On the Licensing of Innovations under Strategic Delegation

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Abstract

This paper uses a three-stage licensing-delegation-quantity game to study the licensing of a cost-reducing innovation by a patent-holding firm to its competitor. It is shown that licensing is less likely to occur under strategic delegation compared to no delegation.

JEL Classification: D45; L10; L20

Key Words: licensing; strategic delegation

1. Introduction

A patented innovation provides the innovator opportunities to reap a reward on his or her investment in research and development. For an outside independent research lab, this reward may be realized through licensing its innovation to the producing firms. For an inside firm, it may keep its innovation for its own use and gains an advantage in competing with its competitors. It may also license the innovation to its competitors. Several important reasons have been advanced in the literature as to why a firm may want to license an innovation to its competitors, covering both the profit motive and the strategic incentive. For example, Gallini (1984) points out the incentive for an incumbent to license to a potential entrant so as to reduce the likelihood of the latter developing a better technology; Katz and Shapiro (1985) regard the incentive to license as an integral part of a firm's R&D decision in evaluating the profitability of a R&D project; Eswaran (1994) explores the possibility that licensees can serve as a barrier to entry; Lin (1996) shows that licensing in the form of a fixed fee may serve as a facilitating device for collusion among competitors.

Despite the presence of these and other possible incentives for a firm to license an innovation to its competitors, it remains well recognized by economists that licensing of innovations between competing firms does not happen very often. The most pronounced explanation for this is the existence of asymmetric information. Potential licensees lack detailed information about an innovation to assess its value prior to licensing so that they are not willing to pay the desired amount of compensation demanded by the patent-holder. Other known reasons include the innovating firm's unwillingness to share pertinent information with competitors that might have bearing on other related and ongoing R&D, and the fact that it may be costly or impossible for the licensor to monitor how the potential licensee uses the licensed innovation, including the monitoring of its output produced using the licensed innovation and the possibility that it might re-license to other firms.[1] The goal of this paper is to point to another potentially important reason for the lack of licensing of innovations between competing firms. It has to do with the widely recognized fact of separation of ownership and control in the modern corporation and the delegation of some decision making from owners to managers.

We consider licensing as part of a delegation-licensing-quantity game. Our game involves three stages and two competing duopoly firms. The first stage is the delegation stage. In this stage, owners of the two firms decide simultaneously incentive contracts for their managers. The second stage of the game is the licensing stage in which the patent-holding firm chooses a licensing contract for its innovation and the other firm decides whether to accept the contract offer. The third stage is the quantity competition stage in which the firms' managers engage in an output competition. The main result of this paper is that licensing is less likely to occur under strategic delegation than under no delegation.

The delegation-licensing-quantity game studied here combines two strands of literature. One is the strategic delegation literature. The most influential early work in this literature includes Fershtman and Judd (1987) and Sklivas (1987). The other is the licensing literature. A seminal paper on technology licensing is Arrow (1962). Kamien (1992) contains an excellent review of this literature. A paper closely related to the present paper is Saracho (2002), who studies licensing by an independent research lab to an oligopoly under strategic delegation. Her model has the same three stages as in our model. Her focus, however, is on the comparison of fixed-fee licensing and royalty licensing and her main result extends the finding in Wang (1998) and Kamien and Tauman (2002) that royalty can be superior to fixed fee for the patentee.

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[1] See Shapiro (1985) for an account of these reasons.

2. Model Setup

The impact of strategic delegation on licensing is most transparent in the context of a homogeneous good Cournot duopoly with a linear demand and constant unit cost of production. Assume the (inverse) market demand function is given by $p = a - Q$, where p denotes price and Q represents industry output. With the old technology, both firms produce at constant unit production cost c ($0 < c < a$). The cost-reducing innovation by firm 1 creates a new technology that lowers its unit cost and any licensee's unit cost by the amount of ε. For simplicity, our focus is on non-drastic innovations (i.e., $\varepsilon < a - c$).[2]

Our game takes place in three stages: delegation, licensing, and quantity competition, respectively. In the first stage, the firms' owners decide simultaneously their incentive contract for their managers. In the second stage, firm 1 (the patent-holder) chooses a licensing contract and firm 2 decides whether to accept firm 1's offer. In the third stage, the firms' managers simultaneously choose their output levels.

The output choice stage is essentially the standard Cournot game except that the managers are not profit maximizers but rather that they maximize a weighted sum of profit and revenue. The delegation stage determines the firms' incentive parameters for their managers, these parameter values determine the weights assigned to profit and revenue in the manager's optimization problem. The values of these parameters will depend on the firms' unit costs of production. That is, the incentive parameters are

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[2] An innovation is drastic if the post-innovation monopoly price is equal to or below the pre-innovation competitive price. That is, $\varepsilon \geq a - c$. See, for example, Kamien (1992) for definition. As it is well established that a drastic innovation will not be licensed to competitors by the patent-holding firm in the case of no delegation (e.g., Katz and Shapiro, 1985), in our model with strategic delegation such an innovation will certainly not be licensed.

functions of the firms' cost levels. The actual levels of the firms' unit costs are determined by the licensing stage. To solve the delegation game, one need only find the solution to the last (output) stage of the game.[3] The payoffs to all participants (both owners and managers) in the three stage game are realized in the last stage of the game.

3. The Delegation-Licensing-Quantity Game

We start by solving the output stage of the three stage game. For convenience, we then move on to solve the delegation problem and finally the licensing problem.

Quantity Competition

To study the managers' output choices in the last stage of the game, we assume that firm 1 has an unspecified constant unit production cost of c_1 and firm 2 has an unspecified constant unit production cost of c_2. These marginal cost levels will be determined by the first two stages of the game. Throughout the paper subscripts 1 and 2 denote for firms 1 and 2, respectively.

Firm 1's profit function is represented by $\Pi_1 = (a - q_1 - q_2 - c_1)q_1$ and its revenue function is $R_1 = (a - q_1 - q_2)q_1$. The manager for firm 1 chooses q_1 to maximize a weighted sum of its profit and revenue, namely,

$$O_1 = \alpha_1 \Pi_1 + (1 - \alpha_1) R_1, \qquad (1)$$

where α_1 is the incentive (weight) parameter chosen by firm 1's owner in the first two stages of the game. Similarly, the manager for firm 2 chooses q_2 to maximize

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[3] It is implied that one need not solve the second (licensing) stage game first in order to solve the first (delegation) stage game. Hence, the order of the first two stages is actually irrelevant to the final solution of the game.

$$O_2 = \alpha_2 \Pi_2 + (1-\alpha_2) R_2, \tag{2}$$

where α_2 is the incentive parameter chosen by firm 2's owner in the first two stages of the game, Π_2 = $(a - q_1 - q_2 - c_2)q_2$ and R_2 = $(a - q_1 - q_2)q_2$ represent respectively firm 2's profit and revenue. As in Fershtman and Judd (1987), we allow α_1 and α_2 to take any value.

Maximizing O_1 in (1) with respect to q_1 gives firm 1's quantity reaction function:

$$q_1 = \begin{cases} (a - \alpha_1 c_1 - q_2)/2 & \text{if } q_2 \le a - \alpha_1 c_1 \\ 0 & \text{if } q_2 > a - \alpha_1 c_1 \end{cases} \tag{3}$$

Similarly, maximizing O_2 in (2) with respect to q_2 yields firm 2's quantity reaction function:

$$q_2 = \begin{cases} (a - \alpha_2 c_2 - q_1)/2 & \text{if } q_1 \le a - \alpha_2 c_2 \\ 0 & \text{if } q_1 > a - \alpha_2 c_2 \end{cases} \tag{4}$$

These reaction functions have the usual interpretation of first-order conditions. That is, a firm's optimal output response is one in which its marginal benefit is equal to marginal cost; and when marginal cost is always higher than marginal benefit the firm's optimal choice of output is zero.

The intersection of the reaction functions (3) and (4) gives the firms' equilibrium quantities in the third stage of the game as a function of choices made in the first two stages of the game. As in the standard Cournot model with unequal marginal costs, the intersection point of the reaction functions may be either an interior point with both quantities positive or a boundary point with one firm producing zero. In our model, firm 1 as the innovating firm will always be at least as efficient as firm 2 (i.e., $c_1 \le c_2$) and the only possible corner solution is where firm 2 produces zero. The third stage output choices as functions of choices in the first two stages of the game are summarized in Lemma 1. (Proofs of all lemmas and propositions are provided in the appendix.)

Lemma 1: Assuming that $\alpha_1 c_1 \le \alpha_2 c_2$, the equilibrium output levels in the quantity game in which firm i's manager maximizes O_i (i = 1,2) are

$$q_1 = \begin{cases} q_1^i \equiv (a - 2\alpha_1 c_1 + \alpha_2 c_2)/3 & \text{if } a + \alpha_1 c_1 - 2\alpha_2 c_2 > 0 \\ q_1^c \equiv (a - \alpha_1 c_1)/2 & \text{if } a + \alpha_1 c_1 - 2\alpha_2 c_2 \le 0 \end{cases} \quad (5)$$

$$q_2 = \begin{cases} q_2^i \equiv (a - 2\alpha_2 c_2 + \alpha_1 c_1)/3 & \text{if } a + \alpha_1 c_1 - 2\alpha_2 c_2 > 0 \\ q_2^c \equiv 0 & \text{if } a + \alpha_1 c_1 - 2\alpha_2 c_2 \le 0 \end{cases} \quad (6)$$

By Lemma 1, if $a + \alpha_1 c_1 - 2\alpha_2 c_2 > 0$ then the third stage quantity game gives an interior solution, given by (q_1^i, q_2^i); and if $a + \alpha_1 c_1 - 2\alpha_2 c_2 \le 0$ then the third stage game gives a corner solution, given by (q_1^c, q_2^c). The assumption $\alpha_1 c_1 \le \alpha_2 c_2$ will be verified later on to be satisfied in equilibrium.

<u>The Delegation Problem</u>

In the delegation stage of the game the two firms' owners simultaneously choose their incentive parameters α_1 and α_2, knowing that the solution for the output stage of the game will be given according to Lemma 1. Let $\pi_1(\alpha_1, \alpha_2)$ and $\pi_2(\alpha_1, \alpha_2)$ denote respectively firm 1's and firm 2's reduced-form profit functions derived based on the solution to the last stage of the game. Maximizing π_1 with respect to α_1 and π_2 with respect to α_2 yield the two firms' incentive parameter reaction functions, as given in the next lemma.

Lemma 2: Firm 1's incentive parameter reaction function in the delegation stage of the game is given by

$$\alpha_1 = \begin{cases} (6c_1 - \alpha_2 c_2 - a)/(4c_1) & \text{if } \alpha_2 \le (a + 2c_1)/(3c_2) \\ (2\alpha_2 c_2 - a)/c_1 & \text{if } (a + 2c_1)/(3c_2) \le \alpha_2 \le (a + c_1)/(2c_2) \\ 1 & \text{if } \alpha_2 \ge (a + c_1)/(2c_2) \end{cases} \quad (7)$$

Firm 2's incentive parameter reaction function (more precisely correspondence) in the delegation stage of the game is given by

$$\alpha_2 = \begin{cases} (6c_2 - \alpha_1 c_1 - a)/(4c_2) & \text{if } \alpha_1 > (2c_2 - a)/c_1 \\ [(a + \alpha_1 c_1)/(2c_2), \infty) & \text{if } \alpha_1 \leq (2c_2 - a)/c_1 \end{cases} \tag{8}$$

Firm 1's incentive parameter reaction curve has three segments. The first line of (7) corresponds to a decreasing segment; the middle line of (7) corresponds to a rising segment (on which $a + \alpha_1 c_1 - 2\alpha_2 c_2 = 0$); and the last line of (7) corresponds to a vertical segment. Regarding firm 2's incentive parameter reaction map, it is important to observe that firm 2's best response on or above the line $a + \alpha_1 c_1 - 2\alpha_2 c_2 = 0$ is not unique. This is because when $a + \alpha_1 c_1 - 2\alpha_2 c_2 \leq 0$ firm 2's profit is zero due to a zero output level. Note that above the line $a + \alpha_1 c_1 - 2\alpha_2 c_2 = 0$, $a + \alpha_1 c_1 - 2\alpha_2 c_2 < 0$. Below the line $a + \alpha_1 c_1 - 2\alpha_2 c_2 = 0$, firm 2's best response is unique and decreasing in α_1.

The intersection of these reaction functions gives the firms' equilibrium values for the incentive parameters as functions of their unit costs of production, as given by the following lemma.

Lemma 3: (a) If $a + 2c_1 \geq 3c_2$, there is a unique equilibrium in the delegation stage of the game, given by

$$\alpha_1 = \frac{8c_1 - 2c_2 - a}{5c_1}, \quad \alpha_2 = \frac{8c_2 - 2c_1 - a}{5c_2}. \tag{9}$$

(b) If $a + 2c_1 < 3c_2$, there is a continuum of equilibria in the delegation stage of the game, given by the set:

$$\mathbf{E} \equiv \{(\alpha_1, \alpha_2): \frac{4c_1 - a}{3c_1} \le \alpha_1 \le \min\{\frac{2c_2 - a}{c_1}, 1\}; \frac{2c_1 + a}{3c_2} \le \alpha_2 \le 1; a + \alpha_1 c_1 - 2\alpha_2 c_2 = 0.\}$$

It may be noted here that the choices of α_1 and α_2 as given by Lemma 3 satisfy the condition in Lemma 1 that $\alpha_1 c_1 \le \alpha_2 c_2$. First, consider the solution given by (9). From (9), $\alpha_1 c_1 - \alpha_2 c_2$ = $2(c_1 - c_2)$, which is less than or equal to zero since $c_1 \le c_2$ with or without licensing. Next, consider the set **E**. On this set, $\alpha_1 c_1 - \alpha_2 c_2 = -(a - \alpha_2 c_2)$, which is less than zero since $a > c_2$ by assumption and $c_2 \ge \alpha_2 c_2$ in equilibrium.

The preceding results lead to the firms' equilibrium quantity choices as functions of levels of marginal costs, as summarized in the following proposition.

Proposition 1: In the quantity competition stage of the three-stage licensing-delegation-quantity game,

(a) if $a + 2c_1 > 3c_2$ then the equilibrium quantities are an interior solution, given by

$$q_1^* = \frac{2}{5}(a + 2c_2 - 3c_1), \quad q_2^* = \frac{2}{5}(a + 2c_1 - 3c_2); \tag{10}$$

(b) if $a + 2c_1 \le 3c_2$ then the following multiple corner solutions in quantity are obtained:

$$\max\{a - c_2, \frac{a - c_1}{2}\} \le q_1^* \le \frac{2}{3}(a - c_1), \quad q_2^* = 0. \tag{11}$$

The interior solution given by (10) has the usual property that each firm's output decreases in its own marginal cost but increases in its competitor's unit cost. Comparing (10) with the well-known solution without delegation, given by $((a + c_2 - 2c_1)/3, (a + c_1 - 2c_2)/3)$, one obtains that industry output is higher under delegation. This confirms the established result that firms under delegation are more aggressive in their output choices than firms under no delegation.

Based on Proposition 1, the firms' reduced-form profit functions are

$$\pi_1^*(c_1,c_2) = \frac{2}{25}(a-3c_1+2c_2)^2, \quad \pi_2^*(c_1,c_2) = \frac{2}{25}(a-3c_2+2c_1)^2 \tag{12}$$

in the case of interior quantity solution, and

$$\pi_1^*(c_1,c_2) = (a-c_1-q_1^*)q_1^*, \quad \pi_2^*(c_1,c_2) = 0 \tag{13}$$

in the case of a corner quantity solution, where q_1^* is given by (11).

The Licensing Problem

In the licensing stage of our three stage game, the patent-holding firm (firm 1) first chooses a licensing contract, then the potential licensee (firm 2) decides whether to accept the offer from the patent holder. The patent-holding firm's objective is to maximize its total income which is the sum of the profit from its own production and the licensing revenue. In the following analysis, we consider three forms of licensing contract: fixed fee only, royalty only, and fixed fee plus royalty.[4] We use F to denote a fixed fee that is independent of the licensee's output level and r to denote a royalty rate per unit of output.

Firm 1's unit cost of production is $c_1 = c-\varepsilon$, firm 2's unit cost of production is $c_2 = c-\varepsilon+r$ when licensing occurs and is $c_2 = c$ when licensing does not occur. Obviously, the royalty rate r cannot exceed the magnitude of innovation (ε). Firm 1 chooses a licensing contract to maximize its total income subject to the constraints that firm 2 is willing to accept the licensing contract and that firm 2's output is non-negative. That is, firm 1 solves the following problem:

$$\text{Max}_{\{r,\,F\}} \qquad \pi_1^*(c-\varepsilon, c-\varepsilon+r) + r\,q_2^*(c-\varepsilon, c-\varepsilon+r) + F \tag{14}$$

s. t.

$$\pi_2^*(c-\varepsilon, c-\varepsilon+r) - F \geq \pi_2^*(c-\varepsilon, c)$$

$$q_2^*(c-\varepsilon, c-\varepsilon+r) \geq 0$$

In (14), the profit functions and firm 2's output function are given by (10)-(13). Under fixed-fee licensing, firm 1 chooses F while restricting r to be zero; under royalty licensing, firm 1 chooses r while restricting F to be zero; under fee plus royalty licensing, firm 1 chooses both F and r.

The next proposition concerns the occurrence of licensing in the equilibrium of the three-stage game under each of the three forms of licensing.

Proposition 2: The equilibrium outcome of the three-stage delegation-licensing-quantity game is given by the following:

(a) under fixed-fee licensing, licensing occurs if and only if $\varepsilon < 2(a-c)/11$;

(b) under royalty licensing, licensing occurs if and only if $\varepsilon < (a-c)/2$;

(c) under fee plus royalty licensing, licensing occurs if and only if $\varepsilon < (a-c)/2$ and will be in the form of royalty only when occurring.

It has been shown in the literature that, for the linear Cournot model without delegation, fixed-

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[4] These three forms of licensing account for almost all licensing in practice. Rostoker (1984) reported that fixed fee alone was used thirteen percent of the time, royalty alone thirty-nine percent, and royalty plus fixed fee forty-six

fee licensing occurs for $\varepsilon < 2(a - c)/3$ and royalty licensing and fee plus royalty licensing occur for any non-drastic innovation (i.e., $\varepsilon < a - c$).[5] Comparing this conclusion with the results in Proposition 2, it follows that in the linear model licensing occurs at most half as likely under strategic delegation compared to no delegation. Moreover, licensing occurs only for small innovations under strategic delegation.[6]

4. Concluding Remarks

This paper has examined licensing of a cost-reducing innovation by a patent-holding firm to its competitor from the profit motive. Under strategic delegation, firms (managers) behave more aggressively than under standard quantity competition, reducing the incentive for the patent-holding firm to license its innovation to the other firm. This is the result of two forces. On the one hand, the cost-reducing innovation (if kept for own use) affords the patent-holding firm a bigger advantage over its competitor under strategic delegation than under no delegation. On the other hand, the potential licensing revenue is smaller due to a smaller potential for profit gain from licensing by the competitor under strategic delegation than under no delegation. Both forces work to reduce the likelihood of licensing under strategic delegation relative to no delegation.

The discussion above also indicates that the main conclusion of this paper that licensing is less likely to occur under strategic delegation than under no delegation should survive extension of the simple homogenous good duopoly model with linear demand to more general settings.

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percent, among the firms surveyed.

[5] See, for example, Katz and Shapiro (1985) and Wang (1998).

[6] The conclusion in Proposition 2(c) extends the result in Wang (1998) and Kamien and Tauman (2002) that royalty is superior to fixed fee for the licensor to the situation with strategic delegation.

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Appendix

Proof of Lemma 1:

For the interior solution given by the first line of (5) and the first line of (6), it is obtained straightforwardly by solving for q_1 and q_2 using the first line of (3) and the first line of (4). This solution also gives us the condition to have an interior solution, namely q_2 is positive (which implies q_1 is positive). For the corner solution, it is important to recognize that under the condition $\alpha_1 c_1 \leq \alpha_2 c_2$ the only possible corner solution is when the intersection point of the firms' quantity reaction curves is on the q_1 axis so that $q_2 = 0$. Using this fact, the corner solution for q_1 is obtained by substituting $q_2 = 0$ into the first line of (3) and is given by the second line of (5).

Proof of Lemma 2:

Based on (5) and (6), the firms' reduced-form profit functions are

$$\pi_1(\alpha_1, \alpha_2) = \begin{cases} (a - c_1 - q_1^i - q_2^i)q_1^i & \text{if } a + \alpha_1 c_1 - 2\alpha_2 c_2 > 0 \\ (a - c_1 - q_1^c)q_1^c & \text{if } a + \alpha_1 c_1 - 2\alpha_2 c_2 \leq 0 \end{cases} \tag{A1}$$

$$\pi_2(\alpha_1, \alpha_2) = \begin{cases} (a - c_2 - q_1^i - q_2^i)q_2^i & \text{if } a + \alpha_1 c_1 - 2\alpha_2 c_2 > 0 \\ 0 & \text{if } a + \alpha_1 c_1 - 2\alpha_2 c_2 \leq 0 \end{cases} \tag{A2}$$

Consider (7) first. Substituting q_1^i and q_2^i given respectively by (5) and (6) into the first line of (A1) and maximizing the resulting profit function for firm 1 with respect to α_1 yield $\alpha_1 = (6c_1 - \alpha_2 c_2 - a)/(4c_1)$, this is the first line of (7). For this solution to represent firm 1's best reaction, the condition in the first line of (A1) must hold. The intersection of the lines $\alpha_1 = (6c_1 - \alpha_2 c_2 - a)/(4c_1)$ and

$a+\alpha_1 c_1 - 2\alpha_2 c_2 = 0$ is $(\alpha_1, \alpha_2) = ((4c_1 - a)/(3c_1), (a+2c_1)/(3c_2))$. Thus, the first line of (7) is proved. Substituting q_1^c in (5) into the second line of (A1) and maximizing with respect to α_1 implies that α_1 = 1. For this to hold, it must be true that $a+\alpha_1 c_1 - 2\alpha_2 c_2 \leq 0$. Replacing a_1 in this inequality by one gives $\alpha_2 \geq (a+c_1)/(2c_2)$. This proves the last line of (7). To show the middle line of (7), it suffices to observe that for the range of α_2 in the interval $[(a+2c_1)/(3c_2), (a+c_1)/(2c_2)]$, firm 1 maximizes its profit function given by the second line of (A1) subject to the constraint that $a+\alpha_1 c_1 - 2\alpha_2 c_2 \leq 0$. For $\alpha_2 \in [(a+2c_1)/(3c_2), (a+c_1)/(2c_2)]$, the above constraint is binding, yielding the middle line of (7).

Next consider (8). The first line of (8) is parallel to the first line of (7) and can be shown similarly by maximizing $\pi_2(\alpha_1, \alpha_2)$ in (A2). For the second line of (8), it suffices to observe that when $a+\alpha_1 c_1 - 2\alpha_2 c_2 \leq 0$ firm 2's profit is zero implying an arbitrary choice of α_2 subject to the constraint that the above inequality holds, which implies that $\alpha_2 \geq (a+\alpha_1 c_1)/(2c_2)$.

<u>Proof of Lemma 3</u>:

(a). Solving the system of equations composed of the first line of (7) and the first line of (8) for α_1 and α_2 gives immediately (9). From (9), $a+\alpha_1 c_1 - 2\alpha_2 c_2 = 6(a+2c_1-3c_2)/5$. Hence, if $a+2c_1 > 3c_2$ then $a+\alpha_1 c_1 - 2\alpha_2 c_2 > 0$ and if $a+2c_1 = 3c_2$ then $a+\alpha_1 c_1 - 2\alpha_2 c_2 = 0$. In either case, firm 1's incentive parameter reaction curve given by (7) and firm 2's incentive parameter reaction map given by (8) have a unique intersection point, that is given by (9). In the case $a+2c_1 > 3c_2$, this intersection point is below the line $a+\alpha_1 c_1 - 2\alpha_2 c_2 = 0$, while in the case $a+2c_1 = 3c_2$, it is right on the line $a+\alpha_1 c_1 - 2\alpha_2 c_2 = 0$.

(b). The proof for Part (a) indicates that if $a + 2c_1 < 3c_2$ then firm 1's incentive parameter reaction curve given by (7) and firm 2's incentive parameter reaction map given by (8) do not have an intersection point below the line $a + \alpha_1 c_1 - 2\alpha_2 c_2 = 0$. In this case they intersect on a line segment of the line $a + \alpha_1 c_1 - 2\alpha_2 c_2 = 0$. Straightforward derivations imply that this line segment corresponds to the set **E**.

Proof of Proposition 1:

(a). Results in (10) follow immediately by substituting (9) into the first line of (5) and the first line of (6).

(b). By Lemma 3(b), if $a + 2c_1 < 3c_2$ the delegation stage of the game results in choices of (α_1, α_2) given by the set **E**. Using (6), it is obvious that on this set $q_2 = 0$. The range of q_1 is obtained by substituting $q_2 = 0$ and the range of values for α_1 in the set **E** into (3). In particular, corresponding to $\alpha_1 = (4c_1 - a)/(3c_1)$ we have $q_1 = 2(a - c_1)/3$ and corresponding to $\alpha_1 = (2c_2 - a)/c_1$ or 1 we have $q_1 = a - c_2$ or $q_1 = (a - c_1)/2$. These values for q_1 confirm the equilibrium range for q_1 in (11).

Proof of Proposition 2:

(a). Based on (10) and (12), both firms produce the quantity of $2(a - c + \varepsilon)/5$ and earn the profit of $2(a - c + \varepsilon)^2/25$ under fixed-fee licensing. If licensing does not occur, based on Lemma 3, an interior solution is obtained if $\varepsilon < (a - c)/2$ and a corner solution is obtained if $\varepsilon \geq (a - c)/2$.

Consider first the case of $\varepsilon < (a - c)/2$. If licensing does not occur, by (12), firm 1's profit is $2(a - c + 3\varepsilon)^2/25$ and firm 2's profit is $2(a - c - 2\varepsilon)^2/25$. The maximum licensing fee firm 1 can charge firm 2 is equal to the difference between firm 2's post-licensing profit and its profit if no licensing occurs. Thus, $F = 2(a-c+\varepsilon)^2/25 - 2(a-c-2\varepsilon)^2/25$, and under fixed-fee licensing, firm 1's total income is $2(a-c+\varepsilon)^2/25+F$. The difference between this and firm 1's profit under no licensing is $2(a-c+\varepsilon)^2/25 + F - 2(a-c+3\varepsilon)^2/25 = 2\varepsilon[2(a-c)-11\varepsilon]/25$, which is greater than zero only for $\varepsilon < 2(a-c)/11$. That is, under fixed-fee licensing and strategic delegation, licensing is profitable for firm 1 for $\varepsilon \in (0, 2(a-c)/11)$ but not profitable for $\varepsilon \in (2(a-c)/11, (a-c)/2)$.

Consider next the case of $\varepsilon \geq (a-c)/2$. In this case firm 2 will produce zero output and make zero profit if licensing does not occur. Thus, the maximum fixed fee firm 1 can charge is equal to $2(a-c+\varepsilon)^2/25$ and firm 1's total income under licensing is $4(a-c+\varepsilon)^2/25$. If firm 1 does not license to firm 2, by (13), its profit is equal to $(a-c+\varepsilon-q_1^*)q_1^*$, where by (11) q_1^* varies from $a-c$ to $2(a-c+\varepsilon)/3$. It is straightforward to verify that $(a-c+\varepsilon-q_1^*)q_1^* > 4(a-c+\varepsilon)^2/25$ for all $q_1^* \in [a-c, 2(a-c+\varepsilon)/3]$ when $\varepsilon \geq (a-c)/2$. That is, under fixed-fee licensing and strategic delegation, licensing is not profitable for firm 1 for $\varepsilon \in [(a-c)/2,\ a-c)$. We have thus proved the assertion made in Proposition 2(a).

(b). By using (10) and (12), firm 1's total income under royalty licensing is

$$M = \pi_1^*(c-\varepsilon, c-\varepsilon+r) + r\,q_2^*(c-\varepsilon, c-\varepsilon+r) = \frac{2}{25}(a-c+\varepsilon+2r)^2 + r\frac{2}{5}(a-c+\varepsilon-3r).$$

Differentiating M with respect to r yields

$$\frac{\partial M}{\partial r} = \frac{2}{25}(9(a-c+\varepsilon)-22r).$$

Non-negative equilibrium output for firm 2 implies that $3r \leq a-c+\varepsilon$. Hence, the feasible choice of r is from zero to $\min\{\varepsilon, (a-c+\varepsilon)/3\}$. It is easy to see that for $r \in [0, \min\{\varepsilon, (a-c+\varepsilon)/3\}]$, $\partial M/\partial r > 0$.

It follows that firm 1's optimal choice of r is equal to $\min\{\varepsilon, (a-c+\varepsilon)/3\}$. The optimal r is equal to ε if $\varepsilon < (a-c)/2$. In this case, licensing occurs and firm 2 produces a positive output in equilibrium. The optimal r is equal to $(a-c+\varepsilon)/3$ if $\varepsilon \geq (a-c)/2$. In this case, licensing does not occur since firm 2 produces zero output in equilibrium. We have thus proved Proposition 2(b).

(c). Obviously, the licensor's optimal F is such that the first constraint in (14) holds in equality. Solving for F from this equality and substituting it into the objective function in (14),

$$M = \pi_1^*(c-\varepsilon, c-\varepsilon+r) + r\,q_2^*(c-\varepsilon, c-\varepsilon+r) + \pi_2^*(c-\varepsilon, c-\varepsilon+r) - \pi_2^*(c-\varepsilon, c).$$

By using (10) and (12),

$$M = \frac{2}{25}(a-c+\varepsilon+2r)^2 + r\frac{2}{5}(a-c+\varepsilon-3r) + \frac{2}{25}(a-c+\varepsilon-3r)^2 - \frac{2}{25}(a-c-2\varepsilon)^2.$$

Differentiating M with respect to r yields

$$\frac{\partial M}{\partial r} = \frac{2}{25}(3(a-c)+3\varepsilon-4r) > 0,$$

where the inequality follows from the fact $r \leq \varepsilon < a-c$. Namely, M is a strictly increasing function of r on the interval $[0, \varepsilon]$ of feasible choices for r.

Two implications follow immediately from the fact that the best choice of royalty rate for the licensor is $r = \varepsilon$. One, in the optimal fee plus royalty contract, the optimal fee is zero. This is because firm 2's marginal cost of production is unchanged by licensing and therefore its profit is unchanged. Second, firm 2's equilibrium output $q_2^*(c-\varepsilon, c) = 2(a-c-2\varepsilon)/5$ is positive only for $\varepsilon < (a-c)/2$. These two conclusions together establish the two statements in Proposition 2(c).