

GEOMETRIC COMBINATORICS  
IN DISCRETE SETTINGS

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ABSTRACT

This thesis is a compilation of work in which the author studies geometric configurations in finite fields and the integers modulo  $q$ . The results of this dissertation are threefold. First, we prove a finite field analog of the Furstenberg-Katznelson-Weiss theorem on triangles in  $\mathbb{R}^2$ . Second, we study volume sets in  $\mathbb{F}_q^d$  and discuss some applications to sum-product problems. Finally, we study geometric combinatorics in  $\mathbb{Z}/q\mathbb{Z}$ . We generalize a result of Hart and Iosevich [27] which has applications to sum-product problems. Finally, we show that the  $\mathbb{Z}_q^d$  analogue of a sphere with unit radius is  $q^{d-1}$ -dimensional.