

# Money, output and the payment system: Optimal monetary policy in a model with hidden effort \*

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March 14, 2007

## Abstract

We propose a new explanation for the observed difference in the cost of intraday and overnight liquidity. We argue that the low cost of intraday liquidity is an application of the Friedman rule in an environment where a deviation of the Friedman rule is optimal with respect to overnight liquidity. In our environment the cost of overnight liquidity affects output while the cost of intraday liquidity only redistributes resources between money holders and non-money holders. We show that it is optimal to set a high overnight rate to reduce the incentives to overuse money. In contrast, intraday liquidity should have a low cost to provide risk-sharing.

**Keywords:** Friedman rule; monetary policy; random-relocation models.

**JEL classification:** E31; E51; E58.

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# 1 Introduction

A key puzzle in the economics of payments is the large difference between the cost of intraday liquidity, which is typically very close to zero, and the cost of overnight liquidity, which is typically an order of magnitude higher. In this paper, we propose a new explanation for this pattern. Intraday liquidity should have a low cost so that the return on money is that same as the return on other riskless short-term assets. This is the argument brought forth by Friedman (1969). Overnight liquidity should be costly because overuse of overnight liquidity leads to a reduction in output.

We study a random-relocation economy similar to Champ, Smith, and Williamson (1996). The setting is a two-period lived overlapping generations model where limited communication and random relocation create an endogenous transactions role for fiat money.<sup>1</sup> Agents are *ex ante* identical but before the end of their first period of life receive a ‘liquidity shock’ with some probability. Agents who receive the shock are relocated and the only asset they can carry with them is money. Agents who are not relocated when young may be relocated at the beginning of their second period of life. Agents can exert costly effort to reduce the probability of having to move in their first period of life.

An important feature of our model is that the use of overnight liquidity affects output. We think that this is a natural assumption in the light of the fact that most central banks use short-term interest rates, and often the overnight rate, to steer the economy. We assume a storage technology that transforms period  $t$  goods into period  $t + 1$  goods. The return on storage is affected by the fraction of young relocated agents. For example, relocation may prevent these agents from monitoring the technology. Further, the cost imposed by relocation is not completely internalized by moving agents.

In contrast, the fraction of old relocated agents does not affect the return on storage. The idea is that monitoring has already taken place. This is a standard assumption as

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<sup>1</sup>Economies with spatial separation and limited communication were first studied by Townsend (1980, 1987).

intraday liquidity is thought to be used mainly for the purpose of settlement. Accordingly, in our model intraday liquidity does not serve to transfer resources intertemporally but only affects the distribution of resources between different types of agents.

We show that a planner would like to distort the consumption of young movers but not of old movers. The planner wants to provide incentives for young agents to exert effort to reduce the probability with which they must move. It is possible to decentralize the planner's allocation by setting a high cost of liquidity for young movers and no cost of liquidity for old movers.

The main insight of our paper is that the cost of liquidity should be related to its role in payments or as a substitute for other assets that contribute to output. If liquidity is used only to make payments, then it should have a very low cost so as to share risk between ex-ante identical agents. If liquidity can be used as a substitute for other assets, then it should have a high cost to reduce the incentive to overuse it.

The payments literature has provided several explanations for the low cost of intraday liquidity. Angelini (1998), and Bech and Garratt (2003) argue that a high cost of intraday liquidity can lead to costly delay of payments. Zhou (2000) and Martin (2004) argue that a low cost of liquidity provides risk-sharing when payments need are random. In all of these models, the focus is on the cost of intraday liquidity rather than on the relationship between intraday and overnight liquidity.

In contrast to these earlier papers, we consider an environment in which costly overnight liquidity is necessary to obtain the planner's allocation. In such an environment, the fact that intraday liquidity should have a low cost naturally arises as an application of the Friedman rule. For this reason, our paper is also related to the literature concerned with the optimality of the Friedman rule.<sup>2</sup> In our model a deviating from the Friedman rule is desirable because money is overused if it is "too good an asset". Specifically, excessive use of money reduces output. This argument is similar to Camera (2001), who considers an

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<sup>2</sup>See Bhattacharya, Haslag, and Martin (2005) and the references therein.

economy in which the use of money promotes illegal activities.

Our paper is also related to recent attempts to study optimal monetary policy in environments with hidden effort. For example, da Costa and Werning (2003) use the dynamic taxation approach of Golosov, Kocherlakota and Tsyvinski (2003) in an environment where money enters in the utility function. Waller (2006) considers a similar approach in an overlapping generations model.

To highlight the role of the different elements of our model, we introduce them one at a time. Section 2 describes the basic features of the physical environment. In section 3 we assume that the effort exerted by agents is observable. In this case, the planner does not need to distort consumption to achieve the desired level of effort. The planner's allocation can be decentralized with a low cost for overnight liquidity. Section 4 considers the case where effort is unobservable. With hidden effort, the planner chooses to distort consumption to provide incentives for agents to exert effort. The planner's allocation can be decentralized if the cost of overnight liquidity is high. Finally, in section 5 we introduce intraday liquidity in the model and show that its cost should be zero.

## 2 The environment

We extend a standard random relocation to include an effort decision. There is an infinite sequence of time periods. Dates are indexed by  $t = \dots - 1, 0, 1, \dots$ .<sup>3</sup> The world is divided into two spatially separated locations. Each location is populated by a continuum of agents of unit mass. Agents live for two periods. There is a single, perishable consumption good.

The economy's key friction is a physical restriction keeping the consumption good from moving between the two locations: One unit of the consumption good on Island A cannot be transported to Island B, and vice-versa. In contrast, agents may have to move between

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<sup>3</sup>It is standard in this literature to have an initial period and an initial old generation but to ignore the welfare of that initial generation. To simplify the exposition we choose instead to have no initial period. Bhattacharya, Haslag, and Martin (2006) contrast economies with or without an initial date.

islands. Young agents face uncertainty regarding where they will spend their old age and  $\Pi$  denotes the probability that a young agent will be relocated to the other island. We assume that  $\Pi$  also denoted the fraction of young agents who are relocated. The two islands are symmetric so that the flow of relocated agents to and from an island offset each other, leaving the population unchanged. We refer to relocated agents as movers and the others as non-movers.

## 2.1 Endowments and Technology

At each date a young agent is endowed with  $\omega$  units of the single consumption good. In addition, each young agent is endowed with one unit of time. Old agents receive no endowment. The young agent can divide their time between leisure and effort, denoted by  $l_t$  and  $e_t$ , respectively, and  $l_t + e_t = 1$ .

Effort affects the probability that a person will be relocated. The function  $\Pi(\cdot)$  satisfies  $\Pi' < 0$ ,  $\Pi'' > 0$ ,  $\Pi(0) = \bar{\Pi} \leq 1$ , and  $\Pi(1) = \underline{\Pi} \geq 0$ . Also,  $\lim_{e \rightarrow 0} \Pi' = -\infty$  and  $\lim_{e \rightarrow 1} \Pi' = 0$ . Hence, as effort approaches zero, the marginal effect on the probability of being relocated is large. As effort approaches its maximum level, the effect on that probability vanishes.

There is a storage technology on each island transforming date  $t$  goods into date  $t + 1$  goods. The return on the storage technology depends on the monitoring activity of young agents. Monitoring is costless but only non-movers are able to monitor the technology adequately. Hence, the return on storage is negatively related to the fraction of movers. We assume this return is given by  $x[1 - \Pi(1 - \phi)] > 1$ . The return of each individual project thus depends on the fraction of movers on the island. The higher the mass of movers, the lower is the island's average return.

The expression for the return of the storage technology can be justified as follows: Each individual project has some probability to have a high return and some probability to have a return of zero. Before agents have to make their choice of effort, a financial market opens

on which agents can trade claims on stored goods. Agents have an incentive to diversify the risk attached to their project and choose to hold a diversified portfolio of projects. Ex-post, the projects of movers have a higher probability of having a return of zero so the return of the portfolio of each individual agent depends on the fraction of movers on the island.

## 2.2 Preferences

Agents derive utility from old-age consumption and from enjoying leisure when young. Utility is additively separable in consumption and leisure. Let  $c_t$  denote old-age consumption enjoyed by members of the generation born at date  $t$ ; utility derived from the consumption good is represented by the function  $U(c_t)$ . We assume that  $U(\cdot)$  is twice continuously differentiable and strictly concave; formally,  $U' > 0$  and  $U'' < 0$ . Let  $V(1 - e_t)$  characterize the function mapping leisure into utility, where we apply the constraint on time with  $l_t = 1 - e_t$ .  $V(\cdot)$  is increasing and strictly concave. In other words  $V' \geq 0$ ,  $V'' < 0$ ,  $\lim_{l \rightarrow 0} V' = \infty$ , and  $\lim_{l \rightarrow 1} V' = 0$ .

## 3 An economy with observable effort

In this section, we assume that effort is observable. We show that in this case the planner wants to equalize consumption between movers and non-movers. Then we show how the planner's allocation can be decentralized.

### 3.1 The Planner's problem

We derive the allocation chosen by a planner who can observe the effort exerted by agents. Since all generations are identical, the planner seeks to maximize the expected utility of a young agent in a representative generation.

Since goods cannot be moved between islands, the planner must distribute the goods

available on an island to the agents present. At the planner's disposal are the goods stored on behalf of the date  $t$  generation, denoted by  $s_t$ , and the goods available from the date  $t + 1$  generation's endowment. We focus on stationary allocation so the amount of goods of the date  $t + 1$  generation's endowment that is stored,  $s_{t+1}$ , must be equal to  $s_t$ .

The planner takes into account the fact that agents' efforts affect the probability with which they are relocated and, indirectly, the return on investment. The planner can punish agents who do not exert the desired amount of effort, since it is observable. We assume that the punishment can be severe enough to make agents choose the effort prescribed by the planner.

The planner's problem is given by

$$\max_{c_t^m, c_t^n, e_t, s_t} \Pi(e)U(c_t^m) + [1 - \Pi(e)]U(c_t^n) + V(1 - e_t), \quad (1)$$

subject to

$$\Pi(e_t)c_t^m + [1 - \Pi(e_t)]c_t^n = x[1 - \Pi(e_t)(1 - \phi)]s_t + \omega_{t+1} - s_{t+1}, \quad (2)$$

$$s_t \leq \omega_t, \quad (3)$$

where  $c_t^m$  denotes the quantity of the consumption good allocated to movers,  $c_t^n$  denotes the quantity of the consumption good allocated to non-movers. We let  $\lambda_t$  and  $\mu_t$  denote the lagrange multipliers associated with the first and second constraints, respectively.

The first-order conditions for the planner's problem are

$$\Pi(e_t)U'(c_t^m) - \lambda_t\Pi(e_t) = 0, \quad (4)$$

$$[1 - \Pi(e_t)]U'(c_t^n) - \lambda_t[1 - \Pi(e_t)] = 0, \quad (5)$$

$$\Pi'(e_t)[U(c_t^m) - U(c_t^n)] - V'(1 - e_t) - \lambda_t\{\Pi'(e_t)[x(1 - \phi)s_t - c_t^m + c_t^n]\} = 0, \quad (6)$$

$$\lambda_t[x(1 - \Pi(e_t)(1 - \phi))] - \mu_t - \lambda_{t+1} = 0. \quad (7)$$

All constraints hold with equality because the planner chooses positive levels of consumption, effort, and storage. In a stationary allocation, since  $s_{t+1} = s_t$  and  $x[1 - \Pi(e_t)(1 - \phi)] >$

1, the planner wants to store as many goods as possible. It follows that  $s_{t+1} = s_t = \omega_t$ . In effect, the planner stores all the endowment goods of the young, and distributes the stored goods from the previous generation between old movers and old non-movers on the island.

Equations (4) and (5) imply that  $U'(c_t^m) = \lambda_t = U'(c_t^n)$ . This implies  $c_t^m = c_t^n = c_t$ . In other words, the planner provides the same amount of consumption to movers and non-movers and the risk of relocation is shared perfectly.

With  $c_t^m = c_t^n = c_t$  and  $s_t = \omega$ , equation (6) simplifies to

$$V'(1 - e_t) = -U'(x [1 - \Pi(e_t)(1 - \phi)] \omega) \Pi'(e_t)[x(1 - \phi)\omega]. \quad (8)$$

The LHS of equation (8) is positive and increases from 0 to infinity as effort goes from 0 to 1. Since  $\Pi' < 0$ , the RHS of equation (8) is positive. Hence there will be an interior solution for the effort level. A sufficient condition for the effort level to be unique is that the RHS of equation (8) be decreasing.

We let  $e^*$  denote the effort level chosen by the planner. The planner trades off between the marginal cost of the effort to the individual agent versus the marginal benefit in terms of additional resources available to the society.

### 3.2 A decentralized economy

In this section, we show how the planner's allocation can be decentralized. We assume the existence of a public sector which we call the CB. The CB can issue fiat money at no cost and can choose prices at which it buys and sells goods. We associate the liquidity provided by the CB in this case with overnight liquidity. The CB redistributes any profits in a lump sum fashion. Since effort is observable, we also assume that the CB can punish agents who do not choose the level of effort consistent with the planner's allocation. The CB is subject to the physical restriction that keeps goods from moving across islands, while cash is transportable across the two islands.

The timing goes as follows: First, young agents receive their endowment and invest it in the storage technology. They must also decide how much effort to exert. After the

effort decision is made, agents learn whether they must relocate. The CB opens and agents can exchange fiat money for the claims on stored goods they hold. Then movers relocate. In their second period of life, nonmovers consume the goods to which they have a claim. Movers buy goods from the CB with the money they hold.

Note that the CB does not need to keep track of agents from one island to another. While the CB operates on both islands, the branch on one island does not need to know whom the branch on the other island has traded with. Also, in contrast to the standard random relocation model, there is no need for banks in this economy. It is enough for the central bank to buy and sell goods, as we show below.<sup>4</sup>

Let  $\bar{m}_t$  denote the amount of money given to agents in exchange for  $q_t$  goods. Let  $v_t$  denote the units of goods per unit of money (the inverse of the price level). It follows that  $q_t = v_t \bar{m}_t$ .  $\bar{m}_t$  and  $v_t$  are choice variables of the CB. We assume that the CB buys and sells goods at the same price. Formally, at each date the CB receives goods from young movers worth  $v_t \bar{m}_t$ . The CB also gives good to old movers worth  $v_t \bar{m}_{t-1}$ . The difference, denoted by  $T_t$  is distributed in lump-sum fashion to young agents:

$$v_t \bar{m}_t - v_t \bar{m}_{t-1} = T_t. \tag{9}$$

Each unit of good held by the CB at date  $t - 1$  turns into  $x [1 - \Pi(1 - \phi)]$  units of goods at date  $t$ .

Young non-movers cannot consume any more by obtaining cash from the CB. For this reason, we assume that non-movers do not exchange goods for money at the CB even if they are indifferent between doing so or not. Young movers cannot do worse than getting money from the CB since they would consume nothing in the absence of the cash.<sup>5</sup>

All agents choose effort  $e^*$  since the CB can punish agents who do not choose the level

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<sup>4</sup>See Schreft and Smith (1997) for an exposition of a standard random relocation model. See Haslag and Martin (forthcoming) for more details on the role of the central bank in providing liquidity.

<sup>5</sup>One interpretation for this arrangement is that the CB makes discount window loans to agents, as in Antinolfi and Keister (2006) or Haslag and Martin (forthcoming). However, our arrangement requires less information as the CB does not need to keep track of moving agents, as noted above.

or effort consistent with the planner's allocation. Let  $\Pi^* \equiv \Pi(e^*)$ . The representative young person born at date  $t$  solves the following problem:

$$\max_{c_t^m, c_t^n, s_t} \Pi^* U(c_t^m) + [1 - \Pi^*] U(c_t^n) + V(1 - e_t^*), \quad (10)$$

subject to

$$s_t \leq \omega + T_t, \quad (11)$$

$$c_t^n \leq x [1 - \Pi^*(1 - \phi)] s_t, \quad (12)$$

$$v_t \bar{m}_t \leq s_t, \quad (13)$$

$$c_t^m \leq v_{t+1} \bar{m}_t. \quad (14)$$

Equation (11) states that agents cannot store more than their endowment and the transfer from the CB. Equation (12) indicates that the consumption of non-movers cannot exceed the return on their storage. Equation (13) tells us that the real money balances received from the CB cannot exceed the goods that are offered in exchange. Finally, equation (14) states that the consumption of movers is bounded by the amount of goods obtained in exchange for money from the CB.

If the money supply is constant,  $T$  must be equal to zero. Consumption of movers and non-movers will thus be equal if and only if

$$\frac{v_{t+1}}{v_t} = x [1 - \Pi(1 - \phi)]. \quad (15)$$

This means that the return on money between periods is equal to the return on storage. In other words, this is the Friedman rule. By implementing the Friedman rule, the CB is able to obtain the efficient allocation.

In this economy,  $v_t$  is a policy variable and not a market determined price level. This is because money does not circulate between agents. We thus have a model of "intermediated money" as money only circulates between the CB and the agents. In particular, the steady-state return on money is not equal to one. Martin and Haslag (forthcoming) show in a

related framework, that this economy is the limiting case of an economy in which the money circulate between agents and in which the CB implements the Friedman rule.

## 4 An economy with hidden effort

In the remainder of this paper, we assume that an agent's choice of effort is unobservable. First, we study an agent's effort choice. Next, we characterize the planner's allocation. Finally, we show how the planner's allocation can be decentralized.

### 4.1 Effort choice

First, we consider the choice of effort of an agent who takes  $c^m$  and  $c^n$  as given. We let  $\pi$  denote the probability that a given agent will be relocated, to distinguish it from  $\Pi$ , the relocation probability of the population of young agents. The individual's problem can be written as

$$\max_e \pi(e)U(c^m) + [1 - \pi(e)]U(c^n) + V(1 - e). \quad (16)$$

Taking the partial derivative of the objective function with respect to  $e$  and setting it to zero yields

$$\pi'(e) [U(c^m) - U(c^n)] - V'(1 - e) = 0. \quad (17)$$

Since  $V'(1) = 0$ , it is apparent from equation (17) that when  $c^n = c^m$ , the optimal choice of effort is  $e = 0$ . That is, an agent who is completely insured against the risk of relocation exerts no effort to reduce this risk. Effort will be positive only if  $c^m < c^n$ . Intuitively, if movers receive less consumption than non-movers, consumers will be try to avoid being relocated.

Since the effort level chosen by an agent depends on  $c^n$  and  $c^m$ , we can write  $e = e(c^m, c^n)$  by invoking the implicit function theorem.

**Lemma 1** *Consumer's optimum effort is (i) decreasing in the quantity of consumption good allocated to movers; and (ii) increasing in the quantity of the consumption good allocated to non-movers.*

The proof is provided in the appendix.

## 4.2 The planner's problem

The planner chooses how much to save and how to allocate available goods between movers and nonmovers to maximize the expected utility of a representative generation. Physically, the planner collects endowments from the young on each island. These goods can be consumed by consumers alive in that period or invested in the technology. The planner also collects goods that were invested last period. The planner knows the effect that movers have on the return to the technology. Since the planner can identify movers and non-movers, each type may receive different quantities of goods.

Recall that the planner's problem is

$$\max_{c_t^m, c_t^n, e_t, s_t} \Pi(e_t)U(c_t^m) + [1 - \Pi(e_t)]U(c_t^n) + V(1 - e_t),$$

subject to

$$\begin{aligned} \Pi(e_t)c_t^m + [1 - \Pi(e_t)]c_t^n &= x[1 - \Pi(e_t)(1 - \phi)]s_t + \omega_{t+1} - s_{t+1}, \\ s_t &\leq \omega_t. \end{aligned}$$

In a steady state,  $s_t = s_{t+1} = s \leq \omega$ . The constraints can thus be combined into

$$\Pi(e)c^m + [1 - \Pi(e)]c^n = x[1 - \Pi(e)(1 - \phi)]\omega. \quad (18)$$

We can substitute the effort function from the previous section,  $e_t = e(c_t^m, c_t^n)$ , and take the partial derivatives with respect to  $c^m$  and  $c^n$  to get two first-order conditions:

$$e_m \Pi' [U(c^m) - U(c^n) - \lambda(c^m - c^n + x(1 - \phi)\omega)] - e_m V' + \Pi(e) [U'(c^m) - \lambda] = 0$$

and

$$e_n \Pi' [U(c^m) - U(c^n) - \lambda(c^m - c^n + x(1 - \phi)\omega)] - e_n V' + (1 - \Pi(e)) [U'(c^n) - \lambda] = 0$$

where  $\lambda$  is the Lagrange multiplier on constraint (18) and  $e_m \equiv \frac{\partial e}{\partial c^m}$  and  $e_n \equiv \frac{\partial e}{\partial c^n}$ .

Solve both equations for  $\lambda$  to get

$$\frac{e_m \Pi' [U(c^m) - U(c^n)] - e_m V' + \Pi(e) U'(c^m)}{e_m \Pi' [c^m - c^n + x(1 - \phi)\omega] + \Pi(e)} = \lambda,$$

and

$$\frac{e_n \Pi' [U(c^m) - U(c^n)] - e_n V' + (1 - \Pi(e)) U'(c^n)}{e_n \Pi' [c^m - c^n + x(1 - \phi)\omega] + (1 - \Pi(e))} = \lambda.$$

We can eliminate  $\lambda$  and obtain one equation in  $c^n$  and  $c^m$ . The planner's resource constraint gives us a second equation in  $c^n$  and  $c^m$  so we can solve for these two unknowns. Let  $c^{m**}$  and  $c^{n**}$  denote the level of consumption chosen by the planner. Next we can solve for the effort level:  $e^{**} = e(c^{m**}, c^{n**})$ .

**Proposition 1** *The planner gives less consumption to movers than non-movers:  $c^{m**} < c^{n**}$ .*

**Proof.** Suppose, instead, that the planner chooses  $c^{m**} = c^{n**} = c$ . We can rearrange the expression  $\frac{\Delta_1}{\Delta_2} = \frac{\Gamma_1}{\Gamma_2}$ , to get

$$[x(1 - \phi)\omega] U'(c) \Pi' = V'. \quad (19)$$

The left hand side of this expression is strictly positive. If  $c^{m**} = c^{n**} = c$ , consumers exert no effort and  $V'(1) = 0$ , so equation (19) cannot hold. We know from lemma 1 that some effort will be exerted if  $c^{n**}$  is increased,  $c^{m**}$  is decreased, or both. ■

The intuition is that the planner wants agents to exert some effort to reduce their chance of relocation. Note that the level of effort desired by the planner is different when effort is hidden than when effort is observable,  $e^{**} < e^*$ . When effort is hidden, the planner chooses lower effort because the benefit from increasing effort must be traded-off with the cost of distorting consumption. In other words, if the planner were to choose  $e^{**} = e^*$ , the

cost of a marginal decrease in effort would be of second order while the benefit of improving risk sharing between movers and nonmovers would be of first order.

It is clear from our specification of the planner's problem that both hidden effort and the cost imposed by movers are important features of the model economy. Together, these features can account for why the planner does not want to offer perfect insurance to consumers in this economy. If movers did not impose a cost on the economy, the planner would not be concerned with reducing the mass of movers. It would then be optimal to provide equal consumption to all agents since they are risk-averse and ex-ante identical. Instead, if movers imposed a cost but the probability of moving was exogenous, then the planner would again choose to provide equal consumption to all agents since it would not be possible to mitigate the cost imposed by movers.

### 4.3 The decentralized economy with hidden effort

With hidden effort, the CB can no longer punish agents directly for not choosing the appropriate level of effort. The representative young agent born at date  $t$  solves the following problem:

$$\max_{c_t^m, c_t^n, s_t} \Pi U(c_t^m) + [1 - \Pi]U(c_t^n) + V[1 - e(c^m, c^n)], \quad (20)$$

subject to constraints (11), (12), (13), and (14). Combining the last three equations we can write

$$\frac{c^m}{c^n} = \frac{v_{t+1}}{v_t} \frac{1}{x[1 - \Pi(1 - \phi)]}. \quad (21)$$

The CB can obtain the ratio  $c^m/c^n$  it desires by choosing  $v_{t+1}$  and  $v_t$  appropriately. As  $v_t/v_{t+1}$ , the rate at which the price level changes, increases, the ratio  $c^m/c^n$  decreases because the rate of return of money decreases. This stimulates agents to provide more effort. The CB chooses  $v_t/v_{t+1}$  in order to set

$$\frac{c^m}{c^n} = \frac{c^{m**}}{c^{n**}}.$$

Deviating from the Friedman rule works particularly well in this environment because only movers use money. Hence, while the CB is unable to distinguish movers from non-movers, it knows that changes in the ratio  $v_t/v_{t+1}$  will modify the relative consumption of the two types of agents. In section 5.3 we show that our results extend to an environment where it is difficult to distinguish different types of agents.

## 5 Late movers and intraday liquidity

It is well documented that the liquidity provided by CB overnight is considerably more costly than the liquidity provided intraday.<sup>6</sup> Our model can help shed some light on this pattern.

In order to have both intraday and overnight liquidity provision, we need to modify the pattern of relocations slightly. As above, there is a probability  $\Pi$  that a young agent must relocate and  $\Pi$  is a function of the agent's effort. We call such agents early movers. Agents who are not relocated when young may be relocated at the beginning of their second period of life with probability  $\alpha$ . We call such agents late movers. As above,  $\alpha$  is also the fraction of relocated agents. The effort that affects the probability  $\Pi$  has no effect on  $\alpha$  and for simplicity we make  $\alpha$  exogenous.

We assume that late-movers must leave their island before stored goods pay off. Thus, they may need money for the same reason as early movers. In contrast to early-movers, however, late-movers do not affect the return on the storage technology as the benefits from their monitoring activity are already realized. This distinction between intraday and overnight liquidity is key. It can also be related to the fact that overnight liquidity is used to transfer resources intertemporally while intraday liquidity is not.

The expected utility of a young agent in this economy can be written as

$$\Pi(e)U(c_t^{em}) + [1 - \Pi(e)] \left[ \alpha U(c_t^{lm}) + (1 - \alpha) U(c_t^n) \right] + V(1 - e_t), \quad (22)$$

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<sup>6</sup>See Bech and Garratt (2003), Zhou (2000).

where  $c_t^{em}$  is the quantity of the good consumed by the early mover,  $c_t^{lm}$  is the quantity of the good consumed by a late mover, and  $c_t^n$  is the quantity consumed by a non-mover.

## 5.1 The case with observable effort

When effort is observable, it is straightforward to see that the planner will choose  $c_t^{em} = c_t^{lm} = c_t^n$ . The amount of storage and effort are the same than in section 3.1. With observable effort, the planner does not need to distort the agents' consumption and chooses to insure them perfectly against the risk of relocation.

As in section 4.3, we assume that the CB can impose a punishment high enough so that agents choose effort  $e^*$ . The representative young agent solves

$$\max_{c_t^{em}, c_t^{lm}, c_t^n, s_t} \Pi^* U(c_t^{em}) + [1 - \Pi^*] \left[ \alpha U(c_t^{lm}) + (1 - \alpha) U(c_t^n) \right] + V(1 - e^*), \quad (23)$$

subject to

$$s_t \leq \omega + T_t, \quad (24)$$

$$c_t^n \leq x [1 - \Pi^*(1 - \phi)] s_t, \quad (25)$$

$$v_t \bar{m}_t^{em} \leq s_t, \quad (26)$$

$$c_t^{em} \leq v_{t+1} \bar{m}_t^{em}, \quad (27)$$

$$v_{t+1} \bar{m}_t^{lm} \leq x [1 - \Pi^*(1 - \phi)] s_t, \quad (28)$$

$$c_t^{lm} \leq v_{t+1} \bar{m}_t^{lm}. \quad (29)$$

The last two constraints show that  $c_t^{lm} = c_t^n$ . Then by the same argument as in section 3.2, the CB can set  $c_t^{lm} = c_t^{em} = c_t^n$  by choosing

$$\frac{v_{t+1}}{v_t} = x [1 - \Pi^*(1 - \phi)]. \quad (30)$$

The Friedman rule can achieve the planner's allocation in this environment. Under the Friedman rule, the real cost of liquidity to agents is zero, whether money is needed intraday or overnight.

## 5.2 The case with hidden effort

From the perspective of the planner, late-movers and non-movers are equivalent. Neither type has a negative effect on the return to the storage technology and, conditional on not being an early-mover, the probability of being a non-mover is exogenous. Hence, inspection of the expected utility function given by expression (22) reveals that the planner will choose to set  $c_t^{lm} = c_t^n$ . With  $c_t^{lm} = cn_t$  the planner's problem is the same as it was in section 4. In particular, lemma 1 and proposition 1 both hold.

We can now turn to the problem faced by a CB in a decentralized economy. An representative agent chooses  $c_t^n, c_t^{em}, c_t^{lm}, e_t$ , and  $s_t$  to maximize

$$\Pi U(c_t^{em}) + [1 - \Pi] \left[ \alpha U(c_t^{lm}) + (1 - \alpha) U(c_t^n) \right] + V(1 - e), \quad (31)$$

subject to constraints (24), (25), (26), (27), (28), and (29).

Combining equations (28), and (29), at equality, we get  $c^{lm} = x [1 - \Pi(1 - \phi)] s_t = c^n$ . With  $c^{lm} = c^n$ , this problem is equivalent to the problem of section 4.3. The CB can achieve the planner's allocation by deviating from the Friedman rule. In contrast to the previous section, the real cost of liquidity is not the same intraday and overnight. Intraday, the cost of liquidity is zero since late-movers and non-movers enjoy the same consumption. The real cost of overnight liquidity is strictly positive since  $c_t^{em} < c_t^n$ .

Note that the CB could make  $c^{lm} \neq c^n$ . For example, if the price at which goods are sold is different from the price at which goods are bought in the same period. But the CB prefers to set  $c^{lm} = c^n$ . By buying and selling goods at the same price, the CB's interactions with late-movers resemble a collateralized loan at an interest rate of zero.

This model suggests a reason why central banks provide intraday and overnight liquidity at very different costs. Intraday, the need for liquidity is only related to making payments and no effect on aggregate real activity. Its only role is to allocate available resources. In contrast, the cost of overnight liquidity has implications for aggregate real activity in our model. From the perspective of young agents, money and storage are two ways of

transferring resources between periods. When money is as good an asset as storage, agents have no incentives to avoid holding money. In our environment, this reduces the return on storage because there are too many movers.

The CB can make money a less attractive asset by deviating from the Friedman rule. As money becomes a worse alternative to storage, agents have more incentives to avoid having to hold money. This is beneficial as it increases the return on storage.

### 5.3 Indistinguishable movers

Our model is special in that the planner, and the CB, can perfectly discriminate agents needing overnight liquidity and those needing intraday liquidity. Hence, the only agents affected by a deviation from the Friedman rule are the agents the CB is trying to target. We can modify our model slightly to address this issue. Assume that early-movers and late-movers are indistinguishable. This may happen because the early-movers of generation  $t$  move at the same time as the late-movers of generation  $t - 1$  and the planner cannot tell them apart.

We retain the assumption that the fraction of early-movers affects output while the fraction of late-movers does not. Hence, while the planner would like to choose different levels of consumption for each type of movers, it cannot do so. The planner's problem in this case is given by

$$\max_{c^m, c^n, e} \alpha [U(c^m) + V(1 - e)] + (1 - \alpha) \{ \Pi(e)U(c^m) + [1 - \Pi(e)]U(c^n) + V(1 - e) \},$$

subject to

$$\alpha c^m + (1 - \alpha) \{ \Pi(e)c^m + [1 - \Pi(e)]c^n \} = x [1 - \Pi(e)(1 - \phi)] \omega. \quad (32)$$

Here we have already taken into account the fact that the planner chooses  $s = \omega$ . The first order conditions with respect to  $c^n$  and  $c^m$  are

$$(1 - \alpha) \{ e_m \Pi' [U(c^m) - U(c^n) - \lambda(c^m - c^n + x(1 - \phi)\omega)] - e_m V' + \Pi(e) [U'(c^m) - \lambda] \} - \alpha \{ e_m V' + U'(c^m) - \lambda + \lambda e_m x(1 - \phi)\omega \} = 0 \quad (33)$$

and

$$(1 - \alpha) \{ e_n \Pi' [U(c^m) - U(c^n) - \lambda(c^m - c^n + x(1 - \phi)\omega)] - e_n V' + (1 - \Pi(e)) [U'(c^n) - \lambda] \} - \alpha e_n \{ V' + \lambda \Pi' x (1 - \phi) \omega \} = 0 \quad (34)$$

Note that the first term in each of the two expressions is the first order condition of the planner from section 4.2. At the allocation  $\{c^{n**}, c^{m**}, e^{**}\}$ , the RHS of the above conditions become

$$-\alpha \{ e_m V' + U'(c^m) - \lambda + \lambda e_m x (1 - \phi) \omega \} \quad (35)$$

and

$$-\alpha e_n \{ V' + \lambda \Pi' x (1 - \phi) \omega \}. \quad (36)$$

The sign of these expression is ambiguous. This opens the possibility that when early and late movers are indistinguishable, the planner could choose either more or less consumption inequality as in the case where movers are distinguishable, depending on the sign of these expressions. Intuitively, on the one hand the planner may not want to impose as much consumption inequality because it hurts late movers. On the other hand, because a given amount of consumption inequality provides fewer incentives, the planner may want to impose more consumption inequality. This resembles the standard trade-off between a ‘substitution’ and an ‘income’ effect.

This allocation can be decentralized along the same line as was done previously, by setting the cost of overnight liquidity appropriately. It would, of course, be possible to reintroduce intraday liquidity by assuming that some late movers are distinguishable. The main insight of this section is that the results of our model do not depend on the planner and the central bank being able to perfectly discriminate between early and late movers.

## 6 Conclusion

This paper proposed a new explanation for the puzzling difference between the costs of overnight and intraday liquidity. We argue that the low cost of intraday liquidity is simply

an application of the Friedman rule in an environment where a deviation of the Friedman rule is optimal with respect to overnight liquidity.

Our model has a number of desirable features: The cost of overnight liquidity can affect output so there is a role for a high cost of overnight liquidity. Agents can choose the composition of their asset portfolio between money and storage and, if money is ‘too good an asset,’ money is overused. Intraday liquidity is not used to transfer goods intertemporally, but only to make payments.

With these features, we show that a central bank can implement a planner’s allocation by setting a high cost to overnight liquidity and a low cost to intraday liquidity. We also showed that this result persists when it is difficult to distinguish agents needing intraday liquidity from those needing overnight liquidity.

## 7 Appendix

### Proof of lemma 1

After substituting the  $e(c^m, c^n)$  into the first-order condition and differentiating, we get

$$\pi''[U(c^m) - U(c^n)]de + V''de + \pi'U'dc^m = 0.$$

After collecting terms and rearranging, we can write

$$\frac{de}{dc^m} = -\frac{\pi'U'}{\pi''[U(c^m) - U(c^n)] + V''}.$$

As usual, the denominator is the second-order condition. If  $\pi'' > 0$  and  $V'' < 0$ , the sign of the denominator is negative, ensuring that the second-order condition is satisfied. In other words, the value of  $e$  that satisfies the first-order condition is indeed a maximum.<sup>7</sup> With  $\pi' < 0$  and positive marginal utility, the implication is that  $\frac{\partial e}{\partial c^m} < 0$ . An increase in the quantity of the consumption given to movers will result in less effort by the consumer. As movers get more of the consumption, holding everything else constant, there is less need to use effort to try to indirectly insure themselves against moving.

It is straightforward to show that  $\frac{\partial e}{\partial c^n} > 0$  along the same lines as above.

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<sup>7</sup>So,  $\pi'' > 0$  corresponds to a case in which the impact of effort on the probability of moving is getting algebraically bigger; that is, a smaller negative number.

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