GENERALIZED LOCAL TB THEOREM AND APPLICATIONS .

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ABSTRACT

The Tb theorem, like its predecessor, the T1 Theorem, is an L^2 boundedness criterion, originally established by McIntosh and Meyer, and by David, Journé and Semmes in the context of singular integrals, but later extended by Semmes to the setting of "square functions". The latter arise in many applications in complex function theory and in PDE, and may be viewed as singular integrals taking values in a Hilbert space. The essential idea of Tb and T1 type theorems, is that they reduce the question of L^2 boundedness to verifying the behavior of an operator on a single test function b (or even the constant function 1). The point is that sometimes particular properties of the operator may be exploited to verify the appropriate testing criterion. In particular, it would be presented some results for "square functions" with non-pointwise bounded kernels as well as the motivation that leads us to study such case.

We apply such result to give a proof of the Kato problem and also to prove that the single layer potential associated to a divergence form, t-independent elliptic operator or system in the half-space $\mathbb{R}^{n+1}_+$ is an L^2 bounded operator, more precisely that $t\partial_t^2 S_t : L^2(\mathbb{R}^n) \to L^2(\mathbb{R}^{n+1}_+, \frac{dxdt}{t})$, assuming some appropriate solvability result for the Dirichlet problem $(D)_q$ and the Regularity problem $(R)_p$.