FACILITATING THE DEVELOPMENT OF ELEMENTARY PROSPECTIVE TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE: A CASE STUDY OF A MATHEMATICS TEACHER EDUCATOR'S ACTIONS AND PURPOSES

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ABSTRACT

Prospective mathematics teachers (PTs) need to develop pedagogical content knowledge (PCK) to improve the teaching and learning of mathematics. Therefore, faculty who prepare teachers of mathematics (i.e., mathematics teacher educators [MTEs]), need to help PTs develop PCK in their courses (Marks, 1990; Mason, 2008). Yet, we know very little about the practices of MTEs, especially in relation to developing PCK as these practices are not widely researched, documented, or disseminated (e.g., Bergsten & Grevholm, 2008; Floden & Philipp, 2003).

In this study, I investigated the actions and related purposes that a reflective MTE used to develop PTs' PCK in an elementary mathematics content/methods course. I present a classification of descriptive categories of the 34 identified actions and 10 core purposes based on the four components of PCK (i.e., knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, knowledge of assessment) conceptualized by Magnusson, Krajcik, and Borko (1999).

Findings from this study contribute to the limited literature on practices of teacher educators (e.g., Murray & Male, 2005). This study is a first step towards documenting the actions enacted during instruction of MTEs, which will contribute to a shared knowledge base in mathematics teacher education and ultimately inform the design and implementation of teacher preparation programs.

Chapter I – Introduction

Introduction

In the United States, a national debate has occurred about how to best recruit and prepare future teachers (Boyd, et al., 2008; Levine, 2006). This debate has been fueled by claims that teacher education programs do not sufficiently prepare teachers for the classroom (Grossman, 2008) and studies have highlighted the variability of teacher education programs in the U.S. (e.g., Darling-Hammond, et al., 2000; Levine). This debate and the current political context have elevated pressure on colleges of education to demonstrate evidence that their graduates have the knowledge, skill, and dispositions to be effective in the classroom (National Council for Accreditation of Teacher Education [NCATE], 2006). Levine (2006) claims that university-based teacher education programs are close to losing responsibility for preparing prospective teachers (PTs) if improvements are not made.

Hiebert and Morris (2009) argue that accomplishing improvements in teacher preparation programs will require an extensive research effort as well as a system to accumulate useable knowledge for the field. They write, “Steady and lasting improvements in teacher education rest on building a useful and cumulating knowledge base” (p. 475). Yet, building a knowledge base about the design and implementation of teacher education programs has been challenging. Lampert and Graziani (2009) argue that the field has not identified common criteria for successful teacher education programs nor common goals for what PTs should be able to do if the program is successful and Levine (2006) emphasizes that there is no process for improving their effectiveness. Furthermore, a lack of agreement exists about teacher education curriculum across disciplines (Levine) and little empirical research focuses on the practices of teacher educators (Doerr & Thompson, 2004; Loughran & Berry, 2005; Murray & Male, 2005).

Discussions related to improving the preparation of teachers have been framed broadly in teacher education, but also within specific disciplines such as mathematics education (Conference Board of the Mathematical Sciences, 2001). Yet, a useful knowledge base for mathematics teacher education is lacking, even with years of research on mathematics teaching in grades K-12. For example, there is no shared professional curriculum to prepare PTs of mathematics (Ball, Sleep, Boerst, & Bass, 2009; Zaslavsky, 2007). Moreover, we know very little about the practices of faculty who prepare teachers of mathematics (i.e., mathematics teacher educators [MTEs[1]]), as these practices are not widely documented or disseminated (Bergsten & Grevholm, 2008; Even, 2008; Floden & Philipp, 2003; Hiebert, Morris, & Glass, 2003; McDuffie, Drake, & Herbel-Eisenmann, 2008; Schempp, 1995). In other words, we do not know what content is taught in courses for PTs or how they are taught. Furthermore, we do not know how the practices of MTEs influence the learning of PTs. Without a shared knowledge base, novice MTEs design courses with limited resources and support contributing to the variability across various preparation programs. As a result, there is a demanding and critical need to research MTE practices, to develop a shared professional curriculum, and to build a usable knowledge base for MTEs (Floden & Philipp, 2003).

[1] Jaworksi (2008) defines Mathematics Teacher Educators as "Professionals who work with practicing teachers and/or prospective teachers to develop and improve the teaching of mathematics" (p. 1).

Theoretical Framing for the Study

MTEs are shaped by a variety of experiences including, but not limited to their: doctoral programs; research; professional development opportunities (e.g., professional readings, conferences); teaching experiences at the K-12 and undergraduate levels; university responsibilities such as field supervision; reflection on their practice; collaboration with colleagues; and other education related experiences in their churches or communities. These experiences and opportunities, in turn, influence the development of MTEs' knowledge, competencies, and purposes[2].

Potentially, MTEs develop knowledge about mathematics; how to teach mathematics with various technologies, manipulatives, and activities; pedagogical and assessment strategies; pedagogical content knowledge (PCK); adult learners; curriculum resources; and the community and schools where PTs complete their field placement. In addition, MTEs may have knowledge about practitioner and research literature addressing the teaching and learning of mathematics, theories of teaching and learning, and conducting their own research (Jaworski, 2008). MTEs draw on their knowledge to teach mathematics content/methods courses (arrow A in Figure 1). Therefore, MTE knowledge influences his (or her) teaching actions, which in turn influence the knowledge that PTs develop in mathematics content/methods courses (arrow B). In addition, as a MTE continues to reflect on his (or her) practice and analyze PTs' thinking and learning, he (or she) may continue to develop his (or her) own knowledge bases (arrow C).

[2] I define *purposes* as the intended or underlying goal for the implemented action. This goal may stem from knowledge the MTE has or beliefs he (or she) has about teaching.

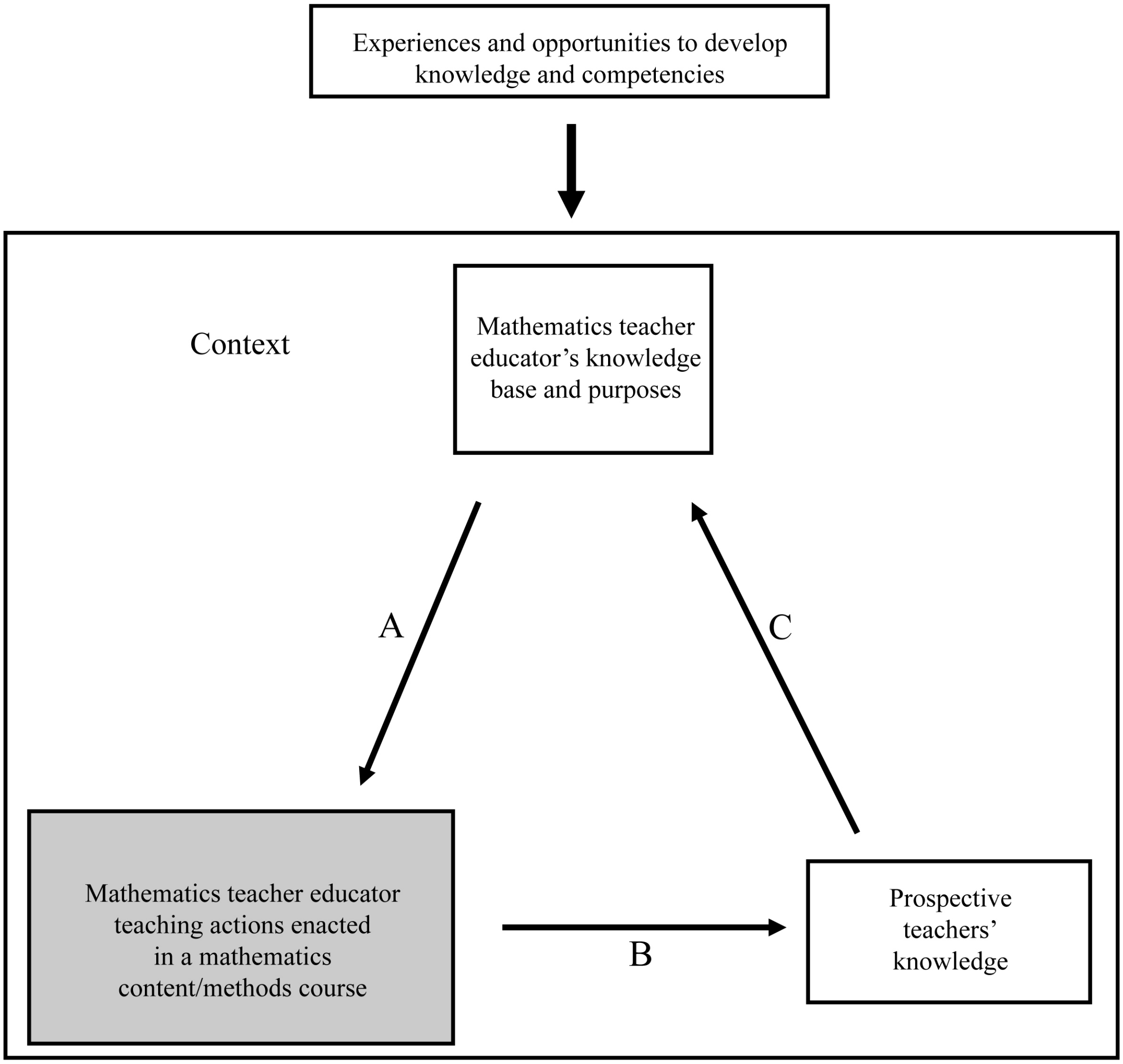


Figure 1. A model of the influence of a mathematics teacher educator's knowledge and practices[3] in a mathematics content/methods course for prospective elementary mathematics teachers.

MTEs have a challenging responsibility as they are expected to not only teach mathematics to PTs, but also help PTs develop a variety of knowledge bases as conceptualized by several scholars (e.g., Grossman, 1990; Hill, Ball, & Schilling, 2008; Magnusson, Krajcik, & Borko, 1999; Shulman, 1986). Therefore, the coursework PTs

[3] I define *practices* as the frameworks, theory, technology, and activities the MTE attends to and draws from to instruct PTs.

experience must help them develop a depth and breadth in content knowledge as well as pedagogy (Adamson, et al., 2003), provide sources and models of effective mathematics teaching, and help PTs develop a vision of what it means to teach mathematics to K-12 students. Moreover, Shulman (1986) argued, "mere content knowledge is likely to be as useless pedagogically as content-free skill" (p. 8). In other words, if the focus is on content or pedagogy, something is missing. Therefore, Shulman stated that teachers need to have the ability to connect content knowledge to lessons and instructional strategies. He coined this type of knowledge as pedagogical content knowledge (PCK) which he described as "the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations" (p. 9). In addition, Shulman defined PCK as:

> A second kind of content knowledge…which goes beyond knowledge of the subject matter per se to the dimension of subject matter knowledge for teaching… Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 9)

Being able to do mathematics and knowing mathematics as a discipline is not sufficient for teaching mathematics effectively. Teachers need knowledge of strategies to draw from that will help them identify students' conceptions and misconceptions for solving various types of mathematics problems. In addition, they need strategies that will help them aid their students in understanding concepts and learning about specific mathematical topics. As a result, PTs need to develop PCK and MTEs need to help them develop PCK in their courses (Marks, 1990; Mason, 2008).

Grossman (1990) built on Shulman's work, further delineating the construct of teacher knowledge. She focused on four central components of PCK including: (1) knowledge and beliefs about the purposes for teaching a subject at different grade levels; (2) knowledge of students' understanding, conceptions, and misconceptions of particular topics in a subject; (3) curricular knowledge; and (4) knowledge of instructional strategies and representations for teaching particular topics. Additionally, Grossman noted that while teacher preparation courses "may be the most logical place for PTs to acquire pedagogical content knowledge, we know very little about the content of methods courses or of professional coursework in general" (p. 13).

Magnusson, Krajcik, and Borko (1999) further modified Grossman's (1990) domains of teacher knowledge (e.g., adding orientation and assessment). Specifically, their model of PCK incorporated five components, centered around a specific subject, including: (1) orientations toward teaching (teachers' knowledge and beliefs about the purposes and goals for teaching specific content [e.g., science, mathematics] at various grade levels); (2) knowledge of curriculum (teachers' knowledge of the goals, objectives, programs, and materials relevant to teaching specific content); (3) knowledge of students' understanding (teachers' knowledge of student misconceptions, approaches, and strategies when subject specific concepts are addressed); (4) knowledge of assessment (teachers' knowledge of activities, procedures, etc. which may be used to assess subject specific concepts as well as the advantages and disadvantages of various assessment techniques); and (5) knowledge of instructional strategies (teachers' knowledge of activities, manipulatives, and different approaches to teaching subject specific concepts).

An, Kulm, and Wu (2004) argues that developing PTs' PCK "should be the most important element in the domain of mathematics teachers' knowledge" (p. 146). Although there has been great attention in conceptualizing and delineating PCK, there has been little research that investigates how MTEs can facilitate the development of PTs' PCK specifically in elementary mathematics content/methods courses. Therefore, in the following section I outline the research purpose and questions that addressed this gap and elaborate on how I focused the study.

Research Purpose and Questions

In this research study, I sought to understand and document what one MTE said and did in her elementary mathematics content/methods course in relation to developing PTs' PCK so that I could create a classification of descriptive categories of actions enacted during instruction. The specific research questions were:

(1) What actions does a reflective mathematics teacher educator use to develop prospective teachers' pedagogical content knowledge in an elementary mathematics content/methods course?

(2) For what purposes does a reflective mathematics teacher educator use the identified actions?

MTEs enact their knowledge and a variety of practices while they teach mathematics content/methods courses in their respective institutions. Some practices may be similar to practices K-12 mathematics teachers use in their classrooms (e.g., modeling, grouping students, questioning, etc.), while other practices are specific to the mathematics content/methods class (e.g., using videos to examine classroom teaching). For the purposes of this study, I focused on the MTE's actions that were specific to teaching PTs (as seen in the shaded portion of Figure 2). For example, describing specific activities PTs may use with grade 1-6 students or discussing the scope and sequence of grade 1-6 mathematics curricula. In addition, I considered the MTE's questions if they were appropriate for PTs, but not elementary students. That is I would include a question such as, "How would you sequence these tasks for third grade students?" but not a question such as, "How did you solve that problem?" Therefore, I solely focused on actions that were specific to instructing PTs.

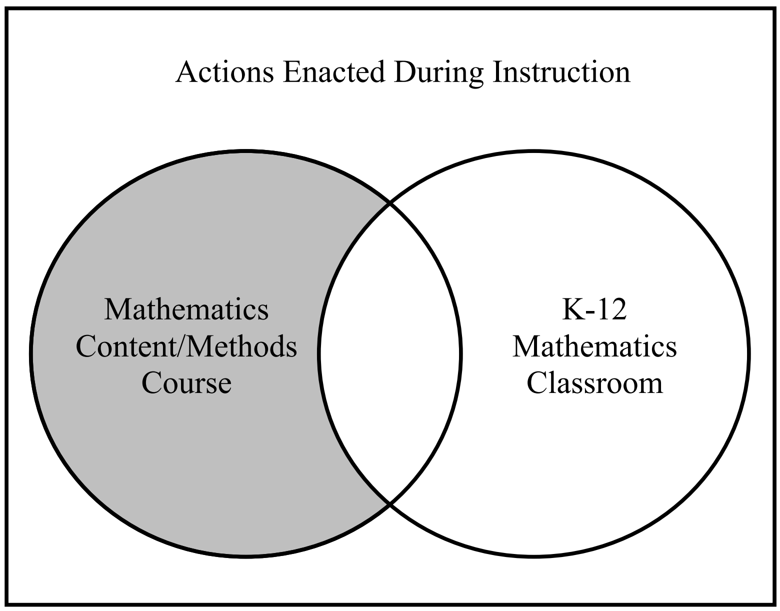


Figure 2. Actions enacted during instruction.

I define *actions* as what the MTE says and does (i.e., writes on the board). MTEs enact a variety of actions during a typical class session such as interacting with an individual PT, small group of PTs, or the entire class. They also engage in a variety of actions outside of the classroom (e.g., designing lessons, grading assignments, or meeting with PTs during office hours). To further focus this study, I only considered the MTE's actions during whole class interactions inside the classroom. I did not analyze the content of the MTE's tasks or activities, but rather her facilitation moves.

Finally, I only considered actions that targeted PTs' PCK. In other words, I did not include actions that targeted developing PTs' mathematical knowledge (e.g., when the MTE asks PTs to find the sample space for a probability situation or asks PTs to present their mathematical solutions on the board), general pedagogical knowledge, or knowledge about equity issues. Specifically, I focused on four PCK components delineated by Magnusson, Krajcik, and Borko (1999) as a lens (see Figure 3) to identify actions enacted during instruction in a mathematics content/methods course.

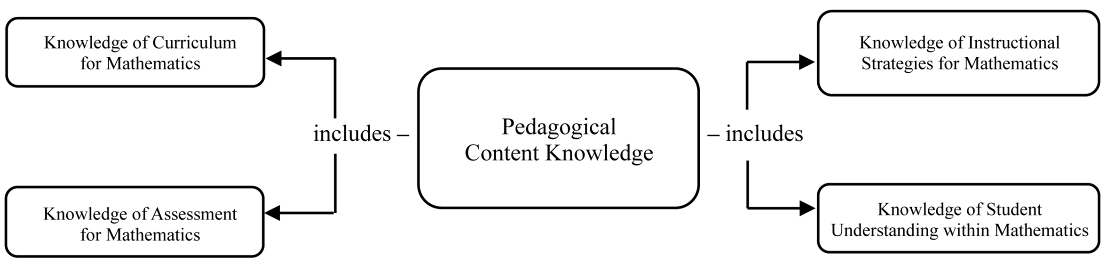


Figure 3. Four components of pedagogical content knowledge I use to create a classification of descriptive categories of one mathematics teacher educator's actions.

To gain a better understanding about what is taught in university mathematics content/methods courses to develop PTs' knowledge of four components of PCK (knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, and knowledge of assessment), this study sought to enhance our understanding about what one MTE said and did in her mathematics content/methods course that was specific to teaching PTs.

Significance of the Study

There have been a number of studies that have examined the classroom practices of K-12 mathematics teachers (e.g., the Inside the Classroom Study and the TIMSS Video Study); however, similar attention has not been given to investigating the practices used in university courses and teacher preparation courses specifically (Zeichner, 2005). A number of recent federal reports and solicitations (Conference Board of the Mathematical Sciences, 2001; National Science Foundation, 2003; Wilson, Floden, & Ferrini-Mundy, 2001) have stated that the preparation of elementary teachers of mathematics needs considerable improvement. But, little empirical evidence exists to inform the mathematics education research community how MTEs prepare prospective elementary teachers in general and in relation to PCK specifically. Research that documents actions of MTEs will ultimately inform the design of programs that prepare prospective mathematics teachers and support novice MTEs as they learn how to teach PTs. The results from this study provide a detailed description of actions enacted by one MTE during instruction to develop elementary PTs' PCK. In addition, the data was used to create a classification of descriptive categories of the identified actions around the four PCK components aforementioned. Moreover, this study is a first step towards documenting the actions enacted during instruction of MTEs, which will contribute to a shared knowledge base in mathematics teacher education. The identified actions from this study could be further investigated to determine their effectiveness, which could potentially be a resource for MTEs to strengthen their practice.

Chapter II – Review of the Literature

The goal of this study was to better understand the actions one MTE enacted during whole-group instruction to develop PTs' PCK in an elementary mathematics content/methods course. Therefore, I reviewed the research literature to inform and frame the study. In this chapter, I discuss the major findings from a review of the research literature related to three primary foci. First, I investigated literature related to facilitating the development of PTs' PCK in mathematics teacher preparation courses. Next, I reviewed the literature on MTEs as well as their practices. Finally, I examined the literature regarding teacher actions at the K-12 level so I could distinguish these actions from actions that were specific to teaching PTs.

Facilitating the Development of Pedagogical Content Knowledge in Mathematics Teacher Preparation Courses

As discussed in Chapter I, teaching mathematics is challenging and requires a teacher who has developed knowledge in several domains. But, the knowledge base for teaching mathematics is broad, encompassing various components of specialized knowledge such as knowledge of mathematical content, knowledge of pedagogy, and PCK—the intersection of both content knowledge and pedagogical knowledge (e.g., Grossman, 1990; Hill, et al., 2008; Magnusson, et al., 1999; Shulman, 1986)).

In a national report focused on PT education, the Conference Board of the Mathematical Sciences (2001) called upon teacher educators to advance the preparation of elementary teachers since "Teaching elementary mathematics requires both considerable mathematical knowledge and a wide range of pedagogical skills" (p. 55). Furthermore, teachers who have strong mathematical knowledge and strong pedagogical knowledge create richer learning experiences for their students (e.g., Ball, Lubienski, & Mewborn, 2001). Additionally, teachers exhibiting PCK know how to select appropriate mathematical tasks, give useful explanations, ask productive questions, and evaluate student learning (Ball, 1990; Conference Board of the Mathematical Sciences, 2001; Ma, 1999b). Thus, PTs should have in-depth knowledge of instructional strategies, curriculum, student understanding, and assessment for planning and teaching mathematics to K-12 students.

Because PCK is unique to the teaching profession, it is underdeveloped in PTs and has become one of many foci in teacher preparation courses. I identified nine publications (see Table 1) where the authors reported on how elementary and secondary PTs' PCK had been enhanced due to specific activities and assignments the PTs completed during their mathematics teacher preparation courses. Five of the studies appeared in peer-reviewed journals (e.g., *Journal for Research in Mathematics Education, Journal of Mathematics Teacher Education, School Science and Mathematics, and Teaching and Teacher Education*) while the others appeared in a conference proceeding and Association of Mathematics Teacher Educators (AMTE) monograph series.

Table 1

Enhancing Prospective Teachers' Pedagogical Content Knowledge Peer-Reviewed Publications

Study	*Title*
Edwards, Özgün-Koca, & Meagher, 2010	Preservice teachers' initial experiences in shifting from "learners/doers of mathematics" to "teachers of mathematics"
Hjalmarson & Suh, 2008	Developing mathematical pedagogical knowledge by evaluating instructional materials
Jenkins, 2010	Developing teachers' knowledge of students as learners of mathematics through structured interviews
Kinach, 2002	A cognitive strategy for developing pedagogical content knowledge in the secondary mathematics methods course: Toward a model of effective practice
Koirala, Davis, & Johnson, 2008	Development of a performance assessment task and rubric to measure prospective secondary school mathematics teachers' pedagogical content knowledge and skills
Steele, 2008a	Building bridges: Cases as catalysts for the integration of mathematical and pedagogical knowledge
Timmerman, 2002	Learning to teach: Prospective teachers' evaluation of students' written responses on a 1992 NAEP graphing task
Tirosh, 2000	Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions
van den Kieboom & Magiera, 2010	Developing preservice teachers' mathematical and pedagogical knowledge using an integrated approach

In all nine publications, the authors describe class activities, assignments, or assessments that PTs completed in order to develop their knowledge. In three cases the authors describe a sequence of mathematical activities that they used in their courses over a period of time (Edwards, Özgün-Koca, & Meagher, 2010; Kinach, 2002; Steele, 2008a; Tirosh, 2000). In the other publications, the authors required PTs to analyze applets and textbooks (Hjalmarson & Suh, 2008), analyze student work and then prepare a lesson plan related to that work (Koirala, Davis, & Johnson, 2008; Timmerman, 2002), and conduct interviews with K-8 students (Jenkins, 2010; van den Kieboom & Magiera, 2010). Below, I begin with a description of this work and the major findings from each of the studies. I then summarize the existing gaps in the literature that the current study addresses.

Edwards, Özgün-Koca, and Meagher (2010), Kinach (2002), and Steele (2008a) each reported on a *sequence of activities* PTs completed over several class sessions that ultimately developed PTs' PCK. Edwards et al. engaged high school PTs in five tasks to move them from students of mathematics to teachers of mathematics. The five tasks were: (a) solve a mathematical task, (b) analyze sample student work of the same mathematical task, (c) create a solution key for the mathematical task, (d) modify the mathematical task, and (e) reflect on the process. The authors reported that the development of PTs' PCK was promoted in two of the five activities (design a solution key and modify the task as an exploration problem) where PTs were required to think and anticipate how high school students would solve the mathematical problem as well as re-design the task so that many approaches to solving the problem could be used.

Similarly, Steele (2008a) reported that participants in a content-focused mathematics methods course (for PTs and practicing teachers) completed and discussed mathematical tasks that were the center of written cases. Participants then engaged in a discussion of the case. By engaging in these two tasks, participants identified both pedagogical issues and mathematical understandings. The participants "had the opportunity to integrate their knowledge of mathematics with their knowledge of pedagogy to create a more powerful learning experience than either activity might have afforded individually" (i.e., than engaging with just the mathematical task or discussing the pedagogical issues surrounding the mathematical example presented in fictitious classrooms) (p. 70).

Like Edwards et al. (2010) and Steele (2008a) who report that their PTs' PCK was enhanced due to completing specific activities in a teacher preparation course, Kinach (2002) also reported using a series of three activities, in a teaching experiment, focused on the mathematical concept of integer addition and subtraction. In this case, PTs shifted their instructional explanations from an instrumental (e.g., describing rules without reasons) to relational (e.g., describing and knowing both what to do and why) understanding of mathematics, teaching, and learning. However, unlike Edwards et al. and Steele where the sequences of activities they reported built upon the PTs first solving a mathematical task, the three activities Kinach posed all asked the PTs to provide an explanation for how to add and subtract integers to an eighth grader learning this mathematical concept for the first time. For the first task, the PTs chose the context for their instructional explanation, but they had to use the number-line context and then an algebra-tile context for the second and third tasks.

Kinach (2002) used a *levels of understanding* framework (Skemp, 1978) to code participants written homework assignments, video-recordings of class discussions, and journals containing instructional explanations, to determine PTs' conceptual difficulties. In this framework, Kinach identified whether PTs' understanding was *instrumental* (i.e., procedural) or *relational* (i.e., advocating mathematical understanding). Analysis from the first task indicated that PTs' explanations were procedural. After task two, the PTs were not satisfied with the number line as justification for integer addition. However, by the conclusion of the teaching experiment, PTs' written explanations became less procedural explanations (i.e., *instrumental*) to more *relational*. Based on the results from this study, Kinach introduced a five-element cognitive strategy for teacher educators to support the development of PTs' PCK. Kinach suggests that teacher educators (a) *identify* PTs PCK, (b) *assess* adequacy of PTs' PCK, (c) *challenge* the adequacy of PTs instructional explanations, (d) ask PTs to explain the concept using a new context so that their PCK begins to *transform*, and (e) delve deeper into PTs' knowledge in order to *sustain* their growth.

Like Kinach (2002), Tirosh (2000) shared several activities around a single mathematics concept. Elementary PTs engaged in activities focused on division of fractions throughout the semester in order to assess and enhance their PCK of rational numbers. At the beginning of the semester, PTs completed a diagnostic questionnaire to assess their PCK. The author also reported several activities the PTs engaged with which focused on fractions (e.g., addition of fractions, class discussions related to fractions as well as general mathematical and pedagogical issues, a practice based task where the PTs were asked to find examples in textbooks that could be represented by the measurement model of division). Additionally, PTs completed a written assignment halfway through the course and an in-class end of course assignment designed to assess the participants' PCK of children's mathematical thinking processes. Results suggested that that PTs' PCK of children's common conceptions of division of fractions was achieved.

Unlike any of the aforementioned studies (Edwards, et al., 2010; Kinach, 2002; Steele, 2008a; Tirosh, 2000) where the authors reported on a sequence of mathematical activities that enhanced PTs' PCK, Hjalmarson and Suh (2008) described two different assignments where elementary PTs were required to evaluate mathematical applets and secondary PTs analyzed mathematics textbooks. Results from the analyses the PTs wrote for the various assignments, indicated that the PTs focused on features beyond surface level features of the various curriculum materials to focusing on student learning and instruction using the materials. This assignment contained "common design characteristics *for* promoting pedagogical content knowledge" (p. 104). In all five of these studies, the authors reported that PTs' PCK was enhanced due to their engagement in the activities reported. However, the authors (with the exception of Hjalmarson and Suh, 2008) did not identify specific components of PCK (e.g., knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, knowledge of assessment)—just that PTs' PCK development in general was promoted.

In two publications, the authors described an assignment (Timmerman, 2002) and a performance assessment task (Koirala, et al., 2008) where PTs analyzed student work and designed resulting mathematical tasks or lesson plans to use with elementary and secondary students. More specifically, Koirala et al. (2008) shared an assessment task and rubric designed and implemented over four years to assess PTs' PCK. PTs were asked to analyze student work and then create a lesson plan based on student needs. Findings reported from 18 secondary mathematics PTs during the second year, demonstrated that the PTs perceived completing the assessment task as beneficial for their future teaching. The authors argued, "The present assessment task and rubric, based on the analysis of student work and the use of that analysis in developing teaching strategies, provide[d] PTs an opportunity to develop pedagogical content knowledge and skills" (p. 135).

Timmerman (2002) shared an end-of-course project elementary PTs completed where she asked them to analyze four student responses, from a 1992 National Assessment of

Educational Progress (NAEP) released item related to graphing, and use the analysis in developing teaching strategies around the mathematical content of the problem. PT written responses included appropriate teaching strategies to facilitate the students' understanding of the mathematical concept. Additionally, this assignment provided the PTs with an opportunity to convey "evidence of their pedagogical content knowledge as they buil[t] connections between students' knowledge and their [own] subsequent teaching practices" (p. 356). In both of the studies reported by Koirala et al. (2008) and Timmerman, assessing student work and then articulating appropriate teaching strategies to engage the fictitious children, provided PTs an opportunity to develop their PCK.

Jenkins (2010) as well as van den Kieboom and Magiera (2010) also asked their PTs to complete mathematical tasks in teacher preparation courses. However, different from the studies mentioned above, PTs in these two studies engaged K-12 students from their field experience sites with the same mathematical tasks they themselves completed in their courses in order to connect their learning of mathematics with their knowledge of teaching and student learning. For example, in the Jenkins study, pairs of PTs in a middle school mathematics methods course conducted structured interviews with three middle school students throughout the semester. Prior to the interview, the PTs: (a) solved the problem, (b) predicted what students would/would not know, (c) anticipated student misconceptions, and (d) modified the original problem to provide more scaffolding during the interview if necessary. Then they conducted the interviews where they presented an open-ended, problem-solving task, posed questions to the middle school student, and asked the student to share his/her thinking using written or verbal communication. After the interview, PTs analyzed interview notes and the student work collected during the interview and shared their findings with their peers regarding middle school students' knowledge of the mathematical content. Jenkins argued that the PTs needed to learn how to conduct student interviews. Therefore, he only analyzed data collected during the third interviews. Based on the analysis of "anecdotal data drawn from the [PTs'] interview reports" (p. 147), Jenkins stated that the interview process, "demonstrates potential for ensuring that PTs acquire a vital piece of PCK, specifically knowledge of how students think and reason about mathematics" (p. 150) which falls under Knowledge of Content and Students, a construct of PCK that Hill, Ball, and Schilling (2008) identify.

Similarly, van den Kieboom and Magiera (2010) also provided PTs with experiences aimed to develop their PCK through interactions with elementary students. In this case, elementary PTs engaged with and discussed mathematical tasks in the content course and then implemented these same tasks with elementary students. Through PT reflection and subsequent MTE feedback on teaching experiences from the PTs' field experience, PTs were provided opportunities to re-examine their mathematical and PCK to focus on student learning prior to their next experience in field. The integrated coursework experience aligned with the field experience both Jenkins (2010) and van den Kieboom and Magiera report, provided the PTs with "authentic opportunities to connect their learning of mathematics with their learning about how to teach mathematics in practice" (p. 186).

As mentioned above, the authors of all nine publications described specific course activities, assignments, or assessments that PTs completed to develop their knowledge. The primary focus for this collection of work involved investigating how these activities, assignments, or assessments influenced PTs' PCK. I highlight four ideas, which were common across all nine articles: (a) PTs were the focus of the studies; (b) specific activities, assignments, or assessments that PTs completed during several class periods were discussed in each of the articles; (c) the authors argued that these activities facilitated the enhancement of PTs' PCK; (d) the authors did not analyze the data based on a PCK framework although most of the activities were related to the PCK components of student understanding and instructional strategies.

First, the authors focused their data collection and analysis efforts on PTs, rather than the MTE. The authors of the nine publications used a variety of data sources to investigate PTs' PCK. In most cases, they used student written or verbal responses to in-class activities or assessment tasks (Edwards, et al., 2010; Hjalmarson & Suh, 2008; Jenkins, 2010; Kinach, 2002; Koirala, et al., 2008; Steele, 2008a; Timmerman, 2002; Tirosh, 2000). In some cases, they interviewed PTs (Koirala, et al., 2008; Tirosh, 2000), video- or audio-recorded course sessions (Kinach, 2002; Steele, 2008a; Tirosh, 2000; van den Kieboom & Magiera, 2010), or collected PTs' written journals (Kinach, 2002; van den Kieboom & Magiera, 2010). In addition, Tirosh (2000) administered a diagnostic questionnaire on PCK and van den Kieboom and Magiera (2010) collected audio records of PTs' interactions with field students. As is evident from the data sources, all of the authors focused their data collection efforts on the PTs rather than the actions of the MTEs in these courses. There was no data collected from MTEs or about MTEs. However, in three publications (Kinach, 2002; Steele, 2008a; van den Kieboom & Magiera, 2010), the authors used transcripts from class sessions including what the instructors said. Yet, their analyses focused on what the PTs said, rather than the MTE.

Second, the activities, assignments, or assessments shared in the studies mentioned above only covered several class periods and not an entire semester. Facilitating the development of PCK should be a primary goal in courses designed to prepare PTs. Karp (2010) argues, “It is not enough to recognize the importance of PCK and skills. It is also necessary to find a place for teaching such knowledge and skills within the framework of the already rather densely packed program of mathematics teacher preparation” (p. 137). The nine publications described above identify a small number of individual activities implemented over a couple of days during the semester that enhance PTs’ PCK.

Third, authors in all nine publications identified activities MTEs could use to facilitate PTs’ PCK. All of the tasks required the PTs to *analyze* either student work (Edwards, et al., 2010; Koirala, et al., 2008; Timmerman, 2002), interactions with children (Jenkins, 2010; van den Kieboom & Magiera, 2010), curriculum materials (Hjalmarson & Suh, 2008), or mathematics (Jenkins, 2010; Kinach, 2002; Steele, 2008a; Tirosh, 2000).

Fourth, none of the authors reported using a PCK lens to analyze the data they collected. The authors of several studies did not acknowledge using a framework to analyze the data (Edwards, et al., 2010; Hjalmarson & Suh, 2008; Steele, 2008a; Timmerman, 2002; van den Kieboom & Magiera, 2010). However, authors who did use a framework did not use a PCK framework as shown in Table 2. The authors of all nine publications claimed that PTs’ PCK was enhanced, but none of the data was analyzed using PCK as a framework.

Table 2

Identified Frameworks Used to Analyze Data Regarding Prospective Teachers' Pedagogical Content Knowledge

Study	*Frameworks*
Jenkins, 2010	Three orientations that teachers have toward listening to students' ideas: *evaluative listening, interpretative listening,* and *hermeneutic listening*
Kinach, 2002	"Used a modification of Perkins and Simmons' *levels of understanding* framework as developed in the Teaching for Understanding Project at the Harvard Graduate School of Education (Perkins and Simmons, 1988; Wiske, 1998)" (p. 55)
Koirala, Davis, & Johnson, 2008	Assessment rubric

Moreover, most of the activities described in these nine publications were related to the PCK components of student understanding and instructional strategies. As I reviewed each of the publications, I identified the PCK components (i.e., knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, and knowledge of assessment) that each activity addressed. (See Table 3.)

Table 3

Pedagogical Content Knowledge Components I Identified the Authors Attending to via the Activities They Reported Prospective Teachers Completing

Study	*Knowledge of instructional strategies*	*Knowledge of curriculum*	*Knowledge of student understanding*	*Knowledge of assessment*
Edwards, Özgün-Koca, & Meagher, 2010	X	X	X	X
Hjalmarson & Suh, 2008	X	X		
Jenkins, 2010			X	
Kinach, 2002	X			
Koirala, Davis, & Johnson, 2008	X		X	
Steele, 2008a	X		X	
Timmerman, 2002	X		X	
Tirosh, 2000	X	X	X	
van den Kieboom, & Magiera, 2010	X		X	

Evident from this table, the activities the PTs completed did not emphasize the opportunity to enhance two components (i.e., knowledge of curriculum and knowledge of assessment) of PCK to the same extent as the other two PCK components (i.e., knowledge of instructional strategies and knowledge of student understanding). In fact, only the series of activities Edwards et al. (2010) described provided the PTs with the opportunity to enhance their knowledge in all four PCK components.

MTEs need images of specific actions MTEs employ during instruction in order to provide the opportunity to promote the development of PTs' PCK. While these publications recognize that PTs' PCK can be enhanced in teacher preparation courses, it is unclear what actions MTEs employ to enhance PTs' PCK. We lack research on what MTEs say and do in their classroom during instruction. In the study I report below, I attend to the gaps from the aforementioned studies. For example, I focus on: (a) MTE actions, rather than an analysis of PT artifacts; and (b) MTE actions across an entire course (over three semesters in which that same course was taught), instead of a small subset of class periods within a semester. Data sources unique to this study included: (a) fieldnotes I took when observing the same course Spring 2008 and Spring 2010, (b) initial interview with the MTE, (c) interviews where the MTE provided commentary on several extended video segments from Spring 2007, and (d) video/fieldnote based interviews. In addition, I use a PCK framework to analyze the data in order to categorize actions one MTE employed to develop PTs' PCK. In the next section, I review the research related to MTEs and their practices.

Mathematics Teacher Educators

As discussed in Chapter I, we know little about the practices of faculty who prepare teachers of mathematics, as these practices are not widely documented or disseminated (Bergsten & Grevholm, 2008; Even, 2008; Floden & Philipp, 2003; Hiebert, et al., 2003; McDuffie, et al., 2008; Schempp, 1995). In this section, I organized the review of the literature on MTEs into three major categories: (a) Professional growth of MTEs, (b) MTE knowledge, and (c) Practices of MTEs. Prior to elaborating on each of these three sections, I begin with a brief description of MTEs.

Who are mathematics teacher educators?

> *"A mathematics teacher educator is someone whose primary work responsibilities and professional contributions are focused on supporting the learning and/or practice of preK-12 teachers of mathematics (elementary, middle or secondary level)" (Reys, 2009, p. 1).*

A variety of individuals (e.g., mathematicians, current and former classroom teachers, graduate students) assume the role of MTEs. These individuals have a variety of degrees (e.g., master's or doctoral degrees in mathematics or mathematics education) and may or may not have any formal training in mathematics education or preparing teachers. Zaslavsky (2009) writes, "Unlike the many kinds of institutionalized (pre-service and in-service) teacher education programmes, there are hardly any teacher educator education programs; thus, becoming a MTE occurs overtime, through ongoing reflection on one's own experiences in facilitating teachers' learning" (p. 108). MTEs are employed at a variety of institutions, such as two-year community colleges, master's degree granting universities or research universities. These individuals are vital to mathematics teacher education as they design and develop learning experiences for PTs. However, in analyzing the syllabi of the courses taught by MTEs, Taylor and Ronau (2006) found a wide variability of goals, objectives, and assignments.

Oesterle and Liljedahl (2009) also provide insight into the variability of MTEs through an examination of two MTEs who taught the same content courses for PTs. During an interview, one MTE stated that PCK is important for PTs to develop during methods courses. However, the second MTE did not believe that attending to PCK was important stating, "mastery of the subject content along with general pedagogical skills are sufficient for teaching mathematics" (p. 1257).

Other indicators of variability of MTEs are documented in publications that describe some of their practices. In recent years, there have been a few publication venues (e.g., AMTE Monograph series), the establishment of an international journal (*Journal of Mathematics Teacher Education)*, and conference venues (AMTE Annual Conferences) for MTEs to share practices they use with PTs. The editors of the National Council of Teachers of Mathematics (NCTM) journal, *Teaching Children Mathematics*, established the Supporting Teacher Learning department as a vehicle for MTEs to share their practices. In this publication, MTEs have shared various ideas about course assignments (Neumann, 2007; Soto-Johnson, IIiams, Hoffmeister, Boschmans, & Oberg, 2007; Timmerman, 2004); course activities (Baek & Flores, 2005; Krebs & Kaller, 2006); ideas to address specific content (Groth, 2006; Hedges, Huinker, & Steinmeyer, 2005); and approaches to help PTs recognize the challenge of teaching elementary mathematics (Chval, 2004; Chval, Lannin, Arbaugh, & Bowzer, 2009; Gee, 2006; Grant, Lo, & Flowers, 2007; Hart, 2006; Stephens & Lamers, 2006). These publications have included some specific activities, assignments, and approaches used by MTEs, but have not included a view of specific actions used to teach prospective mathematics teachers.

The professional growth of mathematics teacher educators. In the last decade, there has been a growing body of research related to the professional growth of teacher educators, and MTEs specifically. Zeichner (2005) reflected on his transition from classroom teacher to university teacher educator. Based on his reflection, he identified specific experiences that help individuals prepare to become future teacher educators. These experiences include: a) teaching a course, b) supervising PTs in field or student teaching placements, and c) engaging in self-study to ultimately improve as a teacher educator. In addition, Zeichner stated that teacher educators need to be familiar with teacher education literature. He explained:

> Ignorance of the literature in teacher education prevents one from potentially seeing one's practice as a teacher educator in new ways that challenge one's existing frameworks and cuts one off from what has been learned by teacher educators in other programs about particular aspects of teacher education such as instructional strategies to accomplish particular purposes. It reduces teacher education to a commonsense activity and is inconsistent with the scholarly norms that universities claim to embrace. (p. 122-123)

In fact, Murray and Male (2005) argued that it may take several years to establish a professional identity as a teacher educator. They also suggested that self-study may help new teacher educators acclimate to the profession and all the responsibilities that come with the job. At a presentation at the 2008 Annual Meeting of the AMTE, Chval, Lannin, and Arbaugh (2008) also described challenges that novice MTEs face as they transition from classroom teacher to university MTE. For example, they argued that MTEs struggle with assuming a professional identity as a MTE and developing critical knowledge bases for teaching PTs.

Several experienced MTEs have conducted self-studies for the purpose of reflecting on their professional growth as MTEs. Tzur (2001, 2008) and Krainer (2008) give reflective self-studies on their personal story of growth and development as a MTE. They trace their history of learning mathematics, learning to teach mathematics, learning to teach teachers, and learning to mentor MTEs. Tzur developed a model for conceptualizing his journey as a MTE. The four levels of his model are: (a) learning mathematics as a student would, (b) learning to teach mathematics as a teacher, (c) learning to teach mathematics teachers as a teacher educator, and (d) learning to teach MTEs as a mentor. Similarly, Krainer described critical incidents of his development from his elementary school days to becoming a teacher of mathematics and MTE. He then continued to describe components of his professional journey regarding his involvement with a professional development program as well as his involvement about bringing interdisciplinary (mathematics, science, and technology) change to teaching in his home country, Austria. His reflection spanned more than 25 years. Likewise, Tzur's journey to becoming a reflective practitioner also spanned decades.

Mohammad's (2008) personal reflection on becoming a MTE also originated with her grade school years. She wrote about her personal development as a MTE in Pakistan and how this international context facilitated major challenges to her work as a MTE with its hierarchies of respect and rigid school systems. She charted her personal growth related to dealing with conflict back to her interactions with teachers. She stated in the end that becoming a teacher educator involved being flexible, having a strong moral stance, and constructing a critical approach towards the practice.

Based on the findings from these self-studies, we know that assuming the professional identity of a MTE may take several years and that there are challenges associated with becoming a MTE. Most importantly, these studies show that reflection on practice facilitates the professional growth of MTEs. These reflective self-studies also highlight the lack of institutional and professional support preparing MTEs to work with classroom teachers and PTs. However, there have been recent efforts to design programs (Dolk, den Hertog, & Gravemeijer, 2002; Even, 1999, 2008) and learning experiences (Van Zoest, Moore, & Stockero, 2006) to educate MTEs in how to teach PTs. For example, Even (1999, 2008) described the MANOR project, a professional development program created to educate MTEs and develop materials for the MTEs to use with PTs in

Israel. The project facilitators invited secondary mathematics teachers (with 5-30 years of teaching experience), representing the entire geographical region of Israel who had the "potential to influence mathematics teaching at the school" (p. 62). This preparation program was designed for MTEs to engage in a community of practice where they had the opportunity to "develop knowledge, skills, dispositions and practices situated in the practices of educating practicing teachers" (p. 68). From 1993 to 2003, 75 participants graduated from this formal two-year preparation program. After graduating, some participants continued to engage in monthly forums and bi-annual conferences in order to support their professional development as MTEs.

In a different professional development experience designed for novice MTEs in the Netherlands, Dolk, den Hertog, and Gravemeijer (2002) researched the utilization of multimedia video cases to support MTEs in learning to mathematize and didactize. In addition, the authors investigated how the MTEs learned to use the video cases with prospective elementary teachers. For 10 years, a 16-day professional development course for beginning MTEs had been offered and modified numerous times in order to support novice MTEs. The researchers used various forms of video cases and first noticed that novice MTEs attended to a variety of aspects related to the clip (e.g., teacher actions, student actions, attending to what actually happened in the video, freely interpreting what happened in the clip). In a later iteration of using the video cases with the novice MTEs, the authors observed that the participants preferred to omit discussing their remarks and breakdown of the footage, and instead seemed intent on focusing on the conclusions, based on their knowledge and beliefs. Based on the authors' research, a six-step framework was posed to help MTEs reflect in order to learn from their own classroom experiences and actions. The six cyclical steps in the framework are: (a) observing (having an awareness of multiple facets of teaching), (b) sharing and discussing observations (engaging in critical thinking), (c) analyzing (focusing on important ideas), (d) reflecting, (e) developing narrative knowledge (constructing stories about classroom events), and (f) expanding the personal repertoire and generalizing the situation into a didactic for teacher education (generalize several narratives into a larger practical narrative for teaching).

In a different type of professional development setting, Van Zoest, Moore, and Stockero (2006) articulated various learning experiences of three novice MTEs, with high school teaching experience, teaching a semester long, middle-school methods course under the mentorship of a more experienced MTE. The novice MTEs struggled to find a balance between sharing their former teaching experience with PTs and fostering an environment where the PTs developed an ability to become reflective about various teaching instances and thinking through mathematical problems.

The novice MTEs assumed that modeling teaching they wanted their PTs to emulate would engage PTs in conversation regarding instances of classroom instruction as they felt that such a strategy had been effective in their high school mathematics classrooms. However, the mentor MTE and novice MTEs' goals for showing a video clip to start a conversation on classroom instruction with the PTs were different. The mentor saw the video as an entry point for analyzing teaching (i.e., analytic perspective of a MTE) while the novice MTEs saw the video clip as a way to model what PT instruction in the field experience might look like (i.e., goal similar to that of a classroom teacher). From their participation in this clinical experience, the authors noted "spending more time upfront on the transition to teacher educator would have allowed [the novice MTEs] to participate in the collaboration as teacher educators rather than as experienced school teachers with different students" (p. 145).

With the increased interest in educating MTEs and designing programs to structure the education of MTEs, there is a need for more research in this area in order to attain the ultimate goal of educating MTEs who work with preservice and practicing teachers. The findings described above demonstrate that MTEs need opportunities and resources to develop specific knowledge and competencies in teaching PTs. There have been self studies (Krainer, 2008; Mohammad, 2008; Tzur, 2001), studies of professional development opportunities for MTEs (Dolk, et al., 2002; Even, 1999, 2008), and a study of collaboration between novice MTEs and a more experienced MTE (Van Zoest, et al., 2006). However, there has not been a study where researchers look at images of specific actions MTEs enact during instruction through use of video and fieldnotes over a couple of years. This study is a small scale, single case study that will provide examples of images to enhance MTEs professional growth (e.g., from video and fieldnotes of one MTE's practice over a three year period).

Mathematics teacher educator knowledge. In the last 20 years, researchers have devoted greater attention to knowledge bases K-12 teachers need in order to effectively work with their students. In essence, questions such as what basic knowledge, skills, and competencies teachers should possess have driven the field of teacher education (Cochran-Smith, 2001). However, there has been a lack of attention on understanding what specific knowledge bases teacher educators need (Cochran-Smith, 2003), and there is recognition that teacher and teacher educator knowledge bases differ. For example, in the Learning and Teaching Linear Functions: Video Cases for Mathematics Professional Development, 6-10 (Seago, Mumme, & Branca, 2004), the designers extended Cohen and Ball's (1999) notion of interaction among teachers, students, and curriculum at the classroom instructional level to describe a similar set of relationships with respect to teaching in teacher education (i.e., professional development and leadership development).

Seago et al. (2004) claimed that the relationships related to teaching in school (e.g., teacher/student, student/math, and teacher/math) could be extended to teaching in teacher education. However, in the latter case, the teacher is the teacher educator, the student is the prospective or practicing teacher and the mathematics is the mathematics used in teaching. Carroll and Mumme (2007) extended the *Teaching in Teacher Education* model even further to address leadership development. At this third level, the teacher educator is now the leader of teacher educators, the teacher is the teacher educator, and the math used in teaching is teaching in teacher education. The knowledge for the leader of teacher educators is most complex, as this knowledge involves the work of a classroom teacher as well as that of a teacher educator. Despite this complexity, the model "has been proven useful as a frame for thinking about the relationships among teacher, learner, and content, whether it be in the classroom, professional development, or leadership development" (p. 83).

There have been several studies that have investigated MTE knowledge. I highlight the findings from five studies. Below I expound on Chauvot (2009) and García, Sánchez and Escudero (2007) who studied their own knowledge and created frameworks detailing their knowledge as MTEs. In addition, I elaborate on studies conducted by Doerr and

Thompson (2004), who researched the understandings of MTEs while using multimedia case studies of practice with PTs, and Zaslavsky and Leikin (2004) and Peled and Hershkovitz (2004) who researched MTEs in a professional development setting with inservice teachers.

In a self study, Chauvot (2009) documented her thinking and analysis in a journal, drawing from artifacts collected over three years from methods courses she taught, as well as other sources not related to teaching (e.g., artifacts from her doctoral program, narratives submitted for promotion and tenure, etc.), in order to investigate her knowledge content as well as structure and growth from her doctoral program days through her third year as a MTE-researcher. She drew from numerous frameworks to inform her study (e.g., Grossman, 1990; Leinhardt & Smith, 1985; Ma, 1999a; Olson, 1998; Shulman, 1986) and utilized narrative inquiry as her methodology.

Chauvot (2009) created, revised, and refined a knowledge map, drawing from the aforementioned frameworks, in order to depict her "perception of the knowledge [she has] sought in [her] role as a mathematics teacher educator-researcher" (p. 362). The map contained three main components: (a) knowledge of context (i.e., teaching university courses or mentoring doctoral students); (b) three categories of knowledge as articulated by Shulman (1986) (i.e., subject matter content knowledge, PCK, and curricular knowledge); and (c) knowledge of research, which serves as a unifier to Shulman's categories (i.e., conducting, interpreting and writing).

Through the creation of the knowledge map, Chauvot (2009) articulated several findings about her practices as a MTE that demonstrated uncertainty. For example, her instructional practices for meeting various goals articulated in her syllabus had been unstable and she stated that she was "uncertain of the appropriate instructional strategies to employ within my courses to address mathematics knowledge of students" (p. 367). Additionally, at the conclusion of her doctoral program, she thought that weaving mathematics and pedagogy together to teach PTs was "the" way to help PTs develop conceptual understanding of mathematical concepts under discussion while simultaneously teaching pedagogy about those same mathematical concepts. Her experience with first teaching in a mathematics department, confirmed that teaching the mathematics and then reflecting on the mathematics as a teacher worked well. However, since teaching in the department of curriculum and instruction, where her courses were driven by pedagogy and not the mathematics, the mathematics was often short-changed due to the emphasis on pedagogy. What originally "seemed simplistic in [her] beginning years as a mathematics teacher educator, has become very complex and abstract at this point in [her] professional growth" (p. 367).

García et al. (2007) write that reflection entails an individual consciously engaging in his/her practice and the authors utilized reflection-on-action as they analyzed their MTE actions' after a classroom event, in a mathematics methods course for PTs, had occurred—describing how their knowledge had grown as they engaged in teaching and research. They reflected on the relationship between theory and practice and broadened their initial theoretical ideas. Through reflection and analysis of what was learned, the authors' knowledge grew as they developed as MTEs. The MTEs learned that they need to make use of theory, theory produces certain things that should be evident in MTEs practice, and that their practice is an excellent source for study which has the potential to reveal other issues worthy of research.

Like García et al. (2007), Doerr and Thompson (2004) also investigated MTE knowledge and understandings of MTEs working with PTs. However, unlike García et al., these authors did not study their personal MTE knowledge, but instead studied four experienced MTEs who facilitated preservice secondary mathematics teacher methods courses that used video cases as part of the curriculum with the goal of understanding how and why MTEs "use[d] cases to meet their intentions for the learning of their [PTs]" (p. 178). Using a case study method, which encouraged reflective reasoning by the participants (in this case, 30 PTs across four sites), prompting the MTEs to reflect on actions of particular instances within a class period, the authors articulated four ways the MTEs used artifacts from the video-case study curriculum to support preservice teacher thinking in a methods course. The MTEs supported PTs' thinking in the following ways: (a) the intricacy of the teacher facilitating small group work in the video, (b) the usefulness of clearly articulating the video teacher's lesson reflections, (c) planning and preparing lessons, and (d) the growth that needs to occur for PTs to comprehend and value students' thinking and gear up lesson ideas.

In addition, use of the video case study curriculum by the MTEs provided the opportunity for reflection on their own practice. One MTE stated that he "reflected on his own developing theory for understanding the limitations of [PTs'] subject matter knowledge" while another said that the video case materials "provided him with a resource that he needed to unpack and repackage with his own thinking about professional development" (p. 195). Overall, results indicated "multimedia case studies of practice can serve as a vehicle for revealing the knowledge and practice of teacher educators as they manage the dilemmas of supporting [PTs'] professional development" as well as providing MTEs with the opportunity "to reflect on their understandings of the professional development of their [PTs]" (p. 199).

Regarding the professional development context for inservice teachers, Zaslavsky and Leikin (2004) drew on theory—combining Jaworski's (1992, 1994) teaching triad and Steinbring's (1998) model of teaching and learning mathematics—in order to conceptualize the learning processes of MTEs who helped facilitate a professional development program for Israeli inservice secondary mathematics teachers. Like García et al. (2007), reflection was also a key issue in this study regarding the development of the MTEs. The 3-layer model the authors constructed offers a lens through which to examine the interplay between the learning processes (e.g., with regards to collaboration, planning a lesson, or mentoring a less experienced MTE) of the different members of the professional development program (i.e., mathematics teachers, MTEs, and MTEs' educators). Likewise, the model was used to analyze MTEs' linear growth through different learning experiences (e.g., mathematics teacher → MTE → MTE-researcher) similar to what Tzur (2001) and Krainer (2008) relayed in their self-studies.

Similar to Chauvot (2009) and García et al. (2007), Peled and Hershkovitz (2004) also studied their own knowledge. During the course of the study, the researchers focused on a specific issue implemented with one group (i.e., PTs and/or inservice teachers), reflected on the MTE actions that transpired, and then posed new tasks and tried them with a new cohort. Analysis indicated that the MTEs learned they "need[ed] to create new tasks that will involve teachers in further analysis of the nature of these answers" (Peled & Hershkovitz, 2004, p. 319). The authors concluded by reporting on additional insights, they as MTEs, garnered from the evolving phases. These insights included the

MTEs recognizing the role and importance of their actions, that they needed to be receptive to their students, and the importance of selecting and using challenging tasks. Pedagogical insights that surfaced included the role of reflection and the importance of being sensitive to the preservice (or inservice) teachers as well as highlighting the importance of making teachers aware of the difference between applying a mathematical model without thought versus first examining a problem situation during problem solving.

These studies demonstrate the importance of this work and developing specific knowledge for MTEs. Although this study is not focused on MTE knowledge, there is an implicit connection in that the MTE's knowledge/knowledge of specific activities is going to influence the actions that they enact.

Practices of mathematics teacher educators. Another significant role that MTEs engage in is researching and learning from their own practice as teacher educators. Some studies involve MTEs researching their teaching by focusing on investigating ways of facilitating PTs' development of mathematics knowledge with the goal of understanding and guiding the PTs' practice as well as exploring what is happening in the course and how it can be improved (e.g., Blanton, 2002; Bowers & Doerr, 2001; Kinach, 2002; Taplin & Chan, 2001). Other MTEs study their own practice with the goal of strictly improving it.

Cady, Hopkins, and Hodges (2008) engaged in lesson study as a means to improve MTE practice one lesson at a time. The authors reported on the reflective process of a lesson study two MTEs conducted to improve their teaching of place value to prospective and inservice teachers. Such improvements included rethinking pedagogical moves the

MTE made and adding additional activities to enhance participants' understanding of place value. Throughout the process, the MTEs learned about content as well as about teaching the topic. Some of the benefits the MTEs reported as a result in engaging in lesson study included: (a) increasing their PCK regarding place value, (b) making sense of their practice, (c) reflecting on the relationship between their instruction and the participants' understanding, and (d) attending to the research, classroom observations, and students' reflections to refine or expand the lesson. The MTEs also noted that lesson study was extremely time consuming, but it was beneficial to have another MTE in the room. They also observed that the time spent reflecting on the lessons and their resulting professional growth was invaluable.

As discussed below, other researchers also reported improvement to the overall course design as a result of collaborating with colleagues. They engage in collaboration in order to improve themselves as teacher educators or improve the design of tasks embedded in the lessons they have designed.

Course design. At the University of Delaware, Hiebert and colleagues (e.g., Berk & Hiebert, 2009; Hiebert, Morris, Berk, & Jansen, 2007; Hiebert, et al., 2003) have planned teaching experiences for PTs in their elementary teacher education program. These teaching experiences center around two learning goals for their PTs: (a) to become mathematically proficient and (b) to prepare to learn to teach for mathematical proficiency. Additionally, they identified parallel learning goals for themselves, the MTEs. They believed the goals they identified to support the knowledge needed to improve teaching (i.e., the goals identified for the PTs) were the same goals to support teacher educators (Jansen, Bartell, & Berk, 2009) in generating knowledge needed to improve teacher preparation and that attending to the identified goals during planning would help contribute to a shared knowledge base for MTEs (Hiebert, Morris, & Glass, 2003).

During the lesson planning phase, Hiebert and colleagues discussed specific activities as well as what they anticipated would happen at certain points so they could plan specific discussions that needed to occur during the lesson. This more explicit approach helped the team to focus on how to enhance PTs' thinking and learning. Hiebert and colleagues have been planning lessons in this fashion for nearly 10 years. Each year, the instructors of the courses inherit these plans and add to the accumulation of wisdom and effectiveness of the plans.

In addition to collaboratively writing lesson plans to use in their teacher preparation courses for elementary PTs, Hiebert and colleagues have developed frameworks to help guide their course design. In a recent article, they introduced a model that focused on improving the preparation of elementary PTs (Berk & Hiebert, 2009). Three key principles define the model. The first principle requires that teacher education programs identify explicit learning goals. By committing to a set of goals, the authors argue that MTEs are able to "treat the course as an object of study, thereby generating cumulating and sharable knowledge" (p. 341). The second principle suggests that MTEs make revisions to their courses based on PTs' learning. The final principle advocates that MTEs preserve the history of their lesson plans (e.g., note improvements and revisions made) for other MTEs to access and improve. Through the authors' engagement with the model they proposed, their teacher education program for prospective elementary teachers has improved.

Hiebert and colleagues conceptualized a second framework to guide the design and enactment of mathematics teacher education courses for PTs (Hiebert, et al., 2007). This framework outlined a set of four skills they claim would aid PTs in learning from their own teaching. The skills were: (a) define learning goals for the mathematical topic, (b) attend to student responses related to the defined learning goals, (c) make hypotheses regarding how instruction is supporting students' learning, and (d) make revisions to improve instruction. The authors did not claim that these skills were the only competencies that PTs should have, but that these skills may benefit PTs, over time, as they embarked to improve their teaching effectiveness.

Unlike Hiebert and colleagues who had been documenting the evolution of their elementary teacher preparation program for nearly a decade, Heaton and Mickelson (2002) and Steele (2008b) reported findings from studies analyzing their own classrooms over a period of less than one year. The researchers engaged their students in semester-long learning experiences in order to help the PTs become independent learners of what they needed to know and provided the PTs with opportunities to make generalizations about teaching.

Heaton and Mickelson (2002) reported on their collaborative effort to teach statistical investigation to two different cohorts (22 students each) of prospective elementary teachers. The teacher educators reported on two projects the PTs engaged in over a semester course with the goal that the PTs would garner some enthusiasm for statistical investigation. The MTEs learned much about the process of inquiry learning and its shortcomings. They learned that PTs need multiple opportunities to be learners of statistical investigations rather than the one activity they did during the course, as the first project alone did not "provide sufficient opportunity for preservice teachers to develop the depth of statistical content knowledge or experience with the process of reasoning with data needed to teach children the process of statistical investigation" (p. 54). As a result of this observation, during the second cohort, the MTEs played a more active role in helping the PTs plan the lesson they taught to their field students. In addition, through teaching and research, the MTEs learned that "much related to the process and content of statistical investigation, as a tool for learning other content, was not obvious to beginning teachers" (p. 54).

Steele (2008b) had 25 prospective and practicing teachers (grades K-12) in his advanced mathematics methods course focused on geometry and measurement in the middle grades. He invited his students to analyze his teacher moves to facilitate their participation in the course as well as connect what they were learning to their future practice. Participants identified facilitation moves the MTE employed, that K-12 teachers would also use, such as revoicing, wait time, and not providing specific answers to mathematical questions PTs posed so PTs would continue to think about the posed question. The resulting conversations "allow[ed] teachers to move from the particulars of their own learning to generalities about teaching" (p. 117). Additionally, this activity allowed the MTE to reflect on his own practice in order to provide insight into his own practice.

Like Steele (2008b), Dixon, Andreasen, and Stephan (2009) investigated the pedagogical practices and role of a MTE. They conducted a teaching experiment involving 16 PTs in a content course for elementary PTs "to understand how [PTs] develop mathematical understandings of number concepts and operations with an underlying goal of examining the establishment of social and sociomathematical norms" (p. 46). The authors found that the MTE was explicit in articulating her expectations to PTs at the beginning of the course. However, as the semester progressed, the MTE took on the role of the student to model her expectations regarding norms she deemed important in order to shift the responsibility for sustaining the norms from the instructor to the students. Additionally, they found that the MTE needed to be proactive in creating opportunities for the negotiation of norms to occur. She needed to plan for opportunities where PTs could partake in conversations where norms could be negotiated. Likewise, she needed to be flexible if her plans did not occur as intended.

In each of the studies aforementioned, the MTEs observed that their personal reflection provided the opportunity to enhance the course design of the teacher preparation course they studied. In addition, Dixon et al. (2009), Heaton and Mickelson (2002), and Steele (2008b) observed that reflection played a role in the growth of their own teaching as well as the overall enhancement of the designed learning experiences PTs came in contact with in the teacher preparation course.

Task design. As previously discussed, some researchers described activities, assignments, or assessments PTs completed in teacher preparation courses in order to enhance their PCK (Edwards, et al., 2010; Hjalmarson & Suh, 2008; Jenkins, 2010; Kinach, 2002; Koirala, et al., 2008; Steele, 2008a; Timmerman, 2002; Tirosh, 2000; van den Kieboom & Magiera, 2010). However, the authors did not share what the instructors garnered from engaging PTs in these tasks or anything about the MTE's practice other than the task he (or she) implemented in a teacher preparation course. Other MTEs, in addition to sharing specific tasks their PTs complete, share advice that other MTEs may draw on to enhance their own practice (Castro, 2006; Goodell, 2006). Furthermore, other MTEs articulate frameworks they used to guide their design of tasks used with PTs (Chval, Lannin, & Bowzer, 2008; Liljedahl, Chernoff, & Zazkis, 2007; Van Zoest & Stockero, 2008). I expound on these studies below.

For example, Castro (2006) designed specific activities to help her prospective elementary teachers learn to use curriculum materials. The results indicate that the two-course sequence provided "students with at least some conceptions of curriculum materials to enable them to use these materials in skillful ways" (Castro, 2006, p. 20). The author outlined a framework for use in PT education programs, which included four components: (a) exposure to curriculum materials, (b) help students develop a critical eye of curriculum, (c) task selection and modification, and (d) incorporating manipulatives to support student understanding. She posed this framework for MTEs to use in their teacher preparation programs to expose PTs to different curriculum materials.

Goodell (2006) prompted her secondary mathematics PTs to reflect on episodes from their field experiences in class. Goodell asked her students to write about a critical incident that happened in field. Over four years, 347 papers were submitted and Goodell examined them to identify what critical incidents the PTs encountered and what they learned from these experiences. Four major categories (teaching and classroom management, student factors, relationships and professionalism, and school policies and procedures) emerged from the PTs' papers. Through her self-study, Goodell learned how to foster her students' reflection so they learned from their reflection on critical incidents. In addition, she learned "that merely providing the opportunities for [her] students to reflect [did] not ensure that they [were] learning anything...It [was] important to monitor the class discussion, and keep them focused on mathematics teaching as much as possible" (p. 241). She concluded that the critical incident reflection assignment was effective in facilitating the "preservice teachers' learning about teaching mathematics for understanding" (p. 243).

Both Castro (2006) and Goodell (2006) described tasks that positively influenced PTs' learning. Additionally, they both shared what they learned from their studies (i.e., framework and advice) to support other MTEs. Van Zoest and Stockero (2008) shared a series of activities they use when instructing PTs . They collaborated to design a 7-step task sequence model for use in their middle-school mathematics methods course. Their goal was to "provide opportunities for preservice teachers to develop their abilities to pose good mathematical questions and tasks, design accurate and useful mathematical explanations, represent ideas carefully and translate among mathematical representations, interpret students' ideas, and respond productively to students' mathematical questions" (p. 48).

The design of the model indicated that each task explicitly integrate the knowledge gained from previous tasks. An original mathematical problem is posed in the methods course and then students engage in seven activities (seven steps in the model) centered on this one mathematical task. This model provided scaffolding that supported PTs' development and the MTE's goal of getting the PTs to maintain a focus on student thinking when faced with real students. In their work together, Van Zoest and Stockero (2008) have had the opportunity to focus and build on PTs' thinking and development in order to best support their students' learning. This collaboration has provided the opportunity for the MTEs to reflect on their own learning and to improve their own practice.

Similarly, Chval, Lannin, and Bowzer (2008) used a task design framework when planning tasks to engage their PTs. They acknowledged that they wanted their teacher preparation courses for elementary PTs to be research based, practice based, and problem based. As a result, they used a framework related to effective learning environments (Bransford, Brown, & Cocking, 2000) to guide the design of specific tasks for PTs. The four perspectives in their framework included: (a) community, (b) learner, (c) assessment, and (d) knowledge. The authors purposefully designed tasks that addressed these four components. They described one task they designed to address these four components while building on PTs' prior knowledge and helping the PTs make connections to their instructional practice as future teachers.

Liljedahl, Chernoff, and Zazkis (2007) introduced a usage-goal framework to guide task design and analysis for use in their mathematics teacher education program. These followed a four-phase recursive task design process (predictive analysis, trial, reflective analysis, and adjustment). Changes to the original task design were influenced by their reflections on PTs' engagement with the task in class. With each cycle of the task design process, the MTEs' knowledge of the task (both mathematical and pedagogical) became more refined. As a result, they tweaked the design of the task to better address their new insight into the affordances the task may have for future teachers who engage in the task.

In a theoretical piece, Peled (2007) stated that she designed tasks to address psychological, curricular, epistemological and/or pedagogical goals during implementation. She also noted that sometimes tasks were planned ahead of time to address the different goals, but other times an issue arose in class and a task was designed to address that specific issue.

MTEs who have written about task design highlight the importance of purposeful and thoughtful tasks for PTs and the helpfulness of frameworks to guide the design process. Peled (2007) noted that it was vital that MTEs were explicit with their PTs regarding the MTEs' purpose for engaging the PTs in various tasks. She stated, "the effect of our work as teacher educators will depend on our habit to use such tasks, on seizing the right opportunities to introduce them or construct new ones, and on making teachers aware of our own pedagogical reasons in introducing them" (p. 378).

In the following section, I describe studies that focus on instructional practices enacted in K-16 classrooms.

Teacher Instructional Practices

For more than a decade, researchers have reported results from studies that have identified various instructional practices[4] teachers employ in K-16 classrooms. Some of the identified instructional practices are specific to a certain context (e.g., different curricula, collaborative learning environments) while other practices are more general and geared towards helping teachers enhance and improve their overall teaching practice. In this section, I discuss examples of studies that involve K-12 teachers as well as university educators (including MTEs), based on self-report survey data and classroom observations. My intent is not to provide a comprehensive review of every study that relates to studying teachers' instructional practices, or to document all possible events of

[4] Some researchers use the term "instructional strategies" (House & Telese, 2008; Rieg & Wilson, 2009; Staples, 2007; Weiss, Pasley, Smith, Banilower, & Heck, 2003) while others use the term "instructional practices" (Drummond, 2002; Herbel-Eisenmann, Lubienski, & Id-Deen, 2006; Newton, 2009), although these terms were not specifically defined.

these studies, but rather to use the research literature as a basis to attend to differences between instructional practices reported at the K-12 and 13-16 levels.

K-12 instructional practices. In 2003, Horizon Research, Inc. issued the report, *Looking Inside the Classroom,* articulating factors that shaped instruction in K-12 mathematics and science classrooms. In 18 months, 31 researchers observed 364 mathematics and science lessons across 31 sites representing classrooms across the nation. Various components were analyzed within the observed lessons including factors that influenced teachers' selection of instructional practices. The authors reported that despite the fact that many factors examined in the study appeared to have some influence on instructional practices, the factors that most frequently influenced teachers' selection of instructional practices were: (a) teachers' background, knowledge, and experience and (b) teachers' beliefs (e.g., subject, effective pedagogy, students). In addition, the authors reported other factors that directly influenced instructional practice selection of teachers nationally including: (a) particular characteristics of students in their classes (about 50%); (b) professional development activities (about 31%); and (c) collaboration with colleagues (18%) (Weiss, et al., 2003).

Unlike Weiss et al. (2003), House and Telese (2008) did not personally visit classrooms, but instead analyzed data from the 2003 Trends in Mathematics and Science Study (TIMSS). Here, the authors investigated trends in mathematics achievement and classroom instructional practices that were significantly associated with algebra achievement among 13-year-old students (about 4200 from Japan and 7800 from the U.S.) from TIMSS, as self reported by the participants. Students completed a 14-question Likert scale questionnaire with respect to classroom instructional practices they engaged with in their mathematics class. Results from the study indicated several classroom instructional practices that were significantly associated with high algebra achievement as well as practices that corresponded to lower algebra achievement (see Table 4).

Table 4

Classroom Instructional Practices Related to Algebra Test Scores (House & Telese, 2008)

	Japan	*United States*
High scores	Students practiced mathematical operations without using a calculator	Students frequently wrote equations and functions to represent relationships
	Students decided on their own procedures when solving complex problems	Students frequently reviewed their homework
	Students explained their answers	Students used calculators
	Students worked problems on their own	Students worked problems on their own
Low scores	More frequent use of cooperative learning activities	More frequently worked on fractions and decimals
	More frequently related what they were learning in mathematics to their daily lives	Had a quiz or test during class
	More frequently used calculators	Engaged in cooperative learning activities
		Interpreted data in tables, charts or graphs
		Frequently listened to the teacher give a lecture-style presentation
		Related what they were learning in mathematics to their daily lives

Both Weiss et al. (2003) and House and Telese (2008) analyzed data sources and identified instructional practices used in mathematics (and science) classrooms as perceived by the teacher and student respectively. However, other researchers have reported results from smaller scale studies regarding instructional practices in mathematics classrooms. Herbel-Eisenmann, Lubienski, and Id-Deen (2006) as well as Staples (2007) investigated instructional practices of one mathematics teacher.

Herbel-Eisenmann et al. (2006) observed an eighth grade teacher for one week in both her Integrated I and traditional Algebra I course focusing on various instructional practices she employed. The purpose of this week-long classroom observation was to determine whether the instructional practices reported by the students via surveys were evident during instruction. Moreover, the research team investigated whether different instructional strategies were used in these courses. As a result of the student responses on the surveys, significant differences (in means) between the two courses were reported for: group work; lectures at the board or overhead most of the class; calculator use in class; and time spent working on a single mathematics problem. Meanwhile, significant differences between the two groups during the week of observation were: small group work; whole-class, teacher-led interactions; teacher lecture; and individual student use of graphing calculators. The researchers concluded that the teacher's instructional practices differed considerably between the two curricular contexts.

Staples (2007) reported on the instructional practices used by Ms. Nelson, an experienced teacher in a ninth grade pre-algebra class during one academic year. Staples identified three components to describe the role that Ms. Nelson played to support collaborative inquiry in her classroom. These components were: (a) supporting students in making contributions, (b) establishing and monitoring a common ground, and (c) guiding the mathematics. Staples then provided a detailed explanation of corresponding instructional practices associated with each component (see Table 5).

Table 5

Teacher's Role in Supporting a Collaborative Learning Environment and the Corresponding Instructional Practices

Supporting Students in Making Contributions	*Establishing and Monitoring a Common Ground*	*Guiding the Mathematics*
Eliciting Student Ideas Request and press Providing time Giving participation points	*Creating a Shared Context* Establishing prerequisite concepts Verbally marking Affording multiple opportunities to access ideas	*Guiding High-level Task Implementation* Modifying tasks Providing "food for thought" Ongoing assessing and diagnosing
Scaffolding the Production of Student Ideas Representing Providing structure Extending	*Maintaining Continuity over Time* Using the board over time Keeping the purpose salient Pursuing discrepancies Grounding, then building	*Guiding with a Map of Students' Algebra Learning* Attending to "pressure points"
Creating Contributions Expanding what counts Demonstrating the logic Linking	*Coordinating the Collective* Positioning students for collective work Controlling the flow	*Guiding by Following: "Going with the Kids"* Flexibly following a student's thinking Keeping students positioned as the thinkers and decision-makers

Note: Taken from page 173 in " Supporting whole-class collaborative inquiry in a secondary mathematics classroom," by M. Staples, 2007, *Cognition & Instruction, 25*(2/3).

This study "illuminate[d] the teacher's role, as well as detailed analyses of deliberate pedagogical actions that support collaborative work" (p. 213).

In this study, I create a classification of descriptive categories of actions one MTE enacted during instruction to develop PTs' PCK, by examining her practice in a mathematics content/methods course for elementary PTs. Thus, when the MTE employed practices reported on in this section (e.g., students explained their answers, students frequently wrote equations and functions to represent relationships, flexibly following a student's thinking, pursuing discrepancies), I did not attend to the actions the MTE enacted corresponding to implementing those practices in her classroom. Actions the MTE employed during the moments previously mentioned are not specific to teaching PTs as these are also actions that K-12 teachers employ in their classrooms. With this specific focus, I only considered actions specific to teaching PTs, which I expand on in the next section.

University educator instructional practices. Other researchers have reported results of instructional practices employed at the university level. For example, Drummond (2002), compiled a collection of 12 best practices based on his 20 years of college teaching (see Table 6). Many of the best practices articulated are supported by other researchers (Dewey, 1916; Marzano, Pickering, & Pollock, 2001).

Table 6

Drummond's (2002) Core Set of Best Practices in Teaching

Best practice	*Description*
Lecture practices	Effective ways to present new information orally to fit differences in learners
Discussion group triggers	Effective ways to present a common experience to engage a group in a discussion
Thoughtful questions	Formulate questions that foster engagement and confidence
Reflective responses to learner contributions	Establish mutually beneficial communication by reflective listening
Rewarding learner participation	Support learner actions with effective, well-timed positives
Active learning strategies	Foster active, constructive participation
Cooperative group assignments	Assign formal cooperative tasks
Goals to grades connections	Goals, objectives, measures, criteria and grades agree
Modeling	Represent openness, learning, and trust
Double loop feedback	Promote the awareness of how one learns to learn
Climate setting	Regulate the physical and mental climate
Fostering learner self-responsibility	Transfer responsibility for discovering, planning and evaluating much of their learning

Documenting teacher educators' perceived effectiveness and frequency of use of instructional practices, Rieg and Wilson (2009) surveyed 123 faculty and filtered out 55 teacher educators (e.g., elementary education, special education, mathematics, english, health and physical education) from two state universities in Pennsylvania to report on instructional and assessment practices teacher educators employed in their classrooms.

In this study, the participants reported that the three most effective instructional practices were: in class application and problem solving activities, pair or small group discussion, and brainstorming. Moreover, the teacher educators also ranked these three practices as the most frequently used in their classrooms. Five other instructional practices (i.e., role play, student debates, field trips, guest speakers, and videoconferencing) were also rated as effective by the participants, but they indicated they rarely used these practices (Rieg & Wilson, 2009). Rieg and Wilson (2009) note that despite the fact that teacher educators were aware of effective instructional practices, they did not always put them into practice in their courses and “students deserve to be taught in ways that actively engage them in the learning process, are student-centered, and evaluate their learning using a variety of measures” (p. 292-293).

Both Drummond (2002) and Rieg and Wilson (2009) report instructional practices that teacher educators perceived as effective; however they were not PCK specific. Additionally, they both list specific instructional practices employed in university courses, but not solely by MTEs or solely specific to teaching university students—as most of these practices K-12 teachers also incorporate in their classroom to provide the opportunity for student learning. However, Newton (2009) reports on instructional practices several MTEs employ in courses for elementary PTs.

As part of a larger study related to PTs’ motivation to work with fractions, Newton (2009) analyzed the time three MTEs spent on specific instructional practices while they taught a unit on fractions to elementary PTs in five sections of a mathematics course. All the instructors used the same teaching notes and curriculum to teach the mathematics course; however their levels of experience varied. Two instructors were novice MTEs (Instructor A previously taught the course one time and Instructor C did not have experience teaching the course) while Instructor B had taught the course for 11 years.

Despite common planning materials, predictable differences existed between the instructors. Instructor C was the most traditional in instruction style and used whole group instruction 76% of the time, to model or lecture PTs. Even though Instructor B used whole group discussion 67% of the time, during these discussions PTs posed ideas and the MTE encouraged PT participation. Instructor A frequently posed problems that PTs solved while working in small groups. Overall, strategies used by Instructors A and B aligned more with reform-based practices while Instructor C tended to be more traditional. As a result, the differences noted between their students (e.g., anxiety level) were not surprising. Specific instructional practices identified by Newton appear in Table 7.

Table 7

Instructional Practices Employed by the Mathematics Teacher Educators

Teacher activity related to understanding	*Teacher activity related to discourse*	*Teacher activity related to being teacher-directed*
Posing high level question or task	Request an alternative method	Modeling with technology
Elaborating on a high level question or task	Request an elaboration of a PT's response	Modeling without technology
Responding with a question back to the PTs		Lecturing mathematics content
Responding with evaluation or feedback		

Many of the researchers explicitly identified instructional practices that teachers and teacher educators were knowledgeable about and employed in their classrooms. Some of the practices included: transferring responsibility for learning to the student (in small group or discussion settings) (Drummond, 2002; House & Telese, 2008; Newton, 2009; Rieg & Wilson, 2009; Staples, 2007); modeling (e.g., mathematics)(Drummond, 2002;

Herbel-Eisenmann, et al., 2006; Newton, 2009; Staples, 2007); and engaging students as learners through questioning (Drummond, 2002; Newton, 2009; Staples, 2007). Additionally, I would classify all of the instructional practices mentioned in the publications as general pedagogical knowledge (i.e., practices a teacher or teacher educator of any subject discipline might employ); none of the instructional practices were specific to PCK (i.e., instructional practices a MTE would employ to develop PTs' PCK).

Furthermore, the instructional practices identified were not described in sufficient detail so that other teachers or teacher educators could implement them in any meaningful way. Only in one study (Staples, 2007) did the author include transcripts from class sessions including what a high school teacher said. However, the context of this study was analyzing instructional practices the high school teacher (and not a university instructor) employed to organize a collaborative learning environment in a mathematics class, which I classify as a general pedagogical instructional practice and not PCK instructional practice. MTEs need images of specific instructional practices (which I refer to as actions) to employ during instruction in order to provide the opportunity to promote the development of PTs' PCK. This is a gap in the literature on teacher instructional practices, as none of the authors articulate what university instructors say or do. Furthermore, reviewing this research made me sensitive to the amount of detail I needed to provide when describing images of specific actions one MTE employed during instruction in order to provide the opportunity to promote the development of PTs' PCK. In the study I report below, I focus on MTE actions employed to develop PTs' PCK rather than general pedagogical instructional practices, as described above.

Hiebert and Stigler (2004) note, "Most teachers learn to teach by growing up in a culture, watching their own teachers teach, and then adapting these methods for their own practice" (p. 13). MTEs must learn instructional practices that are specific to teaching PTs and differ from teaching K-12 mathematics students as researchers have noted that teaching K-12 students is different than teaching PTs (Ball, et al., 2009; Zeichner, 2005). It is essential that MTEs have opportunities to observe and learn from other MTEs so that they are equipped with vivid descriptive examples of instructional practices that specifically support the learning of PTs.

Summary

This research study investigated the actions one MTE employed to develop PTs' PCK in an elementary mathematics content/methods course. Thus, I focused my review of the research literature related to facilitating the development of PTs' PCK in mathematics teacher preparation courses, on MTEs as well as their practices, and teacher actions at the K-12 level so I could distinguish these actions from actions that were specific to teaching PTs.

Studies related to MTE practices aim to provide better learning opportunities for PTs learning to teach mathematics (e.g., Berk & Hiebert, 2009; Castro, 2006; Goodell, 2006; Heaton & Mickelson, 2002; Hiebert, et al., 2003; Steele, 2008b). Yet, research related specifically to MTE practices employed in the context of teaching PTs has been minimal. Overall, addressing actions enacted during instruction specific to educating PTs can be used to inform the design of future courses for PTs as well as contribute to building a knowledge base of mathematics teacher education.

Chapter III – Method

Design of the Study

As demonstrated in the review of the literature, there is considerable variation in course design, teaching practices, and content of courses taught for PTs at the elementary, middle school, and secondary levels. Ultimately, the field needs to develop a coherent system for preparing teachers of mathematics, but also MTEs. This will require a research agenda related to the specific practices of MTEs. To begin the conceptualization of this work, I conducted a focused, small-scale qualitative study to document actions one reflective MTE employed while teaching a mathematics content/methods course to prospective elementary teachers. The research questions were:

(1) What actions does a reflective mathematics teacher educator use to develop prospective teachers' pedagogical content knowledge in an elementary mathematics content/methods course?

(2) For what purposes does a reflective mathematics teacher educator use the identified actions?

After reviewing the literature on the development of PCK in teacher preparation courses, MTEs, and K-16 teacher instructional practices, I determined that a conceptual framework related to MTE actions enacted during instruction did not exist. As a result, I used the beginning stages of grounded theory in order to create descriptive categories and corresponding actions under the four components of PCK (i.e., knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, and knowledge of assessment) one MTE employed during instruction to develop elementary PTs' PCK.

Grounded theory is a method where theory is built from the data (Charmaz, 2005; Corbin & Strauss, 2008; Creswell, 2007; Glaser & Strauss, 1967). However, developing theory is a complex and often lengthy endeavor. Thus, my goal for the end product of this study was to contribute to the beginning basis for theorizing actions one MTE employed in her elementary mathematics content/methods course that were specific to teaching PTs. In other words, my aim was not to discover a theory from data, but to engage in developing a foundation that would support further theorizing by generating descriptive categories and corresponding actions, around the four components of PCK, from the data.

These categorical descriptions were purposefully created to describe actions one MTE employed, which I identified as specific to enhancing PTs' PCK. The completion of this description phase is a basic step to conceptual ordering which is a "type of analysis [that] is a precursor to theorizing" (Strauss & Corbin, 1998, p. 20). Extending this study, subsequent analysis could be completed in order to conceptualize categories, which would then contribute to the generation of theory. Subsequent verification of the resulting theory would be verified throughout the process of research. However, for this study, I identify and describe the actions one MTE used to develop PTs' PCK in an elementary mathematics content/methods course so that other MTEs may learn from and build on the results in order to strengthen their practice. Data collection and analysis occurred simultaneously, with each informing the other throughout the study.

In this chapter, I describe the theoretical framework, case study method, context of the study, data sources, and data collection and analysis for the study.

Theoretical Framework

I drew on four PCK components delineated by Magnusson, Krajcik, and Borko (1999), as described in Chapter I, as a lens for actions I attended to during instruction, one MTE used to develop PTs' PCK in a mathematics content/methods course. I drew on the framework, shown in Figure 4, to guide data collection and analysis as described below.

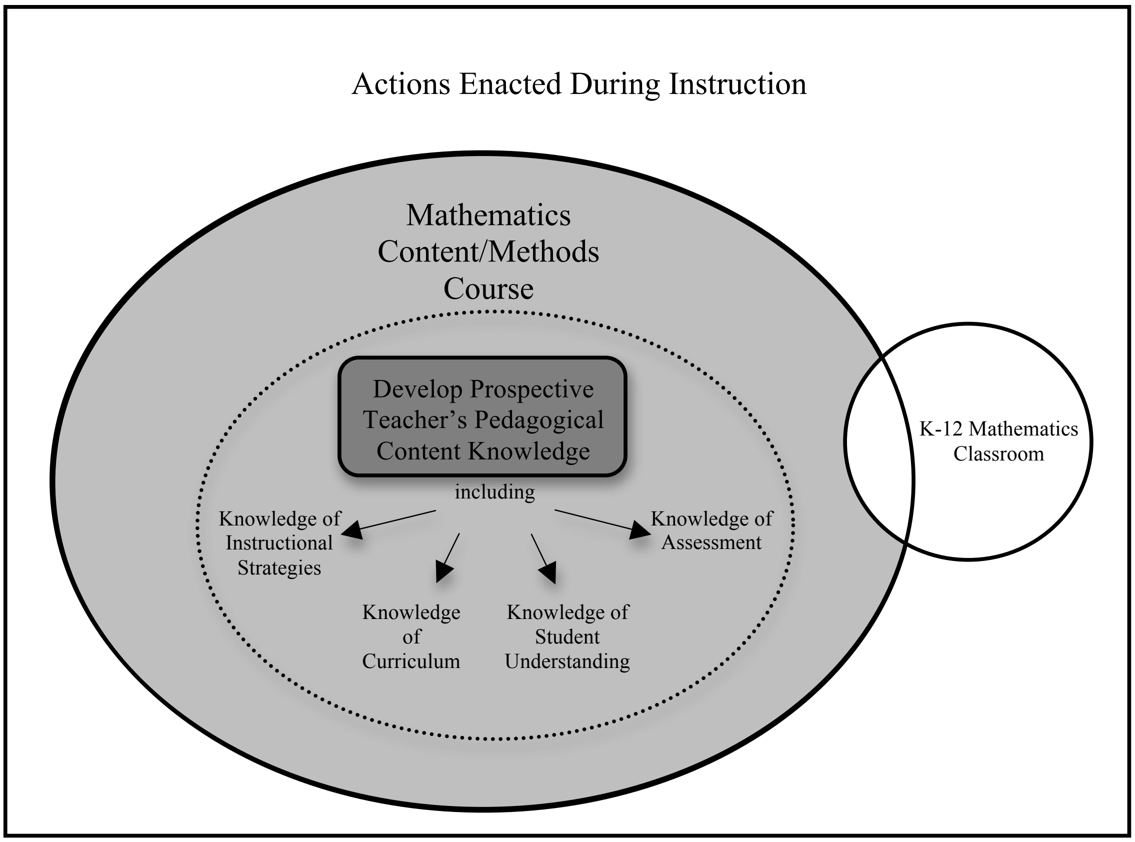


Figure 4. Model of actions enacted during instruction by a mathematics teacher educator that are focused on enhancing prospective teacher's pedagogical content knowledge.

In other words, I focused on the actions, specific to teaching PTs, a reflective MTE enacted during instruction in a mathematics content/methods course. I did not focus on particular tasks or activities the MTE used, but rather specific facilitation moves she used during whole-group instruction to develop PTs' PCK and engage them in learning. More specifically, I only considered MTE actions that were specific to the mathematics content/methods classroom and not the K-12 mathematics classroom. It is important to note that I chose to focus on one MTE's *actions* rather than *strategies* for a couple of reasons. First, the word, *strategies,* has multiple meanings. One of those meanings is "a plan, method, or series of maneuvers or stratagems for obtaining a specific goal or result" (dictionary.com). Based on this definition, a study investigating strategies would require an understanding of the MTE's plan prior to instruction which was not the case. Second, I did not use this term because *instructional strategies* is one of the four PCK components as previously defined in Chapter I. To gain a better understanding about what is taught in university mathematics content/methods courses to develop PTs' knowledge of four components of PCK (knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, and knowledge of assessment), this study sought to enhance our understanding regarding what one MTE said and did in her mathematics content/methods course that was specific to teaching PTs.

Case Study Method

A case study method was selected because the research questions were exploratory in nature, the control of behavioral events was not required, and the focus of the study was on contemporary events. This research method enabled me to try "to illuminate a decision or set of decisions," regarding actions one MTE used in her mathematics methods course, specific to teaching PTs (Yin, 2003, p. 12). Yin further articulates:

> The case study as a research strategy comprises an all-encompassing method – covering the logic of design, data collection techniques, and specific approaches to data analysis. In this sense, the case study is not either a data collection tactic or merely a design feature alone (Stoecker, 1991) but a comprehensive research strategy. (p. 14)

Further narrowing the method of design for this study, I utilized a single-case study in order to identify actions enacted during instruction one MTE employed in her mathematics content/methods course to develop PTs' PCK, as this is something that had not been previously researched and/or reported in the literature. However, there is growing attention regarding practices MTEs employ in their methods course and the need for further research in this area, as articulated earlier. Thus, this single-case was designed to be a *representative* case of actions enacted during instruction MTEs employ. The results from this case will inform the mathematics education research community about the actions one MTE employed in her classroom to develop PTs' PCK.

Additionally, the design of this study is an *instrumental case study*. I chose one MTE, looking broadly at how she teaches, but primarily attending to the actions she utilizes to develop PTs' PCK when teaching her mathematics content/methods course. Moreover, I utilized an *instrumental case study* design because I was interested, not in this specific single-case participant, but in understanding various actions specific to teaching PTs that MTEs use to develop PTs' PCK. In other words, this "particular case is examined mainly to provide insight into an issue…The case is of secondary interest, it plays a supportive role, and it facilitates [the] understanding of something else…the choice of case is made to advance understanding of that other interest" (Stake, 2005, p. 445). It is important to acknowledge that other cases of MTEs' actions enacted during instruction to enhance and develop PTs' PCK cannot be understood until we understand the complexities of the actions one MTE employs. Patton (2002) stated, "you can always build larger case units out of smaller ones," but if you only have program data, you cannot go back and construct individual cases (p. 447). As a result, at the conclusion of this study, other case studies and larger scale video studies can be conducted to identify actions other MTEs enact during instruction to develop PTs' PCK. For example, further investigations may involve MTEs from different types of institutions (e.g., community colleges, master's granting, etc.), geographic locations, grade bands (e.g., middle school, high school, etc.), and types of courses (mathematics, secondary methods, etc.). It is my hope that this single-case study will serve as the foundation for a long-term research agenda.

Context of the Study

Researcher self-reflection. My experiences as a young child influenced my desire to pursue a career in mathematics teacher education. As a young child I would spend time watching my father in his role as a college professor who taught mathematics and mathematics content/methods courses. Through these observations, I became interested in not only how to teach mathematics, but also how to teach future teachers of mathematics in effective ways. The pursuit of this goal was the driving force in the decisions, activities, and experiences I have chosen over the last 25 years to equip me with the knowledge, tools, and skills for success.

When I began my Ph.D. program at the University of Missouri, I looked for opportunities to teach prospective mathematics teachers, but also to research MTEs so that I could learn how to teach university courses more effectively. During this time, I interned in four different mathematics methods courses taught by different faculty members, participated in discussions about teaching these courses, taught one course for elementary PTs two times, and observed others teach these courses. Throughout this process, I became interested in researching the practices of MTEs. Moreover, I was surprised at the lack of resources and publications that were available to support my transition into the role of MTE.

My experience teaching high school mathematics for nearly 10 years, as well as my limited experience teaching PTs, influenced the design of this study. First, I was familiar with planning and preparing lessons, implementing them, and reflecting on the results. I recognized that enacted lessons often differ from the plan for a variety of reasons. Unexpected events (e.g., a question posed by a student or a student mathematical error) may change the direction of a lesson if the teacher decides to pursue the student's thinking. This experience and knowledge strengthened the study as it helped me recognize the complexity of selecting actions enacted during instruction, which were specific to teaching PTs, and their related purposes when I interviewed the MTE and analyzed the video and fieldnotes from the class.

Working with other teachers in high schools and MTEs in higher education as well as researching teachers in grades 6-12 through my graduate research assistantship, I also learned that teachers are often passionate about teaching and use teaching practices that differ from my own. Based on my observations of teachers' practices, I recognized that students respond positively to a variety of teaching practices and that teaching is complex. Thus, I have expanded my own knowledge about teaching through these experiences.

Although my specific experiences related to teaching and researching teaching strengthened my perspective for the study, they also created the possibility that I would bring my own biases about teaching PTs when collecting and interpreting data. Therefore, as I collected and analyzed data, I purposefully and intentionally drew on the literature related to facilitating the development of PTs' PCK in mathematics teacher preparation courses, MTEs, and teacher instructional practices. Additionally, I focused on identifying MTE actions enacted during instruction in a mathematics content/methods course that fell into the four PCK components delineated by Magnusson, Krajcik, and Borko (1999) to guide my interpretations rather than base my interpretations solely on my experiences.

Participant. "Some writers now advocate that researchers interview peers with whom they have already established relationships (Platt 1981; Segura 1989) and that researchers make use of everyday situations in which they are involved (Stanley and Wise 1983)" (Ellis & Berger, 2003, p. 471). In this study, I did just that. The participant of this study was someone whom I, the researcher, already had a working relationship with regarding a common course we had both taught. Below, I articulate our relationship.

Dr. Kathryn Chval is one of many MTEs I have had the opportunity to work with since studying to become a MTE. One of my first teaching internships—the only one at the elementary level—was with Kathryn, during the Spring 2008 semester where I observed her teach 22 class sessions. During each of the lessons, I took detailed fieldnotes on the activities the class engaged with each day as well as questions Kathryn posed and connections to elementary classrooms she described. With no experience teaching mathematics to students in grades 1-6 in U.S. schools, I was unaware about what students learned in elementary school, the academic capability of elementary students, the pre-requisite knowledge of students at various grade levels, major mathematical concepts taught at each grade level, etc. As a result, anytime Kathryn made reference to something related to teaching young children (e.g., typically in third grade, students are able to…), I noted this in my notes with the expectation that I would then have a bank of ideas, relevant to teaching elementary school, to draw from when I taught prospective elementary teachers.

Throughout the semester of my internship, I noticed Kathryn saying and doing different things that I did not consider using to teach PTs (e.g., asking PTs to analyze her teaching), which differed from how I taught secondary mathematics. This internship experience prompted my desire to learn more about how to teach PTs because I recognized I was a true novice and I did not know how to teach them (e.g., what to say and do with PTs in a mathematics content/methods course). I began to read teacher education literature—especially articles MTEs had written about their practice as I wanted to become not just a good MTE, but a great MTE. I would not have thought about other specific details and purposes, unique to teaching PTs, which Kathryn employed if I had not read research on teacher education (e.g., Loughran, 2006; Van Zoest, et al., 2006).

Attending to the details of what Kathryn said and did in the mathematics content/methods course began to intrigue me and foster a desire to attend more to the different facilitation actions she utilized to teach prospective elementary teachers. I was trying to figure out what it was that she did that was different from teaching K-12. I was also looking for a bank of ideas that I could use as a future MTE. Through reading teacher education literature, reflecting on my internship notes, and watching Kathryn teach, I generated ideas that seemed specific to teaching PTs. This teaching internship in an elementary mathematics content/methods course left a lasting impression that has shaped my research interest in mathematics teacher education.

During the same semester, I also attended weekly planning meetings with Kathryn and the other instructors of the course. At these meetings, I was able to ask questions about Kathryn's practices, purposes, and decisions. The following spring (Spring 2009), I was an instructor for one of the sections of this course. I also attended the weekly planning meetings with the other instructors. However, this time, my motivation for asking questions was elevated, as I was now responsible for teaching my own section. My interest in Kathryn's approaches grew.

During Spring 2010, I again taught my own section and attended the weekly instructor planning meetings, but also observed Kathryn teach her section for a second time. I decided to observe her teach again for several reasons. One reason was that I frequently heard her comment the previous year, "I did this different than originally planned and the students seemed to grasp the concept so much better than in previous years" or "I need to change this activity for next year." A second reason was that some lessons had been altered from two years ago due to "tweaking" to enhance the opportunity for students to learn. As a result, I wanted to observe the changes that Kathryn implemented with those altered lesson plans from when I observed her two years prior, with the hope of picking up new actions I could try with my students. Due to the teaching schedule, I was able to observe her teach one day, reflect on the lesson she taught, and then teach my section the following day. This time, my focus changed from connections she made to grade 1-6 classrooms to specific actions she used to instruct and engage PTs in the mathematical concept(s) of the day.

Over the past three years, the teaching internship with Kathryn has left a lasting impression on me and has continued to foster my desire to continue to learn how teaching PTs is different from teaching K-12 students. Throughout the semesters I taught elementary PTs, I would frequently go back to my internship notes to recall actions Kathryn enacted in class that were different from teaching K-12 students. Actions that were obvious to me I recorded in my notes, but if I were to go through and empirically analyze the data—what were other ideas that were not so obvious? This desire to learn more about specific actions Kathryn employed in her mathematics content/methods course during whole group enactment of instruction was the foundation of this study.

Over the course of meeting with Kathryn on a weekly basis for three semesters, I recognized that she was a MTE that was purposeful about what she said and did in her classroom. Moreover, she devoted a tremendous amount of time each week to reflect on how her instruction influenced her students' participation and performance. Kathryn's efforts have been recognized by her students who consistently rated her teaching performance at the highest levels. For example, during her first six years at the University of Missouri, she taught the elementary mathematics content/methods course seven times with an average course evaluation of 4.9 out of 5.0, far exceeding the department average for undergraduate courses. In addition, in January 2011, Kathryn received the *Early Career Award* from the AMTE, awarded to an individual within the first 10 years after receiving their doctoral degree, recognizing her contributions in teaching, service, and scholarship.

As demonstrated above, the selection of a single case was critical. Based on the review of the literature, selecting a MTE who was not reflective or purposeful about her teaching or used a rather limited number of practices would not be advantageous for the purposes of the study. Moreover, the selection of a novice MTE would be problematic. As a result, I selected a participant who was experienced, had a record of successful course evaluations, and had demonstrated a reflective and purposeful nature.

Addressing the ethical dilemma. It is important to note, that until this study, my relationship with Kathryn had been strictly in the form of a mentee seeking advice and knowledge on how to teach PTs from a more experienced and highly regarded MTE. As a result, the majority of our conversations had centered on my questions about and Kathryn's corresponding responses related to teaching elementary mathematics content/methods courses. However, for this study, our relationship was entering new territory as Kathryn directed this dissertation study. I chose Kathryn as my dissertation chair because she had the intellectual capital and experience to direct this work and was highly qualified due to her familiarity of teacher education literature. The selection of Kathryn as the dissertation chair was not a matter of convenience, but due to the fact that this work was unique to the literature based on MTEs and she was most knowledgeable of this literature base.

The design for this study was unique as I was "studying up" (Priyadharshini, 2003). In other words, I studied a more experienced MTE while I was just a novice MTE. Most of the research done in mathematics education is "studying down," where the researchers are more experienced than the individuals they research (e.g., a MTE studying PTs or a researcher studying classroom teachers or K-12 students). Based on this unique design, I had to proactively establish procedures to deal with issues (e.g., conflict) to best protect and address matters that might arise along the way. The following structures were put in place so that I would feel safe and comfortable if something would go wrong prior to the completion of the study as well as ensuring that data analysis was my interpretation/work and not that of my participant's interpretation. The structures put into place at the beginning of the study included:

- Kathryn did not see the interview protocols so she was not aware of the questions that would be asked of her. The other committee members reviewed the protocols and provided advice about the content of them.
- Pre-planned meetings—Prior to beginning data collection, weekly meetings were to be scheduled (as well as make-up dates) between Kathryn and myself throughout the anticipated data collection period. By scheduling regular meetings prior to data collection, it was anticipated that this would address the potential that if for some reason a pre-planned meeting was canceled (e.g., sickness, an emergency came up—I can't meet today), make-up dates would already be in place so that new meeting times were not negotiated because of an unexpected cancelation of the regularly scheduled meeting.
- Using the members of the committee periodically throughout the study in the following ways: (a) When questions arose regarding issues with data analysis or conceptualization of the data, I would pose my questions to members of the committee instead of Kathryn. By using my committee members in this way, I anticipated that data collection would not be tainted due to any discussions Kathryn and I may have had regarding further data collection or data analysis; (b) If the situation arose where Kathryn and I did not agree on whether an action enacted during instruction was specific to teaching PTs, all of the

> committee members would decide whether the action was indeed specific to teaching PTs. If the committee could not reach consensus, then it was agreed that both perspectives would be put in the dissertation for the reader to decide whether the action enacted was specific to teaching PTs; (c) If there was disagreement in how the MTE's actions were described between Kathryn and myself, two other committee members were identified at the onset to act as mediators of those negotiations; and (d) If Kathryn struggled with the intellectual analysis of her teaching practice and she said that she could not direct the work anymore, then a committee member had been designated to step in and direct the study.

The structures described above were put in place to protect the study design so that the trustworthiness of the data would not be questioned. In addition, the following structures were also employed throughout the study.

Participant only. During the data collection phase, Kathryn was solely the participant and I did not go to her regarding any questions I had about tweaks for future interviews or issues regarding data analysis. I met weekly with one of my committee members to discuss data collection and analysis progress. When questions did arise regarding issues with data analysis or conceptualization of the data, I posed my questions to two different members of the committee instead of Kathryn. By using my committee members in this way, data collection was not tainted due to any discussions Kathryn and I might have had regarding further data collection or data analysis.

The role of the committee members. I used the members of my committee periodically throughout the study in other ways as well. Occasionally throughout the data analysis phase, I touched base with a couple of my committee members with questions regarding how I conceptualized the data (e.g., organization, wording of various actions to include all snapshots I tagged under a certain action) as well as various data analysis issues (e.g., actions I thought might be pedagogical in nature and not PCK). These committee members were available to discuss my ideas and challenge my thinking.

By using my committee members in these ways and assigning the role of *participant only* to Kathryn during the data collection phase, Kathryn did not have any knowledge of how I was conceptualizing the data during data collection. If she had been aware of how my analysis was progressing, when asked a question during an interview, she may have thought, "I need to answer the question in this way because Cynthia needs that piece of information to address ____." However, this predicament never transpired, keeping the collected data untainted from participant inside knowledge of ongoing data analysis.

The design of this study did pose some potential ethical dilemmas because of the existing relationship I had with the research participant and the authority that she had been granted as my work supervisor, intern supervisor, and dissertation chair. As a result, the committee and I tried to identify, ahead of time, what those dilemmas could be and how they would be handled if they were to occur as previously articulated. However, by Kathryn assuming the role as participant during data collection, this eliminated many of the potential issues that had been identified as potentially problematic prior to the start of data collection. No other issue arose during the study.

The methods course. The mathematics content/methods course Kathryn instructs is the second of a two-course sequence, where the focus is on the content and complexities of teaching geometry, measurement, probability, and statistics in elementary schools, for prospective elementary teachers in the Teacher Development Program in the College of Education at the University of Missouri. The design of the two-course sequence is built on current research that centers on student learning, standards-based curriculum materials, and the study of effective classroom interactions and teaching. Kathryn's syllabus states:

> The course has been designed around the principles and standards outlined in the MU College of Education Conceptual Framework Design Model, the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000), and the Missouri Standards for Teacher Education Programs (MoSTEP). Course activities will focus on mathematics, equity, curriculum, teaching, learning, assessment, technology, and participation in a professional community.

Course participants engage in mathematical tasks related to the content of the course and examine instructional materials, tools, and practices designed to support a classroom community intent on developing deep conceptual and procedural understanding of mathematics.

Data Sources

To investigate Kathryn's actions and purposes, I used multiple data sources. Individual sources of data consisted of videotaped lessons from the elementary mathematics content/methods course Kathryn taught Spring 2007 as well as fieldnotes I took when observing the same course in Spring 2008 and Spring 2010. Snapshots of Kathryn's practice were captured and analyzed from these three data sources. In the videotapes, a snapshot entailed: (a) a speaking turn in the video from Kathryn, (b) dialogue between the PTs and Kathryn, or (c) a phrase articulated by Kathryn in a lengthy speaking turn. The snapshots I captured in the video ranged in length from 10 seconds to a couple of minutes. Individual snapshots I captured in the fieldnotes included a picture of: (a) material I documented which Kathryn wrote or drew on the board or (b) verbatim (or nearly so) quotes I documented Kathryn speaking that were specific to instructing PTs.

Additional sources of data included an initial interview with Kathryn, interviews where Kathryn provided commentary on several extended video segments from Spring 2007, as well as video/fieldnote based interviews. I elaborate on the nature of each of these data sources below. The alignment of the data sources with the research questions is highlighted in Table 8.

Table 8.

Data Sources and Identifiers Used in the Study Aligned With the Research Questions

Data source description	*Data source identifier*	*RQ1*	*RQ2*
Class videorecordings Spring 2007	Video Day#	X	
Fieldnotes Spring 2008	FN2008 Day#	X	
Fieldnotes Spring 2010	FN2010 Day#	X	
Initial MTE interview	InitialInterview		X
Extended video talk interviews	EVT#		X
Video/fieldnote based interviews	VDFNInt#		X

Videotapes of mathematics teacher educator teaching course. During the Spring 2007 semester, a graduate student videotaped Kathryn teach 21 class sessions of the elementary mathematics content/methods course totaling 26+ hours. This taping included whole group discussions and small group discussions with the camera following the MTE for the duration of the class period. Kathryn wore an audio recorder around her neck in order to capture her discourse with individuals and groups.

Fieldnotes. During Spring 2008 and Spring 2010, I observed Kathryn teach the elementary mathematics content/methods course for PTs, 22 and 24 lessons respectively. During each of the lessons, I took detailed fieldnotes related to Kathryn's questions and practices. While most of the fieldnotes were descriptive, some were reflective. These notes included ideas, impressions, and speculation about what happened during instruction related to the actions Kathryn enacted. There were a couple of purposes related to identified actions documented in the fieldnotes. These notes served as a data source documenting Kathryn's enacted instruction over two separate years, providing an evolving picture of her teaching practice than just the videotapes could on their own. Additionally, the fieldnotes provided initial hunches I garnered, during classroom observations, regarding actions enacted during instruction that were specific to teaching PTs, which Kathryn employed over many classes.

It is important to note that all of the classroom based data (i.e., videotapes and fieldnotes) were collected prior to the design of the study. Thus, the focus of this study did not influence what Kathryn said and did during her instruction.

Individual interviews with the mathematics teacher educator. I conducted semi-structured interviews, where an outline of questions prepared ahead of time to be explored (Kvale & Brinkmann, 2009; Merriam, 1988), to help identify Kathryn's actions and purposes. The interview questions were designed using the research questions and the theoretical framing of the study as a guide.

Initial interview. One semi-structured interview, approximately one-hour in length, (see Appendix A) was designed to uncover background information on Kathryn regarding her journey to becoming a MTE, why she became interested in teaching PTs, as well as what actions she employed during instruction that were specific to teaching PTs (e.g., K-12 teaching issues, analyzing tasks, engaging PTs to think like teachers, etc.). Before conducting the initial interview, I drafted the interview protocol and then piloted it with three different MTEs. The aim of this pilot was to determine the appropriateness of the interview questions as well as to refine the questions. I also sought to assess the structure and clarity of the interview questions. This process led to some amendments in the interview protocol.

I structured this interview to garner ideas of potential initial codes for actions enacted during instruction Kathryn used to teach elementary PTs. From questions posed to Kathryn (e.g., regarding how her actions address her course goals, actions specific to teaching PTs which she learned from colleagues and/or professional development experiences) and her ensuing responses, I identified several purposes that Kathryn had regarding actions she employed specific to teaching PTs.

In a recent publication I elaborated on in Chapter II, Chval and colleagues (Chval, Lannin, & Bowzer, 2008) describe a framework they use to guide the design of their tasks. This framework involves four perspectives of effective learning environments (i.e., community, learner, assessment, and knowledge). Additional questions in the semi-structured interview centered on practices Kathryn used in her mathematics content/methods course to attend to the four different components of the framework.

Kathryn's responses from this interview helped me identify snapshots of her practice from video clips and fieldnotes to use in future interviews described below.

Extended video talk segment interviews. Three different video clips, approximately 20 minutes in length, were selected for these three interviews (see Appendix B) I refer to as the "Director's Commentary." I chose clips spaced out over the semester long course which provided Kathryn the opportunity to discuss actions that were specific to the different mathematical content she taught as well as different actions she employed at different periods during the semester (e.g., the beginning of the semester, a day several weeks into the semester after she and the class had the opportunity to get to know each other). Similar to how a director of a movie provides commentary about scenes as he watches the movie, Kathryn paused the video periodically throughout the interview and talked about what she viewed in the video as well as her purpose for employing the actions she discussed. At the end of each interview, I asked several clarifying questions regarding her commentary on the 20-minute video clip.

One purpose for incorporating this data source was for Kathryn to talk about her purpose(s) and/or what she was hoping to address in class about learning to teach. A second purpose was for Kathryn to think out loud about her practice so that she could share her purpose(s) for different actions enacted during instruction she incorporated into three different lessons. Being a novice MTE, I recognized that I might not have had the experience to recognize actions that Kathryn employed. However, this method of data collection enabled the more experienced MTE to talk uninterrupted and potentially raise actions enacted during instruction that I did not consider as enhancing PTs' PCK nor specific to teaching PTs. This data collection method provided important information to aid me in developing a descriptive framework focused on Kathryn's actions.

Video/fieldnote based interviews. Snapshots from video clips and fieldnotes were the content for these two interviews (see Appendix C for the semi-structured interview protocol). I preselected snapshots from video clips and fieldnotes where I identified actions Kathryn used to develop PTs' PCK. During the interview, I summarized the actions I identified from fieldnotes and played video clips of identified actions to Kathryn. I then asked her to talk about her purpose(s) for employing the identified actions. Prior to this interview, I had purposes for several actions, obtained from the initial interview and/or the extended video talk interviews, but there were other actions in which I had not identified corresponding purposes. As a result, I used these two interviews to gather Kathryn's purpose(s) for these specific actions.

These data sources (archival records, interviews, and direct observation) are three of the six most commonly used in conducting case studies as articulated by Yin (2003). In addition to individual sources of evidence for the case study, I attended to three principles Yin (2003) articulates should be incorporated into a case study investigation in order to substantially increase the quality of the study. These three principles

> Include the use of (a) multiples sources of evidence (evidence from two or more sources, but converging on the same set of facts or findings), (b) a case study database (a formal assembly of evidence distinct from the final case study report), and (c) a chain of evidence (explicit links between the questions asked, the data collected, and the conclusions drawn). (p. 83)

Analysis included the triangulation of multiple sources of data (i.e., videotapes, fieldnotes, and interviews with the MTE) in order to report credible findings.

Data Collection and Analysis

Data analysis occurred throughout the study as "analysis begins with the first … document [addressed]. Emerging insights…direct the next phase of data collection, which leads to refinement or reformulation of one's questions, and so on. It is an interactive process throughout … producing believable and trustworthy findings" (Merriam, 1988, pp. 119-120). Throughout the study, I analyzed the snapshots of practice I captured and the corresponding categories I assigned to the snapshots. Frequently, the themes of the snapshots I identified changed categories as I progressed in data collection. The actions I created from the snapshots of practice I initially identified, in turn helped shape subsequent interviews to garner Kathryn's purpose(s) for the actions I identified from the snapshots of her practice specific to teaching PTs. I transcribed all of the interviews, and analyzed the data via triangulation of multiple data sources and data collection methods in order to create a classification of descriptive categories of actions and claim credible findings.

As mentioned in Chapter I, to guide my analysis, I focused on actions, I conceptualized, Kathryn employing in the mathematics content/methods course, which she used to develop PTs' PCK (see the shaded portion [section A] in Figure 5). I defined actions as what the MTE said and/or did (i.e., wrote on the board) during the enactment of the lesson during whole-group discussion in her mathematics content/methods course. This included the way Kathryn engaged PTs in a discussion about mathematical misconceptions K-12 students make or a discussion of the scope and sequence of K-12 mathematics. This did not include lesson planning, discussion of homework problems, or solving mathematics problems as a class.

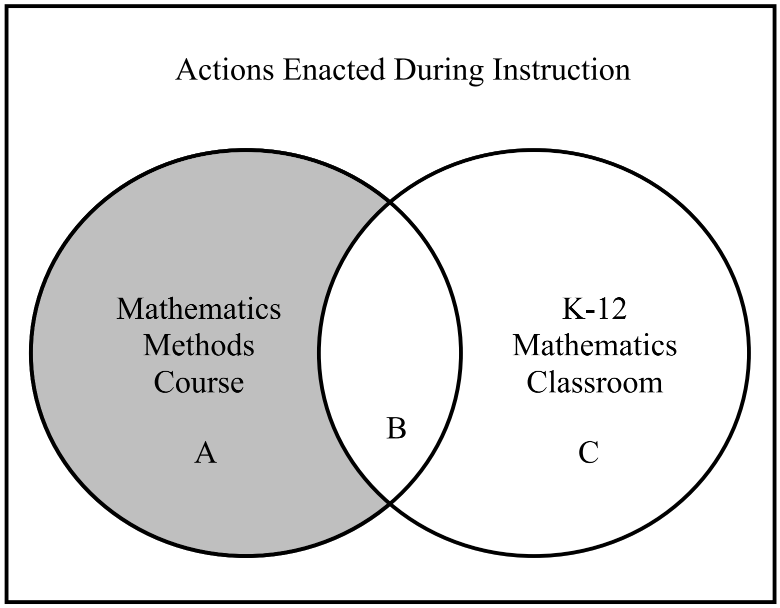


Figure 5. Actions enacted during instruction.

I also considered actions enacted during instruction that were common when instructing both PTs and K-12 students (e.g., questioning). See section B in Figure 5. When an action fell in this category (e.g., a question posed during instruction), I asked myself whether the question would be asked solely in a mathematics content/methods course (i.e., specific to teaching PTs). For example, questions such as: a) What mathematical ideas do you want to highlight regarding the student work posted on the board? or b) What order do you want to discuss the student work that is on the board? were questions that would only be asked in a mathematics content/methods course and thus fell in section A of Figure 5. However, a question such as “How did you solve the mathematics problem?” was ignored since this question would also be asked in a K-12 classroom.

I describe the data analysis process in eight phases below.

Phase 1. In the first phase of data analysis, I pared 26+ hours of video from Spring 2007 down to just under 15 hours of video. During this phase, I eliminated all video that contained: (a) one-to-one MTE interaction with individuals or small groups, (b) small group interaction, and (c) students working individually on tasks. In general, I tagged segments where Kathryn addressed the entire class or there was MTE/PT interaction during whole-group discussion. In the segments that I identified as potential places to identify MTE actions specific to teaching PTs, I kept in mind the definition I had created of *actions*, which was "what the MTE says and does during a specific instance of instruction during the enactment of the lesson during whole-group discussion in a mathematics content/methods course that is specific to teaching PTs." I identified the time from the video and wrote a detailed description of the actions Kathryn employed, organizing them chronologically in a table by video.

Phase 2. In the second phase of data analysis, I captured screen shot images of actions Kathryn employed specific to teaching PTs from Spring 2008 and Spring 2010 fieldnotes. Additionally, I identified snapshots from the video segments I tagged in Phase 1. Throughout this phase, I categorized (and re-categorized) the snapshots I tagged from the fieldnotes and videos into one of the four PCK components (i.e., knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, knowledge of assessment). I assigned each snapshot to one PCK component (i.e., there was no double coding—assigning a snapshot to two or more different PCK components). I used the HyperResearch qualitative data analysis software program (ResearchWare, 2007) to code the video and fieldnote snapshots of practice under the four PCK components.

During this phase, I coded the data in order to lay a foundation for categorizing identified actions. Several examples of actions and their respective corresponding codes are listed in Table 9. This table provides one example of each PCK component from Kathryn's practice, from the videotapes and fieldnotes.

Table 9

Potential Codes From the Data

Data source	*Description of instance/ transcript of instance*	*Potential code*
Video Day2	"One mistake I just made—I modeled with manipulatives in a way no one could see. What should I do instead?"	Knowledge of instructional strategies
Video Day11	"Now we will look at another piece of the textbook. The unit that you just looked at was the first week of fifth grade. The lesson was the first week of fifth grade."	Knowledge of curriculum
FN2010 Day18	A common misconception with kids is that they think the slant height and the perpendicular height of a slanted parallelogram is the same.	Knowledge of student understanding
FN2008 Day4	Student work, two-digit plus one-digit addition, (Carol) is displayed on the board. Students are asked to solve a new problem posed by Kathryn (26 + 15). Kathryn asks PTs what Carol would do if we gave her the two-digit plus two-digit addition problem. Kathryn says, "we really don't know – we don't have evidence of this (Carol completing a two-digit plus two-digit addition problem). Be careful you don't over-extend."	Knowledge of assessment

As I conducted my analysis in Phase 1 and Phase 2, I kept detailed memos (Corbin & Strauss, 2008) articulating ideas for the evolving coding dictionary. I used memos to capture initial thoughts, questions, and ideas for subsequent categories for actions enacted during instruction specific to teaching PTs. As I formed categories from the data, I referred back to my memos as a way to reflect on the actions I identified. I also kept a reflective journal documenting my inexperience as a MTE as well as what I learned about teaching PTs through collecting and analyzing the data. Documenting my thoughts helped me to systematically separate data from my experiences teaching the same course. This process also helped me determine questions to ask Kathryn during subsequent interviews to address my research questions. Here also, I documented and drew on my own life experiences that were closely related to Kathryn's, as I could not completely separate myself from this study given our relationship and my prior teaching experiences.

Phase 3. Coding the initial interview transcript, I analyzed and identified several purposes that Kathryn had regarding snapshots I identified in Phases 1 and 2. I sorted the identified purposes into one of the four PCK components. I began to identify specific actions Kathryn employed, and corresponding purposes, from the snapshots of practice under each of the four PCK components. Themes of actions began to emerge and I wrote descriptions of those themes for the various identified actions. Through several iterations of sorting the snapshots under each of the four PCK components, I created a coding dictionary shown in Appendix D to define each action and illustrate an example of each action with a sample from the data.

Phase 4. After coding all of the video snapshots of practice tagged in Phase 1, I tallied the codes for each PCK component (e.g., knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, knowledge of assessment) for the 21 videos. From the results, I split the 21 videos into thirds (video 1-7, video 8-14, and video 15-21) and chose three rich 20-minute segments (a segment that had multiple codes around at least three PCK components) for Kathryn to talk about—

one segment from each third. The three clips I chose were initially tagged as clips of interest during Phase 1 for the extended video segment interviews and were also part of Kathryn's practice for the three years I had data. The first extended video segment involved a whole-class discussion on finding patterns in a multiplication table as well as patterns that grade 1-6 students would find in the same table (17-minutes in length) and was heavily coded for knowledge of instructional strategies. The second clip was a whole-class discussion about analyzing student work (22-minutes in length) and was heavily coded on knowledge of curriculum. The third clip focused on how to measure a line in inches using a ruler (23-minutes in length), which was heavily coded for knowledge of student understanding.

I was purposeful when I selected the extended video segments. I did not want to select a clip that only took place in 2007. I assumed that if Kathryn engaged the PTs in a specific discussion every year, then she would have a specific purpose for her actions.

I analyzed the extended video talk segment transcripts for corresponding purposes to actions I identified (see Appendix D). Many of the actions I identified, Kathryn articulated purposes for in the extended video talk interviews without being prompted. However, at the end of this phase, there were several actions in each of the four components of PCK in which I did not have corresponding purposes. Analysis from these extended video clip talks helped me answer my second research question as well as modify and solidify actions and corresponding purposes for those identified actions that I created and modified in Phase 3.

Phase 5. During Phase 5, I conducted two video/fieldnote based semi-structured interviews with Kathryn to identify purposes for actions I had not yet identified. An outline of questions focused on specific actions were prepared ahead of time and guided the interview (Kvale & Brinkmann, 2009; Merriam, 1988). During the first interview, I described actions and showed several short video clips of Kathryn's practice around enhancing PTs' knowledge of instructional strategies and student understanding so that she could articulate her purpose for the actions I identified. For the second interview, I focused on actions around enhancing PTs' knowledge of curriculum and assessment.

At the conclusion of this phase, I had a purpose, articulated by Kathryn, for every action I identified her employing to develop PTs' PCK in an elementary mathematics content/methods course.

Phase 6. I further refined the categories and actions under each of the four PCK components by "working back and forth between the themes and the database [the original videotapes and fieldnotes along with the transcripts from the interviews with Kathryn] until they establish[ed] a comprehensive set of themes" (Creswell, 2007, pp. 38-39). Cresswell (2007) referred to this process as inductive data analysis. During this phase, I went back through all of the snapshots of practice, verifying that they were coded appropriately (and multi-coded where applicable). In other words, I reviewed all snapshots originally coded and verified the assigned code I applied using HyperResearch. Three different doctoral students coded two days of fieldnotes as well as snapshots of Kathryn's practice I randomly selected. I selected 64 actions (20% of the total snapshots identified from each of the four PCK components) using a random number generator. I then verified that all 34 actions were represented by at least one of the 64 snapshots of data that I randomly selected. I also randomly selected one day of field notes from 2008 and 2010. Discrepancies were resolved through adjustment of the description of actions.

Phase 7. Codes and the descriptive framework of Kathryn's actions I identified continued to be refined, in the course of multiple passes through the data. Collaboration with Kathryn took place in order to solidify descriptions of actions as well as the categories identified under each of the four PCK components.

Phase 8. After writing a description of all actions and articulating Kathryn's purpose(s) for employing those actions, I collated all of the identified instances where Kathryn articulated her purpose for various actions. I sorted the instances to identify themes of purposes Kathryn had for why she employed the identified actions in her elementary mathematics content/methods course. Through several iterations of sorting articulated instances of purposes, I identified core purposes Kathryn stated; see Appendix E for a subset of Kathryn's core purposes with an example of each purpose from the data, per relevant PCK component. Reliability was confirmed with two doctoral students (via selecting 20% of the statements of purposes I identified Kathryn articulating during individual interviews using a random number generator). All discrepancies were resolved and clarified.

Once I completed the analysis phase, the end product was a case study of Kathryn's actions, and corresponding purposes, enacted during instruction specific to teaching an elementary mathematics content/methods course that were used to develop PTs' PCK. In the subsequent chapter, I provide a "thick description" (Patton, 2002) of actions and corresponding purposes one successful MTE employed in her elementary mathematics methods course, which are specific to teaching PTs.

> A thick description does more than record what a person is doing. It goes beyond mere fact and surface appearances. It presents detail, context, emotion, and the webs of social relationships that join persons to one

> another. Thick description evokes emotionality and self-feelings. It inserts history into experience. It establishes the significance of an experience, or the sequence of events, for the person or persons in question. In thick description, the voices, feelings, actions, and meanings of interacting people are heard. (Denzin, 1989, p. 83, as cited in Patton, 2002, p. 503)

Summary

Research on MTEs has been largely conducted by MTEs themselves researching their own professional growth, personal knowledge needed in teacher education, and various aspects of the courses they teach in order to ultimately improve their own practice as teacher educators and enhance the opportunity for their students to learn. However, we know little about MTEs' practices inside their own classroom. A single-case study method allowed for insight into the instructional practices (i.e., actions) one MTE utilized in her mathematics content/methods course specific to teaching PTs. The analysis of videotapes, fieldnotes, and interviews with the MTE allowed for innovative work and the creation of a classification of descriptive categories of instructional practices one MTE employed. In the next chapter, I present my findings

Chapter IV – Findings and Discussion

In this chapter I present the findings from this qualitative study of one MTE. I analyzed video recordings of class sessions; fieldnotes of mathematics content/methods class sessions; as well as transcripts of an initial MTE interview, extended video talk interviews, and video/fieldnote based interviews to answer the following research questions:

(1) What actions does a reflective mathematics teacher educator use to develop prospective teachers' pedagogical content knowledge in an elementary mathematics content/methods course?

(2) For what purposes does a reflective mathematics teacher educator use the identified actions?

I identified a total of 384 snapshots from Kathryn's practice during the data analysis phase. Snapshots from Kathryn's practice that occurred in multiple years (e.g., Kathryn provided directions on how to make a homemade spinner in 2007, 2008, and 2010) were coded for only one of the years the snapshot occurred (i.e., 2007 and *not* 2008 nor 2010). Thus, this example represented 1 out of 384 snapshots. Additionally, if Kathryn started an activity (e.g., asked PTs to sequence a set of tasks according to grade level) during one class session and then continued the conversation the following class session, I only coded this situation as one snapshot.

I identified snapshots related to all four PCK components, yet the percentage of snapshots for each component varied (i.e., instructional strategies, 40.6%; curriculum, 29.9%; student understanding, 19.0%; and assessment, 10.4%). I organize the remainder of the chapter into three main sections. First, I present findings (i.e., identified actions and their corresponding purposes) related to each of the four main components associated with the PCK framework introduced in Chapter I (i.e., instructional strategies, curriculum, student understanding, and assessment). Next, I provide a synthesis of purposes Kathryn articulated corresponding to the identified actions. Finally, I conclude with a discussion of the findings.

Enhancing Prospective Teachers' Knowledge of Instructional Strategies

Based on three years of data, Kathryn most frequently attended to enhancing PTs' PCK of instructional strategies. Based on the analysis of these snapshots of practice, I identified five major categories in relation to what Kathryn said or did to develop PTs' PCK of instructional strategies. The categories include: (a) mathematical activities; (b) instructional tools; (c) teaching mathematics; (d) organizing mathematics lessons; and (e) emphasizing language. Table 10 summarizes the actions for each of these five major categories. Then I elaborate with more specific descriptions of these five categories. Additionally, I articulate Kathryn's purposes for employing the identified actions in her mathematics content/methods course.

Table 10

Summary of Actions Kathryn Employs to Develop Elementary Prospective Teachers' (PTs') Pedagogical Content Knowledge of Instructional Strategies

Categories of instructional strategies	*Actions*
Mathematical activities	Engages PTs in mathematical activities they may use in their future classrooms
	Describes or mentions activities PTs may use with grade 1-6 students
Instructional tools	Explains how to make homemade manipulatives for use in future classrooms or use common items in place of "catalog manipulatives"
	Shares tip(s) about teaching with instructional tools (e.g., manipulatives, calculators, rulers)
Teaching mathematics	Advises PTs on what to physically give to children when teaching various mathematical concepts
	Advises PTs on what to say, write, and/or have students do when teaching various mathematical concepts
Organizing mathematics lessons	Selects and sequences student work for a whole class discussion
	Articulates typical structure of a whole lesson
	Discusses ideas to attend to, consider, and/or draw from when designing mathematics lessons
Emphasizing language	Prompts PTs to articulate directions to mathematical tasks so students in grades 1-6 will have access to the task
	Explains the importance of connecting verbal and written language with visual aids
	Encourages PTs to be thoughtful about the language they use (e.g., when asking questions or writing mathematical tasks)

Mathematical activities. Kathryn introduces mathematical activities in two primary ways: (a) she *engages PTs in completing mathematical activities,* and (b) she *describes or mentions mathematical activities*. In both of these situations, Kathryn introduces activities that PTs may use with students in grades 1-6 and activities they will perceive as relevant to their future teaching practice. In the following paragraphs, I illustrate these two types of actions.

Kathryn introduces specific mathematical activities and tasks for PTs to complete that potentially would be appropriate for grade 1-6 students. For example, she asked PTs to use a piece of wire that is 24 cm long to create a rectangle. After comparing different rectangles the PTs made with their wires, she asked the PTs to determine the area and perimeter of different rectangles built with the wires that are 24 cm long [FN2010 Day17]. In another example, Kathryn engaged the entire class in playing an enhanced version of Product Bingo, a game included in the Grade 4 Math Trailblazers curriculum [Video Day1]. In both of these examples, all PTs engaged in an activity that they could potentially use to teach mathematical concepts (e.g., area and perimeter, multiplication facts, or probability) in the elementary grades. Kathryn used these activities because PTs view them as relevant to their future practice. Furthermore, she not only used them to teach PTs about teaching mathematics, but also to teach them the mathematics that the PTs need to learn. In other words, she had them do the activities to learn the mathematics [InitialInterview; VDFNInt1].

As demonstrated above, in some cases Kathryn introduces mathematical activities and expects her students to complete them, which takes a significant amount of class time. However, in other situations, she invests less time and *describes or mentions mathematical activities PTs may use with grade 1-6 students* rather than asking the PTs in her course to complete them. When engaging in this action, she draws from her experiences as a student teacher, classroom teacher, supervisor of PTs in the field, professional development facilitator, MTE, and researcher. For example, on one occasion, Kathryn described a mathematical game she used while she was a student teacher in a fifth grade classroom. She described,

> So when I was teaching, I was actually student teaching, I was being evaluated. I did this Battleship game on the board for graphing. So kids have trouble when they have to graph on a two coordinate plane where you go over first and then up. The Battleship game really helped them. I also did another game where it is Connect Four. Those were the two games where they really got the idea of going over first and then up. [Video Day3]

Here, Kathryn briefly introduced the idea of using games such as Battleship and Connect Four to attend to the mathematical concept of graphing on a coordinate plane. She mentioned these in passing and did not go into great detail about this activity and did not describe how she implemented this activity in her class. She explained, "Some of these things I'm describing are ways that you could teach it and so I'm not investing a huge amount of time [articulating details of the activity]" [VDFNInt1].

At other times though, Kathryn provided more elaborate descriptions of mathematical activities PTs could eventually use. In this next example, Kathryn articulated how PTs could help students build meaning for the terms, "certain" and "impossible," when they taught probability in their future classrooms. She did this by sharing a snapshot of what she liked to do with young children:

> I think you need to have those explicit conversations about what is impossible and then what is certain. So what I like to do for certain, for young children is I come in with a brown bag and I put three orange tiles in and then I put my hand in and ask what is the probability that I am going to get an orange one. That was all I put in so of course that is what I am going to get. What is the probability I am going to get a blue one? That is impossible; I am not going to get a blue one because I didn't put any in there. So having that conversation about the extremes, don't lose sight of how important that is with young kids. [Video Day3]

This is a simplistic activity that does not need a great deal of explanation for PTs to implement in their future classrooms. This is in contrast to the previous example involving more complex games and concepts that would require additional effort to prepare PTs to use them. However, both of these examples illustrate how Kathryn describes mathematical activities PTs may use with their grade 1-6 students.

In the previous two examples, Kathryn drew from her classroom experience. In the next example, she described an activity she read about in professional literature. She shared an activity that integrated science and mathematics from the Teaching Integrated Mathematics and Science (TIMS) curriculum in order for elementary students to investigate estimating odd-shaped areas. Kathryn described,

> There is another nice science experiment where the kids take different types of paper towels to determine which ones are more absorbent. So they take eyedroppers and they put water on the paper towels to see which ones spread out more and then they calculate the area of the spread. So that is a nice science experiment that you could tie in with this idea that it is not going to be an exact area, it is going to be a pretty close estimate for the area of that spread. [Video Day18]

Again, this is another case where Kathryn did not invest time for the PTs to do the experiment or even to model the experiment. Thus, she was not preparing PTs to actually use this activity. However, she introduced an idea that PTs could eventually pursue with their own elementary students with additional effort and planning.

At times, Kathryn described a series of activities that built on one another to scaffold student learning about a mathematical concept. For example, while Kathryn was facilitating a discussion about units of measurement, she mentioned activities related to measuring objects with nonstandard units, communicating the need for standard units, and then measuring with standard units. More specifically, she explained,

> Standard units are needed to communicate effectively. So the reason we don't talk about paper clips is why? There are different sizes of paper clips. So you can even do an activity where you have one group measuring the same objects in the room, some are going to use the longer paper clips, one group is going to use the smaller ones and one is going to use the medium ones. Then when you compare the answers at the end and even though they all measured in paper clips, you will get them to move towards standard units. Paper clips are not going to be sufficient any more. [Video Day19]

On this occasion, Kathryn described an activity PTs could use in their future classrooms to introduce the idea of measurement and to help elementary students recognize the need for standard units of measurement. She then continued to explain that after elementary students have had experiences measuring objects using paperclips and other non standard units of measure, they could measure objects with premeasured strips of paper (e.g., one inch strips). She continued,

> So now I did some cut outs. This would maybe be a next step. In my cut outs, I did it using a paper cutter and blue pieces of paper; I made blue strips that are 1 inch long. So in order for kids to see what I am talking about, I also took and drew a blue line across that, I am talking about that length, I am not talking about up and down for example. So that line is 1 inch. You can then have kids measure things with inches. So you have gone from paper clips and now you want to standardize it. These are inches and you would have certain things around the room labeled and you would know how long they are so you can assess them. They are going to put the slips of paper next to each other and we say, these are inches and I am measuring in inches. So this would be the next step from non-standard to standard. Then I have some that are on red paper and these are half inches. So I might give them some of those and talk about how long certain things are in half inches. They would lay those down. These are all things that need to be developed before you get into this ruler idea or conversations or anything like that…You want things perfectly measured ahead of time so that when they first measure whatever it is, it is going to be the whole inch and you give them the blue slips of paper. Then you want something that will go to a half inch and you give them the red slips. Or you might start with the whole, but they are going to do it in halves. So lets say I have a four inch line, but you give them the half inch, they will see that it is eight halves, or eight half inches. They should already have a pretty good idea of what a half is before that. [Video Day19]

This transcript illustrates Kathryn's description of how to sequence several activities related to measuring length prior to introducing a standard foot ruler. She had several purposes for sharing ideas of how to introduce young children to standard units of measurement. She "want[s] them to see what one is. The unit is really critical. And so if we're going to measure in inches, we want them to have a sense of what one inch is or one centimeter is or one gallon or whatever it is" [EVT#3]. In addition, Kathryn also wanted PTs to engage young children in measuring objects. She recollected,

> I want them to know that if you're going to teach measurement, you want kids to actually measure. You know, it's like, in the textbooks, when they do weight. They show this picture of a car and say, oh, this is so many tons. How ridiculous is that? If you want kids to have a sense of weight, you want them to hold things. How much does this weigh? The same thing with length or any kind of measurement. You want kids to actually do the measuring, um, so they experience that, and get a good sense of that. [EVT#3]

Through Kathryn's description of these activities, she emphasized the idea that students need to physically measure objects, many different objects and with various units of measurement.

Throughout the semester, PTs complete mathematical activities that would be appropriate for grade 1-6 mathematics students as described above. Yet, typically Kathryn modified these activities in ways that she would not for elementary students. In other words, if she were to use these activities with elementary students, she would not introduce them the same way. Thus, Kathryn did not want PTs to think they could just replicate what she did with elementary students. As a result, after PTs completed a mathematical activity, Kathryn shared specific tips PTs should implement if they engaged their grade 1-6 students in the same activity they themselves completed in class. For example, Kathryn suggested when grade 1-6 students find the median of a set of data for the first time, to have an odd number in the set of data [FN2008 Day10], yet she did not do this with the PTs. In another snapshot of Kathryn's practice, she said, "If you were doing Roller Derby [a game from the Connected Mathematics curriculum], you would want them [students] to do it and keep track of the data" [FN2008 Day4], but Kathryn did not document the data that the PTs generated. In other words, have the elementary students keep track of the results of their rolls of the dice. Kathryn did not invest a huge amount of time-sharing these tips [VDFNInt1], however, she mentioned advice that could potentially enhance the mathematical experience and mathematical discussion with grade 1-6 students.

Instructional tools. Throughout her course, Kathryn provides opportunities for PTs to use manipulatives (e.g., pattern blocks) and other instructional tools (e.g., dice, rulers, protractors) to solve problems. Yet, this is not unique for PTs as she would use a similar approach with grade 1-6 students. I identified two actions within this category that are specific for PTs: (a) she *explains how to make homemade manipulatives for use in future classrooms or use common items in place of "catalog" manipulatives* and (b) she *shares tip(s) about teaching with instructional tools (e.g., manipulatives, calculators, rulers)*. I provide examples of each of these actions below.

The first action Kathryn attends to in her course every year is explaining how to make a homemade manipulative. Within the first few minutes of Kathryn's first class, she introduces a game called, Product Bingo (even before she introduces herself). This game involves a spinner with the numerals 2-9 on individual sections. Kathryn displays her "homemade" spinner on the overhead projector. [She acknowledges that many of her students will have access to Smartboard technology in their future classrooms; yet some will not have that luxury.] After stating the directions to Product Bingo, she proceeded to explain directions for making a homemade spinner. In 2007, she suggested,

> You can spend $5 buying a spinner or you can make one. I go with making my own. So here's mine and I use a paperclip. So if you're using it on an overhead, if you use a regular pen [to hold the paperclip], it will slide on the glass and might damage the glass. The best pencil to use is an eyeliner pencil...or you could use an overhead pen that doesn't work anymore, that's soft enough on glass. So just change your paperclip so you have a pointer. [Video Day1]

Kathryn also provided additional ideas for making cheap manipulatives for number cubes and angles. Additionally, she shared ideas for where PTs may pick up free materials at local building supply stores, willing to donate scraps for educational purposes, which could then be used for classroom sets of manipulatives (e.g., tiles). She also provided the idea to have students bring in coins (i.e., one penny, nickel, dime, and quarter) from home instead of purchasing overhead coins or play money out of educational catalogs.

Kathryn's purpose, for sharing tips for how to make or acquire homemade manipulatives for use in class originates from her teaching experiences early in her career in under-resourced classrooms in Chicago. She recollected,

> I worked in urban schools where they don't have these things and so I'm assuming some of my students will go and work in schools that don't have the resources we're talking about. And so I don't want them to dismiss that and say, um, well, I don't have spinners so I'm not going to do that. So I do give them some ideas for how they can make [manipulatives for their classroom]…I [also] want to give them some other strategies for um, how they can either make or get free manipulatives and don't think they have to use the store bought kind. [VDFNInt1]

On the first day, Kathryn encouraged PTs to keep a list of different strategies she shared in class or they garnered from the six hours they spent in their field placements on a weekly basis. Specifically, she asked that they identify approaches that they could use in their future classroom if they accepted a position in a school that did not have the financial resources to purchase commercial manipulatives for grade 1-6 students to use.

In addition to providing advice about creating or acquiring manipulatives, Kathryn also *shares tip(s) about teaching with instructional tools (e.g., manipulatives, calculators, rulers)*. After identifying the snapshots for this action, I identified four themes within this sub-category: (a) what grade 1-6 students need to know when using instructional tools, (b) what teachers should attend to when referring to instructional tools during instruction, (c) how teachers could use instructional tools to assess student's mathematical knowledge, and (d) what are appropriate instructional tools to introduce various mathematical concepts. I illustrate each of these below.

In one of Kathryn's lessons about teaching the concept of area and perimeter, she pulled out geoboards (see Figure 6) as a potential instructional tool that PTs could use. She explained that elementary students may have difficulty using geoboards to determine areas and perimeters of shapes created on them. For example, Kathryn stated,

> Here you have to be careful that the kids aren't counting just like if it is on a grid [referring to an earlier lesson about errors elementary students make when they count units on grid paper]. They should not just count the lines or pegs, it is the distance between the pegs. There are five pegs there, but the distance is four. [Video Day18]

Here, Kathryn shared a specific example of how elementary students may struggle with using an instructional tool. She shared an error that PTs may encounter if they decide to use geoboards with their future students. She clearly communicates that it is challenging to use instructional tools effectively.

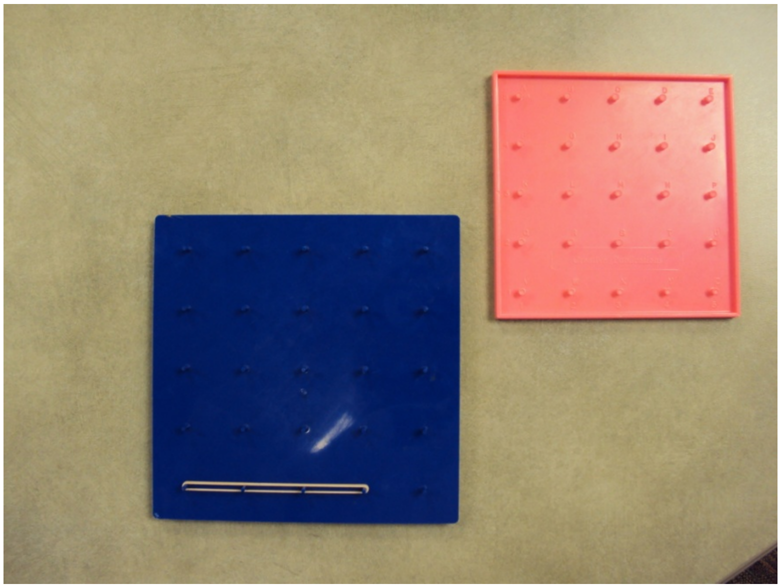


Figure 6. Geoboards.

On another occasion, a discussion in class ensued around using rulers to measure lengths of objects and Kathryn stated, “Kids need to know to start at zero, as some rulers start the edge at zero and some do not” [Video Day17]. Again, in this snapshot of Kathryn’s practice, she shared ideas for PTs to be attentive to when their future students measure with rulers. She wanted to convey several messages about instructional tools such as: (a) they have to be used purposefully, (b) some instructional tools are better than others for specific situations, (c) some instructional tools may be ineffective for some situations, (d) it takes a great deal of planning to use them effectively, and (e) children may be able to solve problems with manipulatives, but when they are removed they may not have the same success [EVTInt#3].

Kathryn also described a common situation involving instructional tools and school supply lists. She distributed a variety of rulers on each of the tables in the classroom, including some worn rulers that no longer provide accurate measurements if a student measured from the edge of the ruler. She explained that often times, “ruler” appears on school supply lists with the expectation that parents will buy them for their children. The message was "any old ruler" would be sufficient. Yet, Kathryn explained that if a ruler was on a school supply list, in a class of 25 students, there might be 20 different versions of rulers that show up the first day of class. Some rulers would distract students as they include pictures of the latest fads or celebrities. But more importantly, the zero mark would appear in different places (i.e., at the edge of the ruler or not), the rulers would have different increments (i.e., tenths of inches, eighths of inches, or sixteenths of inches), and they would not all include both metric and English systems. If this were the case, it would be a nightmare to teach elementary students how to measure lengths with various versions of rulers in the classroom [Video Day17]. Kathryn knows this from experience. She elaborated on her purpose for sharing this tip about rulers, "You really need to think about what tools you are using and don't make assumptions that any old ruler will work...I want them to know that there are a lot of ineffective tools" [EVT#3].

The second theme Kathryn attends to when *sharing tips about teaching with instructional tools* has to do with what PTs should attend to when referring to instructional tools during instruction. For example, she makes the point that it is important that all of the manipulatives be the same size. She explained that a teacher should not use square tiles or cubes that differ in size to demonstrate a mathematical concept where it is important that the edges line up flush with each other (a mistake she made in class – using square tiles of different sizes thus distorting the figure that was created) [FN2008 Day15]. Kathryn highlighted this instructional mistake as well as other instructional mistakes (either mathematics related or general teaching) she made throughout the semester, such as the one just described. She encouraged her students to analyze her teaching and identify mistakes she made, which the PTs should not make when teaching grade 1-6 students [Video Day2]. Another example occurred while the PTs were engaged in a mathematical task in which they had to flip multiple coins and dice simultaneously. The PTs desired to distinguish between the different coins and dice on the same toss, yet they did not identify strategies for doing this. As a result, Kathryn suggested that stickers could be placed on the objects, different types of coins could be used, or different colored dice could be provided to each group [FN2010 Day4]. These were ideas that the PTs had not identified on their own regarding how they could distinguish between the multiple coins and dice during one toss.

A third theme Kathryn attends to when *sharing tips about teaching with instructional tools* has to do with how teachers could use the instructional tools to assess students' mathematical knowledge. She articulated how PTs may use two different colored (blue and yellow) interconnected circles or plastic plates (see Figure 7) to assess different classifications of angles (e.g., right, acute, obtuse) quickly in class. She modeled this tip by asking students to hold up the "blue right angle like an earring" [FN2010 Day23]. Here, Kathryn held up the two connected plates so the blue right angle faced the class and the yellow right angle faced her. Kathryn then walked around the classroom to assess the angles the PTs created as they could do with their future students.

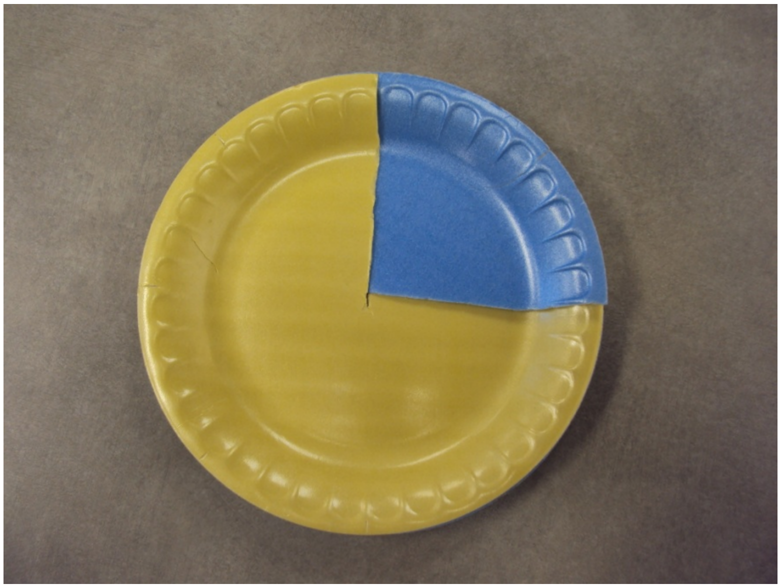


Figure 7. Two interconnected plates used to assess various angle classifications.

A second tip Kathryn shared with the PTs regarding an assessment technique they could use in their future classrooms involved using the calculator as a recording device rather than just a calculation device. Kathryn explained that she has had “kids use calculators before they use paper and pencil because they have to develop number sense and in Kindergarten, kids can’t write numbers. They can talk about 1000, but can’t physically write it. But they can push buttons” [Video Day21]. In addition, she stated, “Many Kindergarten teachers use calculators as a recording device and with special ed[ucation] students. They can’t write the number, but they can tell you the answer by entering it in.” This tip was shared because from Kathryn’s experience, many PTs believe the calculator is used to perform computations and that is the only way a calculator is to be used in the classroom [VDFNInt1]. However, here, Kathryn also shared how this tool may be used to help the PTs find out what their students know mathematically if writing is difficult for their young students.

The final theme Kathryn attends to when *sharing tips about teaching with instructional tools* has to do with what are appropriate instructional tools to introduce various mathematical concepts. For example, some ideas Kathryn suggested throughout the semester include: (a) to use protractors where the center is indicated with a hole rather than on the edge of the protractor (see Figure 8) [FN2010 Day24], (b) for students to use square tiles to build square numbers when first learning square numbers [Video Day2], (c) for students to all use the same type of ruler in a class [Video Day17], (d) to give elementary students strips of paper one inch long to measure items in order to show the students the need for a ruler for measuring items [FN2010 Day20], and (e) to use cubes to talk about mathematical concepts such as even and odd [Video Day2].

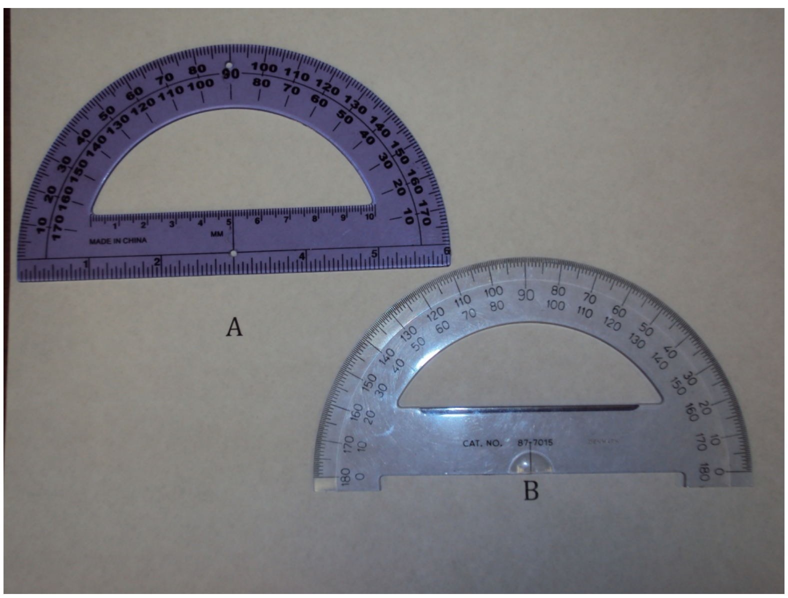


Figure 8. A protractor where the center is indicated via a hole [A] and a protractor where the center is not indicated with a hole [B].

Kathryn articulated several purposes for why she shared ideas for which instructional tools PTs could use to teach various mathematical concepts. For example, she stated, “They wouldn’t have thought that they could use manipulatives to talk about concepts like even and odd” [EVT#1]. In addition, she wanted the PTs to make thoughtful choices about which instructional tools they provided their students. She further emphasized this idea in the third extended video talk interview when she was watching an episode where she had just selected a PT to go up to the overhead to explain to the class how to measure a line in inches with a ruler. The PT asked Kathryn, “Do you want me to bring my own ruler?” [Video Day17]. Kathryn stoped the video here and explained,

> There’s this principle that’s related to the tools that you use in classrooms … and so you really need to think about what tools you are using and don’t make assumptions that any old ruler will work or any old calculator; I can use these plastic whatever. So which manipulatives you use for which purposes is really critical. And that is a big idea here that I am trying to emphasize. [EVT#3]

In the five examples described above, Kathryn emphasized details about specific instructional tools that PTs should attend to if they plan to use them in their future classrooms. Additionally, she provided ideas for which manipulatives may be assets during activities focused on various mathematical concepts.

Teaching mathematics. Snapshots of Kathryn’s practice that fit in this category are parsed into two actions: (a) *advises PTs on what to physically give to children when teaching various mathematical concepts* and (b) *advises PTs on what to say, write, and/or have students do when teaching various mathematical concepts*. The actions I identified in this category capture snapshots of Kathryn’s practice where she makes suggestions as to what PTs could say, do, or provide to grade 1-6 students to enhance childrens’ mathematical learning experience.

The first action, *advises PTs on what to physically give to children when teaching various mathematical concepts*, Kathryn provided ideas for tangible objects grade 1-6 students should have access to when learning various mathematical concepts. Some examples include: (a) When teaching volume, give students objects such as cubes [FN2008 Day16]; (b) “You want to give kids an angle [with sides] long enough where kids can put the protractor on it” (see Figure 9) [FN2010 Day24]; (c) “Would you do this [provide centimeter grid paper to use with tiles that are one square inch] with first graders? No, you’d give them grid paper the same size as the squares” [FN2010 Day15]; and (d) “Physically give the children the wire so they get a sense of what is this perimeter. They need to build stuff with the perimeter.” [Video Day16]. Kathryn explained her purpose for explicitly stating these ideas:

> When you teach, the materials that you give children is especially important… I want them to be purposeful in making decisions on what they’re actually giving the kids. Like even the math task assignment, even a worksheet they might design. I want them to be really thoughtful about how they design that worksheet. And so, you know, manipulatives. If you pick the wrong manipulative, it will be a disaster. [VDFNInt1]

In the above snapshots of Kathryn’s practice, she made suggestions about what PTs may give to children so they physically had a sense of the mathematical concepts under discussion (e.g., wire and perimeter, cube and volume). In addition, she emphasized that it was critical for PTs to select the manipulatives purposefully indicating that just using manipulatives would not necessarily lead to successful teaching.

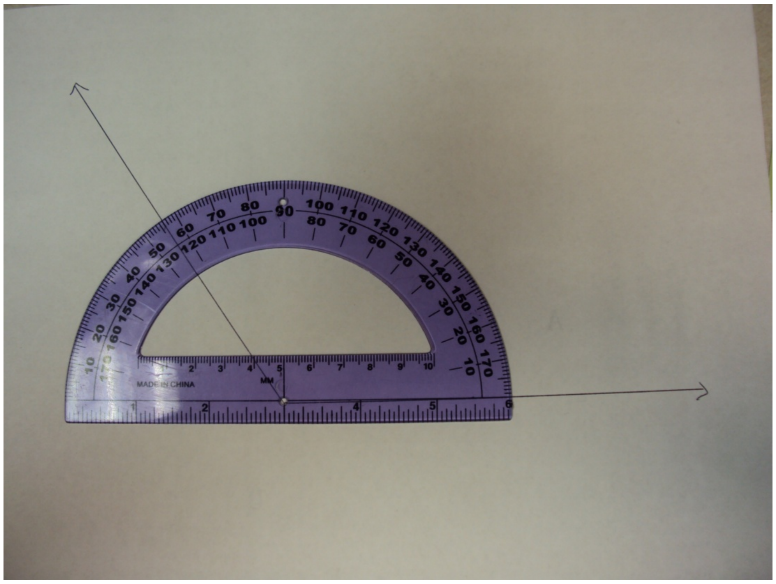


Figure 9. An example where the angle is longer than the protractor.

The second action I identified under the category of making suggestions about mathematics teaching is that Kathryn *advises PTs on what to say, write, and/or have students do when teaching various mathematical concepts*. Kathryn explained her purpose for this action: "They [PTs] don't have the knowledge to think about what are more creative ways I could teach [the mathematical concept under discussion]. So I give them some specific examples that they could do to help kids build meaning for what's going on [mathematically]" [EVT#3]. I identified two themes from the snapshots tagged under this action. The themes include Kathryn articulating: (a) teaching strategies the PTs could implement during instruction and (b) specific examples that PTs could engage their students to help them develop various mathematical concepts and strategies to enhance their overall mathematical understanding.

Kathryn described various mathematical teaching approaches PTs could use to attend to mathematical concepts under discussion. In one instance, she provided ideas for how PTs could develop a series of related problems that would help students identify patterns and develop number sense. For example, Kathryn wrote a series of problems on the whiteboard that she posed to a struggling third grader. Kathryn shared,

> If I want kids to see the check process, I don't just tell them how to check, I give them a bunch of problems and see if they can see a pattern. So lets say I do 23 – 19, 24 – 19 and 25 – 19. So on Friday nights I am working with a third grader in here actually, who is really struggling. And so I gave her a bunch of these problems this past Friday night and she was able to see this is four, this is five, this is six and she is like oh there is a pattern. Yes what a shocker. So what patterns do you see and what if you were to add these two numbers, what would you get? Oh, 23. What if you add these two? 24. So she starts seeing how the check kind of works before I have given her this is what a check is and this is how you do it. Because then it is just a rule. But at first you can do it through patterning. So they kind of get this idea that there is a way to check this. [Video Day9]

In the excerpt above, Kathryn explained how she worked with the third grader to help her discover the wrong answer she provided. Kathryn not only verbally described how she helped this child, but she wrote the mathematical expressions, 23 – 19, 24 – 19, and 25 – 19 vertically on the board as well as the answers to the three computation problems, in order to visually show the PTs the pattern she anticipated the child would also notice. Here, Kathryn shared a process PTs could implement in their classroom to help grade 1-6 students visually see patterns, identify their own errors, and develop number sense. Kathryn states, "You can use a visual" [VDFNInt1]. In this situation, Kathryn emphasized the importance of using visuals to not only support the learning of grade 1-6 students, but also PTs. In addition, Kathryn implemented this strategy because she did not think PTs would create it on their own. She reflected,

> This is a big thing I believe in really strongly. This idea of patterns. I did it earlier in the semester with product bingo and can they look at the times tables, and say, what are the patterns, I noticed, where are there even numbers, which numbers are here, which numbers are not here and I think that if we asked kids, if we gave them a variety of related problems and asked them to analyze them and what patterns do you see, I think that really helps children build, build some nice strategies that they can use. So

> here I show them that, can they give them, you know, three very, three related problems, they all subtract 19 from 23, 24, 25. Do they see some patterns going on? I'm introducing them to this strategy. They would not have come up with this on their own. [EVT#2]

On another occasion, Kathryn mentioned an instructional approach that may help grade 1-6 students be successful learning mathematics. In this case, she explained how she helped a student develop mental images of subtraction problems to help this child transition into the symbol system. She shared,

> I started with having her draw pictures of a story, and we have been doing subtraction stories. So I gave her an example; a clown is at a circus and he is holding 10 balloons and three fly away because they are helium filled balloons. How many is the clown still holding? So, she could get an image of this. She said, yes I can see this. And I am helping her develop mental images with pictures of things like this. [Video Day15]

Here, the PTs get an idea of an approach they could potentially use with future students to help them visualize mathematical situations.

Another similar snapshot of Kathryn's practice included her describing an approach she used with her young son.

> K: A few years when [my youngest son] was in Kindergarten, the public schools did this thing where if you went to summer school, you get 100 bucks….He got his 100 dollar Visa card and spends 19 dollars on his first purchase. So I said how much do you have left? He is going into first grade now and he says, "I have 91 dollars left." So what did I say? Did I say you are wrong? [Shakes her head.] I said, "What is 100 minus 9?" What did he say?
> S: 91.
> K: He said, "91. That other one was wrong, that can't be 91." He said, "That one is 81." What did I say to him?
> S: You gave him an easier problem.
> K: I didn't give him just an easier problem…I gave him the problem to his answer he first gave me. I created that problem. The researchers call this a contrasting case and so in giving him this case, he was able to identify his own error and correct it immediately. Did I have to show him anything? No. Did I have to tell him he was wrong? No, he figured it out all by himself. This is a very effective strategy that you can use, but creating

> contrasting cases off the top of your head is sometimes very hard. It takes some practice and it is not easy. [Video Day9]

Kathryn specifically stated that this was a strategy (i.e., creating contrasting cases when students provide incorrect answers) that PTs could use in their future classrooms. Kathryn shared this personal story of her youngest son because she did not think PTs knew a variety of strategies to handle wrong answers. She commented,

> I think they don't know how to handle wrong answers. And what do we see teachers do? We see teachers either ignore them, or re-direct, like, "Oh, Cynthia, thank you for sharing. Kathryn, what do you think?" And so you want to give them strategies for how to deal with wrong answers. … So here I'm giving them this strategy. I don't tell the child that he's wrong. I don't say, "Try again." There's no one else to interact with to say, "What did you get?" which I think is a common strategy. I give him a problem that would result in the answer he's provided. And think that's a very good strategy and he's going to figure out on his own that his answer is wrong. So he identifies his own mistake. [EVT#2]

Yet another example of Kathryn sharing a technique that PTs may use during instruction included the "let's pretend strategy." In this example, Kathryn stated, "Let's pretend. I like pretending in elementary school. Let's pretend the line was six inches long" [Video Day19]. In this example, a PT had just shared how she converted 8 ¼ inches into feet. However, the method she used (there are 48 one fourth inches in a foot so the line is 33/48 of a foot) did not make sense to the majority of the class. As a result, Kathryn raised this "let's pretend" idea (that the line the class is measuring is no longer 8 ¼ inches) and she guided the students through converting the new measurement. She posed the following questions. What if the line were six inches long? Eight inches long? Nine inches long, etc.? Kathryn used the "let's pretend" strategy to help the PTs build meaning for one of their peers' strategies, but also to introduce them to a strategy they could use with their future elementary students.

A final strategy I describe involves Kathryn discussing common student errors through the use of fictitious children who make mathematical errors. She explained,

> I create children and I put [incorrect student work] up and I will say, "I had this student last year and they did this graph. What can you tell me that is wrong?" That is how I do mistakes so it is nobody in the room. It is fictitious kids. So that seems to work. [Video Day7]

In this example, Kathryn modeled an approach that she wanted the PTs to use in their future classrooms. She wanted "them to put wrong work up on the board…and when you're *starting* to do that, you can't put a kid on the spot. You've got to build an environment where kids are willing to share their wrong answers" [EVT#3]. Thus, she modeled how PTs may use fictitious children who make mathematical errors to address incorrect mathematical answers in their future classroom. More importantly, she provided a strategy that PTs may use at the beginning of the year to build an environment where talking about mistakes is a classroom norm that is comfortable for the children. As demonstrated in the quote, Kathryn wanted the children to become comfortable sharing their own mistakes, but she knows that this will require strategies to build this type of learning environment and creating work from fictitious children who make mathematical errors is one of those strategies.

In the five examples described above, Kathryn shared teaching approaches that the PTs may draw from to teach various mathematical concepts. The personal experiences she relayed may also help PTs attend to misconceptions, error patterns, wrong answers, or incorrect mathematical thinking grade 1-6 student's display. Furthermore, through the scenarios she described, she provided the PTs with ideas they may someday adapt during mathematics lessons they instruct.

The second theme Kathryn attends to when *advising PTs on what to say, write, and/or have students do when teaching various mathematical concepts* is articulate specific examples PTs could use to help students in grades 1-6 develop various mathematical concepts and strategies to enhance their overall mathematical understanding. Throughout the semester, Kathryn suggested some ideas such as: (a) "Students should physically cut out triangles" to see that the area of a triangle is half the area of a rectangle "because it is difficult for students to visualize in their heads" [Video Day16]; (b) Students should "make angles with [their] hands" [FN2008 Day21]; (c) Students should "make an estimate before putting the protractor down – why? There are two numbers on the protractor" [FN2008 Day21]; (d) Students should "fold paper in half [so that it] makes a 45 degree angle" which they may use to estimate the measure of various angles [FN2010 Day10]; and (e) Students should visually represent the concept of median (as described in the Connected Mathematics curriculum). That is, students write the numerically organized class data on a strip of grid paper—one number in each square. Fold the strip of paper in half and "Look for the number in the middle of the fold" — that is your median [FN2008 Day10].

Throughout the semester, Kathryn provided PTs with advice, similar to the examples provided above, that they could engage grade 1-6 students in and help them enhance their understanding of mathematical concepts. She offered the advice described above because the PTs "don't have the knowledge to think about what are more creative ways…they could [employ] to help kids build meaning for what's going on" mathematically [EVT#3].

Organizing mathematics lessons. I identified three actions Kathryn addresses in relation to what PTs should consider as they plan and organize their mathematics lessons: (a) *selects and sequences student work for a whole class discussion*; (b) *articulates typical structure of a whole lesson*; and (c) *discusses ideas to attend to, consider, and/or draw from when designing mathematics lessons*.

Kathryn *selects and sequences student work for a whole class discussion.* Sometimes, she made an executive decision regarding which mathematical representations to display on the board. However, she then engaged the PTs in a discussion about her decisions. An example of this scenario is seen in the following excerpt. Kathryn said, "I made a decision for which [graphs you created] to put up [on the board] and now what mathematical ideas do you want to highlight and what order [do you want] to discuss [them]?" [FN2010 Day8]. In this situation, Kathryn turned the table on her students. She started with "*I* made a decision," but then she said, "Now what mathematical ideas do *you* want to highlight." She transferred the role of teacher to the PTs. In other words, what would they do if they were the teacher in this situation? She also focused them on considering the mathematical ideas that are important to highlight.

Kathryn was purposeful in these situations. When asked specifically about why she engaged PTs in conversations focused on selecting and sequencing student work, she responded,

> I want them to know that they have to be purposeful in selecting which kids are going to present and in what order are they going to present. And then also what's the mathematics that needs to come out of the discussion. And so not every child can present every day and so a teacher, you know, one of my big things in the class is you're constantly making decisions. A teacher's about making decisions and you're making them on the fly. And so here, you know, as they're circulating, as they're watching students work, engaging students in questions, they need to make decisions on

> what work they are going to have presented and what order…So you could make a decision based on the mathematics. And you could make it based on representations. And what are different representations that people are using. So there's this idea that, a, I want you to be purposeful. I don't want you to just randomly pick on people and I think that happens a lot. Or, whom ever's hand went up first we'll call on that person. So I want them to be more purposeful. I want them to think about the math. [VDFNInt1]

On another occasion, Kathryn talked explicitly with PTs about the order she asked four PTs to present their solution strategies. Kathryn asked the PTs to determine the area of an obtuse, scalene triangle on grid paper. The four ways the class found the area of the triangle included: (a) draw a rectangle around the triangle and subtract the area of two triangles (that were not the original triangle) from the area of the large rectangle; (b) make the obtuse scalene triangle into a right triangle and subtract the smaller right triangle from the area of the larger right triangle; (c) create a parallelogram out of the obtuse scalene triangle, find the area of the parallelogram, and half of the area is the area of the original triangle; and (d) find the base and height of the triangle and calculate the area of the triangle using the ½bh formula.

Kathryn asked her students, "Why did I have number one go first? I had a sequence here." In this situation, Kathryn talked explicitly with the PTs about how she organized the sequential presentation of the mathematics displayed on the board. She said that you want to make public all of the different ways students solved the posed mathematical problem so that you can compare the different methods of solution, but you also want to consider the order in which those ideas are presented [FN2010 Day18].

During another lesson, Kathryn provided color photographs of representations created by first grade students (see Figure 10). She asked the PTs to analyze the work and determine what each child knows mathematically. She also asked them to think about what mathematics they would highlight in a discussion [FN2008 Day12]. In addition to asking PTs to think about sequencing PTs' work from their class, Kathryn also provided them with authentic work from elementary students to highlight the importance of this idea.

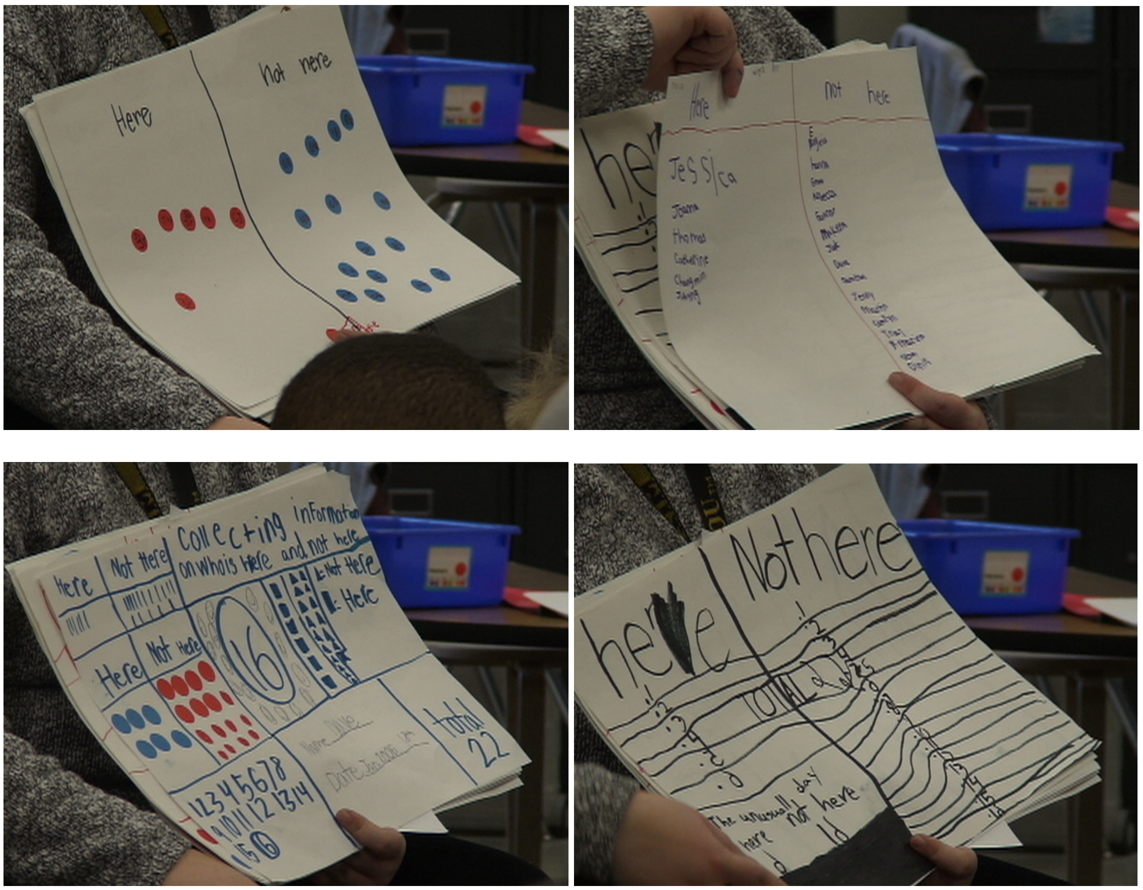


Figure 10. First grade student work prospective teachers analyze in class.

Kathryn engaged PTs in these tasks because:

> A huge part of what I talk about is how kids learn and how can we structure tasks. How can we structure our lessons so that kids are engaging? If kids are not participating, if they are not engaging, then they are not learning. [InitialInterview]

Kathryn was explicit with the PTs about the typical structure of a lesson, the second action I identified. In 2008, she wrote on the board,

> In a typical math lesson, the teacher first "launches" the lesson by introducing a task for the students to work on. Second, the teacher then provides time for the students to work on the task (i.e., "investigation" part of the lesson). At this time the teacher circulates in order to identify misconceptions, student strategies, and important ideas that need to be discussed. Third, the teacher facilitates a discussion of the math ideas and students share their work. [FN2008 Day16].

Kathryn wrote this on the board to prompt the PTs to analyze how the last two lessons were launched in her class. She shared this typical structure (i.e., launch, work, discuss) with the PTs because:

> When they write about their experiences in elementary school, they typically will talk about how they saw mathematics as an independent activity and that the teacher would talk and then they would practice individually at their seats, and then the teacher would collect their papers and grade them. So that's the image of mathematics they walk into my class with. And so I want to give them a different image. At the same time, they don't understand what other possibilities could look like. So I try and help them understand or have images of what a different organizational structure than what they've experienced would look like. [VDFNInt1]

In addition, she stressed that "closure is the most critical piece" of a lesson [Video Day21] and it is the "number one thing that is skipped…where misconceptions can be addressed" [FN2008 Day3]. Kathryn emphasized that closure to a lesson was very important and she provided ideas on how closure might look in the classroom by sharing a video of her teaching a sixth grade lesson [Video Day21]. Overall, she wanted the PTs to get the idea that "You want kids to solve math problems in every math class. And you want to talk about the important mathematics, and so how are you going to do that?" [VDFNInt1].

Kathryn *discusses ideas to attend to, consider, and/or draw from when designing mathematics lessons*, the third action I identified. Kathryn wants PTs to complete mathematics problems they intend to use with their students. To illustrate why this idea is important, Kathryn gave the PTs four worksheets that had a variety of area tasks on them (see Appendix F for examples of the worksheets). Kathryn asked the students to sequence the four worksheets and to justify their decisions (as discussed above). After the discussion, Kathryn asked the PTs to complete the four worksheets. The problems on the sheets varied in difficulty and one sheet included a complex figure (see Figure 11) that only a few students could solve. She then asked the PTs, “Why did I have you do this?” She explicitly stated to the PTs, “Do the problems before giving [tasks] out [to students]. Don’t make [the] assumption, I can just print it off and give it to kids” [FN2010 Day20].

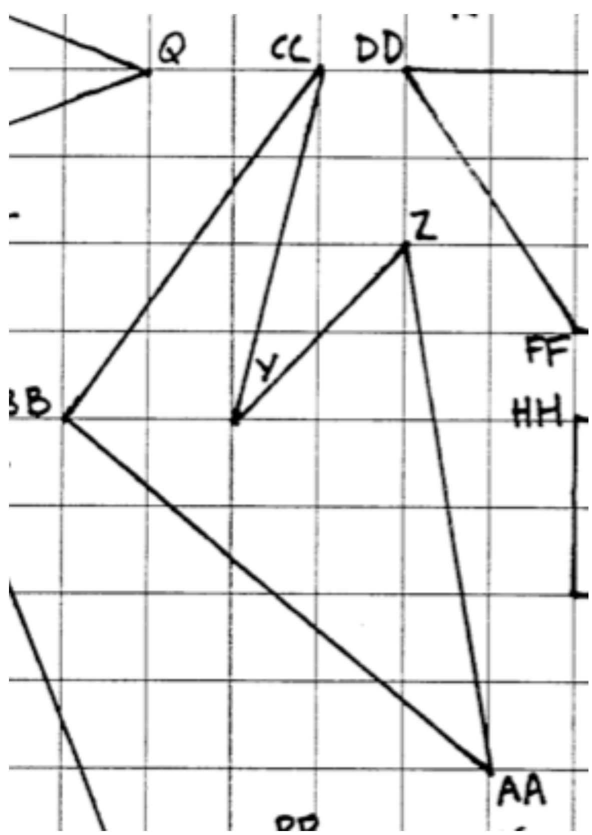


Figure 11. Polygon that challenged the prospective teachers.

When I asked Kathryn to talk about her purposes for attending to this idea, she responded,

> I think the [PTs] come from practice where in high school they would have done the odds or they would have done the evens and it wasn't a purposeful selection of problems. And I think we're in an age where they'll just go to the internet and they'll just print stuff off and it looks cute so lets just do this and they are not giving consideration to the mathematics. So what's the math I want to teach or what is the math I want kids to learn here. What are the underlying mathematical ideas…I think what happens is teachers will randomly select problems, especially novice teachers, and they'll get themselves into trouble because they didn't do the problems. They didn't anticipate where kids would get confused by language issues or what the task is asking…I want to break them of those practices, that when you're teaching you need to think about what the math is and you need to actually do the math problems you are going to give your kids to do. [VDFNInt1]

Kathryn wanted PTs to analyze the mathematics in a task before they made a decision to use it with children. She wanted them to think about the prior knowledge needed to access the task as well as what mathematics they wanted the students to learn from the task. With the endless supply of tasks on the Internet, she wanted PTs to be careful what they use with children.

Emphasizing language. The final category Kathryn attends to in order to develop PTs' knowledge of instructional strategies is actions related to language. In this category, Kathryn highlights the need for PTs to choose appropriate vocabulary and language to pose questions, provide directions, and describe mathematics to grade 1-6 students. In this category, I identified three actions from snapshots of her instruction: (a) *prompts PTs to articulate directions to mathematical tasks so grade 1-6 students will have access to the task*, (b) *explains the importance of connecting verbal and written language with visual aids*, and (c) *encourages PTs to be thoughtful about the language they use (e.g., when asking questions or writing mathematical tasks)*. I illustrate each of these actions below.

First, Kathryn had the PTs consider the language they would use in directions. For example, she asked the PTs to discuss in their groups how they would explain to a fifth grade class how to use a ruler to measure a line in inches [Video Day17]. After the PTs discussed this idea in groups, Kathryn selected a volunteer to come to the overhead projector and teach the class (as if they were fifth graders) how to measure a line. She also asked PTs “Who would like to volunteer to explain how to use a protractor with fourth graders—talk about it in your group what you would say” [FN2010 Day24]. In both of these instances in the mathematics content/methods course, Kathryn prompted the PTs to think about the language they would use to teach fourth or fifth grade students how to use these measurement tools (i.e., ruler or protractor). She challenged a PT belief that “teaching elementary mathematics is easy” [EVT#3]. Kathryn stated, “I think they think, how hard could this be? You know, you’ve just got to explain it clearly and you’re a good teacher, And…[teaching fifth graders how to measure with a ruler to a fraction of an inch] is a lot more complex than you would think” [EVT#3].

Another example of this action was when Kathryn asked PTs how they would describe various mathematical concepts or definitions to grade 1-6 students. Several examples include: (a) “How would you describe what area and perimeter are to a third grader?” [FN2010 Day15], (b) “Explain to a child how you’d find the area of any trapezoid and come up and write your explanation on the board.” [FN2010 Day19], and (c) articulate exactly what you (PT) are going to say to a child who has a misconception about place value [Video Day6]. In all of these cases, the PTs have a difficult time identifying appropriate language for elementary children.

Kathryn also emphasized that PTs should *connect verbal and written language with visual aids.* Frequently in 2010, Kathryn said to the PTs, "Give explanations verbally and written – look at the percent of instructions [that] are given verbally [to children] – percent is really high" [FN2010 Day16]. In 2010, Kathryn emphasized the importance of PTs verbalizing as well as physically writing the mathematics under discussion in the classroom for grade 1-6 students. Kathryn explained that this idea was especially important for students who were English language learners [EVT#3].

Kathryn *encourages PTs to be thoughtful about the language they use (e.g., when asking questions or writing mathematical tasks),* the third action related to language that I identified. She argued, "We need to be really careful with what we say" [FN2008 Day2] and "Be thoughtful with language—revoice what [grade 1-6 students] say" FN2010 Day16]. This action was also apparent when Kathryn prompted PTs to edit mathematical definitions that were written on the board by their peers if the definition did not make sense [FN2010 Day5].

To encourage her students to complete reading assignments from practitioner journals for teachers of mathematics, Kathryn informed the PTs that pairs would be randomly selected to facilitate discussions of the professional articles that they read during the course of the semester. As a result, the PTs were responsible for preparing discussion questions prior to coming to class. As the facilitators led the first discussion, a PT wrote their discussion questions verbatim on the board. After the facilitators finished the article discussion, Kathryn asked the class to analyze the questions the facilitators posed and determine what was problematic with them, again emphasizing the importance of language. The first time Kathryn asked the class to analyze the questions, there was silence. Kathryn then asked the class, "How many yes/no questions" were posed? One PT responded that they were all yes/no questions. Kathryn followed this comment up with the statement that I "want [you] to start to think about questions you ask kids – some yes/no questions are okay." [FN2010 Day1; FN2010 Day2]. Many of the questions the PTs asked their peers the first several times were yes/no questions and questions that were low level (i.e., had one word answers, answers were verbatim in the article). However, Kathryn was explicit with the PTs regarding her purpose for engaging them in this activity. She stated in class, "[I] want you to start thinking about the questions you ask" [FN2010 Day2].

Kathryn argues that "Language is important, and you know, the bigger idea here is: What are these meanings that we use with mathematical words?" [EVT#3]. Kathryn drew on her experience researching English language learners as she talked about the difference between everyday meanings and mathematical meanings of words such as round or similar. She emphasized that teachers need to pay attention to the meanings that kids associate with these words and think of various ways to help them make a connection to the really special meaning in mathematics regarding what those words mean. As teachers, PTs need to emphasize that the mathematical meaning is very different from other meanings. They also need to attend to what is said in the classroom and recognize language issues that occur in the mathematics classroom.

I identified 12 actions from 156 snapshots related to enhancing PTs' knowledge of instructional strategies as well as corresponding purposes that Kathryn employs in her elementary mathematics content/methods class. Kathryn teaches the second course, of a two course sequence for prospective elementary teachers, where the focus is on the content and complexities of teaching geometry, measurement, probability, and statistics in elementary schools. The purpose of this course is to develop a deeper understanding of the mathematical content focus of the course and to critically examine content and issues of the complexities in *teaching* and learning these fundamental mathematical concepts in elementary schools. Thus, the focus of this course is heavier on teaching (where in the first elementary mathematics content/methods course, the focus is heavier on student understanding). Due to the emphasis on teaching in this course, Kathryn invests a significant amount of time throughout the semester by providing advice to PTs on teaching tips as may be seen from the actions *advises PTs on what to say, write, and/or have students do when teaching various mathematical concepts* and *shares tip(s) about teaching with instructional tools (e.g., manipulatives, calculators, rulers)* encompassing almost a quarter of all of the snapshots identified under the component of instructional strategies. Additionally, it makes sense that the largest numbers of snapshots (156 out of 384 or 40.6%) are identified under the component of instructional strategies since one of the course goals is to provide an emphasis on teaching.

An action where only a couple of snapshots were identified was the action *selects and sequences student work for a whole class discussion*. However, this action is also extremely time consuming for Kathryn to enact during the semester (e.g., PTs need to complete the mathematics, then Kathryn selects some samples of PT work to be shared publicly, then a discussion ensues as to why Kathryn selected the work presented to the whole class). Thus, the emphasis/time she commits throughout the semester in the class is not represented via the percentage in the table. It is interesting to note that there were no snapshots identified in 2007. One explanation could be that the snapshots I identified in

2008 and 2010, Kathryn did not engage the PTs in corresponding activities in 2007. A second explanation for why I did not identify any snapshots of the action in 2007 could be that Kathryn spent more time analyzing fictitious student work with the class and answering questions PTs posed about their field placement teaching experiences. Both of these assignments were time consuming, thus, leaving limited time for emphasis on other actions Kathryn employed throughout the semester.

Enhancing Prospective Teachers' Knowledge of Curriculum

Kathryn employs specific actions to develop PTs' PCK related to elementary mathematics curriculum materials. Based on an analysis of the data, I identified three major categories: (a) scope and sequence of curriculum materials; (b) nature and content of curriculum materials and resources; and (c) the analysis and enhancement of curriculum materials. Table 11 summarizes the actions for each of these three major categories.

Table 11

Summary of Actions Kathryn Employs to Develop Elementary Prospective Teachers' (PTs') Pedagogical Content Knowledge of Curriculum

Categories of curriculum	*Actions*
Scope and sequence of curriculum materials	Articulates grade levels that specific mathematical ideas are taught
	Identifies the order that mathematical ideas are taught
	Selects worksheets to engage PTs in sequencing tasks for grade 1-6 students regarding important mathematical concepts
Nature and content of curriculum materials and resources	Provides examples from curriculum resources
	Shares and demonstrates supplemental curriculum resources
	Addresses vocabulary in curriculum resources
	Articulates differences in design and specific features in mathematics curriculum materials
	Identifies topics in mathematics curriculum materials that are (a) presented in inadequate ways or (b) enacted in unproductive ways
Analysis and enhancement of curriculum materials	Asks PTs to compare what was done in class with how curriculum materials are written for grade 1-6 students
	Prompts PTs to analyze the mathematics in curriculum materials or activities completed in class
	Prompts PTs to consider how they could alter materials to enhance students' mathematical experience

Below, I describe Kathryn's specific actions and purposes for using the identified actions in her mathematics content/methods course to facilitate PTs' PCK specifically related to curriculum materials.

Scope and sequence of curriculum materials. I identified three major actions related to scope and sequence: (a) *articulates grade levels that specific mathematical ideas are taught*, (b) *identifies the order that mathematical ideas are taught*, and (c) *sequences tasks to engage students in important mathematical concepts*. Frequently, throughout the semester, Kathryn articulated specific mathematical concepts, skills, or ideas that correspond to specific grade level(s) or grade bands. Some examples of what Kathryn has specifically said are: (a) Fifth grade is learning about prime numbers [FN2008 Day1]; (b) The traditional algorithm [for addition] is introduced in the local school district in fourth grade. It used to be introduced in second grade [Video Day7]; (c) In fourth grade, angles are introduced [FN2008 Day16]; and (d) Congruent is a third grade concept [Video Day17]. The previous examples are all mentioned when Kathryn facilitated discussions regarding each mathematics topic. In these instances, the comments that Kathryn made about the specific grade levels was not in response to a question posed by a PT.

However, at times, PTs did want to know the grade when particular mathematical topics were introduced. In one class session with the PTs, Kathryn directly answered a PT's question as to when percentages were taught. An example of this may be seen in the following excerpt.

> S: When do they [students] learn percentages?
> K: Percentages? It is typically middle school.
> [Video Day6]

The PTs had just watched and engaged in a small group discussion of the video, *Chances Are*. In this video of a third grade classroom, the teacher in the video engages her class in a probability lesson where they learn about certain, unlikely, and impossible events, where the events fall on a probability line, and what numbers should be assigned to the different positions on the probability line. Throughout the lesson, the teacher and students mostly refer to simple fractions (e.g., one-half) when discussing the position of an event on the probability line. The teacher never once mentions the word percentage although some students do say percentage.

In the example above, Kathryn does not articulate one specific grade, but instead, provides a grade range (middle school) that the topic of percentages is addressed. Another example of where Kathryn provides a grade range is in the following description. Kathryn and her students are engaged in a whole group discussion about the mathematical probability of events. Kathryn posed the formula for how to find the theoretical probability of an event and mentions to the class, “You wouldn’t do this [theoretical probability] with really young kids, but possibly with fifth or sixth graders” [FN2008 Day4]. This statement is not prompted by a student, but initiated by Kathryn during a whole group discussion of the mathematical topic, probability.

A final way Kathryn engages in the action of *articulates grade levels that specific mathematical ideas are taught*, is not by stating a specific grade level, but by referring to general terms such as “children,” “young children,” or “elementary school.” For example, Kathryn made statements such as: (a) Sample space is an important concept taught in elementary school [Video Day2]; (b) We want children to represent numbers in multiple ways [FN2010 Day10]; (c) Mathematicians may consider degenerate cases, but you wouldn’t do this with young children [FN2008 Day16]; and (d) One thing we ask children to do is sort shapes [FN2010 Day21]. In these examples, Kathryn did not assign a specific grade level, but instead used terms that indicate children in grades 1-6.

Kathryn employs this action, (i.e., *articulates grade levels that specific mathematical ideas are taught*), numerous times throughout the 2007, 2008, and 2010 semesters and she has various purposes for employing this action in her mathematics content/methods course. Her purposes include: (a) to earn credibility with her students so they do not see her as being removed from grade 1-6 classrooms, but that she does know what goes on in schools [EVT#1]; (b) point out to PTs why the content they are studying is relevant to them as future elementary mathematics teachers [EVT#1]; (c) articulate how children's mathematical learning progresses over the elementary years and beyond [EVT#3]; and (d) attend to comments from past students who complained they did not know the mathematical content taught at various grade levels [EVT#1].

The second action related to scope and sequence has to do with Kathryn *identifying the order that mathematical ideas are taught*. Sometimes Kathryn articulated a sequence of two different mathematical concepts. An example of this occurred in class when Kathryn discussed measuring angles with a protractor. Kathryn said, "Focus kids on estimation first before introducing the protractor" [FN2010 Day24]. Another example from a class session follows.

> So experimental probability, sometimes we do this with young children to develop probability. So we are going to spend a few days over the next few class periods doing some experiments. But that is usually how probability is introduced to kids; they are flipping coins or rolling dice. And they maybe are not going to come with a theoretical probability, but as the more data you collect, so if every pair is collecting data and as a class you then combine it into a table and organize it for them, they are probably going to get pretty close as a class to this idea of theoretical probability. [Video Day3]

Here, Kathryn stated that introducing students to theoretical probability naturally follows engaging students in collecting data to determine the experimental probability. She articulated the sequence that mathematical concepts are taught/introduced to grade 1-6 students.

Sometimes Kathryn embedded mathematical vocabulary, which could be introduced to students, and did not state, explicitly, first teach this, and then teach that. For example, in one mathematical discussion about whether a blue square rotated and/or flipped is still a blue square, she stated, “And this is where you would introduce congruent” [Video Day13]. Kathryn made a statement about where PTs could introduce a new vocabulary term while building meaning for a mathematical concept.

In the previous three examples, Kathryn initiated the idea of sequence; however, in a final example Kathryn answered a PT’s question about the sequence involved in teaching linear measurement. Kathryn stated,

> We talked about measuring with non-standard units first. And then when you get to where it’s more or less, you’re going to do more or less. So is it more or less than five inches? More or less than six inches? You’re not getting into is it five and a half inches yet. You’re just having them get this idea is it more or less. A little bit more or a little bit less. [Video Day20]

On this occasion, Kathryn recommended a sequential order of mathematical ideas related to linear measurement.

Kathryn not only discusses the sequencing of concepts within a specific grade level, but also *sequencing topics over the course of several grade levels.* Kathryn spends several class periods each semester engaging PTs in discussions about this important idea. For example, she selected one task related to data from each grade level in the Investigations in Number, Data, and Space curriculum [FN2010 Day13]. She then asked her students to determine the grade level for each task and to explain their reasoning for their decision. Kathryn also gave the PTs several area tasks that had differing levels of difficulty and asked the students to determine which order they would give them to elementary students [Video Day18]. Typically, each table selected a different sequence, which led into a heated debate of which sequence was best. Invariably, they turned to Kathryn and would ask, “What is the right answer?” Through these activities, PTs develop a better sense of how a mathematical topic is developed over time and what kids know or should have been exposed to when they reach a new grade level.

Kathryn shared a variety of purposes for *identifying the order that mathematical ideas are taught* in her course. First, she wanted to build credibility with her students. She knew that her students wanted to understand what mathematical topics were taught at each grade level and she ensured that she discussed that idea in her course. In addition, she wanted the PTs to walk out of class knowing that mathematical “ideas build over time” [EVT#3]. She elaborated, “You’re [the PTs] getting a small, you’re going to teach kids a little section of that. That little line of the continuum” [EVT#3]. This comment was prompted during extended video talk interview #3, when Kathryn remembered a misconception that surfaced during a lesson in 2007. The PTs had been reading articles from *Teaching Children Mathematics* that provided suggestions for teaching measurement. The authors argued that children should begin measuring with nonstandard units and then progress to more sophisticated ways of measuring with specific tools. Unfortunately, some of the PTs assumed that the entire sequence, starting with nonstandard units and then progressing to measuring with a ruler to a fraction of an inch would take place at one grade level. As a result, Kathryn provided a very direct response about how the PTs would teach measurement in the one grade level that they would be assigned to teach.

Kathryn articulated a final purpose that addresses why she employs actions centered on articulating scope and sequence of various mathematical concepts to her students. She reflected,

> I want them to have this idea, you know, that certain topics or content are taught at specific grade levels. But I also want them to think about the

> articulation. And so I want them to think about if I'm going to teach measurement in third grade, what prior knowledge do kids bring. And, you know, what would be the expectation for me in third grade to prepare them for fourth grade. You know, sometimes I get a sense from them that they think they are going to teach everything. You know, instead of thinking of it as a learning progression or, you know, development or whatever. So if I'm going to teach fractions in third grade or fourth grade, I'm going to teach everything about fractions. Now, what's the little piece of fractions that you are going to teach and how does that build on what they've already learned and where they are going. Um, so I think that articulation issue is really important. [EVT#3]

This quote illustrates that Kathryn wanted the PTs to know that particular mathematical topics are taught at specific grade levels. But more importantly, she wanted them to know that there is a progression that mathematical concepts should be taught in order to enhance student learning and understanding of those mathematical ideas. She wanted them to understand that they should know more than just the grade level they teach. They will need to understand what mathematics is taught before and after the grade level that they will eventually teach.

The final action I identify, *selects worksheets to engage PTs in sequencing tasks for grade 1-6 students regarding important mathematical concepts,* Kathryn has PTs sequence elementary curriculum tasks to enhance their knowledge of the order elementary students would engage with tasks centered around various topics (e.g., data, area). She provided PTs with an opportunity to identify how a first grade curriculum differs (e.g., content, format, and language) from a fourth grade curriculum. For example, Kathryn posed five tasks, around the mathematical concept of data, from the same curriculum for PTs to investigate across grades one through five. PTs were asked to predict the grade level of each worksheet as well as compare and contrast the content and organization of the five worksheets [FN2010 Day13]. The PTs collaborated in small groups to sequence the worksheets by grade level and then Kathryn brought the class together to engage them in a whole class conversation to discuss the grade level the PTs assigned to the various worksheets as well as their reasoning for the designated grade level for the various worksheets.

Through this task, Kathryn exposed PTs to examples of mathematics curriculum materials from various elementary grade levels so that they saw how mathematical ideas are developed across multiple grade levels. Additionally, Kathryn reflected that tasks related to curriculum materials provide opportunities for PTs to be more critical of mathematical materials they will encounter in the future [InitialInterview].

Nature and content of curriculum materials and resources. In this category, Kathryn engages in five major actions, which include: (a) *provides examples from curriculum resources*, (b) *shares and demonstrates supplemental curriculum resources*, (c) *addresses vocabulary in curriculum resources*, (d) *articulates differences in design and specific features in mathematics curriculum materials*, and (e) *identifies topics in mathematics curriculum materials that are (a) presented in inadequate ways or (b) enacted in unproductive ways*. The first action I describe is where Kathryn *provides examples of curriculum resources* (e.g., activity packets, tasks, etc.) to the PTs, which they may use with their future students. She does this throughout the semester as she introduces different mathematical topics (e.g., data analysis, measurement, etc.). For example, Kathryn explained that when protractors are introduced to students, the "expectation is that kids can read, measure, and draw angles with a protractor—you might start with this [page]" [FN2010 Day24]. Kathryn then distributed page 31 from *Maneuvers with Angles,* which included a picture of a protractor on top of an angle as well as a picture of a 50-degree angle for students to practice measuring until they agree that the angle measure is in fact 50 degrees.

Kathryn summarized one of her purposes for this action as follows,

> I know that when they get out there, that they have access to curriculum materials from all over the place. They have stuff from the Internet. They're probably going to have books on their shelf that a teacher left behind. They're going to have their [district-adopted] curriculum. They're going to make these decisions about what they use and some of it's going to be good stuff and some of it's going to be not so good. And so how do you help them make good decisions with that. [InitialInterview]

Kathryn wanted PTs to be critical of curriculum materials that they have access to and make educated decisions about what curriculum materials to use in their classroom. She helped the PTs become familiar with different types of curriculum materials throughout the semester by sharing pages that were related to the mathematical topics the PTs study.

Kathryn also *shares and demonstrates supplemental curriculum resources* (e.g., virtual manipulative websites, resources Kathryn gathered over the years). For example, Kathryn pulled up virtual manipulative websites and demonstrated how they worked as well as provided ideas for ways PTs could use the websites in class or as an individual tutor for students [FN2010 Day10]. In addition, she shared a database that listed children's literature books related to specific mathematical concepts (e.g., *How Big is a Foot*) [Video Day20].

When Kathryn shared different curriculum resources, she would frequently say, "We don't know where you'll be teaching or what curriculum you will have access to" [Video Day11]. The bigger idea that Kathryn wanted PTs to understand as she shared various mathematics curriculum resources was that,

> When [the PTs] get out there, they have access to curriculum materials from all over the place. They have stuff from the Internet. They're

> probably going to have books on their shelf that a teacher left behind. They're going to have their curriculum. They're going to make these decisions about what they use and some of it's going to be good stuff and some of it's going to be not so good. How do you help them make good decisions with that? [InitialInterview]

The third action Kathryn attends to, *addresses vocabulary in curriculum resources*, I identified several themes from the snapshots. The first theme is where she clarifies vocabulary that is articulated in the curriculum versus the word PTs use in class as the following excerpt illustrates:

> K: Borrowing, the bad word. But yes we borrow on these. Do we borrow on all of them? Do all of them require borrowing?
> S: Yes.
> K: Yes. Ok…what is that called now? We don't say borrow anymore.
> S: Renaming.
> K: …Renaming… do we use that with subtraction?
> S: No, I thought renaming was setting up.
> K: So what is it used for in subtraction then?
> [Video Day9]

In this discussion, Kathryn recognized that the PTs use the word "borrow" based on their experiences as elementary students. She explained that that term is no longer used in current mathematics curriculum materials. Another example happened when Kathryn stated, "We don't say diamond in math class, we say rhombus" [FN2008 Day16].

A second theme Kathryn attends to is where she shares different definitions from several textbooks. For example, definitions for length, width, kite, and trapezoid are not uniform across different textbooks. She stated that when teaching these mathematical ideas, PTs should be aware that different definitions exist in curriculum materials, should know the different mathematical definitions for the terms their students are expected to learn, and emphasize the definition they use in their future classrooms [FN2010 Day22].

Kathryn also mentioned terms or ideas that were not included in mathematics curriculum materials. For example, Kathryn mentioned that some curriculum materials do not use the words “dice” or “candy” due to some state adoption rules that prevent the use of these terms [Video Day7]. However, in this next example, Kathryn mentioned a specific mathematical idea that was often missing in elementary mathematics curriculum materials.

> K: [A] second [strategy to introduce probability] is discuss things that are impossible because it is often not in the curriculum. So why is impossible important? Who cares? Why discuss impossible, why is impossible important?
> S: To get them to know what is not likely (inaudible) what is likely.
> K: Ok, what is the difference between not likely and impossible?
> S: Impossible means it can’t happen.
> K: Can’t happen. And not likely means?
> S: Not likely.
> K: Right but there is a possibility that it could happen, right? So most curricula don’t include impossible. Why have they eliminated it? Does that mean it is not important? When we start working with probability with older children and they are actually calculating probabilities, what is the probability of something that is impossible?
> S: Zero?
> K: Zero. And what is the probability of something that is certain?
> S: One?
> K: One. And what about everything in between?
> S: They are decimals or fractions.
> K: Yes, they are decimals or fractions. So it has to be between zero and one. So all probabilities lie between zero and one. It is pretty important that we don’t forget about those [probabilities that are] impossible and those [that are] certain. We talk about the extremes of probability, and a lot of times you’ll see curriculum where they don’t, they just talk about things that are in between. You probably should frame it in terms of what the extremes are for probability.
> [Video Day5]

In this excerpt, Kathryn explained that the idea about *impossible* probabilities was not always included in elementary mathematics curriculum materials; however, it is an important mathematical concept for students to engage with as they are introduced to the concept of the probability of an event.

Kathryn talked about these ideas because she wanted PTs to pay attention to specific vocabulary in the curriculum materials that the PTs would use. In addition, she had encountered PTs who thought in absolutes (e.g., there is just one definition for trapezoid, they should just use one name to refer to something, etc.) and that they were "going to see these other terms and that's something [the PTs] should look at when [they] take a job" [EVT#2].

The fourth action Kathryn employs in the category of the nature and content of curricular materials and resources is *articulates differences in design and specific features in mathematics curriculum materials*. Kathryn shared information about various features of different mathematics curricula available to teachers—even from math textbooks printed one hundred years ago. Some of the information she shared had to do with how information was presented in texts as well as specific sources the curricula materials include. For example, Kathryn stated, "In mathematics curricula 100 years ago, students first did long addition and then short addition" [FN2010 Day8]. See Figure 12. She made this statement when she was leading a class discussion on different ways students add two and three-digit numbers and how in a class of 25 students, there may be several different ways students add numbers correctly—which may not have been the case 100 years ago. Likewise, Kathryn explained that different textbooks present different algorithms for students to learn [Video Day7]. In addition, some mathematics curriculum materials (e.g., Investigations) have specific computer software incorporated into lessons, while others do not [Video Day17].

$$\begin{array}{r} 74 \\ +\ 58 \\ \hline 120 \\ 12 \\ \hline 132 \end{array} \qquad\qquad \begin{array}{r} {}^{1} \\ 74 \\ +58 \\ \hline 132 \end{array}$$

(long addition) (short addition)

Figure 12. Long addition and short addition as was taught 100 years ago.

Another way Kathryn engages in the action is by sharing the overall concept design of different mathematics curricula. For example, she shared how one curriculum series is written so that science is interwoven throughout the series. She said,

> Ok, so this curriculum uses science as a context to teach mathematics. So there is science throughout the whole book. So it isn't a complete integrated math and science curriculum because it doesn't cover all of the science curriculum. But it uses science as a context to teach math. [Video Day11]

She also mentioned a mathematics curriculum program written by Russians who introduce mathematical content (e.g., addition, subtraction, multiplication, division, algebra, geometry, etc.) through the context of measurement [Video Day20]. Likewise, Kathryn articulated differences in curriculum materials available in the educational market for PTs to draw from as they plan lessons for their future students. One reason she described features from many different curriculum materials in class was due to criticism in the local community, which the PTs had heard about. She explained,

> There have been some claims that math educators are [advocates] for particular curriculum and not for other curriculum. And there have been criticisms that we only, that MU specifically, and maybe other [higher education] institutions, only prepare our teachers to teach the NSF funded curricula. And that's never been my position. Um, I use curriculum materials from lots of different places and lots of different features. And so I don't advocate for particular curricula like when you get to a school system you should use this kind. I never do that. I do pick on flaws…that I

> see in curriculum materials. And I say explicitly…that I don't think there's a perfect curriculum out there. There isn't going to be a curriculum that's going to meet the need of every child you're going to teach. [VDFNInt2]

When Kathryn shared sources with PTs, she frequently stated that she had no idea which curriculum resources they would have access to their future classrooms. Furthermore, curriculum resources vary considerably, thus she shared resources that were currently used by teachers around the country.

The final action I identify is where Kathryn mentions mathematical topics in curriculum materials that are presented in inadequate ways. For example, she stated in her class, "Weight is something that is taught so poorly in this country" [Video Day12] and "Place value is taught poorly" [Video Day 15]. These are strong statements. So I repeated them to Kathryn in the second video/fieldnote interview and asked her why she made statements such as these. She was very articulate in clarifying why she purposefully made these statements to the PTs.

> I think the purpose is not just to blindly accept what's in their curriculum materials and you know, can you make modifications that make sense based on the content you have to teach. And so if the only thing in the unit about weight is you know, pictures of things, bring in objects. Bring in scales. Have kids weigh things. Have them compare things. And I think those are really big gaps in curriculum materials and so that's why I make those comments. [VDFNInt2]

She explained that she wanted to "tug at their hearts … and push some buttons, …get to them before they adopt those practices" [VDFNInt2]. Here, she commented on how PTs are idealistic and think that when they are in the classroom, they are going to be good teachers and it is not going to be okay for them to say that a lesson they taught did not go well.

Kathryn also identifies topics in mathematics curriculum materials that are enacted in unproductive ways. For example, she stated, “Probability (and geometry) is not taught in schools – teachers skip it” [Video Day3]. Teachers constantly make decisions on what mathematics to teach and what content to skip. She explained to PTs that teachers omit portions of curriculum for a variety of reasons such as they do not always understand the mathematics or they choose to teach the lessons in the order they are presented in their textbook. The PTs agreed that it was not acceptable to skip content based on a lack of knowledge.

To further articulate the importance of what content to teach and skip, Kathryn shared graphs from research that illustrated the redundancy of mathematical content in mathematics textbooks for grades K-9 (e.g., 100% of textbook pages include new content for kindergarten, but only 30% for eighth grade—most of which is at the end of the textbook) (Flanders, 1987). In addition, she shared a graph illustrating the decisions two different teachers made regarding which lessons they selected to teach out of a textbook (e.g., one teacher taught the lessons in sequential order and stopped 58% of the way through the book while the other teacher made different choices about which lessons to teach) (Chval, Chávez, Reys, & Tarr, 2009).

The PTs recognized that they did not know all of the mathematical content they needed to teach. This became a conflict for the PTs as they recognized that this lack of knowledge could ultimately do a disservice to children. When Kathryn articulated mathematical concepts in curriculum materials that were omitted or not taught well in schools, she tried to implicitly make the point with her students that they need to learn the mathematics they will teach. They should not skip content in their curriculum materials if they do not understand the mathematics or the concepts are located in the back of the book. She said, “Elementary teachers go into the profession because they love kids and want to do the best thing, make a difference with kids and so [I] tug at that a little bit” [VDFNInt2].

Analysis and enhancement of curriculum materials. Kathryn asks her students to analyze, compare, and enhance the curriculum materials. Specifically, I identified that Kathryn: (a) *asks PTs to compare what was done in class with how curriculum materials were written for grade 1-6 students*, (b) *prompts PTs to analyze the mathematics in the curriculum materials or activities completed in class*, and (c) *prompts PTs to identify ways to enhance curriculum materials*. Below I describe these three actions.

First, Kathryn *asks PTs to compare activities they complete in class with how it is written in elementary curriculum materials*. For example, Kathryn’s students played the Roller Derby game in class (a game where the leader rolls two dice and participants add the total and remove one marker from the column that corresponds to the total from the dice). After the PTs finish playing the game, Kathryn showed them the version in curriculum materials. She asked how her design was different. She inquired,

K: What did I do when I designed the task?
S: You included all of them [numbers 1-12].
K: Yes, I included them.

Kathryn prompted the PTs to recall how the board was designed for grade 1-6 students.

K: Which numbers are on the board?
S: Two through 12.
K: Two through 12, so did they have one?
S: No.
K: No. So impossible wasn’t included in the game. So why did I add that?
S: To try to trick us.
K: To try and trick you? No.
S: See if we understood it.
K: See if you understood what?
S: What was impossible and what was not.

> K: Yes, I wanted to include impossible. The curriculum only has two through 12 and I modified it to include the impossible so there would be a discussion of it. You are right, most kids put their chips on the impossible because they are not thinking about is that a possible outcome or not. So we need to do that a little bit more in our tasks that we create.
> [Video Day5]

In this example, Kathryn not only prompted the PTs to compare how they engaged in an activity which they could implement with their future students, but she also prompted them to determine what was different with the board they used versus how the board was originally created for elementary students. By asking the PTs to consider the two versions of the game, Kathryn attended to enhancing the PTs' knowledge of curriculum.

On other occasions, PTs completed a modified task from curriculum materials in class, but then for homework, they completed the task as written in the curriculum materials. Kathryn then engaged the PTs in a discussion by asking them, "How did I change what was in [the] curriculum materials?" For example, she asked the PTs to put five squares together to create as many different shapes as possible. Kathryn did not tell the PTs that 12 different shapes were possible and asked the PTs to explain how they knew they found them all. However, in the written curriculum, the directions required students to only find five different shapes, which was not as challenging. Kathryn and the PTs engaged in a discussion about the differences between the two problems and how Kathryn's version opened up the problem space by not telling the PTs the number of shapes. Kathryn explained that she was not sure "if they would know how to do that" (present the written product in a way that was more engaging for the students) [VDFNInt2]. She believed that using these curriculum comparison tasks was a better pedagogical approach than just saying to the PTs, "Well, I could have done it this way or that way."

A second action was when Kathryn *prompts PTs to analyze the mathematics in curriculum materials*. For example, Kathryn explicitly asked the PTs, "What [mathematical] concepts are in Chapter One of [Maneuvers with] Rectangles?" [Video Day15]. In this whole group discussion, Kathryn asked the PTs to analyze and identify the mathematics that elementary students would engage with as they complete the tasks written in the curriculum materials. In another situation, the PTs looked at a lesson from the Math Trailblazers curriculum that was written for fifth grade students where there was a story about a conservation club that was collecting data on bats (e.g., how many they captured and how many were tagged and released). Kathryn asked the PTs to look at the explore questions that went with the story and asked the PTs, "What do you notice about the difficulty of [the] items?" [Video Day11].

In both of these examples, Kathryn prompted the PTs to identify the mathematics in specific excerpts of curriculum materials. Kathryn wanted to help PTs develop the practice of examining mathematical tasks from curriculum materials for the purpose of identifying the concepts that are addressed in those tasks. As described earlier, she was concerned that the PTs would randomly select curriculum resources from multiple sources. Selecting tasks without considering the mathematical content in them would be problematic. She also wanted the PTs to consider the mathematical complexity that was expected of elementary students. For example, the questions in the "bats" lesson involve variables and PTs were often surprised to find "algebra" in elementary mathematics curriculum (personal communication with Kathryn).

Finally, Kathryn *prompts PTs to consider how they could alter materials to enhance students' mathematical experience*. In the examples provided above, Kathryn enhanced tasks from curriculum materials and asked the PTs to compare the two different versions. She used this approach to prepare PTs to make their own enhancements. She frequently stated a hope she had—"Enhancing curriculum is something hopefully you would do in the future—lots of ways to do things better—what enhancements can you make to make lessons better?" [FN2010 Day16]. In addition, she prompted the class to consider, "How can you open the problem so it's more challenging?" [FN2010 Day17] or "How could you enhance the lesson?" [FN2008 Day3]. The purpose Kathryn articulated for this action was that she wanted PTs to be critical of curriculum. "Even curriculum that's been well developed…so I think we need to continue to push the curriculum envelope to say, how can we do this better" [VDFNInt2].

In order to attend to enhancing PTs' knowledge of curriculum, I identified 11 actions from 115 snapshots of Kathryn's practice as well as corresponding purposes that she employed in her elementary mathematics content/methods class. Almost half of the actions Kathryn employed in relation to the curriculum component focused on the first two actions listed in the category of "scope and sequence of curriculum materials." In these snapshots, Kathryn shared information regarding the grade level and order that various mathematical ideas are taught in elementary school.

Enhancing Prospective Teachers' Knowledge of Student Understanding

The third area of PCK that Kathryn employs specific actions to develop PTs' knowledge is student understanding. I identified four major categories: (a) sample grade 1-6 student mathematical answers; (b) predicting grade 1-6 student mathematical responses; (c) grade 1-6 student mathematical misconceptions or error patterns; and (d) mathematical concepts that are abstract or confusing for grade 1-6 students. Table 12 summarizes the actions for each of these four major categories.

Table 12

Summary of Actions Kathryn Employs to Develop Elementary Prospective Teachers' (PTs') Pedagogical Content Knowledge of Student Understanding

Categories of student understanding	*Actions*
Sample grade 1-6 student mathematical answers	States atypical grade 1-6 student answer/thinking to mathematical concept under discussion
	States incorrect grade 1-6 student answer to mathematical concept under discussion that relays the student's incorrect mathematical understanding about the topic under discussion
Predicting grade 1-6 student mathematical responses	Prompts PTs to predict mathematical answers/strategies grade 1-6 students come up with/predict how grade 1-6 students will solve posed mathematical problems (which were posed to the PTs or PTs discussed as whole class)
Grade 1-6 student mathematical misconceptions or error patterns	Shares misconceptions and/or error patterns grade 1-6 students (and teachers) have regarding mathematical concept(s) under discussion
Mathematical concepts that are abstract or confusing for grade 1-6 students	Articulates mathematical concepts that are abstract for grade 1-6 students
	Articulates language issue grade 1-6 students may have that is interfering with grade 1-6 student's understanding the mathematics under discussion
	Describes examples of mathematical connections that grade 1-6 students may not make

Below I describe Kathryn's actions and purposes for employing the identified actions in her mathematics content/methods course to facilitate PTs' PCK specifically related to student understanding.

Sample grade 1-6 student mathematical answers. Kathryn engages in two actions under this category: (a) *she provides examples of atypical grade 1-6 student approaches/thinking to mathematical concepts under discussion* and (b) *she states incorrect grade 1-6 student answers*.

During one class, Kathryn shared a situation from a lesson she taught to a fifth-grade class. During this lesson, Kathryn asked the fifth grade students to create a rectangle that had a width of five centimeters using a wire that was 24 centimeters long. Kathryn relayed that most students in the class bent the wire into a rectangle. However, she did not anticipate that two boys would get out a pair of scissors and cut the wire. But when they did, they each found a rectangle with a width of five in two different ways. One boy took the piece of 24-centimeter wire and cut it in half to get two 12-centimeter pieces of wire. He then cut five centimeters off both of the 12-centimeter lengths. A second boy cut off five centimeters and then another five centimeters from the 24 centimeter piece of wire. Then, he took half of the 14-centimeter piece of wire to get the two seven centimeter pieces of wire [FN2008 Day16]. When I asked Kathryn why she shared the thinking of these two boys with PTs, she identified multiple purposes. She reflected,

> K: When they are elementary teachers, they're going to encounter…what kids do and say, or if they give children opportunities to make their thinking visible…they're going to see answers they did not anticipate. They're going to encounter situations where they don't know how to respond. They're going to encounter situations where they don't know what the child means. And I think that too often they enter my class thinking that in math there's one answer and there's one way to do it. And I want them to leave my class realizing that that's not the case. That sometimes there is only one answer, but there are multiple ways to get that answer. There are multiple representations that a child could use to help them think about that problem.…I want to make sure that that's a part of my class. And I want them to know that kids are very creative and come up with lots of interesting things that they won't anticipate. So when I refer to the wire problem, and what the children do with that, you know, that was in a, I was teaching, I think a 5th grade class maybe and I didn't anticipate kids to take scissors out and actually physically cut the wire. Why, I do not know. I guess I didn't think they could open up their desk and all of a sudden scissors would be there. Most classrooms I worked with in the city of Chicago, scissors would have been something controlled.
> C: In a cupboard.

> K: In the cupboard and not necessarily kids would have scissors in their desk. So this was a teacher that obviously was able to manage his classroom. And so here those kids gave me a teachable moment. I think that when I tell that story, I kind of talk about how I didn't anticipate this, kids doing this. But at the same time, those children were very creative and that gives me examples to say, I was, instead of punishing them for cutting the wire, I was like, wow, this is terrific. This is a great way to think about this problem. And it was better than just folding the wire. Physically cutting really helps the other children see what was going on with that problem. So, you know, that's an example that I think is a pretty powerful one. That gets at those kind of purposes. [VDFNInt1]

In this example, not only does Kathryn share two fifth grade boy's approaches to the problem, but she also admitted her surprise about these approaches—a method of thinking that she had not anticipated.

Not only does Kathryn provide examples of atypical grade 1-6 student approaches or thinking, but she also *states incorrect grade 1-6 student answer(s) to mathematical concepts under discussion*. In the following example, a PT had just volunteered to come up to the overhead and demonstrate for her colleagues how she would teach fifth grade students how to read a ruler (that is marked in 16^ths^ of an inch). Kathryn prepared a line 2 ¼ inches long on an overhead projector. The PT started to describe to her classmates how to measure the line with a ruler in inches and then Kathryn interrupted the PT's explanation with an incorrect answer she had seen fifth grade students share in class.

> S1: So I want to make sure to line up the zero mark and then I have a little over two inches and it goes to the fourth tick mark. Does anyone know what that is? [long pause] Hmm.
> K: Okay. Ready? I observed this exact problem last year in a 5th grade room. Okay? And the first answer. A kid raises his hand. You called on the first kid and he says, the teacher says, how long is that line? And what does he say? He says 2.4.
> S2: That's what I would've said. (Several other students nod and say "I agree.")
> K: Now how are you going to rectify? Now you are the teacher. You're in the moment—2.4, it happens every year I watch this lesson.
> S: Two and four tenths.

S: of an inch.
[Video Day 17]

In this example, Kathryn stated an incorrect answer that a fifth grade student provided in class where the same length of line was being measured. However, it was evident by the long pause in Kathryn's class that the PTs were not certain what the correct answer was. This was further confirmed when one PT said, "That's what I would've said" and several other PTs agree with that student. A discussion ensued about what each tick mark represented on the ruler and the PTs did become convinced that the line was indeed 2 ¼ inches long. Kathryn further commented that teaching fifth graders how to measure with a ruler is not easy. "They have had a lot of good fraction experience to prepare them for that type of application of fractions, but still, it's not very easy." She further articulated dilemmas to the PTs by stating,

> K: Some of the fraction rulers, I mean inch rulers here are in 10^{ths}, some are in 16^{ths}. So they differ. So what if you've got one table, you've got one kid that brought in his ruler from home because the school said for everybody to go buy a ruler on the school supply list. And so you've got two kids at this table doing 10^{ths} and at this table they're doing 12^{ths} and 16^{ths}. How is that going to help you teach this lesson? It'll be terrible. It'll be just terrible. So yeah, so what do I see? I see every year in the fifth grade classrooms that I go observe this lesson. Kids are going, well that's two inches and one centimeter. Or that's 2.4 inches.
> [Video Day17]

Here, Kathryn posed these two incorrect student answers for several reasons. She recollected,

> Well usually my number one reason is because *those answers are in my class*. And so my number one reason is it's a way for me to address wrong answers of my students in a way that doesn't put them on the spot. I mean, I had people in the room that would have said those exact same, even though I've seen 5th graders say those two answers, I had students in this class that were thinking that. And that's and said that out loud. So, um, so I also encourage them to use that strategy. *I encourage them when they are teaching to have fictitious children who make mistakes*. They're based on

> the mistakes that they are seeing. So here I'm modeling, even though I'm not explicit about this one because, you know, there's a fine line here. But you know, if they were like to get inside my head, I'm doing something that I want them to do. *I want them to put wrong work up on the board*. Um, and when you're starting to do that you can't put a kid on the spot. You've got to build an environment where kids are willing to share their wrong answers. And so, you know, in my professional development of teachers, *I'm always introducing that strategy* to them, so they'll put an example up on the board and say, you know a child from last year did this or a child from the other class did this and what do you think about this? And you know, *so half the kids in the room have that answer but you're not picking on anybody in the room. I think it's a great teaching strategy*. I use it here with them. Um, but they're so many, there are so many students in this class that are really afraid—have such high math anxiety, they don't you know, they feel really uncomfortable because they don't know the math and for them it's an issue of pride. They don't want to look stupid in front of their peers and so *I think it's a safer way to get at wrong answers in the room*. [EVT#3]

To summarize, in this excerpt, Kathryn stated several purposes for why she posed incorrect grade 1-6 student answers in class. Her reasons include, but are not limited to: (a) the PTs have these incorrect answers in her class—it's a safe way to address wrong answers in the room without picking on individual PTs, (b) encourage the PTs to use the teaching strategy of using fictitious children who make mathematical mistakes, and (c) encourage PTs to put incorrect student work on the board in their future classrooms.

Predicting grade 1-6 student mathematical responses. From the very beginning of the semester as well as throughout the semester, Kathryn asked PTs to *predict what grade 1-6 students will say* to a posed mathematical problem or concept currently under discussion in the mathematics content/method class. Kathryn acknowledges that she uses this approach quite a bit because she wants PTs to keep thinking about kids when they plan and teach lessons. Kathryn admits that many times when she asks the PTs to predict what grade 1-6 students would say, the question is rhetorical, but she keeps posing the questions because PTs need to keep thinking about children. Kathryn elaborated based on her observations of lessons that PTs teach in their university field experience in local elementary schools while they are enrolled in her course,

> They're so focused on themselves that they don't think about kids. And they focus on what they're going to do. And I try and get them to think about the kids. And so don't just think about the questions you pose, think about the answers you're going to get and how are you going to respond to those answers. Now with no teaching experience, I recognize that they're not very good at this. They don't know what to expect…but I keep throwing [those questions] out there. [EVT#3]

Kathryn recognized the PTs lack experience in elementary classrooms, but she reiterated that she wanted the PTs to consider their students in addition to their own teaching practice or how they felt.

Sometimes, when Kathryn prompted PTs to predict grade 1-6 student answers, she reiterated the answer to the question herself after the PTs offered their suggestions. An example of this is seen when Kathryn engaged the PTs in the game Roller Derby—a game where two dice are rolled, the sum is computed, and if an individual has a counter on the number of the sum that is rolled, one counter may be removed. She asked the PTs to predict where elementary students put their 12 counters on the game board that has the numbers 1 through 12. She stated,

> K: So let me ask you this question, I have now done this game in Kindergarten, first, second and third grade now. What do you think the number one answer is? …How kids put their distribution down?
> Ss: One on each.
> K: They put one on each. [Video Day2]

At other times, Kathryn answered the question before the PTs have an opportunity to answer the posed question as seen in the next example.

> K: Some of the big measurement ideas that you need to have, these are kind of your goals and things you are assessing. First you must include a number and a unit. So if I am measuring how long this is with paper clips, it is 20 paper clips. Twenty is my number and paper clips is my unit. So

> you have to have this idea that our measurements have a number and a unit. You may compare two measurements if the same unit is used. So, if I say that this table is this line here and that is 30 inches and this table here is five feet. Which one is longer? The five feet or 30 inches?
> S: Five feet.
> K: But what are the kids going to say? They are going to say this one because the number is bigger. Now, I cannot make those comparisons because my unit is different. These are the big ideas of measurement that take a lot of time to develop. This is not something that you are just going to tell them. You are going to develop this by doing a lot of different tasks. [Video Day19]

In this next example, Kathryn provided the PTs the opportunity to make their own mathematical predictions about typical fifth grade responses for a mathematical situation by having them engage in the mathematical task themselves. First, Kathryn gave the PTs three squares and asked them to create all the possible shapes with the three squares (given specified constraints). Then the PTs worked in groups to find the number of different shapes they could make with four squares where the side of one square must be flush with the side of another square (i.e., the corner of one square may not solely touch the corner of another square nor may just half of the side of one square solely touch half of the side of another square). Kathryn then prompted the PTs to make a prediction for how many different shapes were possible using five squares. See Table 13.

Table 13

The Number of Different Shapes That May Be Made With 3, 4, and 5 Squares

Number of squares	*Number of different shapes*
3	2
4	5
5	?

She said to the PTs,

> K: Ok so now we [haven't done] this problem, but I want a prediction. So kids often are looking for patterns, so we are going to have some guesses for how many shapes we are going to find with five squares. What are your guesses and predictions? So you are not actually finding them yet, you are just guessing. [Video Day13]

At this point, there was a whole class discussion where PTs made predictions about how many different shapes could be formed using five congruent squares. They started by stating their predictions, but then Kathryn prompted them to provide answers they thought elementary students might pose. The predictions, with justification they provided included: (a) 10 (doubled five); (b) six (difference of one); (c) eight (difference of three between the two and five in the "number of different shapes" column, so add the difference of three to the five); and (d) seven (2 + 5 in the "number of different shapes" column). However, the PTs did not provide all of the answers/describe all of the patterns that Kathryn had observed school children provide. The PTs were still missing two responses that grade 5 students gave when she posed this problem to them. Kathryn decided that she would state the two patterns that students saw that the PTs had not identified. She commented,

> K: You are missing a couple other ones that kids come up with…There are two right answers in my brain that you are trying to figure out…I will just tell you. Ok, 3 + 2 = 5, so 4 + 5 = 9. Ok? This is a sophisticated child; let me see if I can remember it—2 × 2 + 1 = 5 so 5 × 2 + 1 = 11. Yes, sign that kid up. That kid is doing some thinking. So, this one wasn't in the chart and says if I double two and add one I get five, so I am going to double five which is 10 and add one which is 11.

Kathryn's purpose for sharing predictions fifth graders articulated is that she "want[s] them to know that kids are very creative and come up with lots of interesting things that [the PTs] won't anticipate" [VDFNInt1].

Throughout the semester, Kathryn asked PTs to predict grade 1-6 student answers in various contexts. For example, she asked PTs to predict what a student would say when she engaged the PTs in analyzing student work—specifically, "What do we hope here" when the subtraction problems 25 – 21 and 103 – 99 are posed [FN2008 Day13]. Here, Kathryn shared that she would expect a child to solve these two subtraction problems without using the traditional algorithm. She also shared a study that posed two rectangles with dimensions labeled on some of the sides and the mathematical question was to find the perimeter of each figure. One rectangle had two adjacent sides labeled as 9 and 13 while the other rectangle had all four sides labeled as 8, 12, 8, 12. Kathryn asked the PTs to predict what elementary students would do [FN2010 Day14]. In both of these examples, Kathryn prompted PTs to predict how elementary students would solve the posed mathematical problems, which was different from predicting how grade 1-6 students would place 12 counters on a game board to play a game or would predict which length (e.g., five feet or 30 inches) was longer. However, all five of these examples support the variety of ways Kathryn asked PTs to predict grade 1-6 student responses in order to encourage the PTs to think about kids and what kids will say and do.

Grade 1-6 mathematical misconceptions or error patterns. Kathryn *shares misconceptions and/or error patterns grade 1-6 students have regarding mathematical concept(s) under discussion.* Kathryn shared these misconceptions by engaging PTs in analyzing student work. For example, PTs analyzed student work where a child subtracted two-digit and three-digit numbers. They were then asked to complete two additional subtraction problems as if they were the child. In one case, the error pattern was that the fictitious child was always subtracting the smaller digit from the larger digit no matter where it was placed (e.g., top or bottom if the subtraction problem was written vertically). Kathryn said, "You will see this if you teach subtraction—very common" [FN2010 Day10]. Kathryn engaged PTs in analyzing student work because PTs think it is relevant for them as future teachers [InitialInterview]. In addition, she wanted PTs to be aware of misconceptions and look for misconceptions; she wanted them "to actually look at student thinking, analyze student work, [and] try and figure out what [these children are] thinking" [VDFNInt1].

A second way Kathryn used student work was to not only identify mathematical misconception(s) in the sample student work, but to also emphasize potential teacher moves PTs could implement to attend to the misconception(s) under discussion. In the following excerpt, George, a fictitious student, is always borrowing whether he needs to or not and Kathryn prompted the PTs to think of what the teacher could have done to facilitate this misconception. (See Ashlock, 2006, for student work.)

> K: Ok, what about George?
> S: He always borrows whether he needs to.
> K: Always borrows whether he needs to or not. Ok. What can create such a thing?
> S: Just giving the rules of borrowing.
> K: Just giving the rules of borrowing is one, but there is a second thing here that can create this kind of misconception.
> S: Not knowing when you really need to?
> K: Not knowing when you really need to, but there is a thing that can facilitate it. There is something that a teacher can do that can actually facilitate this misconception.
> S: Maybe not going over problems that you don't need to.
> K: Right exactly. So the teacher gives George a hundred problems that he has to borrow every time, so what does he create in his mind? I am always going to borrow. So when you are teaching just a rule and then you just give problems that follow that rule, then they over generalize and it

> becomes problematic and those misconceptions are formed. Ok, so this has happened to George, now what are you going to do? Tell him? No George, you don't always borrow.
> S: Give him examples where you are not going to borrow so he can see the difference between the two.
> K: Ok, so the problems you select for children to do is very important. Don't underestimate the task where you select what you give kids. Don't go by just what is in the textbooks because they don't necessarily always think about this. So you have to analyze the problems you are giving kids and be careful in your selection of them.
> [Video Day9]

In this situation, Kathryn was explicit with the PTs regarding how they needed to be purposeful when selecting practice problems for grade 1-6 students to complete. A consequence of not carefully selecting problems for students to complete may foster a misconception and cause grade 1-6 students to over generalize a mathematical concept—in this case, always borrowing whether necessary or not. Here, Kathryn explicitly articulates a strategy she wants PTs to use when they are teachers—to purposefully select problems PTs intend to have grade 1-6 students complete [VDFNInt1].

Kathryn also provides specific mathematical misconceptions grade 1-6 students have around the mathematical tasks PTs complete in class. Some examples include: (a) When listing all possible outcomes when flipping three coins, some students think HHT and THH are the same [FN2010 Day4]; (b) If students finding the perimeter of a shape in the form of steps (see Figure 13), then they will count the diagonal (x) from one step to the next as one unit, instead of counting the legs (y and z) as two units [FN2008 Day16]; and (c) Students think the slant height (s) is the same as the altitude (a) for a parallelogram (see Figure 14) [FN2010 Day18].

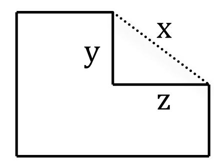


Figure 13. Students count x rather than y and z when finding the perimeter of this polygon.

Figure 14. Students assume the slant height (s) is the same length as the altitude (a).

When Kathryn was asked why she provided examples of these mathematical misconceptions while the PTs learned these concepts, she stated, "Some of the [PTs] have these misconceptions, so I blame children when I know that half of my kids have these same misconceptions" [VDFNInt1].

Mathematical concepts that are abstract or confusing for grade 1-6 students. In this category, Kathryn engages in three actions: (a) *articulates mathematical concepts that are abstract for grade 1-6 students*, (b) *articulates language issues grade 1-6 students may have that will interfere with understanding mathematics*, and (c) *describes examples of mathematical connections that grade 1-6 students may not make*.

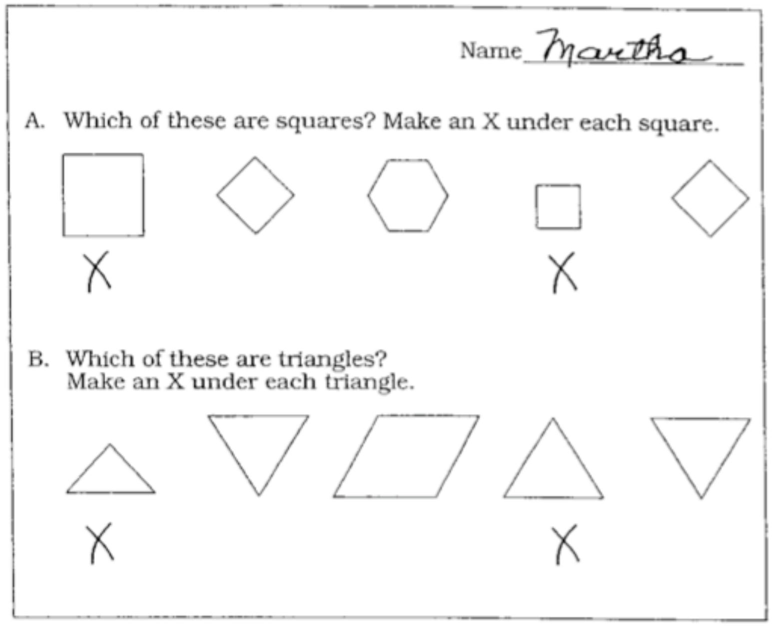
In the first action, *articulates mathematical concepts that are abstract for grade 1-6 students*, Kathryn specifically uses the word "abstract" based on her earliest experiences teaching PTs. She recollected,

> [PTs] would use these words abstract and concrete. And they had very funny ideas about what that meant. And so…concrete meant "the

> manipulatives" and abstract meant, "no manipulatives." And, it was really interesting to see what they would think about these words. So I wanted to kind of challenge that idea about what abstract means…. if a child has no meaning for these things, it doesn't matter if they have a physical object in their hand. So one of the things that I'm constantly doing is saying, this might be abstract for the child. They might not know what this means. They might not know what this table means. They might not know what this manipulative means [represents]. [VDFNInt1]

Kathryn admitted that in the past, she had not been explicit with PTs about her purpose for using the word "abstract," but that was something that she hoped to be more explicit about in the future.

Kathryn attends to mathematics that is abstract for grade 1-6 students by asking PTs to analyze student work. In one sample of student work from Ashlock (2006), the fictitious child placed an "x" beside all of the figures that were squares and all of the figures that were triangles. The fictitious child only marked the figures that had a specific orientation (see Figure 15). During whole group discussion, Kathryn stated that the orientation was throwing the child off and asked the PTs what they would do to help correct this child's misconception. Several students offered ideas. For example, one student said you could "Take a water bottle and turn it on its side – [it's] still a water bottle" and another student suggested "Take manipulatives and paper and rotate—cut out shapes and place on top of each other." Kathryn responded with "Square and triangle are more abstract" [FN2008 Day18].



Note. ASHLOCK, ROBERT B., ERROR PATTERNS IN COMPUTATION: USING ERROR PATTERNS TO IMPROVE INSTRUCTION, 9th Edition, © 2006. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ

Figure 15. An example of a fictitious child's work.

Kathryn also mentioned mathematical representations that were abstract for grade 1-6 students regarding the mathematics PTs engaged with throughout the semester. For example, the PTs engaged in a task where they flipped three coins and then represented all of the possible outcomes, using different representations. After the PTs placed the different representations the class created (e.g., pictures, lists, diagrams) on the white board, there was a whole class discussion. During the discussion, Kathryn said that representing all of the possible outcomes in a table (one representation that a PT put on the whiteboard) "[wa]s more abstract than the list for kids" (another representation a PT shared on the whiteboard) [Video Day3].

Another example where Kathryn articulated that certain mathematical problems posed for grade 1-6 students were more abstract was when PTs analyzed and sequenced, by level of difficulty, four different worksheets involving various geometrical shapes. The geometrical shapes on all four sheets were drawn either on grid paper or dot paper. However, in this example, a PT posed a geometrical shape, without the corresponding dimensions, (see Figure 16) that an elementary student the PT was working with could not determine the area. Kathryn said that this was a typical question that third graders are asked to solve on the state assessment exam and that "This [shape] is more abstract because it doesn't have the grid" [Video Day18].

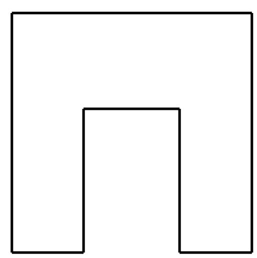


Figure 16. An abstract figure for third graders to find the area because it does not include a grid inside the shape.

The second action Kathryn engages in is where she *articulates language issues grade 1-6 students may have that will interfere with understanding mathematics.* An example of this action was seen when Kathryn shared a personal story of an interaction she had with one of her children when they were toddlers. She shared, "My kid is in a car seat and I say, what is 5 + 3?" Kathryn goes on to say that her child could not do that problem because he had never heard the word *plus*. However, her child could answer the next question Kathryn posed to him which was, "You ate five cookies. I give you three more. How many cookies do you have?" [FN2008 Day13].

Kathryn's purpose for engaging in this action was to encourage PTs to bring up contextual situations that grade 1-6 students are familiar with in order to address the mathematical concepts under discussion. She stated, that "Giving kids contextual situations that they have experiences with where they can make sense and develop number sense, develop operations … through them" was a way for young children to learn various mathematical concepts. In the example with her toddler, she explained that he did not know what equal or plus meant because he had not experienced them; however he was able to solve addition situations involving familiar contexts in which he could relate [VDFNInt1].

The third action I identified in this category was when Kathryn *describes examples of mathematical connections that grade 1-6 students may not make.* There were several ways that illustrated this action. First, at times she explicitly described mathematical situations where grade 1-6 students experienced struggle or confusion. For example, in geometry, students need to be able to visually see the rotation of a figure and see how if you get a duplicate of the original shape by rotating the shape less than 360 degrees, then the figure has rotational symmetry. But, some students struggle with this concept of rotational symmetry—where geometric shapes must have the exact same orientation [FN2008 Day22]. In another example, Kathryn explained that elementary students might struggle with visualizing where to cut a geometric shape to re-organize it into another shape. Here, young children would need to physically make the cut to turn one shape (e.g., parallelogram where no two sides are perpendicular) into a second shape (e.g., rectangle) to discover that the area of a parallelogram is base time height [FN2010 Day18].

A second way Kathryn engages in this action is by attending to language that may confuse students. For example, at the beginning of the semester, Kathryn asked the PTs for the definition of a prime number. One definition that was shared included the term "one and itself" referring to the factors of a prime number. Kathryn asked, "Is one and itself a good definition" for a prime number? [FN2010 Day1]. Further, she stated that this definition is "really confusing for young children." When asked about her purpose for emphasizing this point to PTs, Kathryn emphasizes the PT belief that children should not be confused and that a good teacher explains things in clear and precise ways. Kathryn shared,

> I do know from assessing my students that to them, it's like a mortal sin to confuse children. They think that they're not going to confuse children. And when they define what a good teacher is, [they say] someone that explains things clearly. So if you were to say, you confused a child today. Oh, they would think, you know, that would be like a knife in the heart. So I think, I'm not sure, even though maybe it's not the right word that I kind of push their buttons when I use that word [i.e., confuse]. So if we think about the definition of prime, or one and itself… so here there's the language issue, so I do talk about when we use language that's confusing. And "one and itself" is a phrase that I think is very, would be very confusing to a lot of children, not just ELL kids, but I think a lot of kids. [VDFNInt1]

In this excerpt, Kathryn expresses her uncertainty of whether "confusing" is the right word to say to PTs, but her point is that many words used in a mathematical context are also used in other contexts in other ways (e.g., round—circle, oval, round in boxing, rounding up cattle, etc.). Sometimes words used in the mathematical context do not make sense to grade 1-6 students because they are familiar with using the words in a nonmathematical context and so they may have trouble understanding the specific mathematical meaning of the word.

Third, Kathryn *describes examples of mathematical connections that grade 1-6 students may not make*. She discussed that some of the PTs may encounter this situation in their future classrooms. During one lesson, she said,

> I was working with a girl on Friday here who is in third grade now and she is proficient at skip counting. That has been done in the earlier grades so she can go 5, 10, 15, and 20. She cannot tell you what time it is on the clock though. I said what if the big hands on a three, how many minutes is that? Oh I have no idea. What is five times three? I have no idea. But she can tell you 5, 10, 15. So she hasn't put together that idea how skip counting is related to multiplication. So that will be a big idea when you are working with this. [Video Day2]

Here, Kathryn provided a specific example from her experience of tutoring an elementary school child where the child had an issue of connecting skip counting with multiplication. Kathryn inserted this example while the PTs looked for patterns in a multiplication chart. She further asked the PTs, "What if kids do not see that the first row goes up by twos and the second row goes up by threes and so on—what if kids do not make that connection?" Here, Kathryn shifts from using an example of a child she worked with to posing a scenario that the PTs may encounter in their future classrooms.

Under the component of student understanding, I identified four categories from 73 snapshots, encompassing seven actions as well as corresponding purposes that Kathryn employed in her elementary mathematics content/methods course. From the snapshots I identified and coded for the student understanding component, approximately the same number of snapshots were identified for the four categories of student understanding.

Enhancing Prospective Teachers' Knowledge of Assessment

Of the total snapshots of Kathryn's practice tagged from over three non-sequential years, only 10.4% attended to enhancing PTs' knowledge of assessment—the PCK component I identified Kathryn attending to the least. The actions I identified are parsed into four categories, which include: (a) what content to assess, (b) the philosophy behind assessment, (c) strategies for how to assess and (d) the role of evidence. Table 14 summarizes the actions for each of these four major categories.

Table 14

Summary of Actions Kathryn Employs to Develop Elementary Prospective Teachers' (PTs') Pedagogical Content Knowledge of Assessment

Categories of assessment	*Actions*
What content to assess	Articulates content, as well as assessments/assessment items, that students are assessed on (e.g., standardized tests, classroom tests)
The philosophy behind assessment	Raises issues for PTs to consider or provides the purpose regarding various issues surrounding assessment that may potentially guide assessment decisions in the classroom
Strategies for how to assess	Articulates strategies that could be employed by the PTs to gather information about student understandings or misunderstandings as well as how standardized test items are graded/assessed and questions to consider prior to gathering information about student understandings or misunderstandings
The role of evidence	Attends to student work (or lack thereof) to gather information about student understandings or misunderstandings

Below, I describe specific actions and corresponding purposes Kathryn attends to in her class to develop PTs' knowledge of assessment.

What content to assess. Kathryn *identifies mathematics content and describes mathematics assessment items from* standardized tests. For example, Kathryn commented,

> The way this is assessed on a MAP [Missouri state assessment] test would be something like multiple choice; I am going to measure the distance between Columbia and St. Louis. Am I going to do it in inches, feet, miles or in gallons? Well, I am going to do it in miles because of the distance that is involved. So the idea of what is appropriate for certain lengths. So is it appropriate to measure this distance here in feet, in inches, millimeters? What would be the appropriate unit I would want to use for this distance? [Video Day19]

Kathryn shares this specific potential test item, as well as other test items throughout the semester, because she wants to encourage the PTs to know more than just a list of math topics that are covered on high-stake state assessments, but rather to inquire and become familiar with the specific items around measurement (or other mathematical concepts) elementary students are asked to complete on the annual state assessments [VDFNInt2].

The philosophy behind assessment. The second category Kathryn attends to under assessment is where she *raises issues for PTs to consider or provides the purpose regarding various issues surrounding assessment that might guide assessment decisions in the classroom*. One issue she raised is whether grade 1-6 students should show work when solving computational problems, which could be computed mentally (e.g., 103 – 99). During one class period, she raised the point that we want children to know when to use algorithms and when it may be more advantageous to complete a problem mentally. She explained,

> We would hope kids would look at this problem [103 – 99] and never use an algorithm. This is really silly, isn't it? We wouldn't want kids to use an algorithm here. Algorithms are ok if they [students] understand what they mean, but we want kids to know when to use them. [Video Day10]

Kathryn proceeded to share an experience from her work facilitating professional development. She recollected,

> I used to go around the country giving talks to third grade teachers and I would ask the following question: How many of you would mark that wrong? [Here Kathryn refers to a child that just writes "4" for the answer to the problem 103 – 99.] And guess what they would tell me? One hundred percent would say yes, that is wrong. It drives me crazy. I don't want a kid to use an algorithm for that problem, I want them to look at it and say it is four. I don't want them to have to use an algorithm for that [problem]. They [the teachers] said, they [the students] didn't show their work. What work? I can do that in my head, I don't need to show work. But they expected kids to show that [Kathryn displays the traditional algorithm on the board] for that problem. That is pretty silly. So when should kids use algorithms and when should they be able to do it in their head? [Video Day10]

In this excerpt, Kathryn raised the issue with PTs regarding whether grade 1-6 students needed to show all of their work all of the time or if there were appropriate situations where solving mathematics mentally would be acceptable. Kathryn explains to PTs that requiring students to show all their work will lower scores on some standardized tests. For example, she stated, “Teachers that made you write it down because they want you to show your work, well you get killed on a test [Iowa Test of Basic Skills Computation Test] like that. So it’s little things like that [PTs] don’t realize. I think it’s important for them to understand that” [VDFNInt2]. The issue of students always showing work may potentially be an issue when students complete classroom tests or standardized tests and may alter future assessment decisions PTs make.

Another assessment issue Kathryn raised with the PTs involved timed tests. She asked her students, “How do you feel about the tests kids take in math every day? What are they? Timed tests. Are you going to implement them? How do you feel about that?” [FN2008 Day15]. When I specifically asked Kathryn why she raises the issue with her PTs of whether grade 1-6 students should take timed tests or not, she reflected,

> I state things like, well, are you, every assessment that you design, are you going to let every child finish. Yes. And they’re adamant. Oh yeah. Okay. So, how long would you make the assessment? So that everybody could finish? So let’s say it takes your longest student an hour. When is the first student going to be done? You know, and so, so these are really hard questions for them. They’re not quite sure. They just know that I’m not going to do, I’m not going to do this to kids. Okay, are you going to give timed tests? You know, and they feel pretty strongly you should give timed tests in terms of kids knowing their facts, but yet I’d let everybody finish it. So they’re not quite sure what to do with all of this. This is what I’d call a discrepant event for them. They really don’t know how to make sense of this. Because they, one of their, another one of their buttons is fairness. They really don’t, they really think that everything should be fair and that’s one of their buttons. [VDFNInt2]

Here, Kathryn states that she wants to create a discrepant event for her PTs, but ultimately, she wants them to know that:

> Tests tell you very little information on any given day. They tell you very specific information. They can tell you good information. But we shouldn't make assumptions… if they're not using the data to inform their instruction, then it's probably a waste of their time. [VDFNInt2]

Strategies for how to assess. In this category, Kathryn *articulates strategies that could be employed by the PTs to gather information about student understandings or misunderstandings*. Some of these strategies provide efficient ways for PTs to assess whether their students are completing posed mathematical problems accurately. A couple of examples of strategies that Kathryn shared include: (a) Take a transparency [of a specific angle] and place the transparency over an angle a student just drew to quickly assess the accuracy of the angle the child drew [FN2010 Day24], and (b) Use a post-it note to quickly assess whether kids constructed a right angle, acute angle or obtuse angle [FN2008 Day21]. Kathryn's purpose for sharing these simple strategies is two fold. She says that PTs need strategies for both grading and assessing. She commented,

> You've got 30, you've got 25 kids in the room that are drawing an angle and you want to know if they do it accurately. Whether I'm grading it while you're teaching the lesson, or you're grading it after the lesson. I'm not going to go and measure all those darn things with the protractor. That would take too much time. So grading and time, you know, it can be a time consuming thing. So are there strategies that can help you do it more efficiently. And so if I create a transparency that's got the angle on it, drawn already, I just put it on top of it and I can tell how close are these kiddos at drawing this angle. So that's a strategy for grading, I would say, as opposed to a strategy for how to assess. Um, so I think you need both. Because I think you need strategies for how to assess, but you also, grading can be overwhelming and so they need strategies for how they deal with that challenge. [VDFNInt2]

Kathryn also shared the idea of conducting a one-on-one interview with a child in situations when they wanted to determine if a child really knew the mathematics under discussion or if he/she was cheating—due to not showing work as requested [Video Day10]. In other words, conducting a one-on-one interview is one way to determine whether a child understands the mathematics or whether the child copied the correct answer from another student.

Two final examples Kathryn shared related to strategies for assessing are examples of how PTs should *not* structure exam items. For example, Kathryn shared information about a released standardized test item with PTs and then discussed how students might be at a disadvantage when it comes to answering the posed question. For example, many computation problems are written horizontally on standardized tests, but some kids only see similar computation items written vertically in class [Video Day6]. Thus, Kathryn argued that grade 1-6 students should solve computation problems that are written both horizontally and vertically. Kathryn also shared student work from an open response item on a state mathematics assessment. Kathryn described to the PTs that this child was given pattern blocks to help complete the test item. However, the child chose to complete the posed problem by tracing the manipulative on the paper but had to stop because she ran out of space. There was not enough room on the answer sheet for her to use the manipulative the way she intended [Video Day12]. In these two examples of test items that Kathryn shared, she was implicitly addressing assessment strategies that PTs should attend to—in class, write computation problems both vertically and horizontally for children and second, provide enough space for children to solve posed test questions.

The role of evidence. The final component Kathryn attends to under assessment is where she *attends to student work (or lack thereof) to gather information about student understandings or misunderstandings*. Kathryn engaged PTs in analyzing student work where fictitious children make various mathematical errors. She prompted the PTs to: (a) complete several posed problems as if they were the child making the same mathematical error, (b) describe the mathematical problems that were given to the child, (c) determine which problems the child solved correctly and which problems the child did not solve correctly, (d) describe the child's error pattern, and (e) think of strategies that the PTs could use to help this child correct his/her error pattern (Ashlock, 2006).

Kathryn engaged the PTs in conversations around the various samples of student work she had them analyze. Frequently she made the following comments regarding the importance of having evidence from the child to back up how the PTs thought the child would solve a newly posed problem. For example, Kathryn made the following comments after she posed a new mathematical problem (e.g., a two-digit plus two-digit number) slightly different from the original problem (e.g., the sample student work consisted of the child adding the digits of a two-digit plus one-digit number incorrectly). She asked the PTs, "What would the child do?" However, the PTs had no evidence of what the child would do to solve this type of problem since all of the examples provided in the student work sample were two-digit and one-digit addition problems. Comments from Kathryn included: (a) "We really don't know – we don't have evidence of this – be careful you don't over-extend" [FN2008 Day4], (b) "We don't know what [this child] would do" [FN2008 Day13], (c) "We don't know – no evidence" [FN2010 Day9], (d) "We don't know – we could guess, but kids do funny things" [FN2010 Day8], and (e) "The point is – what do we have to do when students give answers – we need to ask questions to make sure we know how she's thinking" [FN2010 Day3].

In the five aforementioned examples, Kathryn was very explicit with the PTs that they needed to gather evidence about how a child may solve a posed problem rather than make an assumption based on some tangential information the PTs had garnered from student work the child had completed around similar mathematical problems. Kathryn explained,

> Kids make wrong answers for lots of different reasons. It could be just a careless error. It could be flawed thinking. I mean, they could really have no clue and it was a guess. I mean, we really don't know. Um, and sometimes when we ask a question, they'll even realize their own mistake and they'll correct it. So they knew what was going on, but it might have just been a careless error. They didn't write something down right. And so I think we make judgments all the time. I think teachers do this. [VDFNInt2]

Kathryn continued to say that too often she made pre-mature judgments about the mathematical capability of a child to solve problems. She personally reflected,

> I think I do this too often. I think I've gotten better over the years, but, where I'll look at a piece of student work and I'll make assumptions about what that kids knows. And I don't want them to do that. I want them to, so we'll look at student work and I'll say, well, that could be a possibility, but we just don't have evidence for that. [VDFNInt2]

Here, Kathryn's personal experience from when she made pre-mature assumptions about a child's mathematical knowledge without asking the child questions, had prompted her to articulate the importance of using evidence to make decisions about the mathematical capability of a child from student work. Additionally, she stressed the importance of posing additional problems or questions to a child after the teacher had analyzed the child's work. Using these questions or problems can help teachers clarify the child's mathematical thinking. Kathryn articulated her purpose for this specific action,

> It could be this or it could be that, but we don't have the evidence. So we would have to go back to [the child] and pose another problem or ask a question or whatever and so that's part of their practice that I want them to do. I want them to ask kids questions. I want them to ask kids, "How did you think about that?" I think there's this idea that good teachers ask questions. But I don't, I think it's important to be explicit about this idea about evidence. And I don't hear that too much in the discourse—t hat the reason we ask questions, is to gather evidence. That as a teacher I can make better informed decisions. [VDFNInt2]

In order to attend to enhancing PTs' knowledge of assessment, I identified four actions from 40 snapshots as well as corresponding purposes that Kathryn employed in her elementary mathematics content/methods course. It is important to note that the low number of snapshots I identified correlating to this PCK component is not representative of the amount of class time Kathryn spends attending to developing PTs' knowledge of assessment. As aforementioned for other actions that I identified limited numbers of snapshots for, Kathryn spends a significant amount of class time attending to various assessment related activities (e.g., providing PTs sample state assessment questions from third grade, fourth grade, and fifth grade for the PTs to analyze). Kathryn spent nearly one whole class period in 2007 and 2010 on this activity. Thus, despite the fact that I only tagged 10.4% of the snapshots as related to assessment, this is not in actuality representative of the amount of class time each semester Kathryn devoted to developing PTs' knowledge of assessment.

In the previous sections, I embedded identified purposes to correspond with specific actions. In the next section, I synthesize all purposes I identified from the data.

Kathryn's Purposes Corresponding to Identified Actions

Kathryn articulated the purposes for her actions (i.e., what she said and did) in her elementary mathematics content/methods course during the interviews. I identified Kathryn's purposes in two ways: (a) during interviews when I posed direct prompts about Kathryn's purpose(s) and (b) when she made unprompted statements that conveyed her purpose(s) (e.g., "I want them to know," "What I'm trying to teach them," "The big idea here I am trying to emphasize").

It is important to note that the purposes Kathryn articulated, and which I identified, were solely spoken in the interviews (i.e., initial interview, extended video talk interviews, and video/fieldnote interviews). I did not ask Kathryn for her purposes for every snapshot and action I identified. Therefore, Kathryn may have other purposes that she did not identify in the interviews. In addition, the purposes she did identify may correspond with other PCK components; however, this was not captured in the data sources.

I did engage in a thorough process to determine corresponding purposes for identified actions. For example, in the extended video talk interviews, Kathryn articulated why she did certain things when she was reflecting on the video clips she viewed. I asked Kathryn, "Can you articulate your purpose for why you bring up ____ with the PTs? Could you share why you ____. Why did you do that? What was your purpose for that? Throughout the semester, you ____. What is your purpose for _____?" In addition, I directly asked Kathryn her purposes centered around various snapshots of her practice in video/fieldnote interview #1 and #2. During the video/fieldnote interviews, I asked Kathryn, "Something that you frequently do is _____. For example, you will _____. Can you talk a little bit about your purpose for _____?" From analyzing the statements that Kathryn made about why she made the 34 specific actions discussed above, I identified 10 core purposes and the PCK components in which they corresponded (see Table 15).

Table 15

Alignment of 10 Core Purposes Identified From Kathryn's Practice to the Number of Actions Used to Develop Prospective Teachers' (PTs') Pedagogical Content Knowledge

Core purpose identifier	*Purpose*	*Number of actions per core purpose*	*Pedagogical content knowledge component*
A	Wants PTs to know strategies (i.e., that PTs will not come up with) to teach mathematics	15	Knowledge of assessment Knowledge of curriculum Knowledge of instructional strategies Knowledge of student understanding
B	Wants PTs to be thoughtful/purposeful/make good decisions about teaching mathematics	14	Knowledge of assessment Knowledge of curriculum Knowledge of instructional strategies
C	Wants to attend to PTs' mathematical beliefs and challenges they will face as teachers	10	Knowledge of assessment Knowledge of curriculum Knowledge of instructional strategies Knowledge of student understanding
D	Wants PTs to know different ways students in grades 1-6 think mathematically and the importance of language use with them	8	Knowledge of assessment Knowledge of instructional strategies Knowledge of student understanding
E	Wants to support PTs' learning of mathematical content (i.e., math errors students in grades 1-6 make are also errors the PTs make in the elementary content/methods course)	6	Knowledge of curriculum Knowledge of instructional strategies Knowledge of student understanding
F	Wants PTs to view course content as relevant to their future mathematics teaching practice	6	Knowledge of assessment Knowledge of curriculum Knowledge of instructional strategies Knowledge of student understanding
G	Wants PTs to be aware of the different nature and content of mathematics curriculum resources and instructional tools	6	Knowledge of curriculum Knowledge of instructional strategies
H	Wants PTs to know the mathematical content that grade 1-6 students learn	4	Knowledge of curriculum Knowledge of instructional strategies
I	Wants PTs to view her as a credible teacher of mathematics	3	Knowledge of curriculum Knowledge of instructional strategies
J	Wants PTs to focus on what students will do mathematically rather than what they would do as the teacher	3	Knowledge of student understanding

Kathryn identified these 10 core purposes in relation to multiple actions (3 to 15). Kathryn referred to three purposes more than the others: (a) Kathryn *wants PTs to know strategies (i.e., that PTs will not come up with) to teach mathematics*—15 actions—purpose A; (b) Kathryn *wants PTs to be thoughtful/purposeful/make good decisions about teaching mathematics*—14 actions—purpose B, and (c) Kathryn *wants to challenge/attend to PTs' mathematical beliefs and address challenges they will face as teachers*—10 actions—purpose C. She also referred to these purposes in relation to different PCK components. The examples in Table 16 illustrate how Kathryn articulated these three prevalent purposes via the different PCK components. The text represented in *italics* highlights the words I interpreted to align with the identified purpose.

Table 16

Examples of Kathryn's Three Most Prevalent Core Purposes

Core purpose	*Pedagogical content knowledge component*	*Example*
Wants PTs to know strategies (i.e., that PTs won't come up with) to teach mathematics	Knowledge of assessment	The teachers that made you write it down because they want you to show your work, well you get killed on a test like that. So *it's little things like that they don't realize*. I think it's important for them to understand that. [VDFNInt2]
	Knowledge of curriculum	I think often times they would look at these tasks and say, "Oh, this is just a worksheet for kids to do." It's just, not very exciting and you go step A, step B and so I want it to become where that's not the case. That kids are really actively thinking and that, you know, the math becomes alive to them. So, here they don't know I'm modeling. I'm not saying, I'm going to model how to do this. Because that was a word you used, but in a sense *I am modeling how they haven't seen the problem yet*. But I take them through an activity and it's going to be through reflection, but I'm going to be modeling how they could make something, you know, that looks pretty boring in written text to come alive [VDFNInt2]
	Knowledge of instructional strategies	They *don't really have any access to any other ideas* for teaching 5th grade, you know fractions of an inch when I'm measuring, so I want to give them some ideas that are going to help them know how to teach this content with kids making sense of it. [EVT#3]
	Knowledge of student understanding	I'd go over and ask a child, now show me how you got this and I want to see how they are counting around that figure. So *these are strategies that I want them to use when they are teachers*. [VDFNInt1]

Core purpose (continued)	*Pedagogical content knowledge component*	*Example*
Wants PTs to be thoughtful/purposeful/ make good decisions about teaching mathematics	Knowledge of assessment	We make judgments all the time. I think teachers do this. I think I do this too often. I think I've gotten better over the years, but, where *I'll look at a piece of student work and I'll make assumptions about what that kid knows. And I don't want them to do that*. [VDFNInt2]
	Knowledge of curriculum	What I'm trying to teach them is we have to be very careful of the problems that we select to give children to do. Because kids are not going to do every problem in a textbook. There are too many. So teachers make choices everyday on which problems kids are going to do and *I want them to learn that they need to select carefully*. [EVT#2]
	Knowledge of instructional strategies	*I want them to be purposeful in making decisions on what they're actually giving the kids*. Like even the math task assignment, even a worksheet they might design. I want them to be really thoughtful about how they design that worksheet. And so, you know, manipulatives. *If you pick the wrong manipulative, it will be a disaster*. [VDFNInt1]
Wants to attend to PTs' mathematical beliefs and address challenges they will face as teachers	Knowledge of assessment	And they're talking about content that I know is not for specific grade levels, is not on the MAP test. But *they have this idea* that it is. And maybe that's how they'd solve the problem, but the kids they're teaching at the age levels they're teaching wouldn't solve it that way. *So I want to kind of take that on a bit* and just say, well let's, actually analyze these tests. And so I give them you know, sample items that say, well what is the math here? And that's a practice that I would want them to consider when they give these standardized tests. So I want them, actually after they've administered them. What's on these tests? What are they actually asking kids to do? [VDFNInt2]
	Knowledge of curriculum	*Elementary teachers, they go into the profession because they love kids and want to do the best thing*, the best, make a difference with kids and *so I'm tugging at that a little bit*. [VDFNInt2]
	Knowledge of instructional strategies	Manipulatives can be ineffective. I think they come in with these glorious ideas that manipulatives will always be great. Smartboard will always be wonderful. Groups will always be wonderful. And so *I want to hit those misconceptions pretty hard. I consider those misconceptions of teaching. Groups don't always work, manipulatives don't always work. You have to decide which manipulatives to use. And sometimes the overhead is still better than the smartboard*. Um, and so which technology do you use when, when do you want to use chart paper if you want to save it. You'll probably use chart paper, but if you want to put it up for a short period of time, you can use the smartboard. So questioning why those things work, when those things work and why do you use them [EVT#1]
	Knowledge of student understanding	The point that kids might represent an answer in different ways and in some cases, that might be okay. There might be one way to represent an answer. Because *I do think they [PTs] come in thinking there's one right math answer and there's only one way you can write it. And so I want to kind of tackle that issue* to say, now this could be okay if. You know, and so forth. [VDFNInt1]

The remaining seven core purposes (see Appendix E) correspond to 3 to 8 actions and across 1 to 4 PCK components. The purpose where Kathryn *wants PTs to view course content as relevant to their future mathematics teaching practice* was the only other purpose that Kathryn articulated for actions across all four PCK components.

As demonstrated in the data above, Kathryn is purposeful regarding what she says and does in her elementary mathematics content/methods course. She does not just do activities for activities sake, but she has a purpose for the actions she employs. Moreover, each action does not have a sole purpose. Appreciating Kathryn's 10 core purposes distributed across numerous actions and various PCK components demonstrate the complexity of preparing PTs to teach mathematics.

Discussion

As discussed in Chapter II, some authors have described specific activities that MTEs use in methods courses (e.g., Castro, 2006; Goodell, 2006; Van Zoest & Stockero, 2008). However, the findings presented contribute new insights into MTEs' actions and purposes in relation to developing PCK. While this work was unique to the existing literature in several ways (e.g., it was not a self-study; it was longitudinal; the methods involved analyzing videos of a MTE's practice; and it utilized the PCK framework outlined by Magnusson et al., 1999), the major contribution is the conceptualization of one MTE's actions and purposes, which I summarize below. Moreover, I will highlight three big ideas that extend the literature: (a) Kathryn's actions in relation to the four PCK components were interdependent, (b) Kathryn's purposes directed her actions, and (c) Kathryn's actions related to all four PCK components were tied to elements of teaching mathematics in grades 1-6.

In response to the first research question, my findings indicate that Kathryn employed 16 categories of 34 actions in seeking to develop PTs' PCK. She utilized five categories of actions to support PTs' knowledge of instructional strategies and four categories for each of the other three components (i.e, knowledge of curriculum, knowledge of student understanding, and knowledge of assessment). In response to the second research question, Kathryn had 10 core purposes that aligned with her actions. Tables 17-20 summarize Kathryn's 34 actions aligned to their respective categories along with the core purposes. This classification is new to the literature and thus, makes a major contribution to the existing knowledge base of mathematics teacher education.

Table 17

Actions Kathryn Employed By Descriptive Category and the Core Purposes She Articulated for Actions Identified Under the Pedagogical Content Knowledge Component of Instructional Strategies

Categories of instructional strategies *Mathematics teacher educator teaches prospective teachers about*	*Actions*	*Core purposes*
Mathematical activities	Engages PTs in mathematical activities they may use in their future classrooms	Wants PTs to know strategies (i.e., that PTs will not come up with) to teach mathematics Wants PTs to be thoughtful/purposeful/make good decisions Wants to attend to PTs' beliefs and challenges they will face as teachers Wants PTs to know different ways students in grades 1-6 think and the importance of language use with them Wants to support PTs' learning of mathematical content (i.e., math errors students in grades 1-6 make are also errors the PTs make in the elementary content/methods course) Wants PTs to view course content as relevant to their future practice Wants PTs to be aware of the different nature and content of curriculum resource and instructional tools Wants PTs to know the mathematical content that grade 1-6 students learn Wants PTs to view her as credible
	Describes or mentions activities PTs may use with grade 1-6 students	
Instructional tools	Explains how to make homemade manipulatives for use in future classrooms or use common items in place of "catalog manipulatives"	
	Shares tip(s) about teaching with instructional tools (e.g., manipulatives, calculators, rulers)	
Teaching mathematics	Advises PTs on what to physically give to children when teaching various mathematical concepts	
	Advises PTs on what to say, write, and/or have students do when teaching various mathematical concepts	
Organizing mathematics lessons	Selects and sequences student work for a whole class discussion	
	Articulates typical structure of a whole lesson	
	Discusses ideas to attend to, consider, and/or draw from when designing mathematics lessons	
Emphasizing language	Prompts PTs to articulate directions to mathematical tasks so students in grades 1-6 will have access to the task	
	Explains the importance of connecting verbal and written language with visual aids	
	Encourages PTs to be thoughtful about the language they use (e.g., when asking questions or writing mathematical tasks)	

Table 18

Actions Kathryn Employed By Descriptive Category and the Core Purposes She Articulated for Actions Identified Under the Pedagogical Content Knowledge Component of Curriculum

Categories of curriculum *Mathematics teacher educator teaches prospective teachers about*	*Actions*	*Core purposes*
Scope and sequence of curriculum materials	Articulates grade levels that specific mathematical ideas are taught Identifies the order that mathematical ideas are taught Selects worksheets to engage PTs in sequencing tasks for grade 1-6 students regarding important mathematical concepts	Wants PTs to know strategies (i.e., that PTs will not come up with) to teach mathematics Wants PTs to be thoughtful/purposeful/make good decisions Wants to attend to PTs' beliefs and challenges they will face as teachers Wants to support PTs' learning of mathematical content (i.e., math errors students in grades 1-6 make are also errors the PTs make in the elementary content/methods course) Wants PTs to view course content as relevant to their future practice Wants PTs to be aware of the different nature and content of curriculum resource and instructional tools Wants PTs to know the mathematical content that grade 1-6 students learn Wants PTs to view her as credible
Nature and content of curriculum materials and resources	Provides examples from curriculum resources Shares and demonstrates supplemental curriculum resources Addresses vocabulary in curriculum resources Articulates differences in design and specific features in mathematics curriculum materials Identifies topics in mathematics curriculum materials that are (a) presented in inadequate ways or (b) enacted in unproductive ways	
Analysis and enhancement of curriculum materials	Asks PTs to compare what was done in class with how curriculum materials are written for grade 1-6 students Prompts PTs to analyze the mathematics in curriculum materials or activities completed in class Prompts PTs to consider how they could alter materials to enhance students' mathematical experience	

Table 19

Actions Kathryn Employed By Descriptive Category and the Core Purposes She Articulated for Actions Identified Under the Pedagogical Content Knowledge Component of Student Understanding

Categories of student understanding *Mathematics teacher educator teaches prospective teachers about*	*Actions*	*Core purposes*
Sample grade 1-6 student mathematical answers	States atypical 1-6 student answer/thinking to mathematical concept under discussion	Wants PTs to know strategies (i.e., that PTs will not come up with) to teach mathematics Wants to attend to PTs' beliefs and challenges they will face as teachers Wants PTs to know different ways students in grades 1-6 think and the importance of language use with them Wants to support PTs' learning of mathematical content (i.e., math errors students in grades 1-6 make are also errors the PTs make in the elementary content/methods course) Wants PTs to view course content as relevant to their future practice Wants PTs to focus on students (rather than themselves)
	States incorrect grade 1-6 student answer to mathematical concept under discussion that relays the student's incorrect mathematical understanding about the topic under discussion	
Predicting grade 1-6 student mathematical responses	Prompts PTs to predict mathematical answers/strategies grade 1-6 students come up with/predict how grade 1-6 students will solve posed mathematical problems (which were posed to the PTs/PTs are discussion as whole class)	
Grade 1-6 student mathematical misconceptions or error patterns	Shares misconceptions and/or error patterns grade 1-6 students (and teachers) have regarding mathematical concept(s) under discussion	
Mathematical concepts that are abstract or confusing for grade 1-6 students	Articulates mathematical concepts that are abstract for grade 1-6 students	
	Articulates language issue grade 1-6 students may have that is interfering with grade 1-6 student's understanding the mathematics under discussion	
	Describes examples of mathematical connections that grade 1-6 students may not make	

Table 20

Actions Kathryn Employed By Descriptive Category and the Core Purposes She Articulated for Actions Identified Under the Pedagogical Content Knowledge Component of Assessment

<table>
<tr><th>Categories of assessment

Mathematics teacher educator teaches prospective teachers about</th><th>Actions</th><th>Core purposes</th></tr>
<tr><td>What content to assess</td><td>Articulates content, as well as assessments/assessment items, that students are assessed on (e.g., standardized tests, classroom tests)</td><td rowspan="4">Wants PTs to know different ways students in grades 1-6 think and the importance of language use with them

Wants PTs to view course content as relevant to their future practice

Wants PTs to know strategies (i.e., that PTs will not come up with) to teach mathematics

Wants PTs to be thoughtful/purposeful/make good decisions

Wants to attend to PTs' beliefs and challenges they will face as teachers</td></tr>
<tr><td>The philosophy behind assessment</td><td>Raises issues for PTs to consider or provides the purpose regarding various issues surrounding assessment that may potentially guide assessment decisions in the classroom</td></tr>
<tr><td>Strategies for how to assess</td><td>Articulates strategies that could be employed by the PTs to gather information about student understandings or misunderstandings as well as how standardized test items are graded/assessed and questions to consider prior to gathering information about student understandings or misunderstandings</td></tr>
<tr><td>The role of evidence</td><td>Attends to student work (or lack thereof) to gather information about student understandings or misunderstandings</td></tr>
</table>

There have been other studies such as self studies (e.g., Tzur, 2001), professional development opportunities for MTEs (e.g., Dolk, et al., 2002), as well as collaboration between novice MTEs and a more experienced MTE (Van Zoest, et al., 2006) which provide resources for MTEs to reflect on their practice and educate MTEs who work with practicing and inservice teachers. However, this study extends our knowledge of practices one MTE enacted during instruction by providing images of specific actions and related purposes to develop PTs' PCK. Other researchers have reported on pedagogical practices MTEs employed (e.g., Dixon, et al., 2009; Steele, 2008b); however, these authors reported MTE practices that attended to the PTs' general pedagogical knowledge.

Based on the analysis of the data, I propose the following model (see Figure 17) for conceptualizing and organizing the identified actions of one MTE. This figure demonstrates the relation between Kathryn's purposes and the categories of actions, by PCK component.

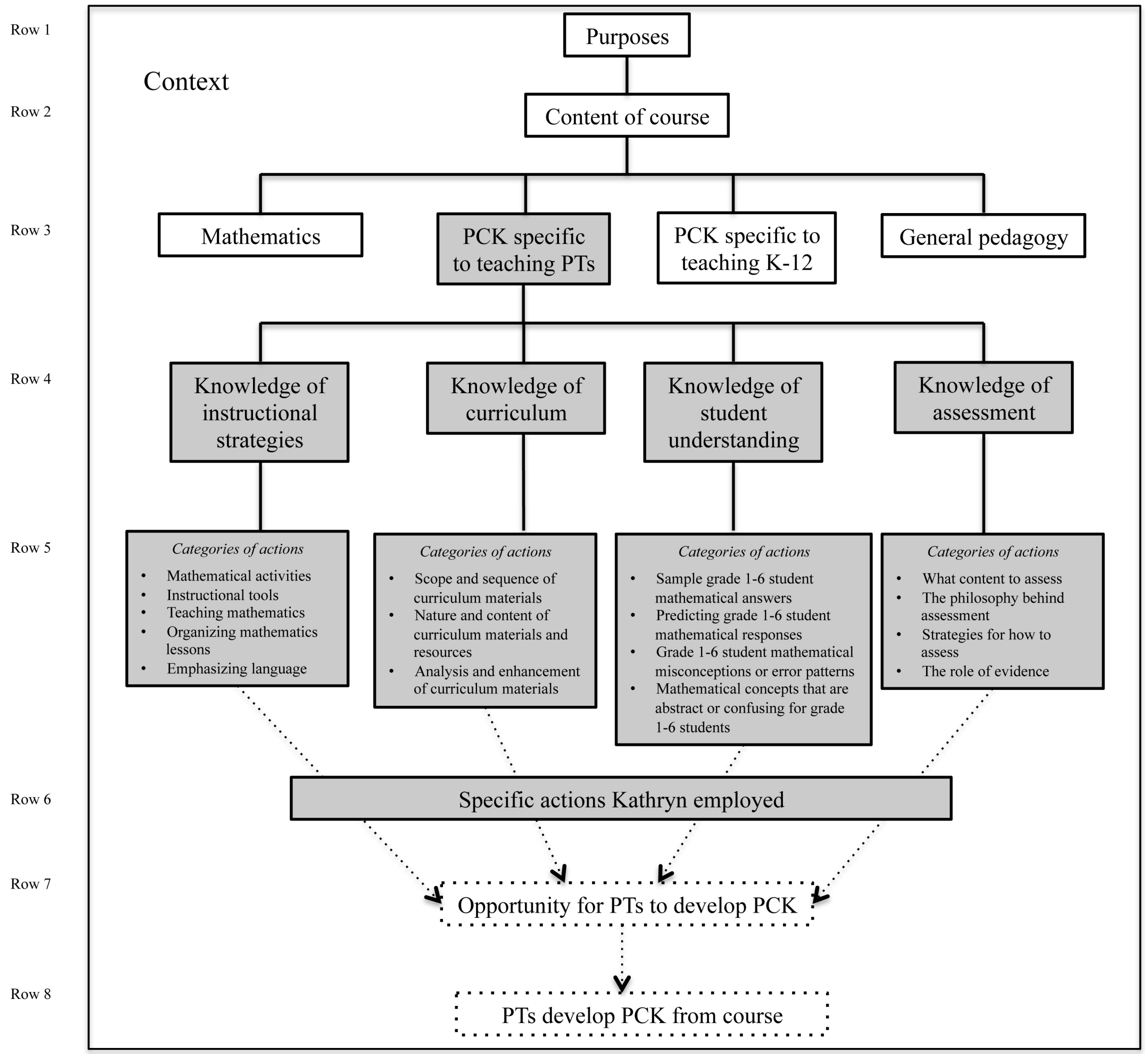


Figure 17. Conceptualization of actions Kathryn employed.

In the model, Kathryn's purposes (row 1) directly influence the content she chooses to implement in her course (row 2). This representation is similar to the representation noted in Figure 1, where a MTE draws on his (or her) knowledge base and purposes to enact actions in a mathematics content/methods course. In her course, Kathryn works to develop a variety of knowledge bases including mathematics, PCK, and general pedagogy (row 3). However, for the purpose of this study, I only focused on actions specific to facilitating PTs' PCK (shaded box in row 3) unlike other researchers (e.g., Drummond, 2002; Rieg & Wilson, 2009) who reported instructional practices related to general pedagogical knowledge.

In row 4, I delineate the four PCK components (i.e., knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, knowledge of assessment) as conceptualized by Magnusson et al. (1999). This delineation helped me to investigate Kathryn's actions in relation to them. However, Magnusson et al. state, "that pedagogical content knowledge represents an integrated knowledge system" (p. 117). In other words, these components are not discrete or independent from one another.

Aligned with these four PCK components, I identified 16 categories of actions (row 5), which were identified from 34 specific actions Kathryn employed (row 6). I hypothesize that Kathryn's actions not only provided the opportunity for PTs to develop PCK (row 7), but that in fact they did develop PCK as a result of their participation in the course (row 8). This depiction is similar to the red arrow shown in Figure 1 where a MTE's actions influence the knowledge that PTs develop in mathematics content/methods courses. However, in Figure 17, these two boxes (row 7 and 8) are dashed and unshaded because the relationship between Kathryn's actions and the PTs' PCK was beyond the scope of this study. Finally, it is important to note that all of Kathryn's purposes and subsequent actions, as characterized in Figure 17, were employed in the context of an elementary mathematics content/methods course for PTs as this has important implications, which I elaborate on in Chapter V.

Kathryn's actions in relation to the four PCK components were interdependent. Kathryn's actions did not address the four PCK components in isolation. For example, she did not design specific lessons related to different components. Rather Kathryn wove the four components of PCK (row 4 in Figure 17) into her course throughout the semester similar to the analogy of the interwoven strands of a rope used to represent the mathematical proficiencies in *Adding It Up* (see page 117, National Research Council, 2001, for more information).

Throughout the semester Kathryn focused on all four components by incorporating various actions. I saw evidence of this when I identified snapshots from the same class period that fell in multiple PCK components. Likewise, snapshots of the same action were seen at various stages during the semester. For example, for the action, *articulates grade levels that specific mathematical ideas are taught,* under the category of scope and sequence of curriculum materials, I shared instances from Kathryn's practice from various days throughout the semester (e.g., FN2008 Day1, Video Day2, FN2008 Day4, Video Day6, Video Day7, FN2010 Day10, FN2008 Day16, Video Day17, and FN2010 Day21). Additionally, for the action, *shares tip(s) about teaching with instructional tools (e.g., manipulatives, calculators, rulers)* I discussed episodes throughout the semester (e.g., Video Day 2, FN2010 Day4, Video Day 17, FN2008 Day17, Video Day18, FN2010 Day 20, Video day 21, FN2010 Day 23, FN2010 Day 24). Thus, Kathryn integrated actions that fostered the development of PTs' PCK throughout the course as well as provided the opportunity for PTs to develop multiple PCK components during a single class period. Heaton and Mickelson (2002) found that PTs need multiple opportunities to be active learners of mathematical concepts. Similarly, Kathryn afforded multiple opportunities, throughout the semester, for PTs to develop various components of PCK.

This study makes contributions to our understanding of how multiple PCK components may be interwoven throughout a semester. Unlike other researchers (e.g., Koirala, et al., 2008; Steele, 2008a; Timmerman, 2002; van den Kieboom & Magiera, 2010) who shared activities PTs engaged with attending to only two PCK components (e.g., knowledge of instructional strategies and knowledge of student understanding), this study focused on actions aligned to all four PCK components. By focusing on all four components, PTs were provided the opportunity to develop each component.

Furthermore, while Edwards et al. (2010) described a series of activities where PTs engaged in all four PCK components, this study also extends our knowledge by providing specific images of facilitation moves enacted throughout a course to develop all four components of PTs' PCK. This study provides details on actions a MTE utilized unlike other researchers (Edwards, et al., 2010; Hjalmarson & Suh, 2008; Jenkins, 2010; Kinach, 2002; Koirala, et al., 2008; Steele, 2008a; Timmerman, 2002; Tirosh, 2000; van den Kieboom & Magiera, 2010) who solely provided details on specific activities they implemented in teacher preparation courses and did not provide any detail regarding actions employed by the MTE.

Kathryn's purposes directed her actions. As demonstrated in the data earlier in this chapter, Kathryn articulated specific purposes for what she said and did in her course. Moreover, she stated during individual interviews that every learning experience she designed addressed specific purposes. In other words, she did not just engage PTs in activities for activities sake, but had a purpose for everything she planned. In Figure 17, Kathryn's purposes (row 1) guide the actions (row 3) she uses in her course. Furthermore, as seen in Tables 17-20, Kathryn's 10 core purposes align to all 34 identified actions.

In the review of the literature, I identified few publications (e.g., Van Zoest, et al., 2006) that referenced MTEs articulating purposes that guided specific instructional decisions in a teacher preparation course. Thus, the articulation of purposes that Kathryn drew on when engaging PTs in learning experiences constitutes a major contribution of this study to building a usable knowledge base for MTEs (Floden & Philipp, 2003).

Although Kathryn articulated purposes for each of her actions, she also admitted that she did not always share her purposes with PTs. During the times when Kathryn was explicit with the PTs, these instances follow what other researchers (Dixon, et al., 2009; Peled, 2007) recommend, which is that it is vital for MTEs to be explicit with their purposes for the activities they engaged the PTs with in the course. However, on occasion she did not share her purposes with the PTs as she referred to these instances as "covert operations." Ultimately, Kathryn conveyed that she was purposeful in the learning experiences she created in her classroom and that her purposes drove what she said and did in her mathematics content/methods course.

Kathryn's actions related to all four PCK components were tied to elements of teaching mathematics in grades 1-6. Whereas several researchers (e.g., Kinach, 2002; Timmerman, 2002; Tirosh, 2000; van den Kieboom & Magiera, 2010) report activities they implemented with elementary PTs to enhance their PCK, none of the examples shared were designed to enhance all four PCK components. This study provides detailed actions and corresponding purposes that align to solely teaching mathematics to grade 1-6 students. Furthermore, this study delineates a model (Figure 17) for conceptualizing and organizing actions Kathryn implemented with elementary PTs.

Similar to other researchers (e.g., Castro, 2006; Timmerman, 2002) who used classroom-based artifacts at the elementary level to engage elementary PTs in learning experiences, Kathryn also used classroom-based artifacts in her course. For example, she used student work, videos, curriculum materials, and sample assessment items to implement actions aligning with all four PCK components. Using these grade 1-6 classroom artifacts, Kathryn was able to make relevant connections to teaching mathematics to grade 1-6 students.

Above, I talked about each of the 34 actions in isolation as depicted in Tables 17-20, where each of the 34 actions are listed separately. However, this theme of using classroom based artifacts to develop PTs' PCK was throughout the discussion that took place around multiple actions across all four components. For example, to develop PTs' knowledge of assessment, Kathryn used student work to teach PTs about *the role of evidence* and she used standardized test items to teach them about *what content to assess*. Kathryn also used student work to teach PTs about *grade 1-6 mathematical misconceptions or error patterns* and *predicting grade 1-6 student mathematical responses* under the PCK component of knowledge of student understanding. Likewise, Kathryn used standardized test items to teach PTs about the scope and sequence that *specific mathematical* ideas are taught as well as *the order that mathematical ideas are taught* under the PCK component of knowledge of curriculum.

In addition, similar to other MTEs (e.g., Grant, et al., 2007; Stephens & Lamers, 2006) who have shared their practice regarding approaches they implemented to help PTs recognize the challenge of teaching elementary mathematics, Kathryn also employed actions with the purpose of attending to challenges the PTs would face as teachers. For example, Kathryn challenged a PT belief that "teaching elementary mathematics is easy" [EVT#3] when she asked the PTs to describe how they would teach fifth graders how to measure with a ruler to a fraction of an inch [Video Day17] and fourth graders how to measure with a protractor [FN2010 Day24]. She challenged this PT belief when she *prompts PTs to articulate directions to mathematical tasks so students in grades 1-6 will have access to the task*. Kathryn states, "I think they think, how hard could this be? You know, you've just got to explain it clearly and you're a good teacher, And…[teaching how to measure with a ruler or a protractor] is a lot more complex than you would think" [EVT#3]. Thus, this study makes a contribution to identifying learning experiences PTs engaged with that addressed challenges they will face in teaching grade 1-6 students.

Santagata and van Es (2010) state,

> Preservice teachers enter teacher education programs with ideas about what they should learn. They want to learn how to plan a lesson along with strategies to manage students and deliver content to students. They are persistent in wanting to know 'what to do.' (p. 119)

Several of Kathryn's purposes attend to giving the PTs what they want (e.g., wants PTs to view course content as relevant to their future practice). Additionally, several of Kathryn's purposes attend to challenging PTs' beliefs and expanding their vision for what it means to teach grade 1-6 students (e.g., wants PTs to focus on students [rather than themselves]). As is evident from multiple actions I identified Kathryn employing, many of her actions attend to sharing ideas on how to teach various mathematical concepts and address PTs desire to "know what to do" (e.g., *describes or mentions activities PTs may use with grade 1-6 students*, *shares tip(s) about teaching with instructional tools*). All of the examples Kathryn shared came from relevant issues teachers of grade 1-6 students would encounter. Thus, this study provides specific images of how Kathryn provides learning experiences for PTs where the PTs feel they are learning about issues that are relevant to their future teaching career, while simultaneously providing them the opportunity to develop their PCK.

Throughout the course of this study, I examined what Kathryn said and did in her elementary mathematics content/methods course over three non-sequential years. Findings from this study contribute to the literature on practices of teacher educators, which is limited (Doerr & Thompson, 2004; Loughran & Berry, 2005; Murray & Male, 2005). Thus, this study contributes a resource for mathematics teacher education regarding practices that might support MTEs in facilitating the learning of prospective (and practicing) teachers. I elaborate on this important idea in Chapter V, where I discuss the limitations, implications, and significance of the study.

Chapter V – Limitations, Implications, and Significance

The purpose of this study was to create a classification of descriptive categories of actions and the corresponding purposes a reflective MTE used to develop PTs' PCK in an elementary mathematics content/methods course. As discussed in Chapter IV, this work makes a significant contribution to the literature as it establishes a foundation in which to conceptualize MTE actions and purposes and suggests future research and activities for preparing and supporting MTEs. In this chapter, I discuss the limitations, implications, and significance of the study.

Limitations

Although the study had a strong research design, I acknowledge two limitations as well as a caution. First, I analyzed existing data that was not collected based on the research questions or theoretical framing. When the 2007 video recordings and Spring 2008 and 2010 fieldnotes were collected, these data sources were not based on the research questions. If that had been the case, I likely would have captured more snapshots in my fieldnotes representing specific components of PCK. I acknowledge this limitation, but also consider it a strength of the study because Kathryn's actions during the three semesters in which data were collected were not influenced by the research questions. In addition, I addressed this limitation in two ways during the design of the study: (a) I analyzed the videos of Kathryn's lessons from 2007 and (b) I designed all of the interviews based on the research questions and theoretical framing. These individual interviews provided opportunities for Kathryn to articulate what she said and did in her teacher preparation course as well as her corresponding purposes based on watching videos together and examining artifacts captured in the fieldnotes. Findings from this study contribute research regarding one MTE's practice and contribute to the need for building a usable knowledge base for MTEs (Floden & Philipp, 2003).

A second limitation was my lack of experience with performing the role of a MTE. I was a novice MTE who did not have the same knowledge or experience with teaching PTs that Kathryn had. As a result, there was a concern that I would not be able to accurately discern her purposes or classify her actions. However, I used several procedures and approaches to address this limitation. For example, I used a variety of data sources. I piloted my interview protocol with other experienced MTEs. In addition, I verified and confirmed findings, results, and conclusions, based on data, with other individuals. I also drew on the expertise of Kathryn and my dissertation committee members.

In addition, because I was a novice MTE, I coded the snapshots I identified based on how I perceived them, but various snapshots may have aligned with additional actions that I did not identify. I tried to code the actions cleanly, but Kathryn may have tried to develop multiple components of PCK per identified snapshot. In other words, a more experienced MTE may have identified multiple codes for individual snapshots. I chose to go with a cleaner cut approach of qualitative research and I tried to put each snapshot in one category and action even though the snapshot may have addressed other categories and actions.

Focusing on a rich data set for one MTE using a single-case study method enabled me to provide in-depth details of actions one MTE used. Although these results provide us with new insights, I caution the readers about the extent to which they generalize these findings without further research as the components of a MTE's practice potentially vary significantly across different settings (e.g., at other grade levels such as middle and secondary, in other types of institutions, in courses with different content emphasis, etc.) and based on the experiences of MTEs (e.g., number of years of MTE experience, teaching experience at the level in which they are preparing teachers to teach, etc.). However, since the goal of this study was to create a classification of descriptive categories of actions one MTE utilized to develop and enhance elementary PTs' PCK, broadening the number of participants will be the next step to test and refine actions MTEs employ specific to instructing PTs. It is also important to note that Kathryn employed additional actions that were not related to developing PTs' PCK. For example, Kathryn engaged PTs in actions around unpacking her practice to help transition PTs from being a student to analyzing teaching through the lens of a teacher. However, these actions were thrown out because they were related to mathematics, general pedagogy, etc., and that was not the focus of this study.

Implications

The knowledge base for teaching mathematics is complex and encompasses mathematical knowledge, pedagogical knowledge, and specialized knowledge (i.e., PCK) that is the intersection of the two (e.g., Ball, Thames, & Phelps, 2008; Shulman, 1986). Pedagogical content knowledge is important to foster and should be a key feature of teacher preparation programs (Borko, et al., 1992) as this balance between integrating pedagogy and content knowledge "should be the most important element in the domain of mathematics teachers' knowledge" (An, et al., 2004, p. 146). As a result, MTEs need to help foster this special knowledge in their courses. In Chapter IV, I provided a thorough description of what one MTE said and did in her course. Findings from this study are new to the literature and at this point, it is unclear how representative these findings are in relation to actions and corresponding purposes other MTEs implement in their courses because there are not researchers that have reported on actions MTEs used to develop PTs' PCK. In this section, I present a number of implications for future research, PT education, and the preparation and support of MTEs based on the results from this study.

Implications for future research. Hiebert and Morris (2009) "believe that steady and lasting improvements in teacher education rest on building a useful and cumulating knowledge base" (p. 475). The field has models of PCK (e.g., Grossman, 1990; Magnusson, et al., 1999), but not models of how to facilitate it. Often, the primary focus within graduate education is preparing researchers rather than university instructors (Austin, 2002). Thus, in this study I presented a model for conceptualizing and organizing actions that might assist MTEs in facilitating the learning of PCK for prospective (and practicing) teachers. This model was created to represent categories of actions an elementary-focused MTE utilized. However, further research is needed to test and refine the model with a larger sample of MTEs from different backgrounds and contexts (e.g., novice MTEs, MTEs with no K-12 teaching experience, MTEs teaching middle and secondary methods courses, MTEs teaching the single methods course for PTs, MTEs teaching one of multiple methods course PTs complete, etc.).

Little is known how coursework helps PTs to acquire PCK (Kinach, 2002). This suggests that we need to research whether MTE actions (row 6 of Figure 17) facilitate the development of PTs' PCK (row 7 of Figure 17) as well as what PCK PTs actually develop during their teacher preparation courses (row 8 of Figure 17). Results from this study and future efforts will inform researchers who develop instruments to measure PTs' PCK as well as the influence of actions on the development of PTs' PCK. By engaging in research such as this, the effectiveness of various programs and approaches will be documented and address the call suggested by Floden and Philipp (2007).

The results from this study suggest a larger research agenda. Potential future studies should include investigations that consider actions and purposes other MTEs employ who teach courses with a different content focus as well as courses designed to prepare PTs to teach a different grade band (e.g., grade 4-8). Additional research efforts could target ideas such as: How are Kathryn's actions and corresponding purposes similar to or different from other elementary MTEs? Secondary MTEs? How might the classification of descriptive categories of actions (as well as the actions and corresponding purposes) be different if participants teach a mathematics content/methods course where it is the sole course that prospective elementary teachers take or if it is the first, second, or third mathematics content/methods course PTs take in a three-course sequence? Furthermore, how do we take findings from this work and future work to create research-based materials that will provide MTEs with images of teaching PTs, which may help develop MTEs' knowledge in relation to actions and purposes for teaching PTs? Finally, and in relation to this study, how do we prepare MTEs to focus efforts on developing PCK?

As described in Chapter IV, the purposes Kathryn articulated directed her actions. These results from this study are a first step to informing the mathematics education field about potential purposes of MTEs. What purposes do other elementary MTEs have? Secondary MTEs? MTEs teaching a similar course? MTEs teaching a different course? If one goal is to improve the preparation of MTEs and another goal is to identify what should be taught in mathematics content/methods courses for PTs, then MTEs need to share purposes they have which inform what they say and do in their teacher preparation courses if the purposes of MTEs direct their actions. This is especially important for novice MTEs to be aware of purposes more experienced MTEs have for engaging PTs in various learning experiences as researchers (e.g., Van Zoest, et al., 2006) have reported a difference in purposes (i.e., mentor MTE's purpose versus novice MTE's purpose) for implementing the same activity in a methods course.

Moreover, we need to determine whether MTE actions influence actual change in PTs' knowledge, beliefs, and teaching practice. Additionally, in order to be prepared to educate their future students, PTs need to have sufficient amounts of subject matter knowledge and pedagogical knowledge (Adamson et al., 2003). Thus, we need to study whether PTs were able to apply and/or draw from the PCK they learned from their mathematics content/methods course in their student teaching or first several years of teaching. Also, we need to research what PCK beginning elementary teachers draw from when teaching mathematics in elementary classrooms. Findings from this new research would inform the design of teacher preparation programs regarding what PTs should be learning in the courses.

Implications for prospective teacher education. Earlier publications (e.g., Koirala, et al., 2008; Timmerman, 2002; Tirosh, 2000; van den Kieboom & Magiera, 2010), highlighted MTEs' activities that attended to enhancing PTs' knowledge of instructional strategies and knowledge of student understanding. However, only Edwards et al. (2010) engaged PTs in a sequence of activities that were designed to enhance all four PCK components. This raises the question of what happens when PTs do not have the opportunity to learn the other two PCK components (i.e., knowledge of curriculum and knowledge of assessment) and suggests that MTEs will need to consider what actions can develop PCK in teacher preparation courses.

In addition, due to the variability in teacher preparation programs that researchers report (e.g., Ferrini-Mundy & Floden, 2007; Floden & Philipp, 2003), it is highly likely that there are many MTEs that do not think about actions in terms of enhancing PTs' PCK. As a result, we need to get serious as a mathematics education community about identifying what should be in teacher preparation courses. As a field, how do we identify priorities? What do we most need to teach in mathematics content/methods courses? How do we identify core experiences for PTs so that they are prepared to enter the classroom? What MTE actions will help PTs develop knowledge, skills, and dispositions related to these priorities?

Implications for preparing and supporting mathematics teacher educators.

> *Mathematics teacher education involves a wide range of activities in which teacher educators facilitate the learning of prospective and practising teachers. However, the notion of educating teacher educators for their professional task is relatively new and thus the practices that might support their learning and development are not well understood. (Goos, 2009, p. 214)*
>
> *We all have much to learn about teaching. It is only through improving the education of teacher educators that we will be able to improve teacher education systematically. (Van Zoest, et al., 2006, p. 146)*

The comment by Goos (2009) raises several questions about how can universities help MTEs prepare to teach PTs. For example, what materials (e.g., videos, written cases, written resources) are necessary to help prepare and support MTEs? What preparation do MTEs need in order to successfully teach PTs? MTEs teaching courses for certain grade bands where they have no experience (e.g., a MTE with grade 7-12 teaching experience teaching an elementary mathematics content/methods course for PTs) are not necessarily going to have the PCK they need to develop their PTs' PCK as they prepare them to teach elementary students. How do we help these MTEs develop this PCK?

Several researchers (e.g., Edwards, et al., 2010; Steele, 2008a; Tirosh, 2000) have found that PTs' PCK can be enhanced in teacher preparation courses. However, in these publications, it was unclear what actions MTEs employed to enhance PTs' PCK. Thus, this study joins other resources (Mason & Johnston-Wilder, 2006) including instructor's Manuals (Reys, Lindquist, Lambdin, Smith, & Suydam, 2004; Van de Walle, 2007) for supporting MTEs in the planning phase for teaching PTs. Findings from this study can serve as a resource for MTEs as they instruct PTs in an undergraduate elementary mathematics content/methods course. For example, the 34 actions identified above may help MTEs plan instructional activities to engage PTs in opportunities to foster their PCK, which the MTEs had not previously thought to do. Additionally, the identified actions and corresponding purposes may also help MTEs develop activities and facilitate discussions that may challenge and expand PTs' initial understandings of what it means to teach mathematics to grade 1-6 students. Finally, findings demonstrate the complexity of this work (e.g., multiple purposes, multiple PCK components in addition to other knowledge bases such as mathematics, general pedagogy, etc.) and so how can we take this new information, about actions and purposes one MTE used, to better prepare MTEs?

All of the actions I identified Kathryn employing across the four PCK components were tied to elements of teaching mathematics to students in grades 1-6. Researchers (e.g., van Driel, de Jong, & Verloop, 2002) in other disciplines (e.g., science education) have focused their work on understanding topic-specific PCK. Hence, this finding has implications for MTEs. If MTEs want to develop PTs' PCK for teaching mathematics to students in grades 1-6, MTEs need to use illustrations, artifacts, and mathematical representations from grade 1-6 classrooms. Additionally, MTEs need to have PCK specific to teaching PTs. This PCK will look different for MTEs who teach elementary PTs versus MTEs who teach secondary PTs. In other words, PCK is topic and grade band specific. As a result, MTEs need to have PCK specific to the content and grade bands taught in their mathematics methods courses. In other words, MTEs need to develop their own PCK in relation to courses they will teach.

As described in Chapter IV, the purposes Kathryn articulated directed her actions. MTEs should consider their purposes (row 1 of Figure 17) and the PCK components (row 4 of Figure 17) they want to develop when selecting tasks and/or activities. Additionally, they should consider the actions (row 6 of Figure 17) they will use to implement the chosen tasks and/or activities. Throughout the design process for developing learning experiences for PTs, more experienced MTEs should collaborate with novice MTEs regarding their thought process behind various purposes for why they implement certain tasks and not others with PTs. Through this type of communication, novice MTEs may garner purposes they had never previously thought about.

In my own anecdotal reflections during data collection, I identified several purposes Kathryn articulated for various snapshots for which I had not anticipated. For example, during the first video/fieldnote interview, I asked Kathryn about her purpose for the action I had identified as *encourages PTs to be thoughtful about the language they use (e.g., when asking questions or writing mathematical tasks)* and had referred to a scenario

Kathryn articulated in 2007 [Video Day 15]. She shared a story about giving her own children math problems when they were in their car seats. One problem was, "You have five cookies and I give you three more cookies, how many cookies do you have?" Her children could answer this question, but her children could not answer the question, "What does five plus three equal?" I thought Kathryn was going to say her purpose for sharing the scenario with PTs was that her children could not do the second problem because they had not heard the word "plus" or "equals" before and that choosing appropriate vocabulary is essential. However, Kathryn did not articulate this as her purpose, but that she was attending to a PT belief where they think you first need to do naked number problems like 5 + 3 and then you introduce word problems. I was surprised by this purpose. From this experience, I realized that novice MTEs may make assumptions about purposes from watching videos of other MTEs' practice and as a result, it would be important to have discussions about purposes to prepare them to teach mathematics/methods courses.

Significance

This study contributes to a call to better equip MTEs (Li, 2009) by highlighting what one MTE said and did. Additionally, findings from this study provide images of specific MTE practices and actions in relation to four PCK components, which informs the mathematics education research community and contributes to the research literature regarding how PTs are taught in their teacher preparation courses (e.g., Edwards, et al., 2010; Kinach, 2002; Steele, 2008b). More specifically, this study is the first of its kind in mathematics education that identifies actions and corresponding purposes of a MTE throughout an entire course. Implications from this study could be foundational for a new research agenda in mathematics teacher education. Furthermore, the 10 core purposes identified from this study could inform teacher educators from other disciplines (e.g., Science, English, etc.); thus, findings from this study contribute to both the mathematics education and the broader teacher education research community.

Another noteworthy contribution of this study was the “Director’s Commentary” Interview method. I refer to this method as the “Director’s Commentary” as this method is often used as an extra resource for consumers who purchase DVDs of recent films. In this case, the director of the film provides commentary about scenes as he watches the movie. I applied this idea in this study through the use of the extended video talk interviews, in which I played a video displaying a significant portion of one of Kathryn’s classes (approximately 20 minutes in length). Rather than posing interview questions or asking Kathryn to comment on short clips, Kathryn talked about her actions and purposes while she watched the extended clip. She controlled the computer and made decisions about when she wanted to pause the clip to talk without distraction. As the researcher, I played the role of quiet observer. When Kathryn had finished her comments, I followed up with questions based on comments Kathryn had made when she watched (and paused) the selected video clip.

The Director’s Commentary method differed from other video-based interviews identified in the literature such as stimulated recall interviews (Calerhead, 1981). During stimulated-recall interviews, the researcher selects a specific clip for the interviewee to view and then respond to related questions. In other words, the interviewer directs the participant on what to focus on by choosing a particular video clip and asking him (or her) to respond to the clip they just watched. However, in the Director’s Commentary method, the interviewee decides what to focus on rather than the researcher. With this method, Kathryn guided the discussion based on sensitivity to particular aspects of her teaching. Thus, I relinquished the power of *noticing*, "characterized as consisting of three parts: attending to noteworthy events, reasoning about such events, and making informed decisions on the basis of the analysis of these observation," to Kathryn (van Es, 2011, p. 135). As a result, I garnered data that I would not have acquired if I had solely guided the interview by choosing shorter video segments for Kathryn.

This study joins others (e.g., Dolk, et al., 2002; Even, 1999, 2008; Van Zoest, et al., 2006) in providing new insights regarding content and images of practices to enrich the preparation of MTEs—practices MTEs could attend to in order to enrich the learning experience of PTs. The conceptualization presented in Chapter IV highlights examples of MTE actions and purposes that other MTEs may draw on as they reflect on their own actions and purposes. Goos (2009) argues:

> Research in mathematics teacher education as a distinctive field of inquiry has grown substantially in status over the past ten years, as evidenced by the establishment of an international journal (Journal of Mathematics Teacher Education), the appearance of edited books (e.g., Lin & Cooney, 2001), the commissioning of the 15th ICMI Study on the professional education and development of teachers of mathematics (Ball & Even, 2008), and publication of the first International Handbook on Mathematics Teacher Education (Wood, 2008). Notwithstanding this progress, research on the development of mathematics teacher educators is still in its infancy. (p. 210)

This study contributes to the conversation among those involved in the education of future K-12 mathematics teachers. This sharing of actions and purposes one MTE employed in her elementary mathematics content/methods course can ultimately enhance the education of MTEs as well as PTs.

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APPENDIX A

FACILITATING THE DEVELOPMENT OF ELEMENTARY PROSPECTIVE TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE: A CASE STUDY OF A MATHEMATICS TEACHER EDUCATOR'S ACTIONS AND PURPOSES
Interview Instrument

Say to participant: Thank you again for participating. I am interested in the things you do during instruction in your elementary mathematics methods course that are specific to teaching prospective teachers. Additionally, I am interested in why you choose to implement the things you identify. During this interview, I will ask you questions about what you say and do when teaching the mathematics methods course to prospective teachers.

Start the audio-recorder.

Say to the participant: Do I have your permission to record this interview?

Initial Interview Questions

1. Talk a little about the journey you have traveled to become a MTE and why you have become interested in teaching PTs. [For participant's background.]
2. You have eight course goals stated in your syllabus (show the eight listed to the participant). Describe three things you do during instruction (specific to teaching PTs) that addresses any of the goals you list.
    a. Follow-up with: Describe other things you enact during instruction that specifically addresses any of the stated goals in the syllabus?
    b. Do you have other course goals that are not listed here? If so, can you describe some of those goals?
3. What have you learned about teaching elementary PTs from colleagues?
4. What have you learned about teaching elementary PTs from your own professional development experiences (e.g., reading research)?
5. Do you ____? Describe that for me.
    - Address K-12 teaching issues
    - Address K-12 student mathematics issues
    - Address K-12 classroom management issues
    - Address assessing K-12 students

- Address PTs' field experience
- Address state standards/NCTM principles and standards
- Analyze teaching practice
- Analyze tasks
- Analyze curriculum
- Engage PTs to *think like teachers*
- Engage PTs to *think like students*
- Analyze student work
- Challenge PTs' beliefs about mathematics
- Engage PTs in making sense of the mathematics they will someday teach
- Create experiences that challenge PTs' own understanding of mathematics

6. What is one thing you have implemented/felt you improved on at a later time in your class due to something you did (intentionally or unintentionally) on a bad day or poorly implemented the first time?
7. What is your favorite thing to engage PTs in? [extended video talk segment selection]
8. What actions are you taking that are different now that you are teaching PTs than when you taught elementary students?
    a. Are your purposes different? If so, could you talk about that?
9. Reflecting back to when you started teaching PTs, could you talk on your professional journey when you thought, I can do that in a mathematics course but not a mathematics content/methods course?

APPENDIX B

FACILITATING THE DEVELOPMENT OF ELEMENTARY PROSPECTIVE TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE: A CASE STUDY OF A MATHEMATICS TEACHER EDUCATOR'S ACTIONS AND PURPOSES
Interview Instrument

Say to participant: I've identified a ___ minute clip that demonstrate several actions you enact during instruction that are specific to teaching PTs. While you are watching the clip, I would like you to talk about the purpose/what you are hoping to address about learning to teach, through your instruction.

Start the audio-recorder.

Say to the participant: Do I have your permission to record this interview?

Participant watches identified clip, pausing the clip periodically to verbally express her purpose/what she is hoping to address about learning to teach, through her instruction.

Extended Video Talk Interview Questions

At the end of the clip, after the participant has no further comments regarding what she just saw—potential follow-up questions to probe more into her "why/purpose."

- Could you articulate your purpose for why you bring up ____ with the PTs?
- Could you share why you ____. Why did you do that? What was your purpose for that?
- Throughout the semester, you ____. What is your purpose for _____?

APPENDIX C

FACILITATING THE DEVELOPMENT OF ELEMENTARY PROSPECTIVE TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE: A CASE STUDY OF A MATHEMATICS TEACHER EDUCATOR'S ACTIONS AND PURPOSES
Interview Instrument

Say to participant: I have identified several different snapshots of your practice that demonstrate actions you enact during instruction that are specific to teaching PTs. I will describe what happened in the fieldnotes or I'll describe what happened in video clips from 2007. If I do not provide a clear description for you and you want to see the fieldnotes so you can get some context around it or you want to see the video clip that I am revering to, let me know and I will share the corresponding snapshot of practice I am referring to.

Start the audio-recorder.

Say to the participant: Do I have your permission to record this interview?

Video/Fieldnote Based Interview Questions

- Something that you frequently do is _____. For example, you will _____. Can you talk a little bit about your purpose for _____?

APPENDIX D

CODING DICTIONARY

Categories of instructional strategies *Mathematics teacher educator teaches prospective teachers about*	*Actions*	*Examples*
Mathematical activities	Engages PTs in mathematical activities they may use in their future classrooms	Kathryn asked PTs to take a piece of wire that was 24cm long, create a rectangle, and then determine the area and perimeter of different rectangles. [FN2010 Day17]
	Describes or mentions activities PTs may use with grade 1-6 students	"There is another nice science experiment where the kids take different types of paper towels to determine what are more absorbent. So they take eyedroppers and they put water on the paper towels to see which ones spread out more and then they calculate the area of the spread. So that is a nice science experiment that you could tie in with this idea that it is not going to be an exact area, it is going to be a pretty close estimate for the area of that spread." [Video Day18; FN2010 Day17]
Instructional tools	Explains how to make homemade manipulatives for use in future classrooms or use common items in place of "catalog manipulatives"	To make your own spinner, use a paperclip and eyeliner pencil or overhead pen. [Video Day1]
	Shares tip(s) about teaching with instructional tools (e.g., manipulatives, calculators, rulers)	When using geoboards with children, you "have to be careful that the kids aren't counting just like if it is on a grid. They should not just count the lines or pegs, it is the distance between the pegs. There are five pegs there, but the distance is four." [Video Day18]
Teaching mathematics	Advises PTs on what to physically give to children when teaching various mathematical concepts	When teaching volume, give students a cube. [FN2008 Day16]
	Advises PTs on what to say, write, and/or have students do when teaching various mathematical concepts	One example Kathryn shared with PTs to teach mathematical concepts under discussion was that "Students should physically cut out triangles" to see that the area of a triangle is half the area of a rectangle "because it is difficult for students to visualize in their heads." [Video Day16]

Categories of instructional strategies continued *Mathematics teacher educator teaches prospective teachers about*	*Actions*	*Examples*
Organizing mathematics lessons	Selects and sequences student work for a whole class discussion	"I made a decision for which [graphs you created] to put up [on the board] and now what mathematical ideas do you want to highlight and what order to discuss." [FN2010 Day8]
	Articulates typical structure of a whole lesson	The "number one thing that is skipped – closure to a lesson where misconceptions can be addressed." [FN2008 Day3]
	Discusses ideas to attend to, consider, and/or draw from when designing mathematics lessons	Kathryn had the PTs complete several worksheets where they needed to find the area of various figures. She then asked the PTs, "Why did I have you do this?" She explicitly states to the PTs, "Do the problems before giving [worksheets] out [to students]. Don't make [the] assumption, I can just print it off and give it to kids." [FN2010 Day20]
Emphasizing language	Prompts PTs to articulate directions to mathematical tasks so students in grades 1-6 will have access to the task	Kathryn asked PTs to discuss in groups what they would say if they were going to teach 5th graders how to measure with a ruler. [Video Day17; FN2010 Day20]
	Explains the importance of connecting verbal and written language with visual aids	"Give explanations verbally and written – look at the percent of instructions [that] are given verbally [to children] – percent is really high." [FN2010 Day16]
	Encourages PTs to be thoughtful about the language they use (e.g., when asking questions or writing mathematical tasks)	Kathryn asked the PTs how they found the perimeter of a shape and one responded by stating, "counting squares." Kathryn replied with, "If a child said counting squares, why would you not want to ignore it? Be thoughtful with language – revoice what they say." [FN2010 Day16]

Categories of curriculum *Mathematics teacher educator teaches prospective teachers about*	*Actions*	*Example*
Scope and sequence of curriculum materials	Articulates grade levels that specific mathematical ideas are taught	"Fifth grade is doing prime numbers." [FN2008 Day1]
	Identifies the order that mathematical ideas are taught	"Focus kids first on estimation before introducing the protractor." [FN2010 Day24]
	Selects worksheets to engage PTs in sequencing tasks for grade 1-6 students regarding important mathematical concepts	Kathryn posed five tasks, around the mathematical concept of data, from the same curriculum for PTs to investigate across grades one through five. PTs were asked to predict the grade level of each worksheet as well as compare and contrast the content and organization of the five worksheets. [FN2008 Day12; FN2010 Day13]
Nature and content of curriculum materials and resources	Provides examples from curriculum resources	Kathryn handed out a couple of activities from curriculum that PTs could do with kids around measuring lines. [Video Day19]
	Shares and demonstrates supplemental curriculum resources	Kathryn shared a virtual manipulative website that rolls dice. [FN2008 Day2]
	Addresses vocabulary in curriculum resources	"We don't say borrow anymore…[we now say] renaming." [Video Day9]
	Articulates differences in design and specific features in mathematics curriculum materials	"This curriculum [Math Trailblazers] uses science as a context to teach mathematics. So there is science throughout the whole book." [Video Day11]
	Identifies topics in mathematics curriculum materials that are (a) presented in inadequate ways or (b) enacted in unproductive ways	"Weight is something that is taught so poorly in this country." [Video Day12]
Analysis and enhancement of curriculum materials	Asks PTs to compare what was done in class with how curriculum materials are written for grade 1-6 students	PTs are asked to compare how Kathryn designed the game board of an activity completed in class with how the game board was designed for 1-6 students. [Video Day5]
	Prompts PTs to analyze the mathematics in curriculum materials or activities completed in class	"What [mathematical] concepts are in Chapter 1 of [Maneuvers with] Rectangles?" [Video Day15]
	Prompts PTs to consider how they could alter materials to enhance students' mathematical experience	"How can you open the problem so it's more [mathematically] challenging." [FN2010 Day17]

Categories of student understanding *Mathematics teacher educator teaches prospective teachers about*	*Actions*	*Examples*
Sample grade 1-6 student mathematical answers	States atypical 1-6 student answer/thinking to mathematical concept under discussion	One boy took a piece of 24cm wire and cut it in half to get two 12cm pieces of wire. He then cut 5cm off both of the 12cm lengths. A second boy cut off 5cm and then another 5cm from the 24cm piece of wire. Then he took half of the 14cm piece of wire to get the two 7cm pieces of wire [FN2008 Day16]
	States incorrect grade 1-6 student answer to mathematical concept under discussion that relays the student's incorrect mathematical understanding about the topic under discussion	Every year Kathryn observes kids measure a line in inches in classrooms. For example, a line that is 2 ¼ inches long, 5th grade "kids are going, well that's 2 inches and 1cm or that's 2.4 inches." [Video Day17]
Predicting grade 1-6 student mathematical responses	Prompts PTs to predict mathematical answers/strategies grade 1-6 students come up with/predict how grade 1-6 students will solve posed mathematical problems (which were posed to the PTs/PTs are discussion as whole class)	Kathryn asked PTs to predict what a student would say when she engaged then in analyzing student work—specifically "What do we hope [a child would do] here?" when the problems 25 minus 21 and 103 minus 99 are posed [FN2008 Day13]
Grade 1-6 student mathematical misconceptions or error patterns	Shares misconceptions and/or error patterns grade 1-6 students (and teachers) have regarding mathematical concept(s) under discussion	When Kathryn engaged PTs in analyzing student work, she articulated how the student work the PTs were analyzing was representative of what 1-6 students do. For example, student work where a child is subtracting 2-digit numbers and 3-digit numbers, the error pattern is that the fictitious child is always subtracting the smaller digit from the larger digit no matter where it is placed. Kathryn said, "You will see this if you teach subtraction—very common" [FN2010 Day10]
Mathematical concepts that are abstract or confusing for grade 1-6 students	Articulates mathematical concepts that are abstract for grade 1-6 students	The PTs engaged in a task where they flip three coins and then represent all of the possible outcomes, using different representation. After the PTs placed the different representation the class created (e.g., pictures, lists diagrams) on the white board, there was a whole class discussion. During the discussion, Kathryn said that representing all of the possible outcomes in a table (one representation that a PT put on the whiteboard) "is more abstract than the list for kids" (another representation a PT shared on the whiteboard) [Video Day3]
	Articulates language issue grade 1-6 students may have that is interfering with grade 1-6 student's understanding the mathematics under discussion	Kathryn shared a personal story of an interaction she had with one of her children when they were in a car seat. She shared, "My kid is in a car seat and I say, what is 5 + 3." Kathryn goes on to say that her child could not do that problem because he had never heard the word plus before. However, her child could answer the next question Kathryn posed to him which was, "You ate five cookies. I give you three more. How many cookies do you have?" [FN2008 Day13]
	Describes examples of mathematical connections that grade 1-6 students may not make	In geometry, kids need to be able to visually see the rotation of a figure and see how if you get a duplicate of the original shape by rotating the shape less than 360 degrees, then the figure has rotational symmetry. But, kids struggle with this concept of rotational symmetry—where geometry shapes must have the exact same orientation [FN2008 Day22]

Categories of assessment *Mathematics teacher educator teaches prospective teachers about*	*Actions*	*Examples*
What content to assess	Articulates content, as well as assessments/assessment items, that students are assessed on (e.g., standardized tests, classroom tests)	Kathryn shared a big idea about measurement that takes time to develop and she shares how the idea of “one unit may be more appropriate than another” might be assessed on a standardized test. Kathryn stated, “The way this is assessed on a MAP test would be something like multiple choice; I am going to measure the distance between Columbia and St. Louis. Am I going to do it in inches, feet, miles or in gallons? Well, I am going to do it in miles because of the distance that is involved. So the idea of what is appropriate for certain lengths. So is it appropriate to measure this distance here in feet, in inches, millimeters? What would be the appropriate unit I would want to use for this distance?” [Video Day19]
The philosophy behind assessment	Raises issues for PTs to consider or provides the purpose regarding various issues surrounding assessment that may potentially guide assessment decisions in the classroom	An assessment issue Kathryn raised with the PTs involved timed tests. She asked her students, “How do you feel about the tests kids take in math every day? What are they? Timed tests. Are you going to implement them? How do you feel about that?” [FN2008 Day15]
Strategies for how to assess	Articulates strategies that could be employed by the PTs to gather information about student understandings or misunderstandings as well as how standardized test items are graded/assessed and questions to consider prior to gathering information about student understandings or misunderstandings	Take a transparency and place the transparency over an angle a student just drew to quickly assess the accuracy of the angle the child drew [FN2010 Day24]
The role of evidence	Attends to student work (or lack thereof) to gather information about student understandings or misunderstandings	Sometimes, Kathryn posed a new mathematics problem for the PTs, to predict how the child would solve the problem, slightly different from the student work the PTs analyzed. Kathryn would say, “We don’t know what [the fictitious child] would do” [FN2008 Day13]

APPENDIX E

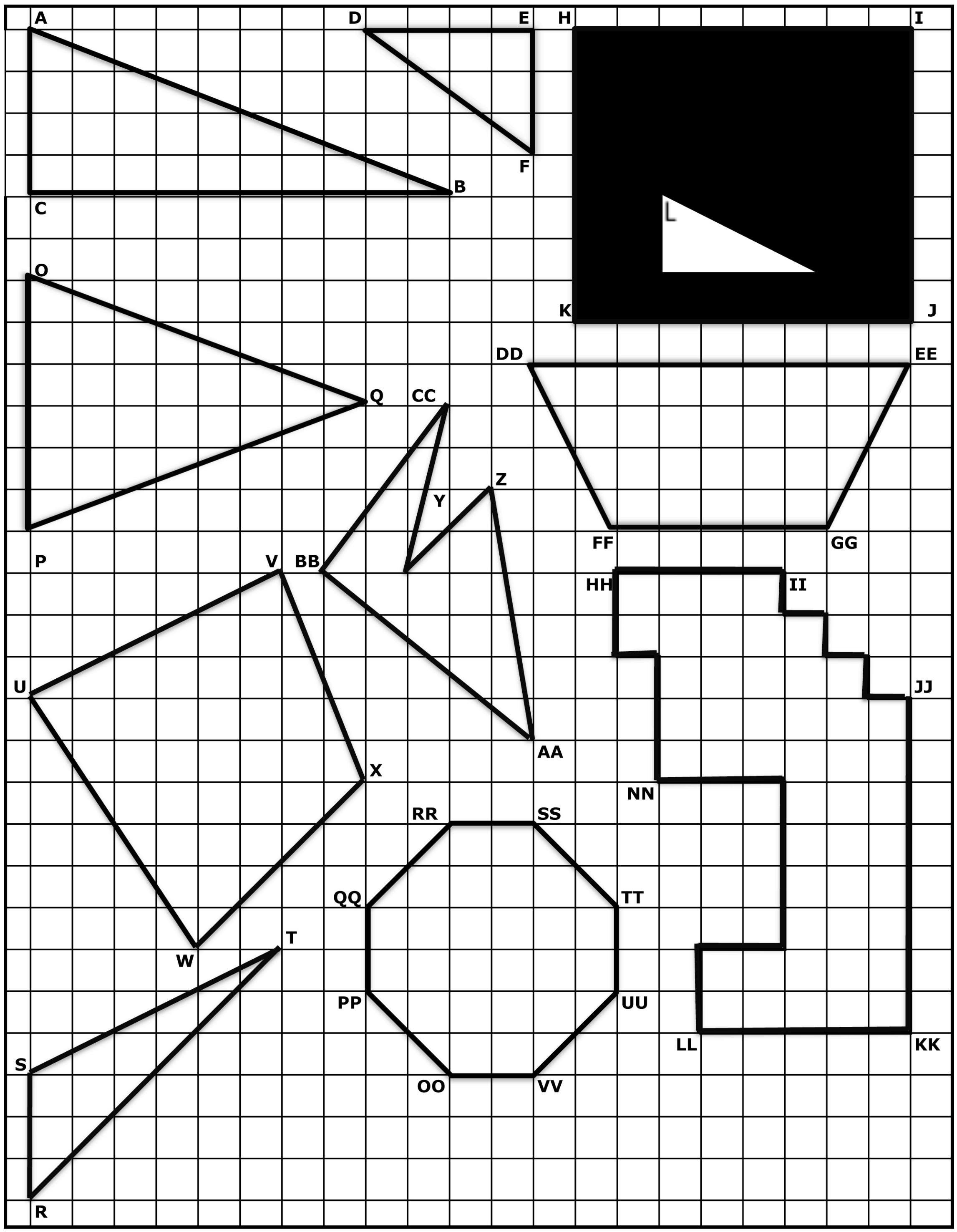
SEVEN CORE PURPOSES

Core purpose	*Pedagogical content knowledge component*	*Examples*
Wants PTs to know different ways students in grades 1-6 think mathematically and the importance of language use with them	Assessment	*You have to be really careful on the language that we use*. [VDFNInt2]
	Instructional strategies	Math vocabulary is very important and I'm not minimizing that in any way. Um, *but we've got to pay attention to the meanings that kids bring to these words and how do we help them. The everyday meaning*, you know, like the rounding example and they've heard that word for a long time, but now here we've got a really special meaning in math of what this means. And um, it's very different from the other meanings you've had. We've got to emphasize that. [EVT#3]
	Student understanding	I want them to know that *kids are very creative and come up with lots of interesting things that they won't anticipate*. [VDFNInt1]
Wants to support PTs' learning of mathematical content (i.e., math errors students in grades 1-6 make are also errors the PTs make in the elementary content/methods course)	Curriculum	They have to sequence the area tasks. That's always interesting. Every year they come up with all these different sequences for different reasons and again, there, *I'm worried they can't do the math*. They can't find the area of a complex figure, but, so I have that really complicated one in the middle that's pretty hard problem to solve. So here, they think it's a teaching thing, but again I'm sneaking the math in. [InitialInterview]
	Instructional strategies	*If it's mathematics they need to learn, then you need to have them do it*. [VDFNInt1]
	Student understanding	*What I'm getting at there is the mathematics misconceptions that exist in the room*. [InitialInterview]
Wants PTs to view course content as relevant to their future mathematics teaching practice	Assessment	What do I want them to know? *That [standardized] tests tell you very little information on any given day. They tell you very specific information. They can tell you good information*. But we shouldn't make assumptions. [VDFNInt2]
	Curriculum	So, *trying to consistently point out why this is relevant to them as future teachers*. Especially, early on in the semester. Um, they probably think mathematically there's nothing here, um, I know what an even is, I know what an odd number is, so why are we talking about this? So *I want to make sure they know that this is relevant to what they are learning*. [EVT#1]
	Instructional strategies	*That's why I start with games. Cause that's what they walk into class wanting* and I've got to convince then otherwise that they need other things. So that's why I start with games. [InitialInterview]
	Student understanding	When I do the Ashlock student work piece, *they think it's relevant*. [InitialInterview]

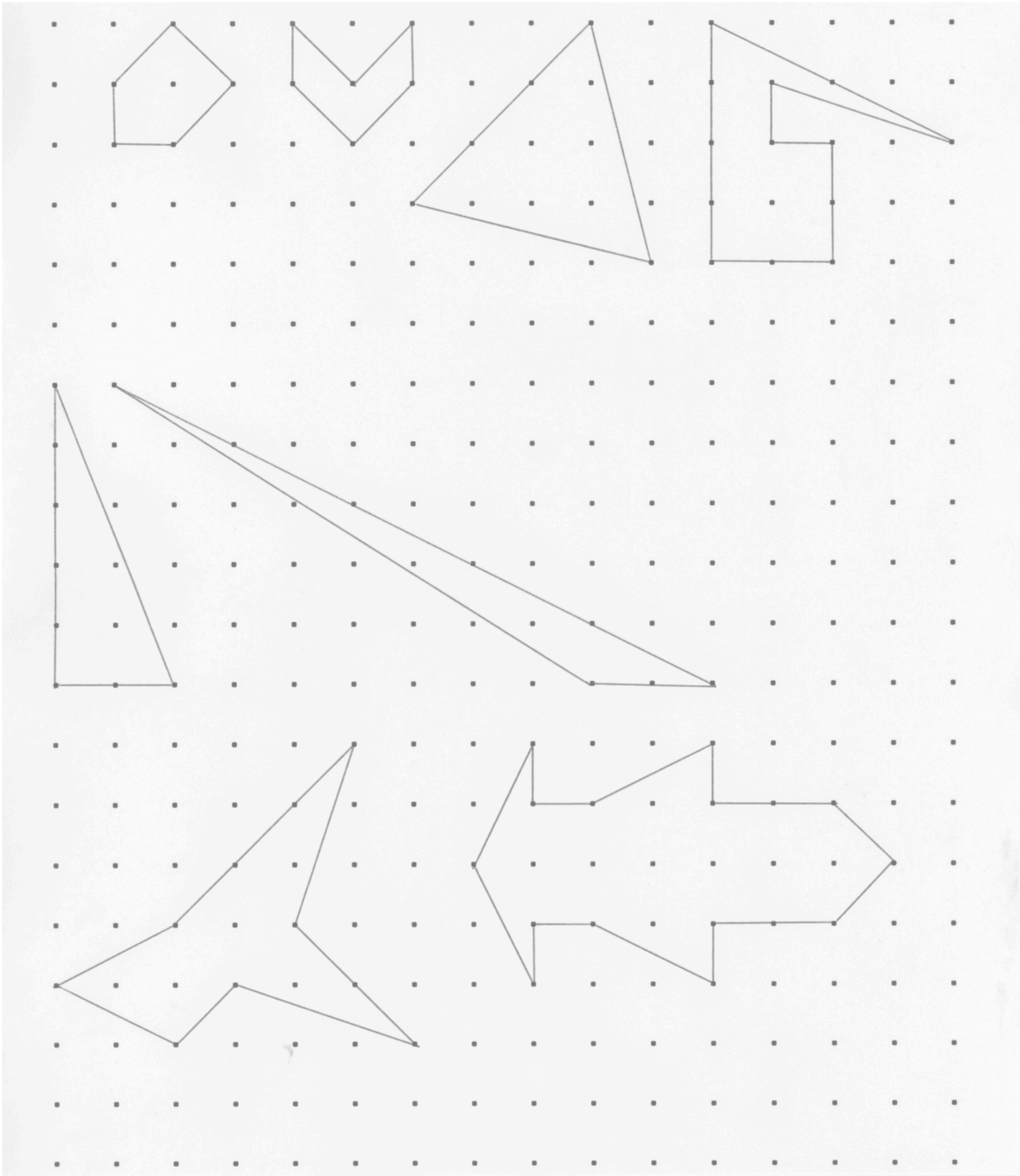
Core purpose	*Pedagogical content knowledge component*	*Examples*
Wants PTs to be aware of the different nature and content of mathematics curriculum resources and instructional tools	Curriculum	*I don't know what curriculum you will have access to when you become a teacher* and so this semester you will see examples from many different curriculum. [VDFNInt2]
	Instructional strategies	I worked in urban schools where they don't have these things and so *I'm assuming some of my students will go and work in schools that don't have the resources we're talking about*. And so I don't want them to dismiss that and say, um, well, I don't have spinners so I'm not going to do that. Um, so I do give them some ideas for how they can make. [VDFNInt1]
Wants PTs to know the mathematical content that grade 1-6 students learn	Curriculum	*I want them to have this idea, you know, that that certain topics or content are taught at specific grade levels*. But I also want them to think about the articulation. And so I want them to think about if I'm going to teach measurement in 3rd grade, what prior knowledge do kids bring. And, you know, what would be the expectation for me in 3rd grade to prepare them for 4th grade. You know, sometimes I get a sense from them that they think they are going to teach everything. You know, instead of thinking of it as a learning progression or, you know, development or whatever. So if I'm going to teach fractions in 3rd grade or 4th grade, I'm going to teach everything about fractions. No, what's the little piece of fractions that you are going to teach and how does that build on what they've already learned and where they are going. Um, so I think that articulation issue is really important. [EVT#3]
	Instructional strategies	So *there's things you can do with young children to help them build the sense of this idea of unit*. Um, and so I'm making that point here. [EVT#3]
Wants PTs to view her as a credible teacher of mathematics	Curriculum	The other purpose there, a lot of my stuff right at the beginning of the year is that they see me as a competent person and I know what I'm talking about, and so, I don't want them to see me as a person that's removed from classrooms at the university and I don't know what's going on. So *I want to convey to them that I know when certain things are taught*. [EVT#1]
	Instructional strategies	Teaching kids to use a tool and use it well is not easy. It's not about just explaining line this up or whatever, there's fractions embedded in this. There's measurement embedded in this. There's these different measurement scales that we use, um. So, um, you know, *I tie it back to these lessons that I've observed so that I have credibility with them*. [EVT#3]
Wants PTs to focus on what students will do mathematically rather than what they would do as the teacher	Student understanding	Kids see things very differently than we do. And we have to give them the opportunity. And so, you know, I think I use that strategy a lot because *I want them to start thinking about kids. I want them to start anticipating what kids will do and say*. [EVT#3]

APPENDIX F

THREE AREA TASKS PROSEPCTIVE TEACHERS
ENGAGE WITH AND SEQUENCE FOR GRADE 1-6 STUDENTS



Source: Created by David Page



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VITA

Cynthia E. Taylor is currently Assistant Professor of Mathematics Education in the Department of Mathematics at Millersville University in Millersville, Pennsylvania. She earned the following degrees and certificates: B.S. in Mathematics Education from Indiana University of Pennsylvania, Indiana, Pennsylvania (1998); M.S. in Natural Science from Rensselaer Polytechnic Institute, Troy, New York (2002); Supervisory Certificate in Mathematics Education from Millersville University, Millersville, Pennsylvania (2006); and Ph.D. in Curriculum and Instruction with an emphasis in Mathematics Education from the University of Missouri, Columbia, Missouri (2011). Cynthia's research interests include teacher preparation, teacher knowledge, and teacher practices.

Cynthia grew up in Lancaster, Pennsylvania where she taught high school mathematics for nearly 10 years. Additionally, she taught 10-14 year olds for one year at South Charnwood High School, Markfield, England on a Fulbright Teacher Exchange Scholarship. During her doctoral studies, she instructed undergraduate prospective teacher general education and mathematics education courses, facilitated professional development with mathematics teachers, and participated in research projects that allowed her to develop professional development materials, study beginning mathematics teachers, and study mathematics teachers' knowledge.