal to one half of the sum of the bases by the altitude; and many other examples in mensuration. Why not make use of ratio and similar figures to connect geometry and trigonometry. This idea does not mean the theoretical is subservient to the practical, but that the two go hand in hand.

1.Tinsley, Individual Instruction in Math. N. E. A.1905. Page 466.

Work in construction problems should be emphasized. Do not be content with solving only, but have the constructions carefully worked out on paper with ruler, compass, squared paper and protractor. In construction work all figures unless meeting special conditions should be general as this gives breadth of view. At this point give plenty of numerical problems based upon the present day needs of the student and having a real interest to him. Geometry is a new thing to the student and the concrete numerical problem will hold his interest until he gets a better knowledge of geometry. The High School age is the age at which the boy or girl likes to be doing something. First impressions should be made use of A real need of the problem being shown gives a strong incentive to work. The work should be grouped about types. We should always bear in mind that definitions, rules, axioms, and propositions are not ends but aids to an end as usually shown, but if we can show at the same time that there is a real need for the problem, so much more is gained. We are also to remember that a proposition or definition is merely some one's way of stating a result derived from observation or a course of reasoning. Though some of the suggestions seem contrary the teacher should not do with out a text. The text is an economical basis for assigned work. The mathematical collection of facts should be exemplified in the practical world. Model making, another from of paper folding, is worthy of consideration, if we actually construct instead of merely supposing. In[1] mensurational geometry the balance can be used to an advantage if one is careful to use homogeneous material, or squared paper finds a use in determining laws of area. Drawing to scale, a

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1. J. C. Packard, Co-ordination of Mathematics and Physics, School Review 1903. Page 791.

practical problem that confronts the contractor and builder, measurement of inaccessible distances, a study of parallelogram of forces, inclined plane, or images in a plane mirror together with tl elements of surveying are some of the features of the new geometry. In addition to paper folding the constructions actually used in the manual training department or the building of a house mean more, especially in solid geometry than all the drawings you can make. Instead of computing by increasing the sides of an inscribed polygon use the vernier and deduce concrete problems from the laboratory apparatus concerning the area of a circle, volume of cylinder, and cone, as found in circular mils or the diameters of wires and the common base ball. A visit to a building to study architectural form not only adds an interest, but "develops a sense of the beautiful in geometrical construction and a love for symmetry." This stirs up an interest and a love for the subject and destroys mechanical and routine work and instead gives us brain work. This widens the scope of geometry by taking it out of the field of pure deduction as some of the propositions can be assumed or algebraically treated. A slight reference to the historical development of parts of the field of geometry or to an incident concerning a proposition can only add a human interest.

Some writers[1] extending the plan based upon the definition for an axiom would build up an individual geometry. This permits the child to draw from his own resources and arrange his own geometry. Following this plan many propositions, corollaries, postulates, and axioms would be accepted without question. Students who begin deductive geometry have had no experience with logical deductions as such. They have been in the habit of

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1. E.H. Moore, Foundations of Math. School Rev. 1903. Page 521.

convincing themselves by all the resources at their command, that certain statements are or are not valid. It is bewildering to them to prove a statement by means of certain other statements, and to be careful not to let intuition enter into the proof in the slightest degree. Not only is the subject new but the activity is new. The proof that right angles are equal[1] is foolish to a student fourteen years old. The proofs in the beginning are not strictly deductive. If each child is permitted to draw upon his own experiences he will then have an opportunity to enrich his own field, and while his set of propositions might differ from his neighbor's the different "discussions arising would serve to make clearer the function of axioms in geometry"[1]. Our field while limited by this idea for deduction, makes wider the field for observational,[2] inventional, heuristic, and constructive geometry. This plan omits the thoroughly abstract and relegates the subject to the status of the beginner. Instead of attempting to define geometry, line, angle, and other terms give the student a chance to use his knowledge of these same principles. In the industrial world the natural way to do a thing is not to explain the implement and its method of operation but to put the implement to use and learn by using.

In a measure[3] the mathematical teacher of to-day is an experimenter, his class room his laboratory, and the new methods his apparatus. In a measure this permits the work to be individual.

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1. Phillips and Fisher, Elements of Pl. and Sol. Geom. Page 9.

2. Using each child's experience.

3. Jones, Mathematical Conference, N. E. A. 1905. Page 466.

A few[1] schools have tried correlation with success but a greater number seemed to have failed in the plan. Co-ordination has a place if rightly presented. The work in crystallography, optics and botany have elements in common with elementary geometry. Astronomy has principles that are found in solid geometry. The work can be made real and a human interest aroused with problems from mechanical drawing, domestic science and architecture. Several of the texts seem to realize that there should be a correlation between geometrical and physical solids by an attempt to make the figures in solid geometry more attractive to the eye and more real, by using color, making them more easily interpreted. When we come to a problem, as the construction of a triangle when the three sides are given, why not use the problem of the engineer of bracing as used by bridge builders?

Mathematics is in a large measure a subject calling for rare imagination and observation. Many[2] mathematical concepts are the direct product of observation. Mathematics differ from the other subjects in that scientific deductions can be made with certainty from observations, which are not always possible in other subjects particularly in physics and astronomy. Since observations in mathematics can be verified it is a good subject for observation. By following the plan of observation one can exclude all that is not immediately useful to the execution of the ends the child had in view. It will be easier for the student to make his own fundamental principles growing out of his experiences, than to learn them from a text book.

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1. Bradley Polytechnic and Lincoln, Nebraska High School.

2. W. E. Story, Unification of Mathematics in Curriculum School Review 1903. Page 832.

By making the applications deal with familiar objects in which the student is interested he readily gets the desirable concepts, because concepts of variation in intensity of sunshine, temperature, rainfall, the area of a field, and the yield per acre are things that appeal to the student's viewpoint. It is a true education that gets a man to work in an interesting way to him, and the only way to do so is to vitalize his task by relating it to some form of reality. The test of this plan as it may well be of any other is, does it bring out the full working power of every student.

Mr. Chase[1] in his article in School Science and Mathematics presents rather a condensed view of some of the present day thinkers. He would not follow what seems to be the idea of most of the text book writers on geometry. Instead of learning a series of definitions he would merely assume that they were known. Assume that a student knows a straight line, or circle, and let him draw lines to a straight edge and circles with dividers. Drawing a line through the centre is the diameter, while another perpendicular to the diameter at the center is the radius. Questions that arise as to how many arcs or angles are formed, and as how to they compare in size. After testing with compass, cut out and compare by superposition. From this one can lead to degrees, right angles, radii, adjacent angles, vertical and bisected angles. By drawing concentric circles the students easily see that the sides of any angle have nothing to do with its size. By drawing figures with ruler and compass, and then using the idea of paper folding, such propositions as the sum of two adjacent angles, having their exterior sides in the same straight line is a straight angle, the

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1. Chase, How Geom. Should be Learned. Sch. Sci. and Math. '08. P.399.

the angular magnitude about a point on one side of a straight line is equal to two right angles, or by drawing a triangle and measuring the angular magnitude of the angles of the triangle, and the angular magnitude about a point on the same side of a straight line, the sum of the angles of a triangle is two right angles, are easily deduced, and easily followed. By following this plan most propositions will not be presented until there is a need evident then it is easy to fit the proposition and construction into the student's experience.

After[1] an axiom is learned it should be used in the further work. A principle learned to lay aside is not worth learning. The theorem should be so well learned and understood as to be recognized in different relations, as the right angle is a right angle not only in the text but also the corner of the book, the carpenter's square and other places. Such a proposition as " Two triangles are equal if two sides and the included angle of the one are equal respectively to two sides and the included angle of the other."[2], means more to the live boy who sees in it a possibility of measuring inaccessible objects or distances between points not accessible or that cannot be measured on a straight line. This not only adds an interest but also does away with technicalities. He is also putting into practice the tool that he has acquired. The aim ceases to be how best prepare the boy for college but how best help him to realize upon present investments. This prevents our submitting rigorous geometry to immature inexperienced minds.

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1. G. C. Shutts, Hints on Teaching Geometry. Sch. Sci. and Math. 1907, Page 107.

2. Wentworth, Plane and Solid Geometry. Page 35.

MEANS. In addition to a closer study of the child his equipment should be extended. For[1] a class just entering geometry a small equipment of ruler, compass, scissors, protractor, parallel ruler, squared paper, drawing board, and a T-square as well as the text will be very advantageous. In addition to wooden prisms, pyramids, cylinders and cones, pasteboard for construction and sand for testing capacity, a set of drawing instruments, a draughtman compound curve and similar objects should be placed in the student's reach.

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1. J. C. Packard, Co-ordination of Mathematics and Physics. School Review 1903. Page 798.

Criticism.

DEFINITION. The definitions of the subject or science given above are in most cases not clear, are abstract and in several cases wrong, or so badly overlap other subjects as to be actually confusing to the student. For example can we agree that it is a science according to the definition commonly accepted? For most purposes it is a science but in some respects also an art. The axiom of parallels is wrong according to the current definition of axioms. If we would permit maxima and minima, limits, and incommensurables, the definiteness or exactness of some elementary problems might be questioned as in the metricity of the diagonal of the unit square.

Again do the definitions as given by the texts cited earlier give a mental picture? Most psychologists think that we should work from the concrete to the abstract. Do these definitions furnish a good start? Is the question of the attitude of the student permitted here?[1] An axiom is often defined as a self evident truth or a straight line as being the shortest path between two points.[2] This mixing of terms is often more confusing than the redundancy in definitions. It is only necessary to take a few of these definitions in the texts and then notice some of the things called for in "introductions" and tendencies as shown by the late discussions and writings, to illustrate the point in question.

In Phillips and Fisher's Elements of Geometry the only definition for space attempted gives but a single property of space, extension. Halstead in Elements of Geometry defines a

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1. B. F. Brown, Math. In Sec. Schools. Sch. Rev. 1902 Page 292.
2. Phillips and Fisher, Elements of Geometry. Page 4.

straight line as "a line that pierces space evenly, so that a piece of space from along one side of it will fit any side of any other portion." He also defines a magnitude as being" anything which can be added to itself so as to be double." One can readily see that the mathematicians themselves are uncertain in their own mind as how to define certain terms. Phillips and Fisher define a circle as being "a plane figure bounded by a line all points of which are equally distant from a point from within called the center." Halstead defines a circle: "If a sect[1] turns about one of its end points, the other end point describes a curve which is called a circle." He also defines an arc as being a part of a circle while Wentworth and most all other writers of elementary geometries define an arc as being a part of the circumference. Dodge[2] gives the same definition to the circle as Halstead. A serious trouble confronts us when we go to find the area of the circle, especially when we remember that Mr. Halstead beleives in such rigorous proofs from the beginning as he has often said in his various discussions. A more extended study will show other striking definitions, some of which will be taken up later.

To define elementary geometry as treating of lines and surfaces or by similar statements is not clear as some of our work in drawing is the same, and the High School student in Chemistry has found that a study of crystallography is nothing more than a study of lines, angles and surfaces. A. L. Baker[3] says to define

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1. Segment of a line.

2. Dodge, Elements of Geography, Page 4.

3. A.L.Baker, Purposive Geometry, Sch. Sci. and Math. 1906. P. 511.

"Geometry as the science of space simply stuffs with words and deludes him[1] into thinking he has learned something." The space concept is beyond the ready realization of most students of the tenth and eleventh grades. To define space in the eleventh grade in the course in physics is so difficult that most books and teachers do not attempt a definition. Starting then with an abstract indefiniteness cannot lead the student anywhere; if he is driven he goes blindly and is much confused.

The principal objection to the definition of what is geometry is that it disobeys the subject mathematics, which calls for definitions in clear definite terms. In Beman and Smith's New Plane and Solid Geometry how is one to distinguish between geometry and drawing or painting, or art when the authors say that Geometry is the study of form. In the study of archaeology one is ever aware that form is an emphatic principle to be ever kept evident. And then again geometry does not stress form as this definition would indicate, instead it is a secondary object, as a careful study of the text will reveal. A rectangle, a triangle or any other figure may have a hundred different forms as to length, breadth or position and still meet the primary conditions laid down for definitions. The student never pays any attention as to whether a triangle is narrow, broad, short, high or low. Instead we note whether the triangle or rectangle meets certain requirements as whether the triangle is right, isosceles or scalene and the length, breadth and the right angle of the rectangle. Not a single geometry makes use of the elementary principle of similarity of right triangles as early used in trigonometry. Whether it is

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1. The child.

necessary to make use of this principle in geometry or not is not the question at this point, but if geometry is a study of form why should not this elementary principle receive at least passing notice by one or more writers on geometry?

Again the definition "Geometry is the science of space", unless you modify the generally accepted meaning of space, means nothing. Our study of elementary geometry deals generally with metrical elements, as the non-metrical usually belongs to projective geometry, a subject found in the college curriculum. And again the science of space is treated in a manner in physics, astronomy, non-Euclidean geometry and other subjects. The fifth definition "Science which treats of the properties, the construction and measurement of geometrical figures," is not clear as the word "geometrical" bears with it no meaning to the ordinary student. Possibly the most serious objection to the definitions as a whole is their generality. Each definition grasps at the whole thing. While it is true that the chils gets the concept "house" before the percept "door", "window", "wall", or "other parts", the concept "house" is a principle that he is most familiar with from the standpoint both of sight and kinaesthetic sensations, while geometry as usually presented comes to him at the best in a vague way, and entirely outside of his experiences. He is given the abstract, logical reasoning of the mature Greek philosophers at the age when he is immature and concrete.

The eighth definition, "Geometry is the science which treats of the properties of geometrical magnitudes," is hard for the young boy or girl to find consistent with the teacher's statement that a point has no size, a line nothing but length, and a surface, no thickness.

Definitions of the subject matter are no more confusing than the definitions and statements concerning other parts of the elementary books in geometry. The order of development of the thought is not consistent, with the subject matter, and demonstrational, constructive, and mensurational geometry are mixed together in a confused mass.

Geometry is truly logical hence hard for the student. To make it meet the student's needs it must be made concrete and real in a measure. In order to make it real it is not advisable to vary from the truth as "an axiom is a truth assumed as evident"[1] or "an axiom is a statement admitted to be true without proof"[2], when an axiom is an assumption and may be proved or not approved according to whatever you have in mind. Most of the axioms if true in Euclid are false in non-Euclidean geometry or some other field of mathematics. A thing that may be self evident to one person may be far from self evident to another. To the arithmetical student it is evident that one is the only number that cubed will give one, but not so to the algebraic student who is acquainted with the equation $X^3 - 1 = 0$.

PURPOSE. In Mr. Warren's[3] geometry, he states emphatically that the pupil is led to draw for himself conclusions which afterwards form the subject of the propositions. This plan if followed probably would be very valuable, but in no place does he leave a problem without stating the conclusion conclusively.

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1. Phillips and Fisher, Elements of Geometry. Pages 1 and 4.
2. Wentworth, Plane and Solid Geometry. Page 4.
3. Warren, Experimental and Theoretical Geometry.

He does, however, treat in a good manner possibilities in the use of the compass and squared paper, with other helps.

Bush and Clarke[1] leave nothing for the student in the interpretation of the problem. The work states that it hopes to arouse interest, inspire confidence, and develop ability to discover hidden truths, and then proceeds to make clear all the facts, to such an extent that the reasoning is lost in the memorizing.

In one of the geometries used largely in Missouri schools a straight line is defined as "the shortest path between two points".[2] Smith ably points out that "line is not distance."[3] It is also evident from higher mathematics that the line is not necessarily the shortest. The concept line is elementary and the definition might be easily omitted.[4]

VALUE. A four years course in mathematics where the elements are introduced early would be worth while, because some parts of geometry and trigonometry are more elementary than portions of algebra, and then the student has an opportunity to use these elements in the more abstract work in algebra. Then too, the large per cent of boys and girls who never take the advanced work

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1. Bush and Clarke, The Elements of Geometry.
2. Phillips and Fisher, Elements of Geometry. Page 4.
3. Smith, The Teaching of Elementary Mathematics. Page 258.
4. Pascal:

   a. Do not attempt to define any terms so well known in themselves that you have no clearer terms by which to explain them.

   b. Admit no terms which are obscure or doubtful, without definition.

   c, Employ in definitions only terms which are perfectly well known, or which have already been explained.

in the High School mathematics would have an opportunity of knowing the elements of the more advanced work. If this plan were followed mathematics would probably not be in such disrepute among the electives.

The paper folding, drawing to a scale, use of squared paper have psychological advantage as it gives a student an opportunity to use the hands as well as the eye and the ear. The very fact that he can work these principles out with material things increases the interest and reveals to the student a beauty in the real usefulness. It is possible to go to the extreme in this kind of work the same as in any other kind of work.

Co-ordination should not be such that the work is on the plane of the work in the kindergarten or such as that it is confused with the work in drawing. One may become so interested in the mechanical as to lose sight of the theoretical. Again there is a possibility that the geometry class may become a physics class. It was suggested that students, in a large measure, make their own assumptions, which we can agree with provided of course that we do not permit this plan to waste time and allow the child to go astray. If the work is too much on the individual plan we lose the incentive to interest that a class recitation may give, and prevent the student from learning from his fellow studemt.

METHOD.

Halstead[1] would have us believe that all geometry should be very rigorous in proof, and not to spend too much time on the concrete except as will be readily seen to have a bearing upon the work in hand. He recites from Sophus Lie, "Geometry shall in its different stages so far as possible be established purely geometrically," or Veronese's " The best method suited to teaching

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1. G. B. Halstead, Teaching of Geometry. Educa. Rev. 1902 P. 456.

geometry is the geometric method since it flows from the constitutive process of intuition of space." Halstead himself says, "The student is most likely to become a sound geometer who is not introduced to the notion of numerical measures until he has learned that geometry can be developed independently of it altogether." This notion is subtle and highly artificial from a purely geometrical point of view and its rigorous treatment is difficult.

As an argument to this it might be said we are not trying to make teachers or geometers out of our students, but men and women who are to take part in a varied life in the world. It[1] is both psychologically and logically impossible to bring a child up on such rigorous work as is suggested by Mr. Halstead. If we make the work as rigorous as Mr. Halstead would have us to we might make in a life time one mathematician, Fine as he did, but never reach the masses.

Improvement will have to come along with a more rational formulation and rigorous sequence and by those who look at it from the standpoint of common sense or ordinary experience. It will have to be presented in a way that calls for the experience and the intelligence of the student and thus help to make his mathematics more vital to him. The teaching of mathematics must have enough of the useful that can be readily applied to the plans of every day life to make it worth while or it cannot hope to meet with the highest success.

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1. J. Dewey, Psychology and Logic in Teaching Geometry, Educational Review 1903. Page 387.

MEANS.

"In the discussion of the teaching of geometry the belief has been emphasized that geometry should be taught not as a collection of settled facts to be learned, even though the facts of geometry that are taken up in a first course have in the main been settled for thousands of years, but as a set of phenomena to be investigated scientifically. Geometry is a living and growing science; if it is taught so th at the pupil himself makes some discoveries he must ask questions, he must scrutinize the various possibilities of the topic. Questions may be raised that are too difficult for an elementary course or that open the door for some little account by the teacher of work in geometry, ancient and modern, beyond the scope of the course. The pupil will then come to the end of the course in geometry with possibilities of study still unexhausted, perhaps with some problems still unsolved, and with hearsay knowledge of important lines of geometric study different from those he has followed. He will not regard geometry as a cast iron subject whose sum total is recorded in the book he has studied, but as a large and growing field of which he is a part; and he should look forward with pleasure to obtaining a deeper and more critical insight into the part he has already studied, as well as to extend his knowledge to other parts of subjects.[1]"

The problem that teacher and text-book needs to solve is to teach the student to have his senses alert for possible problems, whose field embraces the principles of geometry.

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1. J. W. A. Young, The Teaching of Mathematics, Page 287.

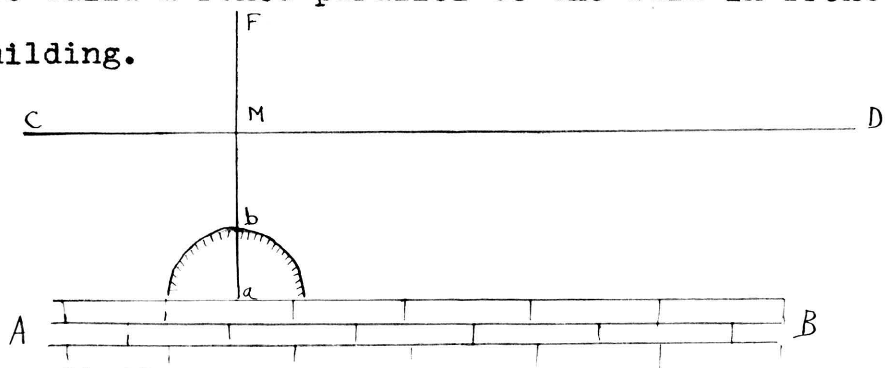
If the student is brought to see that there is a relation between many every day problems and the principles of geometry, he has secured an effective tool for future leverage, and at the same time felt a vital interest in his surroundings, and in his assigned lessons. He sees the need of knowing the principles and realizes the possibility of correctly interpreting many common place problems. Thus will education tend to help one to understand his environment.

Chapter II.

A series of type problems
presented to a class in
Geometry in the Mound City,
Missouri High School
during the year 1909-1910.

Problem:

To build a fence parallel to the walk in front of the school building.



Given the walk AB.

To construct the fence CD parallel to the walk.

Place the straight edge of the protractor along the straight edge of the walk, and mark on the walk the centre of the edge of the protractor, also thake the middle point on the curved side of the protractor, that is the point at the 90 degree angle. Stretch a fine wire EF tight across the two points <u>a</u> and <u>b</u>. EF is perpendicular to AB. When one straight line meets another straight line so that the two angles formed are equal, each is a right angle, and the line is perpendicular to the given line.[1] Choose the point M in the same manner at the distance from the walk at which the fence is to be built and the line CD is constructed perpendicular to the line EF at point M in the same manner as EF was constructed perpendicular to AB.Since CD is perpendicular to EF it is parallel to AB. Two straight lines perpendicular to a third straight line are parallel.[2]

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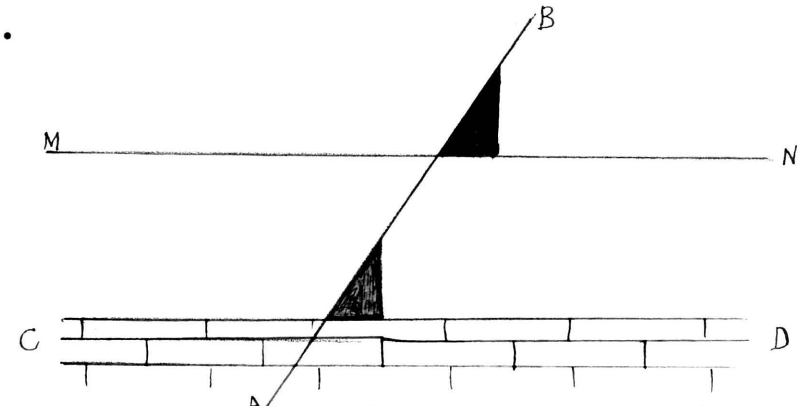
1. Wentworth, Plane Geometry, page 11, Sec. 63.
2. Wentworth, Plane Geometry, Page 24, Prop. X.

This problem was presented by the teacher as were several others in the beginning of the course.

Problem:

To build a fence parallel to the walk in front of the school building.



Use a straight stick with two equal wooden triangles, called a set square. The triangles have their respective angles equal. Let AB represent the set square. Place AB as a transversal on the walk CD so that the lower edge of the upper triangle is exactly above the edge of the walk. Then along the lower edge of the lower triangle draw the straight line MN. MN is the line desired. Two straight lines in a plane are parallel, if no matter how the transversal is placed that the exterior-interior angles are equal.[1] Therefore MN is parallel to the fence AB. If two straight lines in a plane are cut by a transversal, and the exterior-interior angles are equal, the straight lines are parallel.[2]

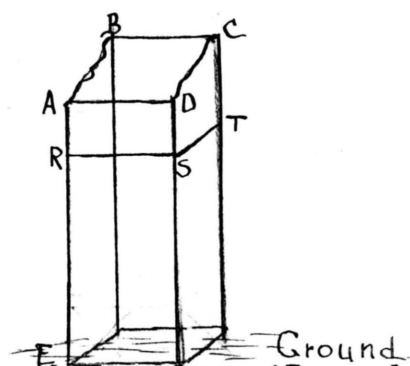
Other proofs were submitted.

1.Bush and Clarke, The Elements of Geometry, Page 23.

2.Wentworth, Plane Geometry, Page 28, Prop.XV.

Problem:

To saw off a post so that the edges of the post are perpendicular to the plane of the top.



To draw a line RS on the surface AB, and ST on the surface DT as a guide for the saw.

At point R on AE, with protractor erect perpendicular RS.[1] Since RS is perpendicular to AE it is also perpendicular to DS. A straight line perpendicular to one of two parallel lines is perpendicular to the other.[2] At point S in DS erect perpendicular to DS in the plane DT. If desired continue the process about the post. Then the plane RST is perpendicular to the edges and if sawed through the plane RST, the top of the post will be level with the ground, or the plane of the top perpendicular to the edges of the post.

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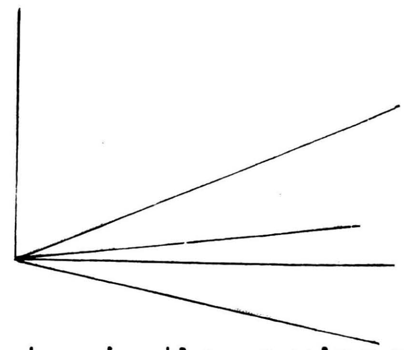
1. Wentworth, Plane Geometry. Page 25, Prop. XI.

2. Wentworth, Plane Geometry, Page 11, Sec. 64.

This problem was suggested by Dr. Meriam.

Problem:

To find a north south line.



Set a vertical post. From ten in the morning until two in the evening take readings every fifteen minutes. The readings are the lenghts of the shadows. At night determine a north south line through the foot of your perpendicular post by means of the North Star. Verify this line with a compass. Then with a protractor draw an east west line. Verify this with a compass.

Table of readings.

Hour	Lenght of shadow.
10:00	68 cm.
10:15	64
10:30	60
10:45	57
11:00	54
11:15	51
11:30	48
11:45	46
12:00	45
12:15	43
12:30	42
12:45	44
1:00	48
1:15	52

1:30	53.5
1:45	57
2:00	61

The shadows were not the same length. Did the shortest line or shadow coincide with the line determined by the North Star? The shadow on the north south line is the shortest shadow. The perpendicular is the shortest line that can be drawn from an external point to a straight line.[1] Considering the rays of light as lines recall that of two unequal lines drawn from a point in a perpendicular the greater cuts off the greater distance from the foot of the perpendicular.[2]

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1. Wentworth, Plane Geometry. Page 21, Prop. VII.

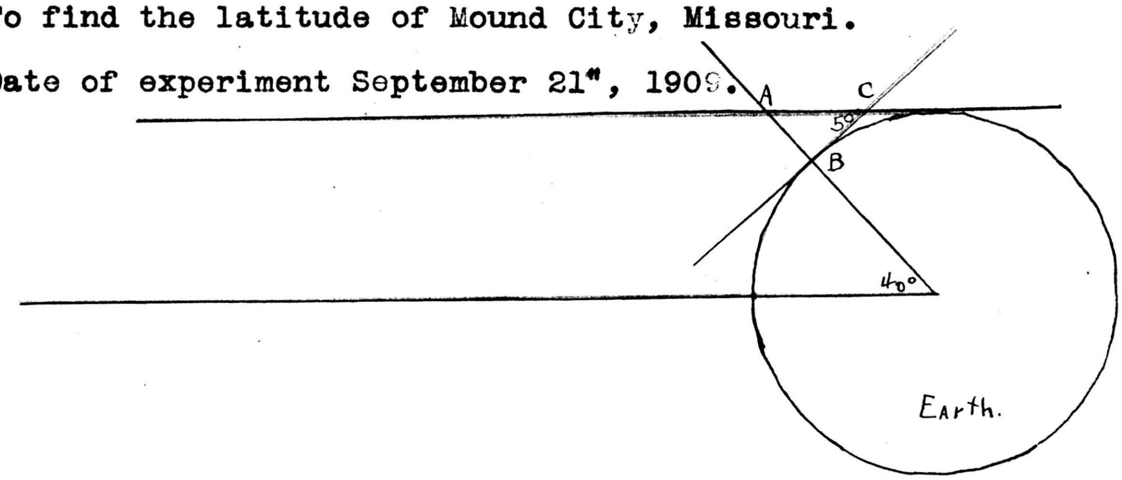
2. Wentworth, Plane Geometry. Page 23, Sec. 102.

Problem suggested by the Principal of the High School. Taken from work in Physical Geography.

Problem:

To find the latitude of Mound City, Missouri.

Date of experiment September 21", 1909.



On the 21" of September set a vertical stake in the ground, so as to use the north south determined in the preceeding problem. Use the north south line for our reading. Readings taken every fifteen minutes from nine thirty in the morning until two thirty in the evening. To take a reading a centimeter ruler was laid on the ground along the shadow, and from the end of the shadow to the top of the perpendicular stake draw a fine wire. With a protractor measure the angle which the wire makes with the ground, that is its projection[1] upon the plane of the ground. From the drawing the angle formed is readily seen to be the complement of the angle[2] that measures the latitude.

Table of readings.

Hour	Angle.
9:30	35 degrees.
9:45	38
10:00	39
10:15	41
10:30	42
10:45	43
11:00	44

11:15	45
11:30	46
11:45	47
12:00	48
12:15	49
12:30	50
12:45	48
1:00	47
1:15	46
1:30	45
1:45	44
2:00	42
2:15	40
2:30	37

At twelve thirty o'clock by the watch, which was noon by sun time we found the angle to be 50 degrees,[3] which made the latitude reading 40 degrees. The correct latitude of Mound City, Missouri is about 40 degrees and 10 minutes, the error due to inaccuracies in the instruments used and in the manipulation of the apparatus.

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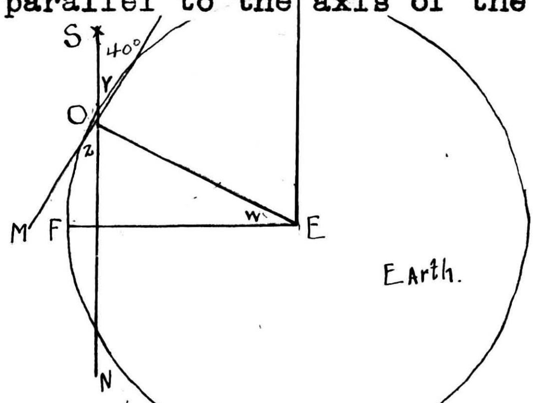
1. Wentworth, Plane Geome try, Page 162, Sec. 374.
2. Wentworth, Plane Geometry, Page 13, Sec. 76.
3. Wentworth, Plane Geometry, Page 26, Prop. XII.

Problem taken from work in Physical Geography class.

Problem:

To find the latitude of Mound City by observing the North Star. Prove that the latitude of the observer is measured by the altitude, angular distance above the horizon. For practical problems the direction of the North Star from the observer is considered parallel to the axis of the earth.



EF bisects the chord ON also arc OFN.[1]

Angle Z equals the angle Y.[2]

Angle Z is measured by one half arc OFN[3] or by arc OF.

Angle W is measured by arc OF.[4]

Therefore angle W = angle Z, = the angle Y,[5] the latitude of Mound City, Missouri. By using the protractor the angle Y is found to be 40 degrees.

O = observer, S = North Star, MO = tangent to surface.

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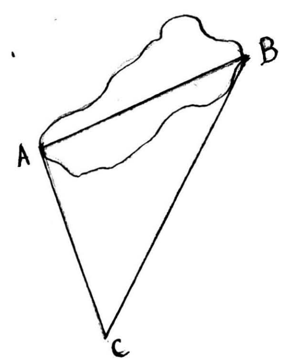
1. Wentworth, Plane Geometry, Page 82, Prop. V.
2. Wentworth, Plane Geometry, Page 18, Prop. IV.
3. Wentworth, Plane Geometry, Page 105, Prop. XIX.
4. Wentworth, Plane Geometry, Page 102, Art. 288.
5. Wentworth, Plane Geometry, Page 6, Art. 34, Ax. 1.

Problem furnished by the teacher.

Problem:

To measure the length of a pond west of the school building.



Given a pond AB.

To find the length of the pond AB.

From an accessible point C measure line CB and line CA, and with the protractor the angle ACB. Line CB is 270 feet, line CA 400 feet and angle ACB 45 degrees. Construct a triangle to scale, using one mm. for one foot. Draw line AC 270 mm. long and at point C construct an angle equal to angle ACB,[1] 45 degrees, and draw line CB 400mm. Connect points A and B. Measure the line AB, which is 290 mm. long and you have the length of the pond as 290 feet. Two triangles are equal if two sides and the included angle of the one are equal to the two sides and the included angle of the other.[2] The triangle reduced to scale that represents the lines measured on the earth's surface and your drawing are equal triangles.

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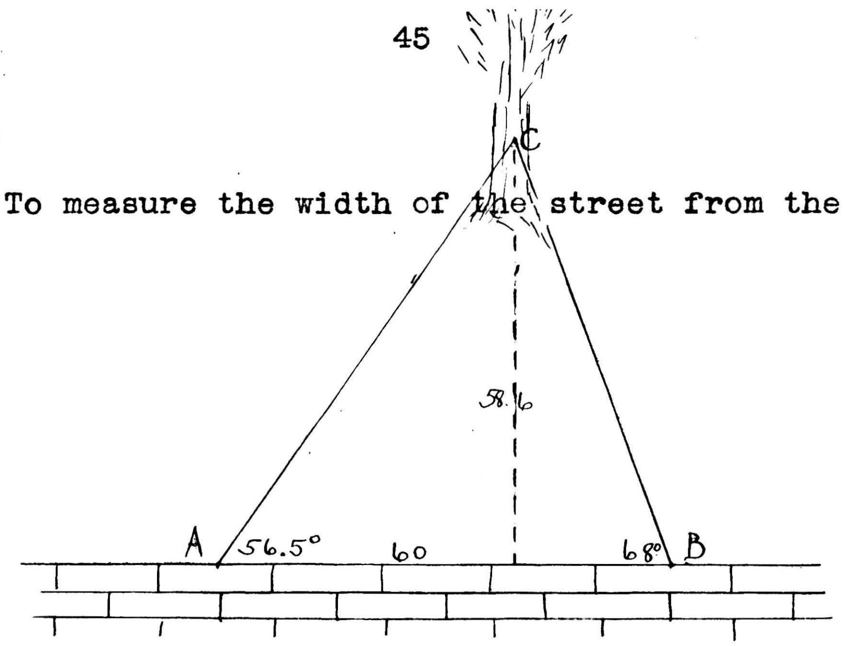
1. Wentworth, Plane Geometry, Page 116, Prop. XXVI. This proposition may be used later.

2. Wentworth, Plane Geometry, Page 35, Prop. XXI.

Problem found by a student.

Problem:

To measure the width of the street from the sidewalk.



Given the sidewalk AB and a tree C, on the opposite side of the street. To find the width of the street.

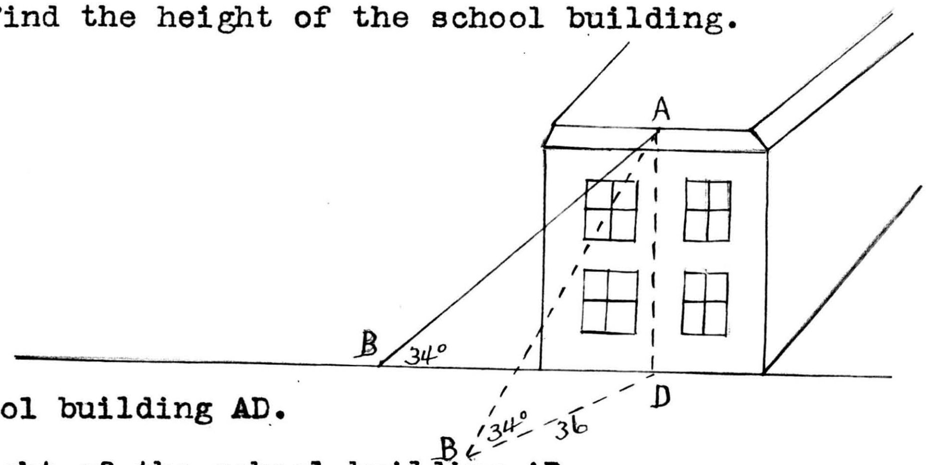
Measure along the walk AB a distance of 60 feet and at the point A and B at the extremities of the line AB measure the angles the walk makes with a line drawn from the point A to the tree C and also the line BC with the walk AB. Use the protractor to find the angles. The angle at A is 56.5 degrees and at B 68 degrees. On a sheet of paper lay off a line AB 60 mm. long and at the extremities construct the angles respectively 56.5 and 68 degrees. Measure the perpendicular distance from the point C where the lines met. This line is 58.6 mm. Therefore the width of the street is 58.6 feet. Actual distance measured with a metre stick is 58.8 feet. The difference between the measured distance and the computed distance is accounted for by the uneveness of the surface of the walk and in the inaccuracies in the use of the instruments. Similar triangles may be used later in the course on the same problem.

Wentworth, Plane Geometry, Page 34, Prop. XX.

Problem found by student.

Problem:

a, To find the height of the school building.



Given: the school building AD.

To find the height of the school building AD.

Measure on the ground along a straight line BD a distance of36 feet. At B find with the protractor the angle that the line AB, a line drawn from the point B to the top of the building, makes with its projection upon the plain of the ground, the line BD. The angle ABD is 34 degrees. Draw a triangle to scale. The distance AD is found to be 27 feet. Therefore the building is 27 feet high.[1]

b, To find the height of the tree at the southeast corner of the yard.[2]

c, To find the height of the standpipe.[2]

d, To find the height of the flag pole.[2]

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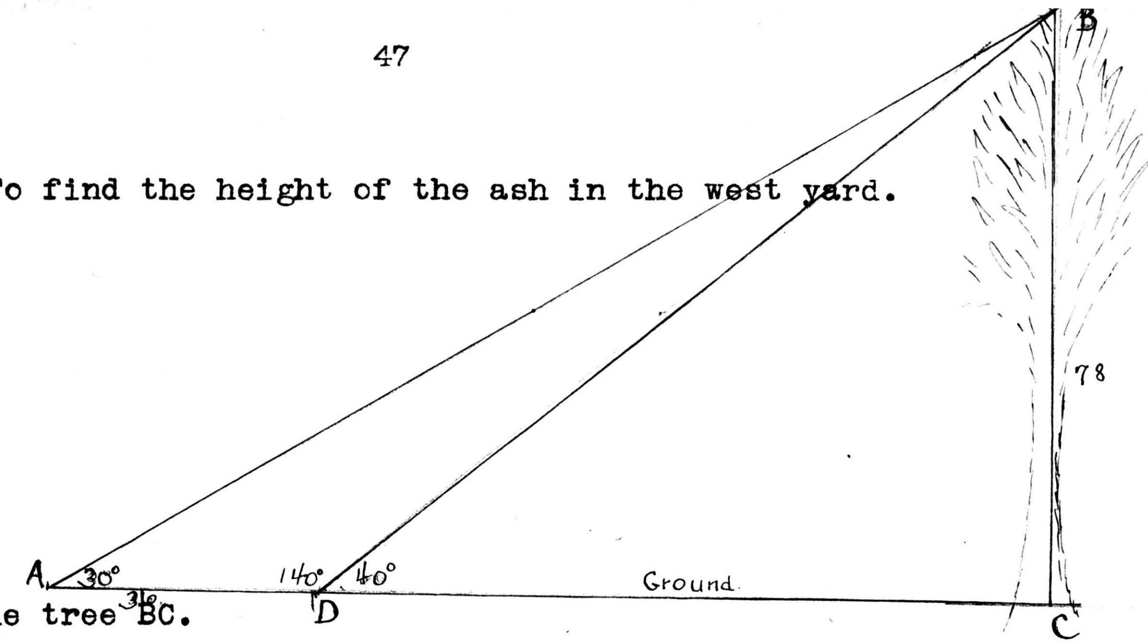
1. Wentworth, Plane Geometry, Page 34, Sec. 142, Cor. 3.

2. see problem a, supra.

Problems suggested by various students.

Problem:

To find the height of the ash in the west yard.



Given: the tree BC.

To find the height of the tree BC.

Measure the line AD that is one segment of the line that extends from D on the line DC. The points A and D being any two points that may be on any line that passes through the foot of the tree. At A and D measure angles 1 and 2 with protractor. Then angle 3 is known. AD is 36 feet, and angles 1 and 2 equal respectively 40 and 30 degrees. Then angle 3 is 140 degrees.[1] Construct a triangle to scale using 1 mm. for 1 foot. On a straight line take AD 36 mm. long and at A construct angle equal to 30 degrees and at D construct angle equal to 140 degrees. Extend thesides of these angles until thay meet in a point, and from that point draw a perpendicular[2] to the straight line AD extended. This perpendicular is 78 mm.[3] Therefore the tree is 78 feet high.

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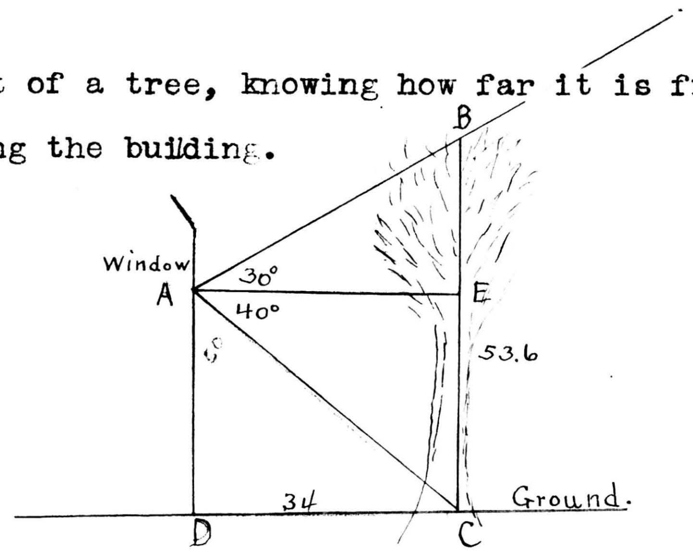
1. Wentworth, Plane Geometry, Page 13, Sec. 77.

2. Wentworth, Plane Geometry, Page 11, Sec. 64.

3. Wentworth, Plane Geometry, Page 34, Prop. XX.

Problem:

To find the height of a tree, knowing how far it is from the building without leaving the building.



Given: a tree at a distance of 34 feet from the building.
To find the height of the tree.
From an upstairs window of the school building find with the protractor the angles EAB and CAE. Lay off on paper a straight line equal to AE, 34 feet, which is the same as DC,[1] the distance of the tree from the buiding. Construct the angles EAB and CAE. Angle EAB is equal to 30 degrees and angle CAE is equal to 40 degrees. At E erect a perpendicular with a protractor.[2] Extend this perpendicular until it cuts the lines AB and AC. This point C will represent the foot of the tree. Then CB, 53.6 feet, is the height of the tree.[3] The figure is composed of two right triangles.

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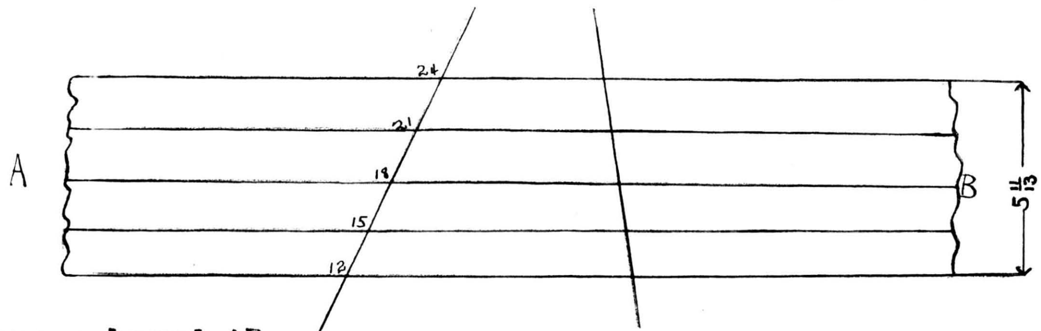
1. Wentworth, Plane Geometry, Page 49, Sec. 180.
2. Wentworth, Plane Geometry, Page 11, Sec. 64.
3. Wentworth, Plane Geometry, Page 34, Sec.142, Cor. 3.

Problem:

To divide a given board into four strips equal in width.



Given: a board AB.

To divide the given board into four strips each equal to the others in width.

Lay a meter stick or carpenter's square across the board, placing the stick or square so that the edge of the board is at the point 12 on the measuring stick and the point 24 is at the other edge of the board. Then mark the points at the places marked 15, 18, and 21 on the measuring standard. Place the measuring standard at some other point on the board and repeat the manipulation.

Then draw straight lines[1] through the pairs of points taken at the 15, 18 and 21 points.

These lines will divide the board into four equal strips[2]

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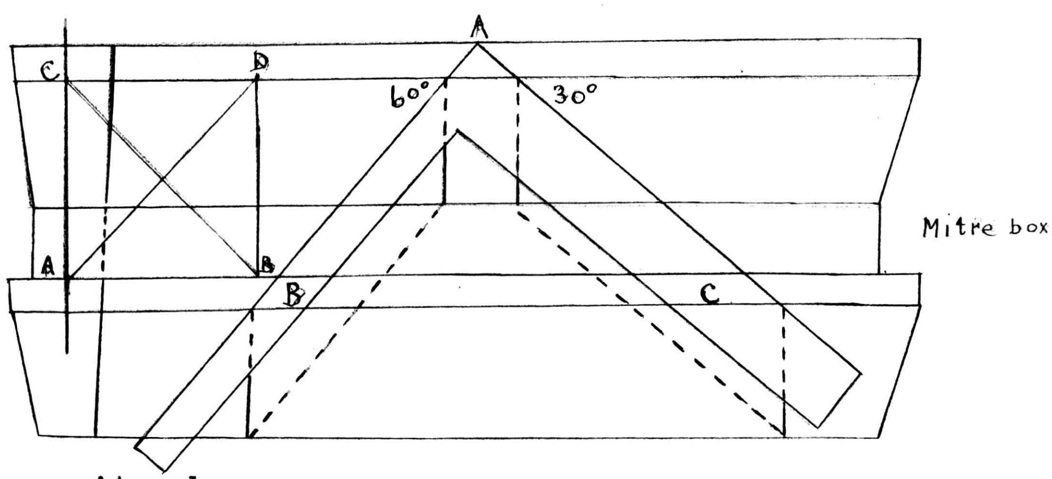
1. Wentworth, Plane Geometry, Page 8, Sec. 44 and Sec. 51.
2. Wentworth, Plane Geometry, Pgae 54, Prop. 38.

Taken from School Science and Mathematics by one of the students.

Problem:

To make a mitre box for the 30, 45, and 60 degree angles.



Given: a mitre box.

To sa w for the 30, 45 and 60 degrees angle.

To saw for the 45 degree angle.

Lay off along one edge a distance CD equal to the width of the box, BD. Draw AC perpendicular to CD[1] at C and draw the lines AD and CB. The triangle CBD is a right triangle with a right angle at D, and since BD and CD are equal, the angles opposite, angles BCD and DBC, are equal.[2] The sum of the angles of a triangle is equal to 180[3] degrees and if angle CDB is equal to 90 degrees the sum of the angles BCD and DBC is 90[4] degrees and if they are equal, each id 45 degrees. In like manner the angles CAD and CDA are each equal to 45 degrees.

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1. Wentworth, Plane Geometry, Page 11, Sec. 64.
2. Wentworth, Plane Geometry, Page 36, Prop. XXII.
3. Wentworth, Plane Geometry, Page 32, Prop. XVIII.
4. Wentworth, Plane Geometry, Page 33, Sec. 135, Cor. 6.

Problem given to a student by a carpenter in town.

To saw for the 30 and 60 degree angles.

Measure the width of the box AC. With a carpenter's square used as a diagonal twice as long as the width of the box and one end at A mark the other end at B. Then angle ABC is equal to 30 degrees.[1] If the leg of a right triangle is one half of the hypotenuse the angle that is opposite that leg is 30 degrees. The other arm of the square extends in the direction AM at right angles to AB. The sum of the angles EAB, BAM and MAF is 180 degrees[2] and if the sum of the angles EAB and BAM is 120 degrees the remaining angle MAF is equal to 60 degrees. Angle EAB and angle ABC are equal.[3]

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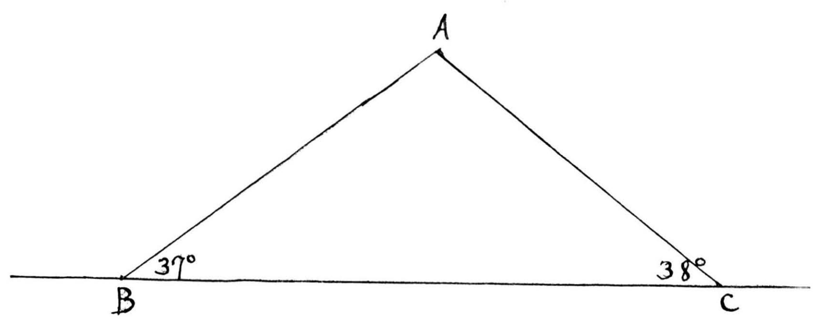
1. Wentworth, Plane Geometry, Page 66, Ex. 22.
2. Wentworth, Plane Geometry, Page 32, Prop. XVIII.
3. Wentworth, Plane Geometry, Page 26, Prop. XII.

Problem:

Two men with guns located at B and C along a counter BC have the position of their guns marked. Each shoots and hits the target A. How may they determine most readily, which, if either, had the advantage?



With a protractor measure the angle ABC and angle ACB. Angle ABC is found to be 37 degrees and angle ACB is found to be 38 degrees. By drawing a triangle to scale it would be very easy to determine the distance of each from the target, and as the distance BC could be easily determined.[1] An easier way is to measure only the angles, and then if two angles of a triangle are unequal, the sides opposite are unequal, and the greater side is opposite the greater angle.[2] C had the advantage.

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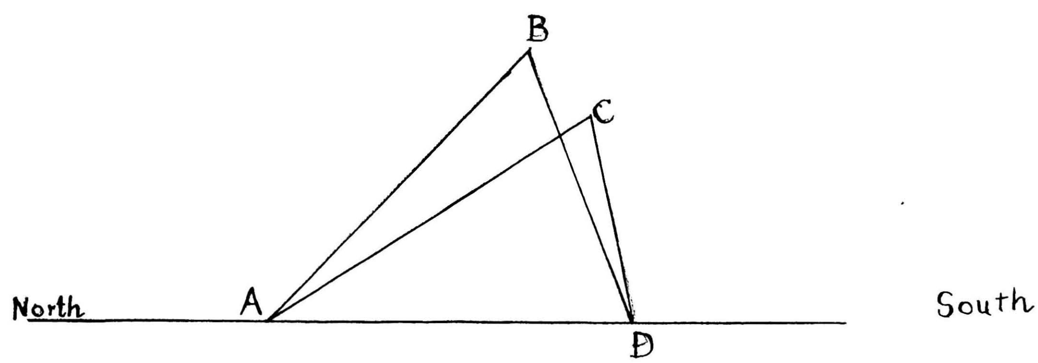
1. Wentworth, Plane Geometry, Page 34, Prop. XX.
2. Wentworth, Plane Geometry, Page 41, Prop. XXVII.

Problem:

Two boats on a still day started at the same time from the same point on a lake. If the boats have the same rate of speed, and travel the same length of time, but in the beginning take different directions which will be the farther from some point on a north south line?



Let A represent the starting point, and B and C the points after sailing the same length of time at the same rate of speed. If AD represents the north south line how do the distances BD and CD compare?[1] What can be said about two triangles which have two sides equal, but the included angle of one greater than the included angle of the other?

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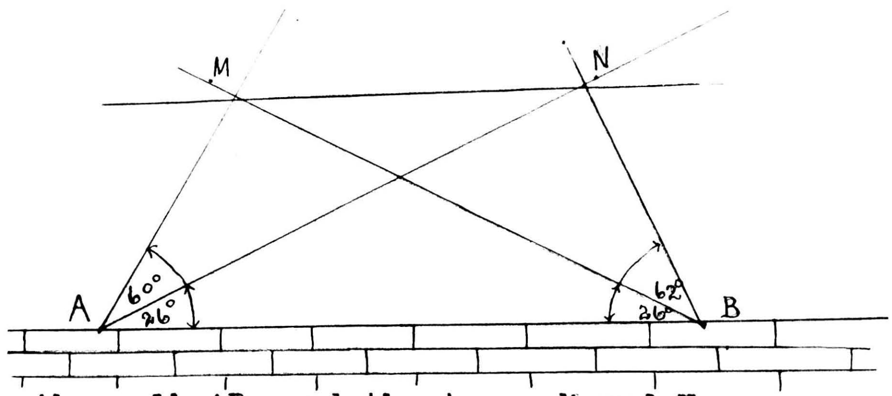
1. Wentworth, Plane Geometry, Page 42, Prop. XXVIII.

Problem:

To find the distances between two inaccessible points, as the distance between two trees on opposite sides of the street without leaving the sidewalk.



Given: the walk AB, and the trees M and N.

To find distance MN.

First measure on the side walk some known distance, 80 feet, and with protractor measure the angle that the line AM makes with AB, and find its value, 60 degrees, and the angle that the walk makes with the line AN some known angle, as 26 degrees. Also with the protractor measure the angles ABM, 26 degrees, and angle ABN, equal to 62 degrees.

Then lay off on paper a line AB 80 mm. long and at end A construct an angle of 60 degrees and an angle of 26 degrees, and at B construct an angle equal to 62 degrees and an angle equal to 26 degrees. These two triangles determine two fixed points, where their sides meet as M and N.[1] MN[2] is the distance required. Measuring MN it is found to be 45 mm. and then we know that the distance between the two trees is 45 feet.

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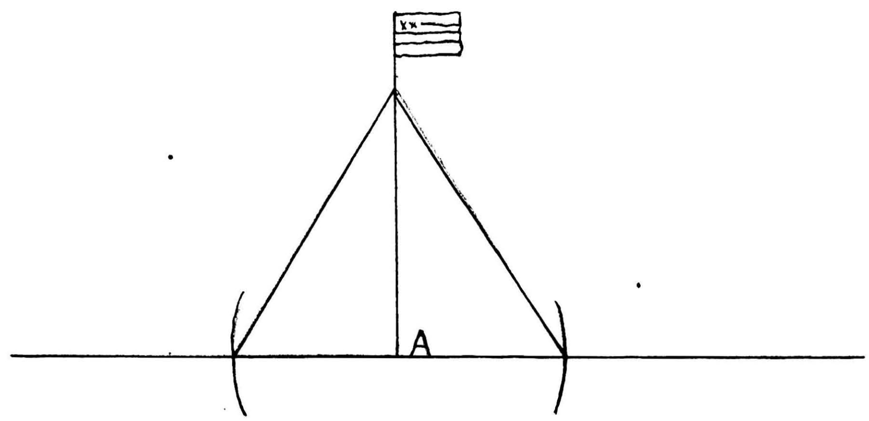
1. Wentworth, Plane Geometry, Page 34, Prop. XX.

2. Wentworth, Plane Geometry, Page 8, Def. 50 to 53.

Problem:

To erect a perpendicular flag staff on a roof when you have neither square or level, provided the roof is level.



Suppose A is the point on the roof at which the staff is to be erected. Place the flag staff in position, and with the base of the staff as a centre and any convenient radius describe a circle.[1] Then with any known distance, a given stick, place one end at some point in the circumference, and with another stick of the same length and one of its end points in the circumference bring the two free ends of the sticks to a position where they touch the flag staff. Then the flag staff will be perpendicular to the roof.[2] The points chosen must lie on the straight line that passes through the foot of the flag staff.

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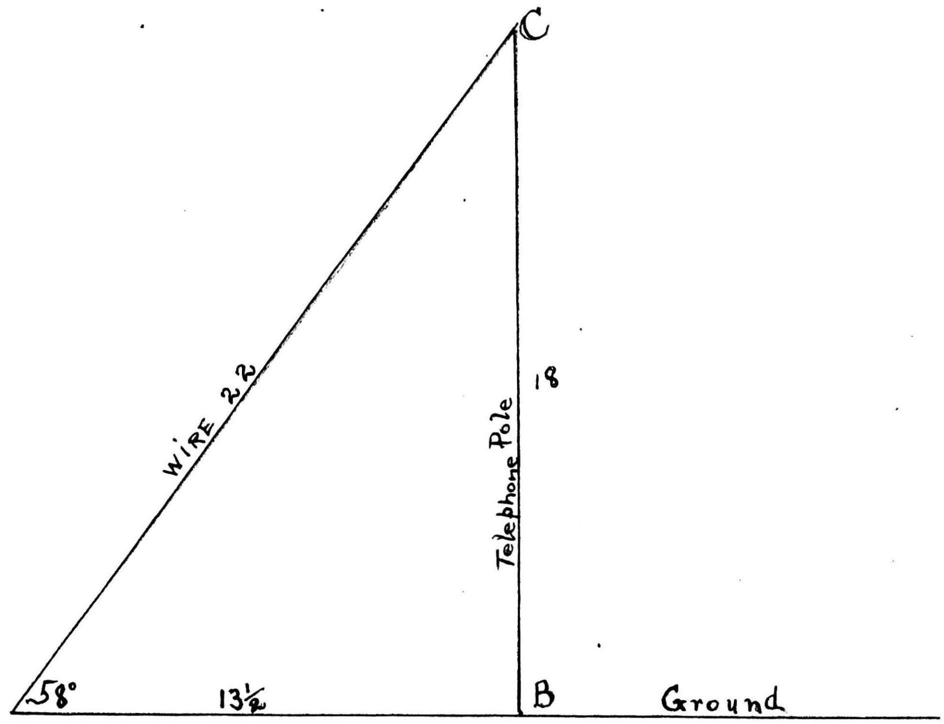
1. Wentworth, Plane Geometry, Page 75, Def. 216.

2. Wentworth, Plane Geometry, Page 19, Prop. V.

Problem furnished by a student.

Problem:

A telephone is set perpendicular to the ground, and a wire is fastened to it 18 feet from the ground and then to a stake 13 1/2 feet from the foot of the pole, used as a guy wire. Show by drawing to scale, how long the wire; and what angle the wire makes with its projection on the ground.



Draw a straight line AB 13 1/2 (1/2 cm,) long and at B[1] erect a perpendicular CB and measure on CB the distance 18 (1/2 cm,) Then draw AC. Measure AC and find it to be 22 (1/2 cm.), hence the wire is 22 feet long.[2] Measuring the angle CAB with a protractor it is equal to 58 degrees.

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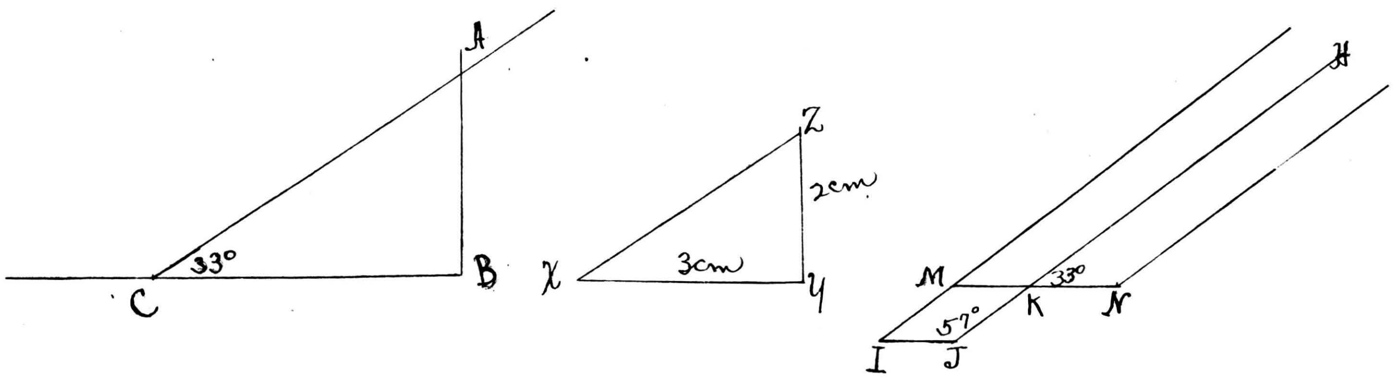
1. Wentworth, Plane Geometry, Page 113, Prop. XXII.

2. Wentworth, Plane Geometry, Page 34, Prop. XX.

Problem suggested by a student.

Problem:

The school house building is 48 feet wide and the roof has a rise of two feet to three measured horizontally. By drawing a plan to scale construct the size of the angle at which the carpenters must cut the rafters. Also allowing 18 inches for the projection of the rafters at the eaves, compute the length of the rafters.



Draw a straight line CB to represent one half of the top of the ceiling and draw AB perpendicular to CB at B using the protractor. If B is the cntre of the building the perpendicular will go throug the ridge row of the house.[1] By drawing a straight line 3" long as XY and a perpendicular YZ at Y 2" long and then connecting X and Z we have the angle ZXY, which measured with the protractor is 33 degrees. Then by constructing on the figure an angle at C equal to 33 degrees and extending CA to cut AB the length of the rafter CA is found to be 27.8 feet. Then adding the 18 inches, the length of the rafters is 29.3 feet. To find the angle to cut the rafter construct angle HKN equal to 33 degrees. Then angle JKM[2] equals 33 degrees and angle KJI is equal to 57 degrees, the angle required.[3]

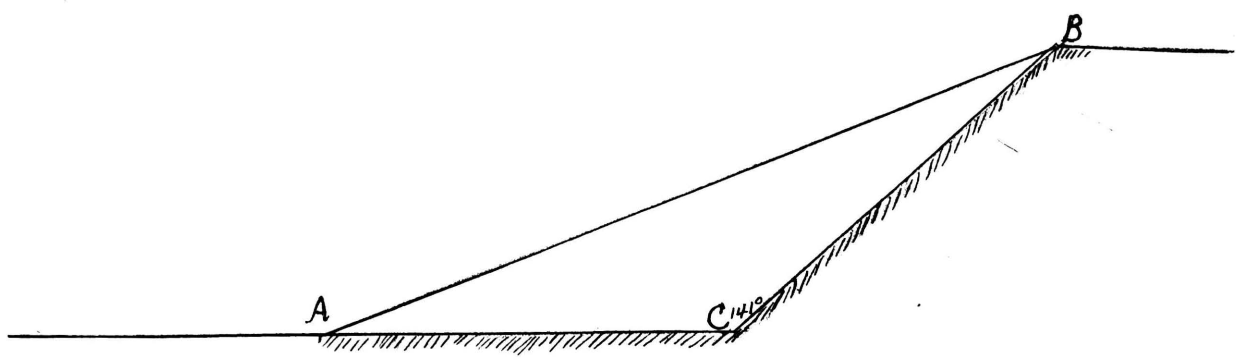
---

1. Wentworth, Plane Geometry, Page 37, Cor. 2, Sec. 149.
2. Wentworth, Plane Geometry, Page 18, Prop. IV.
3. Wentworth, Plane Geometry, Page 29, Prop. XVI.

Problem furnished by teacher.

Problem:

Near Mound City a railroad embankment stands on a horizontal plane; and we desire to know the distance from a point in the plane to the top of the embankment.



Select a point C at the foot of the embankment lying in the same vertical plane as A and B, and measure the distances AC and CB and the angle BAC. By measurement find BC equal to 36 feet and AC equal to 32 1/2 feet and angle ACB on protractor is 141 degrees. Construct straight line AC 32 1/2 mm. long and with protractor construct angle ACB, 141 degrees. Draw BC = 36 mm. AB measures 65 1/2 mm. making the distance AB equal to 65 1/2 feet.[1]

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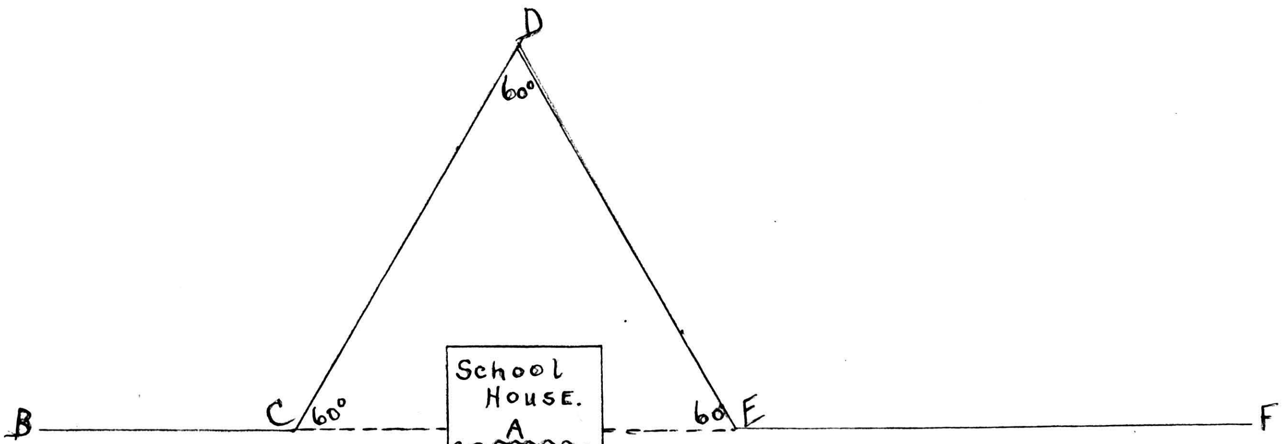
1. Wentworth, Plane Geometry, Page 35, Prop. XXI.

Problem:

To lay out a straight line when some insurpassable object intervenes.



Suppose A to be the school house and BCEF the straight line to be determined. At point C construct angle 1[1] = 60 degrees and lay off DC any convenient distance, at least long enough to see beyond the buiding, and at D construct angle 3 = 60 degrees and draw DE = DC and at E construct angle 2 = 60 degrees, and draw EF. Then BCEF is a straight line[2], and CE = DC or DE[3]. Distance from B to F is BC plus CD or (DE) plus EF.

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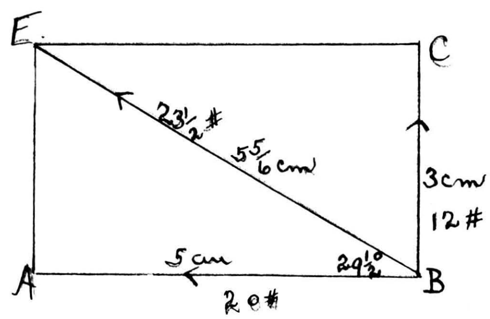
1. Wentworth, Plane Geometry, Page 115, Ex. 135.
2. Wentworth, Plane Geometry, Page 8, Sec. 46.
3. Wentworth, Plane Geometry, Page 36, Prop. XXII.
   Plane Geometry, Page 37, Prop. XXIII.
   Plane Geometry, Page 37, Art. 148, Cor. 1.
   Plane Geometry, Page 40, Ex. 9.

Problem:

Two forces act upon a body at right angles, one with an intensity of 20 pounds and the otherwith an intensity of 12 pounds. Show by drawing to scale the direction and the intensity of the resultant.



Let AB be 5 cm. (1 cm, = 4#) and AC be 3 cm. Draw AC perpendicular to AB. Complete parallelogram ACDB[1] and draw diagonal AD; measuring we find AD equal to 5 5/6 cm.[2] Then the resultant will be 23 1/2 pounds. Proof verified by using the dynanometers. The direction is determined by the angle between one of the forces and the resultant, which can be measured with the protractor, and is found to be 29 1/2 degrees. See angle 1.

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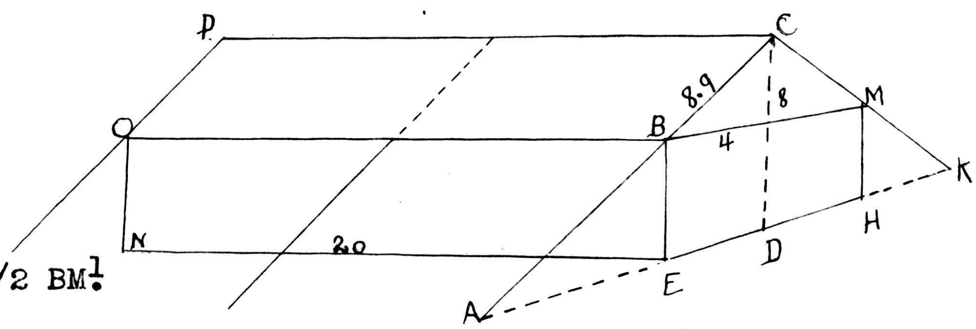
1. Wentworth, Plane Geometry, Page 47, Sec. 166.
2. Wentworth, Plane Geometry, Page 113, Prop. XXII.

Problem:

In constructing a wall tent, with the height of the wall 4 feet, to the ridge row 12 feet and the width of the tent 8 feet; What must be the dimensions of the roof canvas and what must be the length of the side guy ropes? How much canvas will be required altogether barring allowance for seams; if length of the tent is 20 ft'



BE = 1/2 BM.[1]

CE squared plus BE squared = DC squared, (4 squared plus 8 squared = 8.9 squared).[2]

Dimensions of roof canvas must be 20 feet by 8.9 feet for each side or 2 pieces 20'x8.9' for both sides. CE:BC = CD:AC;[3] (8:12 = 8.9:13.3 or length of side guy rope is 13.3 feet.

To find amount of canvas:

Roof = 2 pieces 20'x 8.9'x 1 square foot = 356 square feet.[4]
Sides and ends = perimeter x height = 56 feetx4'xl square foot. equals 224[4] square feet. Area of gable BCM is 1/2 BMxCE or 8'/2x 8'xl square foot = EW[5] square feet and two ends = 64 square feet. Total canvas 644 square feet. Approx.

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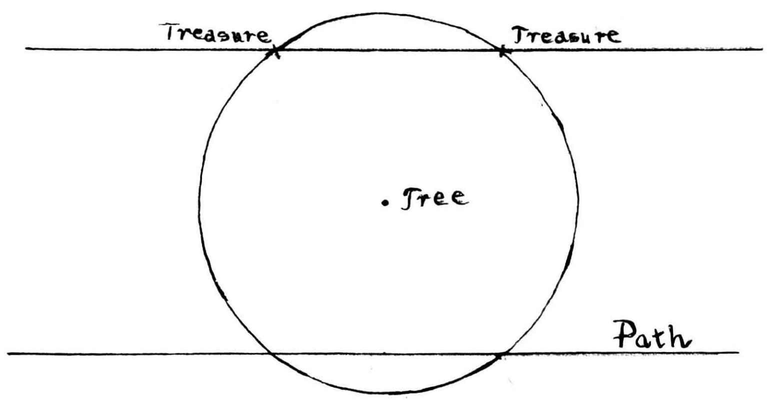
1. Wentworth, Plane Geometry, Page 37, Art. 149, Cor. 2.
2. Wentworth, Plane Geometry, Page 162, Prop. XXVIII.
3. Wentworth, Plane Geometry, Page 153, Prop. XXI.
4. Wentworth, Plane Geometry, Page 187, Prop. III.
5. Wentworth, Plane Geometry, Page 189, Prop. V.

Problem:

Pirates buried treasure 75 feet from a certain tree, and 100 feet from a straight path which passed the tree at a short distance. Show how to locate the treasure.



If the treasure is 75 feet from a certain tree, it must lie on the circumference of a circle whose radius is 75 feet.[1] If 100 feet from a path it must lie on a line parallel to the path at a distance of 100 feet.[2] There are two, one or no solutions depending upon whether the circumference cuts the straight line in two points, or whether it is tangent to it or whether it lies entirely outside of its field.

This a good problem in loci.

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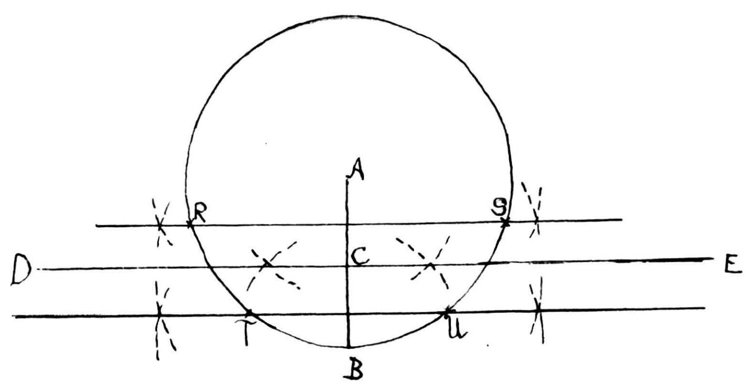
1. Wentworth, Plane Geometry, Page 75, Art. 216.

2. Wentworth, Plane Geometry, Page 24, Art. 103.

Problem:

A straight railway passes two miles from a town. A place is four miles from town and one mile from the railway. To find by construction the places that answer this description.



Let A represent the town. With center A and a radius to represent AB; four miles in length describe a circle. Points must be on the circumference.[1] Construct perpendicular DE to the line AB, bisecting at C. Let DE represent the railroad. Draw parallels to DE on either side, at a distance from DE equal to one half AC, one mile.[2] Let these parallels cut the circle's circumference in the points R, S, T and U. Then any one of these points will satisfy the given condition. Therefore there are four solutions.[3]

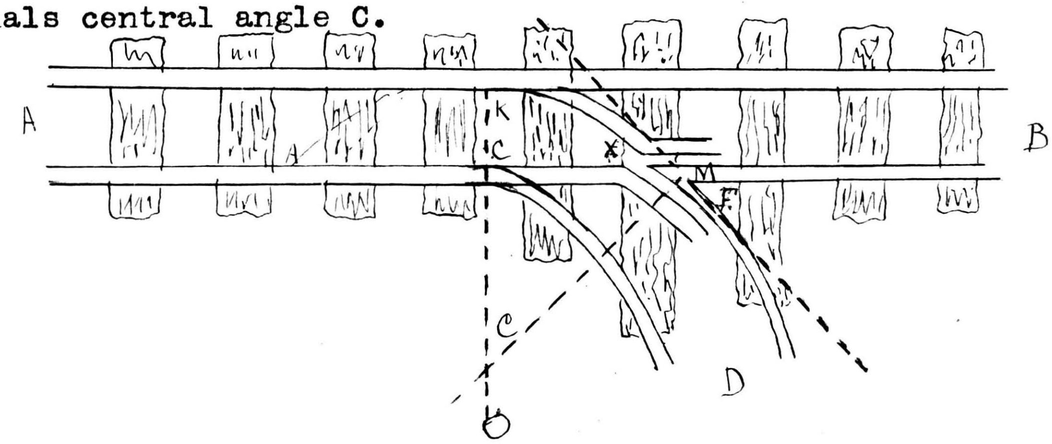
---

1. Wentworth, Plane Geometry, Page 75, Art. 216.
2. Wentworth, Plane Geometry, Page 24, Art. 103.
3. Wentworth, Plane Geometry, Page 44, Art. 158.

Problem taken from the text.

Problem:

In laying out a 'turnout' or a switch, on a railroad track, a 'frog' is used at the intersection of the two rails to allow the flanges of the wheels moving on one rail to cross the other rail. The angle of the frog that must be selected for any place depends upon the control angle of the curve of the two tracks. If one track is straight and the other curved, prove that the angle F of the frog equals central angle C.



Given: Straight track AB and curved track CD.

To prove, angle C equals angle F. Draw OK perpendicular to AB, and OM perpendicular to tangent at M. Then angle F equals angle X.[1] Angle X is measured by 1/2 arc AM.[2] OC is perpendicular to AB. Radius is perpendicular to tangent at point of contact. Therefore arc AK and arc MK equal 1/2 arc AM, Radius perpendicular to a chord bisects the subtended arc. Therefore angle X is measured by arc MK. But angle C is measured by arcKM.[3] Therefore angle X equals angle C, measured by the same arc. Therefore angle F equals angle C. Magnitudes equal to equal magnitudes are equal.

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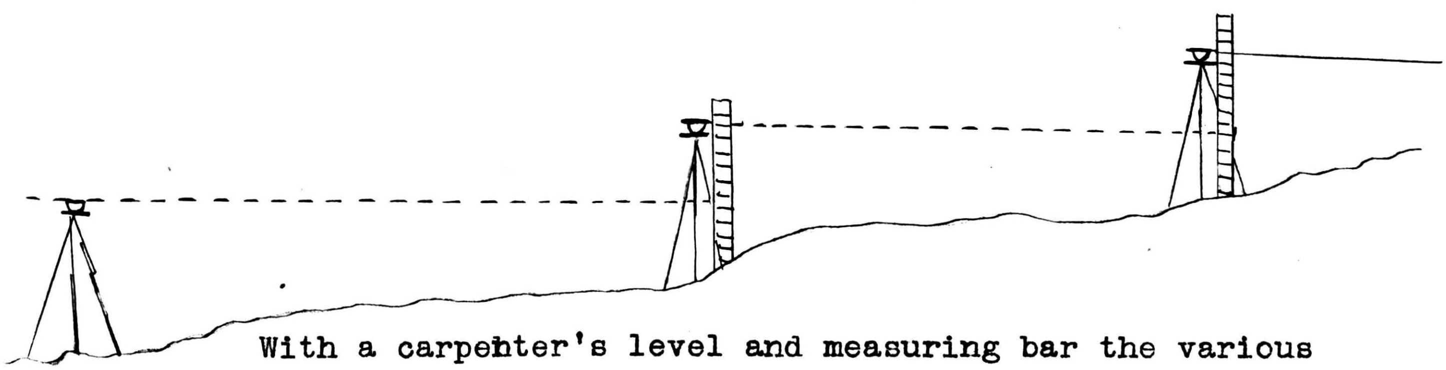
1. Wentworth, Plane Geometry, Page 18, Prop. 4.
2. Wentworth, Plane Geometry, Page 105, Prop. XIX.
3. Wentworth, Plane Geometry, Page 102, Art. 288.

Problem:

To find the number of feet that the Grammar School building; and the number of feet that the High School building are above sea level, also the angle of elevation of the High School from the Grammar School. State Street is 825 feet above sea level.



With a carpenter's level and measuring bar the various heights of successive points were established. The level is set with the bar of the sigths over the point that you are leveling from, and the instrument is in level. Find the height of the sighting bar with the bar. Put down the height as measured on your measuring bar as the first reading. Next at some convenient distance set the measuring bar and sighting through the level move the sighting bar up or down on the measuring rod until it is on a level with the level. The height of the sliding bar on the measuring bar is the second reading. The difference added to the original height gives the elevation of the point. If a second reading is less than your difference is negative and must be subtracted from the previous reading or elevation. Comtiue the process.

Following is a table of the level readings fromState Street to the Grammar School:

After the nineteenth reading the elevations concern the High School.

Stations.	Height of level.	Height of rod.	Difference.	Elevation.
1	4' 4 3/4"	1' 8"	2' 7 3/4"	827' 3/4"
2	4' 4 1/2"	1' 4 1/2"	3'	830' 7 3/4"
3	4' 5 1/8"	4 5/8"	4' 1/2"	834' 8 1/4"
4	4' 3 1/2"	2 1/8"	4' 1 3/8"	838' 9 5/8"
5	4' 4 3/4"	1"	4' 4 3/4"	843' 1 3/8"
6	4' 4"	1"	4' 3"	847' 4 3/8"
7	4' 4 1/2"	1"	4' 3 1/2"	851' 7 7/8"
8	4' 4 3/4"	1"	4' 2 3/4"	855' 8 5/8"
9	4' 4 3/8"	1"	4' 3 3/8"	859' 9"
10	4' 4 1/4"	1"	4' 3 1/4"	864' 1/4"
11	4' 5 3/8"	1"	4' 4 3/8"	868' 4 5/8"
12	4' 4 1/4"	1"	4' 3 1/4"	872' 7 7/8"
13	4' 5 3/8"	1"	4' 4 3/8"	877' 3/8"
14	4' 5 5/8"	3' 1 1/8"	1' 4 3/4"	878' 5 1/8"
15	4' 3 3/4"	4' 6 5/8"	- 3 3/8"	878' 1 3/4"
16	4' 4 3/8"	1' 10 11/16"	3' 5 11/16"	881' 7 7/16";
17	4' 5 1/2"	4 1/16"	4' 1 7/16"	885' 8 7/8"
18	4' 4 1/4"	2 5/8"	4' 1 5/8"	889' 10 1/8"
19	4' 5 1/4"	4 3/8"	4' 7/8"	893' 1"
20	4' 4 1/2"	8 15/16"	3' 8 9/16"	896' 8 9/16"
21	3' 11 3/4"	2' 8"	1' 3 3/4"	897' 11"
22	4' 5 3/8"	5 1/8"	4' 1/4"	901' 11 7/16"
23	4' 6"	8 5/8"	3' 9 3/8"	905' 8 13/16"
24	5' 6 1/4	8 1/4"	4' 10"	910' 6 13/16"
25	4' 5 7/16"	8 3/4"	3' 8 11/16"	914' 3 1/2"
26	4' 4 1/8"	5 7/8"	3' 10 1/4"	918' 1 3/4"
27	4' 2 7/8"	10"	3' 4 7/8"	921' 6 5/8"
28	4' 5 3/16"	6 7/8"	3' 10 5/16"	925' 8 5/16"
29	4' 6 3/8"	8 11/16"	3' 9 11/16"	929' 6".

30	3' 7"	2' 2"	1' 5"	930' 11"
31	4' 2"	2' 6 5/8"	1' 7 3/8"	932' 6 3/8"

At the reading of the 19" station we have the elevation of the Grammar School to be 893' 1" and at the close we have the elevation of the High School to be 932' 6 3/8".

Lines AB, CD, and EF are parallel[1]; their alternate interior angles being equal[2]. A plumb line was used to get the level exactly over the point at which the preceding reading was taken.

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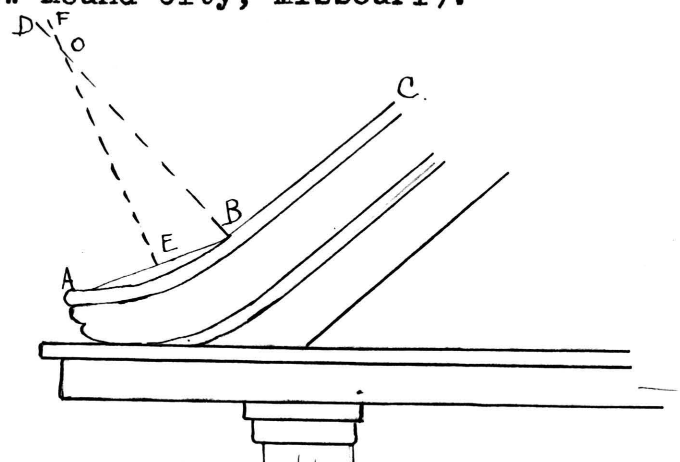
1. Wentworth, Plane Geometry, Page 24, Prop. X.

2. Wentworth, Plane Geometry, Page 26, Prop. XII.

Problem:

To draw easement of cornice tangent to rake cornice BC at B and passing through A. (Data taken from local building, Dr. Jesse Scott's bungalow Mound City, Missouri).



Draw BD perpendicular to BC and draw AB chord to arc AEB. Draw Ef perpendicular bisector of AB, meeting BD at O. With O as a centre and OB as a radius draw an arc. This arc passes through the point A and is tangent to BC at B. O is equidistant from A and B.[1] All points in perpendicular bisector of a line are equidistant from its extremities.

Therefore, the arc with centre O and radius OB passes through A.[2] Since BD is perpendicular to BC, Bc is tangent to arc AB at B.[3] A line perpendicular to a radius at the point of tangency, the extremity of the radius is the tangent to the circle at that point.

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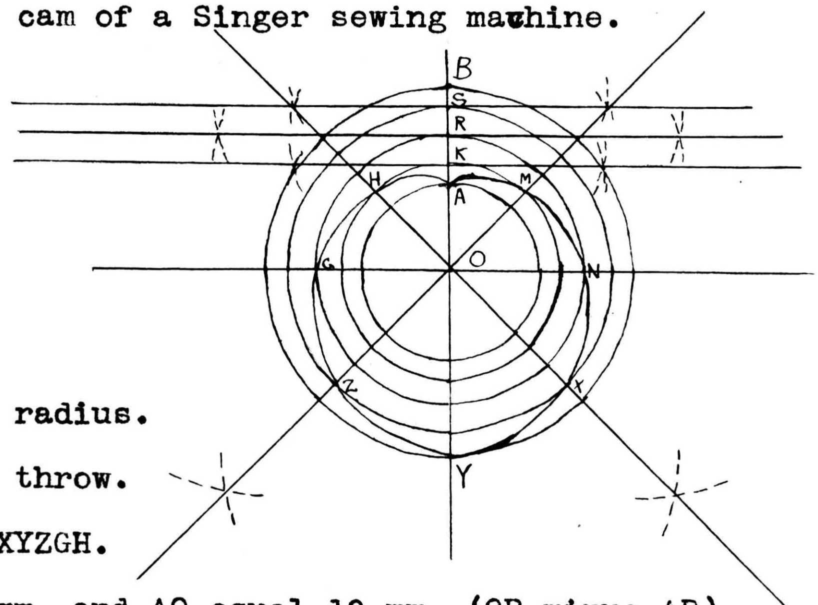
1. Wentworth, Plane Geometry, Page 45, Prop. 30.
2. Wentworth, Plane Geometry, Page 75, Art. 216.
3. Wentworth, Plane Geometry, Page 86, Prop. IX.

Problem:

To construct a pattern for a uniform motion cam with a throw of 14 mm. and a maximum radius of 24 mm. A cam is a wheel having its axis of revolution out of its centre of figure. This problem based upon the cam of a Singer sewing machine.



Given: OB = 24 mm. the radius.

AB = 14 mm. the throw.

To construct curve AMNXYZGH.

On BY take OB equal WRmm. and AO equal 10 mm. (OB minus AB) At point O draw GN perpendicular to BY and bisecting the angles.[1] Then divide AB up into equal parts.[2] Through AKRS draw circles with O as a centre. The circles will be concentric. Begin at A mark the points where the consecutive concentric circles and the consecutive rays that bisect the angles intersect, and through these points draw a smooth curve. This curve will be a solution to the problem.

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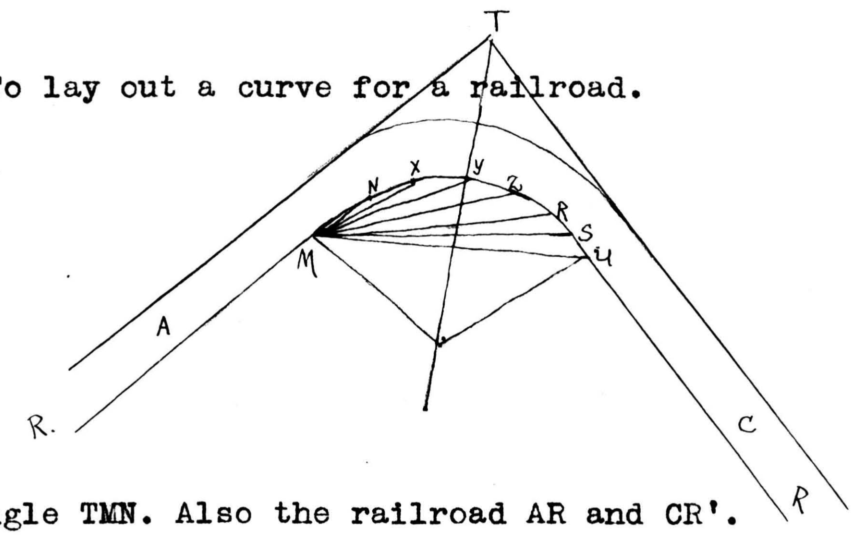
1. Wentworth, Plane Geometry, Page 115, Prop. XXV.
2. Wentworth, Plane Geometry, Page 114, Prop. XXIII.
   Wentworth, Plane Geometry, Page 117, Prop. XXVIII.

Problem:

To lay out a curve for a railroad.



Given: angle TMN. Also the railroad AR and CR'.

Let the chords MN, NX, XY, YZ, ZR, RS and SU each be 100 feet.

To lay off these chords suppose the continuat on of the railroad RRV to AT would be a tangent to the curve of the railroad. Then angle TMN is laid off from the tangent MT, and 100 feet is measured on lime MN thus locating point N, then angle NMX is laid off and arc with centre at N and radius with 100 feet is made to intersect MX, thus fixing point X. In the same way each successive point is found by laying off the proper angle and finding the intersection of its side with radius arc when the radius is 100 feet, whose centre is the stake last set. Since the curve is to be the arc of a circle, the angles TMN, NMX, XMY, YMZ, successive angles are all equal. Each angle is an inscribed angle, hence measured by one half of its arc[1] and each is equal to one half of the central angle MOX, XOY, YOZ and successive angles for central angles measured by subtended equal chords are equal,[2] and these chords were each one hundred feet.

---

1. Wentworth, Plane Geometry, Page 102, Prop. XVII.

2. Wentworth, Plane Geometry, Page 80, Prop. III.

Problem takenfrom School Science and Mathematics. May, 1909.

Problem:

In the railroad problem show that the sum of the 'deflection angles' TMN, NMX, XMY, and succeeding angles equals one half of the 'intersection angle' ATC, the angle between the tangents at the ends of the road curve. Show that the intersebtion angle equals the central angle subtended by the curve. For diagram see preceding problem.

MT equals TU.[1], OM equals OU.[2], Then OT is the perpendicular bisector of MU.[3], In triangles OMP and MTP; MP = MP, common; angle MPO equals angle MPT.[4] Angle MOU is measured by arc MYU,[5] and angle MOY,[6] one half of MOU, is measured by one half arc MYU.[7] But angle TMP is measured by one half arc MYU. Therefore angle TMP equals angle MOP, measured by one half the same arc. Therefore triangles OMP and MTP are equal.[8] Therefore OM = MT,[9] also OU = TU, Therefore angle MTO = angle MOT,[9] also in like manner angle UTO equal angle UOT.[9] Therefore angle MTO plus angle OTU equal angle MOT plus angle TOU.[10] Q. E. D.

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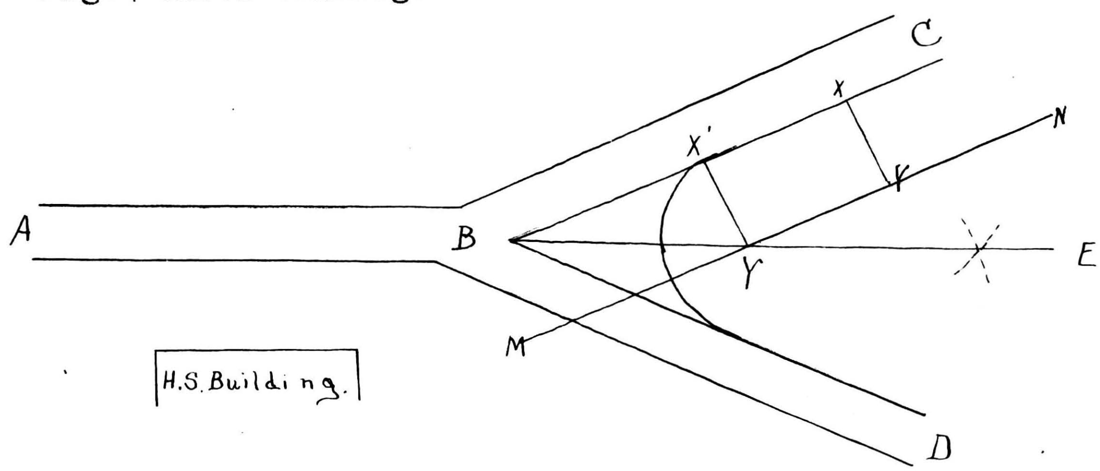
1. Wentworth, Plane Geometry Page 89, Prop. XII.
2. Wentworth, Plane Geometry, Page 75, Art. 217.
3. Wentworth, Plane Geometry, Page 45, Prop. XXX.
4. Wentworth, Plane Geometry, Page 15, Art. 82. Cor. 1.
5. Wentworth, Plane Geometry, Page 102, Art. 288.
6. Wentworth, Plane Geometry, Page 82, Prop. V.
7. Wentworth, Plane Geometry, Page 105, Prop. XIX.
8. Wentworth, Plane Geometry, Page 34, Art. 142, Cor, 3.
9. Wentworth, Plane Geometry, Page 31, Art, 128.
10. Wentworth, Plane Geometry, Page 6, Art. 34, Ax. 2.

Problem:

To draw the connecting curve, such as the line of the front of a building, of established radius 60 feet, or turn of street between 6" and 7" streets in front of the Mound City High School building.



Given: the streets AB, BC and BD. To find line of front of building of 60 foot radius. Bisect the angle DBC[1] with line BE. Erect perpendicular XY to BC[2] and lay off distance equal to 60 feet, and at Y draw MN perpendicular to XY at Y. The point of intersection of MN and BE will be the centre of the circle, that will be tangent to the streets, with radius 60 feet. This meets the requirements 60 feet from the streets and also equal distant from each street.

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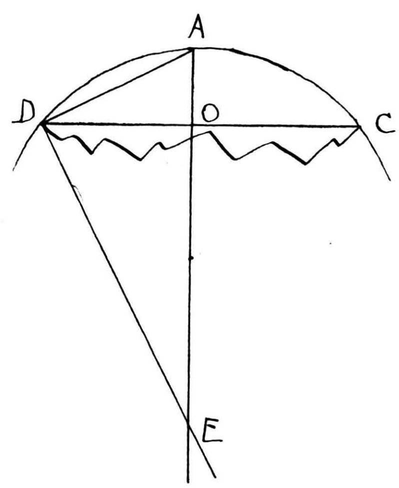
1. Wentworth, Plane Geometry, Page 46, Prop. XXXI.
2. Wentworth, Plane Geometry, Page 112, Prop. XXI.

Problem:

Given a fragment of a water wheel. Construct a new wheel of its size. DC was found by measurement to be 4' and AO to be 1'. Compute the diameter.



Given segment of water wheel, or water tank, to find the diameter. Let ADC be the segment, to find the diameter AE.

Lay off a chord DC equal to 4' and at its mid point erect perpendicular AO equal to one[1] foot. Draw AD. At D draw DE perpendicular to AD.[2] Then AE will be the diameter.

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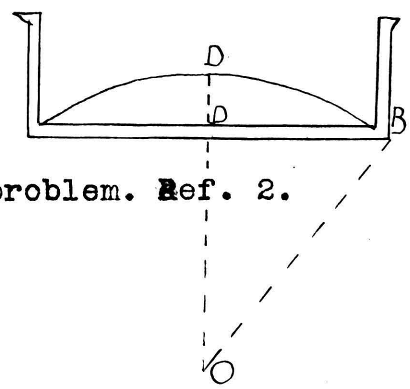
1. Wentworth, Plane Geometry, Page 113, Prop. XXII.
2. Wentworth, Plane Geometry, Page 102, Prop. XVII,
   Page 103, Sec. 290, Cor, 1.

Problem:

Having given the radius, 4.8 feet of a circular cinder pit, whose bottom is spherical on a radius of 8 feet. To find how high the centre of the pit is above the outside edge.



See preceding problem. Ref. 2.

Also:

BD = 4.8'

OD = 8'

Then OB is equal to the square root of the square of OD minus the square of BD.

$OB^2 = 8^2 - 4.8^2$ [1]

OB = 6.4'

Then AB = AO - OB.

Or AB = 8' - 6.4' or 1.6'

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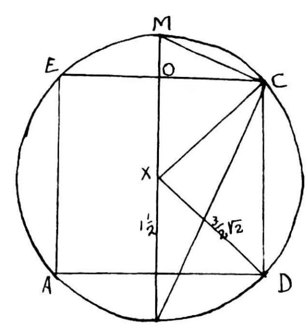
1. Wentworth, Plane Geometry, Page 194, Prop. X.

2. See the preceding problem.

This problem furnished by the Principal of the Grammar School.

Problem:

In making an Indian Club, pieces of Holly are glued to the faces of a square piece of maple. This is turned up in a lathe, the Holly producing oblong, oval light spots on the surface of the club. The thickness of the maple is three inches. I n order to get a club in which these oval spots just touch at the greatest thickness of the club; how thick must the piece of Holly be taken?



Given: cross section AECD of an Indian club with CD or AD equal to 3 inches.

To find OM.

OC and OX equal 1 1/2"

Then $XC^2 = OC^2$ plus OX^2 [1]

$XC^2 = 9/4$ plus $9/4 = 4\ 1/2$.

XC = 3/2 times the square root of 2.

But XM = XC = 3/2 times the square root of 2 [2]

OM = XM - OX.

OM = 3/2 times the square root of 2 minus 1 1/2 = $3/2(\sqrt{2} - 1)$ [3]

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1. Wentworth, Plane Geometry, Page 162, Prop. XXVIII.
2. Wentworth, Plane Geometry, Page 75, Def. 217.
3. Wentworth, Plane Geometry, Page 6, Sec. 34, Ax. 9.

This chapter presents types of the problems that were used in the course. The strictly computational problem has been omitted intentionally. That type of problem is so mumerous both in arithmetical and algebraic work that it is not necessary to stress it here. There was, howe\ver some work done in mensurational geometry. These problems are principally demonstrational or constructional. Not all the problems presented in the coubse are given in the paper. Many elementayy problems are omitted because they employ the same principles in the proof or are somewhat similar in idea.

The problems are taken from the experience of the teacher and student, together with problems that were found in books and magazines. The student's problem was emphasized. The students were encour:aged to make problems and to do so incidentally. The problems are in a measure stated as the student found the problem. The proofs are in a large measure the proof presented by the student. No problems appear on some of the books of the plane geometry, but similar work was done. It would be impossible to incorporate all of the problems used, as different students worked out different problems. Because we are not able to enclose all of the problems the preceding ones are selected as types. The strong student was able to do on an average more than one problem a day in addition to looking up the principles. The average student could about do one problem a day. It being understood that often many exercises growing out of the problem were required. By one problem a day is meant one line of thought.

The next chapter will offer a discussion on the work, as results of the method appeared in the Mound City High School.

Chapter III.

A brief discussion of the work as carried out in a class in Mound City, Missouri, High School. The work had for a basis the series of practical problems, of which types are presented in chapter II.

The bases of geometry are statements, assumptions, definitions, and theorems. To this we add our empirical knowledge of facts and mathematical data previously gained. Axioms, definitions and theorems are understood only in the experiences and possible development of the student. Each student's equipment, at the approach to geometry, is materially different. Therefore statements in geometry must be very definite, that is, mean something to the individual who expects to use them.

"Again, geometry must not be isolated from other branches of mathematics, as if it were something quite apart and different in kind; each must be taught with due regard to the whole, giving to, and receiving from the others. It is one of the advantages of the proposed system that artificial barriers between the parts will to a large extent be removed.[1]" Realizing that most fields of knowledge, and mathematics in particular, are an outgrowth of experience, and that the mathematical principles as taught in the ordinary High School must be applied before their use is evident to the average individual, the work as outlined in the preceding chapter was planned and carried out in the tenth grade geometry class in the Mound City, Missouri, High School during the school year 1909-1910.

The work as outlined is expected to show only types that are characteristic of the work done. Not all of the problems that were given or that were developed are shown. These problems were not given as they could have been to more advanced or more mature students or as might have been given to any other class. The work

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1. Harrison, Practical Plane and Solid Geometry, Page 6.

is an attempt to meet the interpretation of existing conditions and existing material. Other localities present other material, both in mental development and problem material that is accessible to the student body. The problems do not include all of the facts, but set forth the type of ideas worked out by the class. The purpose has been to show what might be done with a geometry class of tenth grade students in the public High School in order to get the highest development of each individual student, practically and culturally. The important thing is not whether the student has learned all the facts connected with the subject matter, but whether he has learned to select and organize the facts with which he has dealt into a form convenient for use.

The field of real problems may be divided into mensurational, demonstrational or requiring a discussion, and constructional or requiring a construction. The first field, mensurational, is such an easy one and so full of real problems in measurement as found in arithmetic and algebra, that it was thought advisable to emphasize the other two fields, demonstrational and constructional.

Believing thoroughly that this plan will develop accuracy in conception; accuracy in expression; and accuracy in manipulation, the following discussion is offered.

Bearing in mind that geometry had its birth in empirical knowledge as developed by the Egyptians and Babylonians, and later brought to its present logical, theoretical form by the mature thinking of philosophical Greeks, an attempt has been made to precede the mature ideas as presented by the text, by the experimental knowledge that may be gained in the practical problems of life.

The work then necessarily began with the simplest fact that had already been made the property of each student. No time was wasted in attempting to define terms, the meaning of which were already clearer than any definition could make it. Since the definition of a straight line in books is confusing and the student's idea is clear, there is no need to confuse the student by taking out his conception and planting another. Nor is it well to accept the definition of "axiom as a truth assumed as self-evident"[1] as the same truth is not self-evident to all minds, and some of these "self-evident" truths are not true absolutely. The concrete idea is possible. Fold paper, and you realize a straight line or angle. A walk gives opportunity for the observance of straight lines, the boundary of the school lot; the walks of parallel lines, the angles of the lot, right angles and various other ideas. This was followed by a drawing to scale, using co-ordinate paper, of the school house and school lot; a determining of the ratio of lengths by use of diagonal scale and dividers gave the correct conception, taught observation, developed an interest and applied the truth in a usable way. In addition the student had not been discouraged by rigor or a confusion of unintelligible words. The balance was used for determining laws of area and squared paper for the same purpose. This gave each child his own axioms and definitions.

Problems may be found easily in mechanical drawing, physics, in the questions of force and motion, strength of materials, structure of bridges and other buildings, as roof trusses, machinery in motion, of cutting and embanking and other engineer-

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1. Phillips and Fisher, Elements of Geometry, Page 1.

ing problems. Graphical representation of statistics at once suggests itself. Economic and industrial problems may be graphically plotted, as well as the location of a river by latitude and longitude. Under symmetry use common objects as the capital Latin letters or the snow crystal. One can find problems in crystallography, botany, optics, surveying and domestic science. The theorem of two triangles with two sides and the included angle, suggests method of bracing as used by bridge builders or of surveying in finding inaccessible distances.

The collection of problem material has been carried on by teacher and pupil. If the plan is followed by any one, it should be followed only as a suggestive one. No plan is good unless the teacher employs his own individuality, and if the teacher's individuality is poor the best plan may be a failure. The material was collected in a natural manner and in the ordinary affairs of life.

In the work at home, in the laboratory, agricultural and natural science, in books read and other ways was our list added to. A current copy of the Scientific American, School Science and Mathematics, and similar magazines furnished a source of problem material. An excursion trip after school hours or on Saturday to a building in process of erection, a study of the blue print, and the examining of the data of the city engineer gave many mensurational problems.

Books as Osterhout's[1] or King's[2] furnished interesting material for the student interested in agriculture. In like manner books

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1. Osterhout, Experiment with Plants.

2. King, Physics of Agriculture.

upon architecture, engineering, drawing, manual training and other subjects were found to be a fertile source for suggestion. The students were encouraged to find and formulate problem material. At the beginning of the course a few excursions for observation were taken, to the flouring mill, cement block factory, brickyard, oil tanks, canning factory, department store, and other industries. A study of the drainage system of the Missouri River bottom, and an examination of the railroad track construction, an economical plotting of garden and farm plots, as to drainage and to fencing convinced the student that problems were real things existing in the material world and mot merely in books. After a week or two of training of this kind, where material for geometry, history and agricultural classes were worked up, the student was led to discover problems for himself and in a short time many interesting problems of real worth to the student were suggested. As an example, a country boy, wanted a plan by which he could determine the point of division on an evener and also how to arrange the guy wires on his telephone line; another, who had a tendency to constructive work wanted a plan by which he could make a release cam for a pump that he was making. Girls were interested in mural decoration, and wished to study with a view to economical and symmetrical arrangement of their homes.

When problem material furnished by the students was scarce the teacher supplied the needs or used the class period to develop problems. The student was urged to read books for suggestions, and types of problems and then to make a problem that would naturally grow out of his suggestion. The student was encouraged to select his material from that encountered in actual life. Book problems were discouraged - and if used at all for

problem material to be used only as a suggestive guide. Part of the material was found to be unsuited, but by experimentation it was readily determined what part of the material appealed to the needs, capacity and interest of the High School student. The problems in carpentry, masonry or surveying appealed to some and lacked interest to others. This difference is due to the inherent taste, former training and present environment. In order that the boy or girl may appreciate the fact that there is a problem he must know something about the material. If he could not appreciate the problem, we attempted to interest him in the material leaving him to invent the problem rather than attempting an explanation of the 'why' or 'wherefore' of the problem. The various occupations; carpentry, manual training, laboratory, drawing, masonry, blacksmithing, navigation, newspaper and others present a varied list of problems that are based upon the principles of elementary geometry.

Not all the problems found are adapted to all High Schools. Some of the problems are too complex, indefinite, or call for material that cannot be readily obtained or associated with the student's experiences - that is, the technical matter is beyond his knowledge and interests. One of the problems encountered in this plan is the elimination of this material, and the deciding of how much it may be used in a non- technical High School. Sometimes problems overlap; and again the problems may not bridge the gap between one state of knowledge and a later state. It is difficult to get a series of problems that follow in order of development. And again members of the class are wont to follow their own inclination and find it difficult to do intensive study upon problems that do not have, as they think, an immediate value to them. There is a tendency for the pseudo-practical problem to creep into

the work, due to the great amount of literature on popular mechanical constructions distributed at the present and to ignorance of true knowledge. The limitations of the field can be lessened by any wide awake teacher, who will supply the proper literature for a guide, and correct directions for formulating real problems.

The problem should be short, simple and definite. It should be short because the average High School student is not willing to spend several hours on some long tedious problem. It is necessary that the problem should be short as a student at this age is not expected to be able to hold many or diverse ideas in his mind for a very great length of time. If the problem is not elementary, though one student may understand the situation, the others fail to get the idea, hence class demonstration is lost as far as value to the class is concerned. This would keep the plan from being economical and also at the same time destroy interest and dull the enthusiasm of even the brighter students. Although a problem is short and elementary it needs must be definite or it leaves the student in a confused state, and prevents him from organizing his knowledge. In addition to being short, simple and definite it is necessary that it be economical and worth while. Again the experience of the child must be kept in mind. A tenth grade student is not mature enough to attack a complex problem. If the problem is not definite he is at a loss to know where to begin his solution. "Any new knowledge offered to a child must be met by old ideas closely related to it, if it is to be well comprehended and appreciated."[1] "The healthy growth of the mind is just

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1. Mc Murry, The Method of the Recitation, Page 81.

in proportion to the activity of thoughts on the study of outward objects."[1]

The problem should be one in which the student can be made to feel an interest. Unless he does feel an interest in the task set, his work will be more or less mechanical, because he has no motive for working out the problem set for him, but if interesting the capacity broadens to meet the demand.

The text in geometry adopted by the Board of Education in the Mound City Independent School District, Wentworth's Plane Geometry, was accessible to each student in the class. In addition several copies of the current texts in geometry were put in the library and on the teacher's desk. Students were encouraged to look through these books and make a comparative study of at least two of the books. This comparative study, as a matter of fact, is limited by the age of the student, yet the propositions growing out of parallel lines furnishes an easy comparative study especially as shown in the adopted text and in Bush and Clarke's[2] Geometry. It is necessary to have access to a text in order to have a source for defining terms that may arise, and also to give an evidence of stability to the work. Again it puts down adverse criticism to the plan, and helps to carry out the idea. The text should be used as a ready and repeated reference. Also after the idea that the problem teaches has been developed the proposition bearing upon the problem should be discussed, in order to give a chance for logical, theoretical demonstration and organization. As a type of what is meant see problem

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1. Emerson, Natural History of the Intellect, Page 12.
2. Bush and Clarke, The Elements of Geometry. Pages 23 to 29 inc.

on page 44 in Chapter II. After this problem had been thoroughly worked out together with other problems in which equal triangles were stressed, a short study was given to the exposition of equal triangles as given in one or other of the texts. Not always did we use the adopted text, in fact probably no more so than any other except for definition and suggestion as to order of development and line of thought. This is necessary for economical reasons. It prevents confusion on account of indefiniteness, which is evident with students of this age.

The students in a short time became actively interested, and found problems where before they had no thought that a real problem could exist. They soon came to use their tools of geometrical principles as readily as one of the German students used his German vocabulary and more naturally.

After the teacher or student had discovered a problem he began to study the problem in the light of geometric facts to see whether or not it was adapted to school room use, and also to see if it was a special or general truth. As a type of the possible development of a problem the following concrete incident is given. Each problem, because of its subject matter, called for a different treatment, but as a possibility of treatment, problem on page 45 in Chapter II discussing the method of finding the width of the street without crossing the street was suggested by Mr. Alkire, one of the boys in the class, one morning, when we had a fifteen minute period devoted to finding problems. A similar problem had confronted his father a few days previously when he had desired to build a swinging gate across a stream of water. After the problem had been suggested those that were interested in the problem or that had no other work assigned went to work to find

a solution. Various methods were suggested, one of which was to tie a weight to a string, throw weight across the street and pull the weight to the edge of the opposite walk, note the place where the string is held, pull in the string and then measure the string. This method was discarded as suggestions often were because no geometrical principle supporting the method could be found and because it would not be feasible for great distances, nor for measuring heights, which in the meantime some one had suggested was a problem similar to the horizontal distance. One suggestion led to another until Mr. Ramsey suggested plotting a triangle with a distance along the walk and angles at the extremities, which was carried out, by constructing to scale a triangle and erecting a perpendicular from the vertex of the triangle which represented the position of the tree that stood on the opposite edge of the street. For the theoretical proof we then took up the problem of two triangles being equal if the two angles and included side of one are equal to the two angles and included side of the other. After this the student was taught to get as general statement of his problem as possible. Sometimes this idea was so new to the student in that field that he found difficulty in getting a general formula.

As an example of this difficulty a constructive problem in surveying or engineering, while appealing to the ordinary boy sometimes lacked interest to the girl, because she knew nothing about the problem or its condition or had any desire or interest in the knowing. She could be easily interested by field excursion or by field experimentation, and by showing her that this at least added to her cultural knowledge if she had no practical need. Or by showing her the relation of the problem to wall paper

design or the idea of interior decoration or the arrangement of a home. Even if this did not arouse an interest, the engineering material of the boys was more real and held out to her greater possibility in conception than the abstract enigma found in the text. She had a chance to try and find out, she could measure and verify.

In the work no attempt was made to make the theoretical subservient to the practical but rather that the student could be led to see an association between his acquired knowledge and what he is gaining. In so doing the student is brought to realize that he is actually doing something, and that the study is worth while.

Some Problems that cannot be Determined in one School Year in one School.

1. Determining the field in which the problems are best suited to the interests and experiences of the student.

No two classes in geometry in the same district will be composed of students with the same material knowledge, local surroundings, interests or ambitions. This is probably more true for different towns. In various localities the curriculum is so arranged that certain branches of study precede others, thus changing the material knowledge of the student, and again age and previous training is a vital factor.

2. Whether interest and knowledge of the boys and girls relative to the problem material are different and if so why.

Some cities have separate High Schools for the boys and girls, consequently courses of study are different. With the class in Mound City, the town is so small that the boy and girl in a

large measure enjoyed a common interest and common knowledge of most of the activities in the town.

3 How much supplementary explanation the teacher must give.

In this class as little supplementary explanation as possible was given. In some schools this would have to be modified owing to the arrangement of the course of study, and to the plan followed in teaching arithmetic in the seventh and eighth grades, also amount of work in mechanical drawing, and in general previous knowledge.

4. Can the student stand the examinations.

Many questions arise relative to the plan and many cannot be answered. One asked as often as any other, "can the student stand the examinations?", is best answered by the result obtained by the class. The Sophomore class in the High School was divided according to their seats in the study hall, one following the plan outlined in Chapter II the other following the text in the usual manner. In this division there could be no evident difference in the equipment of the students. The two classes were held in the same building but under different teachers. The average standing of the two classes upon work in the text did not differ materially, while on an examination of interpretation the former class was far ahead, averaging nearly twelve per cent above the other class and stating their ideas in a more definite and economical manner. Results showed that the class not only had as much actual text material, but in addition a working knowledge of their geometrical principles.

5. Is it expedient.

The plan has been markedly successful. "A student's ability to prove a proposition is no assurance that he knows it. The test is as to whether he knows it is whether he can use it."[1] The student learned to do. He saw a relation between life in school and life outside of school. It is well to bear in mind that "From the standpoint of the child, the great waste in school comes from his inability to utilize, within the school, the experiences he gets outside, and to apply in daily life what he learns in school."[2] As to whether all classes would arrive at this result is not possible to say with only one class. A different teacher, in a different locality, with a different student might get different results.

"We should bear in mind that real progress should not be measured by the ground apparently covered, but rather by what the child actually gets in such a way as to make it his own."[3] Probably nine tenths of the work learned in school is soon forgotten. Would it not be wiser to learn well the part of the text that is covered? The facts learned in school are not the most important feature of the work but it is the attitude of the mind, the habits of study, that the student forms, which are important. He is prepared then to go out into the world, not merely to remember what he learned in school, but to put into practice the habits which he has formed. The objection to this plan may be overcome by results. An educational plan is better judged by its results under fair and sufficient trial than by any other standard.

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1. McMurry, The Method of the Recitation, Page 160.

2. Dewey, Psychology.

3. Osgood, by Hart, Sch. Sci. and Math. 1905. Teach. of Geom. P. 649

If the fundamental principles of geometry are taught by practical methods, so that the knowledge acquired may be real and usable, the subject becomes real and interesting. Faith is strengthened by some concrete verification or test of the results, thought out, and on the other hand, measurements and experiments may often give the student the key to the demonstration or point out new possibilities.

Some Detail Explanation of the Plan.

Each of the two classes in geometry spent six months in the work. The adopted text was Wentworth's Plane Geometry, Revised. One of the classes followed rather carefully the text, covering completely the first four books, with the exception of some of the exercises on limits, and the original exercises which were carried out in part. The class that followed the plan of 'Real Problems' had access to many texts and particularly the adopted text. It did not attempt to work all the exercises or to follow the general plan of the text, but rather to approach the problems along the lines of known and elementary material, no matter where found in the text. The class in the text book devoted on an average the same amount of time to preparation and recitation work as nearly as could be determined. The 'Real Problem' class worked out quite a number of the original exercises, finding them as a rule easier than the class in the text book especially in the method of attack in the solution.

Chapter II gives only type problems. The better students handed in, neatly and carefully worked out about one hundred and twenty problems, the average student about one hundred, while the student below the average worked as low as eighty. To say the student worked one hundred and twenty problems does not convey the

whole answer, because he often worked out several exercises covering different fields of knowledge, on the same problem as the problem on page 46 in Chapter II or on principles of symmetry.

Results as given by examinations which are mentioned above show that the plan gave a greater grasp of the situation, a clearer interpretation of the problem and a more economical working basis. The examinations were conducted jointly by the two teachers and the results seemed to indicate that the real problem plan had given greater results in both the ability to think and to do.

The plan has been of great interest to the teacher. The plan called for much work on the part of the teacher, as there was no collection of problems published[1] or no discussion of the plan to be found in literature. The students entered upon the work with much interest. One of the difficulties that presented itself at first was the adjustment of the work with other subjects so as not to permit the student to neglect his other subjects as he was wont to do. The Principal of the High School and the patron of the district became interested in the problem and were a help to the school in furnishing problems and in establishing a closer relation to the activities of the student.

The Advantages of this Method.

MOTIVE. The students work with a fixed purpose. The geometrical problems growing out of individual experiences are worked with an amount of interest not to say eagerness, which artificial problems can never inspire. The problem becomes a vital question to him because it is made up out of data based upon his experiences.

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1. A partial list has been published this year by School Science and Mathematics, 1909 - 1910.

It is real and he sees a need for the solution hence works for the end. This definite aim or problem causes the student to centre his attention on some main idea, and thus furnishes him a proper motive for active thinking. It makes him conscious of the course he is to pursue. He knows definitely what he is to accomplish. "A definite aim furnishes a motive for effort[1]" A good aim becomes a standard, both for the teacher and the student for judging the value of facts. When all are conscious of a fixed and definite aim, they have a standard by which they can determine whether or not a certain fact is worth their attention at that particular time. No attempt is made to cover all of the ground given in the text, nor to study all the material in the portion studied. Instead of beginning at the first of the book and taking each lesson as it is given in the text, the student learns to select the material which he needs to furnish him information on the problem, which he has before him as is shown by problem on page 49 in Chapter II. It makes the student study the method of selection. It is in this way that the student learns to take what at first appears a wilderness of unrelated facts and organize them into a consistent body of knowledge. The student is placed in a situation so that he looks for the significance of facts.

NATURAL METHOD.

The material and the method follow the laws of mental condition as a result of growth, that is the student not the subject matter is the centre of the educational process. Instead of giving the student some logical truths for future use, it makes geometry an instrument in the process of

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1. McMurry, The Method of the Recitation, Page 113.

living in the present, thus giving a more useful instrument or tool for future leverage. The student gets the viewpoint of the people who developed the science. He can be made to feel an interest in his work. The student as the business man, is more interested in his work when he has some definite purpose in view. If the student is confronted by a definite problem he is led to feel an interest in the facts of history, because they are the means by which he is able to study the problem before him, The systematic application of the psychological factor 'suggestion' strenghhened the interest and the desire to know, thus making the work easier for both teacher and student.

UNITY.

This method is an aid to real correlation, and helps in unifying the student's mathematical knowledge and activity, and helps to solve the question of relatives values in school principles. Problems that have no value in actual life either in fact or principle can be omitted or placed in their relative position with regard to other material. The student learns to actually use his mathematics. He correlates arithmetic, algebra, geometry, physics, machanics and empirical knowledge in a way that convinces him that there may be a relation between school life and life outside of school. The rigor of the solutions is in no way lessened, for a student has not proved a problem nor understood it until he has assimilated the facts concerning it.

UTILITY.

The practical utility gives a good basis for work. He does not have to wait for an opportunity to apply his principles and facts. He has already tried the problem.

GENERAL TRAINING. This method develops observation, skill in discrimination, skill in making inferences, individual initiative, and capacity to get the correct idea or conception of axioms and definitions, in such a manner as to make them usable. It also tests the student's ability to recognize the theorem in different relations.

Disadvantages of the Method.

ORGANIZATION. One of the decided disadvantages is the the fact that the problems overlap each other in material, thus destroying an economic principle unless used with extreme care.

MATERIAL. Since the idea is new there is no compiled data or or material thus the teacher has to collect and organize his material. This difficulty would soon remove itself if the plan was followed for a few years or by several teachers. The fact that the material has not been collected necessitates a reconstructed environment in the school which must be brought about in a careful way so as not to destroy other valuable features connected with the other courses offered in the school.

METHOD. The experiences of the students and the teachers are so widely different, and the experiences of the individual students differ so much that it is difficult to follow class teaching. The plan unless watched becomes an individual one. Under the experience of the teacher might be grouped the following:

1. Teachers are not efficient; are not acquainted with object matter nor with the subject matter, that is do not know how to formulate problems and arrange same according to ability of student.

2. Lack of familiarity on part of the teacher of previous training of student in method and subject matter.

3. Lack of observamce on the part of the teacher of the interests of the students.

4. Lack of interest, on part of teacher in the subject.

5. Lack of strong personality in the teacher.

6. Conservative community.

7. Superintendent may object, if text is not completed.

8. Patrons may object.

"In this discussion of the teaching of geometry, the belief has been emphasized that geometry should be taught not as a collection of settled facts to be learned, even though the facts of geometry that are taken up in a first course have in the main been settled for thousands of years, but as a set of phenomena to be investigated scientifically. Geometry is a living and growing science; if it is taught so that the pupil himself makes some discoveries, he will feel this life; to make discoveries he must ask questions, he must scrutinize the various possibilities of the topic. Questions may be raised that are too difficult for an elementary course, or that open the door for some little account by the teacher of work in geometry, ancient and modern, beyond the scope of the course. The student will then come to the end of the course in geometry with possibilities of study unexhausted, perhaps with some problems still unsolved, and with hearsay knowledge of important lines of geometric study different from those he has followed. He will not regard geometry as a cast iron subject whose sum total is recorded in the book he has studied, but as a large and growing field of which he knows a part; and he should look forward with pleasure to obtaining a deeper and more critical insight into the part that he has already studied, as well as to

extend his knowledge to other parts of the subject. This anticipation would be appropriately met by suitable courses in elementary geometry in the earlier collegiate years.[1]"

1. J. W. A. Young, The Teaching of Mathematics, Page 287.

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Emerson, Natural History of the Intellect.

Halstead, Elements of Geometry.

Harrison, Practical Plane and Solid Geometry.

Hopkins, Plane Geometry.

Iles, My Class in Geometry.

King, Physics of Agriculture.

McMurry, The Method of the Recitation.

Osterhout, Experiments with Plants.

Phillips and Fisher, Elements of Geometry.

Sanders, Elements of Plane and Solid Geometry.

Schultze and Sevenoak, Plane and Solid Geometry.

Shutts, Elements of Plane and Solid Geometry.

Smith, The Teaching of Elementary Mathematics.

Spencer, Inventional Geometry.

Warren, Experimental and Theoretical Geometry.

Wells, Plane and Solid Geometry.

Wentworth, Plane and Solid Geometry.

Young, The Teaching of Mathematics.

Personel of the Class.

Alkire, Ed.

Cottier, Edith.

Childers, Hazel.

Cooksey, Mary.

Davis, Floy.

Dillon, Nellie.

Groves, Cora.

Greiser, Charles.

Jackson, Robert.

Karns, Orman.

Kennish, Emma.

King, Dolly.

King, Mamie.

Porter, Floyd.

Ramsey, Lauriston.

Robinson, Roscoe.

Shutts, Paul.

Strobel, Gertrude,

Smith, Myrtle.

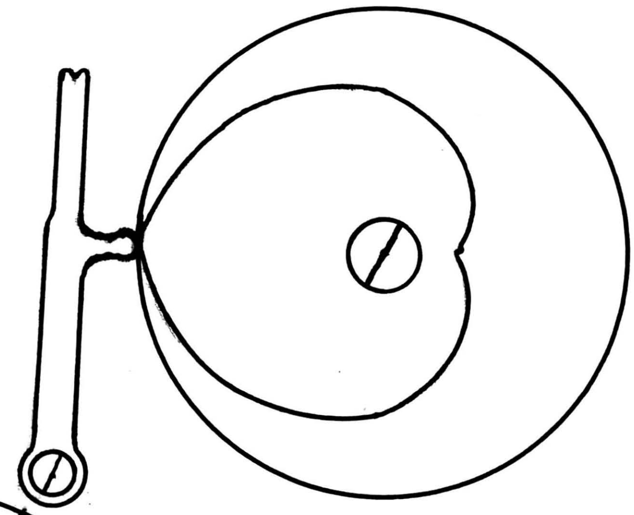
Stubbs, Eva.

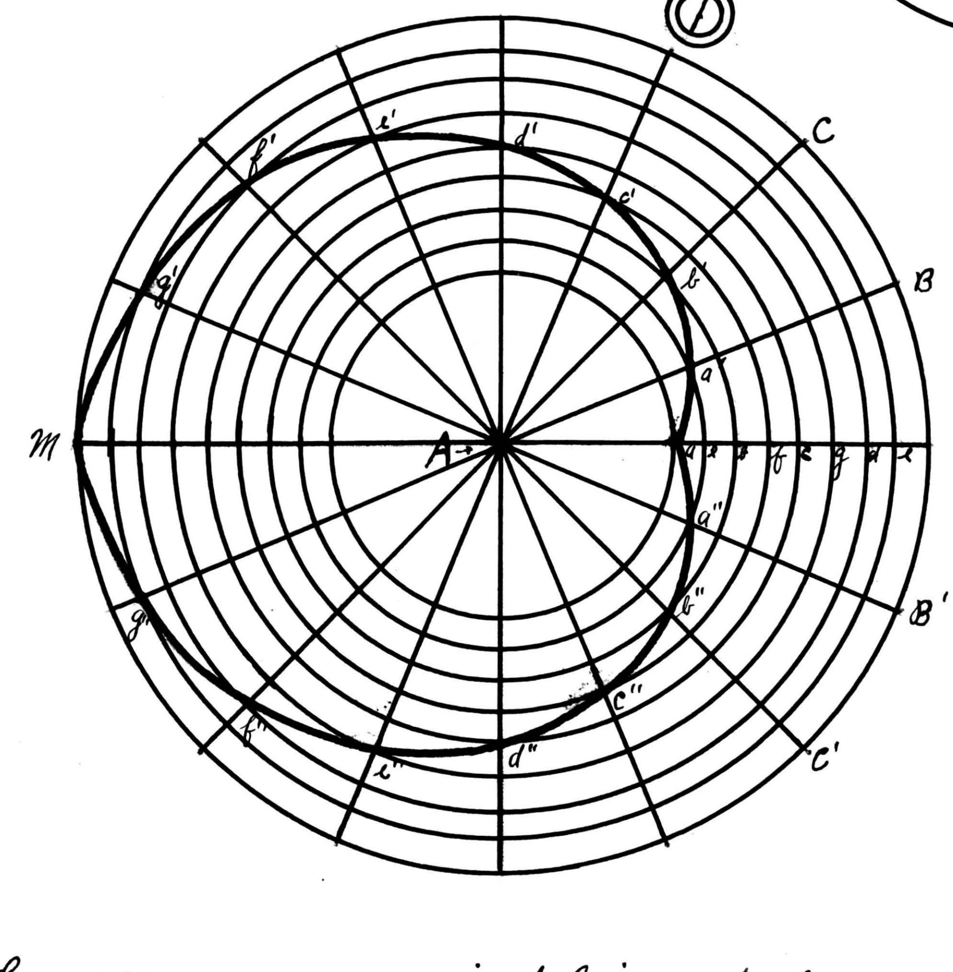
Young, Warren.

This set of papers among the best in drawing and general work.

Problem:-

To construct a cam as found on the Singer Sewing Machine.





Lay off any convenient line and at some point erect perpendicular, bisect the angles formed and their angles. Construct any circumference, the angles will all be equal. Central angles

Warren Young

of the same are equal.[1] Next lay off the chosen throw to a scale; and also the radius. Divide the throw into as many parts as there are angles. Next construct concentric circles whose centres are at A, with radius equal to a, e, b, f, c, g, d. Then beginning at a, mark line from intersection of circumference and radius. The line formed is the curve of the cam.

Points a', a," b' b" are equal distances from the radius. Curve a' b' c' etc is symmetrical curve with respect to AM and a" b" c" etc.[2]

---

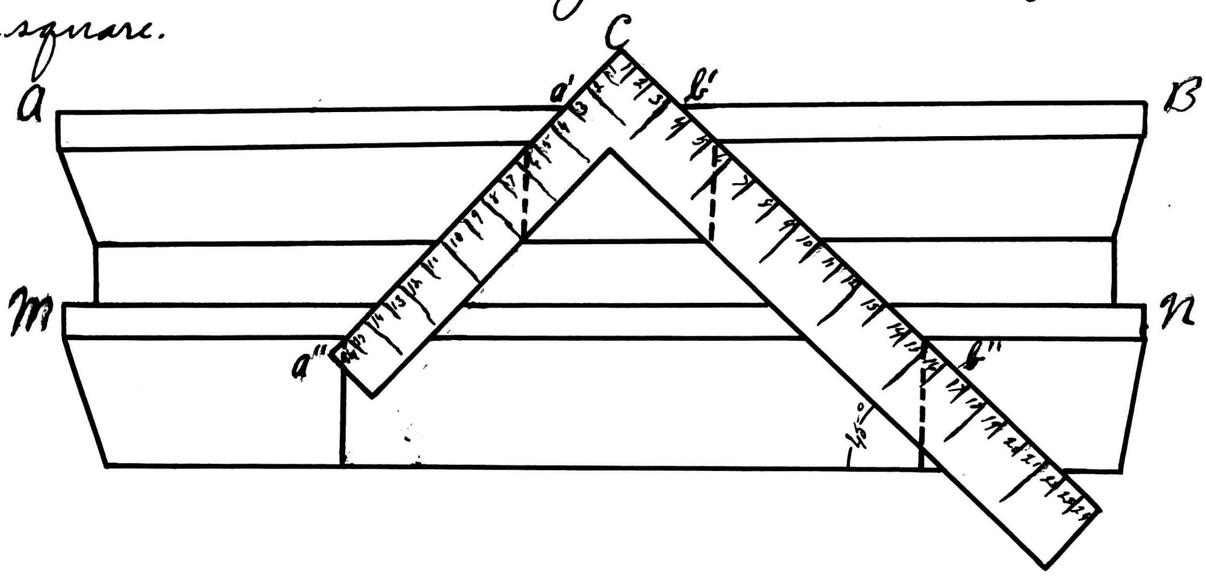
1 Wentworth, Plane Geometry, Page 78, Prop. 11.

2. " " " 60 Art. 209.

Warren Young

Problem:-

To construct a 45° angle on a mitre box by means of a square.



Place square on the mitre box with corresponding lengths of both legs on one edge of the mitre box as AB. Then Ca' and Cb' are equal, which makes angle a'Cb' an isosceles triangle.[1] If the sides are extended, angle a"Cb" will be an isosceles triangle, for the sides AB, MN etc of the mitre box are parallel. Angl a"Cb" is a right angle,[2] and if a perpendicular is dropped from C to MN the right angle is bisected which makes angles of 45°. Thus, a line drawn along the sides of the square will make angles of 45°.

---

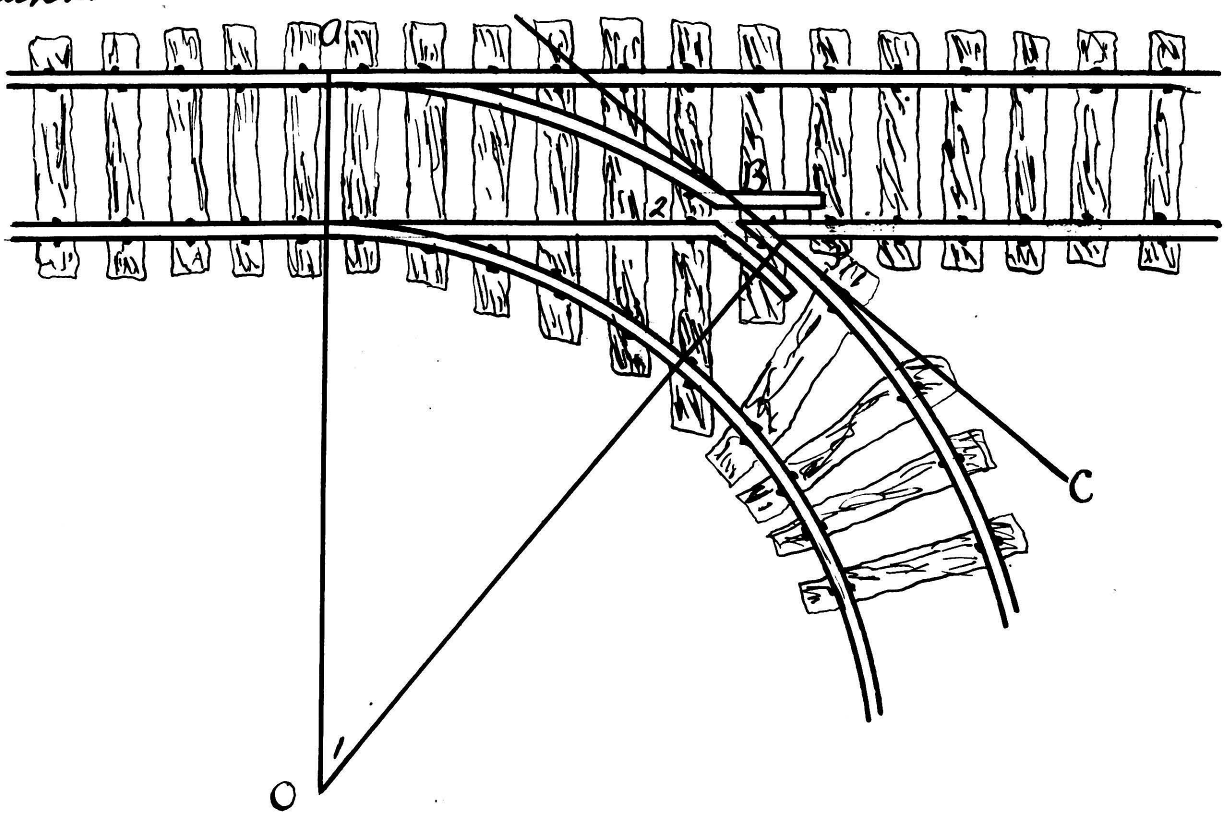
(1) Wentworth, Plane Geometry Page. 34, Prop. XXII.

(2) " " " " 11. Art. 63.

Warren Young

Problem:-

To prove that the "frog" angle of a railroad switch is equal to the central angle formed by lines drawn from centre of curvature to the tangents of the straight and curved tracks.



From point O draw radii OA and OB. The central angle 1 is measured by arc AB(3). Angle 2 is measured by ½ arc AB(4). Therefore these two angles are equal. The verticle angles 2 and 3 are equal(2). Therefore angle 1 equals angle 3. Magnatudes which are equal to the same magnitudes or equal magnatudes are equal to each other(1).

(1.) Wentworth, Plane Geometry, Page 6, Ax. 1
(2.) " " " " 18, Prop. IV.
(3.) " " " " 100 " XVI.
(4.) " " " " 105 " XIX.

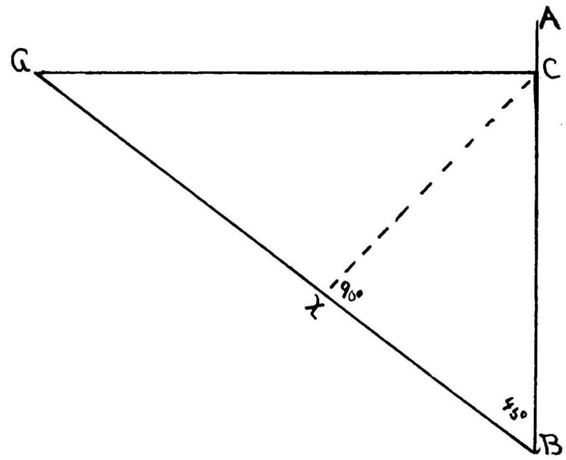
Warren Young.

This set of papers show neatness and care.

Idea good. Explanations usually clear.

Problem;

The distance from AC at which a ship passes a light house A, when moving in the direction BC, is obtained by observing the moment when the direction of the light house makes an angle of 45° with the course of the ship, and again when it makes an angle of 90°, the distance the ship has gone between the two observations being noted. Show how to compute the distance at which the light house is passed.



Given: A lighthouse A and a ship moving along AB in the direction of BC. The distance from the light house at which the ship passes the light house is found by noting the moment the ship makes an angle of 45° with the light house or the ∠CBA and also the time the ship makes an angle of 90° with the light house or ∠CXB.

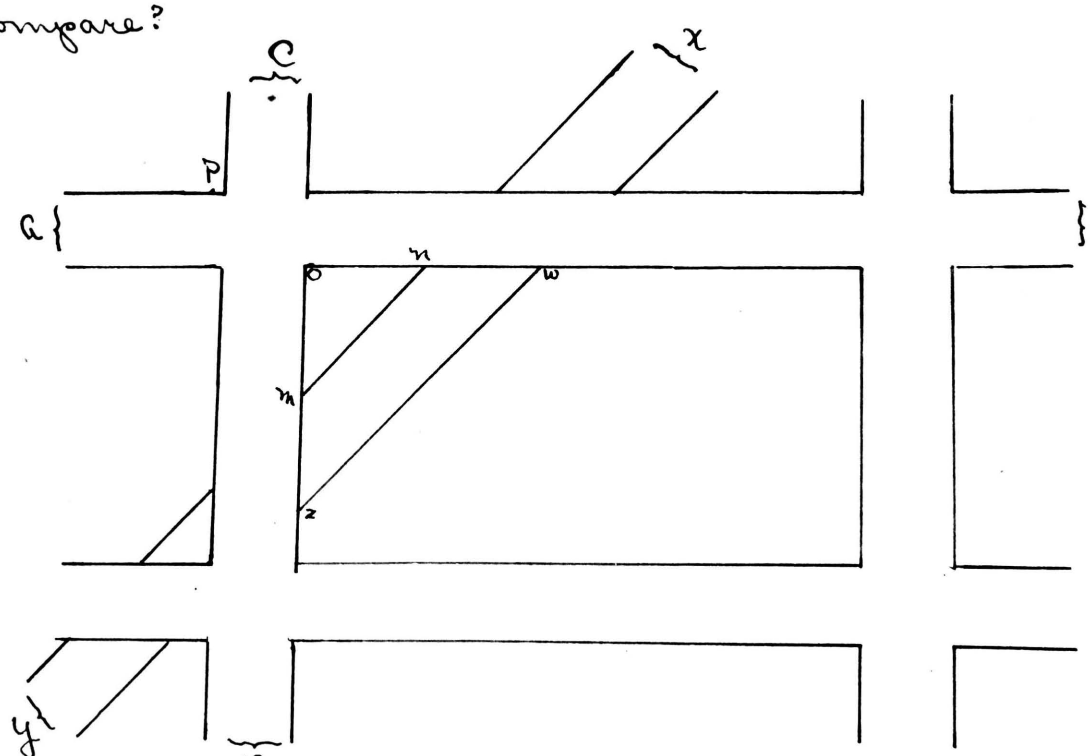
To Find the distance at which the ship passes the light house or BC.

Proof. Construct a line CX perpendicular to the line AB. Then triangle CXB is an isosceles right triangle. ∠CXB is a right angle by construction. Angle XBC is equal to 45 degrees by hypothesis. Therefore ∠XCB must equal 45 degrees since there are 180° in a triangle. Let each side of the isosceles right triangle CXB = a. For the sides are equal.[3] The hypotenuse of a right triangle equals the sum of the squares of the other two sides.[4] Therefore $a^2 + a^2 = 2a^2$. Taking the square root of $2a^2$ we get $a\sqrt{2}$, the length of BC or the distance at which the ship passes the light house, in terms of a.

Hazel Childers

Problem;

A boulevard cuts Eighth and Savannah Streets at equal distances from the corners. How do the angles at the junction of the boulevard with Eighth and Savannah Streets compare?



Given: Two streets Eighth (AB) and Savannah (CD) intersect at right angles in the points P and O, and cut by the boulevard xy at equal distances from the corners, in the points mn and zw.

To compare the size of the angles where the boulevard cuts the two streets, namely ∠s Omn and Onm.

Proof. The triangle Omn is isosceles[1] for by the Hypothesis the distances from the corner to the cutting line of the boulevard are equal. Since the triangle Omn is isosceles the angles Omn and Onm are equal.[2]

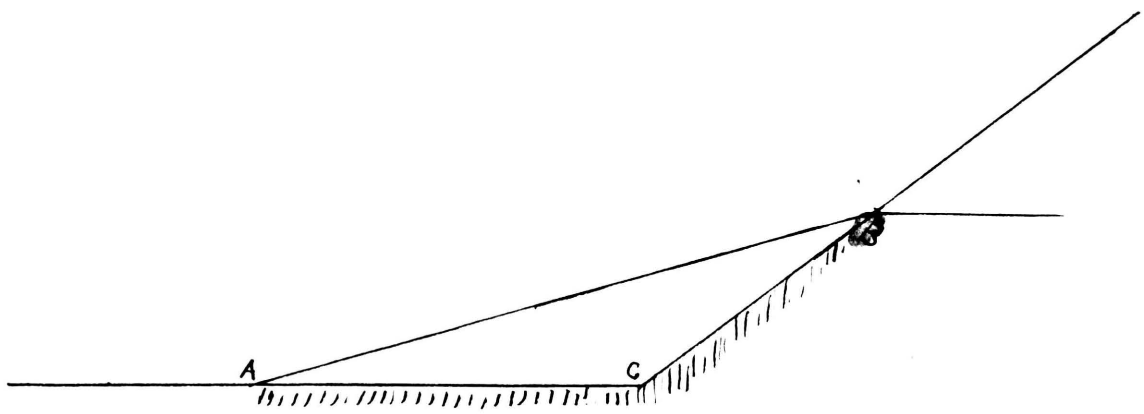
---

1. Phillips and Fisher, Plane Geometry, Page 33, Art. 69 Definition.
2. Phillips and Fisher, Plane Geometry, Page 34, Prop. XVIII.

Hazel Childers

Problem;

Near Mound City a railroad embankment stands on a horizontal plane; and we desire to know the distance from a point in the plane to the top of the embankment.



Select a point C at the foot of the embankment lying in the same vertical plane as A and B, and measure the distances AC and CB and the angle BAC. By measurement find BC equal to 36 feet and AC = 32 ½ feet and angle ACB on protractor is 141 degrees. Construct straight line AC 32½ mm. long and with protractor construct angle ACB, 141 degrees. Draw BC = 36 mm. AB measures 65½ mm. long making the distance AB equal to 65½ feet.[1]

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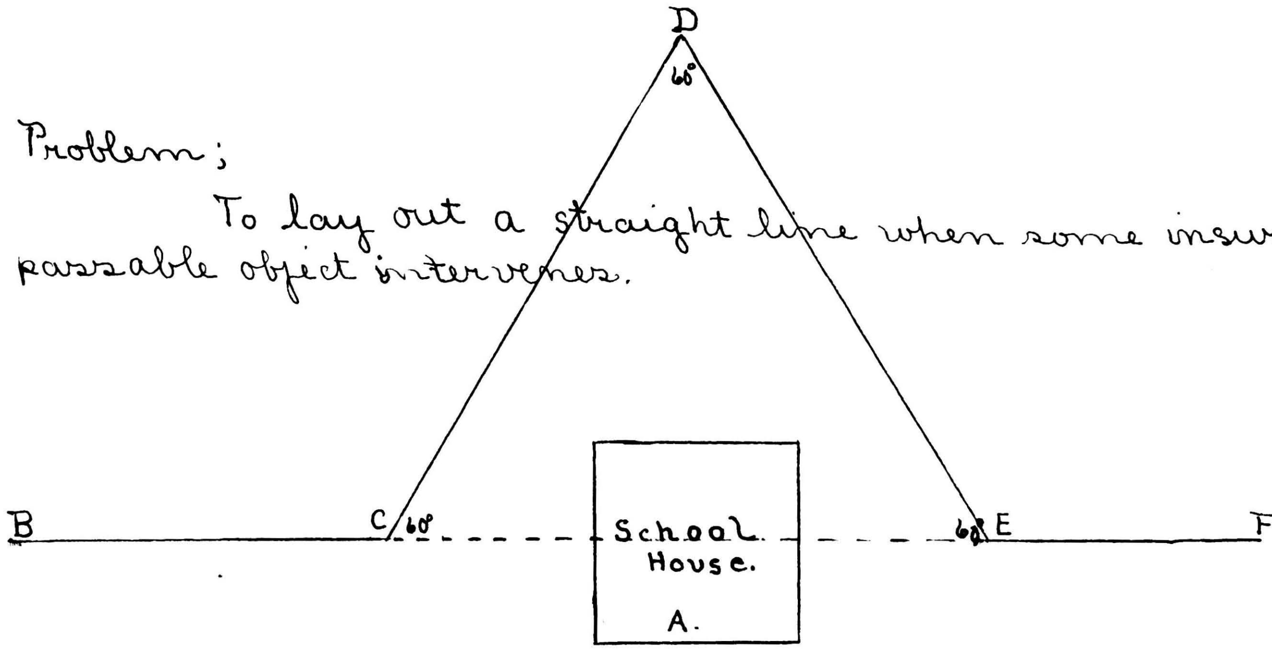
1. Wentworth, Plane Geometry, Page 35, Prop. XXI.

Hazel Childers

Problem;

To lay out a straight line when some insurpassable object intervenes.



Suppose A to be the school-house and BCEF the straight line to be determined. At point C construct angle[1] = 60 degrees and lay off DC any convenient distance, at least long enough to see by the building, and at D construct angle[3] = 60 degrees and draw DE = DC and at E construct angle 2 = 60 degrees, and draw EF. Then BCEF is a straight line[2], and CE = DC or DE. Distance from B to F is BC plus CD or (DE) plus EF.

---

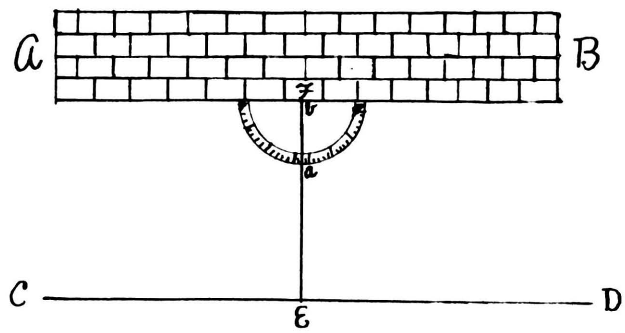
1. Wentworth, Plane Geometry, Page 115, Ex. 135.
2. Wentworth, Plane Geometry, Page 8, Sec. 46.
3. Wentworth, Plane Geometry, Page 36, Prop. XXII.
   Plane Geometry, Page 37, Prop. XXIII
   Plane Geometry, Page 37, Art. 148, Cor. 1.
   Plane Geometry, Page 40, Ex. 9.

Hazel Childers.

This set of papers belong to one of the best in interpretation, and general idea.

Problem:

To build a fence parallel to a walk in front of the school house.



Given: The walk A B.

To construct a fence parallel to the walk.

Place the straight edge of the protractor along the straight edge of the walk, and mark on the walk, the centre of the protractor. Also on the curved edge of the protractor take the middle point. Stretch wire E F tight across the two points a and b. E F is perpendicular to A B. When one straight line meets another straight line so that the two angles formed are equal, each is a right angle and the line is perpendicular.[1] Repeat the process at E. C D is perpendicular to E F. C D is also parallel to A B. Two straight lines perpendicular to a third straight line are parallel.[2]

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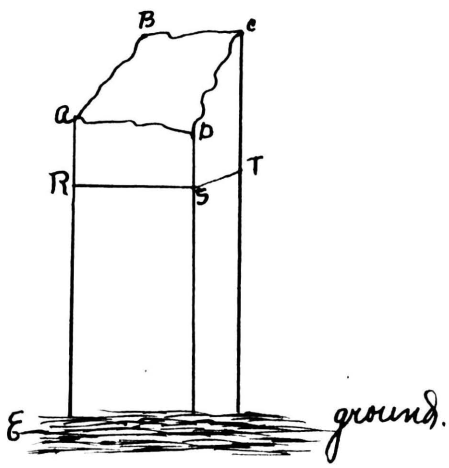
1. Wentworth, Plane Geometry, Page 11, Section 63.
2. Wentworth, Plane Geometry, Page 24, Proposition X.

Gertrude Strobel.

Problem:

To saw off a post so that the edges of the post are perpendicular to the plane of the top.



Given: A post.

To draw a line RS on surface AB and ST on the surface DT as a guide for the saw.

At point R on AE, with a protractor erect a perpendicular, RS.[1] Since RS is perpendicular to AE it is also perpendicular to DS. A straight line perpendicular to one of two parallel lines is perpendicular to the other.[2] At point S in DS erect a perpendicular to DS in the plane DT. If desired continue the process about the post. Then the plane RST is perpendicular to the edges and if sawed through the plane RST, the top of the post will be level with the ground, or the plane of the top will be perpendicular to the edges of the post.

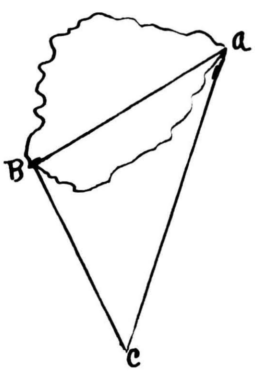
---

1. Wentworth, Plane Geometry, Page 25, Proposition XI.
2. Wentworth, Plane Geometry, Page 11, Section 64.

Gertrude Strobel.

Problem:

To measure the length of a pond west of the school building.



Given: A pond.

To find the length of the pond.

First take any point near the pond, from which both ends of the pond may be seen. At this point, c, set up the surveying instrument. Sight to each end of the pond, a and B. In measuring the angle formed by the lines ac and Bc it is found to be 45°. When the sides of the angle are measured, the side Bc is found to be 270 feet and ac, 400 feet.

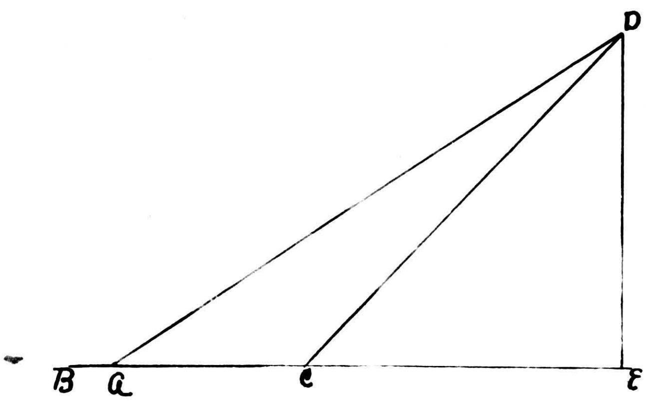
In the drawing use the scale 1 millimeter = 10 feet. Upon paper construct an angle, with a protractor, equal to 45°.[1] Then draw BC 27 millimeters and AC 40 millimeters long. Connect a and B,[2] measure this line with a centimeter ruler and it is found to be 29 millimeters long. Since the scale, 1 millimeter = 10 feet is used, the length of the pond is 290 feet.

1. Wentworth, Plane Geometry, Page 116, Proposition XXVI.
2. Wentworth, Plane Geometry, Page 35, Proposition XXI.

Gertrude Strobel.

Problem:

To find the height of the school building.



Take any point near the school building from which the top of the building can be seen and sight to the top of the building. With a protractor the angle formed by the line sighted to the top of the building and the ground is measured and found to be 47°. From this point, C, measure off any convenient length as 20 ft. At the end of this line, A, sight to the top of the build-ing. This angle is found to be 33.5°.

In the drawing use the scale 1 millimeter = 1 ft. Upon paper draw BC any length. At A construct with a protractor an angle equal to 33.5°. Upon BC take AC equal to 20 millimeters. At B construct an angle equal to 47°. Produce the sides DB and DC until they intersect at D. From D drop a perpendicular to AC,

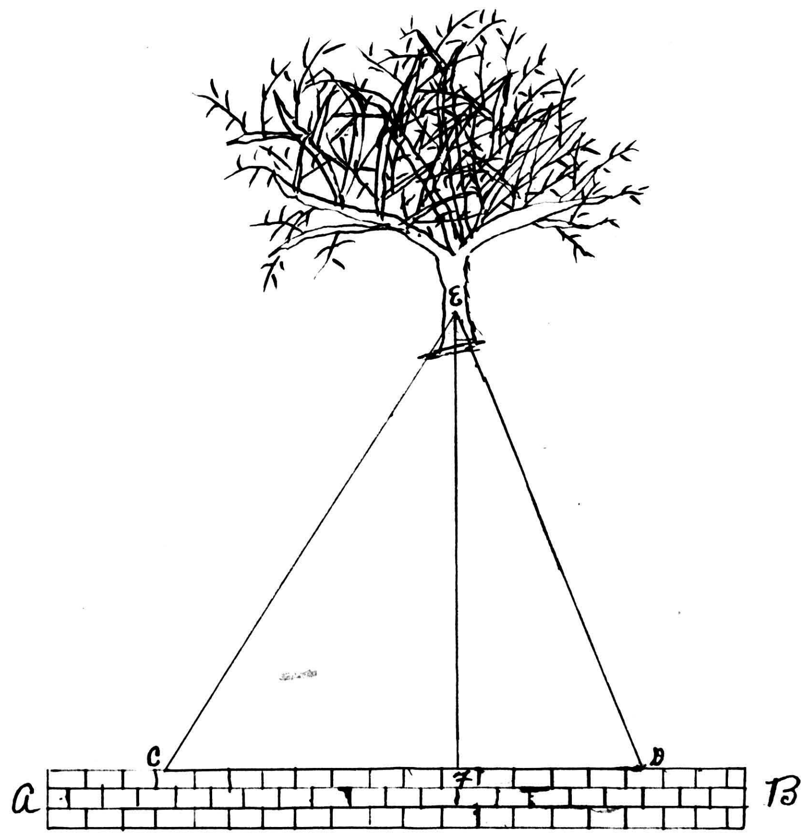
produced, at E. Measure this line with a centimeter ruler and it is found to be 43 millimeters long. Since the scale 1 millimeter = 1 ft. is used, the height of the building is 43 ft.

Wentworth, Plane Geometry, Page 34, Section 142, Cor. 3.

Gertrude Strobel.

Problem:

To measure the width of the street from the sidewalk.



To measure the width of the street.

Upon the sidewalk, A B, measure off any convenient length as 60 feet, which is represented by line C D. From C sight to a tree which is across the street and find the angle formed by a line from C to the tree and the sidewalk. By measuring this angle with a protractor it is found to be 56.5°. At D repeat the process and this angle is found to be 68°.

In the drawing use the scale 1 millimeter = 1 foot.

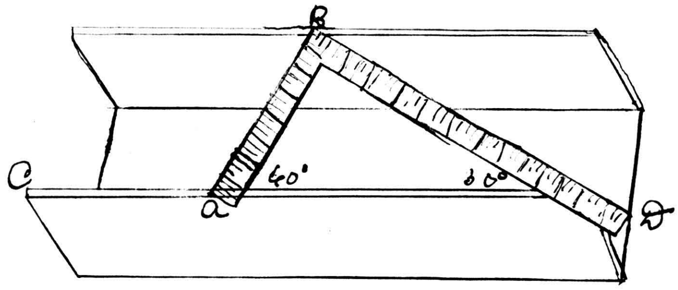
On AB take CD 60 millimeters long. At C construct an angle, with a protractor, equal to 56.5°; and at D construct an angle equal to 68°. Produce the sides of the angles until they intersect at E. Draw EF from E perpendicular to CD. EF represents the width of the street. Measure the line EF with a centimeter ruler and it is found to be 58.6 millimeters. As the scale used is 1 millimeter = 1 ft., then the width of the street is 58.6 ft.

Wentworth, Plane Geometry, Page 34, Proposition XI.

Gertrude Strobel.

This set of papers belong to an average student.

Problem: To construct a 60° and a 30° angle on a mitre box.



First measure off a distance twice the width of the mitre-box as a transversal, and mark the line on the box thus made. I also marked the line on the other side of the square ⊥ to it. If the hypotenuse of a right triangle is double the shorter leg, one acute angle will be double the other. Hence one angle is 30° and one 60°. If the length of side AB is placed along side of the box and a line is drawn from the extremity of the line to the vertex of the triangle, then each angle is 60°, for an equiangular triangle is formed. In an equiangular triangle each angle is one half of 2 right angles or 60°.(1)

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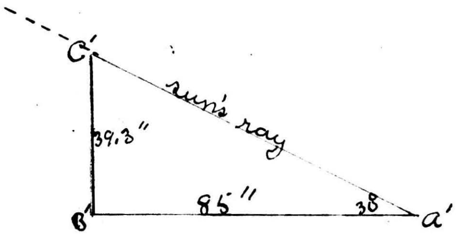
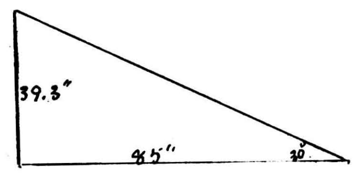
1. Wentworth Plane Geometry. Page 33. Cor. 7.

Cora Groves, Jr.

Problem: To find the altitude of the sun.





At some time in the day when the sun is shining measure the shadow made by a stick that is perpendicular to the earth. Also note the time the experiment was made. In this problem the stick used was a meter-stick 39.3 inches long, and the shadow cast was 85 inches long. The experiment was made at 1:20 o'clock. A drawing was then made to the scale of ½ centimeter equals one inch. The two extremities of the lines were then connected and the angle measured and was found to be 30° angle. So the sun's rays of that day were on a 30° angle. Figure out the length of the sides A'B', B'C' and the altitude will be the length of B'C'. These two triangles are equal because two sides and the included angle of the one are equal, respectively, to two sides and included angle of the other.

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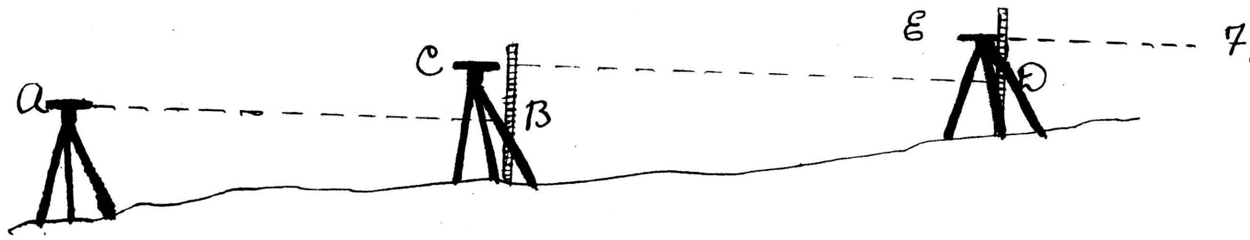
1. Wentworth Page 35. Prop. XXI.

Cora Groves, Jr.

Problem:- To find the height of the High School building above sea level by leveling.



Given:- The county engineer's reading at State street, which is 825 feet above sea level.

To Find:- The height of the Grammar and High School buildings.

The method of leveling is this: the level is set with the bar of the sights over the point you are leveling from and the instrument leveled. The rod is simply a straight stick with inches and feet marked on it. Put down the height of the bar as the first reading. Next have some person walk ahead of the level with the rod at the greatest distance possible. Then sight through the level and have the person at the rod move the sliding block so that the bottom side coincides with the sighting bar. Put down the height of the block and subtract from the first reading, and add the sum to the height figured out before. Move the level up where the rod stood, level and proceed as before. If you measure down a hill you must subtract from the sum found before. Following is a list of the level readings for finding the height of the schools from State street.

Stations.	Height of Level.	Height of Rod.	Difference.	Height.
1.	4' 3 3/4"	1' 8"	2' 7 3/4"	827' 7 3/4"
2.	4' 4 1/2"	1' 4 1/2"	3' 00"	830' 7 3/4"
3.	4' 5 1/8"	4 5/8"	4' 0 1/2"	834' 8 1/4"
4.	4' 3 1/2"	2 1/8"	4' 1 3/8"	838' 9 3/8"
5.	4' 4 3/4"	1"	4' 3 3/4"	843' 1 1/8"
6.	4' 4"	1"	4' 3"	847' 4 3/8"
7.	4' 4 1/2"	1"	4' 3 1/2"	851' 7 7/8"
8.	4' 3 3/4"	1"	4' 2 3/4"	855' 8 5/8"
9.	4' 4 3/8"	1"	4' 3 3/8"	859' 9"
10.	4' 4 1/4"	1"	4' 3 1/4"	860' 0 1/4"
11.	4' 5 3/8"	1"	4' 4 3/8"	866' 4 5/8"
12.	4' 4 1/4"	1"	4' 3 1/4"	872' 7 7/8"
13.	4' 5 3/4"	1"	4' 4 3/8"	877' 0 3/8"
14.	4' 5 5/8"	3' 1 1/8"	1' 4 3/4"	878' 5 1/8"
15.	4' 3 1/4"	4' 6 5/8" *	3 3/8"	878 1 3/4"

The readings for the height of the Grammar School are the first fifteen and the following 4 from the corner of the yard to the building.

	4' 4 3/4"	3 1/2"	4' 1 1/4"	882' 6 3/8"
	4' 2 1/4"	1' 6 5/8"	2' 7 5/8"	885' 20"
	4' 3 1/8"	9"	3' 6 1/8"	888' 8 1/2"
	3' 6"	7 1/2"	2' 10 1/2"	891 6 5/8"
16.	4' 4 3/8"	1' 10 11/16"	3' 5 11/16"	881' 7 7/15"
17.	4' 5 1/2"	4 1/15"	4' 1 7/16"	885' 8 7/8"
18.	4' 4 1/4"	2 5/8"	4' 1 5/8"	889' 10 1/8"

Station.	Height of Level.	Hight of Rod.	Difference.	Height.
19.	4' 5$\frac{1}{4}$"	4$\frac{3}{8}$"	4' $\frac{7}{8}$"	893' 10"
20.	4' 4$\frac{1}{2}$"	8$\frac{15}{16}$"	3' 8$\frac{9}{16}$"	896' 8$\frac{9}{16}$"
21.	3' 11$\frac{3}{4}$"	2' 8"	1' 3$\frac{3}{4}$"	897' 11$\frac{3}{16}$"
22.	4' 5$\frac{5}{8}$"	5$\frac{1}{8}$"	4' 0$\frac{1}{4}$"	901' 11$\frac{7}{16}$"
23.	4' 6"	8$\frac{5}{8}$"	3' 9$\frac{3}{8}$"	905' 5$\frac{13}{16}$"
24.	5' 6$\frac{1}{4}$"	8$\frac{1}{4}$"	4' 10"	910' 6$\frac{13}{16}$"
25.	4' 5$\frac{7}{16}$"	8$\frac{3}{4}$"	3' 8$\frac{11}{16}$"	914' 3$\frac{1}{2}$"
26.	4' 4$\frac{1}{8}$"	5$\frac{7}{8}$"	3' 10$\frac{1}{4}$"	918' 1$\frac{3}{4}$"
27.	4' 2$\frac{7}{8}$"	10"	3' 4$\frac{7}{8}$"	921' 6$\frac{5}{8}$"
28.	4' 5$\frac{3}{16}$"	6$\frac{7}{8}$"	3' 10$\frac{5}{16}$"	925' 8$\frac{5}{16}$"
29.	4' 6$\frac{3}{8}$"	8$\frac{11}{16}$"	3' 9$\frac{11}{16}$"	930' 10$\frac{11}{16}$"
30.	3' 7"	2' 2"	1' 5"	932' 3$\frac{11}{16}$"
31.	4' 2"	2' 6$\frac{5}{8}$"	1'★ 7$\frac{3}{8}$"	934 8$\frac{5}{16}$"

Lines AB, CD and EF are parallel[1]; because they have their alternate interior angles equal[2]. A plumb line was used to get the bar sight exactly perpendicular to the point on the ground. The height of the Grammar School is 891 feet. 6$\frac{5}{8}$ inches, and the High School 934 feet, 8$\frac{5}{16}$ inches.

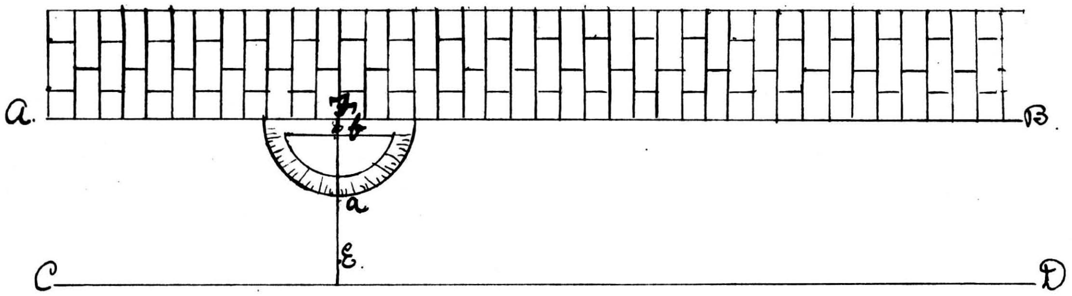
---

1. Wentworth Page 24 Prop. X.
2. Wentworth Page 26 Prop. XII.

★ The reading for the height of the rod was greater than the level reading so the difference must be subtracted from the sum before.

Cora Groves, Jr.

Problem:- To build a fence parallel to a walk in front of the schoolhouse.



Given:- The walk AB.

To Construct:- A fence parallel to the Walk.

Place the straight edge of the protractor along the straight edge of the walk, and mark on the walk the centre of the protractor, Also on the curved edge of the protractor take take the middle point. Stretch wire EF tight across the two points a and b. EF is perpendicular to AB. When one straight line meets another straight line so that the two angles formed are equal, each is a right angle and the line is perpendicular.[1] C.D is ⊥ to EF. C.D is also parallel to AB. Two straight lines perpendicular to a third straight line are parallel.[2]

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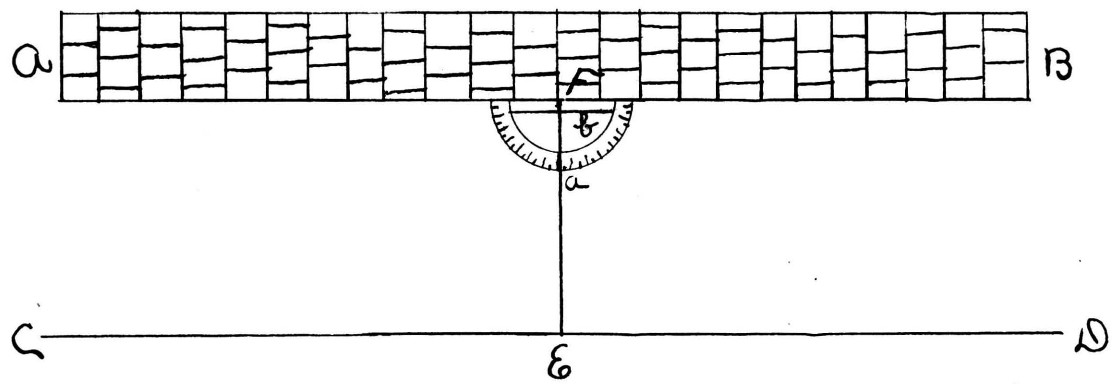
1. Wentworth's Plane Geometry. Page 11, Sec. 63.
2. Wentworth's Plane Geometry. Page 24. Prop. X.

Cora Groves, Junior.

This set of papers belong to an average student.

Problem:

To build a fence parallel to a walk in front of the school-house.



Given: The walk AB

To Construct a fence parallel to the walk.

Place the straight edge of the protractor along the straight edge of the walk, and mark on the walk, the center of the protractor. Also on the curved edge of the protractor like the middle point. Stretch the wire EF tight across the two points a and b. EF is perpendicular to AB. "When one straight line meets another straight line so that the two angles formed are equal, each is a right angle and the line is perpendicular"[1] Repeat the process at E. CD is ⊥ to EF, CD is also ∥ to AB. Two straight ⊥ to a third st. line are parallel.[2]

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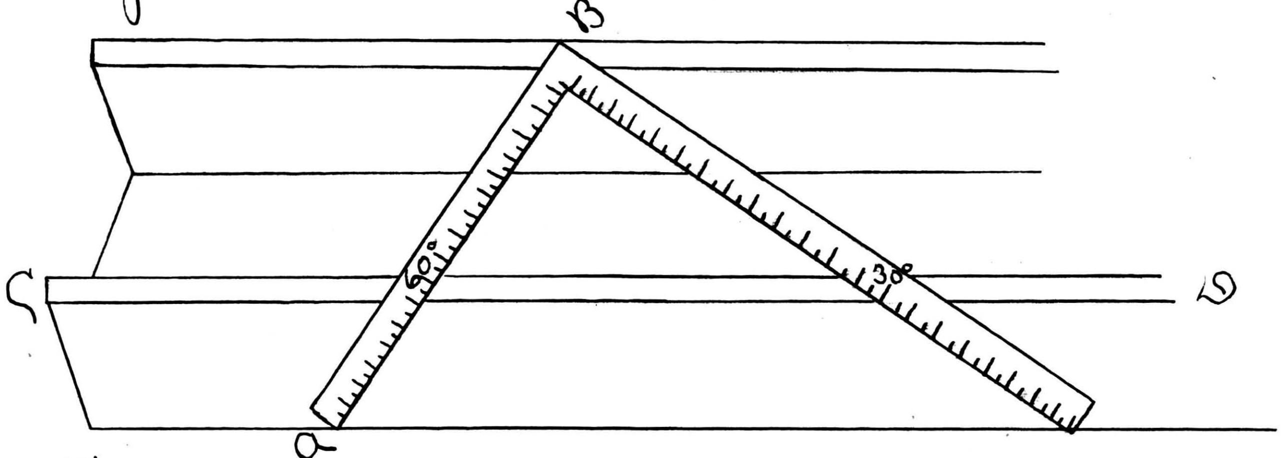
1. Wentworth Plane Geometry P. 11, Sec. 63.
2. Wentworth Plane Geometry P. 24, Prop. X

Mayme King.

Problem

To construct a 60° and a 30° angle on a mitre box.



First measure off a distance twice the width of the mitre box as a transversal, and mark the line on the box thus made. I also marked the line on the other side of the square perpendicular to it. If the hypotenuse of a right triangle is double the shorter leg, one acute angle will be double the other. Hence one angle is 30° and one 60°. If the length of side aB is placed along side of the box and a line to the vertex of the triangle, and each angle is 60°, for an equiangular triangle is formed. In an equiangular triangle each angle is 1/3 of 2 right angle, or 60°.[1]

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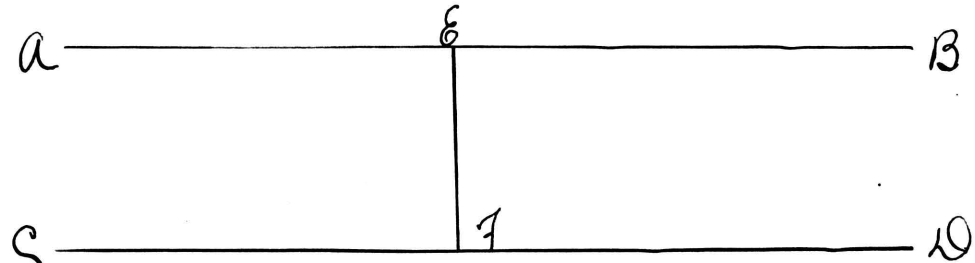
1. Wentworth Plane Geom. P. 33, Cor. 7

Mayme King

Problem: A surveyor losing his compass wishes to establish an east west line, knowing a formerly established east west line what does he do?



The surveyor knows the established east west line A B; at some point on line A B he erects a perpendicular line and at a given distance on this line he erects a line perpendicular to it.[1] Then A B is parallel to C D[2]. Therefore C D is the east west line.

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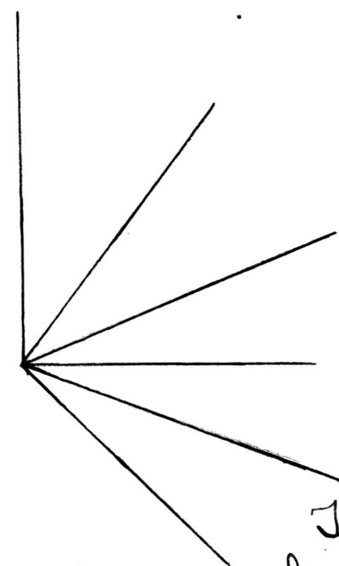
1. Wentworth Page 24, Prop. X
2. Wentworth Page 25, Prop. XI

Mayme King.

Problem:

To find a north south line.



First we set a vertical post in the ground 23 inches high. From nine in the morning until three in the evening readings were taken every fifteen minutes. The readings were the length of the shadows. At night a north south line was determined by the north star. With a protractor and a compass an east and west line was established.

Table of readings:

Time	Length of shadows.
9:00	92 cm
9:15	85 cm
9:30	79 cm
9:45	73 cm
10:00	65 cm
10:15	62.5 cm
10:30	58 cm
10:45	55 cm
11:00	52 cm
11:15	49 cm
11:30	46 cm
11:45	46 cm
12:00	48 cm
12:15	50 cm

Mayme King

Time	Length of shadows.
12:30	49 cm
12:45	48 cm
1:00	49 cm
1:15	52 cm
1:30	53.5 cm
1:45	57 cm
2:00	61 cm
2:15	66 cm
2:30	71 cm
2:45	76.5 cm

The shadows were not the same length. Did the shortest line or shadow coincide with the line determined by the north star? (yes) The north south line was found to be the shortest line. The perpendicular is the shortest line that can be drawn from a given external point to a given straight line.[1] Only two equal straight lines can be drawn from a given point to a straight line, and of two unequal lines, the greater cuts off the greatest distance from the foot of the perpendicular.[2]

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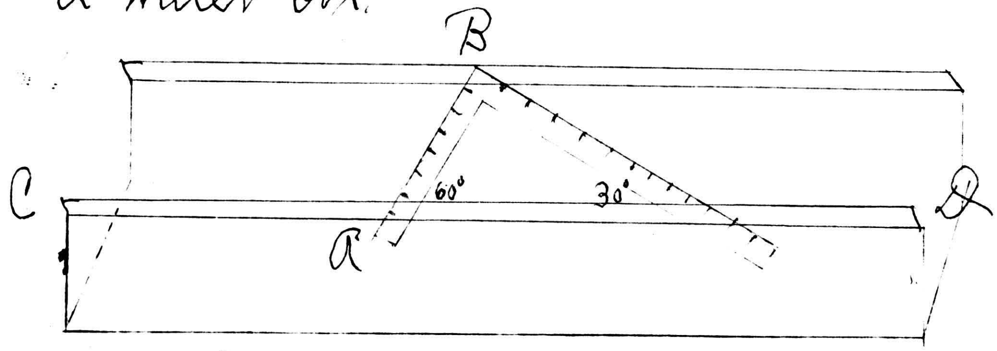
1. Wentworth, Page 21. Prop. VII
2. Wentworth, Page 23. Sec. 102.

MAYME KING.

Several papers here are selected from various students' work, among the best papers in neatness. Not corrected or worked over, but just as received from the student the first time.

Problem:- To construct a 60° and a 30° angle on a miter box.

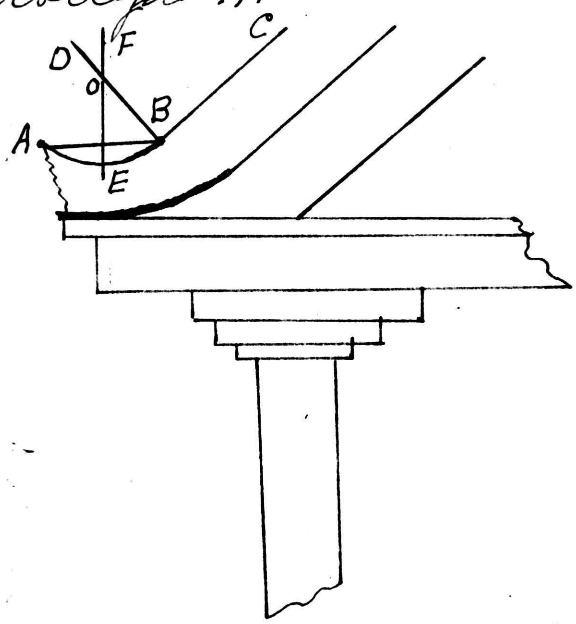


First measure off a distance twice the width of the miter box as a transversal and mark the line on the box thus made. I also marked the line on the other side of the square perpendicular to it. If the hypotenuse of a right triangle is double the shorter leg one acute angles will be acute angle will be double the other. Hence one angle is 30° and the other 60°. If the length of of side AB be placed along side of box and a line drawn from extremity of line to the vertex of the triangle, and each angle is 60°, for an equiangular triangle is formed. In an equiangular triangle each angle is equal to 1/3 of 2 right angles or 60°[1]

1 Wentworth Plane Geometry P.33, Cor. 7

Lawrston Ramsey

Problem: To draw easement of cornice tangent to rake cornice BC at B, and passing through A.



Given: st. cornice BC and pt A.

To construct: a semicircle tangent to BC at B, and passing through pt A.

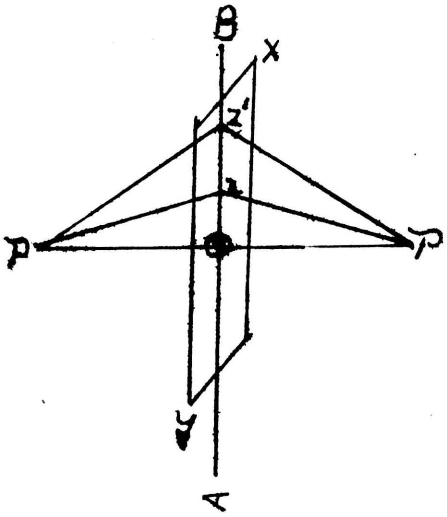
Proof: Draw cord AB. Draw ⊥ bisector EF of AB. Then EF passes through centre of circle of which AB is a cord. (Ref: 1:). Erect BD ⊥ to BC at pt B. Then BD passes through centre of circle. (Ref: 2:). Hence O intersection of BD and EF is centre of circle. With OB as a radius describe arc AEB. Then arc AEB ~~passes~~ through A. (Ref. 3.) Also BC is tangent to arc AEB. (Ref. 4.) Hence arc AEB is required easement of cornice.

References: Phillips and Fisher's Geometry.

No(1)	page	68,	prop.	162.
"(2)	"	74,	cor.	171.
"(3)	"	50,	Prop	103.
and	"	62.	Def.	146,
"(4)	"	73.	Prop.	170.

Mary Cooksey.

If any object is placed before a mirror its image appears to be as far behind the mirror as the object is in front.



Given - mirror XY and object P whose image is P' each appearing to be equally distant from XY..

To prove - XY is the locus of all points equally distant from two given points PP' not in XY.

Join P and P'. Let O be the point of intersection of PP' with the plane of the mirror XY. Through O in the plane of the mirror XY construct AB a perpendicular bisector to the line PP'. Then OP = OP'; PZ = P'Z; PZ' = P'Z'. But AB lies in the plane of the mirror XY. Therefore XY is the locus of all points equally distant from two given points PP'.

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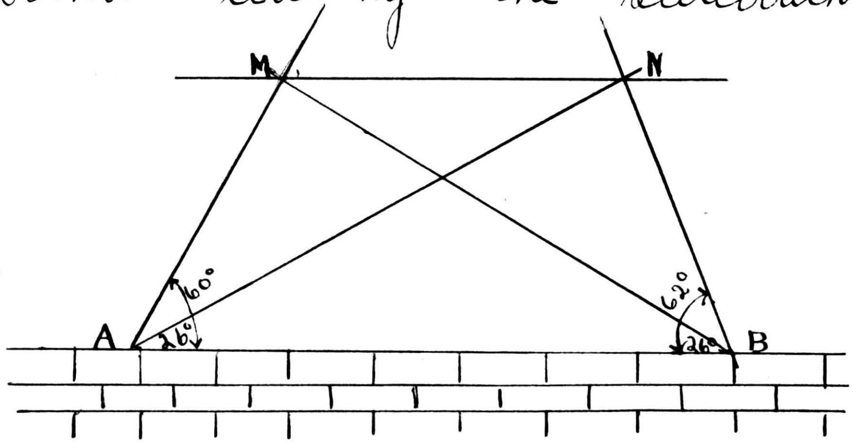
1. Phillip's and Fisher's Plane and Solid Geometry, Page 50. Proposition 103.

Orman Karns.

Problem;

To find the distance between two inaccessible points, as the distance between two trees on opposite sides of the street without leaving the sidewalk.



Given: the walk AB, and the trees M and N.
To find the distance MN.

First measure on the sidewalk some known distance, 80 feet, and with protractor measure the angle that the line AM makes with AB, and find its value, 60 degrees and the angle that the walk makes with the line AN some known angle, as 26 degrees. Also with the protractor measure the angles ABM, 26 degrees, and angle ABN, equal to 62 degrees.

Then lay off on paper a line AB 80 mm long and at end A construct an angle of 60 degrees and an angle of 26 degrees, and at B construct an angle equal to 62 degrees and an angle equal to 26 degrees.

These two triangles determine two fixed points, where their sides meet, as M and N. MN is the distance required. Measuring MN it is found to be 45 mm. and then we know that the distance between the two trees is 45 feet.

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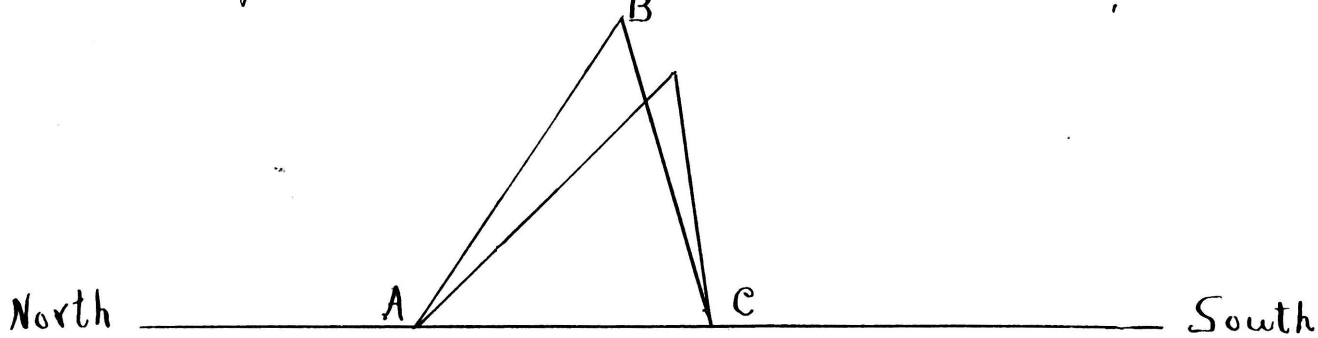
1. Wentworth, Plane Geometry, page 34, Prop XX.
2. Wentworth, Plane Geometry, page 8, Def. 50 - 53.

Robert Jackson

Problem;

Two boats on a still day started at the same time from the same point on a a lake. If the boats have the same rate of speed, and travel the same length of time, but in the beginning take different directions, which will be farthest from some point on a north-south line?



Let A represent the starting point, and B and C the points after sailing the same length of time at the same rate of speed. If AD represents the north-south line how do the distances BD and CD compare? What can be said about two triangles that have two sides equal, but the included angle of one greater than the included angle of the other?

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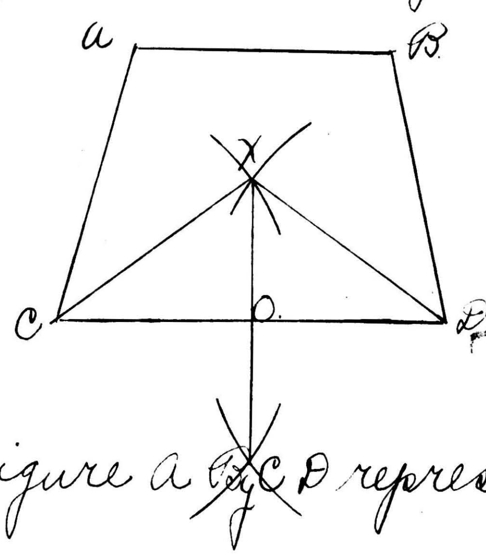
1. Wentworth, Plane Geometry, Page 42, Prop XXVIII

Robert Jackson

Problem.

First a board must be beveled on two sides to fit into the gable of the house; how will the carpenter find the center of the board if he has only a straight edge.



Given;-

The figure a B C D representing the board.

To find;-

The center of the board.

Proof;

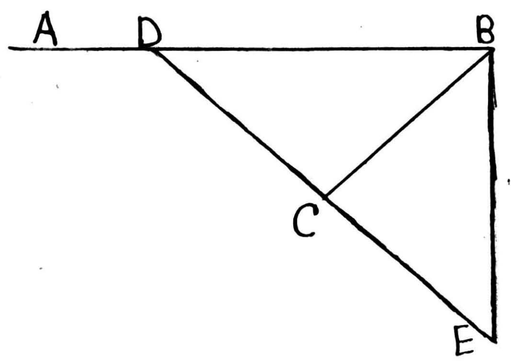
Bisect the straight line C D. Use C and D as centres and with a radius greater than one half of C D describe arcs which intersect at X and Y. Join the points X and Y. The line XY will be the perpendicular bisector of the line C D, bisecting it at the point O. Draw the radii C X and D X. The triangles thus formed are right triangles and they are equal. Therefore the line OC = line OD. This would be true by actual measurements.

Phil. and Fisher. P. 24, art. 42. Phil. and Fisher. P. 50, art 101.

" " " " 51, " 104.

Paul Shutts.

Problem; Justify the following method by which a surveyor may lay out a line perpendicular to AB at B. Select any point C, 50 feet from B. With one end of the 50 foot tape at C, swing the other end to D, in a line with A and B, using a lining pole at D to aid in lining it from A, if necessary, with one end of the tape still at C, swing the other end to locate E, in a line with D and C. Then BE is the required perpendicular.



Given; the line AB, with the line BE meeting it at B. C was chosen 50 ft. from B. BC was swung about C as a center until it cut AB at D, and swung about C as a centre until it cut the line DC extended, at E.

Prove; BE is the required perpendicular.

CD = CE = BC. (1)

$\angle CDB = \angle CBD$. (2)
$\angle BEC = \angle CBE$.

$\angle CDB + \angle DBC + \angle BCD + \angle ECB + \angle CBE + \angle BEC$ = 4 rt. angles. (3)

$\angle DCB + \angle BCE$ = 2 rt. $\angle$s. (4)

$\angle CDB + \angle DBC + \angle CBE + \angle BEC$ = 2 rt. $\angle$s.

$\angle CDB + \angle DBC + \angle CBE + \angle BEC = 2\angle DBC + 2\angle CBE$.

$2\angle DBC + 2\angle CBE$ = 2 rt. $\angle$s.

$\angle DBC$ + CBE = 1 rt. $\angle$.

$\angle DBE$ = 1 rt. $\angle$.

∴ BE is perpendicular to AB.

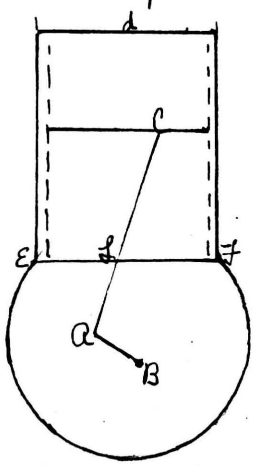
Phillips and Fisher's Plane Geometry.

(1) Construction.
(2) Page 34, art. 70, Proposition XVIII.
(3) Page 30, art. 57, Proposition XV.
(4) Page 12, art. 22, Proposition III.

Myrtle Smith

Problem;

In constructing a gas engine the piston D, which is in the form of an inverted cup, is five inches in inside diameter; the crank AB is 5" between the centers of the pivots and the connecting rod AC is 17" between the centres of the pivots. How far from the mouth of the cup must the pin C be adjusted in order that the connecting rod may just clear the edge of the cup at E and F, the diameter of AC being 1"?



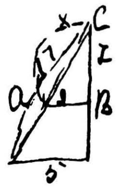
Given: a gas engine whose piston D, which is in the form of an inverted cup, is 5 inches in inside diameter; the crank AB, 5" between the centers of the pivots; and the connecting rod AC 17" between the centers of the pivots, and the diameter of AC being 1".

To find: how far from the mouth of the cup the pin C must be adjusted so that the connecting rod AC may just clear the edge of the cup at E and F.

As the crank AB revolves about B as a center,

Ho. Davis.

the figure ABC assumes the form of a rt. Δ. Then since EF = 5 inches and AC = 1 inch, EF - AC = 4 inches. The line of centers bisects EF. ∴ EL = 2 " . EL is now ∥ to AB.[1] ∴ The right Δs ECL and ACB are similar[2], and their homologous sides are proportional[3].



$\therefore \frac{5}{2} = \frac{17}{x}$. $x = \frac{34}{5}$. In the right Δ ABC

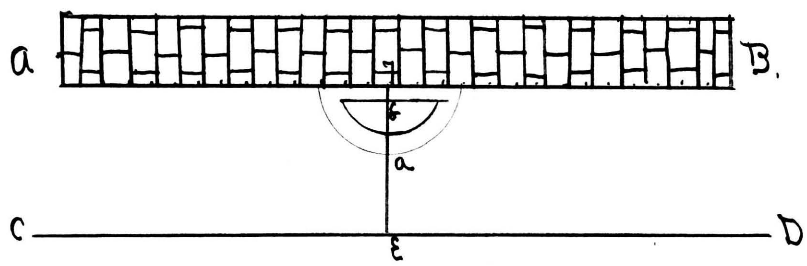
$\left(\frac{34}{5}\right)^2 = (2)^2 + Z^2$.[4] $Z = 6.49+$

---

1. See Phillips and Fisher, Page 16, Sec. 33, Prop. VI.
2. " " " " , " 101, " 262, Prop. III.
3. " " " " , " 101, " 261, Definition.
4. " " " " , " 121, " 305, Prop. XV.

Problem:

To build a fence parallel to a walk in front of the school house.



given: The walk a B.

To construct a fence parallel to the walk.

Place the st. edge of the protractor along the straight edge of the walk, and mark on the walk, the centre of the protractor. Also on the curved edge of the protractor take the middle point. Stretch wire E F across two points a and b. E F is perpendicular to a b. When one straight line meets another straight line so that the two angles formed are equal; each is a rt. angle and the line formed is perpendicular.[I] Repeat the process at E. C D is ⊥ to E F, C D is also ⊥ to a B. Two st. lines ⊥ to a third st. line are parallel.[II]

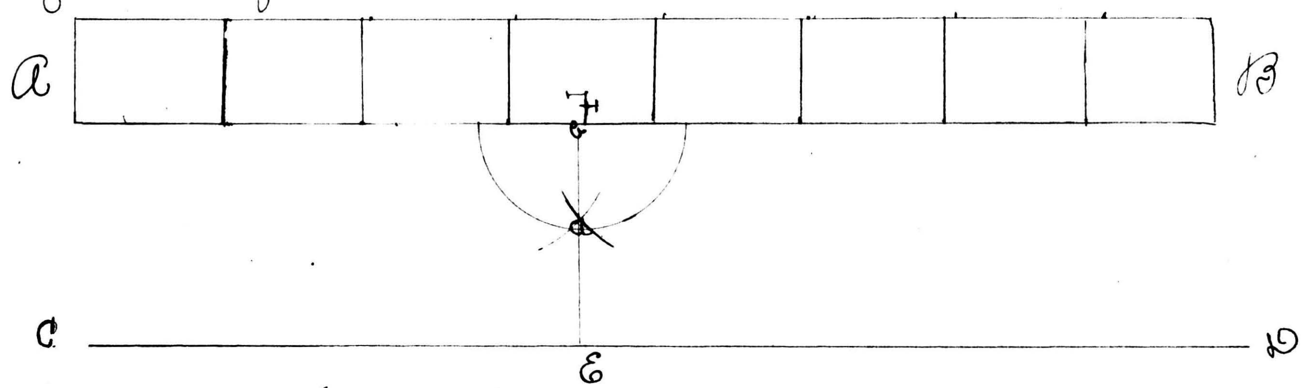
I. Wentworth Plane Geometry P. 11, Sec. 68.

II " " " " 24, Prop. X.

EVA STUBBS.

Problem:

To build a fence parallel to a walk in front of the school house.



Given: the walk AB.

To construct a fence parallel to the walk.

Place the straight edge of the protractor along the straight edge of the walk, and mark on the walk the centre of the protactor. Also on the curved edge of the protractor, take the middle point. Stretch wire EF tight across the two points a and e.

EF is perpindicular to AB.

When one straight line meets another straight line so that the two angles formed are equal, each is a right angle and the line is perpendicular.[1]

Repeat the process at E. CD is perpendicular to EF. CD is also parallel to AB. Two straight lines perpendicular to a third straight line are parallel.[2]

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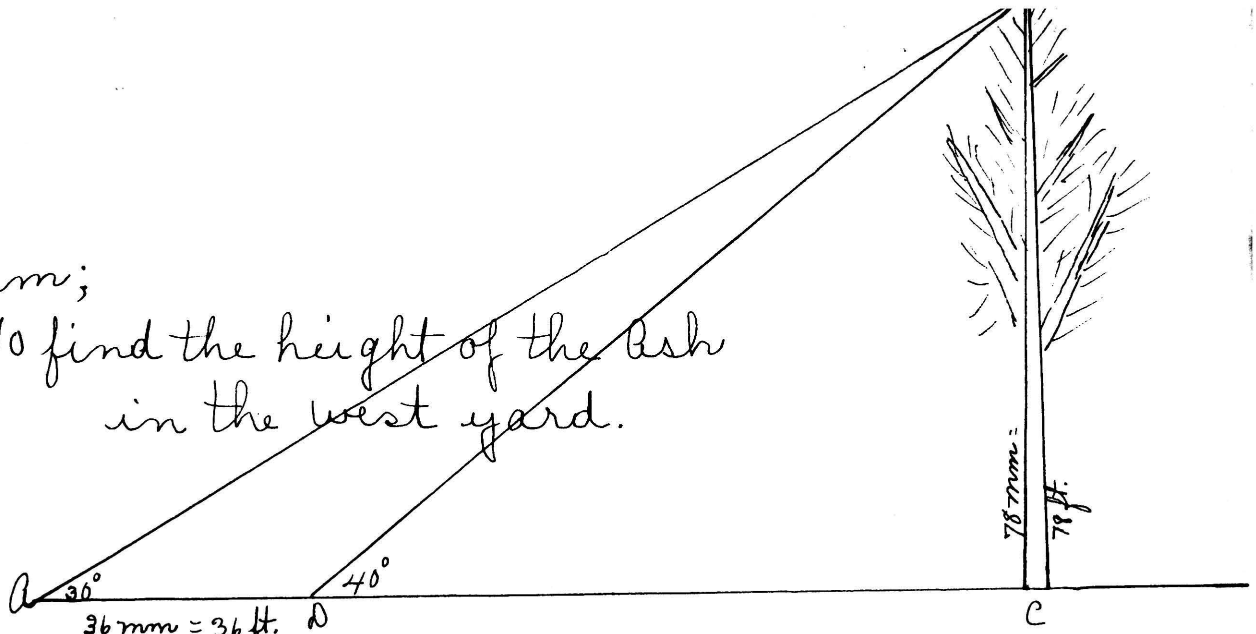
1. Wentworth Plane Geometry page 11 Sec. 63
2. " " " " " 24 Prop 10.

Nelle Dillon

Problem;

To find the height of the Ash in the west yard.



Given: the tree BC.

To find the height of the tree BC.

Measure the line AD that is one segment of the line that extends from D on the line DC. The points A and D being any two points that may be on any line that passes through the foot of the tree. At A and D measure angles 1 and 2 with protractor. Then angle 3 is known. AD is 36 feet, and angles 1 and 2 equal respectively 40 and 30 degrees. Then angle 3 is 140 degrees.[1] Construct a triangle to scale using 1 mm. for 1 foot. On a straight line AD 36 mm. long and at A construct angle equal to 30 degrees and at D construct angle equal to 140 degrees. Extend the sides of these angles until they meet in a point, and from that point draw a perpendicular[2] to the straight line AD extended. This perpendicular is 78 mm.[3] Therefore the tree is 78 feet high.

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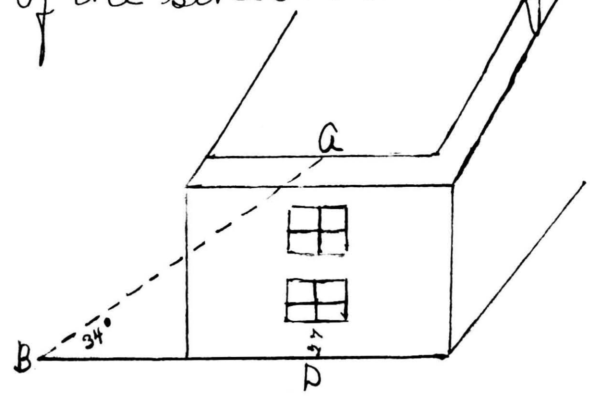
1. Wentworth, Plane Geometry, page 13, Sec. 77.
2. Wentworth, Plane Geometry, page 11, Sec. 64.
3. Wentworth, Plane Geometry, page 34, Prop. XX.

Edith Cottier

Problem;

a, To find the height of the school building.



Given: the school building, aD.

To find the height of the school building, aD.

Measure on the ground along a straight line BD a distance of 36 feet. At B with the protractor of the angle that the line aB a line drawn from the point B to the top of the building, makes with its projection upon the plane of the ground, the line BD. The angle aBD is 34 degrees. Draw a triangle to scale. The distance aD is found to be 27 feet. Therefore the building is 27 feet high.[1]

b, To find the height of the tree at the southeast corner of the yard.[2]

c. To find the height of the stand pipe.[2]

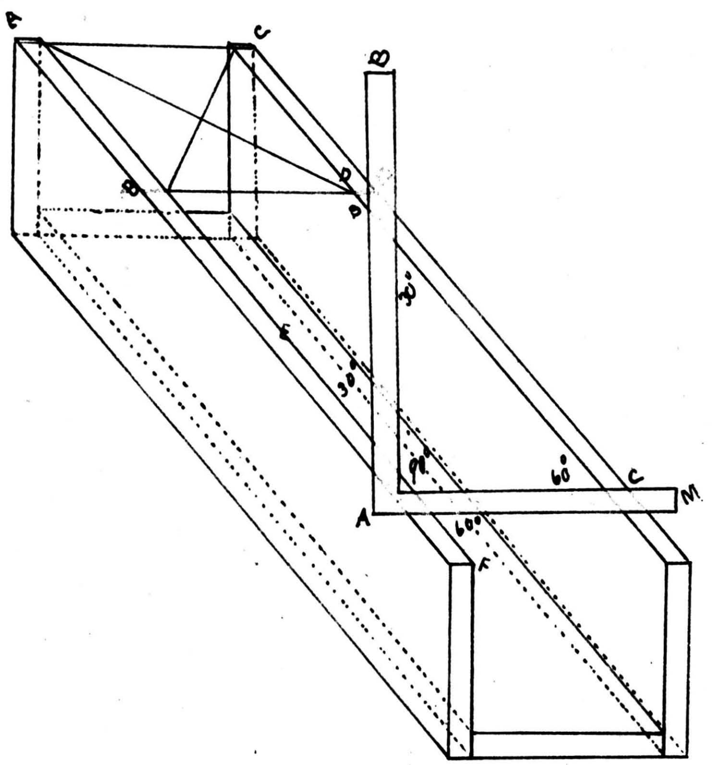
---

1. Wentworth, Plane Geometry, page 34, Sec. 142, Cor. 3.
2. See problem a, in lesson VIII.

Edith Cottier

Problem;

To make a mitre box for the 30, 45, and 60 degree angles.



Given a mitre box.

To saw for the 30, 45, and 60 degree angles.

To saw for the 45 degree angle.

Lay off along one edge a distance CD equal to the width of the box, BD.

Draw AC perpendicular to CD' at C and draw the lines AD and CB. The triangle CBD is a right triangle with a right angle at D, and since BD and CD are equal, the angles opposite, angles BCD and DBC, are equal.[2]

The sum of the angles of a triangle is equal to 180 degrees[3] and if angle CDB is equal to 90 degrees the sum of the angles BCD and DBC is 90[4] degrees and if they are equal each is 45 degrees. In like manner the angles CAD and CDA are each equal to 45 degrees.

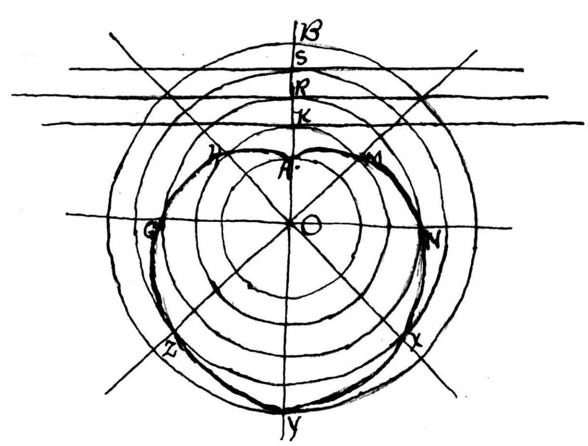
---

1. Wentworth, Plane Geometry, page 11, Sec. 64.
2. Wentworth, Plane Geometry, page 36, Prop. XXII.
3. Wentworth, Plane Geometry, page 32, Prop. XVIII.
4. Wentworth, Plane Geometry, page 33, Sec. 135, Cor. 6.

Ed. Allen

Problem;

To construct the pattern for a uniform motion cam with a throw of 14 mm. and a maximum radius of 24 mm. A cam is a wheel having its axis of a revolution out of its centre of figure. This problem based upon the cam of a Singer sewing machine.



Given; OB = 24 mm. the radius.
AB = 14 mm. the throw.

To construct curve AMNXYZGH.

On BY take OB = WR m m. and AO = 10mm; (OB - AB).

At point O draw GN perpendicular to

Floyd Porter,

BY and bisecting the angles.[1]
Then divide AB up into equal parts.[2]
Through AKRS draw circles with O as a center. The circles will be concentric. Begin at A mark the points where the consecutive concentric circles and the consecutive rays that bisect the angles intersect, and through these points draw a smooth curve. This curve will be a solution to the problem.

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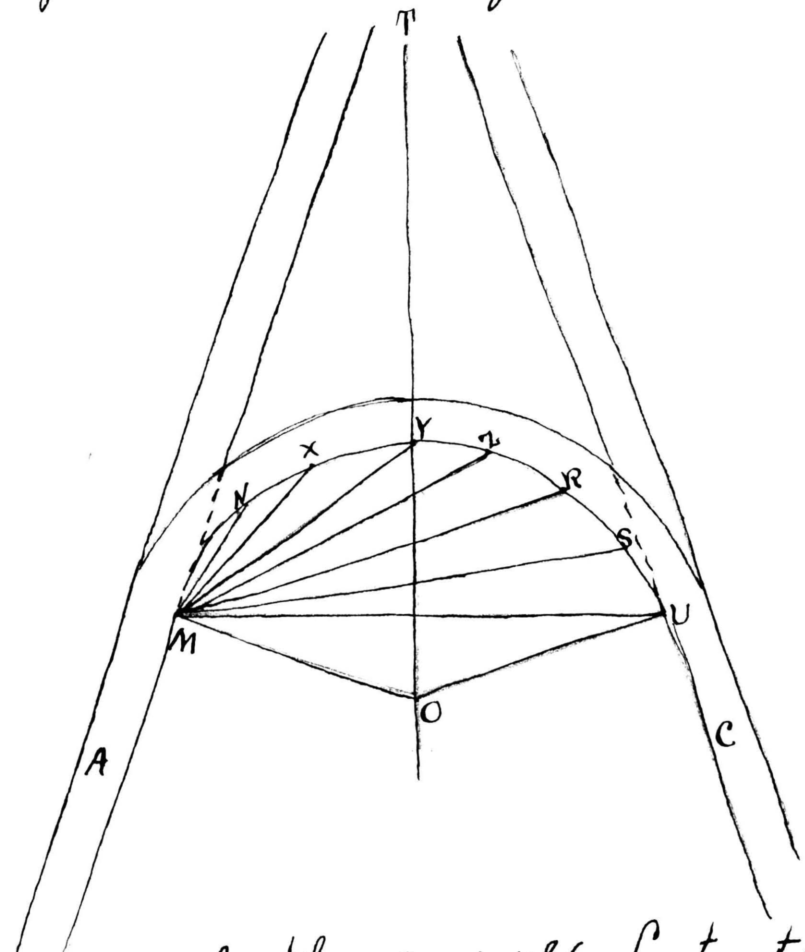
1. Wentworth, Plane Geometry, Page 115, Prop. XXV.
2. Wentworth, Plane Geometry, Page 114, Prop. XXIII.
   Wentworth, Plane Geometry, Page 117, Prop. XXVIII.

Problem;

To lay out a curve for a railroad.



Let ABC equal the curve. Let the chords
MN, NX, XY, YZ, ZR, RS, and SU each be 100
feet. To lay off these chords suppose the
continuation of the railroad RR to AT,
would be a tangent to the curve of the
railroad. Then angle TMN (given) is
laid off from the tangent MT, and 100
feet is measured on line MN thus locat-
ing point N, then angle NMX is laid off
and arc with centre at N and radius
with 100 feet is made to intersect MX, thus
fixing point X. In the same way each

Floyd Porter.

successive point is found by laying off the proper angle and finding the intersection of its side with radius arc when the radius is 100 feet, whose centre is the stake last set. Since the curve is to be arc of a circle, the angles TMN, NMX, XMY, YMZ successive angles are all equal. Each angle is an inscribed angle, hence measured by $\frac{1}{2}$ of its arc;[1] and each is equal to $\frac{1}{2}$ of the central angle MOX, XOY, YOZ and successive angles for central angles measured by subtended by equal chords are equal,[2] and these chords were each 100 feet.

---

1. Wentworth, Plane Geometry, Page 102, Prop XVII

2. Wentworth, Plane Geometry, Page 80, Prop. III.

Problem;

In the railroad problem above show that the sum of the 'deflection angles' TMN, NMX, XMY, and succeeding angles equals one half of the 'intersection angle' ATC, the angle between the tangents at the ends of the road curve. Show that the intersection angle equals the central angle subtended by the curve.

For diagram see problem above.

MT=TU[1], OM=OU[2], Then OT is the perpendicular bisector of MU[3]; In triangles OMP and MPT; MP=MP, common; angle MPO= angle MPT[4] Angle MOU is measured by arc MYU[5] and angle MOY[6] $\frac{1}{2}$ of MOU, is measured by $\frac{1}{2}$ arc MYU. But angle TMP is measured $\frac{1}{2}$ arc MYU[7] Therefore angle TMP= angle MOP, measured by $\frac{1}{2}$ same arc. Therefore triangles OMP and MTP are equal[8] Therefore OM=MT[9] also OU=TU, Therefore angle MTO= angle MOT[9], also in like manner angle UTO= angle UOT[9]

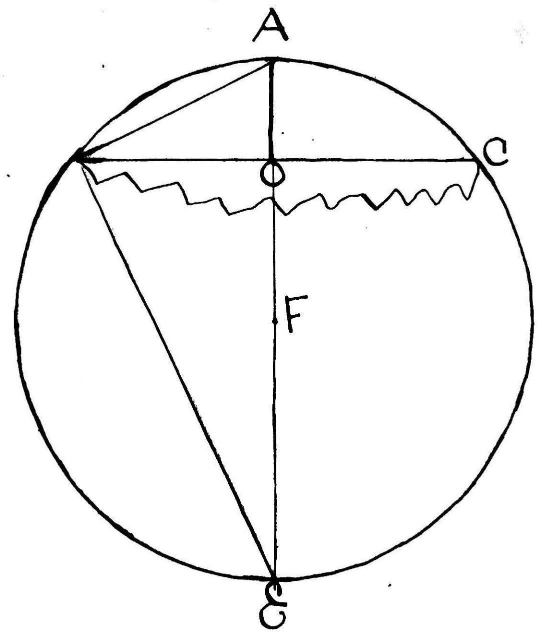
Floyd Porter

Therefore angle MTO plus angle OTU = angle MOT plus angle TOU.[10] Q. E. D.

---

1. Wentworth, Plane Geometry, Page 89, Prop. XII.
2. Wentworth, Plane Geometry, Page 75, Art. 217.
3. Wentworth, Plane Geometry, Page 45, Prop. XXX.
4. Wentworth, Plane Geometry, Page 15, Art. 82, Cor. I.
5. Wentworth, Plane Geometry, Page 102, Art. 288.
6. Wentworth, Plane Geometry, Page 82, Prop. V.
7. Wentworth, Plane Geometry, Page 105, Prop. XIX.
8. Wentworth, Plane Geometry, Page 34, Art 142, Cor. III.
9. Wentworth, Plane Geometry, Page 31, Art. 128.
10. Wentworth, Plane Geometry, Page 6, Art. 34, Ax. 2.

Problem: Given a fragment of a water wheel. Construct a new wheel of its size. DC was found by measurement to be 4' and AO to be 1'. Compute diameter.



Given: A segment of a water wheel (or water tank. Let ADC be the segment.
Find: The diameter AE.

Lay off a chord DC = to 4' and at the middle point erect ⊥ AO[1] = to 1'. Draw AD. At D draw DE ⊥ to AD.[1] Then extend AO until it cuts DE as at the pt. E. Then AE will be the diameter.[2]

$AE = OE + 1$. $\overline{AE}^2 = \overline{OE}^2 + 2OE + 1$.

$\overline{AE}^2 = 5 + \overline{DE}^2$. $\overline{OE}^2 + 2OE + 1 = 5 + \overline{DE}^2$. $\overline{DE}^2 = \overline{EO}^2 + 4$.

$\overline{OE}^2 + 2OE + 1 = 5 + \overline{EO}^2 + 4$. $2OE = 8$.

$OE = 4. \quad AE = 5.$

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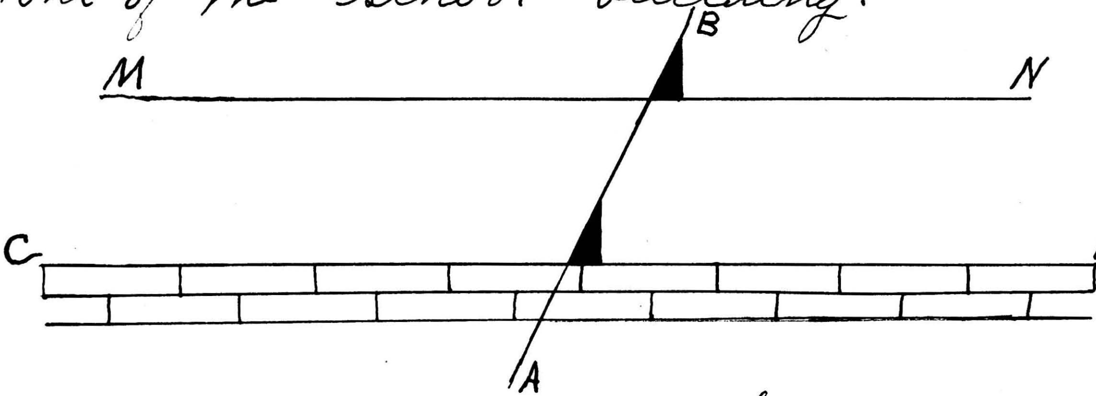
① Wentworth, page 113, Proposition XXII.
② Wentworth, page 102, Proposition XVII, paragraph 103, Section 290, Cor. I.

Dolly King.

Problem;

To build a fence parallel to the walk in front of the school building.



Use a straight stick with two equal wooden triangles, called a set square. The triangles have their respective angles equal. Let AB represent the set square. Place AB as a transversal on the walk CD so that the lower edge of the upper triangle is exactly above the edge of the walk. Then along the lower edge of the lower triangle draw the straight line, MN. MN is the line desired. Two straight lines in a plane are parallel, if no matter how the transversal is placed the exterior-interior angles are equal.[2] Therefore MN is parallel to the fence AB. If two straight lines in a plane are cut by a transversal, and the exterior-interior angles are equal, the straight lines are parallel.

Other proofs were submitted.

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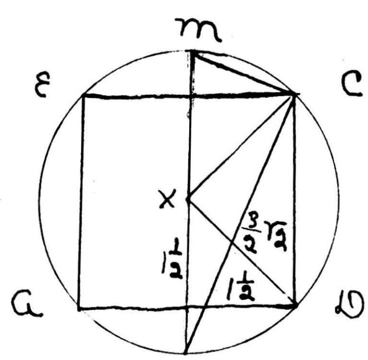
1. Wentworth, Plane Geometry, page 28, Prop. XV.
2. Bush + Clarke, Plane and Solid Geometry, page 23.

Problem; In making an Indian Club, pieces of holly are glued to the faces of a square piece of gum. This is then turned up in a lathe, the holly producing oblong, oval light spots on the surface of the club. The thickness of the gum is three inches. In order to get a club in which these oval spots just touch at the point of the greatest thickness of the club; how thick must the piece of holly be taken?



Given: the cross section AECD of an Indian Club with CD or AD = to 3".

To find Om.

OC and $OX = 1\frac{1}{2}$ ①

Then $\overline{XC}^2 = \overline{OC}^2 + \overline{OX}^2$

$\overline{XC}^2 = \frac{9}{4} + \frac{9}{4}$ or $\frac{18}{4}$

$XC = \frac{3}{2}\sqrt{2}$

But $Xm = XC = \frac{3}{2}\sqrt{2}$ ②

$Om = Xm - OX = \frac{3}{2}\sqrt{2} - 1\frac{1}{2} = \frac{3}{2}\sqrt{2} - \frac{3}{2} = \frac{3}{2}(\sqrt{2} - 1)$ ③

Wentworth's plane geometry, page 162, Proposition XXVIII.
Wentworth's plane geometry, page 75, Definition 217.
Wentworth's plane geometry, page 6, Section 34, Axiom 9.

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