ABSTRACT

This thesis concerns the study of a class of second order quasilinear elliptic differential operators. For $1 < p < \infty$, the model equation we consider is:

$$(1) \qquad \mathcal{L}(u) = -\Delta_p u - \sigma |u|^{p-2} u.$$

Here the *potential* σ is a function (or distribution), and the differential operator $\Delta_p u$ is the *p-Laplacian*. Such operators are said to have '*natural growth*' terms. When $p = 2$, the operator reduces to the linear time independent Schrödinger operator.

We will study the operator under minimal conditions on σ, where classical regularity theory for the operator $\mathcal{L}$ breaks down. Our focus will be on two heavily studied problems:

1. An existence and regularity theory for positive solutions of $\mathcal{L}(u) = 0$, under the sole condition of *form boundedness* on the real-valued potential σ:

$$(2) \qquad |\langle |h|^p, \sigma \rangle| \le C \int_\Omega |\nabla h|^p \, dx, \qquad \text{for all } h \in C_0^\infty(\Omega).$$

Here σ is assumed to lie in the *local* dual Sobolev space $L^{-1,p'}_{\text{loc}}(\Omega)$, and the pairing in display (2) is the natural dual pairing.

2. The *pointwise behavior* of fundamental solutions of the operator $\mathcal{L}$. Here we will be concerned with positive solutions of $\mathcal{L}(u) = 0$ with a prescribed isolated singularity.

The techniques developed to attack these two related problems will be quite different in nature. The first problem relies on a study of the doubling properties of nonnegative functions satisfying a weak reverse Hölder inequality, along with certain weak convergence arguments. The second problem is approached via certain nonlinear integral equations involving Wolff's potential, and makes use of tools from non-homogeneous harmonic analysis.