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Mean value formulas for differential operators

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The main goal of our research is to discuss Mean Value Formulas for solutions of partial differential equations of a certain form (see 1 below) where A is a symmetric, positive definite, real-valued matrix. Such equations arise when modeling various steady state (time independent) phenomena. A mean value formula is a precise way to relate the value of the solution at a point c to the values taken in a neighborhood of that point. A neighborhood being simply a region around the point c. Mean Value Formulas have a rich history stretching back to the early 1800 when the famous mathematician C.F. Gauss initiated their study by considering the case when the matrix A is the identity. This corresponds to the Laplace equation (see 2 below). In this case, the neighborhood around a point c is a sphere. The more general settings we discuss correspond to: integer powers of the Laplacian, the anisotropic (direction dependent) model and the Lame system of isotropic elasticity. We were able to establish mean value formulas in each of these cases. There are many aspects of the project that lend themselves to future study. This includes considering other types of partial differential equations, such as the heat equation and Helmholtz equation, and proving converses to the Mean Value Theorems already established, obtaining the most general version of the Mean Value Formula, valid for all solutions to the Lame system.