

The Evans function, the Weyl-Titchmarsh function, and the Birman-Schwinger  
operators

Alim Sukhtayev

Dr. Yuri Latushkin, Dissertation Supervisor

ABSTRACT

We focus on the spectral stability of travelling wave solutions of partial differential equations. First, we use the Gohberg-Rouche Theorem to prove equality of the algebraic multiplicity of an isolated eigenvalue of an abstract operator on a Hilbert space, and the algebraic multiplicity of the eigenvalue of the corresponding Birman-Schwinger type operator pencil. Next, we apply this result to discuss three particular classes of problems: the Schrödinger operator, the operator obtained by linearizing a degenerate system of reaction diffusion equations about a pulse, and a general high order differential operator. We study relations between the algebraic multiplicity of an isolated eigenvalue for the respective operators, and the order of the eigenvalue as the zero of the Evans function for the corresponding first order system.

We also describe relations between the Evans function, a modern tool in the study of stability of traveling waves and other patterns for PDEs, and the classical Weyl-Titchmarsh function for singular Sturm-Liouville differential expressions and for matrix Hamiltonian systems. Also, for the scalar Schrödinger equation, we discuss a related issue of approximating eigenvalue problems on the whole line by that on finite segments.