Access Price and Vertical Control Policies for a Vertically Integrated Upstream Monopolist when Sabotage is Costly

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Abstract

Input price and novel vertical control regulations are derived for a vertically integrated upstream monopolist when the monopolist can engage in non-price discrimination against a downstream rival. The paper extends the literature on sabotage in network industries by characterizing welfare-optimal regulatory policy with a realistic set of policy tools when sabotage can be undertaken, at a cost, by a monopoly access provider who also competes in a differentiated products Bertrand retail duopoly. Welfare optimal regulation balances the conflicting goals of reducing non-price discrimination and stimulating efficient production levels downstream. The regulator can induce the first best in limited cases when the downstream rival is efficient relative to the downstream affiliate of the monopoly access provider, non-price discrimination by the integrated monopolist is sufficiently costly and downstream competition is not too intense. Otherwise, the regulator faces a trade-off between reducing the double markup problem by pricing access low while imposing restrictions on the control the monopoly input provider can exercise over its retail affiliate, versus pricing access high and allowing unrestricted vertical control within the vertically integrated firm in order to deter non-price discrimination. Discrimination costs and competition intensity determine which policy is optimal.

Keywords: Regulation, Vertical integration, Non-price discrimination, Sabotage.

JEL Codes: L42, L43, L51.

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Many network industries have an upstream monopolist that sells an essential input to downstream firms. Examples include electricity markets where a firm owning transmission lines sells access to generating companies, telecommunications where a local operator sells access to a competitive long distance market or to other communications providers such as wireless firms or providers of high speed internet services,[1] cable telephony where a local cable monopolist sells access to firms providing long distance and local voice services,[2] and software markets where Microsoft provides a key resource, the Windows operating system, while also competing with other firms for other software components (media players, for example) that seek compatibility with Windows.

The access offered to downstream firms by an upstream monopolist consists of two parts, an access charge and quality of access. Often the upstream monopolist is regulated in terms of the access charge while the downstream market is deemed sufficiently competitive and is therefore unregulated. There is a large literature on access charges and the regulation thereof to optimize social welfare (Armstrong et al. 1996; Laffont and Tirole 1990 and 1994; Bustos and Galetovic 2007; Reiffen 1998; Vickers 1995; Weisman 1995 and 1998; Weisman and Kang 2001). In an unregulated market, quality of access probably would not be a concern as the upstream monopolist could extract monopoly profits via access charges. Even with regulated access charges an unintegrated upstream monopolist still would probably not want to degrade access quality since doing so may be costly and does not generate additional revenues for the monopolist.

However, access price regulation may create incentives to degrade the quality of access when the upstream monopolist is affiliated with one of the downstream competitors and thus receiving profits from retail markets. Absent the ability to extract monopoly profits via access pricing, the upstream monopolist may degrade the quality of access to its downstream rivals to increase its own retail profits and possibly foreclosure its rivals. This type of non-price

[1] Digital Subscriber Line (DSL) is a family of technologies that provide digital data transmission over the wires of a local telephone network. See http://en.wikipedia.org/wiki/DSL.

[2] Voice Over Internet Protocol (VOIP) is a recent technology, where voice is carried through the internet. See http://en.wikipedia.org/wiki/Voip.

discrimination by an upstream monopolist has been labeled "sabotage" in the literature. Sabotage can take various forms such as favoring the affiliated firm's customers over the rivals' customers in terms of voice quality or bandwidth they receive, making rival software incompatible, making the customer switching process from the affiliate's network to the rival network costly and lengthy, and increasing the technical support waiting times for rivals' consumers. Hence there may be a need for regulation of access quality along with regulation of access prices.

The literature on sabotage has focused on theoretical possibilities of when access quality degradation may or may not occur. The upstream monopolist's incentives to sabotage vary as relative efficiencies of the downstream affiliate and rival change as well as with the degree of competition. Although it is well-established that it may be possible to manipulate regulated access charges in order to decrease sabotage and thereby enhance overall welfare (Ordover, Sykes, and Willig 1985; Economides 1998; Sibley and Weisman (SW) 1998; Mandy 2000; Mandy and Sappington 2007), there have been no attempts to derive welfare-optimal regulatory policies for a vertically integrated upstream monopoly when the upstream monopolist can sabotage its downstream rivals.

The present paper fills this gap in the literature by bringing the study of optimal regulatory policies for a vertically integrated upstream monopolist to a setting that may involve sabotage. Existing regulatory policies have long acknowledged the tension between access price regulation designed to encourage efficiency downstream and the potential that such regulation may encourage sabotage. Policies intended to maximize welfare and deter sabotage in network industries include marginal cost pricing of access, restrictions on vertical control between the monopolist and its downstream subsidiary, nondiscrimination requirements, performance monitoring of input quality, penalties for violations of nondiscrimination requirements, and limitations on the tools employed by regulators.[3] "Vertical control" in

[3]All of these policies are present in contemporary telecommunications regulation. On May 31, 2000 the US Federal Communications Commission (FCC) adopted the proposal of the Coalition for Affordable Local and Long Distance Service (CALLS) to rapidly move access charges toward the marginal cost of supplying access (FCC, 2000). Section 272 of the Telecommunications Act of 1996 restricts the vertical control a Bell

this setting means the extent to which the upstream monopolist can align the objective of its downstream affiliate with the objective of the overall firm. Despite the explicit vertical control policies used in practice, formal modeling of vertical control as a policy parameter has not previously appeared in the literature.

The regulator in our model is endowed with limited but realistic policy tools. Regulation of the downstream market and regulatory price discrimination are not permitted. Instead, the regulator sets a nondiscriminatory uniform access price and a level of vertical control. Limited internal control is due to external restrictions imposed by the regulator on the organization of the firm, not because of partial ownership or agency problems. In this paper the control of nonprice discrimination to maximize welfare is the primary concern, so the Ramsey pricing aspects of covering fixed costs and the mechanism design issues associated with asymmetric information are both ignored. The access price and vertical control policies modeled here are representative of the tools used under the US Telecommunications Act of 1996 to regulate local network access for long-distance suppliers once a Bell company enters the long-distance market.[4]

We find that optimal access price and vertical control policies vary depending on the relative efficiency of the downstream suppliers and the intensity of downstream competition. The regulator may be able to achieve the first best welfare outcome even with limited regulatory tools if there is an efficient downstream rival selling a product that is a close or

company can exert over its long distance affiliate by requiring that the affiliate operate independently and maintain separate books from the Bell operating company; have separate officers, directors, and employees; have separate credit arrangements; and conduct all transactions with the parent company at arm's length. This section also requires that the Bell company not discriminate in the provision of services between its affiliate and other suppliers of long distance. To enforce nondiscrimination, the FCC has required substantial performance monitoring agreements and pre-specified fines for violations as a condition of a Bell company's entry into long-distance markets (for example, see FCC 1999). Regulatory tools are limited, however. Price discrimination and Baron-Myerson (1982) style incentive mechanisms are rarely used in practice, despite their established theoretical advantages.

[4]In the case of vertically integrated providers in electricity markets, the US Federal Energy Regulatory Commission (FERC) does not currently require "corporate" or "structural" unbundling of transmission and generation services. FERC Order No. 890 (Docket Nos. AD05-17-000 and RM05-25-000) only requires "functional" unbundling. That is, FERC requires that employees engaged in transmission functions operate separately from employees of energy affiliates and marketing affiliates. However, several members of the industry have urged FERC to take more decisive steps towards corporate unbundling (FERC 2007).

perfect substitute to the downstream affiliate's product. This is accomplished by pricing access below its marginal cost to overcome the "double margin" problem. The upstream monopolist does not engage in sabotage in this setting provided the relative efficiency of the rival is large enough and sabotage costs increase rapidly with the level of sabotage. In contrast, we show it is not possible to achieve the first best when the downstream affiliate of the access monopolist is an efficient producer.

We explicitly derive the optimal access price and vertical control policy for the important case of equally efficient downstream firms and a simple sabotage cost specification. If the access monopolist's sabotage cost increases relatively slowly with the level of sabotage then it is welfare-optimal to price access above marginal cost and allow full vertical control. On the other hand, if sabotage cost increases relatively rapidly with the level of sabotage then it is welfare-optimal to price access below marginal cost and restrict vertical control. The latter provides greater inducement to increase downstream output when the monopolist's cost of sabotage can be more heavily relied upon to control the level of sabotage.

Hence we find a complementarity between the access price and vertical control policy parameters. Full vertical control is optimal when a high access prices is needed for control of sabotage, whereas limitation on vertical control is optimal when exogenous sabotage costs are sufficiently high to make a low access price viable from a welfare perspective. It is never optimal in this setting to price access at marginal cost because then the level of vertical control is effectively eliminated as a policy tool, having no impact on welfare. Thus we cannot rationalize some current regulatory practices that aim to both price access at marginal cost and impose restrictions on vertical control.

Our results formally verify a conjecture that has appeared in the literature (see, for example, Laffont and Tirole 2000, Chapter 4), that it is sometimes optimal to price access above marginal cost in order to deter sabotage. However, in this case full vertical control provides the maximum inducement to increase downstream output, so our model refutes the notion that restrictions on vertical control are generally part of an optimal sabotage-deterrence policy. In general, the welfare effects of vertical control depend on whether the upstream access margin is positive or negative.

Section 1 presents the model and Section 2 derives the downstream equilibrium. Section 3 identifies the situations in which the regulator can achieve the first best and the policies that do so. Then section 4 studies the second best when the downstream firms are equally efficient. Section 5 concludes.

1 The Model

The downstream market is a differentiated products Bertrand duopoly where the representative consumer has a symmetric quadratic utility function for the outputs of the affiliate $\left(q^d\right)$ and the rival (q^r):

$$U\left(q^d, q^r\right) = \alpha q^d + \alpha q^r - \frac{1}{2}\left[\beta\left(q^d\right)^2 + 2\gamma q^d q^r + \beta\left(q^r\right)^2\right] \tag{1}$$

for $\gamma \in [0, \beta]$. This utility function gives rise to the following linear inverse demand functions and the corresponding demand functions:

$$p^i\left(q^i, q^j\right) = \alpha - \beta q^i - \gamma q^j \tag{2}$$

for $i, j = d, r$ $(i \neq j)$ and

$$q^i\left(p^i, p^j\right) = \frac{1}{\beta}\left[\frac{\alpha}{1+\theta} - \frac{p^i}{1-\theta^2} + \frac{\theta p^j}{1-\theta^2}\right], \tag{3}$$

where $\theta = \frac{\gamma}{\beta} \in [0, 1)$.

The parameter θ reflects the degree of product homogeneity in this setting. When $\theta = 0$, the products of the affiliate and the rival are fully differentiated in the sense that the demand for each firm's product is not affected by the competitor's price or output level. When $\theta \to 1$, the products of the affiliate and the rival become homogeneous, because each firm's demand declines as its own price rises at the same rate that its demand increases as the price of its competitor rises.

We assume that marginal costs are constant but asymmetric, sabotage raises the marginal cost of the rival, and adopt the standard assumption in the literature on network industries that the downstream technology is fixed coefficients. Let $c^d, c^r \geq 0$ denote downstream marginal costs, a denote the regulated price of access, and $s \geq 0$ the degree of sabotage. Then the downstream profits are

$$\pi^r = [p^r - c^r - a - s] \, q^r \tag{4}$$

$$\pi^d = \left[p^d - c^d - a\right] q^d. \tag{5}$$

Let c^u denote the marginal cost of the upstream monopolist and $K(s)$ the cost to the upstream monopolist of engaging in s units of sabotage. We assume $K(0) = K^{'}(0) = 0$ and $K^{'}(s), K^{''}(s) > 0$ for $s > 0$, so the cost of the first increment of sabotage is zero but sabotage costs are strictly increasing and strictly convex in the degree of sabotage. In parts of the analysis we will assume K is sufficiently convex to ensure that second order conditions are satisfied. Then the upstream profit is

$$\pi^u = (a - c^u)\left(q^d + q^r\right) - K(s). \tag{6}$$

We assume throughout that any upstream viability constraint can be met with non-distorting transfers, so upstream fixed costs are ignored and the marginal cost of access may even exceed the access price (i.e., $a < c^u$ is a feasible regulatory policy).

As mentioned above a new innovation in our model is the explicit parameterization of vertical control as a policy tool. We use the vertical control parameter $\lambda \in [0, 1]$ to measure the degree of influence exercised by the upstream monopolist over the choices of its downstream affiliate. Letting IA denote "Integrated Affiliate" and IP denote "Integrated Parent," we specify their objectives as

$$\pi^{IP} = \pi^u + \pi^d \tag{7}$$

$$\pi^{IA} = (1 - \lambda)\pi^d + \lambda \pi^{IP} = \pi^d + \lambda \pi^u. \tag{8}$$

If $\lambda = 0$ (zero vertical control) the affiliate maximizes only its own profit π^d, while if $\lambda = 1$ (full vertical control) the affiliate maximizes the profit of its upstream parent. Note that this vertical control specification is not the usual agency approach to modeling separation of ownership and control. The IP has full information in our model, but is still unable to perfectly control the IA (when $\lambda < 1$) because of legal restrictions on the control mechanisms that can be used (for example, the restrictions that are imposed by Section 272 of the Telecommunications Act of 1996, and the various rules promulgated by the FCC to implement this legislation).

Aggregate welfare is the sum of consumer's surplus and profit. Taking the utility function (1) as monetarily valued gross consumer surplus, welfare is

$$\begin{aligned} W &= CS + \pi^u + \pi^d + \pi^r \\ &= U\left(q^d, q^r\right) - \left[c^r + c^u + s\right] q^r - \left[c^d + c^u\right] q^d - K(s). \end{aligned} \tag{9}$$

There is perfect and complete information in our model except for simultaneity in the downstream price choices. The timing of decisions is as follows. First the regulator chooses an access charge and control policy (a, λ) to maximize W. This is observed by all three firms. Next the IP chooses a level of sabotage s to maximize π^{IP}. This is observed by both downstream firms. Finally the downstream firms simultaneously choose prices (p^r, p^d) to maximize π^r and π^{IA}, respectively. The equilibrium concept is subgame perfection. We use backward induction to study the regulator's choice of welfare optimal access charge and vertical control given the outcomes in the downstream and upstream markets. This is the first formal modeling of vertical control as a regulatory policy and provides insight into the extent to which current regulatory practice is optimal.

2 Downstream Equilibrium

First rewrite IA's profit function as $\pi^{IA} = (p^d - c^d - a)q^d + \lambda[(a - c^u)(q^d + q^r) - K(s)]$. The first order conditions for an interior equilibrium are:

$$\frac{\partial \pi^r}{\partial p^r} = \frac{1}{\beta}\left[\frac{\alpha}{1+\theta} - \frac{2p^r}{1-\theta^2} + \frac{\theta p^d}{1-\theta^2} + \frac{\hat{c}^r}{1-\theta^2}\right] = 0 \tag{10}$$

$$\frac{\partial \pi^{IA}}{\partial p^d} = \frac{1}{\beta}\left[\frac{\alpha}{1+\theta} - \frac{2p^d}{1-\theta^2} + \frac{\theta p^r}{1-\theta^2} + \frac{\hat{c}^d}{1-\theta^2}\right] = 0, \tag{11}$$

where $\hat{c}^r = c^r + a + s$ and $\hat{c}^d(\theta) = c^d + a - \lambda(a - c^u)(1-\theta)$ are "effective" marginal costs for the rival and affiliate, respectively. Note that the affiliate's effective marginal cost depends on θ and also when $a - c^u$ and λ are positive, and $\theta < 1$, the effective marginal cost of the affiliate is lower than marginal cost $(c^d + a)$, reflecting the fact that bigger output increases access revenues for the parent. The notation $\hat{c}^r$ and $\hat{c}^d$ is used throughout the paper to denote effective marginal cost. To ensure that we have an interesting problem, we assume throughout that α exceeds both effective marginal costs for all values of a and s under consideration.

Taking into account corner outcomes (i.e., price pairs for which one firm is foreclosed), the augmented demand functions are:

$$q^d(p^d|p^r) = \begin{cases} 0 & \text{if } p^d \geq \alpha(1-\theta) + \theta p^r \\ \frac{1}{\beta}\left[\frac{\alpha}{1+\theta} - \frac{p^d}{1-\theta^2} + \frac{\theta p^r}{1-\theta^2}\right] & \text{if } \frac{p^r}{\theta} - \frac{\alpha(1-\theta)}{\theta} \leq p^d \leq \alpha(1-\theta) + \theta p^r \\ \frac{\alpha}{\beta} - \frac{p^d}{\beta} & \text{if } p^d \leq \frac{p^r}{\theta} - \frac{\alpha(1-\theta)}{\theta} \end{cases} \qquad (12)$$

and

$$q^r(p^d|p^r) = \begin{cases} 0 & \text{if } p^r \geq \alpha(1-\theta) + \theta p^d \\ \frac{1}{\beta}\left[\frac{\alpha}{1+\theta} - \frac{p^r}{1-\theta^2} + \frac{\theta p^d}{1-\theta^2}\right] & \text{if } \frac{p^d}{\theta} - \frac{\alpha(1-\theta)}{\theta} \leq p^r \leq \alpha(1-\theta) + \theta p^d \\ \frac{\alpha}{\beta} - \frac{p^r}{\beta} & \text{if } p^r \leq \frac{p^d}{\theta} - \frac{\alpha(1-\theta)}{\theta} \end{cases} \qquad (13)$$

Again taking into account corner outcomes the reaction functions are:

$$p^d(p^r) = \begin{cases} \alpha(1-\theta) + \theta p^r & \text{if } p^r \leq \frac{\hat{c}^d - \alpha(1-\theta)}{\theta} \\ \frac{\alpha(1-\theta) + \theta p^r + \hat{c}^d}{2} & \text{if } \frac{\hat{c}^d - \alpha(1-\theta)}{\theta} \leq p^r \leq \frac{\alpha(1-\theta)(2+\theta) + \theta\hat{c}^d}{2-\theta^2} \\ \frac{p^r}{\theta} - \frac{\alpha(1-\theta)}{\theta} & \text{if } \frac{\alpha(1-\theta)(2+\theta) + \theta\hat{c}^d}{2-\theta^2} \leq p^r \leq \alpha(1-\theta) + \frac{(\alpha + \tilde{c}^d)\theta}{2} \\ \frac{\alpha + \tilde{c}^d}{2} & \text{if } p^r \geq \alpha(1-\theta) + \frac{(\alpha + \tilde{c}^d)\theta}{2} \end{cases} \qquad (14)$$

$$p^r(p^d) = \begin{cases} \alpha(1-\theta) + \theta p^d & \text{if } p^d \leq \dfrac{\hat{c}^r - \alpha(1-\theta)}{\theta} \\ \dfrac{\alpha(1-\theta) + \theta p^d + \hat{c}^r}{2} & \text{if } \dfrac{\hat{c}^r - \alpha(1-\theta)}{\theta} \leq p^d \leq \dfrac{\alpha(1-\theta)(2+\theta) + \theta\hat{c}^r}{2-\theta^2} \\ \dfrac{p^d}{\theta} - \dfrac{\alpha(1-\theta)}{\theta} & \text{if } \dfrac{\alpha(1-\theta)(2+\theta) + \theta\hat{c}^r}{2-\theta^2} \leq p^d \leq \alpha(1-\theta) + \dfrac{(\alpha + \tilde{c}^r)\theta}{2} \\ \dfrac{\alpha + \tilde{c}^r}{2} & \text{if } p^d \geq \alpha(1-\theta) + \dfrac{(\alpha + \tilde{c}^r)\theta}{2}, \end{cases} \tag{15}$$

where $\tilde{c}^d = c^d + a - \lambda(a - c^u)$ and $\tilde{c}^r = \hat{c}^r = c^r + a + s$ are marginal costs when the affiliate and the rival, respectively, use monopoly pricing (in which the affiliate's effective marginal cost does not include $(1-\theta)$ since the rival is foreclosed).

A typical reaction curve of the rival is pictured in Figure 1. The first part of the reaction curve $p^i(p^j) = \alpha(1-\theta) + \theta p^j$ is the region where firm i does not produce and firm j serves the whole market. Actually, firm i's reaction consists of the whole region $p^i(p^j) \geq \alpha(1-\theta) + \theta p^j$ pictured as the shaded area in Figure 1. The second part of the reaction curve $p^i(p^j) = \dfrac{\alpha(1-\theta) + \theta p^j + \hat{c}^i}{2}$ is the region where firms i and j produce and price according to the first order conditions. The third and fourth parts of this reaction curve, $p^i(p^j) = \dfrac{p^j}{\theta} - \dfrac{\alpha(1-\theta)}{\theta}$ and $p^i(p^j) = \dfrac{\alpha + \tilde{c}^i}{2}$ respectively, are the region where firm i forecloses; firm i uses monopoly pricing in the latter case.

Note that the first parts of the two reaction curves cannot intersect, because that would imply marginal costs are so high that both firms produce zero in equilibrium. Similarly, the third parts of the two reaction curves cannot intersect because that would imply both firms price in a way that they both foreclose each other. The possible intersections of these reaction curves give us three different types of equilibria which are examined in detail below. Readers need to keep in mind that these reaction curves are derived when $0 < \theta < 1$. Equilibria when θ takes on extreme values need special attention and are examined later on.

2.1 Interior Equilibrium

For $0 < \theta < 1$ equilibrium prices and quantities are derived from the first order conditions:

$$p^{\star i} = \frac{\alpha(1-\theta)(2+\theta) + \theta\hat{c}^j + 2\hat{c}^i}{4-\theta^2} \tag{16}$$

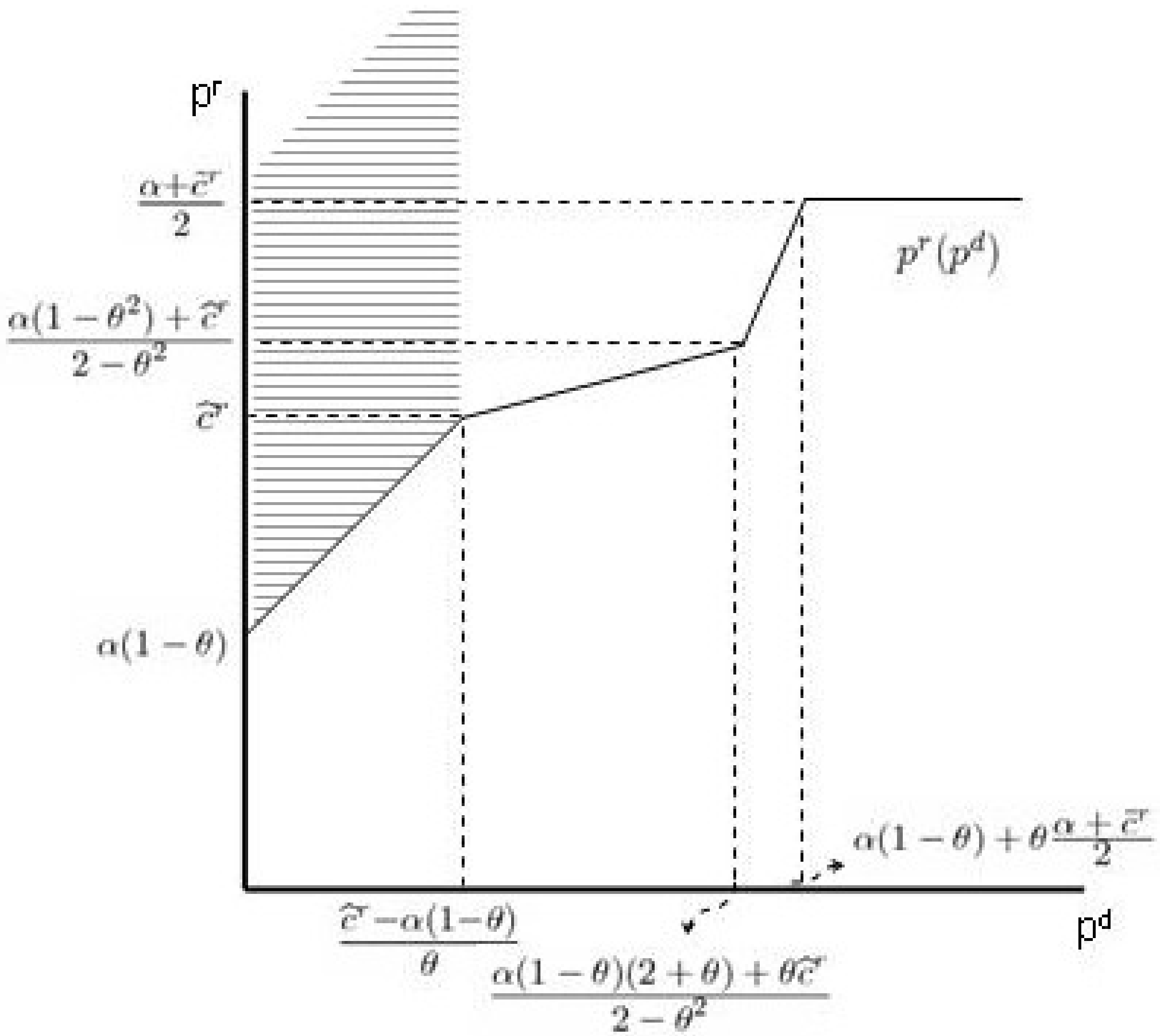

Figure 1: Rival's Reaction Curve.

and

$$q^{\star i} = \frac{\alpha(1-\theta)(2+\theta) - (2-\theta^2)\hat{c}^i + \theta\hat{c}^j}{\beta(4-\theta^2)(1-\theta^2)}, \tag{17}$$

where $i, j = r, d$ $(i \neq j)$. This corresponds to the region where the second portions of the reaction curves intersect. A typical interior equilibrium is pictured in Figure 2. However, for θ large enough and c^r and c^d sufficiently different from each other it is possible for either the rival or the affiliate to foreclose the market.

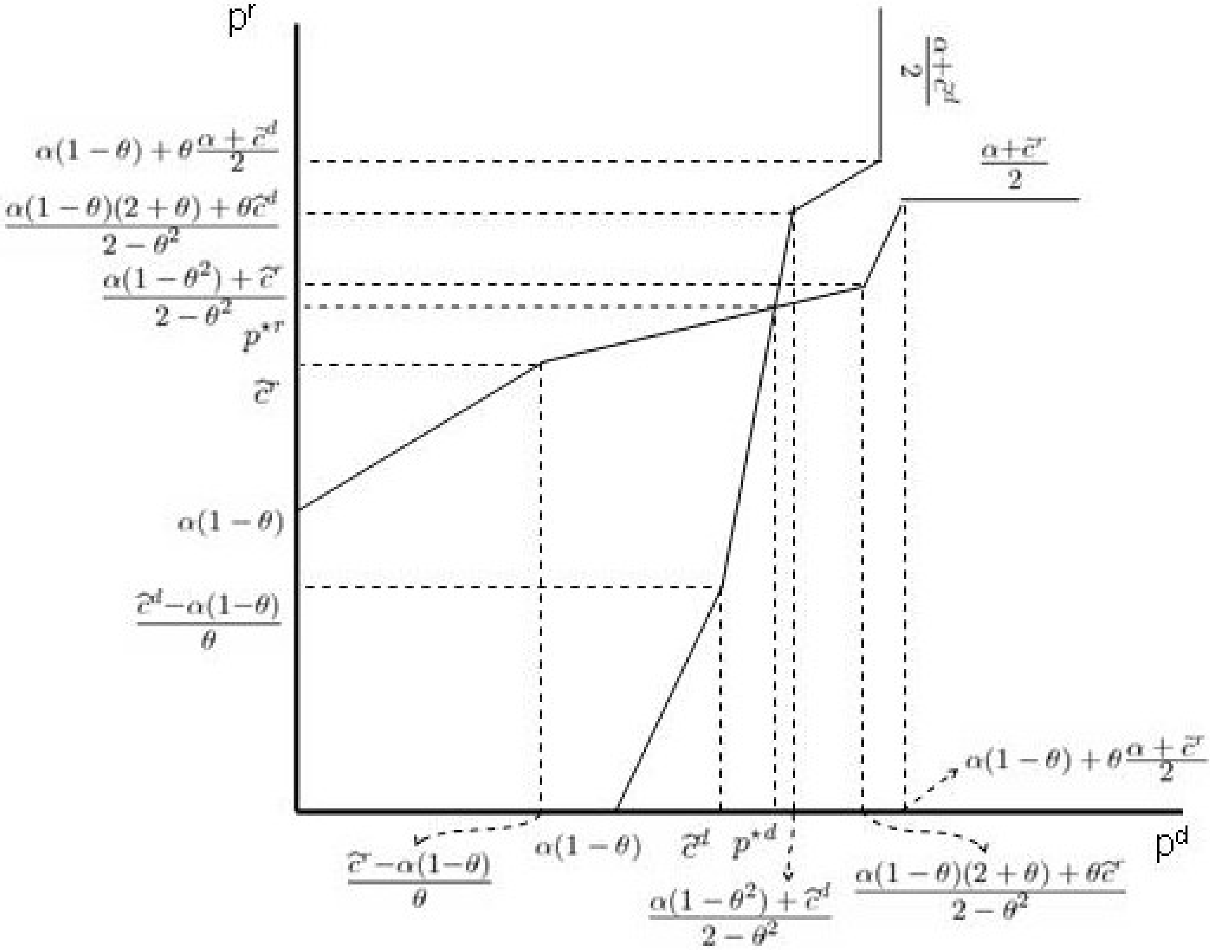

Figure 2: Interior Equilibrium.

2.2 Market Foreclosure (deterrence or blockade)

Suppose $0 < \theta < 1$. Then firm j serves the whole market if

$$\hat{c}^i \geq \frac{\alpha(1-\theta)(2+\theta)+\theta\hat{c}^j}{2-\theta^2}. \tag{18}$$

Given this, the low-cost firm j "deters"[5] by choosing

$$p^{\star j} = \frac{\hat{c}^i - \alpha(1-\theta)}{\theta}, \qquad q^{\star j} = \frac{\alpha - \hat{c}^i}{\gamma}, \tag{19}$$

and firm i prices at $\hat{c}^i$ when

$$\hat{c}^i < \alpha(1-\theta)+\theta\frac{\alpha+\tilde{c}^j}{2}. \tag{20}$$

[5]Even though market participation is not explicitly modeled as requiring incurrance of a sunk cost we borrow the "deterrence" and "blockade" terminology for ease of exposition.

The low-cost firm j "blockades" by choosing the monopoly point

$$p^{\star j} = \frac{\alpha + \tilde{c}^j}{2}, \qquad q^{\star j} = \frac{\alpha - \tilde{c}^j}{2\beta}, \tag{21}$$

and firm i prices at or above $\hat{c}^i$ when

$$\hat{c}^i \geq \alpha(1-\theta) + \theta\frac{\alpha + \tilde{c}^j}{2}. \tag{22}$$

Condition (18) is a threshold point between the interior and market foreclosure equilibria. The converse of (18) would imply that firm i always produces. Typical deterrence and blockade outcomes are pictured in Figure 3 and 4.

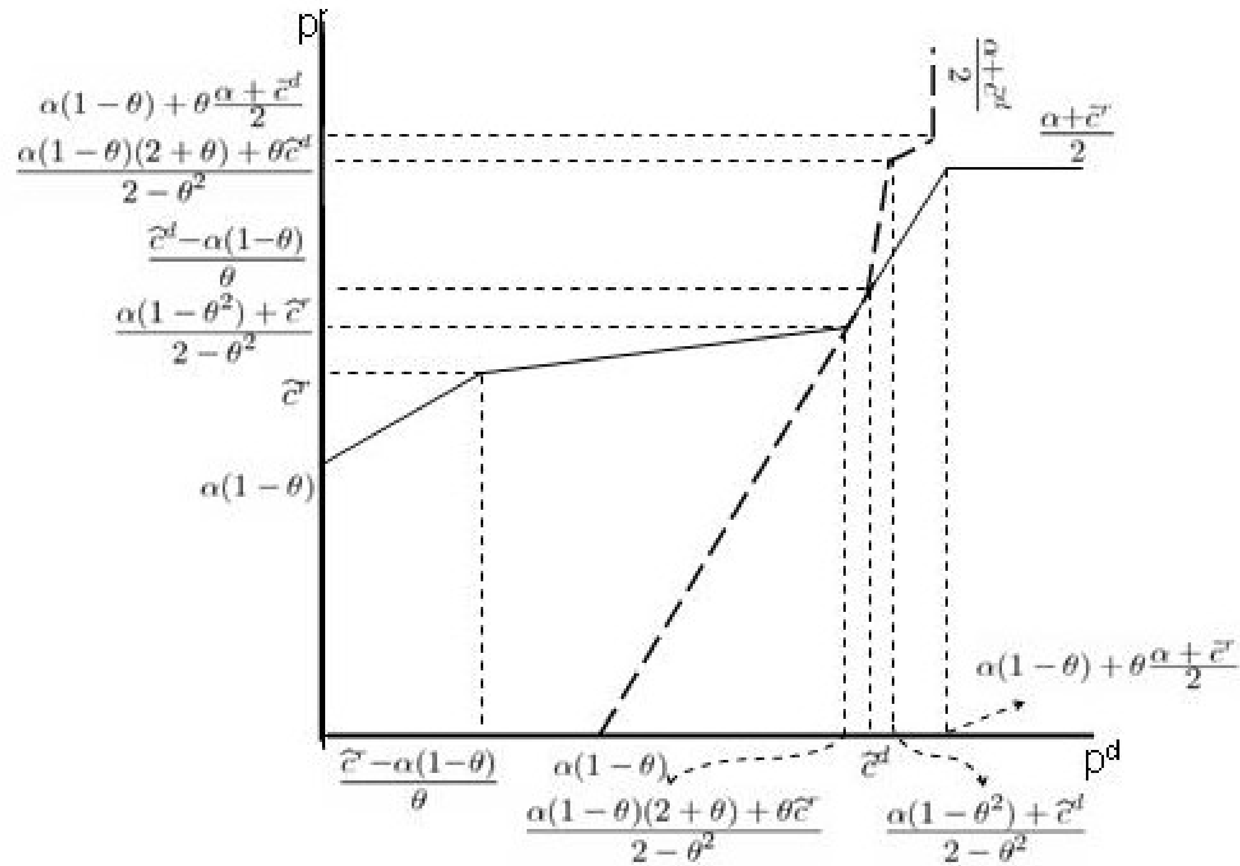

Figure 3: Deterrence by Firm r.

2.3 Local Monopolies

When $\theta = 0$ or $\theta = 1$ reaction curves (14) and (15) are not well-defined. When $\theta = 0$ the products are not substitutes and both the rival and the affiliate face the monopoly inverse

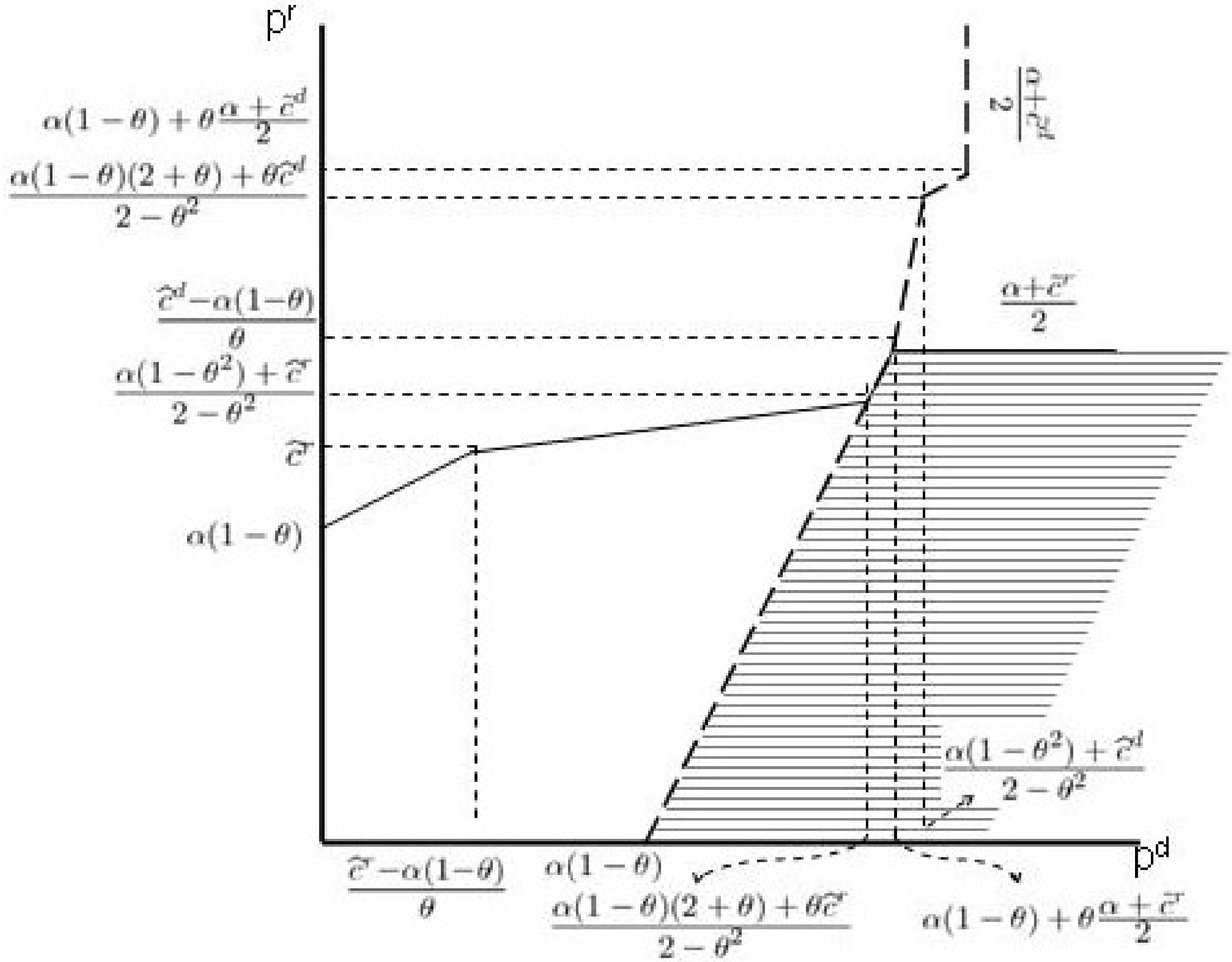

Figure 4: Blockade by Firm r.

demand function $p^i(q^i) = \alpha - \beta q^i$, with equilibrium

$$p^{\star i} = \frac{\alpha + \tilde{c}^i}{2}, \qquad q^{\star i} = \frac{\alpha - \tilde{c}^i}{2\beta}. \tag{23}$$

2.4 Pure Bertrand Competition

When $\theta = 1$ the products are perfect substitutes and a firm with a lower cost supplies the whole market. If $\hat{c}^i < \hat{c}^j$, then

$$q^{\star j} = 0, \qquad q^{\star i} = \max\left\{\frac{\alpha - \hat{c}^j}{\beta}, \frac{\alpha - \tilde{c}^i}{2\beta}\right\} \tag{24}$$

$$p^{\star j} = \hat{c}^j, \qquad p^{\star i} = \min\left\{\hat{c}^j, \frac{\alpha + \tilde{c}^i}{2}\right\}. \tag{25}$$

3 First Best

The regulator can achieve the first best in a limited number of cases, but usually the regulator faces a trade-off between sabotage deterrence and efficient production. This trade-off is explored in the later sections. Before doing so it is necessary to identify those cases in which the regulator is able to achieve the first best. We start by characterizing the first best.

The first observation is that the first best should be sabotage-free. Sabotage is a socially costly activity because it adds production costs sq^r for the rival and adds direct costs $K(s)$ for the IP without adding any gross surplus, as is evident from the aggregate welfare function (9). Recall that in the game tree the IP observes (a, λ) set by the regulator and then chooses the level of sabotage which is then followed by the pricing decisions of the affiliate and the rival. In some cases it is possible for the regulator to manipulate the downstream equilibrium to induce the IP to choose the zero level of sabotage while also inducing downstream firms to produce output levels that achieve productive efficiency.

Definition (derived in the Appendix). *Suppose* $c^i < c^j$ *for* $i, j = r, d$ $(i \neq j)$. *If* $\theta < \dfrac{\alpha - (c^j + c^u)}{\alpha - (c^i + c^u)}$ *then the first best is*

$$p^j = c^j + c^u, \quad q^j = \frac{\alpha(1-\theta) + \theta(c^i + c^u) - (c^j + c^u)}{\beta(1-\theta^2)} \tag{26}$$

$$p^i = c^i + c^u, \quad q^i = \frac{\alpha(1-\theta) + \theta(c^j + c^u) - (c^i + c^u)}{\beta(1-\theta^2)}. \tag{27}$$

If $\theta \geq \dfrac{\alpha - (c^j + c^u)}{\alpha - (c^i + c^u)}$ *then the first best is*

$$q^j = 0, \quad p^j \geq \alpha(1-\theta) + \theta(c^i + c^u) \tag{28}$$

$$p^i = c^i + c^u, \quad q^i = \frac{\alpha - (c^i + c^u)}{\beta}. \tag{29}$$

In all cases $s = 0$ *in the first best.*

Condition $\theta = \dfrac{\alpha - (c^j + c^u)}{\alpha - (c^i + c^u)}$ is a threshold point that defines whether the high-cost firm should produce in the first best. This ratio measures the size of the market for the low-cost firm relative to the size of the market for the high-cost firm. If θ is below this relative market size measure the regulator would like to have the high-cost firm supplying the market because the products are sufficiently different compared to the cost difference to justify offering both products; whereas for θ above this relative market size measure only the low-cost firm should supply as products sold by the two firms are close substitutes compared to the cost difference. That is why when $\theta = 1$ only the low-cost firm should supply and when $\theta = 0$ both firms should supply. Note that the threshold degenerates to $\theta = 1$ if marginal costs are the same, for then there is no tradeoff between productive efficiency and product variety.

The regulator's task is to choose (a, λ) to achieve productive efficiency downstream as described in the first best definition while also inducing the IP to engage in zero sabotage if possible. Differentiating (7) with respect to s yields:

$$\frac{\partial \pi^{IP}}{\partial s} = (a - c^u)\left(\frac{\partial q^r}{\partial s} + \frac{\partial q^d}{\partial s}\right) + (p^d - c^d - a)\frac{\partial q^d}{\partial s} + \frac{\partial p^d}{\partial s} q^d - K'(s). \tag{30}$$

It is assumed throughout that the convexity of K is sufficient to guarantee strict concavity of π^{IP} in s and that the foreclosure level of sabotage is not optimal. Since the first increment of sabotage is costless to the IP,

$$\left.\frac{\partial \pi^{IP}}{\partial s}\right|_{s=0} = (a - c^u)\left(\left.\frac{\partial q^r}{\partial s}\right|_{s=0} + \left.\frac{\partial q^d}{\partial s}\right|_{s=0}\right) + (p^d|_{s=0} - c^d - a)\left.\frac{\partial q^d}{\partial s}\right|_{s=0} + \left.\frac{\partial p^d}{\partial s}\right|_{s=0} q^d|_{s=0}. \tag{31}$$

To achieve the first best the regulator must choose (a, λ) such that (31) is nonpositive and the equilibrium prices and quantities are as specified in the definition of the first best. Each of the following propositions is proven by identifying which (a, λ) value(s) induce the right prices and quantities in equilibrium assuming $s = 0$, and then checking whether (31) is nonpositive at these value(s) of (a, λ).

Proposition 1. *Suppose $\theta = 0$. Then the first best is not achievable regardless of which firm is efficient.*

When $\theta = 0$ the two downstream firms are local monopolies and they use monopoly pricing as in (23). The regulator would like to achieve marginal cost pricing and thus sets the access charge below cost in order to offset the double margin, turning the access markup $(a - c^u)$

negative. The IP's profits consist of access revenues and affiliate's profits. The affiliate's price and quantity do not depend on s from (23). However, the IP is incurring an access deficit since access is priced below its cost and thus chooses to sabotage in order to reduce the rival's output and hence the access deficit.

Proposition 2. *Suppose $\theta = 1$. Then the first best is not achievable when the downstream affiliate is at least as efficient as the rival. Setting $a - c^u = c^r - c^d$ may achieve the first best when the rival is more efficient, depending on $K(s)$ and the rival's relative cost advantage (λ is irrelevant in this case).*

When $\theta = 1$ the two downstream firms are perfect competitors so the lowest cost firm serves the market. With equal marginal costs, both firms use marginal cost pricing and earn zero profits. In this case the regulator would like to set the access charge equal to its cost. At this access charge the IP receives no access revenues and no downstream profits. However an arbitrarily small level of sabotage will foreclose the rival, thereby enabling positive downstream profits for the affiliate, and is almost free to the IP since $K'(0) = 0$. Hence the IP will sabotage when the regulator tries to induce productive efficiency.

Essentially the same argument applies when the downstream affiliate is strictly more efficient than the rival. In this case the affiliate will foreclose the rival and will either match the rival's effective marginal cost or use monopoly pricing. The regulator can only achieve productive efficiency by choosing the access price to equate the rival's effective marginal cost (i.e., the affiliate's price, because this choice of access price drives the rival's effective marginal cost below the affiliate's monopoly price) with the rival's true cost c^r. However, the IP receives no access revenues since the rival is foreclosed, and can increase the rival's effective cost (and hence the affiliate's price and downstream profit) by engaging in sabotage. At least a small amount of sabotage is therefore profitable for the IP since the first increment of sabotage is costless.

The proof of the case when the rival is strictly more efficient is more complicated. With zero sabotage the downstream equilibrium will have the rival serving the whole market. This aligns with the regulator's objective, and the regulator can use the access charge to bring the rival's price to the socially efficient level. In this case the regulator would like to set the access charge below cost. So the parent's profits will only consist of access revenues from the rival (the affiliate is undercut), which are negative. A small amount of sabotage given the regulator's chosen access charge only decreases the IP's profits because the sabotage is costly and from (25) the rival prices at the affiliate's marginal cost which is independent of s. However, if sabotage exceeds the difference in marginal costs $c^d - c^r$ the rival is induced to exit the market and the affiliate takes over. Further increases in sabotage allow the affiliate to charge a higher price and earn higher downstream profits. At very high levels of s it might even be possible for the affiliate to use monopoly pricing. If the affiliate's marginal cost is sufficiently close to the rival's, then a small increase in s will tip the market in the affiliate's favor while sabotage costs are relatively low and thus the IP will engage in a finite amount of sabotage. However, if $K(s)$ sufficiently convex and rival's relative efficiency is large, the sabotage cost incurred when the IP tries to foreclose the rival is larger than the profit the IP can enjoy as the sole downstream producer, and thus the IP will not engage in sabotage. Hence the regulator can achieve the first best when $\theta = 1$ and $c^r < c^d$ provided $K(s)$ is sufficiently convex and the downstream cost difference is large enough to render unprofitable the finite increment of s necessary to foreclose the rival; otherwise the first best is not achievable.

Proposition 3. *Suppose $0 < \theta < 1$ and $c^r \geq c^d$. Then the first best is not achievable.*

If θ is small the first best requires that both firms produce with prices $p^r = c^r + c^u$ and $p^d = c^d + c^u$. It is straightforward to show that there is no value of (a, λ) with $\lambda \in [0, 1]$ that simultaneously equates the interior equilibrium prices to these costs unless $c^d = c^r$. The value of a that achieves this is below c^u when $c^d = c^r$, which creates an incentive for at least a small amount of sabotage, hence the first best is not achievable.

If θ is large the first best requires that the affiliate produces and the rival does not with the affiliate charging $p^d = c^d + c^u$. The downstream equilibrium entails the affiliate either blockading or deterring the rival. As shown in the Appendix, there is no feasible value of (a, λ) that achieves productive efficiency when the affiliate blockades. The regulator may be able to achieve productive efficiency when the affiliate deters. In this case the first best requires access to be priced below its cost. However, the parent can increase its downstream profits by engaging in sabotage and hence increasing the rival's effective marginal cost (i.e., the affiliate's price) and will do so since the first increment of sabotage is free.

Proposition 4. *Suppose $c^d > c^r$. If $0 < \theta < \sqrt[3]{\frac{\alpha - (c^d + c^u)}{\alpha - (c^r + c^u)}}$ then the first best is not achievable. If $\sqrt[3]{\frac{\alpha - (c^d + c^u)}{\alpha - (c^r + c^u)}} \leq \theta < 1$ then choosing (a, λ) to satisfy*

$$\lambda = \frac{c^d + a - \alpha(1 - \theta) - \theta(c^r + c^u)}{(a - c^u)(1 - \theta)}$$

may achieve the first best, depending on $K(s)$ and the rival's relative cost advantage.

When $\theta < \frac{\alpha - (c^d + c^u)}{\alpha - (c^r + c^u)}$ (this implies that $\theta < \sqrt[3]{\frac{\alpha - (c^d + c^u)}{\alpha - (c^r + c^u)}}$), the regulator would like to have both firms supplying the market and charging their respective marginal costs plus the cost of access according to the definition of the first best. This is achieved by setting the access charge below its cost. However, at the regulator's chosen (a, λ) the IP incurs an access deficit and thus would like to reduce it by increasing sabotage. In addition the IP receives higher downstream profits when it sabotages by allowing its affiliate to charge a higher price.

When $\theta > \frac{\alpha - (c^d + c^u)}{\alpha - (c^r + c^u)}$ the regulator would like the rival to serve the whole market charging its marginal cost plus the cost of access. There are feasible values of (a, λ) that accomplish this only when θ is high enough, specifically, when $\theta \geq \sqrt[3]{\frac{\alpha - (c^d + c^u)}{\alpha - (c^r + c^u)}}$ and the rival deters the affiliate by pricing at the affiliate's effective marginal cost. To achieve productive efficiency the regulator needs to set the access charge below its cost. In this case the parent's profits consist of only access revenues from the rival, which are negative.

The IP could reduce the access deficit by sabotaging the rival but the rival is matching the affiliate's marginal cost, deterring it from "entry" and is thus unresponsive to a small amount of sabotage. A "large enough" level of sabotage can reduce the rival's relative cost advantage so much that the affiliate starts producing downstream, in which case the parent's downstream profits as well as sabotage costs $K(s)$ increase. The level of sabotage required for the affiliate to "enter" the market depends on the relative cost difference. Therefore, if the rival holds a relatively small cost advantage and sabotage costs do not increase sharply, the IP will engage in sabotage; otherwise the IP will not engage in sabotage.

In summary, the regulator can achieve the first best when firms are sufficiently close substitutes (including the pure Bertrand case), provided the direct cost of the finite increment of sabotage that would either foreclose the rival or allow the affiliate to "enter" the market is sufficiently high. The remaining cases, when the regulator faces a real trade-off between sabotage and productive efficiency, are examined in the next section.

4 Second Best with Interior Downstream Equilibrium

Collectively, Propositions 1-4 give an exhaustive characterization of when the first best is and is not attainable, and identify the vertical control and access price policies that achieve the first best when it is attainable. What remains is to study the regulator's optimal policies when the first best is not attainable. From Propositions 1-4, there are three such situations:

1. When firms are local monopolies ($\theta = 0$).
2. When firms are equally efficient or the downstream affiliate is the efficient producer.
3. When products are not close substitutes and the rival firm is the efficient producer.

All three cases are quite complicated. Most interest centers on the case with differentiated but competing products and regulatory policies that are not driven by a presumption that one firm is more efficient than the other. Thus we assume $0 < \theta < 1$ and equally efficient firms ($c^r = c^d \equiv c$). We also assume that both the rival and affiliate firms produce positive quantities in downsteam equilibrium. Later we will derive conditions that guarantee participation by both firms. In this case the downstream equilibrium is given by (16) and (17).

Before looking at the regulator's problem in the first stage of the game tree, we must investigate the access monopolist's choice of sabotage given the downstream equilibrium determined by the last stage of the game. Setting (30) to zero defines an interior optimal sabotage choice $s^\star(a, \lambda)$. It has been assumed that the convexity of K is sufficient to satisfy the second order condition, so $\frac{\partial^2 \pi^{IP}}{\partial s^2} < 0$. Differentiating (30) and assuming the downstream equilibrium is given by (16) and (17) (interior equilibrium prices and quantities) yields the comparative statics of an interior optimum for s as:

$$\frac{\partial s^\star}{\partial a} = -\frac{(1-\theta)(-8(1+\theta)+\theta^3(1-\lambda))}{D(4-\theta^2)\partial^2\pi^{IP}/\partial s^2} \stackrel{s}{=} -8(1+\theta)+\theta^3(1-\lambda) < 0 \tag{32}$$

$$\frac{\partial s^\star}{\partial \lambda} = \frac{\theta^3(1-\theta)}{D(4-\theta^2)\partial^2\pi^{IP}/\partial s^2}(a-c^u) \stackrel{s}{=} -(a-c^u), \tag{33}$$

where $D = \beta(1-\theta^2)(4-\theta^2)$.

Proposition 5. *Suppose* $0 < \theta < 1$. *Then, at any interior equilibrium for the sabotage choice: (i) Sabotage is decreasing in the access price, (ii) Sabotage is increasing in vertical control when access is priced below cost and vice versa, and (iii) Vertical control has no effect on sabotage when access is priced at cost.*

Item (i) formalizes the intuition that the input monopolist's temptation to sabotage can be diminished by making upstream production more profitable. When access is priced below cost, an increase in vertical control increases the effective marginal cost of the downstream affiliate (see the definition of $\hat{c}^d$ following (11)), thereby shifting the equilibrium output mix toward the rival and increasing the IP's losses from unprofitable input sales, whence a stronger incentive to diminish the rival's output via sabotage. Item (iii) is at odds with some contemporary policies, for example in telecommunications, that attempt to price access at marginal cost while also attempting to deter non-price discrimination by imposing restrictions on vertical control.[6]

The regulator seeks to maximize the welfare function (9). Letting $c = c^d = c^r$ be the common downstream marginal cost, and differentiating (9) while accounting for the resulting changes in the optimal levels of sabotage and the downstream quantities, yields:

$$\frac{\partial W}{\partial a} = (p^d - c - c^u)\frac{\partial q^d}{\partial a} + (p^r - c - c^u)\frac{\partial q^r}{\partial a} - s\frac{\partial q^r}{\partial a} + \frac{\partial s}{\partial a}\Psi \tag{34}$$

$$\frac{\partial W}{\partial \lambda} = (p^d - c - c^u)\frac{\partial q^d}{\partial \lambda} + (p^r - c - c^u)\frac{\partial q^r}{\partial \lambda} - s\frac{\partial q^r}{\partial \lambda} + \frac{\partial s}{\partial \lambda}\Psi, \tag{35}$$

where

$$\Psi = (p^d - c - c^u)\frac{\partial q^d}{\partial s} + (p^r - c - c^u)\frac{\partial q^r}{\partial s} - s\frac{\partial q^r}{\partial s} - q^r - K'(s) \tag{36}$$

is a multiplier that reflects the effect on welfare of an increment in sabotage. As might be expected since sabotage is a socially costly activity:

Lemma. $\Psi \leq 0$.

The comparative statics of the downstream equilibrium quantities in (34) and (35) are:

$$\frac{\partial q^d}{\partial a} = \frac{-(1-\theta)(2+\theta-\lambda(2-\theta^2))}{D} < 0, \quad \frac{\partial q^r}{\partial a} = \frac{-(1-\theta)(2+\theta(1+\lambda)}{D} < 0 \tag{37}$$

$$\frac{\partial q^d}{\partial \lambda} = \frac{(1-\theta)(2-\theta^2)(a-c^u)}{D} \overset{s}{=} (a-c^u), \quad \frac{\partial q^r}{\partial \lambda} = \frac{-(1-\theta)\theta(a-c^u)}{D} \overset{s}{=} -(a-c^u) \tag{38}$$

$$\frac{\partial q^d}{\partial s} = \frac{\theta}{D} > 0, \quad \frac{\partial q^r}{\partial s} = \frac{-(2-\theta^2)}{D} < 0. \tag{39}$$

It is assumed hereinafter that W is strictly concave in a (this is assured in the special case of K considered below if K is sufficiently convex).

Equation (34) shows that there are two effects of sabotage that cause the regulator to depart from inducing efficient downstream prices $p^d = p^r = c + c^u$ when setting the access price. First, a decrease in the access price causes a sabotage-related increase in production costs of the rival when there is sabotage, in the amount $s\frac{\partial q^r}{\partial a}$. Second, a decrease in the access price causes an increase in sabotage that directly diminishes welfare by $\Psi\frac{\partial s}{\partial a}$. Hence

[6] See the CALLS proposal adopted in FCC (2000) and Section 272(b) of the US Telecommunications Act of 1996.

the last two terms in (34) are nonnegative. These extra costs cause the regulator to set the access price higher than would be optimal in the absence of sabotage, resulting in equilibrium downstream prices above the efficient level.

It is difficult to give more characteristics of the regulator's optimum without more assumptions. An explicit solution for the optimal policy can be obtained when the sabotage cost function takes the specific form $K(s) = ms^2$ for some cost parameter $m > 0$. For convenience and without loss of generality assume $m = \frac{\eta\theta^2}{D(4-\theta^2)}$ for some parameter η. The second order condition for s requires $\eta > 1$. With this specification, setting (30) to zero and substituting from the downstream equilibrium as needed yields

$$s^{\star} = \frac{2\theta(\alpha - c - c^u)(1-\theta)(2+\theta) - (a - c^u)(1-\theta)(8 + 8\theta - \theta^3(1-\lambda))}{2\theta^2(\eta - 1)}. \tag{40}$$

As one can see, optimal sabotage by the parent is positive if and only if $a - c^u < \frac{2\theta(2+\theta)(\alpha - c - c^u)}{8+8\theta-\theta^3(1-\lambda)}$. In some cases it might be optimal for the regulator to set the access charge so high that sabotage is eliminated, at the welfare cost of distorting the downstream equilibrium prices further above the efficient level.

When the presence and severity of sabotage induces the regulator to price access above cost, the regulator can lessen the impact of a high access charge on downstream prices by allowing full vertical control ($\lambda = 1$), which minimizes the affiliate's effective marginal cost $\hat{c}^d = c + a - \lambda(a - c^u)(1-\theta)$ given the positive access margin. Similarly, when access is priced below cost the regulator induces the maximum output-expanding effect of the low access charge by inducing the affiliate to ignore the effect expanded output has on the parent's profits, by fully restricting vertical control ($\lambda = 0$), which again minimizes the affiliate's effective marginal cost given the negative access margin. This ability to manipulate the affiliate via the vertical control parameter is lost when access is priced at marginal cost; if the regulator priced access at marginal cost, a finite improvement in welfare could be obtained by perturbing the access price a small amount (which has a small effect on welfare) and then moving vertical control in the welfare-improving direction.

These observations lead to a complete characterization of the welfare optimal regulatory policy. Setting (34) to zero, substituting for equilibrium levels of sabotage and downstream prices, and evaluating at $\lambda = 0$ and $\lambda = 1$ yields:

$$a^\star - c^u|_{\lambda=0} = 2(\alpha - c - c^u)(1-\theta)\theta \frac{n(0,\eta,\theta)}{d(0,\eta,\theta)} \tag{41}$$

$$a^\star - c^u|_{\lambda=1} = -(\alpha - c - c^u)(1-\theta)\theta(2+\theta)^2 \frac{n(1,\eta,\theta)}{d(1,\eta,\theta)}, \tag{42}$$

where $n(\lambda, \eta, \theta)$ and $d(\lambda, \eta, \theta)$ are polynomials in (λ, η, θ). Equations (41) and (42) give an explicit solution for the optimal access markup as a function of the sabotage cost parameter η and the degree of competition θ under each of the two possible equilibrium vertical control regimes, $\lambda = 0$ and $\lambda = 1$, respectively. The second order condition $\frac{\partial^2 W}{\partial a^2} < 0$ for an interior solution requires that the sabotage cost parameter η be large enough to ensure $d(\lambda, \eta, \theta) > 0$. Positive equilibrium quantities also require that η be sufficiently large. Let $\underline{\eta}$ be the lower bound that ensures second order conditions for $s^\star$ and $a^\star$, and positive equilibrium quantities. Then, for $\eta \geq \underline{\eta}$,

$$a^\star - c^u|_{\lambda=0} \stackrel{s}{=} n(0,\eta,\theta) \tag{43}$$

$$a^\star - c^u|_{\lambda=1} \stackrel{s}{=} -n(1,\eta,\theta). \tag{44}$$

Hence the aforementioned welfare-enhancing effects of regulating vertical control are obtained when $\lambda = 0$ only if $n(0, \eta, \theta)$ is negative. Similarly, $\lambda = 1$ is potentially optimal only if $n(1, \eta, \theta)$ is negative.

$n(\lambda, \eta, \theta)$ is quadratic in η and the roots of n always lies on opposite sides of $\underline{\eta}$. Moreover, $n(0, \eta, \theta)$ is strictly concave in η and $n(1, \eta, \theta)$ is strictly convex in η. Hence (41) is relevant only if η exceeds the larger root of $n(0, \eta, \theta)$ and (42) is relevant only if η lies between $\underline{\eta}$ and the larger root of $n(1, \eta, \theta)$. Let $\bar{\eta}(\lambda, \theta)$ denote the larger root of $n(\lambda, \eta, \theta)$. We have:

Proposition 6. *Suppose the downstream firms are equally efficient ($c^d = c^r = c$) and $0 < \theta < 1$. Suppose further that sabotage costs are given by $K(s) = \frac{\eta\theta^2}{D(4-\theta^2)} s^2$ for some $\eta > \underline{\eta}$. Then there exists a critical level of sabotage cost $\hat{\eta} \in (\bar{\eta}(0, \theta), \bar{\eta}(1, \theta))$ such that* *full vertical control is optimal ($\lambda^\star = 1$) and the optimal access margin is positive and given by (42) for $\eta \leq \hat{\eta}$; and complete restriction of vertical control is optimal ($\lambda^\star = 0$) and the optimal access margin is negative and given by (41) for $\eta \geq \hat{\eta}$. In particular, pricing access at marginal cost is never optimal.*

Hence, depending on the cost parameters and degree of competition, it may be optimal to price access below marginal cost and fully restrict vertical control, or it may be optimal to price access above marginal cost and allow full vertical control. The regulator would like to set $a^\star < c^u$ to induce output expansion, but this may create more sabotage than it is worth in welfare terms. Access must be priced above marginal cost in order to deter sabotage when sabotage is not very costly to the saboteur ($\eta \leq \hat{\eta}$). When sabotage is more costly ($\eta \geq \hat{\eta}$) the regulator can induce more efficient output levels by pricing access below marginal cost. If sabotage considerations cause the regulator to price access above marginal cost then it is optimal to allow full vertical control because this places the most weight on c^u rather than $a^\star$ in the IA's objective, thereby minimizing effective downstream costs. Alternatively, if sabotage costs are high then the output-expanding benefit of pricing access below marginal cost exceeds the welfare cost of sabotage. In this case it is optimal to fully restrict vertical control because this places the most weight on $a^\star$ rather than c^u in the IA's objective, again minimizing effective downstream costs and inducing the maximal downstream output for the particular access price chosen. It is never optimal in this setting to price access at marginal cost. Doing so neutralizes the regulator's ability to induce output expansion through manipulation of vertical control.

The foregoing discussion included no explicit consideration of whether the optimal choice of sabotage is positive or is the corner solution at zero. One might expect that the equilibrium level of sabotage decreases as sabotage costs increase and that sabotage reaches the corner solution at zero for very high levels of sabotage cost. Interestingly, this is not true in general. To understand when a corner solution for sabotage occurs, we must decompose the total change in sabotage into the partial effect of a change in the access charge a and the partial (direct) effect of a change in sabotage cost η:

$$\frac{ds^\star}{d\eta} = \frac{\partial s^\star}{\partial a^\star}\frac{\partial a^\star}{\partial \eta} + \frac{\partial s^\star}{\partial \eta}. \tag{45}$$

From (40), $\frac{\partial s^\star}{\partial a^\star}$ and $\frac{\partial s^\star}{\partial \eta}$ are unambiguously negative (when $s^\star$ is positive). However, $\frac{\partial a^\star}{\partial \eta}$ is negative at least for some values of η (but it can be be shown by example that it is not always negative). Ambiguity in the response of sabotage to an increase in sabotage cost arises because the regulator may decrease the access price fast enough to offset the direct effect ($\frac{\partial s^\star}{\partial \eta}$) of the cost increase. For low levels of sabotage cost the regulator resorts to high access charges as a way of curtailing sabotage. The regulator may set the access charge high enough so that it completely eliminates sabotage even though sabotage cost is very low.

This can be directly seen from equation (40). It is clear from (40) that sabotage is positive when the access margin is negative (recall $\underline{\eta} \geq 1$). Hence the $\lambda^\star = 0$ regime always has positive sabotage in equilibrium. When $\lambda^\star = 1$, $a^\star - c^u$ from (42) can be substituted into (40) to obtain

$$s^\star \stackrel{s}{=} k(\eta, \theta), \tag{46}$$

where $k(\eta, \theta)$ is linear and increasing in η. Therefore starting from the point where sabotage cost is very high, the regulator prices access below cost and the IP chooses a positive level of sabotage. As sabotage cost falls, the regulator might price access sufficiently above cost to ensure that the IP chooses zero sabotage. This threshold value of η at which sabotage becomes zero, denoted by $\tilde{\eta}(\theta)$, varies with θ. It can be shown by numerical examples that, depending on the magnitude of θ, the IP might engage in zero sabotage whenever access is priced above cost (Figure 5), sabotage might be positive despite a positive access margin when the sabotage cost parameter takes on medium values and then turn zero when sabotage becomes very cheap (Figure 6), or sabotage might be always positive (Figure 7). In any case, $s^\star$ is always increasing at $\hat{\eta}$ because, from (40), $s^\star$ is strictly larger when the access margin is negative than when it is positive.[7]

[7] All three of these examples are for general demand and marginal cost paremeters $\alpha - c - c^u$ and β. Results shown in Figures 5-7 only depend on the (η, θ) parameter space.

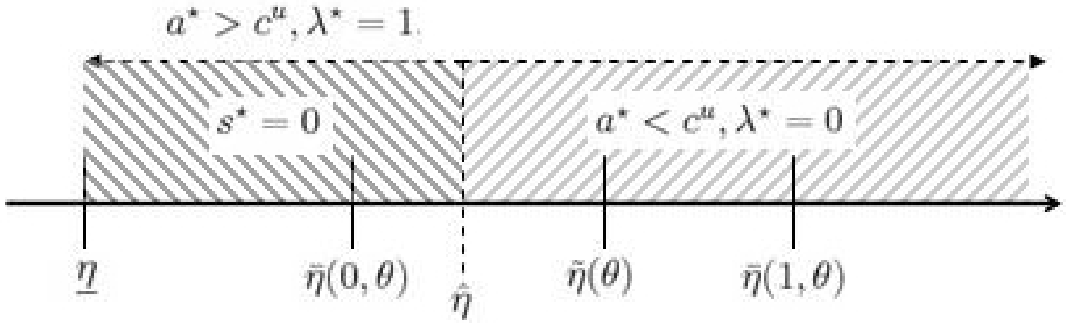

Figure 5: Optimal Access Charge, Vertical Control and Sabotage when $\theta = 0.2$.

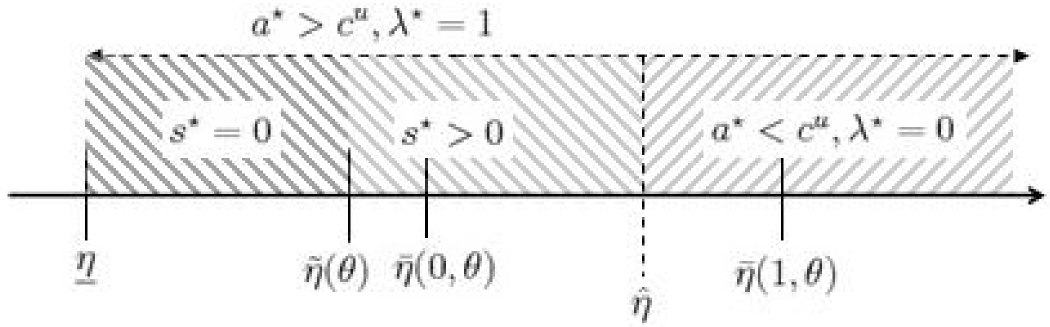

Figure 6: Optimal Access Charge, Vertical Control and Sabotage when $\theta = 0.7$.

One interesting observation is that the optimal access charge $a^\star$ is constant for all $\eta < \tilde{\eta}(\theta)$. This is because, when sabotage is cheap, the only reason the regulator prices access above cost is to deter the IP from engaging in sabotage. When sabotage is zero there is no reason for further increases in the access charge; such increases only hurt downstream efficiency. Therefore the regulator leaves the optimal access charge unchanged once sabotage cost falls below $\tilde{\eta}(\theta)$.

A simple numerical example gives an indication of the magnitudes involved. Suppose $\alpha - c - c^u = 1$, $\beta = 0.3$, and $\theta = 0.3$. Then the first best upstream access mark-up is $a^\star - c^u = -0.7$ and the first best welfare level is $W = 2.56$. Figure 8 shows the second best equilibrium outcomes as functions of the sabotage cost parameter. The discontinuities in the graphs clearly display the switch from full vertical control and positive access margins to zero vertical control and negative access margins at the critical level of sabotage cost $\hat{\eta}$.

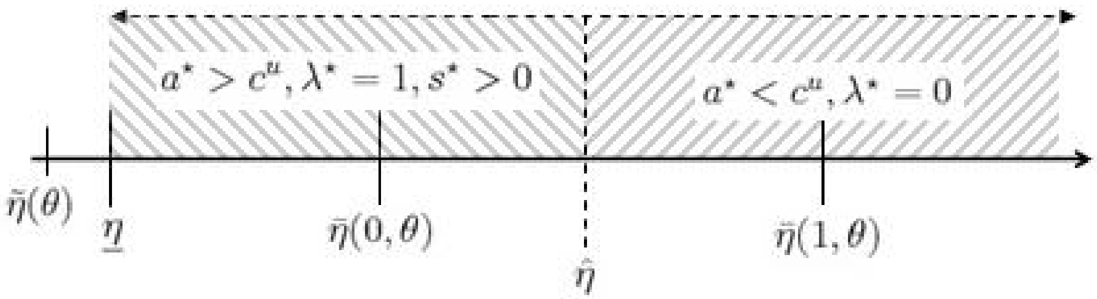

Figure 7: Optimal Access Charge, Vertical Control and Sabotage when $\theta = 0.96$.

The level of sabotage is quite low when sabotage costs are low because the regulator is aggressively trying to counter the parent's incentives by pricing access high. Sabotage levels jump up drastically as the regulator switches to the full vertical restriction regime by pricing access below cost. Not shown in this graph, sabotage levels decrease monotonically and approach zero as sabotage costs grow large.

Access margins fall rapidly as the regulator is able to rely more on sabotage costs for sabotage deterrence. The upper curves in the last graph show the output produced by the affiliate. As shown here downstream output levels are increasing fast, consistent with rapidly falling access charges. Welfare is increasing as the drastic fall in access charges more than offsets increases in sabotage when sabotage costs are in the medium range. Welfare approaches the first-best level as sabotage costs grow large. Note, however, that at the relatively large sabotage costs depicted in the graphs (relative to the demand-cost margin)[8] the possibility of sabotage has severe welfare consequences since welfare is still only 82% of the first best level.

5 Conclusion

This is the first formal study of restriction on vertical control, distinct from vertical ownership, as a policy tool. Restricting vertical control appears to be a useful weapon in some settings. It gives the policy maker a new way of inducing output-expansion and can thereby

[8]When $\eta = 140$ the marginal cost of a unit of sabotage at $s^\star$ is 0.32, which is about third of the demand-cost margin.

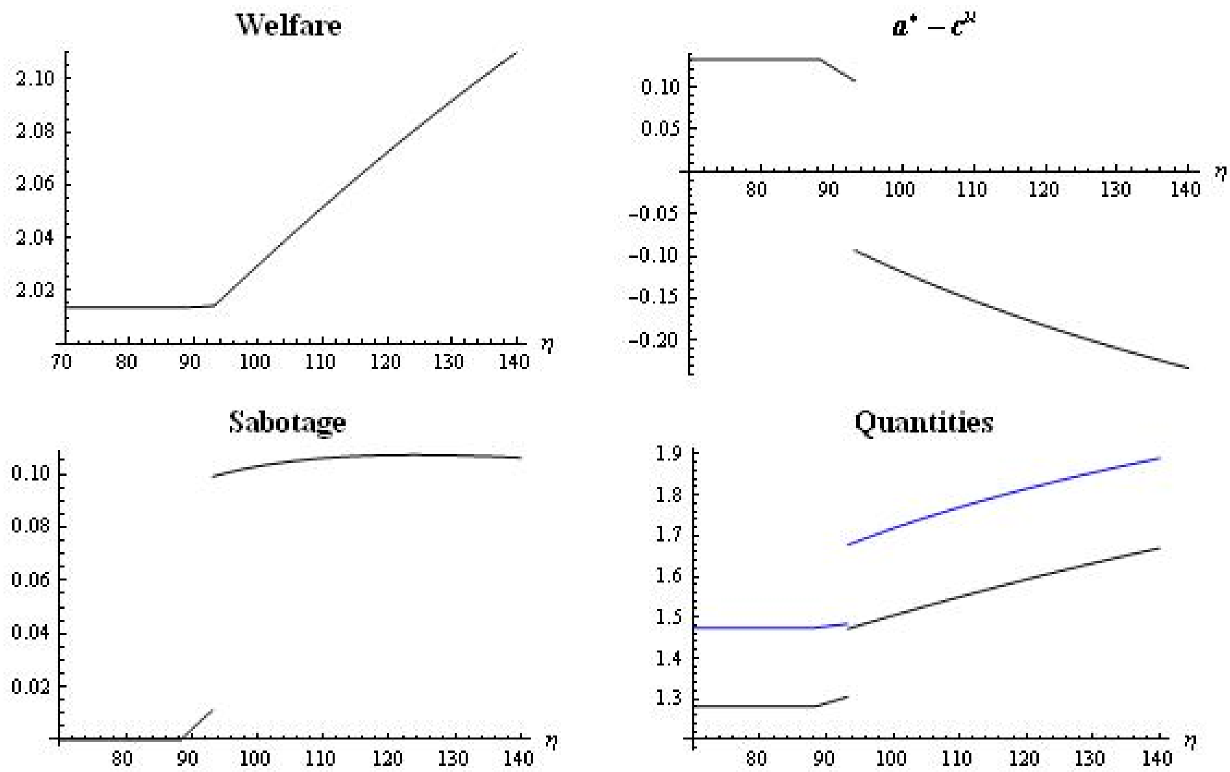

Figure 8: Sample Equilibrium Outcomes Across Sabotage Cost η.

complement access pricing policy when pricing policy alone is inadequate because of potential for non-price discrimination. Manipulation of vertical control can be beneficial in inducing prices closer to the first best in two settings. First, if products are close substitutes and the rival firm is more efficient than the affiliate, the regulator would like the rival to serve the whole market and price at marginal cost. The regulator can achieve this by setting the access charge below cost and allowing for partial vertical control. Doing so brings the rival's price to the first-best level and at the same time keeps the affiliate out of the market. Second, if the downstream firms have identical costs but differentiated products, then for any access price that balances sabotage deterrence with output-expansion the regulator can induce maximal output-expansion by minimizing effective downstream costs through the choice of vertical control. The cost minimum is obtained by allowing a high degree of vertical control when access is priced above marginal cost, and by severely restricting vertical control when access is priced below marginal cost.

Sabotage deterrence is the reason it may be optimal to price access above marginal cost. If the regulator is able to influence costs of sabotage by monitoring and penalties, then the regulator can possibly set sabotage costs high enough to make restrictions on vertical control ($\lambda^\star = 0$) and access pricing below marginal cost ($a^\star < c^u$) the optimal policy. Whether this is optimal depends on the magnitude of the cost of monitoring. If monitoring costs are low, then the policy of substantial performance monitoring pursued by the FCC as a condition for Bell company entry into long-distance markets (see, for example, FCC 1999), accompanied by restrictions on vertical control as required by Section 272 of the Telecommunications Act of 1996, may be an optimal use of limited regulatory tools. Note, however, that Proposition 6 suggests such a policy should also include access prices below marginal cost rather than the marginal cost pricing of access embraced by the FCC (FCC, 2000).

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6 Appendix

Definition of First Best. Differentiating (9) with $s = 0$ and setting it to zero yields:

$$\frac{\partial W}{\partial q^i} = \alpha - \beta q^i - \gamma q^j - (c^i + c^u) = 0 \qquad i, j = d, r. \tag{47}$$

The first best values of q^i are found from the above expression:

$$q^i = \frac{\alpha(1-\theta) + \theta(c^j + c^u) - (c^i + c^u)}{\beta(1-\theta^2)}. \tag{48}$$

This is positive iff

$$\theta < \frac{\alpha - (c^i + c^u)}{\alpha - (c^j + c^u)}. \tag{49}$$

If $c^i < c^j$ then (49) always holds. If $c^i > c^j$ and $\theta \geq \dfrac{\alpha - (c^i + c^u)}{\alpha - (c^j + c^u)}$ then $q^i = 0$ and q^j is found from (48). First best prices are found from the inverse demand functions.

Proof of Proposition 1. Suppose $\theta = 0$. Downstream prices are given by (23). The regulator would like to achieve zero sabotage as well as productive efficiency by pricing at $p^d = c^d + c^u$ and $p^r = c^r + c^u$. If the parent does not sabotage, setting $a = c^r + 2c^u - \alpha$ and $\lambda = \frac{c^r - c^d}{c^r + c^u - \alpha}$ is required to equate (23) with these first-best prices. Then $(a - c^u) = c^r + c^u - \alpha < 0$.[9] Now we must check whether the IP will indeed choose $s = 0$ given the outcome downstream and the regulator's chosen (a, λ). The IP's profits are:

$$\pi^{IP} = (a - c^u)(q^r + q^d) + (p^d - c^d - a)q^d - K(s). \tag{50}$$

As seen from (23) and the definition of $\tilde{c}^i$, s only affects q^r. Thus

$$\left.\frac{\partial \pi^{IP}}{\partial s}\right|_{s=0} = -\frac{a - c^u}{2\beta} > 0, \tag{51}$$

so the IP will choose a positive level of sabotage when the regulator tries to induce first best retail prices. The first best is not achievable.

Proof of Proposition 2. Suppose $\theta = 1$. The first best requires that there be no sabotage. Assuming $s = 0$, the $\theta = 1$ condition implies that the effective downstream marginal costs are $\hat{c}^d = c^d + a$ and $\hat{c}^r = c^r + a$. There are three cases:

Case 1: $c^d = c^r = c$. The downstream market is pure Bertrand competition with equal marginal costs, so the rival and the affiliate both price at the common effective marginal cost and split the market:

$$p^{\star^d} = p^{\star^r} = c + a, \quad q^{\star^d} = q^{\star^r} = \frac{\alpha - c - a}{2\beta}. \tag{52}$$

In order to achieve efficiency, the regulator must set $a = c^u$ and hope that the IP does not sabotage. The parent's downstream and upstream profits are both zero in this case. However, with $a = c^u$, if the parent does sabotage then the rival drops from the market and

[9] If $c^r > c^d$ then $\lambda < 0$ and the regulator cannot achieve productive efficiency downstream.

the affiliate's Bertrand equilibrium price is the rival's effective marginal cost, $p^d = c + c^u + s$, and quantity is $q^d = \frac{\alpha - c - c^u - s}{\beta}$. The parent's profit in this case is

$$\pi^{IP}|_{s>0} = \pi^u + \pi^d = (p^d - c - c^u)q^d - K(s) = s\frac{\alpha - c - c^u - s}{\beta} - K(s). \tag{53}$$

Since $\alpha - c - c^u$ is strictly positive and $K'(s)$ is arbitrarily small for s small, a small increase in s starting from zero pays off for the parent:

$$\frac{\partial \pi^{IP}}{\partial s} = \frac{\alpha - c - c^u - 2s}{\beta} - K'(s) > 0. \tag{54}$$

So the first best is not achievable.

Case 2: $c^d < c^r$. In this case the affiliate serves the whole market and either price matches the rival's marginal cost or blockades it as in (25). To achieve productive efficiency the regulator would like $p^d = c^d + c^u$ and $p^r \geq c^d + c^u$. If the parent does not sabotage, the regulator can only achieve the first best pricing when the affiliate prices (deters) at $p^d = c^r + a$. In this case the regulator must set $a = c^d + c^u - c^r$ to make the affiliate's price first-best, and then $(a - c^u) = c^d - c^r < 0$. It can be verified that $c^r + a < \dfrac{\alpha + c^d + a - \lambda(a - c^u)}{2}$ so the affiliate does not blockade when the regulator chooses a in this way. Now we must check the parent's incentives to sabotage. The IP's profits in this setting are:

$$\pi^{IP} = q^d(p^d - c^d - c^u) - K(s) = s\frac{\alpha - c^d - c^u - s}{\beta} - K(s). \tag{55}$$

Thus:

$$\left.\frac{\partial \pi^{IP}}{\partial s}\right|_{s=0} = \frac{\alpha - c^d - c^u}{\beta} > 0, \tag{56}$$

and the first best is not achievable.

Case 3: $c^d > c^r$. If the parent does not sabotage then the rival serves the whole market and either deters or blockades the affiliate. Just as in case 2, the regulator can only achieve productive efficiency when the rival deters with $p^r = c^d + a$. To achieve this the regulator must set $a = c^r + c^u - c^d$ ($a - c^u = c^r - c^d < 0$). Now we must check the parent's incentives to sabotage.

The IP's profits require careful examination. If sabotage s exceeds $(c^d - c^r)$ then the rival will drop from the market and the affiliate will serve the whole market. If s is even higher the affiliate will be able to blockade the rival. Consequently, the parent's modified profit function is:

$$\pi^{IP} = \begin{cases} (a - c^u)\frac{\alpha - c^d - a}{\beta} - K(s) & \text{if } s \leq \underline{s} = c^d - c^r \\ \frac{\alpha - c^r - a - s}{\beta}(c^r + a + s - c^d - c^u) - K(s) & \text{if } \underline{s} < s \leq \overline{s} \\ \frac{\alpha - c^d - a + \lambda(a - c^u)}{2\beta}\left(\frac{\alpha + c^d + a - \lambda(a - c^u)}{2} - c^d - c^u\right) - K(s) & \text{if } s > \overline{s} \end{cases} \tag{57}$$

where $\overline{s} = \dfrac{\alpha + c^d - 2c^r - a - \lambda(a - c^u)}{2}$. The second part of the profit function is the affiliate's profit when it deters the rival by matching the rival's marginal cost. The third part is the affiliate's profit when it is using the monopoly price from (25). Using the values for a, $\underline{s}$ and $\overline{s}$ we get the following: $\pi^{IP}|_{s<\underline{s}} = (c^r - c^d)\frac{\alpha - c^r - c^u}{\beta} - K(s)$, so $\frac{\partial \pi^{IP}}{\partial s}|_{s<\underline{s}} = -K'(s) < 0$. Also, the profit function (57) is continuous. The derivative of the second part of the IP's profit function evaluated at $\underline{s} = c^d - c^r$ is $\frac{a-c^r-c^u+c^d-c^r}{\beta} - K'(s)$. Although the first term is positive, it's impossible to sign the derivative without specifying the exact functional form of $K(s)$ (the profit function is constant for $s > \overline{s}$). If $c^d - c^r$ is very small then the sabotage costs required to foreclose the rival will be relatively small and the IP will engage in sabotage. For $K(s)$ sufficiently convex and $c^d - c^r$ large enough the profit function will be decreasing for $s > \underline{s}$ and the parent will not engage in sabotage.

Proof of Proposition 3. Suppose $0 < \theta < 1$ and $c^r \geq c^d$. There are two cases depending on the intensity of competition and the cost parameters.

Case 1: Suppose $\theta < \dfrac{\alpha - c^r - c^u}{\alpha - c^d - c^u}$, *or* $c^r + c^u < \theta(c^d + c^u) + \alpha(1-\theta)$. According to the definition of first best the regulator would like both firms serving the market, pricing and producing according to (26) and (27): $p^r = c^r + c^u$ and $p^d = c^d + c^u$. The downstream equilibrium prices when both firms produce are given in (16). Setting these expressions for p^r and p^d equal to $c^r + c^u$ and $c^d + c^u$, respectively, (and imposing $s = 0$ as required for the first best) yields a regulatory policy of $a - c^u = c^u + c^r - \theta(c^d + c^u) - \alpha(1-\theta) < 0$ and $\lambda = \frac{(c^r - c^d)(1+\theta)}{(a-c^u)(1-\theta)}$. This policy is infeasible when $c^r > c^d$ because then $\lambda < 0$.

When $c^d = c^r = c$ then $\lambda = 0$, so we must check whether the parent chooses zero sabotage. The parent's profit function is:

$$\pi^{IP} = (a - c^u)(q^r + q^d) + (p^d - c^d - a)q^d - K(s). \tag{58}$$

Substituting in the equilibrium prices and quantities and taking into account that the first best requires $\lambda = 0$ gives us

$$\begin{aligned}\pi^{IP} = & \frac{(a - c^u)}{\beta(1+\theta)(2-\theta)}(2\alpha - 2(c+a) - s) + \\ & + \frac{((1-\theta)(2+\theta)(\alpha - c - a) + \theta s)^2}{\beta(1-\theta^2)(4-\theta^2)^2} - K(s).\end{aligned} \tag{59}$$

The derivative evaluated at $s = 0$ is:

$$\left.\frac{\partial \pi^{IP}}{\partial s}\right|_{s=0} = -\frac{a - c^u}{\beta(1+\theta)(2-\theta)} + \frac{2\theta(1-\theta)(2+\theta)(\alpha - c - a)}{\beta(1-\theta^2)(4-\theta^2)^2} > 0. \tag{60}$$

The first term is positive since $a < c^u$ is required for the first best and the second term is positive as well since $\alpha - c - c^u > 0$ and $a < c^u$. Thus the first best is not achievable.

Case 2: Suppose $\theta \geq \dfrac{\alpha - c^r - c^u}{\alpha - c^d - c^u}$, *or* $c^r + c^u \geq \theta(c^d + c^u) + \alpha(1-\theta)$ *(note that* $\theta < 1$ *implies* $c^r > c^d$ *in this case).* Then the the first best requires that the affiliate serve the whole market with prices at $p^r \geq \alpha(1-\theta) + \theta(c^d + c^u)$ and $p^d = c^d + c^u$. If the parent does not sabotage there are two possibilities downstream: the affiliate either deters or blockades.

First consider the blockade. In this case the downstream equilibrium entails $p^r \geq c^r + a$ and $p^d = \dfrac{\alpha + c^d + a - \lambda(a - c^u)}{2}$. To make the affiliate's equilibrium price be first best the regulator must choose (a, λ) to satisfy $\lambda(a - c^u) = \alpha - c^d - c^u + a - c^u$. The policy must also satisfy the necessary inequality (22) for the blockade. Substituting for λ and simplifying:

$$a - c^u > \alpha(1-\theta) + \theta(c^d + c^u) - c^r - c^u. \tag{61}$$

At this point the sign of $a - c^u$ is still indeterminate. First observe that $a \neq c^u$, since this would imply $\alpha - c^d - c^u = 0$. Suppose $a - c^u < 0$. We need to make sure that λ is positive, that is, $\alpha - c^d - c^u + a - c^u < 0$. However, $\alpha - c^d - c^u + a - c^u > \alpha - c^d - c^u + \alpha(1-\theta) + \theta(c^d + c^u) - c^r - c^u = \alpha - c^r - c^u + (1-\theta)(\alpha - c^d - c^u) > 0$. This implies that λ is infeasible. Now suppose $a - c^u > 0$. We need to make sure that $\lambda < 1$, that is, $\alpha - c^d - c^u + a - c^u < a - c^u$. However, $\alpha - c^d - c^u > 0$. Thus λ is again infeasible.

Now consider the case when the affiliate deters. The downstream equilibrium prices are $p^r = c^r + a$ and $p^d = \dfrac{c^r + a - \alpha(1-\theta)}{\theta}$. Setting $a = \alpha(1-\theta) + \theta(c^d + c^u) - c^r$ (note that $a - c^u < 0$) is required to achieve the desired pricing of $p^d = c^d + c^u$ and $p^r = c^r + c^u$.

The IP's profit function is:

$$\begin{aligned}\pi^{IP} &= (p^d - c^d - c^u)q^d - K(s) \\ &= \left(\frac{c^r + a + s - \alpha(1-\theta)}{\theta} - c^d - c^u\right)\frac{\alpha - c^r - a - s}{\gamma} - K(s). \qquad (62)\end{aligned}$$

The derivative evaluated at $s = 0$ and at $a = \alpha(1-\theta) + \theta(c^d + c^u) - c^r$ is:

$$\left.\frac{\partial \pi^{IP}}{\partial s}\right|_{s=0} = \frac{\alpha - c^d - c^u}{\gamma} > 0. \qquad (63)$$

Thus first best is not achievable.

Proof of Proposition 4. Suppose $0 < \theta < 1$ and $c^d > c^r$. There are two cases depending on the intensity of competition and the cost parameters.

Case 1. Suppose $\theta < \dfrac{\alpha - (c^d + c^u)}{\alpha - (c^r + c^u)}$, *or* $c^d + c^u < \theta(c^r + c^u) + \alpha(1-\theta)$. In this case the first best requires that both firms serve the market charging $p^r = c^r + c^u$ and $p^d = c^d + c^u$. Downstream prices for the interior equilibrium are given by (16). If the parent does not sabotage, to match the first best and downstream prices the regulator must set $a = 2c^u + c^r - \theta(c^d + c^u) - \alpha(1-\theta)$ (note that $a - c^u = c^u + c^r - \theta(c^d + c^u) - \alpha(1-\theta) < 0$) and $\lambda = \frac{(1+\theta)(c^r - c^d)}{(a - c^u)(1-\theta)} > 0$.

Now we must check whether the parent does indeed choose zero sabotage at the regulator's chosen (a, λ). Since both firms produce downstream, the parent's profit function is:

$$\pi^{IP} = (a - c^u)(q^r + q^d) + (p^d - c^d - a)q^d - K(s). \qquad (64)$$

Downstream equilibrium quantities and prices are found in (16) and (17). After some simplification,

$$\begin{aligned}\pi^{IP} = &\frac{(a - c^u)}{\beta(1+\theta)(2-\theta)}(2\alpha - \hat{c}^r - \hat{c}^d) \\ &+ \frac{1}{\beta(1-\theta^2)(4-\theta^2)}\left(\frac{\alpha(1-\theta)(2+\theta) + \theta\hat{c}^r + 2\hat{c}^d}{4-\theta^2} - c^d - a\right) \\ &* (\alpha(1-\theta)(2+\theta) - (2-\theta^2)\hat{c}^d + \theta\hat{c}^r) - K(s). \qquad (65)\end{aligned}$$

The derivative evaluated at $s = 0$ is:

$$\begin{aligned}
\left.\frac{\partial \pi^{IP}}{\partial s}\right|_{s=0} &= -\frac{a - c^u}{\beta(1+\theta)(2-\theta)} \\
&\quad + \frac{\theta}{\beta(1-\theta^2)(4-\theta^2)^2}(2\alpha(1-\theta)(2+\theta) + \theta^2\hat{c}^d + 2\theta\hat{c}^r - (4-\theta^2)(c^d + a)) \\
&= \frac{1}{\beta(1-\theta^2)(4-\theta^2)}(-(a-c^u)(1-\theta)(2+\theta) + 2\theta p^d - \theta(c^d+a) - \theta\hat{c}^d) \\
&= \frac{1}{\beta(1-\theta^2)(4-\theta^2)}((a-c^u)(1-\theta)(\theta\lambda - 2 - \theta) + 2\theta(p^d - c^d - a)) \\
&> 0. \qquad (66)
\end{aligned}$$

The sign follows because $(a - c^u) < 0$ from above, and $p^d - c^d - a > 0$ given the chosen (a, λ). Thus, first best is not achievable.

Case 2. Suppose $\theta \geq \frac{\alpha - (c^d + c^u)}{\alpha - (c^r + c^u)}$, *or* $c^d + c^u > \theta(c^r + c^u) + \alpha(1-\theta)$. In this case the first best requires that the rival serve the whole market with both firms charging prices as specified in (28) and (29): $p^d \geq \alpha(1-\theta) + \theta(c^r + c^u)$ and $p^r = c^r + c^u$. The downstream equilibrium entails the rival either deterring the affiliate or blockading it, with no sabotage for the first best.

First consider the blockade. If the rival blockades then $p^d \geq c^d + a - \lambda(a - c^u)(1-\theta)$ and $p^r = \frac{\alpha + c^r + a}{2}$. To match the first best price for the rival the regulator must set $a = c^r + 2c^u - \alpha$ (note that $(a - c^u) = c^r + c^u - \alpha < 0$). At this access charge the necessary inequality (22) for the blockade, that is, $c^d + a - \lambda(a-c^u)(1-\theta) > \alpha(1-\theta) + \frac{\theta}{2}(\alpha + c^r + a)$, cannot hold. After simplification this condition is $\lambda > \frac{2c^d - \theta c^r - \alpha(2-\theta) + a(2-\theta)}{2(a-c^u)(1-\theta)}$. However $\frac{2c^d - \theta c^r - \alpha(2-\theta) + a(2-\theta)}{2(a-c^u)(1-\theta)} > 1$ implying that $\lambda > 1$. Thus the regulator cannot achieve the first best in this case.

The regulator may be able to achieve the first best when the rival deters the affiliate. In this case downstream equilibrium prices are $p^d = c^d + a - \lambda(a - c^u)(1-\theta)$ and $p^r = \frac{c^d + a - \lambda(a-c^u)(1-\theta) - \alpha(1-\theta)}{\theta}$ from (19). The regulator must match those prices with first best prices $p^d \geq \alpha(1-\theta) + \theta(c^r + c^u)$ and $p^r = c^r + c^u$ given that the parent does not sabotage. $\lambda = \frac{c^d + a - \alpha(1-\theta) - \theta(c^r + c^u)}{(a-c^u)(1-\theta)}$ is both necessary and sufficient for the first best prices to be deterrence equilibrium prices, provided this value of λ is in $[0, 1]$, the affiliate doesn't produce in equilibrium, and the rival's price is below the monopoly level. The latter two conditions are:

$$\hat{c}^d \geq \frac{\alpha(1-\theta)(2+\theta) + \theta\hat{c}^r}{2-\theta^2}, \qquad (67)$$

$$\hat{c}^d < \alpha(1-\theta) + \theta\frac{\alpha + \tilde{c}^r}{2}. \qquad (68)$$

Together equations (67) and (68) imply that $c^r + c^u - \alpha < a - c^u < (1-\theta^2)(c^r + c^u - \alpha)$. Hence the access charge is set below cost. Furthermore, $0 \leq \lambda \leq 1$ implies $\frac{c^r + c^u - \alpha}{\theta}\left(\theta - \frac{\alpha - c^d - c^u}{\alpha - c^r - c^u}\right) \leq$

$a-c^u \le (c^r+c^u-\alpha)\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right)$. Since $c^r+c^u-\alpha < \frac{c^r+c^u-\alpha}{\theta}\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right)$, the effective lower bound for $a-c^u$ is

$$\frac{c^r+c^u-\alpha}{\theta}\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right) \le a-c^u. \tag{69}$$

Both upper bounds of $a-c^u$ may be relevant, depending on θ. Hence

$$a-c^u \le \min\left\{(1-\theta^2)(c^r+c^u-\alpha), (c^r+c^u-\alpha)\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right)\right\}. \tag{70}$$

The inequality (70) requires more careful examination. When $\theta = \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}$, then $(1-\theta^2)(c^r+c^u-\alpha) < (c^r+c^u-\alpha)\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right)$ and when $\theta = 1$ then $(c^r+c^u-\alpha)\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right) < (1-\theta^2)(c^r+c^u-\alpha)$. Moreover, the first bound is increasing in θ and the second bound is decreasing in θ. Thus there is some threshold value of θ, say $\overline{\theta}$, where those two expressions are equal and for $\theta < \overline{\theta}$ the effective upper bound is $(1-\theta^2)(c^r+c^u-\alpha)$. However, when $\theta = \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}$ the lower bound $\frac{c^r+c^u-\alpha}{\theta}\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right)$ is zero and the upper bound $(1-\theta^2)(c^r+c^u-\alpha)$ is strictly negative and thus the interval for $a-c^u$ becomes degenerate. The effective upper bound $(1-\theta^2)(c^r+c^u-\alpha)$ is increasing in θ and the lower bound $\frac{c^r+c^u-\alpha}{\theta}\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right)$ is decreasing in θ, and they cross at $\theta = \sqrt[3]{\frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}}$. Hence the $a-c^u$ interval is non-degenerate for θ larger this cube root (the other potential upper bound in (70) is always above the lower bound). To summarize, the necessary and sufficient conditions for the regulator to achieve the productive efficiency downstream are:

$$\theta \ge \sqrt[3]{\frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}}, \tag{71}$$

$$\lambda = \frac{c^d+a-\alpha(1-\theta)-\theta(c^r+c^u)}{(a-c^u)(1-\theta)}, \tag{72}$$

$$\frac{c^r+c^u-\alpha}{\theta}\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right) \le a-c^u, \tag{73}$$

$$a-c^u \le \min\left\{(1-\theta^2)(c^r+c^u-\alpha), (c^r+c^u-\alpha)\left(\theta - \frac{\alpha-c^d-c^u}{\alpha-c^r-c^u}\right)\right\}. \tag{74}$$

The regulator is free to choose any value of $a-c^u$ that satisfies (73) and (74) (and there are such values given (71)) provided the corresponding λ is set according to (72).

Now we must check that the parent indeed does not sabotage given the regulator's chosen (a,λ). The parent's profits are:

$$\pi^{IP} = (a-c^u)q^r - K(s) = (a-c^u)\frac{\alpha-(c^d+a-\lambda(a-c^u)(1-\theta))}{\gamma} - K(s). \tag{75}$$

Plugging in the regulator's chosen λ and simplifying yields:

$$\pi^{IP} = (a-c^u)\frac{\alpha-c^r-c^u}{\beta} - K(s). \tag{76}$$

This is strictly decreasing in s, so the parent will not engage in a small amount of sabotage.

However, the IP can increase sabotage to the level where the rival's effective cost advantage diminishes enough for the affiliate to start producing in the downstream equilibrium. At this point, the downstream equilibrium is given by (16) and (17) and the IP's profit is:

$$\pi^{IP} = (a - c^u)(q^r + q^d) + (p^d - c^d - a)q^d - K(s). \tag{77}$$

The parent's downstream profit $(p^d - c^d - a)q^d$ is increasing in s and the downstream aggregate quantity $q^r + q^d$ is decreasing in s. Thus, aside from sabotage costs $K(s)$, the IP gains from sabotage once sabotage is high enough to make the downstream affiliate viable (remember that $a - c^u < 0$). As seen from (67), $s > (1-\theta^2)(c^r + c^u - \alpha) - (a - c^u)$ is required for the affiliate to participate in the downstream market. This s is positive at the regulator's chosen (a, λ). So the key question is whether (76) evaluated at $s = 0$ exceeds the maximum of (77) over s subject to the constraint $s \geq (1-\theta^2)(c^r + c^u - \alpha) - (a - c^u)$. If so, there will be no sabotage and the first best is achievable. If not, sabotage will be positive and the first best is not achievable. Which outcome occurs depends on the specific functional form for $K(s)$ (specifically, how quickly $K(s)$ increases), on θ, and on the size of the rival's cost advantage $c^d - c^r$ (i.e. the size of the sabotage threshold $(1-\theta^2)(c^r + c^u - \alpha) - (a - c^u)$).

The sabotage threshold approaches zero as $\theta \to 1$ or $c^r \to c^d$ at the regulator's choice of $a - c^u$, hence the first best is unachievable for any given K provided θ is "high enough" or $c^r - c^d$ is "low enough." On the other hand, if $K(s)$ is convex enough, the two marginal costs are sufficiently different, and θ is not too close to one then the IP will not engage in sabotage.

Proof of Lemma. Let $U^\star(p^r, p^d) = \max_{(q^r,q^d)} U(q^r, q^d) - p^d q^d - p^r q^r$ be the indirect utility function. Now rewrite the welfare objective using $U^\star(p^r, p^d)$ and the downstream equilibrium profit functions:

$$W(a, \lambda) = U^\star(\hat{p}^r(s^\star(a,\lambda), a, \lambda), \hat{p}^d(s^\star(a,\lambda), a, \lambda)) + \hat{\pi}^{IP\star}(a, \lambda) + \hat{\pi}^r(s^\star(a,\lambda), a, \lambda), \tag{78}$$

where the circumflex denotes downstream equilibrium values:

$$\hat{\pi}^r(s^\star(a,\lambda), a, \lambda) = [\hat{p}^r(s^\star(a,\lambda), a, \lambda) - c^r - a - s^\star(a,\lambda)]\, \hat{q}^r(s^\star(a,\lambda), a, \lambda) \tag{79}$$

$$\begin{aligned} \hat{\pi}^{IP\star}(a, \lambda) &= \hat{\pi}^{IP}(s^\star(a,\lambda), a, \lambda) \\ &= \left[\hat{p}^d(s^\star(a,\lambda), a, \lambda) - c - c^u\right] \hat{q}^d(s^\star(a,\lambda), a, \lambda) \\ &\quad + (a - c^u)\hat{q}^r(s^\star(a,\lambda), a, \lambda) - K(s^\star(a,\lambda)). \end{aligned} \tag{80}$$

Differentiating (78) while accounting for the resulting changes in the optimal level of sabotage and the downstream quantities and prices yields:

$$\begin{aligned} \frac{\partial W}{\partial a} &= U_1^\star(\hat{p}_1^r \frac{\partial s^\star}{\partial a} + \hat{p}_2^r) + U_2^\star(\hat{p}_1^d \frac{\partial s^\star}{\partial a} + \hat{p}_2^d) + \hat{\pi}_1^{IP\star} + \hat{\pi}_1^r \frac{\partial s^\star}{\partial a} + \hat{\pi}_2^r \\ &= [U_1^\star \hat{p}_1^r + U_2^\star \hat{p}_1^d + \hat{\pi}_1^r] \frac{\partial s^\star}{\partial a} + U_1^\star \hat{p}_2^r + U_2^\star \hat{p}_2^d + \hat{\pi}_1^{IP\star} + \hat{\pi}_2^r \\ &= \Psi \frac{\partial s^\star}{\partial a} + U_1^\star \hat{p}_2^r + U_2^\star \hat{p}_2^d + \hat{\pi}_1^{IP\star} + \hat{\pi}_2^r. \end{aligned} \tag{81}$$

It is easy to see from (81) that $\Psi = U_1^\star \hat{p}_1^r + U_2^\star \hat{p}_1^d + \hat{\pi}_1^r < 0$, since $U_1^\star$, $U_2^\star$ and $\hat{\pi}_1^r$ are negative and $\hat{p}_1^r$, $\hat{p}_1^d$ are positive.

Proof of Proposition 6. Rewrite (35) using (38) as:

$$\begin{aligned}\frac{\partial W}{\partial \lambda} &= (p^d - c - c^u)\frac{(1-\theta)(2-\theta^2)(a-c^u)}{D} - (p^r - c - c^u - s)\frac{(1-\theta)\theta(a-c^u)}{D} + \frac{\partial s}{\partial \lambda}\Psi \\ &= \frac{(1-\theta)(a-c^u)}{D}\Gamma + \frac{\partial s}{\partial \lambda}\Psi, \end{aligned} \tag{82}$$

where $\Gamma = (p^d - c - c^u)(2-\theta^2) - (p^r - c - c^u)\theta + \theta s$. We will show first that $a - c^u$ always has the same sign as $\frac{\partial W}{\partial \lambda}$, establishing that $\lambda^\star = 0$ if $a^\star - c^u < 0$ and $\lambda^\star = 1$ if $a^\star - c^u > 0$. The last term $\frac{\partial s}{\partial \lambda}\Psi$ always takes on the same sign as $a - c^u$, since $\Psi \leq 0$ and $\frac{\partial s}{\partial \lambda} \overset{s}{=} -(a-c^u)$ (or $\frac{\partial s}{\partial \lambda} = 0$ at the corner $s^\star = 0$), thus we omit it from further analysis. What remains to show is that Γ is always positive. Substituting for the interior equilibrium values of p^r and p^d and simplifying reveals that Γ is linear in λ and:

$$\Gamma|_{\lambda=1} = \frac{1}{4-\theta^2}[(\alpha - c - c^u)(1-\theta)^2(2+\theta)^2 + 4\theta(a-c^u)(1-\theta^2) + 2\theta s(2-\theta^2)] \tag{83}$$

$$\Gamma|_{\lambda=0} = \frac{1}{4-\theta^2}[(\alpha - c - c^u)(1-\theta)^2(2+\theta)^2 + (a-c^u)(1-\theta)(2+\theta)^2 + 2\theta s(2-\theta^2)]. \tag{84}$$

If $a - c^u > 0$ then (83) and (84) are clearly positive, therefore $\Gamma > 0 \ \forall \ \lambda \in [0,1]$.

If $a - c^u < 0$ the second term inside the square brackets in (83) and (84) turns negative thus we must show that the first term dominates the second in both cases. Recall from Case 1 of Proposition 3 that, if there were no sabotage, the optimal regulatory policy would be $\lambda = 0$ and $a - c^u = -(\alpha - c - c^u)(1-\theta)$. Since the last two terms in (34) are nonnegative and the welfare function is concave in a under the stated assumptions, we know that the possibility of sabotage causes the optimal access charge to be no lower than the first best level. Thus:

$$\begin{aligned}\Gamma|_{\lambda=0} &\geq \frac{(1-\theta)(2+\theta)^2}{4-\theta^2}\left[(\alpha - c - c^u)(1-\theta) - (\alpha - c - c^u)(1-\theta) + \frac{2\theta s(2-\theta^2)}{(2+\theta)^2(1-\theta)}\right] \\ &= \frac{2\theta s(2-\theta^2)}{4-\theta^2} \geq 0. \end{aligned} \tag{85}$$

For the $\lambda = 1$ case when $a - c^u < 0$, if there were no sabotage and the regulator could not choose λ then the optimal access price would satisfy

$$\frac{\partial W}{\partial a} = (p^d - c - c^u)\frac{\partial q^d}{\partial a} + (p^r - c - c^u)\frac{\partial q^r}{\partial a} = 0. \tag{86}$$

Solving for this optimal access price yields $a - c^u = -(\alpha - c - c^u)\frac{(1-\theta)(2+\theta)^2}{5\theta^2+4}$. Again, since the regulator must set the optimal access charge no lower when sabotage is possible compared to when there is no sabotage, we have:

$$\begin{aligned}\Gamma|_{\lambda=1} &\geq \frac{(1-\theta)^2(2+\theta)^2}{4-\theta^2}\left[(\alpha - c - c^u) - (\alpha - c - c^u)\frac{4\theta(1+\theta)}{5\theta^2+4} + \frac{2\theta s(2-\theta^2)}{(1-\theta)^2(2+\theta)^2}\right] \\ &= \frac{(1-\theta)^2(2+\theta)^2}{4-\theta^2}\left[(\alpha - c - c^u)\left(1 - \frac{4\theta(1+\theta)}{5\theta^2+4}\right) + \frac{2\theta s(2-\theta^2)}{(1-\theta)^2(2+\theta)^2}\right] \\ &= \frac{(1-\theta)^2(2+\theta)^2}{4-\theta^2}\left[(\alpha - c - c^u)\frac{(2-\theta)^2}{5\theta^2+4} + \frac{2\theta s(2-\theta^2)}{(1-\theta)^2(2+\theta)^2}\right] > 0, \end{aligned} \tag{87}$$

Therefore $\Gamma > 0 \; \forall \; \lambda \in (0,1]$ when $a - c^u < 0$ (since Γ is linear).

Next we show that the regulator will never choose $a - c^u = 0$. Differentiate (34) with respect to λ:

$$\frac{\partial^2 W}{\partial a \partial \lambda} = \frac{\partial p^d}{\partial \lambda}\frac{\partial q^d}{\partial a} + (p^d - c - c^u)\frac{\partial^2 q^d}{\partial a \partial \lambda} + \left(\frac{\partial p^r}{\partial \lambda} - \frac{\partial s}{\partial \lambda}\right)\frac{\partial q^r}{\partial a} + (p^r - c - c^u - s)\frac{\partial^2 q^r}{\partial a \partial \lambda} + \\ + \frac{\partial^2 s}{\partial a \partial \lambda}\Psi + \frac{\partial s}{\partial a}\frac{\partial \Psi}{\partial \lambda}, \tag{88}$$

where

$$\frac{\partial \Psi}{\partial \lambda} = \frac{\partial p^d}{\partial \lambda}\frac{\partial q^d}{\partial s} + (\frac{\partial p^r}{\partial \lambda} - \frac{\partial s}{\partial \lambda})\frac{\partial q^r}{\partial s} - \frac{\partial q^r}{\partial \lambda} - K''(s)\frac{\partial s}{\partial \lambda}. \tag{89}$$

Using the downstream equilibrium prices, (32), (33), (37), (38) and (39), if $a = c^u$ then $\frac{\partial p^d}{\partial \lambda} = \frac{\partial p^r}{\partial \lambda} = \frac{\partial s}{\partial \lambda} = \frac{\partial q^r}{\partial \lambda} = 0$. Therefore, after simplifying

$$\left.\frac{\partial^2 W}{\partial a \partial \lambda}\right|_{a=c^u} = (p^d - c - c^u)\frac{(2-\theta^2)(1-\theta)}{D} - (p^r - c - c^u - s)\frac{\theta(1-\theta)}{D} + \frac{\partial^2 s}{\partial a \partial \lambda}\Psi \\ \stackrel{s}{=} \frac{1-\theta}{D(4-\theta^2)}(\alpha - c - c^u)(1-\theta)^2(2+\theta)^2 + 2\theta s(2-\theta^2) > 0, \tag{90}$$

since $\frac{\partial^2 s}{\partial a \partial \lambda}$ (from (40)) and Ψ are both non-positive. Equation (90) shows that $a = c^u$ cannot be optimal. If $\frac{\partial W}{\partial a}$ were zero at $a = c^u$, a change in λ would have no effect on W (since $\frac{\partial W}{\partial \lambda} \stackrel{s}{=} a - c^u$) but such a change would make $\frac{\partial W}{\partial a}$ nonzero (since $\frac{\partial^2 W}{\partial a \partial \lambda} > 0$). So, changing a in the same direction as λ then raises W. Hence, the optimum is either $a - c^u > 0$ or $a - c^u < 0$.

Finally, we sketch why the ($\lambda^\star = 1, a^\star > c^u$) regime occurs when $\eta \in (\underline{\eta}, \hat{\eta}]$ and the ($\lambda^\star = 0, a^\star < c^u$) regime occurs when $\eta \geq \hat{\eta}$. It can be shown that $\bar{\eta}(0,\theta) > \bar{\eta}(1,\theta)$ for all $\theta \in (0,1)$. Since the regulator uses the $\lambda = 0$ regime for $\eta > \bar{\eta}(0,\theta)$ and the $\lambda = 1$ regime for $\eta < \bar{\eta}(1,\theta)$ there is a region between $\bar{\eta}(0,\theta)$ and $\bar{\eta}(1,\theta)$ where either regime could be optimal. For $\eta \in (\bar{\eta}(0,\theta), \bar{\eta}(1,\theta))$, the choice of λ is determined by a direct comparison of welfare. It is straightforward but tedious to substitute for downstream equilibrium quantities from (16), sabotage from (40) and optimal access charges from (41) and (42) into the welfare expression to obtain

$$W|_{\lambda=0} = \frac{-(\alpha - c - c^u)^2\Phi(\eta,\theta)}{2\beta(1+\theta)d(0,\eta,\theta)} \tag{91}$$

$$W|_{\lambda=1} = \frac{(\alpha - c - c^u)^2\Pi(\eta,\theta)}{2\beta(1+\theta)d(1,\eta,\theta)}, \tag{92}$$

where $\Phi(\eta,\theta)$ and $\Pi(\eta,\theta)$ are second order polynomials in η. Hence

$$W|_{\lambda=0} - W|_{\lambda=1} \stackrel{s}{=} -\left[\frac{\Phi(\eta,\theta)}{d(0,\eta,\theta)} + \frac{\Pi(\eta,\theta)}{d(1,\eta,\theta)}\right]. \tag{93}$$

It can be verified that this expression is negative at $\eta = \bar{\eta}(0,\theta)$ and positive at $\eta = \bar{\eta}(1,\theta)$. Therefore by the intermediate value theorem there is at least one root of (93) on $\eta \in (\bar{\eta}(0,\theta), \bar{\eta}(1,\theta))$. Moreover, it is possible to show that the derivative of (93) is positive for all $\eta \in (\bar{\eta}(0,\theta), \bar{\eta}(1,\theta))$. Thus the root is unique on this interval.