Uncommitted Couples:
Some Efficiency and
Policy Implications of Marital Bargaining

Saku Aura[*]

University of Missouri-Columbia and CESifo

ABSTRACT: This paper studies married couple's dynamic investment and consumption choices under the assumption that the couple cannot commit across time to not to renegotiate their decisions. The inefficiencies that can arise are characterized. Efficiency properties of different divorce asset division regimes are examined. A stylized common law regime is shown to lead to a fully efficiency in a simple model while it is shown that under community property regime the couple is unlikely to attain full efficiency. The effect of inability to commit across time on the savings level is examined under a tractable special case of the model.

JEL Codes: D10, D91, J12

1 Introduction

This paper studies the implications of inability commit across time for economic efficiency and decision-making. While there exists a vast literature on the effects of the inability to commit to a given solution in many applications (including investment decisions and monetary policy), the implications of this imperfectness have not been studied much in the context of a household. This paper asks how the fact that a (married) couple cannot necessarily credibly commit to a future consumption division affects the decision made today.

The setup of this paper is simple. A couple needs to decide today on how to divide current consumption between spouses and how much to save. They cannot credibly agree on how to divide consumption in the future, since they cannot commit not to renegotiate the current agreement in the future. Furthermore, the "balance of power" in the family might be different in the future, so one spouse might have comparative advantage in the family now that will decay in the future. This potential disparity of "balance of power" is shown to lead to an economic inefficiency within the household since the spouses cannot complete Pareto-improving trades between themselves across periods due to lack of commitment.

This paper remains mostly agnostic about the sources of the difference of "balance of power" across time periods by not explicitly modelling the phenomen in most cases. It is only assumed that for some reason, the relative "welfare weight"[1] in the household's objective function varies across time periods. The reason why current objective function is not necessary aligned with the future objective can be justified e.g. by assuming that the spouses engage in period-by-period bargaining. If, loosely speaking, the relative outside options of the spouses differ from period to period, then welfare weights will differ too. Reasons for nonconstant relative outside options abound: dynamic effects of labor force attachment of spouses (through human capital acquisition), spouse specific educational investments, the details divorce laws, remarriage prospects, health shocks[2] and time-varying differing relative attachment to the community outside the family.

---

[1] This correspond to the ratios of marginal utilities of consumption on a given period between spouses in the optimum solution.

[2] This paper, for most part, does not deal with uncertainty, which is a integral part of health shocks.

A major exception to the agnostic attitude toward the "balance of power" phenomen is taken in the section dealing with divorce. This paper makes two contributions to theory of divorce: First it shows how divorce rules can have an effect on savings for families that stay married through their effect on the marital bargaining. In a tractable repeated Nash-bargaining model (a special case of the general model), this effect can be positive or negative depending on the elasticity of intertemporal substitution of the utility functions. This effect is completely separate from the traditional "insurance against bad outcome" effect of divorce on savings considered in Cubeddu and Ríos-Rull (1997).

The second contribution to the theory of divorce is efficiency comparison between different divorce regimes. It is shown in the section 4 that a stylized common-law property regime attains full efficiency under special conditions while the community property regime is unlikely to lead to full efficiency. Thus the choice between common-law and community property regimes involves potentially an equity-efficiency trade-off.

This paper is a part of the growing literature of models of family that models the family not as a single aligned entity, that can be modelled as if it were an individual agent, but as group of agents whose preferences are not necessarily completely aligned. While this line of inquiry goes back at least to Becker (1973), the papers that initiated the more recent interest in this research topic are papers on Nash bargaining models of family (Manser and Brown 1980, McElroy and Horney 1981) and papers on the efficient contracting models of family (starting from Chiappori 1988). It is interesting to note, that many empirical papers in this literature implicitly assume lack of full commitment across time-periods (and legislative regimes) in household decision-making. Examples of this are papers using the effects of law changes to test the single-utility-function view of the household, like Aura (2001), Duflo (2000) and Lunberg, Pollack and Wales(1996).

This paper can be seen as a simple extension of the framework set out in the series of papers by Chiappori and his coauthors into dynamic setting. Mazzocco (2000) extends similar ideas as this paper, although its focus is different. Mazzocco's analysis of risk is more general than in this paper, but he restricts his analysis of investment decisions into cases where one household member holds all the property rights on the assets.

There are two separate branches of literature that are closely related to this paper. First, a new and a small branch is the one which this paper is

also belongs to: the literature on savings decisions and family bargaining. Three papers are worth mentioning here: Browning (2000), Lundberg and Ward-Batts (2000), and Lundberg, Startz and Stillman (2002). Browning (2001) presents a two-period game-theoretic model, where he shows that under certain assumption, the non-cooperatively made savings and investment decisions yield full Pareto efficiency. Papers by Lundberg and coauthors on the other hand are mostly empirical papers trying to shed light on retirement related issues (like the drop in consumption at retirement) using household bargaining logic and models relatively similar to, although simpler than, the ones presented in this paper.

The other related literature is the growing literature on informal insurance arrangements. Two examples of this are Attanasio and Ríos-Rull (2000) and Ligon, Thomas and Worral (2000). This literature emphasizes the same issue as this paper: the inability to commit across time (and states of the world). However, since most of this literature is on agricultural communities in developing countries, they do not see savings as the most interesting aspect (since the crop cannot be stored indefinitely) and emphasize risk-sharing almost solely. Ligon et al. do consider savings, but while coming close to the results of the first part of this paper, they do not emphasize the role of savings in their model.

The paper is organized as follows: Section 2 presents the basic model. Section 3 analyzes the effect of lack of commitment on the households savings level. Section 4 analyzes efficiency properties of different divorce regimes. Section 5 considers the problem of life-insurance protection for surviving spouses. Section 6 concludes.

2   The basic model and results

Can a married couple commit to a given consumption path and sharing rule across time? One reason to think that the answer to this question might be negative is that the outside options (outcomes, should they divorce) of the spouses might evolve in time. This might make the agreement based on yesterday's balance of power unsustainable today. If this is the case, then in today's decision making the couple has to take into account the effect of today's choices on future decisions. This dynamic linkage, through the fact that future behavior is constrained by the outcome of future renegotiation

process and the fact that today's choices affect this process, is the central attention of the study undertaken in this paper. In order to proceed, this paper makes three further assumptions on the decision making process.

First, the within-period decision-making is assumed to Pareto-efficient with respect to the constraints. This can be seen either as a simple modelling choice or as a statement about possible interactions within a couple. If we assume that no motives of malice guide the decision-making (and even these could be incorporated into the utility functions) it is hard to believe that a partner in a couple would turn down suggested change that would make both partners better off while fully taking into account the constraints of the problem. At least as long as the partners are sufficiently patient and the random divorce risk[3] is sufficiently low, it is a justifiable working assumption to assume that the couple can overcome prisoner's dilemma type problems that they might face.

The second, related assumption, is a very strong assumption on rationality. In the models that follow, it is assumed that in making decisions about savings and investment (these could broadly be viewed to include decisions about human capital investment) the strategic element of each of these decisions in the future is fully understood and incorporated in the decision making. By the structure of the models that follow (say repeated Nash-bargaining), this means that the couple is able to understand the very complicated dynamic effects of current period decisions.

Third important assumption is that of full information. While many of the results presented here are or could be extended to one type of uncertainty (exogenous risks) the issues relating to asymmetric information (hidden knowledge or hidden actions) within marriage are not considered. This can be viewed as an important caveat in modelling families, since it might be argued that one of the spouses often has informational advantage

---

[3]The simplistic view on divorce taken in this paper is the following. Each period two states of the world are possible: under the normal circumstances, the couple continues together if their utility from sticking together under the cooperative regime is higher than in the case of divorce. However, in each period, there is a remote chance of a random shock that irrespective of economic variables makes the couple incompatible with each other (say one partner is caught cheating). The latter (an unmodelled phenomen) is what is called the risk of divorce in this paper. Also, for most of the paper the break-up of relationship is interchangeable with divorce, since unless different legal environments are explicitly modelled, it does not matter whether the couple is legally married or not for the decision making problem.

over families finances (the one who takes care of the day-to-day finances). This possible extension is left to further research.

To illustrate the decision-making problem that the couple[4] faces, when it cannot commit to a time-consistent solution, consider the following problem. Let the world last for T-periods. Assume that there is no uncertainty and that the only decision the couple faces at period t is how much to save and how to divide the current consumption between spouses. Assume that the life-time utility functions of respective spouses are defined as time-separable utilities over their own consumption only:

$$U^m = \sum_{t=1}^{T} u_t^m(c_t^m)$$

$$\text{and } U^f = \sum_{t=1}^{T} u_t^f(c_t^f),$$

where superscripts m and f refer to husband and wife respectively.

The inability to commit means that future behavior is taken into account as a constraint on the constrained Pareto-efficient decision program at every period. Starting from last period this means that at period T the problem that couple solves is:[5]

$$V_T^m(A_T) = \max_{c_T^m, c_T^f} u_T^m(c_T^m)$$

$$\text{subject to } A_T = c_T^m + c_T^f$$

$$\text{and } u_T^f(c_T^f) = V_T^f(A_T),$$

---

[4] The assumption that there are exactly two members of the family whose utility functions are relevant for the decision making will matter for the results that follow. So this paper's results do not necessarily extend to families with teenage or grown-up children or other family members participating in the decision process. The key is that children can be included through the effect of their consumption through parent's utility functions, but not as someone exerting any power in the decision-making.

[5] As long as we assume that there is only one private consumption good the period T maximization is trivial.

for some function $V_T^f(A_T)$ that represent wife's period specific utility level at time T in the optimum.[6] The $V_T^m(A_T)$ and $V_T^f(A_T)$ functions are reduced form representations of the household decision-making process, representing the point in the utility possibility frontier that the household will choose. A leading example of structural form representation that could be characterized by these functions is Nash-bargaining between spouses (being a member of the class of Pareto efficient decision-making processes).

Working backwards, at period $T-1$ the household now has three choice variables, current consumption of respective spouses and the assets level at period T. The inability to commit across time is captured by the fact that they cannot contract on the respective consumption levels of spouses at period T. Instead, they have to take the decision process in the period T as a constraint while making decisions at time $T-1$. One possible way to characterize this constrained decision problem is:

$$V_{T-1}^m(A_{T-1}) = \max_{c_{T-1}^m, c_{T-1}^f, A_T} u_{T-1}^m(c_{T-1}^m) + V_T^m(A_T)$$

$$\text{subject to } A_{T-1} = c_{T-1}^m + c_{T-1}^f + \frac{A_T}{(1+r_{T-1})}$$

$$\text{and } u_{T-1}^f(c_{T-1}^f) = V_{T-1}^f(A_{T-1}) - V_T^f(A_T),$$

where, as earlier, $V_{T-1}^f(A_{T-1})$ represents the sum of the wife's period-specific utilities at period $T-1$ and T in the optimum. The functions $V_{T-1}^m(A_{T-1})$ and $V_{T-1}^f(A_{T-1})$ give us a reduced form representation of the decision making process without commitment.

Using backward induction this leads to following characterization of the problem:

---

[6] This is nothing more than the usual characterization of Pareto-efficient choice, except that for generality it is assumed that the wife's utility level is a function of the wealth holdings of the household at period T. The dependence of V_T^f on the wealth holding makes it possible to characterize general efficient forms of household decision-making (like Nash-bargaining or social welfare function maximization) where an increase of wealth available in the period T will in general have effect on the utilities and consumptions of both spouses.

$$V_t^m(A_t) = \max_{c_t^m, c_t^f, A_{t+1}} u_t^m(c_t^m) + V_{t+1}^m(A_{t+1})$$

$$\text{subject to } A_t = c_t^m + c_t^f + \frac{A_{t+1}}{(1 + r_t)}$$

$$\text{and } u_t^f(c_t^f) = V_t^f(A_t) - V_{t+1}^f(A_{t+1}).$$

The first constraint is the usual budget constraint, where A_t is the remaining life-time wealth of the couple at period t.[7] The second constraint is the usual Pareto-efficiency requirement altered to take into account that the couple is constrained by the process characterizing their decision-making in the future and cannot commit to (potentially better) future consumption allocations that are incompatible with that process. Naturally, a completely equivalent characterization of the process is the maximization of the wife's life-time utility subject to constraint on the husbands utility level. This interchangeability will become useful in providing short proofs for the theorems.

A key assumption in the definition of V_t-functions is that they are functions of the current assets holdings only. Thus, while this will be extended in one direction to handle multiple assets, it precludes complicated dynamic dependencies from past actions. This assumption can be defended on tractability grounds and by its intuitive appeal.[8]

It is important to understand that the above is meant to be a characterization of the optimum in a same way as Pareto-frontier characterizes possible optima in a usual exchange economy setting. This means that while we can study properties of the optimum using this characterization, we cannot derive comparative statics with respect to changes of the economic parameters without specifying the structural process by which the couple arrives into the solution.[9] Thus, without further specification of the process we can

---

[7] All the analysis in this paper would go through when A_t is construed as the net wealth of the couple at time t, and in each period each partners gets an additional amount (possible negative) of income I_t^m and I_t^f respectively. While completely equivalent characterizations of the dynamic budget constraint in the current environment, in the extensions this allows for the respective I_t-processes to be contingent on the divorce or widowhood states. Thus the results do extend to a more general specification of the household's state-contingent budget constraint.

[8] However, this excludes the possibility that the assets division in case of divorce might be affected by e.g. a spending spree prior to divorce by one of the spouses.

[9] A reduced-form way of doing this would be to parametrize the V_t^f-function family.

study types of inefficiencies that can arise and if we find policy intervention that can lead to a first-best solution, we can claim that it will make at least one member of the couple better off. However, without getting into the black box of family decision making, the approach followed in this section cannot be used to say anything about within family redistributive effects. This is an important qualification, since some of efficiency-enhancing policy recommendations of this paper might have huge distributive impacts (like the desirability of common-law divorce asset division regime over community property regime purely on efficiency grounds).

To characterize the efficiency properties and to describe the nature of inefficiencies that can arise because of the lack of commitment across time periods, the following assumption will be made.

Assumption 1 (More wealth is better for both in every period) $V_t^{m\prime} \geq 0$ and $V_t^{f\prime} \geq 0$ for all time periods in the optimum solution.

The justification for assumption 1 is that it bounds the bargaining effect of wealth to not dominate the intuitive effect of more wealth (the expansion of the budget set in the future effect dominates the bargaining effect of more wealth). A problem with assumption 1 is that it is in terms of both $V_t^{m\prime}$ and $V_t^{f\prime}$. While intuitively appealing, this is not completely satisfactory, since only one of the V_t-functions should be taken as fundamental of the problem (i.e. reduced form presentation of the bargaining process), the other being a quantity derived from the optimization solution. While the author feels that assumption 1 is a reasonable restriction on the class of admissible models under many circumstances, this might not be satisfactory to all the readers. Therefore it is also necessary to give fundamental sufficient conditions that would yield Assumption 1. A set of alternative conditions guaranteeing that assumption 1 holds is given as lemma 0.

Lemma 0. Any one of the following assumptions is sufficient for Assumption 1 to be satisfied:

a) Value functions are independent of wealth for one of the spouses: $V_t^{f\prime} = 0$ $\forall t, A_t$ or $V_t^m = 0$ $\forall t, A_t$.

An analogy of this into the exchange economy setting is that while we can study the properties of the Pareto-frontier, in order to say how the equilibrium changes with respect to the outside parameters, we need to make additional assumptions (like price-taking and utility maximization) to have predictions about the effects of parameter changes.

b) Two period world and wealth good for both in second period: let $T = 2$ and let the second period solution be characterized by a sharing rule for wealth: $c_2^m = \psi_2(A_2)$ and $c_2^f = A_2 - \psi_2(A_2)$. Furthermore let $0 \leq \psi_2' \leq 1$.

c) T-period world, with last period described as in b) and letting the series of functions $V_t^f(A_t)$ satisfy following conditions: $V_t^{f'} \geq 0, V_t^{f''} \leq 0$ $\forall t, A_t$ and

$$(1 + r_t)V_{t+1}^{f'}(A) \geq V_t^{f'}(A) \ \forall t, A.$$

d) Like c), but applying the restrictions on $V_t^m(A)$-functions.

e) CRRA-utilities and outside options in repeated Nash-bargaining: Let Assumptions 2-5 hold of the section 3 hold.

f) $V_t^{m'} \geq 0$ and $V_t^{m'} \leq u_t^{m'} \ \forall t$ in the solution; or equivalently $V_t^{f'} \geq 0$ and $V_t^{f'} \leq u_t^{f'} \ \forall t$ in the solution.

Proof: See Appendix.

The point of Lemma 0 is to illustrate that Assumption 1 covers a large class of interesting problems. Some of these alternative conditions need further commenting: first f) is nothing more than restatements of Assumption 1 using envelope theorem to derive more interpretable conditions. It suffers from the same weakness as Assumption 1, it refers to quantities (marginal utilities of consumption) that are defined in the optimum.

Condition a) provides an interesting special case, where one of the spouses, say the husband, has constant outside options. This means, that in the optimum, the wife will attain full efficiency in her consumption plan even though the husband's consumption plan might be distorted.

Condition e) provides an example of a structural model that satisfies the Assumption 1.

Conditions b), c) and d) are the most fundamental. Condition b) uses two-stage budgeting in the last period as the starting point (the sharing rule of assets). This can be viewed as fundamental description of the bargaining and not as an outcome.[10] Conditions c) and d) extend this with relatively

---

[10] Unfortunately this two-stage budgeting does not extend to any other than last period. In all other periods there are three goods, one of which is a public good (assets in the next period).

strong assumptions into T-period setting. However, the wide class of problems that will satisfy Assumption 1 will become evident from the proof of condition c). From that proof we can see that condition c) is just a very strong sufficient condition that can be violated while Assumption 1 still holds.[11] Thus Assumption 1 is rather general.

Definition 1. (Undersaving and oversaving) Define the situation where $u_t^{m\prime} < (1+r_t)u_{t+1}^{m\prime}$ and $u_t^{f\prime} < (1+r_t)u_{t+1}^{f\prime}$ as undersaving and situation where both inequalities are reversed as oversaving.

The justification for the definition of undersaving is that it defines a situation where a both spouses would prefer to transfer current consumption into next period consumption. It is noteworthy, that the definition here applies to consecutive time periods only.

Theorem 1. Let assumption 1 be satisfied. In the optimum there cannot be undersaving nor oversaving.[12]

Proof: Let μ_t and λ_t be the Lagrange-multipliers for the budget constraint and the wife's utility level constraint on the period t-suboptimization. The first order conditions for the optimization are:

$$u_t^{m\prime} - \mu_t = 0$$

$$\lambda_t u_t^{f\prime} - \mu_t = 0$$

$$V_{t+1}^{m\prime} - \frac{\mu_t}{1 + r_t} + \lambda_t V_{t+1}^{f\prime} = 0.$$

---

[11] Condition c) is too strong in two senses. First, it requires that the inequality holds for all values of possible values of A. However, this is only done to avoid a (circular) reference to quantities relating to the optimum. Another sufficient condition is that $(1 + r_t)V_{t+1}^{f\prime}(A_{t+1}) \geq V_t^{f\prime}(A_t)$, where A_{t+1} and A_t are the quantities chosen in the optimum (this form is implied by the for all values of A condition, concavity of $V_t^{f\prime\prime}$ and by the fact that $A_t \geq A_{t+1}$). Even this is not necessary, since this inequality condition is also just a sufficient condition, that may not be satisfied while Assumption 1 still holds.

[12] Note that the non-negativity constraints on consumptions are ignored in the specification. While this is mostly to save on notation, it is not completely without loss of generality. An assumption that would guarantee an interior optimum with respect to consumption is that $\lim_{c_t^q \to 0} u_t^{q\prime}(c_t^q) = \infty$ $\forall t$ where $q \in \{m, f\}$. Otherwise Theorem 1 might not hold for if the non-negativity constraint on consumption are imposed and at least one of these constraints is binding in the optimum.

Using the envelope theorem and the first order condition with respect husband's consumption the intertemporal first order condition can manipulated as:

$$\mu_{t+1} - \lambda_{t+1}V_{t+1}^{f\prime} - \frac{\mu_t}{1 + r_t} + \lambda_t V_{t+1}^{f\prime} = 0 \Leftrightarrow$$

$$u_{t+1}^{m\prime} - \frac{u_t^{m\prime}}{1 + r_t} = (\lambda_{t+1} - \lambda_t) V_{t+1}^{f\prime}.$$

By considering the same problem from wife's perspective (and using relations between the Lagrange coefficients in the two problems), the intertemporal first order condition for the wife can be written as:

$$u_{t+1}^{f\prime} - \frac{u_t^{f\prime}}{1 + r_t} = \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t}\right) V_{t+1}^{m\prime}.$$

By applying the envelope theorem, the dynamic first order conditions can be now written as:

$$u_{t+1}^{m\prime} - \frac{u_t^{m\prime}}{1 + r_t} = (\lambda_{t+1} - \lambda_t) V_{t+1}^{f\prime}$$

$$u_{t+1}^{f\prime} - \frac{u_t^{f\prime}}{1 + r_t} = u_{t+1}^{m\prime}\left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t}\right)\left(1 - \frac{V_{t+1}^{f\prime}}{u_{t+1}^{f\prime}}\right),$$

or equivalently as

$$u_{t+1}^{m\prime} - \frac{u_t^{m\prime}}{1 + r_t} = u_{t+1}^{f\prime}(\lambda_{t+1} - \lambda_t)\left(1 - \frac{V_{t+1}^{m\prime}}{u_{t+1}^{m\prime}}\right)$$

$$u_{t+1}^{f\prime} - \frac{u_t^{f\prime}}{1 + r_t} = \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t}\right) V_{t+1}^{m\prime}.$$

The result is immediate from the above.∎

It is well worth noting, that this result does not carry over to non-consecutive time periods: an counterexample for more general result will be given in the next section. The following are constructed examples of undersaving and oversaving, when Assumption 1 is not satisfied:

Example 1 (Undersaving). Consider a two period world with no discounting. Let both spouses have identical separable log utility functions

over first and second period consumption. Let total lifetime wealth of the couple be 4. Let the outcome of the second period negotiation between spouses be characterized[13] by $c_2^f = 1 - 0.1 * A_2$ and let the first period required utility level for wife be zero (achievable e.g. by consuming 1 each period). In the optimum (approximately) $c_1^m = 1.00$, $c_2^m = .96$, $c_2^f = 1.21$ and $c_2^f = .82$.

Example 2 (Oversaving). Like example 1, but let the outcome of the second period negotiation be characterized by $c_2^f = 2 - 0.1 * A_2$. In the optimum (approximately) $c_1^m = .81$, $c_2^m = .86$, $c_2^f = .57$ and $c_2^f = 1.7$.

Corollary 2. (Multiple goods) Let period t consumption vector be divided into three separate components for each period: c_t^m, c_t^f and c_t^p (husband's consumption, wife's consumption and within household public goods consumption respectively). Theorem 1 holds for between any arbitrary pairs of husband's and wife's private consumption.

Proof of Corollary 2. Similar to theorem 1 with additional notation.∎

Unfortunately, the Euler equation for the household public goods does not yield any interesting economic intuition.

Further interesting extension of the theorem 1 is to include labor supply decisions (or leisure consumption) as one of the choice variables. In the simplest case, where both spouses have a period-specific exogenously given period-specific wages, corollary 2 is all we need analyze this case. However, the labor supply decisions in this dynamic model becomes extremely interesting, when one allows for the the following generalizations:

1) The current period wage is affected by the past labor supply decisions
2) The current period wages of the spouses affect their relative positions in the family decision-making.

Formally these two effects can be taken into account by specifying that $w_t^f = f_t^f(w_{t-1}^f, L_{t-1}^f)$ and $w_t^m = f_t^m(w_{t-1}^m, L_{t-1}^m)$, where w stands for wage and L for labor supply, and that $V_t^f = V_t^f(A_t, w_t^m, w_t^f)$. Three interesting results arise from this extension. First is that the theorem 1 continues to hold for the consumption goods (but not for the labor supplies). The second is the obvious fact that even under full commitment the labor supply

---

[13] Equivalently this can stated as $V_2^f(A_2) = \log(1 - 0.1 * A_2)$. This means that $V_2^{f\prime} < 0$.

decisions must take the dynamic (human capital) effects of the labor supply decisions into account (so marginal disutility of labor is equated with the sum of the wage and the marginal dynamic effect of labor supply to the future wages). The third is that without commitment, labor supply becomes a "strategic" variable: because through future wages it affects future bargaining, in setting the labor supply this effect has to take into account. Thus the model is compatible with the effect put forward in Wells and Maher (1998), where they argue that for strategic reasons the lower-earning capacity spouses might not specialize in home-production because this would diversely affect their future bargaining position.

Theorem 1 and its extension would provide an interesting starting point for empirical investigation. Since Theorem 1 is very robust to additions to the model, it could be used either in panel or repeated cross-section setting to test the theory of efficient decision-making without commitment. The data requirement for this kind of exercise are not simple. Like most other test of new models of household in economics, it would require data on consumption items that are assignable to spouses (pure private goods in the household, or at least goods that have arguable much stronger effect on one spouse's utility). This empirical application is left for further research.

3 Effect of lack of commitment on the level of savings

Beyond the question of efficiency (i.e. would more or less savings be Pareto-improvement), it is interesting to also ask, whether lack of commitment causes the couple to save more or less than in a world where they could commit from day one to a consumption plan. As explained earlier, this is a question that cannot be asked in the most general framework, since this change in environment will almost always include redistribution effects too. Hence, one needs more specific model to answer this question. The model presented in this section will give one answer to this question: in this model, the effect of inability to commit is similar to the effect of as decrease of the assets return. In the end of this section, it is shown how a small change in the assumptions can be used to generate a completely different answer. This counterexample should not be viewed as nothing more than a proof that the result does not admit arbitrary generalizations, since the model presented

in this section still has intuitive appeal as a possible characterization of the household's decision-making process.

This section makes also general point about the feedback of divorce rules to savings behavior. In previous literature (like in Cubeddu and Ríos-Rull 1997) the effect of divorce on the savings behavior is an insurance effect: the savings are positively linked to higher probability of divorce because the marginal utilities of consumption are assumed to be higher in divorce state. This effect could be trivially incorporated to the model of this section, but to save on notation it is omitted. The feedback effect of divorce rules on savings that is presented in this section operates through a different channel: the future divorce property division rules affect bargaining power in the future and this can have (positive or negative) effect on savings in a model where decisions are renegotiated each period.

Assumption 2. (CRRA utilities) Let both spouses have life time utility function that can be written in the time separable CRRA-form:

$$U^m = \frac{1}{1-\theta^m} \sum_{k=1}^{T} (\beta^m)^{k-1} (c_k^m)^{1-\theta^m}$$

$$U^f = \frac{1}{1-\theta^f} \sum_{k=1}^{T} (\beta^f)^{k-1} (c_k^f)^{1-\theta^f}$$

Assumption 3. (Nash-Bargaining with divorce outside options) Let the household decision making process be repeated Nash-bargaining without commitment across time periods, so the period t decision making process can be characterized as:

$$\max_{c_t^m, c_t^f, A_{t+1}} \left(\left(\frac{1}{(1-\theta^m)} (c_t^m)^{1-\theta^m} + V_{t+1}^m(A_{t+1}) \right) - \tilde{V}_t^m(A_t) \right)$$

$$* \left(\frac{1}{(1-\theta^f)} \left(\left(c_t^f \right)^{1-\theta^f} + V_{t+1}^f(A_{t+1}) \right) - \tilde{V}_t^f(A_t) \right)$$

subject to $A_t = c_t^m + c_t^f + \dfrac{A_{t+1}}{(1+r_t)}$,

where $V_{t+1}^m(A_{t+1})$ and $V_{t+1}^f(A_{t+1})$ are the value functions characterizing the utility value of future periods for a given asset level in period $t+1$ and where $\tilde{V}_t^m(A_t)$ and $\tilde{V}_t^f(A_t)$ characterize the outside options of spouses should the negotiation break down and divorce occur. Thus, the outside option functions incorporate all the relevant information on the institutions (e.g. divorce assets division rules) and environment (e.g. remarriage prospects) relevant to spouses should they divorce. Furthermore, let us assume that once divorce happens, it is final.

Assumption 4. Let the outside option functions be of the form:

$$\tilde{V}_t^q(A_t) = \max_{\{\tilde{c}_i^q\}_{i=t}^T} \frac{1}{1-\theta^q} \sum_{k=t}^T (\tilde{\beta}^q)^{k-t}(\tau_k^q \tilde{c}_k^q)^{1-\theta_q}$$

$$\text{subject to } \psi_t^q(A_t) = \sum_{k=t}^T \frac{\tilde{c}_k^q}{\Pi_{j=t}^k(1+r_j)^{j-t}},$$

where $q \in \{m, f\}$. Parameters τ_k^q (typically < 1) presents how much utility loss is there in each period from being divorced. Furthermore assume linear sharing rule of property in case of divorce so that

$$\psi_t^q(A_t) = \begin{cases} \alpha_t A_t, & \text{if } q = m \\ (1-\alpha_t)A_t, & \text{if } q = f \end{cases}.$$

Assumption 5. (Identical discount factor and CRRA parameter) Let $\theta^m = \theta^f \equiv \theta$ and $\beta^m = \beta^f \equiv \beta$.

Theorem 3 Let assumptions 2-5 hold. The inability to commit across periods implies higher wealth holdings in every period after initial period (more savings) if $\theta > 1$. For $\theta = 1$ (log utility) the level of savings is unaffected by the inability to commit. For $\theta < 1$ the inability to commit decreases savings in every period.

Proof: In the appendix.

Theorem 3 provides a special case where inability to commit across time-periods acts analogously as decrease of the return on the asset. Thus, when elasticity of intertemporal substitution is higher than 1 (i.e. $\theta < 1$) the inability to commit decreases savings. When elasticity of intertemporal

substitution is less than 1 (i.e. $\theta > 1$) the inability to commit increases savings.

How good a guide is Theorem 3 for our intuition about the effects of inability to commit to savings level in more general cases? The following is an artificial counterexample to show that changing just one of the assumptions can turn this conclusion around.

Example 3 (Property rights regimes matter for the savings level). Let both spouses have time separable log utilities with no discounting. Let the lifetime wealth of the couple be 4 and let the interest rate be zero. Let the decision be made through repeated Nash-bargaining. Let the first period outside option be characterized like in assumption 3 (with $\tau_1^m = \tau_1^f = \tau_2^m = \tau_2^f = 0.8$). Let the wife have second period outside option equal to zero ($=\log(1)$, meaning that husband has to provide her with $1/0.8 = 1.25$ in the case of divorce in the second period) and hence the husband's second period outside option is to consume the rest of the assets after providing his wife's after divorce. In the solution, $c_1^m = 0.99$, $c_2^m = 0.96$, $c_1^f = 0.83$ and $c_2^f = 1.22$. Thus the wealth at the start of the second period is 2.18, which is larger than what solution with commitment would be ($= 2$, since the efficient solutions involve both consuming 1 in each time period).

Examples 4 and 5 use the models of this section to show that Theorem 1 holds only for consecutive periods even if Assumption 1 is satisfied.

Example 4. (Undersaving) Consider three times repeated Nash-bargaining problem as described in this section. Let $\theta^m = \theta^f = .5$, $\alpha^1 = .75, \alpha^2 = .65, \alpha^3 = .95$ and $\tau_1^m = \tau_1^f = \tau_2^m = \tau_2^f = \tau_3^m = \tau_3^f = .8$. Let the couple have life-time wealth of 6. In the solution, $c_1^m = 1.70$, $c_2^m = .90$, $c_3^m = 1.45$, $c_1^f = .43$, $c_2^f = 1.29$ and $c_3^f = 0.22$. Thus, an increase of consumption in the period 3 for both spouses and a decrease in period 1 consumption for both spouses would be Pareto improving.

Example 5. (Oversaving) Let $\theta^m = \theta^f = 2$, $\alpha^1 = .9, \alpha^2 = .8, \alpha^3 = .95$ and $\tau_1^m = \tau_1^f = \tau_2^m = \tau_2^f = \tau_3^m = \tau_3^f = .8$. Let the couple have life-time wealth of 6. In the solution, $c_1^m = 1.68$, $c_2^m = 1.11$, $c_3^m = 2.11$, $c_1^f = .20$, $c_2^f = .65$ and $c_3^f = 0.24$.

Example 6 below makes a point that is rather general to the class of models considered in this paper: an increase of one partner's (say wife's) outside

16

option in the future can be bad for both partners by making pre-existing dynamic distortions worse. So while in general redistribution towards wife should be good for her this efficiency effect of redistribution effect can dominate the positive future redistribution effect under some circumstances. This observation has an application to changes in pension legislation: say that the rights of non-working spouses (say wives') on their husband's pension are enhanced (like in the Retirement Equity Act of 1984 analyzed in Aura 2001) and this legislation becomes as an unexpected shock. The effects on the couples who are at the "retirement" period are straightforward: the wives benefits on their husbands expense. However, the effect of such a legislation can be welfare deteriorating on young couples: it is possible that for them, both are worse off because this future redistribution makes the pre-existing distortions worse. The opposite might also hold for young couples, the legislation might decrease the pre-existing distortions and make both spouses better off.

Example 6. (Future redistribution can be welfare decreasing for both spouses) Consider twice repeated Nash-bargaining model as in this section, with no discounting and log-utilities. Let $\tau_1^m = \tau_1^f = \tau_2^m = \tau_2^f = .8$ and let the life-time wealth of the couple be 4. The table below characterizes the life-time utilities attained by prospective spouses under three different divorce property division arrangement:

Table 1. Life-time utilities of spouses under different property regimes.

α_1	α_2	U^m	U^f
0.5	0.5	0	0
0.5	0.4	-0.0694	0.0289
0.5	0.25	-0.2461	-0.0517

4   Divorce

This section continues to consider the case where divorce outcomes define the outside options of the spouses. The question asked in this section is whether the family of models considered here can give strong policy recommendations, i.e. whether changes in divorce laws could be used to restore first-best solutions. The answer to this shown to depend crucially on how one views divorce. If divorce is just an off-equilibrium path event, then the first-best solution can be easily restored. However, adding (even small) additional

risk of getting into the divorce state will destroy this conclusion. Perhaps surprisingly it is shown that in general even "divorce insurance"-products could not restore efficiency.

It is interesting to compare the results of this section to the real world existing divorce property division regimes. The two-assets model of this section can be viewed as stylized common-law property regime, where both spouses can own assets while married without them becoming jointly owned. A special case of single assets model is a community property regime where the property accumulated during marriage would be split 50-50 in case of divorce. Generally, community property regime is viewed as more progressive and more "pro-women" or "pro-weaker spouse" (Weitzman 1990). Dnes (1999) argues that a community property regime is likely to be more efficient, since it provides lesser incentives for costly litigation in case divorce than community property and since, at least in England, the discretion that judges have in common-law property regime creates excess uncertainty.[14]

The results of this section are not supportive of Dnes' conclusion (although, they do not consider the effects that Dnes emphasizes). Consider community property regime through the following description of marriage. In the beginning of marriage, the spouses both own some assets. These assets can be covered by prenuptial contract to assign permanent property rights on them. The assets accumulated during marriage[15] are divided 50-50 in case of divorce. Now if the assets are the only thing that matter for relative outside options (like in the examples in the previous sections) there is no reason to expect that the community property regime does not yield the first-best solution in the simple model that has no exogenous divorce risk. However, other factors also can affect the outside options and these factors can lead to a non-constant time path of relative outside options. These factors can include remarriage prospects, attachment to community outside the couple and human capital accumulation that are unlikely to be

---

[14] In the US community property states are: Arizona, California, Idaho, Louisiana, Nevada, New Mexico, Texas, Washington and Wisconsin. In addition, Puerto Rico is a community property jurisdiction. However, since 1970s, all of the common-law states (with the exception of Missisipi) have enacted so called "equitable division" clauses, that make their legislation be somewhere between the two extrememes considered here. In UK, Scotland has community property regime, while Wales and England have common-law regime (Dnes 1999). Many continental European countries have community property regime (Dnes 1999).

[15] In the US, depending on the State, the asset accumulation that is considered community property can include human capital components like acquired degrees (Weitzman 1990).

included (at least perfectly) in the community property valuation. Also, if the community property valuation is not perfectly forward-looking and the pre-marital assets are not divided equally in the prenuptial contract, then this potentially leads to a non-constant path of relative outside options even if the assets are sole factor affecting outside options. In contrast, Theorem 4 states that under a stylized common-law setting the couple can take care off these disparities themselves by trading future property rights to current consumption and attain first-best efficiency.

The point of Theorem 4 is not to say that common-law regime is necessarily more desirable than community property regime. Instead Theorem 4 is meant to highlight the possible efficiency-equity trade-off between these two regimes. Under the common-law regime the couple attains a point in the unrestricted life-time utility possibilities frontier. This point does not necessarily Pareto-dominate the solution under community property. Thus, by forcing the community property rules on the couple the government can possible attain distributional goals, but since this typically involves a departure from the first-best intertemporal and intrapersonal allocation of consumption within the couple, this possible equity gain comes with an efficiency cost. Whether a common-law regime is more desirable than community property regime therefore depends on the magnitudes of these gains and losses and how these gains and losses are weighted in the social objective.

4.1 No-equilibrium path divorce

Consider the environment of Theorem 1 with the following modifications:

Assume that there are more than one asset (without loss of generality, we can assume that there are only two assets), with possibly different rates of rates of return.[16] Assume that what drives the power in decision-making is the outcome for the spouses should they divorce. An obvious structural model having this implication is repeated Nash-bargaining with divorce as outside option.

The decision problem can now be written as:

---

[16] For simplicity, it is assumed that the rates of return are non-stochastic.

$$V_t^m(A_t^m, A_t^f) = \max_{c_t^m, c_t^f, A_{t+1}^m, A_{t+1}^m} u_t^m(c_t^m) + V_{t+1}^m(A_{t+1}^m, A_{t+1}^f)$$

$$\text{subject to } A_t^m + A_t^f = c_t^m + c_t^f + \frac{A_{t+1}^m}{(1 + r_t^m)} + \frac{A_{t+1}^f}{(1 + r_t^f)}$$

$$u_t^f(c_t^f) = V_t^f(A_t^m, A_t^f) - V_{t+1}^f(A_{t+1}^m, A_{t+1}^f),$$

where A_t^m (A_t^f respectively) represents the assets that has stronger effect on the husband's (wife's) utility in the future since he has larger marginal claim on this assets in case of divorce. As always in this paper, assume that there are no liquidity constraints. The first-order conditions characterizing the optimal choice are:

$$u_t^{m\prime} - \mu_t = 0$$

$$\lambda_t u_t^{f\prime} - \mu_t = 0$$

$$\frac{\partial V_{t+1}^m}{\partial A_{t+1}^m} - \frac{\mu_t}{1 + r_t^1} + \lambda_t \frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} = 0$$

$$\frac{\partial V_{t+1}^m}{\partial A_{t+1}^f} - \frac{\mu_t}{1 + r_t^2} + \lambda_t \frac{\partial V_{t+1}^f}{\partial A_{t+1}^f} = 0.$$

Now, make the following two assumptions.

Assumption 6. (Different marginal effects of assets on outcomes) There does not exists a pair of A_t^m, A_t^f for any t such that $\frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} = \frac{\partial V_{t+1}^f}{\partial A_{t+1}^f}$.

Assumption 7. (Equal rates of return) Let $r_t^m = r_t^f \equiv r_t$ for all t.

Assumption 6 can be justified in a case of divorce as outside option Nash-bargaining, where the marginal rights to assets in case of divorce are different, since while increase in each asset has similar effect on the budget set, they will have differing effect on the outside options in the next period.

Theorem 4. Let assumption 6 and 7 hold and let an interior optimum exits. The resulting outcome without commitment is fully efficient.

Proof: The first order conditions can be rearranged to yield:

$$\left(\frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} - \frac{\partial V_{t+1}^f}{\partial A_{t+1}^f} \right) (\lambda_t - \lambda_{t+1}) = 0.$$

Since this is true for all time-periods, this means that $\lambda_t = \lambda_{t'} \quad \forall t, t'$. This combined with the basic dynamic first-order conditions

$$u_{t+1}^{m\prime} - \frac{u_t^{m\prime}}{1 + r_t} = (\lambda_{t+1} - \lambda_t) \frac{\partial V_{t+1}^f}{\partial A_{t+1}^m}$$

$$u_{t+1}^{f\prime} - \frac{u_t^{f\prime}}{1 + r_t} = \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t} \right) \frac{\partial V_{t+1}^m}{\partial A_{t+1}^m},$$

will yield the fully efficient solution:

$$u_t^{m\prime} = (1 + r_t) u_{t+1}^{m\prime}$$

$$u_t^{f\prime} = (1 + r_t) u_{t+1}^{f\prime}. \blacksquare$$

The intuition for theorem 4 comes from incomplete markets analogy. With just one asset, the couple is constrained in the way it can transform current consumption to future consumption. With two assets and assumptions 6 and 7, they have two assets that span the whole space of required transactions.

Three major caveats are in order, before one thinks of Theorem 4 having strong public policy implications. While Theorem 4 seems to be saying that the efficient divorce laws would let the married couple (while still married) decide on who owns what portions of their net worth in case of divorce (by having "his" and "hers" accounts), this does not take into account that moving from a single-asset world (community property) to a stylized common-law regime would imply a redistribution of welfare within family. As is typical, a move from one inefficient regime to a Pareto-efficient regime is not necessarily a Pareto-improvement. Also, consideration of independent divorce risk will alter the conclusion of the Theorem 4.

Third major caveat to Theorem 4 is that unlike many of the theorems in this paper it does not extend straightforwardly to the case where there

is any asset return or other risk (like health shocks) that affects the V-functions. If this extension is taken into account, then one needs to consider the incomplete risk sharing aspect of lack of intertemporal commitment. This question and its implications on the desirability of common-law versus community property regime is left for further research.

An interesting point to note about this model (now for generality, assume that there can be more than two assets) is that the model is consistent with interior optimum without any constraints in cases where one assets return is dominated by some other assets return. This is obvious from the first-order conditions, if the property rights (i.e. the effects on the next period's value functions) are different, an interior optimum with positive holdings of dominated assets is a possibility.

Other interesting point is the interaction of the divorce law-regime and the labor supply decisions. Under pure common-law there is no reason to think that the labor supply will be distorted because of dynamic strategic reasons (that work through human capital accumulation and/or labor force attachment as explained earlier), since the assets can be used to accomplish right bargaining positions for future periods. This is not the case under common-law legislation and thus dynamic strategic effects of labor supply become relevant. An interesting test of the levance of the model would be to use data from the enactment of "equitable division" clauses in the US and see what kind of labor supply effects did these law changes have.

4.2   Independent risk of divorce

The results of theorem 7 changes if one adds little bit of empirical relevance to the model. In real life, divorces do occur. This section takes another simplistic view (ignoring the link of past actions to divorces) by assuming that divorce is a random event that strikes the couple with an exogenous probability. This could be called "suddenly the love died out" view of divorce: in the beginning of each period a random event (no divorce or divorce) is realized. After that the couple renegotiates their allocation (e.g. by Nash-bargaining) still taking into account the divorce outcomes as threat points in their decision making, since they always have the option to divorce if the negotiations breaks down.

Now, for the sake of argument, assume that there are three assets available to the couple: an asset that pays in the case of couple not divorcing

and divorce insurance accounts (which can have also negative balances) for both spouses. A divorce insurance is an insurance product, that will deliver income to each spouse in the case of divorce (regardless of whether divorce happens because of the exogenous shock or because of mutual welfare maximization).[17]

With divorce risk the decision problem can be written as:

$$V_t^m(A_t, I_t^m, I_t^f) = \max_{\substack{c_t^m, c_t^f, A_{t+1}, \\ I_{t+1}^m, I_{t+1}^f}} u_t^m(c_t^m) + (1-p)V_{t+1}^m(A_{t+1}, I_{t+1}^m, I_{t+1}^f) + p\tilde{V}_{t+1}^m(I_{t+1}^m)$$

$$\text{subject to } A_t = c_t^m + c_t^f + \frac{(1-p)A_{t+1}}{(1+r_t)} + \frac{p(I_{t+1}^m + I_{t+1}^f)}{(1+r_t)}$$

$$u_t^f(c_t^f) = V_t^f(A_t, I_t^m, I_t^f) - (1-p)V_{t+1}^f(A_{t+1}, I_{t+1}^m, I_{t+1}^f) - p\tilde{V}_{t+1}^f(I_{t+1}^f)$$

where $\tilde{V}_{t+1}^m$ and $\tilde{V}_{t+1}^f$ represent the indirect utilities of spouses in case of divorce and p is the probability of divorce.

Theorem 5. (Inefficiency theorem) With independent divorce risk, generically the decision will not be efficient.

Proof: Full efficiency requires that

$$u_t^{m\prime} = (1+r_t)u_{t+1}^{m\prime} = (1+r_t)\tilde{V}_{t+1}^{m\prime}$$
$$u_t^{f\prime} = (1+r_t)u_{t+1}^{f\prime} = (1+r_t)\tilde{V}_{t+1}^{f\prime}.$$

This means that full efficiency is a condition on 6 objectives (marginal utilities to be equated). The couple only has five choice variables to achieve this, so generically it cannot do this while satisfying the budget constraint.∎

The sense in which word "generically" is used in Theorem 5 is that starting from a model where the solution of the problem without commitment is fully efficient; by slightly perturbing the problem (e.g. by changing how the divorce threat points affects current bargaining) we will always find solutions that are not Pareto-efficient. Conversely, if we start with a solution

---

[17]The huge moral hazard and assymmetric information problems related to divorce insurance accounts are ignored in this section. The reason why divorce insurance is considered in this section is to illustrate even the perfect insurance markets would not restore full efficiency.

that is not fully efficient, a slight perturbation of the problem will not lead to a solution that is fully efficient.

The basic problem in the model with independent divorce risk is that the divorce insurance is used to do two things: to equate individual marginal utilities between current consumption and tomorrow's divorce state; and to equate the ratios of marginal utilities of spouses between today's consumption and tomorrow's consumption.

The point about divorce insurance in this section was not to be a realistic description of reality. Instead, the divorce insurance was considered to illustrate the following point. With divorce insurance, the couple has set of assets that span the future state-space. Since even with the complete spanning they cannot always reach Pareto-efficient allocation, they generally cannot do that with less complete assets selection.

5 Death

Besides sharing of the current consumption, one of the potential points of contention couples face is how much protection to provide for each spouse in the case of the death of the other spouse. Since providing survivor protection is costly, spouses (even if one allows for altruistic motives) can have differing views on the optimal level of survivor protection. This section illustrates that under complete markets the survivor protection is similar to private consumption. Thus, under the class of models considered here and with complete and perfect markets, there cannot be independent concern about the lack of survivor protection without concern for the economic circumstances of the individual spouses while both partners are alive.[18]

With perfect insurance markets, the decision problem becomes:

---

[18]This does not mean that the view that couples seem to choose insufficient amounts of survivor protection is irrational. The results here depend on several assumptions. The key assumption, that the couples have access to actuarially fair insurance, is especially questionable for a large part of the population.

$$V_t^m(A_t) = \max_{\substack{c_t^m, c_t^f, A_{t+1}, \\ I_{t+1}^m, I_{t+1}^f}} u_t^m(c_t^m) + (1 - p_{t+1}^m - p_{t+1}^f - p_{t+1}^{mf})V_{t+1}^m(A_{t+1}) + p_{t+1}^m \tilde{V}_{t+1}^m(I_{t+1}^m)$$

subject to

$$A_t = c_t^m + c_t^f + \frac{(1 - p_{t+1}^m - p_{t+1}^f - p_{t+1}^{mf})A_{t+1}}{(1 + r_t)} + \frac{p_{t+1}^m I_{t+1}^m}{(1 + r_t)} + \frac{p_{t+1}^f I_{t+1}^f}{(1 + r_t)}$$

$$u_t^f(c_t^f) = V_t^f(A_t) - (1 - p_{t+1}^m - p_{t+1}^f - p_{t+1}^{mf})V_{t+1}^f(A_{t+1}) - p_{t+1}^f \tilde{V}_{t+1}^f(I_{t+1}^f),$$

where p_{t+1}^f is the probability of the wife becoming a widow (and being alive), p_{t+1}^{mf} is the probability of both spouses dying before next period, I_t^f is life insurance protection protecting the wife and A_t is now an annuity that ceases to pay when one partner dies.[19] Note, that for simplicity, the altruistic motives are assumed away from the specification.[20]

Theorem 6. (Limited efficiency of survivor protection) In the optimum, neither spouse would like to trade his or her current period consumption for more survivor protection.

Proof: Follows trivially from the first order condition for the optimum.∎

The key intuition that drives the difference between mortality and divorce risk is that once spouses make their decisions about their consumption allocations at period $t+1$ the survivor protection that they had for that period $t + 1$ is just an sunk investment that did not pay off, while the divorce allocation can be seen as affecting the allocation (e.g. through threat points like in Nash-bargaining).

One of the key assumptions of the partial efficiency result is the ability to adjust the level of life-insurance protection and annuity holdings each period with actuarially fair pricing. A key feature in the real world of these

---

[19] The choice of A_t as an annuity product instead of regular savings account is just to save on notation. As always, the choice of which group of assets to characterize a completely spanned state-space allocation is completely irrelevant for substantive purposes.

[20] If altruistic motives for survivor protection were to be included, then also altruistic motives for current period consumption should be included for avoid biasing the model. With altruistic motives, the interpretation of results would always hinge on whether the altruistic motives between spouses are stronger when spouses are both alive or for survivor protection.

life-contingent insurance products is that the required transactions to do this are not necessarily available. Typically an annuity or life-insurance contract is a long-term contract that cannot be undone in the next period without non-trivial financial penalties.[21] Unfortunately, the analysis of longer term contracts is very complicated in the current setting because a purchase of long term contract implies non-negativity constraints on the future allocations (since the purchase of more life-insurance protection is a possibility in the future but the reverse transaction might not be available). Whether (a modified version of) Theorem 6 holds with long-term contracts is left as an open research question.

6   Conclusion

This paper provides starting-point for further research. The theoretical results identified in this paper could be taken to data to test the model of family decision-making presented here against more restrictive models (single-utility-function view of household) or more general models (non-cooperative models of household). The results could also easily be extended to consider risk more generally than is done in this paper.

The main conclusion of this paper is that at least in theoretical model of family decision-making the inability to commit across time matters for economic efficiency. The use of seemingly dominated assets can be explained as an attempt to overcome the problems related to incomplete commitment. Theorem 3 also provides an added justification why divorce as a phenomen can lead couples to save more than they would otherwise do. This justification has nothing to do with the traditional "saving for the rainy day" argument for increased savings (self-insuring against divorce). Instead, it is shown that the fact that the divorce threat-points affect the balance of power within the family while still married can lead to higher saving through an effect that is analogous to decrease in the return on the assets. In Theorem 4, it is shown that in a simple model a stylized common-law divorce property division regime is likely to lead to an efficient solution. This results can be viewed as an extension of Coase's Theorem: in absence of transaction cost

---

[21] A purchase of annuity is often done in relation of conversion of pension wealth to an annuity stream at retirement. This choice cannot often be undone. Adverse selection concerns are one obvious plausible reason why the perfect access period-to-period markets with actuarially fair pricing are not necessarily very accurate description of the reality for many households.

assigning property rights leads to a Pareto-efficient outcome. However, this results is shown to depend on the assumption of no exogenous divorce risks. Taking into account this caveat and also taking into account the possible differing distributional impact of different divorce regimes means that the superiority of common-law over community property regime in the theoretical model should be taken only as a tentative result. Further research on the optimal divorce property division regimes is clearly needed.

7 Appendix: Proofs of Selected Propositions

7.1 Proof of Lemma 0

Lemma 0. Any one of the following assumptions is sufficient for Assumption 1 to be satisfied:

a) Value functions are independent of wealth for one the spouses: $V_t^{f\prime} = 0$ $\forall t, A_t$ or $V_t^m = 0$ $\forall t, A_t$.

b) Two period world and wealth good for both in second period: let $T = 2$ and let the second period solution be characterized by a sharing rule for wealth: $c_2^m = \psi_2(A_2)$ and $c_2^f = A_2 - \psi_2(A_2)$. Furthermore let $0 \leq \psi_2\prime \leq 1$.

c) T-period world, with last period described as in b) and letting the series of functions $V_t^f(A_t)$ satisfy following conditions: $V_t^{f\prime} \geq 0, V_t^{f\prime\prime} \leq 0$ $\forall t, A_t$ and

$(1 + r_t)V_{t+1}^{f\prime}(A) \geq V_t^{f\prime}(A)$ $\forall t, A$.

d) Like c), but applying the restrictions on $V_t^m(A)$-functions.

e) CRRA-utilities and outside options in repeated Nash-bargaining: Let Assumptions 2-5 hold.

f) $V_t^{m\prime} \geq 0$ and $V_t^{m\prime} \leq u_t^{m\prime}$ $\forall t$ in the solution; or equivalently $V_t^{f\prime} \geq 0$ and $V_t^{f\prime} \leq u_t^{f\prime}$ $\forall t$ in the solution.

Proofs:

a) Direct consequence of envelope theorem and the first order conditions yielding $V_t^{m\prime} = u_t^{m\prime} - \frac{u_{t+1}^{m\prime}}{u_{t+1}^{f\prime}} V_t^{f\prime}$.

b) Under the assumptions, $V_2^m(A_2) = u_2^m(\psi_2(A_2))$ and $V_2^f(A_2) = u_2^f(A_2 - \psi_2(A_2))$. Therefore $V_2^{m\prime} \geq 0$ and $V_2^{f\prime} \geq 0$ iff $0 \leq \psi_2\prime \leq 1$.

c) By induction. Since last period is like period 2 in b) the claim holds for last period. Now, the first order condition of the problem yields $u_{t+1}^{m\prime} - \frac{u_t^{m\prime}}{1+r_t} = (\lambda_{t+1} - \lambda_t) V_{t+1}^{f\prime}$. Using the fact that by induction assumption $V_{t+1}^{f\prime} \leq u_{t+1}^{m\prime}$ this can be manipulated to yield $\frac{u_t^{f\prime}}{1+r_t} \geq V_{t+1}^{f\prime}$. Now using the assumption on the inequalities that V^f-functions will satisfy, concavity of V^f and the fact $A_{t+1} \leq A_t$ yields $u_t^{f\prime} \geq (1+r_t)V_{t+1}^{f\prime}(A_{t+1}) \geq V_t^{f\prime}(A_{t+1}) \geq V_t^{f\prime}(A_t)$. By the calculation done in a) this means that claim holds.

d) Same as c), except with roles of m and f reversed.

e) Follows from the positivity of γ_j and δ_j constants in Lemma A2.

f) Follows from the same calculation as a).

7.2 Proof of Theorem 3

Theorem 3 is proved in by first stating and proving two lemmas.

Lemma A1. Let assumptions 2-4 be satisfied. The Nash-Bargaining problem in period t can be written as:

$$\max_{c_t^m, c_t^f, A_{t+1}} \left(\left(\frac{1}{(1-\theta^m)} (c_t^m)^{1-\theta^m} + \sum_{k=t+1}^{T} (\beta^m)^{k-t}(\gamma_k A_{t+1})^{1-\theta^m} \right) - A_t^{1-\theta^m} \tilde{v}_t^m \right)$$
$$* \left(\frac{1}{(1-\theta^f)} \left((c_t^f)^{1-\theta^f} + \sum_{k=t+1}^{T} (\beta^f)^{k-t}(\delta_k A_{t+1})^{1-\theta^f} \right) - A_t^{1-\theta^f} \tilde{v}_t^f \right)$$

subject to $A_t = c_t^m + c_t^f + \frac{A_{t+1}}{(1+r_t)}$,

for some constants $\tilde{v}_t^m$ and $\tilde{v}_t^f$ and series of constants γ_k and δ_k.

Proof: Consider first the outside for the husband:

$$\tilde{V}_t^m(A_t) = \max_{\{\tilde{c}_i^m\}_{i=t}^T} \frac{1}{1-\theta^m} \sum_{k=t}^T (\tilde{\beta}^m)^{k-t} (\tau_k^m \tilde{c}_k^m)^{1-\theta_m}$$

$$\text{subject to } \alpha_t A_t = \sum_{k=t}^T \frac{\tilde{c}_k^m}{\Pi_{j=t}^k (1+r_j)^{j-t}}.$$

By rewriting the objective as

$$\tilde{V}_t^m(A_t) = \max_{\{\tilde{c}_l^m\}_{k=t}^T} A_t^{1-\theta_m} \frac{1}{1-\theta^m} \sum_{k=t}^T (\tilde{\beta}^m)^{k-t} (\tau_k^m \frac{\tilde{c}_k^m}{A_t})^{1-\theta_m}$$

it is easy to see that $\tilde{V}_t^m(A_t) = A_t^{1-\theta_m} \tilde{V}_t^m(\alpha_t)$. The outside option value function for the wife is handled similarly. The rest of the proof is a simple induction argument. Consider the Nash-bargaining problem in period T:

$$\max_{c_t^m, c_t^f} \left(\left(\frac{c_T^m}{(1-\theta^m)} \right)^{1-\theta^m} - A_T^{1-\theta_m} \tilde{v}_T^m \right) \left(\left(\frac{c_T^f}{(1-\theta^f)} \right)^{1-\theta^f} - A_T^{1-\theta^f} \tilde{v}_T^f \right)$$

subject to $A_T = c_T^m + c_T^f$.

By the similar homogeneity argument as in the case of the outside options, the optimum solution for $\{c_T^m, c_T^m\}$ is just a linear scaling of the optimal solution in the case where $A_T = 1$. The induction step, assuming that the claim holds for period $t+1$, and showing that the claim holds for period t follows using exactly the same homogeneity of the objective function argument as in the period T.

Lemma A2. Let the assumptions 2-4 hold. Consider maximization of the linear combination of life time utilities of spouses (with weight μ_t on wife's utility, this being one characterization of a Pareto-efficient solution subject to constraints):

$$W(A_t, \mu_t) = \max_{A_{t+1}, \{c_i^m, c_i^f\}_{i=t}^t} \frac{1}{1-\theta} \left(\sum_{k=t}^{T} \beta^{k-t} (c_k^m)^{1-\theta} + \mu_t \sum_{k=t}^{T} \beta^{k-t} (c_k^f)^{1-\theta} \right)$$

subject to $A_t = c_t^m + c_t^f + \dfrac{1}{1+r_t} A_{t+1}$

$$c_j^m = \gamma_j A_{t+1}$$
$$c_j^f = \delta_j A_{t+1},$$

where $j = t+1, ..., T$ and where the series of constants γ, δ satisfy the budget constraint for future periods $1 = \sum_{k=t+1}^{T} \left(\frac{\gamma_k + \delta_k}{\Pi_{j=t+1}^{k}(1+r_j)^{j-t-1}} \right)$. For $\theta > 1$ the savings A_{t+1} are higher than in the optimal unconstrained solution. For $\theta = 1$ (log utility) the savings A_{t+1} are always at the first best level. For $\theta < 1$ the savings A_{t+1} are lower than in the optimal unconstrained solution.

Proof: Substitute the constraints for future consumption into the objective and consider the first order condition:

$$c_t^m = \mu_t^{-1/\theta} c_t^f$$
$$c_t^m = (1+r_t)^{-1/\theta} A_{t+1} * \omega_t^{-1/\theta}$$

$$\text{where } \omega_t = \sum_{k=t+1}^{T} \beta^{k-t} \gamma_k^{1-\theta} + \mu_t \sum_{k=t}^{T} \beta^{k-t} \delta_k^{1-\theta}$$

The second part of the claim is immediate from above: in the case of $\theta = 1$ the choice of the coefficients does not affect the savings level.[22] Next consider the case where $\theta > 1$ and consider ω_t as a function of $(\gamma_k, \delta_k)_{k=t+1}^{T}$. Subject to the constraint $1 = \sum_{k=t+1}^{T} \left(\frac{\gamma_k + \delta_k}{\Pi_{j=t+1}^{k}(1+r_j)^{j-t-1}} \right)$, the function ω_t has an unique minimum (since it is a convex function) at the choice $(\gamma_k, \delta_k)_{k=t+1}^{t}$ that correspond to the unconstrained optimal solution of joint utility maximization. This fact, budget constraint and the first order conditions imply that for any feasible choice of $(\gamma_k, \delta_k)_{k=t+1}^{t}$ the wealth holdings in period $t+1$ will be higher than in the unconstrained optimum. For $\theta < 1$, similar reasoning will imply that the capital stock in period $t+1$ will be lower than in the unconstrained optimum (since ω_t now is a concave function).∎

---

[22] Naturally, a separate treatment of the log-utility would confirm this result.

Theorem 3 (from the main text): Let assumptions 2-5 hold. The inability to commit across periods implies higher wealth holdings in every period after initial period (more savings) if $\theta > 1$. For $\theta = 1$ (log utility) the level of savings is unaffected by the inability to commit. For $\theta < 1$ the inability to commit decreases savings in every period.

Proof: Application of Lemmas A1 and A2. By Lemma A1 the problem in any period can be written as Nash-Bargaining with future consumption allocation being a linear transformation of tomorrows wealth. Since any Nash-bargaining solution is Pareto-efficient (with respect to constraints) Lemma A2 applies here for some μ_t. To prove the theorem, it now suffices to notice that under the assumption of equal discount rates and θ-parameters for spouses, any fully Pareto-efficient solution will imply same levels of wealth holdings, so the fact that the fully efficient (with commitment) solution corresponds to a (potentially) different welfare μ_t becomes irrelevant. ∎

References

[1] Attanasio, Orazio P. and José-Víctor Ríos-Rull (2000): Consumption Smoothing and Extended Families. Unpublished mimeograph, University College London.

[2] Aura, Saku (2001): Does the Balance of Power with a Family Matter: the Case of the Retirement Equity Act. IGIER Working Paper no. 202.

[3] Becker, Gary S. (1973): A Theory of Marriage: Part I. *Journal of Political Economy*, Volume 81, Issue 4.

[4] Browning, Martin (2000): The Saving Behaviour of a Two-person Household. *Scandinavian Journal of Economics*, Volume 102, Issue 2.

[5] Chiappori, Pierre-André (1988): Rational Household Labor Supply. *Econometrica*, Volume 56, Issue 1.

[6] Cubeddu, Luis and José-Víctor Ríos-Rull (1997): Marital Risk and Capital Accumulation. Federal Reserve Bank of Minneapolis.

[7] Dnes, Antony W. (1999): Applications of economic Analysis to Marital Law: Concerning a Proposal to Reform the Discretionary Approach to the Division of Marital Assets in England and Wales. *International Review of Law and Economics*, Volume 19.

[8] Duflo, Esther (2000): Grandmothers and Granddaughters: Old Age Pension and Intra-household Allocation in South Africa. Unpublished mimeograph, Massachusetts Institute of Technology.

[9] Ligon, Ethan, Jonathan P. Thomas and Tim Worral (2000): Mutual Insurance, Individual Savings, and Limited Commitment. *Review of Economic Dynamics*, Volume 3.

[10] Lundberg, Shelly, Robert A. Pollak, and Terence J. Wales (1996): Do Husbands and Wives Pool Their Resources? Evidence from the United Kingdom Child Benefit. *Journal of Human Resources*, Volume 32, Issue 4.

[11] Lundberg, Shelly, Richard Startz and Steven Stillman (2001): The Retirement-Consumption Puzzle: A Marital Bargaining Approach. Unpublished mimeograph, University of Washington.

[12] Lundberg, Shelly & Jennifer Ward-Batts (2002): Saving for Retirement: Household Bargaining and Household Net Worth. Journal of Public Economics, forthcoming.

[13] Manser, Marilyn and Murray Brown (1980): Marriage and Household decision making: A Bargaining Analysis. *International Economic Review*, Volume 21, No. 1.

[14] Mazzocco, Maurizio (2000): Household Intertemporal Behavior: a Collective Characterization and Empirical Tests. Unpublished Mimeograph, University of Chicago.

[15] McElroy, Marjorie B. and Mary Jean Horney (1981): Nash-bargained Household Decisions: Toward a Generalization of the Theory of Demand. *International Economic Review*, Volume 22, No. 2.

[16] Weitzman, Lenore J. (1992): Marital Property: Its Transformation and Division in the United States. In Weitzman and Mclean (eds.): *Economic Consequences of Divorce - The International Perspective.* Clarendon Press, Oxford.

[17] Wells, Robin and Maria Maher (1998): Time and Surplus Allocation within Marriage. Unpublished mimeograph, Massachusetts Institute of Technology.