

Mass Customization with Vertically Differentiated Products*

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Abstract

We analyze a duopoly game in which products are initially differentiated in variety and quality. Each consumer has a most preferred variety and a quality valuation. Customization provides ideal varieties but has no effect on product qualities. The firms first choose whether to customize their products, then engage in price competition. We show that in equilibrium either both firms customize, only the higher quality firm customizes, or no firm customizes. Even if customization is costless, the firms might not customize. This happens when the quality difference between the firms is small. We explore how the total welfare changes with the fixed cost of customization. Interestingly, the relationship is not always monotonic. Contrasting with the situation when customization is not feasible, both consumer surplus and total welfare are higher when one or both firms customize.

JEL classification: D43, L13, C72

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1 Introduction

Mass customization is the capability to produce individually tailored products and services on a large scale with near mass production efficiency. Advances in Internet-based information technologies and improvements in manufacturing flexibility have made mass customization a reality in many product categories. For example, National Bicycle Industrial Company custom manufactures high-end bicycles; Dell custom builds notebook and desktop computers with thousands of combinations of features possible; NikeID.com allows consumers to create their most preferred athletic pair of shoes by choosing among many different options for base color, accent color, lace color, etc.

Considerable attention has been paid to mass customization in operations management and information systems studies. However, only a few works have analyzed customization theoretically (e.g., Dewan et al. 2003, Syam et al. 2005, Syam and Kumar 2006, Bernhardt et al. 2007, Alptekinoglu and Corbett 2008, and Alexandrov 2008). Most of the existing literature adopt one-dimensional horizontal differentiation settings of Salop (1979) or Hotelling (1929). Customization enables consumers to get their ideal products represented by their locations in the product attribute space. Firms are symmetrical and make symmetric choices in equilibrium.¹ Even though many important aspects of customization are captured by these studies, important issues have yet to be examined. Casual empiricism indicates that rival firms do not always make the same customization choices (some customize, some do not) and that higher quality/priced firms are more likely to offer customization. The goal of our paper is to incorporate these observations into product customization competition.

We adopt the basic model from the literature that combines horizontal and vertical differentiation (e.g., Economides 1989, Neven and Thisse 1990). Products have two attributes – variety and quality. Each consumer has a most preferred variety and a quality valuation. On the supply side, there are two firms that initially produce single products located at the end points of the variety space. The firms are asymmetrical due to having different qualities. Customization provides ideal varieties for consumers but has no effect on product qualities. The firms play a sequential two-stage game. In the first stage, they simultaneously decide whether to customize their products. In the second stage, they simultaneously choose prices.

The paper closest to ours is Syam et al. (2005). In both papers the consumer space is two-dimensional. Syam et al. (2005) endow products with two horizontal attributes, for which consumers have heterogeneous but independent preferences. The firms are initially maximally differentiated with respect to both attributes. Like in our model, the firms play a two-stage game. They first choose whether to customize both, one, or none of the attributes, then compete in prices. The key difference between the two papers is that Syam et al. (2005) work with symmetric firms and focus on how the possibility of customizing multiple attributes affect customization choices, whereas we work with asymmetric firms and focus on the role of quality in the firms' customization decisions.

Another closely related paper is Bernhardt et al. (2007), in which ex ante symmetrical firms

¹Exceptions are Bernhardt et al. (2007) and Syam and Kumar (2006). In both asymmetric equilibrium can arise. However, such equilibrium always appears in pairs. In contrast, the asymmetric equilibrium is unique in our paper.

acquire different information about consumers and make different customization choices. Similar to our paper, consumer preferences are two-dimensional, corresponding to two attributes of the product, and the second attribute – brand loyalty – cannot be customized. There are two main differences between Bernhardt et al. (2007) and our study. First, Bernhardt et al. (2007) emphasize the cost side of customization, whereas we focus on the strategic effects of customization. Second, brand loyalty is a horizontal attribute, not vertical as quality in our paper is.

Several other papers have studied customization using a one-dimensional consumer space. Dewan et al. (2003) provide a deliberate treatment of customization technology and allow for second-degree price discrimination. Syam and Kumar (2006) highlight the role of standard products in customization competition. Alexandrov (2008) extends Salop’s model in which firms can offer interval-long adjustable “fat” products.²

Our paper as well as Syam et al. (2005) model customization as zero-one decisions, so that all customers of a customizing firm get their most preferred products. In contrast, all the other papers mentioned above treat customization as continuous choices. Both approaches match aspects of reality and have their advantages. With zero-one decisions, more attention can be devoted to the strategic effects of customization. With continuous customization choices, one can focus more on how efficiency considerations determine the range of customization.

Our equilibrium analysis shows that either both firms customize, only the high quality firm customizes, or no firm customizes. The appearance of this sequence of outcomes is monotone in the fixed cost of customization. The key to this result is that the high quality firm always gains more from customization than the low quality firm. As a result, both firms customize when the cost of customization is small, only the high quality firm customizes for intermediate levels of customization cost, and neither firm customizes when the cost is high.

We show that even if customization is costless, each of the three equilibria mentioned above is possible. In particular, both firms choose not to customize when the difference between the firms’ quality levels is small, only the high quality firm customizes for moderate quality differences, and both firms customize when the quality difference is large. The intuition behind this result is as follows. Customization by one or both firms makes the rivals “closer” to each other, thus intensifying price competition. The smaller is the quality difference, the tougher is price competition. In the extreme case in which the quality difference is zero and both firms customize, price competition results in the Bertrand outcome. Therefore, when the quality difference is small, the firms do not customize their products in order to avoid a price war. When the quality difference is large, the firms customize to take advantage of consumers’ desires for ideal varieties. The intermediate case involves customization by the high quality firm only.

We also explore how total welfare changes with the fixed cost of customization. In general, total welfare decreases with the customization cost. However, this relationship may be reversed when the equilibrium switches from customization by both firms to customization by the high quality

²MacCarthy et al. (2003) reviews six categories of mass customization: core customization, post-product customization, mass retail customization, self-customizing products, and high variety push. Fat products can be classified as either self-customizing or high variety push products.

firm only. This is largely due to the savings by the low quality firm from not undertaking costly customization.

Furthermore, we contrast our model with the benchmark model in which customization is not feasible. Both consumer surplus and total welfare are higher when customization is undertaken. However, the firms may find themselves in a prisoners' dilemma situation and end up worse off under customization.

The rest of the paper is organized as follows. In the next section we setup the model. In Section 3 the pricing stage of the game is analyzed. In Section 4 we study the firms' customization choices and investigate how these choices and total welfare are affected by the cost of customization and quality difference. In Section 5 we contrast our model with the benchmark model and examine the effects of customization on consumers, firms, and total welfare. Concluding remarks are provided in Section 6. Proofs of all lemmas and propositions are relegated to the Appendix.

2 The Basic Model

Consider an industry in which products are defined along two characteristics, variety and quality. The former corresponds to horizontal differentiation and the latter to vertical differentiation. Consumer preferences vary along two dimensions. Each consumer has most preferred variety $x \in [0, 1]$ and quality valuation $y \in [0, 1]$. A consumer of type (x, y) derives the following utility from buying one unit of product i :

$$v + yq_i - t|x - x_i| - p_i,$$

where v is a positive constant, t is a preference parameter, q_i is the quality, x_i is the variety, and p_i is the price of product i . We will assume that v is large enough for all consumers to find a product that yields positive payoff in equilibrium. Consumers are uniformly distributed over the unit square $[0, 1] \times [0, 1]$ with a total mass equal to 1.

There are two firms, A and B, operating with zero marginal costs of production. Initially, firm A offers a single (standard) product of quality $q_A = 0$ and variety $x_A = 0$, whereas firm B offers a single product of quality $q_B = q > 0$ and variety $x_B = 1$. That is, firm B is the higher quality firm and the two firms have maximum variety differentiation.³

We will normalize t to 1. This amounts to a monotonic transformation of preferences. The utilities of a consumer of type (x, y) from buying firm A's and firm B's standard products are

$$v - x - p_A \tag{1}$$

and

$$v + qy - (1 - x) - p_B, \tag{2}$$

respectively.

³The assumption that $q_A = 0$ is just a normalization. What matters is the quality difference, which here is represented by q .

Investing $K \geq 0$ into product-customization technology allows a firm to produce a product that exactly matches a given consumer's preferred variety. The utilities of type (x, y) from buying firm A's and firm B's customized products are

$$v - p_A \tag{3}$$

and

$$v + qy - p_B, \tag{4}$$

respectively.

The game involves two stages. In stage 1, the firms simultaneously decide whether to customize their products. These decisions become common knowledge after they are made. In stage 2, the firms simultaneously choose prices, consumers decide which products to purchase, and profits are realized. The equilibrium concept employed is subgame perfect Nash equilibrium. The analysis of consumer choices is straightforward. We, therefore, focus on the firms' choices and proceed using backward induction.

3 Analysis of the Pricing Stage

In this section we investigate the firms' pricing decisions, taking as given their customization choices in the first stage of the game. There are four subgames to consider, corresponding to the following stage 1 scenarios: no firm customizes; firm A (the low quality firm) customizes; firm B (the high quality firm) customizes; and both firms customize. We denote these subgames by "NN", "YN", "NY", and "YY".

3.1 Subgame NN: No Firm Customizes

When no firm customizes, utilities from firm A's and firm B's standard products are given by (1) and (2), respectively. Hence, a consumer of type (x, y) purchases from firm A if and only if

$$v - x - p_A \geq v + qy - (1 - x) - p_B.$$

Therefore, for a given level of quality valuation y , the marginal consumer type in terms of variety x is given by

$$\hat{x}(y) = \frac{1}{2}(1 - qy + p_B - p_A). \tag{5}$$

It follows that the set of consumers who are indifferent between purchasing from firm A and firm B corresponds to a straight line in the unit square. We will refer to this set as the indifference line. This line divides the unit square into two areas representing firm A's and firm B's customers.

The slope of the indifference line (5) equals $-2/q$. An increase in q makes the line flatter. An increase in p_B (and/or decrease in p_A) shifts the line to the right, thereby reducing the market size of firm B. It follows that there are four possible positions for the indifference line. In Figure 1(a) the line intersects the top and bottom sides of the square, in 1(b) the left and bottom sides, in 1(c)

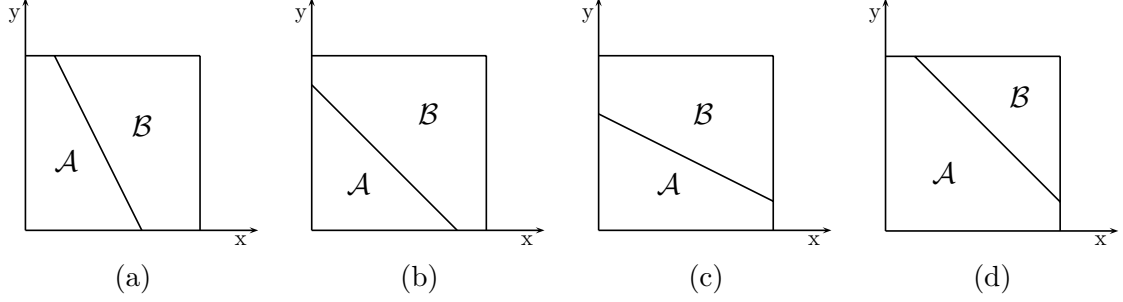


Figure 1: Indifference line and market areas

the left and right sides, and in 1(d) the top and right sides. Market areas served by firms A and B are labeled \mathcal{A} and \mathcal{B} , respectively.

Let $D_A(p_A, p_B)$ and $D_B(p_A, p_B)$ denote the demand functions of firms A and B. The expressions for these functions depend on the position of the indifference line. (The details are provided in the proof of Lemma 1 in the Appendix.) Because the firms' marginal production costs are normalized to zero, their profit functions are

$$\Pi_A(p_A, p_B) = D_A(p_A, p_B)p_A$$

and

$$\Pi_B(p_A, p_B) = D_B(p_A, p_B)p_B.$$

Firms A and B choose their prices p_A and p_B simultaneously to maximize $\Pi_A(p_A, p_B)$ and $\Pi_B(p_A, p_B)$, respectively.

The following lemma presents the equilibrium for subgame NN.

Lemma 1 (Equilibrium prices and profits when no firm customizes). *Suppose neither firm customizes in stage 1, then the equilibrium prices and profits in stage 2 are as follows.*

(i) If $q \leq 3/2$,

$$\begin{cases} p_A^{NN} = 1 - \frac{1}{6}q \\ p_B^{NN} = 1 + \frac{1}{6}q \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NN} = \frac{1}{2} \left(1 - \frac{1}{6}q\right)^2 \\ \Pi_B^{NN} = \frac{1}{2} \left(1 + \frac{1}{6}q\right)^2 \end{cases}$$

(ii) If $3/2 < q \leq 3$,

$$\begin{cases} p_A^{NN} = \frac{1}{8} (1 + \sqrt{1 + 16q}) \\ p_B^{NN} = \frac{1}{8} (-5 + 3\sqrt{1 + 16q}) \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NN} = \frac{1}{q} \left(\frac{1 + \sqrt{1 + 16q}}{8}\right)^3 \\ \Pi_B^{NN} = \left(1 - \frac{1}{q} \left(\frac{1 + \sqrt{1 + 16q}}{8}\right)^2\right) \frac{-5 + 3\sqrt{1 + 16q}}{8} \end{cases}$$

(iii) If $q > 3$,

$$\begin{cases} p_A^{NN} = \frac{1}{3}q \\ p_B^{NN} = \frac{2}{3}q \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NN} = \frac{1}{9}q \\ \Pi_B^{NN} = \frac{4}{9}q \end{cases}$$

It is easy to verify that the corresponding prices and profits in parts (i) and (ii) are equal when evaluated at $q = 3/2$. Similarly, the prices and profits in parts (ii) and (iii) are equal at $q = 3$. Therefore, the equilibrium prices and profits vary continuously when q changes. This is also true for the other three subgames, i.e., in Lemmas 2 – 4.

Lemma 1 deserves some discussion. Case (i) corresponds to Figure 1(a) in which quality difference q is small. In equilibrium, both firms serve consumers with all quality valuations, and each firm attracts consumers closer to its position on the variety interval. Case (ii) corresponds to Figure 1(b). Firm A attracts only consumers who are close to its variety position and have low quality valuations (i.e., small x 's and small y 's). Firm B uses its quality advantage to capture all the other consumers. Case (iii) corresponds to Figure 1(c) in which quality difference q is large. In this case, the low quality firm A competes aggressively and the high quality firm B does better setting a high price and exploiting consumers with high quality valuations. In equilibrium, firm A serves consumers of all variety preferences, and so does firm B. Firm A attracts consumers with low quality valuations and firm B attracts consumers with high quality valuations.

It is worthwhile to note the intuitive outcome implied by Lemma 1 that firm B sets a higher price, serves a larger market area, and earns a higher profit than firm A in the equilibrium of subgame NN. Furthermore, Figure 1(d) does not arise in equilibrium. This is also true for the other three subgames studied next.

3.2 Subgame YN: Firm A Customizes

When only firm A customizes, utilities from firm A's customized product and firm B's standard product are given by (3) and (2). Hence, a consumer of type (x, y) purchases from firm A if and only if

$$v - p_A \geq v + qy - (1 - x) - p_B.$$

For a given y , the marginal consumer type in terms of x is given by

$$\hat{x}(y) = 1 - qy + p_B - p_A, \tag{6}$$

from which we can calculate the demand and profit functions for the firms.

The following lemma presents the equilibrium for subgame YN.

Lemma 2 (Equilibrium prices and profits when firm A customizes). *Suppose firm A customizes in stage 1, then the equilibrium prices and profits in stage 2 are as follows.*

(i) If $q \leq 1$,

$$\begin{cases} p_A^{YN} = \frac{2}{3} - \frac{1}{6}q \\ p_B^{YN} = \frac{1}{3} + \frac{1}{6}q \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{YN} = \left(\frac{2}{3} - \frac{1}{6}q\right)^2 \\ \Pi_B^{YN} = \left(\frac{1}{3} + \frac{1}{6}q\right)^2 \end{cases}$$

(ii) If $q > 1$,

$$\begin{cases} p_A^{YN} = \frac{1}{3}q + \frac{1}{6} \\ p_B^{YN} = \frac{2}{3}q - \frac{1}{6} \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{YN} = \frac{1}{q} \left(\frac{1}{3}q + \frac{1}{6}\right)^2 \\ \Pi_B^{YN} = \frac{1}{q} \left(\frac{2}{3}q - \frac{1}{6}\right)^2 \end{cases}$$

Note that the critical values for the cases in this lemma are different from those in Lemma 1, and that the equilibrium of subgame YN involves only two possible positions of the indifference line. This is due to the fact that the slope of the indifference line (6) is $-1/q$, which is different from that of (5).

Case (i) of Lemma 2 corresponds to Figure 1(a) in which quality difference q is small. Customization enables firm A to overcome its quality disadvantage. Firm A's equilibrium price, market size, and profit are higher than those of firm B. In equilibrium, both firms serve consumers with all quality valuations, and each firm attracts consumers closer to its position on the variety interval. To explain why firm B still attracts consumers with low quality valuations, it suffices to consider consumers with $y = 0$. Such consumers do not gain any extra utility buying from the high quality firm, yet some of them are attracted by firm B because of its low price.

Case (ii) corresponds to Figure 1(c) in which quality difference q is large. In this case, customization does not overcome the quality disadvantage of firm A. In equilibrium, firm A's price, market size, and profit are lower than those of firm B. Firm A serves consumers of all variety preferences and so does firm B. Firm A attracts consumers with low quality valuations and firm B attracts consumers with high quality valuations. These results are similar to case (iii) of subgame NN.

3.3 Subgame NY: Firm B Customizes

When only firm B customizes, utilities from firm A's standard product and firm B's customized product are given by (1) and (4). Hence, a consumer of type (x, y) purchases from firm A if and only if

$$v - x - p_A \geq v + qy - p_B.$$

For a given y , the marginal consumer type in terms of x is given by

$$\hat{x}(y) = -qy + p_B - p_A. \quad (7)$$

The following lemma presents the equilibrium for subgame NY.

Lemma 3 (Equilibrium prices and profits when firm B customizes). *Suppose firm B customizes in stage 1, then the equilibrium prices and profits in stage 2 are as follows.*

(i) If $q \leq 1/2$,

$$\begin{cases} p_A^{NY} = \frac{1}{3} - \frac{1}{6}q \\ p_B^{NY} = \frac{2}{3} + \frac{1}{6}q \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NY} = \left(\frac{1}{3} - \frac{1}{6}q\right)^2 \\ \Pi_B^{NY} = \left(\frac{2}{3} + \frac{1}{6}q\right)^2 \end{cases}$$

(ii) If $1/2 < q \leq 2$,

$$\begin{cases} p_A^{NY} = \frac{1}{4}\sqrt{2q} \\ p_B^{NY} = \frac{3}{4}\sqrt{2q} \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NY} = \frac{1}{16}\sqrt{2q} \\ \Pi_B^{NY} = \frac{9}{16}\sqrt{2q} \end{cases}$$

(iii) If $q > 2$,

$$\begin{cases} p_A^{NY} = \frac{1}{3}q - \frac{1}{6} \\ p_B^{NY} = \frac{2}{3}q + \frac{1}{6} \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NY} = \frac{1}{q} \left(\frac{1}{3}q - \frac{1}{6}\right)^2 \\ \Pi_B^{NY} = \frac{1}{q} \left(\frac{2}{3}q + \frac{1}{6}\right)^2 \end{cases}$$

This subgame leads to qualitatively similar results as subgame NN. Here, firm B's quality advantage is reinforced by customization, pushing the critical values lower compared to subgame NN.

3.4 Subgame YY: Both Firms Customize

When both firms customize, utilities from firm A's and firm B's customized products are given by (3) and (4). Hence, a consumer of type (x, y) purchases from firm A if and only if

$$v - p_A \geq v + qy - p_B,$$

or equivalently,

$$y \leq \frac{p_B - p_A}{q}.$$

The following lemma presents the equilibrium for subgame YY.

Lemma 4 (Equilibrium prices and profits when both firms customize). *Suppose both firms customize in stage 1, then the equilibrium prices and profits in stage 2 are given by:*

$$\left\{ \begin{array}{l} p_A^{YY} = \frac{1}{3}q \\ p_B^{YY} = \frac{2}{3}q \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \Pi_A^{YY} = \frac{1}{9}q \\ \Pi_B^{YY} = \frac{4}{9}q \end{array} \right.$$

In the equilibrium of subgame YY the indifference line is horizontal at $1/3$ from the bottom side of the unit square. That is, firm A serves all consumers with quality valuations less than $1/3$, and firm B serves the rest. The result here is the same as in the standard vertical differentiation model.

4 Equilibrium Customization Choices

In stage 1 of the game the firms simultaneously decide whether to customize their products. This is presented by the following matrix game, where "N" stands for no customization and "Y" stands for customization.

		Firm B	
		N	Y
Firm A	N	Π_A^{NN}, Π_B^{NN}	$\Pi_A^{NY}, \Pi_B^{NY} - K$
	Y	$\Pi_A^{YN} - K, \Pi_B^{YN}$	$\Pi_A^{YY} - K, \Pi_B^{YY} - K$

Recall that K is the fixed cost of customization for each firm. Denote firm A's change in gross profit in the two columns by

$$c_1 = \Pi_A^{YN} - \Pi_A^{NN}, \quad c_2 = \Pi_A^{YY} - \Pi_A^{NY}.$$

They represent firm A's gains in profit from customization given B's customization choice. Denote firm B's change in gross profit in the two rows by

$$r_1 = \Pi_B^{NY} - \Pi_B^{NN}, \quad r_2 = \Pi_B^{YY} - \Pi_B^{YN}.$$

They represent firm B's gains in profit from customization given A's customization choice.⁴ Hence, (N,N) is a Nash equilibrium if $K \geq \max\{c_1, r_1\}$, (Y,N) is a Nash equilibrium if $K \in [r_2, c_1]$, (N,Y) is a Nash equilibrium if $K \in [c_2, r_1]$, and (Y,Y) is a Nash equilibrium if $K \leq \min\{c_2, r_2\}$.

Lemma 5 (Relative gains from customization). *For $i, j = 1, 2$, $c_i < r_j$.*

That is, the high quality firm gains more from customization than the low quality firm. It follows immediately that (Y,N) is not a Nash equilibrium for any value of K . Hence, there is a unique Nash equilibrium except for $K = c_2$ and $K = r_1$. For ease of presentation without affecting the results, we will select (Y,Y) as the Nash equilibrium at $K = c_2$ and (N,Y) at $K = r_1$. Accordingly,

$$\left. \begin{array}{l} (N,N) \\ (N,Y) \\ (Y,Y) \end{array} \right\} \text{ is the Nash equilibrium iff } \begin{cases} K > r_1 \\ K \in (c_2, r_1] \\ K \leq c_2 \end{cases} \quad (8)$$

Because K is non-negative, the above discussion implies that the signs of c_2 and r_1 are crucial for equilibrium analysis. Specifically, when both c_2 and r_1 are negative (N,N) is the Nash equilibrium for any K . When $c_2 < 0$ and $r_1 > 0$, either (N,Y) or (N,N) is the Nash equilibrium, depending on K . When both c_2 and r_1 are positive, the Nash equilibrium can be (Y,Y), (N,Y), or (N,N).

The next proposition summarizes our main results on customization choices.

Proposition 1 (Equilibrium customization choices). *The following holds for the firms' equilibrium customization choices in stage 1.*

- (i) *If $q \leq 0.56$ then the Nash equilibrium is (N,N) for any value of K .*
- (ii) *If $0.56 < q \leq 0.63$ then the Nash equilibrium is (N,Y) for $K \leq r_1$, and (N,N) for $K > r_1$.*
- (iii) *If $q > 0.63$ then the Nash equilibrium is (Y,Y) for $K \leq c_2$, (N,Y) for $K \in (c_2, r_1]$, and (N,N) for $K > r_1$.*

Detailed expressions for the critical values of K (i.e., c_2 and r_1) are provided in the proof of Proposition 1. Figure 2 plots c_2 and r_1 as functions of q .⁵ Depending on the values for parameters q and K , either both firms customize, only the high quality firm customizes, or no firm customizes. Moreover, the appearance of this sequence of outcomes is monotone in the fixed cost of customization. Specifically, starting at the equilibrium when $K = 0$ we always move down the sequence (Y,Y)–(N,Y)–(N,N) as K increases. Proposition 1 also implies that even if customization is costless, the firms might not customize in equilibrium. This happens when the quality difference q is small (≤ 0.56).

It is worth noting that whereas c_2 is an increasing function of q , r_1 is not. As a result, for fixed K , the equilibrium does not follow a certain sequence as q changes. This can be seen from Figure 2.

⁴These gains are dependent on q and can be negative for both firms.

⁵The number 0.56 is an approximate solution to $81\sqrt{2q} = 2(6+q)^2$. The number 0.63 is an approximate value of $81/126$.

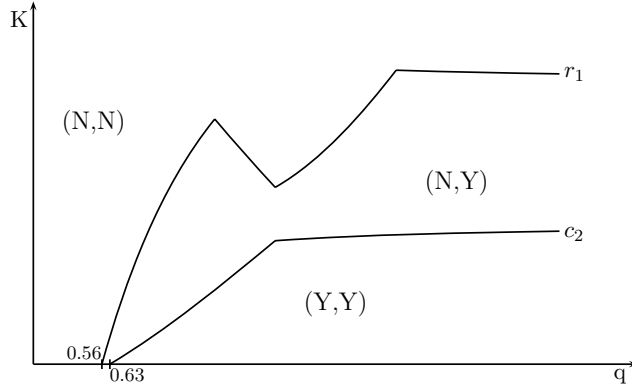


Figure 2: Equilibrium customization choices

For example, as q increases, the equilibrium changes from (N,N) to (N,Y) to (Y,Y) for small values of K , and from (N,N) to (N,Y) to (N,N) to (N,Y) for a range of values of K .

To highlight the effect of q on the equilibrium customization choices, consider $K = 0$. Customization by one or both firms makes the rivals “closer” to each other, thus intensifying price competition. The smaller is the quality difference, the tougher is price competition. In the extreme (hypothetical) case in which $q = 0$ and both firms customize, price competition results in the Bertrand outcome. This intuition is behind the findings of Proposition 1. When q is small, the firms do not customize their products in order to avoid a price war. When q is large, the firms customize to take advantage of consumers’ desires for ideal varieties. The intermediate case involves customization by one of the firms (firm B).

We next explore how total welfare, defined as the sum of consumer and producer surpluses, changes with the fixed cost of customization. Surprisingly, we find that total welfare is not always monotonically decreasing in K (Proposition 2). Because prices are transfers between consumers and firms, total welfare in our model is v plus utility from purchasing the high quality product, minus disutility from consuming a less preferred variety, minus customization costs. That is,

$$W = v + q \iint_{\mathcal{B}} y \, dx dy - (1 - \mathbb{I}_A) \iint_{\mathcal{A}} x \, dx dy - (1 - \mathbb{I}_B) \iint_{\mathcal{B}} (1 - x) \, dx dy - K(\mathbb{I}_A + \mathbb{I}_B). \quad (9)$$

In (9), \mathbb{I}_A and \mathbb{I}_B are indicator functions; $\mathbb{I}_A = 1$ if firm A customizes and zero otherwise, similar for \mathbb{I}_B . The second term is the benefit accrued to consumers who buy from firm B. The third (fourth) term measures the disutility of firm A’s (B’s) consumers from not getting their most preferred varieties. The last term is the sum of the firms’ customization costs.

Proposition 2 (Welfare effects of K). *The following holds for total welfare as a function of K .*

- (i) *For $q \leq 1.53$, total welfare monotonically decreases in K .*
- (ii) *For $q > 1.53$, total welfare is non-monotonic in K . Specifically, it decreases up to some critical value of K at which it jumps up and then continues to decrease.*

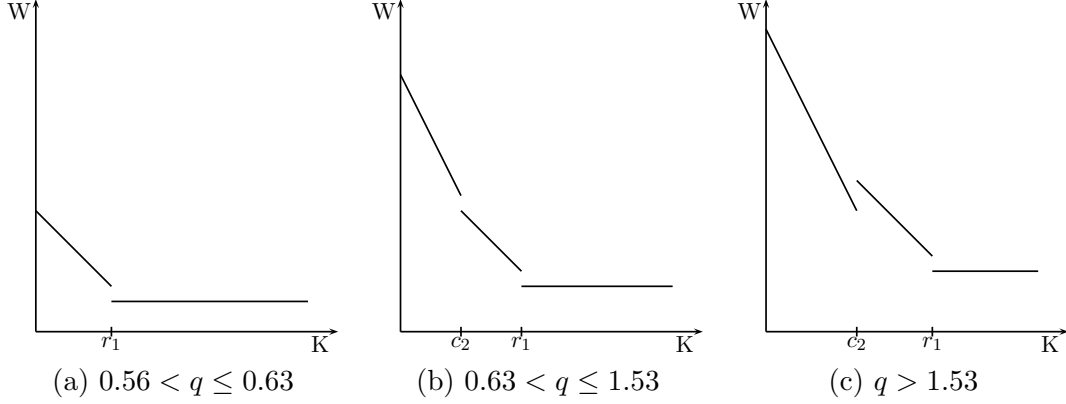


Figure 3: Total welfare

We first provide graphic illustrations for this proposition and then the intuition. By Proposition 1, if $q \leq 0.56$ then the firms do not customize in equilibrium for any value of K . Hence, W is independent of K . If $0.56 < q \leq 0.63$ then firm B customizes for small values of K and neither firm customizes for large values of K . Total welfare is a monotone function in K , as depicted in Figure 3(a).⁶ If $q > 0.63$ then the equilibrium switches from (Y,Y) to (N,Y), and then to (N,N) as K increases. In this case there are two patterns for W , monotonic in Figure 3(b) and non-monotonic in Figure 3(c).⁷

The intuition behind these results, the non-monotonicity result in particular, can be obtained by examining how the individual terms in total welfare change with K . For convenience, let W^{YY} , W^{NY} , and W^{NN} denote total welfare under the three possible equilibria. By (9),

$$W^{YY} = v + q \iint_{\mathcal{B}} y \, dx dy - 2K, \quad (10)$$

$$W^{NY} = v + q \iint_{\mathcal{B}} y \, dx dy - \iint_{\mathcal{A}} x \, dx dy - K, \quad (11)$$

and

$$W^{NN} = v + q \iint_{\mathcal{B}} y \, dx dy - \iint_{\mathcal{A}} x \, dx dy - \iint_{\mathcal{B}} (1-x) \, dx dy. \quad (12)$$

Note that market areas \mathcal{A} and \mathcal{B} are generally different in these expressions for a given q . Consider first what happens at $K = c_2$, where the equilibrium switches from (Y,Y) to (N,Y). According to (10) and (11), total welfare increases by K and decreases by $\iint_{\mathcal{A}} x \, dx dy$. Also, $q \iint_{\mathcal{B}} y \, dx dy$ increases because firm B attracts more consumers when firm A stops customizing. This effect is stronger for higher values of q . When q is small, the two positive effects are not large enough to overwhelm the negative effect. As a result, W jumps down at $K = c_2$ as illustrated in Figure 3(b). When q is high,

⁶The first segment has the slope of -1 and the second segment is horizontal.

⁷In both cases, the first segment has the slope of -2 , the second has the slope of -1 , and the third is horizontal. The number 1.53 is an approximate value of $49/32$.

the two positive effects overcome the negative effect, and W jumps up at $K = c_2$ as in Figure 3(c).

Consider next what happens at $K = r_1$, where the equilibrium switches from (N,Y) to (N,N). From (11) and (12), total welfare increases by K and decreases by $\iint_{\mathcal{B}}(1-x)dx dy$. Moreover, the term $q \iint_{\mathcal{B}} y dx dy$ decreases because firm B stops customizing and loses some consumers to firm A. The two negative effects are stronger than the positive effect, leading to the downward jumps of W in both Figures 3(b) and 3(c). Similar reasoning applies to Figure 3(a).

5 Customization vs. No Customization

In this section we contrast the customization model studied above with the benchmark model in which customization is not feasible. This allows us to investigate how customization affects consumers, firms, and total welfare. Obviously, the benchmark model corresponds to subgame NN, hence the prices and profits are as in Lemma 1.

Proposition 3 is concerned with consumer surplus, which is equal to the first four terms in (9) minus the payments by all consumers.

Proposition 3 (Effect of customization on consumer surplus). *Customization increases consumer surplus.*

Customization enables some or all consumers to get their most preferred varieties, it also increases competition between the firms. Both factors work to improve consumer welfare. Obviously, if both firms choose not to customize in the customization model then the two models yield the same outcome.

Whereas the effect of customization on consumer surplus is unambiguous, it is not so with respect to the firms' profits net of customization costs.

Proposition 4 (Effect of customization on net profits). *Compared to the benchmark,*

- (i) *firm A is worse off and firm B is better off whenever (N,Y) is the equilibrium in stage 1;*
- (ii) *both firms are worse off whenever (Y,Y) is the equilibrium in stage 1.*

It is obvious that firm B is better off when it is the only firm customizing in equilibrium. In this case firm A is hurt by more fierce competition. When q and K are such that both firms customize in equilibrium, the stage 1 game is a prisoners' dilemma game. Customization is the dominant choice for each firm, but intensified price competition destroys the advantages of customization.

The next proposition concerns total welfare.

Proposition 5 (Effect of customization on total welfare). *Customization improves total welfare.*

To summarize, customization always benefits consumers but may hurt the firms. However, the gain by consumers always outweighs possible losses of the firms.

6 Concluding Remarks

The novelty of our paper is the incorporation of difference in product qualities into customization competition. Customization enables firms to take advantage of consumers' desires for ideal varieties. However, it makes firms less differentiated and, therefore, intensifies price competition. This intuition is behind much of the results in the theoretical literature on customization, and is shown here to be valid when there is vertical differentiation.

The most important finding in our paper is that quality does play an important role in firms' strategic decisions concerning customization. We show that customization can occur in equilibrium only when the quality difference is sufficiently large. Moreover, the high quality firm can always reap a larger benefit from customization than the low quality firm. As a result, the high quality firm may be the only firm customizing in equilibrium, whereas the low quality firm never customizes alone. This result is supported by many real-world observations. We do not see customized low quality bicycles and shoes. As another example, Timbuk 2 customizes its messenger bags, whereas lower quality bags made by many other manufactures are not customized.

Both consumer surplus and total welfare are shown to be higher under customization, but the firms often find themselves playing a prisoners' dilemma game and end up worse off. These results are largely consistent with findings in the literature. A new and interesting result in our paper is that total welfare is not always monotonically decreasing in the fixed cost of customization.

Our paper has focused on the strategic aspect of customization. One extension is to make the range of customization a continuous choice variable. Such a model would involve a fairly complex analysis. We believe that the qualitative results of our paper will hold; e.g., the range of customization of the higher quality firm is larger than that of the lower quality firm. Another interesting extension is to introduce dynamics into the model to address the roles of customization and quality in entry deterrence.

Appendix

Proof of Lemma 1. Each case is proven in turn.

- (i) Consider $q \leq 3/2$. Suppose the indifference line (5) intersects the top and bottom sides of the unit square as shown in Figure 1(a). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{2} \left(1 - \frac{1}{2}q + p_B - p_A \right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{2} \left(1 + \frac{1}{2}q + p_A - p_B \right)$$

in this case. The profit maximizing first-order conditions

$$\begin{cases} 1 - \frac{1}{2}q + p_B - 2p_A = 0 \\ 1 + \frac{1}{2}q + p_A - 2p_B = 0 \end{cases}$$

imply

$$\begin{cases} p_A^{NN} = 1 - \frac{1}{6}q \\ p_B^{NN} = 1 + \frac{1}{6}q \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NN} = \frac{1}{2} \left(1 - \frac{1}{6}q \right)^2 \\ \Pi_B^{NN} = \frac{1}{2} \left(1 + \frac{1}{6}q \right)^2 \end{cases}$$

It is left to verify that under these prices the indifference line intersects the top and bottom sides of the unit square. Algebraically, $\hat{x}(1) \geq 0$ and $\hat{x}(0) \leq 1$. Indeed,

$$\hat{x}(1) = \frac{1}{2} (1 - q + p_B^{NN} - p_A^{NN}) = \frac{1}{2} \left(1 - \frac{2}{3}q \right) \geq 0,$$

$$\hat{x}(0) = \frac{1}{2} (1 + p_B^{NN} - p_A^{NN}) = \frac{1}{2} \left(1 + \frac{1}{3}q \right) < 1$$

hold for $q \leq 3/2$. (In fact, $\hat{x}(1) = 0$ when $q = 3/2$.)

- (ii) Consider $3/2 < q \leq 3$. Suppose the indifference line (5) intersects the left and bottom sides of the unit square as shown in Figure 1(b). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{4q} (1 + p_B - p_A)^2 \quad \text{and} \quad D_B(p_A, p_B) = 1 - \frac{1}{4q} (1 + p_B - p_A)^2$$

in this case. The profit maximizing first-order conditions

$$\begin{cases} (1 + p_B - p_A)(1 + p_B - 3p_A) = 0 \\ (1 + p_B - p_A)(1 + 3p_B - p_A) = 4q \end{cases}$$

imply

$$\begin{cases} p_A^{NN} = \frac{1}{8}(1 + \sqrt{1 + 16q}) \\ p_B^{NN} = \frac{1}{8}(-5 + 3\sqrt{1 + 16q}) \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NN} = \frac{1}{q} \left(\frac{1 + \sqrt{1 + 16q}}{8} \right)^3 \\ \Pi_B^{NN} = \left(1 - \frac{1}{q} \left(\frac{1 + \sqrt{1 + 16q}}{8} \right)^2 \right) \frac{-5 + 3\sqrt{1 + 16q}}{8} \end{cases}$$

It is left to verify that under these prices the indifference line intersects the left and bottom sides of the unit square. Algebraically, $\hat{x}(1) \leq 0$ and $\hat{x}(0) \in [0, 1]$. Indeed,

$$\hat{x}(1) = \frac{1}{2} \left(1 - q + \frac{1}{4} \left(-3 + \sqrt{1 + 16q} \right) \right) < 0,$$

$$\hat{x}(0) = \frac{1}{2} \left(1 + \frac{1}{4} \left(-3 + \sqrt{1 + 16q} \right) \right) \in (0, 1]$$

hold for $3/2 < q \leq 3$. (In fact, $\hat{x}(1) = 0$ when $q = 3/2$ and $\hat{x}(0) = 1$ when $q = 3$.)

- (iii) Consider $q > 3$. Suppose the indifference line (5) intersects the left and right sides of the unit square as shown in Figure 1(c). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{q} (p_B - p_A) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{q} (q + p_A - p_B).$$

The profit maximizing first-order conditions

$$\begin{cases} p_B - 2p_A = 0 \\ q + p_A - 2p_B = 0 \end{cases}$$

imply

$$\begin{cases} p_A^{NN} = \frac{1}{3}q \\ p_B^{NN} = \frac{2}{3}q \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NN} = \frac{1}{9}q \\ \Pi_B^{NN} = \frac{4}{9}q \end{cases}$$

It is left to verify that under these prices the indifference line intersects the right and left sides of the unit square. Algebraically, $\hat{x}(1) \leq 0$ and $\hat{x}(0) \geq 1$. Indeed,

$$\hat{x}(1) = \frac{1}{2} \left(1 - \frac{2}{3}q \right) < 0 \quad \text{and} \quad \hat{x}(0) = \frac{1}{2} \left(1 + \frac{1}{3}q \right) > 1$$

hold for $q > 3$. (In fact, $\hat{x}(0) = 1$ when $q = 3$.)

□

Proof of Lemma 2. Each case is proven in turn.

- (i) Consider $q \leq 1$. Suppose the indifference line (6) intersects the top and bottom sides of the unit square as shown in Figure 1(a). The firms' demand functions are

$$D_A(p_A, p_B) = 1 - \frac{1}{2}q + p_B - p_A \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{2}q + p_A - p_B.$$

The profit maximizing first-order conditions

$$\begin{cases} 1 - \frac{1}{2}q + p_B - 2p_A = 0 \\ \frac{1}{2}q + p_A - 2p_B = 0 \end{cases}$$

imply

$$\begin{cases} p_A^{YN} = \frac{2}{3} - \frac{1}{6}q \\ p_B^{YN} = \frac{1}{3} + \frac{1}{6}q \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{YN} = \left(\frac{2}{3} - \frac{1}{6}q \right)^2 \\ \Pi_B^{YN} = \left(\frac{1}{3} + \frac{1}{6}q \right)^2 \end{cases}$$

It is left to verify that under these prices the indifference line intersects the top and bottom sides of the unit square. Algebraically, $\hat{x}(1) \geq 0$ and $\hat{x}(0) \leq 1$. Indeed,

$$\hat{x}(1) = 1 - q + p_B^{YN} - p_A^{YN} = \frac{2}{3} - \frac{2}{3}q \geq 0 \quad \text{and} \quad \hat{x}(0) = 1 + p_B^{YN} - p_A^{YN} = \frac{2}{3} + \frac{1}{3}q < 1$$

hold for $q \leq 1$. (In fact, $\hat{x}(1) = 0$ and $\hat{x}(0) = 1$ when $q = 1$.)

- (ii) Consider $q > 1$. Suppose the indifference line (6) intersects the left and right sides of the unit square as shown in Figure 1(c). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{q} \left(\frac{1}{2} + p_B - p_A \right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{q} \left(q - \frac{1}{2} + p_A - p_B \right).$$

The profit maximizing first-order conditions

$$\begin{cases} \frac{1}{2} + p_B - 2p_A = 0 \\ q - \frac{1}{2} + p_A - 2p_B = 0 \end{cases}$$

imply

$$\begin{cases} p_A^{YN} = \frac{1}{3}q + \frac{1}{6} \\ p_B^{YN} = \frac{2}{3}q - \frac{1}{6} \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{YN} = \frac{1}{q} \left(\frac{1}{3}q + \frac{1}{6} \right)^2 \\ \Pi_B^{YN} = \frac{1}{q} \left(\frac{2}{3}q - \frac{1}{6} \right)^2 \end{cases}$$

It is left to verify that under these prices the indifference line intersects the right and left sides of the unit square. Algebraically, $\hat{x}(1) \leq 0$ and $\hat{x}(0) \geq 1$. Indeed,

$$\hat{x}(1) = \frac{2}{3} - \frac{2}{3}q < 0 \quad \text{and} \quad \hat{x}(0) = \frac{2}{3} + \frac{1}{3}q > 1$$

hold for $q > 1$. (In fact, $\hat{x}(1) = 0$ and $\hat{x}(0) = 1$ when $q = 1$.)

□

Proof of Lemma 3. Each case is proven in turn.

- (i) Consider $q \leq 1/2$. Suppose the indifference line (7) intersects the top and bottom sides of the unit square as shown in Figure 1(a). The firms' demand functions are

$$D_A(p_A, p_B) = -\frac{1}{2}q + p_B - p_A \quad \text{and} \quad D_B(p_A, p_B) = 1 + \frac{1}{2}q + p_A - p_B.$$

The profit maximizing first-order conditions

$$\begin{cases} -\frac{1}{2}q + p_B - 2p_A = 0 \\ 1 + \frac{1}{2}q + p_A - 2p_B = 0 \end{cases}$$

imply

$$\begin{cases} p_A^{NY} = \frac{1}{3} - \frac{1}{6}q \\ p_B^{NY} = \frac{2}{3} + \frac{1}{6}q \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NY} = \left(\frac{1}{3} - \frac{1}{6}q \right)^2 \\ \Pi_B^{NY} = \left(\frac{2}{3} + \frac{1}{6}q \right)^2 \end{cases}$$

It is left to verify that under these prices the indifference line intersects the top and bottom sides of the unit square. Algebraically, $\hat{x}(1) \geq 0$ and $\hat{x}(0) \leq 1$. Indeed,

$$\hat{x}(1) = -q + p_B^{NY} - p_A^{NY} = \frac{1}{3} - \frac{2}{3}q \geq 0 \quad \text{and} \quad \hat{x}(0) = p_B^{NY} - p_A^{NY} = \frac{1}{3} + \frac{1}{3}q < 1$$

hold for $q \leq 1/2$. (In fact, $\hat{x}(1) = 0$ when $q = 1/2$.)

- (ii) Consider $1/2 < q \leq 2$. Suppose the indifference line (7) intersects the left and bottom sides of the unit square as shown in Figure 1(b). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{2q} (p_B - p_A)^2 \quad \text{and} \quad D_B(p_A, p_B) = 1 - \frac{1}{2q} (p_B - p_A)^2.$$

The profit maximizing first-order conditions

$$\begin{cases} (p_B - p_A)(p_B - 3p_A) = 0 \\ (p_B - p_A)(3p_B - p_A) = 2q \end{cases}$$

imply

$$\begin{cases} p_A^{NY} = \frac{1}{4}\sqrt{2q} \\ p_B^{NY} = \frac{3}{4}\sqrt{2q} \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NY} = \frac{1}{16}\sqrt{2q} \\ \Pi_B^{NY} = \frac{9}{16}\sqrt{2q} \end{cases}$$

It is left to verify that under these prices the indifference line intersects the left and bottom sides of the unit square. Algebraically, $\hat{x}(1) \leq 0$ and $\hat{x}(0) \in [0, 1]$. Indeed,

$$\hat{x}(1) = -q + \frac{1}{2}\sqrt{2q} < 0 \quad \text{and} \quad \hat{x}(0) = \frac{1}{2}\sqrt{2q} \in (0, 1]$$

hold for $1/2 < q \leq 2$. (In fact, $\hat{x}(1) = 0$ when $q = 1/2$ and $\hat{x}(0) = 1$ when $q = 2$.)

(iii) Consider $q > 2$. Suppose the indifference line (7) intersects the left and right sides of the unit square as shown in Figure 1(c). The firms' demand functions are

$$D_A(p_A, p_B) = \frac{1}{q} \left(-\frac{1}{2} + p_B - p_A \right) \quad \text{and} \quad D_B(p_A, p_B) = \frac{1}{q} \left(q + \frac{1}{2} + p_A - p_B \right).$$

The profit maximizing first-order conditions

$$\begin{cases} -\frac{1}{2} + p_B - 2p_A = 0 \\ q + \frac{1}{2} + p_A - 2p_B = 0 \end{cases}$$

imply

$$\begin{cases} p_A^{NY} = \frac{1}{3}q - \frac{1}{6} \\ p_B^{NY} = \frac{2}{3}q + \frac{1}{6} \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{NY} = \frac{1}{q} \left(\frac{1}{3}q - \frac{1}{6} \right)^2 \\ \Pi_B^{NY} = \frac{1}{q} \left(\frac{2}{3}q + \frac{1}{6} \right)^2 \end{cases}$$

It is left to verify that under these prices the indifference line intersects the right and left sides of the unit square. Algebraically, $\hat{x}(1) \leq 0$ and $\hat{x}(0) \geq 1$. Indeed,

$$\hat{x}(1) = \frac{1}{3} - \frac{2}{3}q < 0 \quad \text{and} \quad \hat{x}(0) = \frac{1}{3} + \frac{1}{3}q > 1$$

hold for $q > 2$. (In fact, $\hat{x}(0) = 1$ when $q = 2$.)

□

Proof of Lemma 4. The firms' profit functions are

$$\Pi_A(p_A, p_B) = \frac{1}{q} (p_B - p_A) p_A \quad \text{and} \quad \Pi_B(p_A, p_B) = \frac{1}{q} (q + p_A - p_B) p_B.$$

The first-order conditions

$$\begin{cases} p_B - 2p_A = 0 \\ q + p_A - 2p_B = 0 \end{cases}$$

imply

$$\begin{cases} p_A^{YY} = \frac{1}{3}q \\ p_B^{YY} = \frac{2}{3}q \end{cases} \quad \text{and} \quad \begin{cases} \Pi_A^{YY} = \frac{1}{9}q \\ \Pi_B^{YY} = \frac{4}{9}q \end{cases}$$

□

Proof of Lemma 5. Lemmas 1 – 4 imply that there are six regions of q to consider. Lemma 5 follows immediately from the following.

(1) Consider $q \leq 1/2$. By Lemmas 1(i), 2(i), 3(i), and 4,

$$\begin{cases} c_1 = \left(\frac{2}{3} - \frac{1}{6}q\right)^2 - \frac{1}{2}\left(1 - \frac{1}{6}q\right)^2 \\ c_2 = \frac{1}{9}q - \left(\frac{1}{3} - \frac{1}{6}q\right)^2 \\ r_1 = \left(\frac{2}{3} + \frac{1}{6}q\right)^2 - \frac{1}{2}\left(1 + \frac{1}{6}q\right)^2 \\ r_2 = \frac{4}{9}q - \left(\frac{1}{3} + \frac{1}{6}q\right)^2 \end{cases}$$

It is straightforward to show that if $q \leq 0.34$ then $c_1 < c_2 < r_1 < r_2 \leq 0$, and if $0.34 < q \leq 1/2$ then $c_1 < c_2 < r_1 < 0 < r_2$.

(2) Consider $1/2 < q \leq 1$. By Lemmas 1(i), 2(i), 3(ii), and 4,

$$\begin{cases} c_1 = \left(\frac{2}{3} - \frac{1}{6}q\right)^2 - \frac{1}{2}\left(1 - \frac{1}{6}q\right)^2 \\ c_2 = \frac{1}{9}q - \frac{1}{16}\sqrt{2q} \\ r_1 = \frac{9}{16}\sqrt{2q} - \frac{1}{2}\left(1 + \frac{1}{6}q\right)^2 \\ r_2 = \frac{4}{9}q - \left(\frac{1}{3} + \frac{1}{6}q\right)^2 \end{cases}$$

Simple algebra implies that if $1/2 < q \leq 0.56$ then $c_1 < c_2 < r_1 \leq 0 < r_2$, if $0.56 < q \leq 0.63$ then $c_1 < c_2 \leq 0 < r_1 < r_2$, and if $0.63 < q \leq 1$ then $c_1 < 0 < c_2 < r_1 < r_2$.

(3) Consider $1 < q \leq 3/2$. By Lemmas 1(i), 2(ii), 3(ii), and 4,

$$\begin{cases} c_1 = \frac{1}{q}\left(\frac{1}{3}q + \frac{1}{6}\right)^2 - \frac{1}{2}\left(1 - \frac{1}{6}q\right)^2 \\ c_2 = \frac{1}{9}q - \frac{1}{16}\sqrt{2q} \\ r_1 = \frac{9}{16}\sqrt{2q} - \frac{1}{2}\left(1 + \frac{1}{6}q\right)^2 \\ r_2 = \frac{4}{9}q - \frac{1}{q}\left(\frac{2}{3}q - \frac{1}{6}\right)^2 \end{cases}$$

Simple algebra implies that if $1 < q \leq 1.43$ then $c_1 \leq 0 < c_2 < r_1 < r_2$, and if $1.43 < q \leq 3/2$ then $0 < c_1 < c_2 < r_1 < r_2$.

(4) Consider $3/2 < q \leq 2$. By Lemmas 1(ii), 2(ii), 3(ii), and 4,

$$\begin{cases} c_1 = \frac{1}{q}\left(\frac{1}{3}q + \frac{1}{6}\right)^2 - \frac{1}{q}\left(\frac{1+\sqrt{1+16q}}{8}\right)^3 \\ c_2 = \frac{1}{9}q - \frac{1}{16}\sqrt{2q} \\ r_1 = \frac{9}{16}\sqrt{2q} - \left(1 - \frac{1}{q}\left(\frac{1+\sqrt{1+16q}}{8}\right)^2\right)\frac{-5+3\sqrt{1+16q}}{8} \\ r_2 = \frac{4}{9}q - \frac{1}{q}\left(\frac{2}{3}q - \frac{1}{6}\right)^2 \end{cases}$$

It is straightforward to show that $0 < c_1 < c_2 < r_1 < r_2$.

(5) Consider $2 < q \leq 3$. By Lemmas 1(ii), 2(ii), 3(iii), and 4,

$$\begin{cases} c_1 = \frac{1}{q} \left(\frac{1}{3}q + \frac{1}{6} \right)^2 - \frac{1}{q} \left(\frac{1+\sqrt{1+16q}}{8} \right)^3 \\ c_2 = \frac{1}{9}q - \frac{1}{q} \left(\frac{1}{3}q - \frac{1}{6} \right)^2 = \frac{1}{9} - \frac{1}{36q} \\ r_1 = \frac{1}{q} \left(\frac{2}{3}q + \frac{1}{6} \right)^2 - \left(1 - \frac{1}{q} \left(\frac{1+\sqrt{1+16q}}{8} \right)^2 \right) \frac{-5+3\sqrt{1+16q}}{8} \\ r_2 = \frac{4}{9}q - \frac{1}{q} \left(\frac{2}{3}q - \frac{1}{6} \right)^2 \end{cases}$$

Simple algebra implies that if $2 < q \leq 2.74$ then $0 < c_1 \leq c_2 < r_1 < r_2$, if $2.74 < q \leq 2.84$ then $0 < c_2 < c_1 < r_1 \leq r_2$, and if $2.84 < q \leq 3$ then $0 < c_2 < c_1 < r_2 < r_1$.

(6) Consider $q > 3$. By Lemmas 1(iii), 2(ii), 3(iii), and 4,

$$\begin{cases} c_1 = \frac{1}{q} \left(\frac{1}{3}q + \frac{1}{6} \right)^2 - \frac{1}{9}q \\ c_2 = \frac{1}{9}q - \frac{1}{q} \left(\frac{1}{3}q - \frac{1}{6} \right)^2 = \frac{1}{9} - \frac{1}{36q} \\ r_1 = \frac{1}{q} \left(\frac{2}{3}q + \frac{1}{6} \right)^2 - \frac{4}{9}q = \frac{2}{9} + \frac{1}{36q} \\ r_2 = \frac{4}{9}q - \frac{1}{q} \left(\frac{2}{3}q - \frac{1}{6} \right)^2 \end{cases}$$

It is straightforward to show that $0 < c_2 < c_1 < r_2 < r_1$.

□

Proof of Proposition 1. Applying the proof of Lemma 5 to (8) leads to the three conclusions in this proposition.

(i) If $q \leq 0.56$, both c_2 and r_1 are negative. Hence, (N,N) is the Nash equilibrium for any value of K .

(ii) If $0.56 < q \leq 0.63$, $c_2 \leq 0 < r_1$. Hence, (N,Y) is the Nash equilibrium when

$$K \leq r_1 = \frac{9}{16}\sqrt{2q} - \frac{1}{2} \left(1 + \frac{1}{6}q \right)^2,$$

and (N,N) is the Nash equilibrium when $K > r_1$.

(iii) If $q > 0.63$, both c_2 and r_1 are positive. Hence, (Y,Y) is the Nash equilibrium when $K \leq c_2$, (N,Y) is the Nash equilibrium when $K \in (c_2, r_1]$, and (Y,Y) is the Nash equilibrium when $K > r_1$. Here,

$$c_2 = \begin{cases} \frac{1}{9}q - \frac{1}{16}\sqrt{2q}, & \text{if } 0.63 < q \leq 2 \\ \frac{1}{9} - \frac{1}{36q}, & \text{if } q > 2 \end{cases}$$

and

$$r_1 = \begin{cases} \frac{9}{16}\sqrt{2q} - \frac{1}{2} \left(1 + \frac{1}{6}q \right)^2, & \text{if } 0.56 < q \leq 3/2 \\ \frac{9}{16}\sqrt{2q} - \left(1 - \frac{1}{q} \left(\frac{1+\sqrt{1+16q}}{8} \right)^2 \right) \frac{-5+3\sqrt{1+16q}}{8}, & \text{if } 3/2 < q \leq 2 \\ \frac{1}{q} \left(\frac{2}{3}q + \frac{1}{6} \right)^2 - \left(1 - \frac{1}{q} \left(\frac{1+\sqrt{1+16q}}{8} \right)^2 \right) \frac{-5+3\sqrt{1+16q}}{8}, & \text{if } 2 < q \leq 3 \\ \frac{2}{9} + \frac{1}{36q}, & \text{if } q > 3 \end{cases}$$

□

Proof of Proposition 2. Lemmas 1 – 4 together with Proposition 1 imply that there are six regions of q to consider. The two results in Proposition 2 follow immediately.

- (1) Consider $q \leq 0.56$. By Proposition 1, the firms do not customize in equilibrium for any value of K . Hence, W is independent of K .
- (2) Consider $0.56 < q \leq 0.63$. When $K \leq r_1$, only firm B customizes, the prices are as in Lemma 3(ii), and the indifference line intersects the left and bottom sides of the unit square (Figure 1b). Hence,

$$W^{NY} = v + q \int_{\frac{1}{\sqrt{2q}}}^1 \int_0^1 y \, dx dy + q \int_0^{\frac{1}{2}\sqrt{2q}} \int_{\hat{x}(y)}^1 y \, dx dy - \int_0^{\frac{1}{\sqrt{2q}}} \int_0^{\hat{x}(y)} x \, dx dy - K = v + \frac{1}{2}q - \frac{1}{12}\sqrt{2q} - K,$$

where

$$\hat{x}(y) = -qy + \frac{1}{2}\sqrt{2q}.$$

When $K > r_1$, neither firm customizes, the prices are as in Lemma 1(i), and the indifference line intersects the top and bottom sides of the unit square (Figure 1a). Hence,

$$W^{NN} = v + q \int_0^1 \int_{\hat{x}(y)}^1 y \, dx dy - \int_0^1 \int_0^{\hat{x}(y)} x \, dx dy - \int_0^1 \int_{\hat{x}(y)}^1 (1-x) \, dx dy = v + \frac{1}{4}q + \frac{1}{18}q^2 - \frac{1}{4},$$

where

$$\hat{x}(y) = \frac{1}{2} \left(1 - qy + \frac{1}{3}q \right).$$

Evaluating W^{NY} and W^{NN} at r_1 yields $W^{NY} > W^{NN}$. Hence, W decreases in K until r_1 , jumps down at r_1 , and then stays constant (Figure 3a).

- (3) Consider $0.63 < q \leq 3/2$. When $K \leq c_2$, both firms customize, and the prices are as in Lemma 4. Hence,

$$W^{YY} = v + q \int_{\frac{1}{3}}^1 \int_0^1 y \, dx dy - 2K = v + \frac{4}{9}q - 2K.$$

When $K \in (c_2, r_1]$, only firm B customizes, the prices are as in Lemma 3(ii), and

$$W^{NY} = v + \frac{1}{2}q - \frac{1}{12}\sqrt{2q} - K.$$

Finally, when $K > r_1$, neither firm customizes, the prices are as in Lemma 1(i), and

$$W^{NN} = v + \frac{1}{4}q + \frac{1}{18}q^2 - \frac{1}{4}.$$

Evaluating W^{YY} and W^{NY} at c_2 yields $W^{YY} > W^{NY}$. Evaluating W^{NY} and W^{NN} at r_1 yields $W^{NY} > W^{NN}$. Hence, W decreases in K until c_2 , jumps down at c_2 , continues to decrease until r_1 , jumps down at r_1 , and then stays constant (Figure 3b).

(4) Consider $3/2 < q \leq 2$. When $K \leq c_2$, both firms customize, and

$$W^{YY} = v + \frac{4}{9}q - 2K.$$

When $K \in (c_2, r_1]$, only firm B customizes, the prices are as in Lemma 3(ii), and

$$W^{NY} = v + \frac{1}{2}q - \frac{1}{12}\sqrt{2q} - K.$$

Finally, when $K > r_1$, neither firm customizes, the prices are as in Lemma 1(ii), and the indifference line intersects the left and bottom sides of the unit square (Figure 1b). Hence,

$$\begin{aligned} W^{NN} &= v + q \int_{\frac{\Delta}{q}}^1 \int_0^1 y \, dx dy + q \int_0^{\frac{\Delta}{q}} \int_{\hat{x}(y)}^1 y \, dx dy - \int_0^{\frac{\Delta}{q}} \int_0^{\hat{x}(y)} x \, dx dy \\ &\quad - \int_{\frac{\Delta}{q}}^1 \int_0^1 (1-x) \, dx dy - \int_0^{\frac{\Delta}{q}} \int_{\hat{x}(y)}^1 (1-x) \, dx dy = v + \frac{1}{2}q + \frac{1}{4q}\Delta^2 - \frac{1}{6q}\Delta^3 - \frac{1}{2}, \end{aligned}$$

where

$$\hat{x}(y) = \frac{1}{2}(\Delta - qy) \quad \text{and} \quad \Delta = \frac{1}{4} \left(1 + \sqrt{1 + 16q} \right).$$

Evaluating W^{YY} and W^{NY} at c_2 yields $W^{YY} < W^{NY}$ if $q > 1.53$. Evaluating W^{NY} and W^{NN} at r_1 yields $W^{NY} > W^{NN}$. That is, W is non-monotonic in K for $1.53 < q \leq 2$. Specifically, it decreases until c_2 , jumps up at c_2 , decreases until r_1 , jumps down at r_1 , and then stays constant (Figure 3c).

(5) Consider $2 < q \leq 3$. When $K \leq c_2$, both firms customize, and

$$W^{YY} = v + \frac{4}{9}q - 2K.$$

When $K \in (c_2, r_1]$, only firm B customizes, the prices are as in Lemma 3(iii), and the indifference line intersects the left and right sides of the unit square (Figure 1c). Hence,

$$W^{NY} = v + q \int_0^1 \int_{\hat{y}(x)}^1 y \, dy dx - \int_0^1 \int_0^{\hat{y}(x)} x \, dy dx - K = v + \frac{4}{9}q + \frac{1}{9q} - \frac{1}{9} - K,$$

where

$$\hat{y}(x) = \frac{1}{q} \left(-x + \frac{1}{3}q + \frac{1}{3} \right).$$

Finally, when $K > r_1$, neither firm customizes, the prices are as in Lemma 1(ii), and

$$W^{NN} = v + \frac{1}{2}q + \frac{1}{4q}\Delta^2 - \frac{1}{6q}\Delta^3 - \frac{1}{2}.$$

Evaluating W^{YY} and W^{NY} at c_2 yields $W^{YY} < W^{NY}$. Evaluating W^{NY} and W^{NN} at r_1 yields $W^{NY} > W^{NN}$. Hence, W decreases until c_2 , jumps up at c_2 , decreases until r_1 , jumps

down at r_1 , and then stays constant (Figure 3c).

(6) Consider $q > 3$. When $K \leq c_2$, both firms customize, and

$$W^{YY} = v + \frac{4}{9}q - 2K.$$

When $K \in (c_2, r_1]$, only firm B customizes, the prices are as in Lemma 3(iii), and

$$W^{NY} = v + \frac{4}{9}q + \frac{1}{9q} - \frac{1}{9} - K.$$

Finally, when $K > r_1$, neither firm customizes, the prices are as in Lemma 1(iii), and the indifference line intersects the left and right sides of the unit square (Figure 1c). Hence,

$$W^{NN} = v + q \int_0^1 \int_{\hat{y}(x)}^1 y \, dy dx - \int_0^1 \int_0^{\hat{y}(x)} x \, dy dx - \int_0^1 \int_{\hat{y}(x)}^1 (1-x) \, dy dx = v + \frac{4}{9}q + \frac{1}{6q} - \frac{1}{2},$$

where

$$\hat{y}(x) = \frac{1}{q} \left(1 - 2x + \frac{1}{3}q \right).$$

Evaluating W^{YY} and W^{NY} at c_2 yields $W^{YY} < W^{NY}$. Evaluating W^{NY} and W^{NN} at r_1 yields $W^{NY} > W^{NN}$. Hence, W decreases until c_2 , jumps up at c_2 , decreases until r_1 , jumps down at r_1 , and then stays constant (Figure 3c). □

Proof of Proposition 3. Subtracting the firms' equilibrium profits Π_A^{NN} and Π_B^{NN} from W^{NN} calculated in the proof of Proposition 2 yields consumer surplus under the benchmark model,

$$\overline{CS} = \begin{cases} v + \frac{1}{4}q + \frac{1}{36}q^2 - \frac{5}{4}, & \text{if } q \leq 3/2 \\ v + \frac{1}{2}q - \frac{3}{8}(1 + \sqrt{1 + 16q}) + \frac{2}{3q} \left(\frac{1 + \sqrt{1 + 16q}}{8} \right)^3 + \frac{1}{2}, & \text{if } 3/2 < q \leq 3 \\ v - \frac{1}{9}q + \frac{1}{6q} - \frac{1}{2}, & \text{if } q > 3 \end{cases}$$

Let CS denote consumer surplus in the model with customization. It can be obtained by removing the last term in (9) and subtracting away profits Π_A and Π_B . By using the the detailed expressions for W in the proof of Proposition 2 and the profit expressions in Lemmas 1, 3, and 4, we get

$$CS = \begin{cases} v - \frac{1}{9}q, & \text{if } q > 0.63 \text{ and } K \leq c_2 \\ v + \frac{1}{2}q - \frac{17}{24}\sqrt{2q}, & \text{if } 0.56 < q \leq 0.63 \text{ and } K \leq r_1 \\ v + \frac{1}{2}q - \frac{17}{24}\sqrt{2q}, & \text{if } 0.56 < q \leq 2 \text{ and } K \in (c_2, r_1] \\ v - \frac{1}{9}q + \frac{1}{18q} - \frac{2}{9}, & \text{if } q > 2 \text{ and } K \in (c_2, r_1] \\ \overline{CS}, & \text{otherwise} \end{cases}$$

Straightforward algebra implies that $CS \geq \overline{CS}$ holds for any values of q and K . □

Proof of Proposition 4. Each part is proven in turn.

(i) It is obvious that, compared to the benchmark model, firm B is better off when (N,Y) is the equilibrium in stage 1. In this case firm A does worse, because $\Pi_A^{NN} > \Pi_A^{NY}$ holds for $q > 0.56$.

(ii) Both firms are worse off when (Y,Y) is the equilibrium in stage 1. Indeed, for $q > 0.63$ it is straightforward to show that $\Pi_A^{NN} > \Pi_A^{YY}$ and $\Pi_B^{NN} > \Pi_B^{YY}$, implying $\Pi_A^{NN} > \Pi_A^{YY} - K$ and $\Pi_B^{NN} > \Pi_B^{YY} - K$ hold for any $K \leq c_2$.

□

Proof of Proposition 5. This result follows immediately from Proposition 2, Figure 3, and the easily confirmed fact that in Figure 3(c) the horizontal portion is below both decreasing portions.

□

References

- [1] Alexandrov, Alexei, 2008, "Fat Products," *Journal of Economics and Management Strategy*, 17(1), pp. 67-95.
- [2] Alptekinoglu, Aydin, and Charles J. Corbett, 2008, "Mass Customization versus Mass Production: Variety and Price Competition," *Manufacturing and Service Operations Management*, 10(2), pp. 198-203.
- [3] Bernhardt, Dan, Qihong Liu, and Konstantinos Serfes, 2007, "Product Customization," *European Economic Review*, 51(6), pp. 1396-1422.
- [4] Dewan, Rajiv, Bing Jing, and Abraham Seidmann, 2003, "Product Customization and Price Competition on the Internet," *Management Science*, 49(8), pp. 1055-1070.
- [5] Economides, Nicholas, 1989, "Quality Variations and Maximal Variety Differentiation," *Regional Science and Urban Economics*, 19(1), pp. 21-29.
- [6] Hotelling, Harold, 1929, "Stability in Competition," *Economic Journal*, 39, pp. 41-57.
- [7] MacCarthy, Bart, Philip G. Brabazon, Johanna Bramham, 2003, "Fundamental Modes of Operation for Mass Customization," *International Journal of Production Economics*, 85(3), pp. 289-304.
- [8] Neven, Damien, and Jacques-François Thisse, 1990, "On Quality and Variety Competition," J.J. Gabszewicz, J.-F. Richard, and L.A. Wolsy (eds.), in *Economic Decision-Making: Games, Econometrics, and Optimization*, Amsterdam: North-Holland, pp. 175-199.
- [9] Salop, Steven C., 1979, "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, 10(1), pp.141-156.
- [10] Syam, Niladri B., and Nanda Kumar, 2006, "On Customized Goods, Standard Goods, and Competition," *Marketing Science*, 25(5), pp. 525-537.
- [11] Syam, Niladri B., Ranran Ruan, and James D. Hess, 2005, "Customized Products: A Competitive Analysis," *Marketing Science*, 24(4), pp. 569-584.