TELRIC PRICING WITH VINTAGE CAPITAL

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Abstract. This paper studies the effect of technical progress on competitive equilibrium prices in a formal dynamic setting that includes the dynamic effects of business income taxes. The model is designed to facilitate comparison between competitive equilibrium prices and the TELRIC prices that were recently adopted by the FCC for determining universal service subsidies in telecommunications. The equilibrium prices differ from the regulatory prices due to 1) differences in discount factors, 2) differences in the stream of operating costs, and 3) differences in the discounting method applied to the revenue stream. In a calibrated comparison of prices for end-office switching services, we find that the last difference is the most important, and the net effect of these differences is regulated prices that understate competitive equilibrium prices by billions of dollars nationwide in present value terms. We also note that equilibrium prices can be derived without making any assumptions about depreciation methods, contrary to conventional regulatory practice, and that competitive prices cannot be calculated in advance of costs once capital utilization is endogenized.

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"We agree with the petitioners that basing the allowable charges for the use of an ILEC's existing facilities and equipment (either through interconnection or the leasing of unbundled network elements) on what the costs would be if the ILEC provided the most efficient technology and in the most efficient configuration available today utilizing its existing wire center locations violates the plain meaning of the act. It is clear from the language of the statute that Congress intended the rates to be 'based on the cost . . . of providing the interconnection or network element,' (Telecommunications Act of 1996, §252(d)(1)(A)(i), emphasis added by writer) not on the cost some imaginary carrier would incur by providing the newest, most efficient, and least cost substitute for the actual item or element which will be furnished by the existing ILEC pursuant to Congress's mandate for sharing. Congress was dealing with reality, not fantasizing about what might be."

–Opinion, United States Court of Appeals for the Eighth Circuit, in *Iowa Utilities Board III,* Circuit Judge Hansen writing for the Court.

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On August 1, 1996 the Federal Communications Commission (FCC) adopted the *Local Competition Order* as its first major step toward implementation of the Telecommunications Act of 1996. Therein, the FCC concluded that "... prices for interconnection and unbundled network elements ... should be set at forward-looking long-run economic cost," and defined "Total Element Long Run Incremental Cost," or TELRIC, as the version of "forward-looking long-run economic cost" to be used in practice (FCC, 1996, ¶672). The agency's rules implementing this conclusion were initially vacated by the Eighth Circuit on jurisdictional grounds (*Iowa Utilities Board*, 1997), but the Supreme Court remanded that ruling to the Eighth Circuit (*Iowa Utilities Board II,* 1999) which then gave the above-quoted reasons for vacating the rule requiring use of a hypothetical most-efficient network when calculating TELRIC prices (*Iowa Utilities Board III,* 2000).

At the heart of this controversy lies much confusion concerning exactly what costs are recovered by efficient firms that operate in dynamic, risk-laden, competitive markets. The FCC's interest in TELRIC apparently stems from the belief that "... a pricing methodology based on forward-looking, economic costs best replicates, to the extent possible, the conditions of a competitive market" (FCC, 1996, ¶679). Although the competitive standard is appealing to most economists, and the FCC acknowledges the importance of dynamics in its reference to "forward-looking" costs, the dispute persists because of disagreement over whether the FCC's costing methods indeed emulate the forward-looking nature of competitive equilibrium prices. In particular, does the TELRIC paradigm properly account for the effects of technical progress and changes in riskiness on competitive equilibrium prices? The FCC answers "yes," through proper calculation of depreciation rates and the cost of capital (FCC, 1996, ¶686 and ¶703), but it is very far from clear whether the FCC's actual implementation accomplishes this. Indeed, we show that substantial confusion persists in the FCC's recent orders regarding the effect of technical progress on competitive equilibrium prices.

In this paper we clarify exactly what costs are recovered by efficient firms operating in a dynamic competitive market, by using a vintage capital model to explicitly capture technical progress so that a clear and precise dynamic definition of competitive equilibrium prices can be given. The formulation here allows us to give some numbers, whose heretofore absence "bothered" the Eighth Circuit Court during oral arguments in *Iowa Utilities Board III,*[1] that indicate why and by how much the FCC's formula fails to provide sufficient

[1]See p. 46, lines 10-14 of the oral arguments heard on September 17, 1999, as transcribed for the law firm of Kellog, Huber, Hansen, Todd & Evans, PLLC, Washington, D.C.

rates. We find that the FCC's actual depreciation and cost of capital do not reflect the effects of technical progress on competitive equilibrium prices.

After specifying the model in section 1, we formally derive in section 2 the supply correspondence of a perfectly competitive firm with our vintage capital technology. Capital demand, cost, and output supply at each time depend, in general, on the *entire future* sequence of output prices. This dependence reveals the exact meaning of "forward-looking" costs in a perfectly competitive market undergoing technical change. In section 3 we use the factor demand and output supply correspondences to define and derive a competitive equilibrium price sequence.

Next, in section 4 we compare the competitive equilibrium price sequence with prices from the FCC's Hybrid Cost Proxy Model (HCPM), which is the tool the agency has adopted for calculating "forward-looking" costs for purposes of determining universal service subsidies in telecommunications (FCC, 1998 and 1999). We find three distinct differences. First, the discount factor in the model is inconsistent with the cost of capital used for depreciation reserves. This causes an understatement of cost but has an indeterminate effect on the TELRIC prices. It appears this error may be caused partly by needlessly circuitous depreciation calculations. We contend it is unnecessary and overly complicated to consider any depreciation techniques other than those required to calculate tax liabilities. Second, the HCPM overstates operating cost early in the life of an asset but understates operating cost later in the asset's life. Third, when there is technical progress the so-called "levelization" method of calculating per-period payments used by the HCPM substantially understates prices relative to the competitive equilibrium stream of payments. Conversely, levelization overstates prices if the (quality adjusted) cost of capital assets is increasing over time.

Finally, in section 5 we calibrate the model using data on end-office digital switching costs from the HCPM. Together, the differences we identify result in TELRIC price estimates from the FCC's model that understate competitive equilibrium prices by an estimated \$679 thousand in present-value revenues over the economic life of one (large) end-office digital switch, or about 24% of the present value cost of the switching services. Nationwide, this understatement is worth billions of dollars in end-office switching costs *alone*, not counting overhead or even other costs directly associated with end-office switching (such as the distribution frame, power supply, and land), and not counting any costs of tandem switching or any other elements of the local network. This result differs from some earlier criticisms that regulatory costing models incorrectly allocate capital costs over time. Our estimates show that the HCPM will make substantial errors *in present value terms*.

Prior work in this area that underlies the legal dispute includes contributions by Hausman and Kahn. Hausman (1997) uses the option value investment theory of Dixit and Pindyck (1994) to argue that the correct *economic* depreciation rate and the correct *risk-adjusted* cost of capital interact in ways that are not reflected in the FCC's estimates of "forward-looking" costs, with the result that prices are below competitive levels and therefore facilities investment by both incumbents and entrants is discouraged. Kahn (1998, pp. 89-103) contends that the FCC's TELRIC paradigm incorrectly envisions a firm that instantly and costlessly reinvents itself in response to technical progress, and that the result of this unrealistic view is prices that do not grant even an efficient firm an opportunity to recoup its investments. The present paper is focused directly on Kahn's concerns, which are rooted in Fellner's classic (1951) study of the effects of technical progress on cost. We provide both the theoretical underpinnings for these concerns and an indication of the magnitude of the error. In order to focus on technical progress, we assume throughout that the firm is operating in a world of perfect certainty. Thus, we do not consider Hausman's concerns about the interaction of depreciation and riskiness. Once uncertainty is admitted, a higher cost of capital may be required to compensate the firm for its risks, especially any option value that is foregone when the firm makes a sunk investment. However, it is worth noting that the option value and vintage capital effects on prices may work in opposite directions for an asset whose (quality adjusted) capital cost increases over time.

Among the extant models, our model is closest to the basic model studied by Biglaiser and Riordan (2000). Both are vintage capital models in the spirit of Malcolmson (1975) and Nickell (1975, 1978), but both are specialized to capture items of particular interest. The most notable specializations here are the treatment of taxes and the omission of mechanistic (Jorgensonian) depreciation. The main intention of both Biglaiser and Riordan and the present effort is to *use* existing vintage capital tools to study the effects of regulation in a dynamic setting. Biglaiser and Riordan use their model to provide a comprehensive analysis of the effects of technical progress under rate-of-return and price-cap regulation. The present effort uses a similar model to conduct a careful comparison of competitive equilibrium prices with the FCC's TELRIC calculation. A useful extension of the comparisons presented here would be to use the model modifications presented in Biglaiser and Riordan's (2000) Appendix B to incorporate both technical and demand uncertainty, and then use the resulting structure to make comparisons that embody Hausman's (1997) concerns.

Our conclusion, that the FCC's pricing policies as implemented by the HCPM will understate competitive prices when there is technical progress, is reminiscent of Crew and Kleindorfer (1992). They show regulatory depreciation methods may lead to price regulation that does not permit full capital cost recovery for a regulated firm facing competition and technical progress. This pricing shortfall occurs because the introduction of falling competitive prices forces the firm off the regulator's price path. Although this effect may be present in our model, the pricing understatement we identify stems from entirely different sources and is in addition to the revenue shortfall that may occur if competition forces the regulated firm to abandon the regulated price path. Our shortfall occurs because of incorrect discount factors and operating costs, and the use of levelization. Indeed, we show that the regulatory depreciation calculations in the HCPM will lead to *correct* prices if the correct discount factor, economic life, and discounting methods are used, even though the depreciation calculations in the HCPM are unnecessary.

Like Crew and Kleindorfer (1992), Salinger (1998) identifies the distinction between regulatory and economic depreciation as a central flaw in a regulatory pricing regime. However, unlike Crew and Kleindorfer, Salinger's pricing regime is the post-1996 "forward-looking" paradigm, and Salinger argues that market-clearing zero-profit prices must be calculated *before* economic depreciation can be calculated, and hence *before* the components of "forward-looking" cost can be calculated. This leads Salinger to label this process "price-based costs" rather than "cost-based prices." The analysis presented here shows that prices and costs are determined simultaneously except in some special circumstances.

Alleman (1999) makes some of the observations noted here regarding the use of capital budgeting procedures, particularly the irrelevance of regulatory depreciation and the simultaneity of prices and costs. However, Alleman's main concern is the effect of including option value when calculating prices, and he therefore incorporates uncertainty while leaving aside technical progress and equilibrium price determination. In other words, Alleman's model is not an equilibrium vintage capital model of the kind considered here and in Biglaiser and Riordan (2000). Moreover, Alleman's examples are intended only to illustrate the possible relative magnitude of real options effects (Alleman, p. 173), and therefore are not derived from FCC data and do not estimate the aggregate dollar consequences of errors in the HCPM.

Lehman (1999) also evaluates TELRIC prices and concludes that the FCC's prices are too low. His focus is on comparing "embedded" cost methodologies with "forward-looking" methodologies, which differs substantially from the present effort. Nevertheless, certain common themes emerge. In particular, Lehman notes the importance of technical progress, depreciation methods, and the effects of payment "levelization."

1. A Vintage Capital Model with Exogenous Technical Progress

It is necessary to adopt a vintage capital model in order to study the effects of technical progress. The models of Malcolmson (1975) and Nickell (1975; 1978, especially section 7.3) provide a basic framework, but some specializations and additions are needed to adequately and concisely capture the TELRIC paradigm.

Technical progress is introduced through the following cost specification, which is formulated in discrete time to facilitate direct comparison with the FCC's TELRIC calculations. Let $I_\tau(K)$ be the cost of installing $K \geq 0$ units of capacity at time $\tau = 0, 1, 2, \ldots$. This is the cost of K units of vintage τ capital. An investment in K units of vintage τ capital purchases the ability to produce up to K units of output at time $t \geq \tau$ at operating cost $O_{t,\tau}(q)$, for $q \in [0, K]$. We follow Malcolmson (1975, p. 25) and Nickell (1978, p. 40) in assuming investment is irreversible, perhaps due to prohibitive installation and removal costs. In other words, investment in a vintage is sunk. This is a stylized fact of much telecommunications investment and is the basis for Hausman's (1997) concerns about foregone option value. Exogenous, embodied, capital-augmenting technical progress drives down both investment and operating costs for a new vintage of capital at a constant rate $\gamma \in (0, 1)$. Hence $I_\tau(\cdot) = \gamma^\tau I_0(\cdot)$ and $O_{\tau,\tau}(\cdot) = \gamma^\tau O_{0,0}(\cdot)$ for $\tau = 0, 1, 2, \ldots$.[2] These cost savings may be due to increases in the physical productivity of capital in the production of a homogeneous output, as in Nickell (1978, p. 127); decreases in the operating cost of (potentially) physically homogeneous capital; some combination thereof, as in Malcolmson (1975, p. 26); or decreases in the acquisition and installation cost of (potentially) physically homogeneous capital.[3] It is unnecessary to distinguish among

[2]Biglaiser and Riordan (2000) consider the possibility that technical progress affects operating and investment costs at different rates. They find this complicates the analysis because it makes economic life vary by vintage, but suggest it will not affect the ultimate price path much provided the two rates of growth are not too different.

[3]We do not model disembodied technical progress, which would drive prices down over time in a manner similar to embodied technical progress, but which would not affect retirement and replacement decisions as it is not vintage-specific.

these forms of technical progress for the purpose at hand.

Vintage τ capital depreciates at a constant *nominal* rate $\alpha_\tau > 1$, making operating costs higher as more maintenance is needed to keep the vintage operating. Thus $O_{t,\tau}(\cdot) = \alpha_\tau^{t-\tau} O_{\tau,\tau}(\cdot)$ for $\tau = 0, 1, 2, \dots$ and $t = \tau, \tau+1, \dots$. Clearly, α_τ is not an economic depreciation rate in general. Rather, α_τ is a constant-growth-rate parameterization of Malcolmson's (1975, p. 26) operating cost and Nickell's (1978, p. 127) maintenance cost.[4] Nickell separates maintenance from other variable operating costs (e.g., labor) and assumes that the former increases as a vintage ages while the latter may take any time path. Malcolmson does not separate maintenance from other variable operating costs and does not require that the combination increase as a vintage ages. However, both authors adopt the fixed-coefficients technology for each vintage that is implicit in this specification. $\alpha_\tau > 1$ anticipates a generally inflationary environment for other operating costs; or at least maintenance costs that accelerate, as each vintage ages, rapidly enough to offset any decreases in other variable operating costs. Technical progress also manifests in the evolution of the nominal depreciation rate α_τ, which we assume is nonincreasing in τ.

Combining the assumptions of constant rates of technical progress and nominal depreciation yields $O_{t,\tau}(\cdot) = \gamma^\tau \alpha_\tau^{t-\tau} O_{0,0}(\cdot)$ for $\tau = 0, 1, \dots$ and $t = \tau, \tau + 1, \dots$. Here, $I_0(\cdot)$ and $O_{0,0}(\cdot)$ are given initial conditions for the evolution of future costs.

Note that we follow Malcolmson in omitting from the model a Jorgensonian exogenous physical depreciation rate. Although it is possible to include both maintenance costs and mechanistic decay, as in Nickell's model, it is unnecessary to do so and even a bit difficult to imagine mechanistic decay that persists despite arbitrarily large maintenance expenditures.[5]

Since the objective of the TELRIC paradigm is to emulate a competitive equilibrium we depart from Malcolmson and Nickell by assuming price-taking behavior in the product market. A perfectly-informed

[4] The asset is not a "one-hoss-shay" as it has ongoing maintenance costs. A one-hoss-shay has zero maintenance costs until some exogenous date, at which time the maintenance costs jump to infinity. Although the basic one-hoss-shay is relatively easy to study theoretically, it does not comport with the FCC's inclusion of operating costs in the HCPM and does not allow for an endogenous determination of economic life.

[5] Biglaiser and Riordan's recent (2000) application of these models to the study of price cap and rate-of-return regulation includes mechanistic decay, but their operating costs do not vary with the age of the vintage. Thus, their model does not admit maintenance expenditures that can be accelerated to overcome mechanistic decay as a vintage ages. This formulation follows Jorgenson (1963, 1967), but has the perhaps undesirable feature that a price-taking firm will never scrap a vintage unless there is technical progress that drives down output prices. The present formulation does not have this feature, and also permits calibration entirely in terms of cost, rather than physical, parameters. This is convenient later in the paper when we compare TELRIC and competitive equilibrium prices.

price-taking firm faces a known sequence of output prices $\{p_t\}_{t=0}^{\infty}$. At each time t, the firm must choose the number of units of capacity $K_t \geq 0$ to install and a utilization rate $u_{t,\tau} \in [0,1]$ for each vintage $\tau \leq t$. The firm may be endowed with multiple vintages at time 0. Denote the capacities of these initially endowed vintages by $K_{-b}, \ldots, K_{-1}$, where $b \geq 0$ is the number of initially endowed vintages. We assume throughout that the initially endowed capacities are finite. We use the notation introduced above for the costs of these initially endowed vintages, but do not require that their costs be related according to the rate of technical progress γ. Rather, we simply require that I_τ, $O_{\tau,\tau}$, and α_τ be nonincreasing in τ for $\tau \leq 0$. Thus, cash flow at time t includes the following components:

$$\begin{aligned} \text{Investment Cost:} \quad & I_t(K_t) = \gamma^t I_0(K_t), \\ \text{Operating Cost:} \quad & OC_t \equiv \sum_{\tau=-b}^{t} O_{t,\tau}(u_{t,\tau}K_\tau) = \sum_{\tau=-b}^{t} \alpha_\tau^{t-\tau} O_{\tau,\tau}(u_{t,\tau}K_\tau), \text{ and} \\ \text{Sales Revenue:} \quad & R_t \equiv p_t \sum_{\tau=-b}^{t} u_{t,\tau}K_\tau. \end{aligned}$$

Due to their magnitude, income taxes must be incorporated into any model that is used to actually calculate equilibrium prices. This requires calculation of *accounting* depreciation, whereby I_τ is spread over some useful life and then some salvage value is recovered at the end of that life, because accounting depreciation determines, in part, the flow of business income tax obligations, and thereby has significant effects on the timing of cash flows. Note, however, that it is *only* for tax purposes that accounting depreciation is relevant. Without taxes, cash flow in period t is simply $R_t - I_t - OC_t$, and the only depreciation in this expression is the nominal rate of increase α_τ in the operating cost of vintage τ capital. That is, it is not necessary to consider *any form* of accounting or economic depreciation to calculate $R_t - I_t - OC_t$ (i.e, it is unnecessary to spread I_τ over some "economic" life). Indeed, in a world without business income taxes the accounting concept of depreciation is completely irrelevant to a firm's optimal investment plan since, as the cash flow expression $R_t - I_t - OC_t$ indicates, the timing and magnitude of cash flows are not determined by accounting depreciation.

Our introduction of taxes here differs from other models. Malcolmson (1975) and Biglaiser and Riordan (2000) do not consider taxes. Jorgenson (1963) incorporates taxes but does not have vintages of capital and does not carefully distinguish between tax depreciation and mechanistic decay. Nickell (1978, chapter

9) also incorporates taxes but focuses on the relationship between the tax structure and the optimal capital structure. In contrast, since we are not studying uncertainty we take the firm's capital structure as given, and emphasize instead the cash flows created by the U.S. corporate income tax structure.

Telecommunications equipment is depreciated for U.S. Federal income tax purposes by the Modified Accelerated Cost Recovery System (MACRS) (Internal Revenue Code of 1986, as amended, §168; Rev. Proc. 87-56, 1987-2 C.B. 674), according to which the government tax authority exogenously specifies an asset-specific schedule of percentages of the investment that can be expensed each year for tax purposes. Accordingly, let $d_i > 0$ denote the expense percentage for capital of age i, $i = 0, \ldots, T-1$, where T is the tax life of the asset. The vector of percentages is on the unit simplex: $\sum_{i=0}^{T-1} d_i = 1$. For notational ease, set $d_i = 0$ for $i \geq T$. Now let $S_{t,\tau}$ be the salvage price of vintage τ capital at time $t \geq \tau$, and $s_{t,\tau}$ denote the percentage of its vintage τ capital the firm sells for salvage at time $t \geq \tau$. Then salvage revenues at time t are

$$SR_t \equiv \sum_{\tau=-b}^{t} S_{t,\tau} s_{t,\tau} K_\tau .$$

At time t, the percent of vintage τ capital that has not yet been sold for salvage is $1 - \sum_{i=\tau}^{t} s_{i,\tau}$. The firm can only claim depreciation expense on this portion of its vintage τ capital. When salvage of vintage τ capital occurs at time t, the firm may expense the entire remaining depreciation percentage, $\sum_{i=t}^{\tau+(T-1)} d_{i-\tau}$. Thus, depreciation expense at time t for tax purposes is

$$D_t \equiv \sum_{\tau=t-(T-1)}^{t} \left[1 - \sum_{i=\tau}^{t} s_{i,\tau}\right] I_\tau(K_\tau) d_{t-\tau} + \sum_{\tau=t-(T-1)}^{t} s_{t,\tau} I_\tau(K_\tau) \left[\sum_{i=t}^{\tau+(T-1)} d_{i-\tau}\right].$$

The firm must pay tax on both its operating profits and its capital gains from salvage. Letting $\kappa \in [0,1)$ denote the (constant) marginal tax rate, tax cost at time t is

$$TC_t \equiv \kappa[R_t - OC_t + SR_t - D_t].$$

Thus, cash flow at time t is

$$\begin{aligned}
\pi_t &\equiv R_t - OC_t - I_t(K_t) + SR_t - TC_t \\
&= (1-\kappa)[R_t - OC_t + SR_t] - I_t(K_t) + \kappa D_t \\
&= (1-\kappa) \sum_{\tau=-b}^{t} [p_t u_{t,\tau} K_\tau - O_{t,\tau}(u_{t,\tau} K_\tau) + S_{t,\tau} s_{t,\tau} K_\tau] - I_t(K_t) \\
&\quad + \kappa \sum_{\tau=t-(T-1)}^{t} I_\tau(K_\tau) \left[\left[1 - \sum_{i=\tau}^{t} s_{i,\tau} \right] d_{t-\tau} + s_{t,\tau} \sum_{i=t}^{\tau+(T-1)} d_{i-\tau} \right].
\end{aligned} \tag{1}$$

It is worth pausing here to review the timing implicit in these expressions. Any capacity acquired at time t is available for immediate use and the tax expense for the first period is also taken immediately. If some portion of the asset is sold for salvage at time t then the revenue from that sale is received immediately and any taxes on the capital gain are paid immediately, and the portion sold is not available for production at time t. In other words, we have adopted a beginning-of-period placement and removal convention. MACRS and the FCC actually use mid-year conventions. For comparison purposes later in the paper, we use a beginning-of-period version of the FCC's rules. This simplifies the calculations considerably and puts the FCC's rules on the same foundation as the model, without changing the fundamental nature of the comparison.

Solving for equilibrium prices from the profit objective (1) is a complicated general equilibrium problem. If the salvage percentages $s_{t,\tau}$ are choice variables the vintage characteristics of capital require that there be distinct resale prices for each vintage at each time, in which case we have a general equilibrium problem of dynamic simultaneous price determination in the output, new capital, and all secondhand markets. Allowing $s_{t,\tau}$ to be choices is, essentially, reversibility of a complicated form since it is vintage-specific. Applying the irreversibility assumptions of Malcolmson and Nickell to this context avoids much of the complexity and focuses attention on the main issue of output price determination. However, salvage revenues are an unavoidable part of the regulatory landscape, so they cannot be ignored completely.

A simple assumption that avoids the general equilibrium problem, and which seems plausible given the "dearth of secondhand markets" for capital noted by Nickell (1978, p. 40), is that transactions costs are prohibitive, making salvage prices so low that salvage of a vintage occurs only when operating profit becomes negative and remains negative thereafter. If the salvage price is positive at that time then the buyer of the asset must intend to use it for a different purpose, since the asset has no remaining value in the production of the final product under study here. Alternatively, the salvage price (received by the seller) could be negative at that time if there are removal costs. A common accounting approach used by the FCC to specify salvage revenues at that time is to assume that salvage revenues are a fixed percentage $\theta < 1$ of the initial investment. This structure retains the possibility of either positive or negative salvage revenues while also exogenizing those revenues. Formally, define the *economic life* of an investment in K_τ units of vintage τ capital by

$$L_\tau \equiv \sup\{t \geq \tau \colon p_t u_{t,\tau} K_\tau - O_{t,\tau}(u_{t,\tau} K_\tau) \geq 0 \ \exists u_{t,\tau} \in [0,1]\} - \tau + 1, \tag{2}$$

and note that L_τ depends on the *entire* succeeding sequence of prices $\{p_t\}_{t=\tau}^{\infty}$. Then assume the salvage percentages for vintage τ are zero through time $\tau + L_\tau - 1$, one at time $\tau + L_\tau$, and zero thereafter. With these assumptions, (1) becomes

$$\begin{aligned} \pi_t = (1-\kappa) \sum_{\tau=-b}^{t} [p_t u_{t,\tau} K_\tau - O_{t,\tau}(u_{t,\tau} K_\tau) + \theta I_\tau(K_\tau)\chi(t = \tau + L_\tau)] - I_t(K_t) \\ + \kappa \sum_{\tau=t-(T-1)}^{t} I_\tau(K_\tau) \left[\chi(t < \tau + L_\tau) d_{t-\tau} + \chi(t = \tau + L_\tau) \sum_{i=t}^{\tau+(T-1)} d_{i-\tau} \right], \end{aligned} \tag{1'}$$

where $\chi(\cdot)$ is the characteristic function for the condition $(\cdot)$, taking on 1 when $(\cdot)$ is true and 0 otherwise.

The firm seeks to maximize the present value of cash flows (1') over the life of the asset. The discount factor used for this present value is contentious, so we carefully specify its constituent parts. Let s_d be the firm's share of debt financing and s_e be the share of equity financing. Likewise, let k_d be the firm's (before-tax) cost of debt (i.e., the face rate on the firm's bonds) and k_e be the firm's cost of equity. The firm's before-tax weighted average cost of capital is then $r = s_d k_d + \frac{s_e k_e}{1-\kappa}$ and the firm's discount factor, defined as one divided by one plus the after-tax cost of capital, is

$$\delta = \frac{1}{1 + (1-\kappa) r} \in (0, 1). \tag{3}$$

This discount factor embodies debt financing and its preferential tax treatment since we have neither expensed interest nor credited the attendant tax savings in the cash flow expression (1'). The optimization problem is then

$$\max_{\{K_t, u_{t,\tau}\}_{t=0,\tau=-b}^{\infty,t}} \quad \Pi \equiv \sum_{t=0}^{\infty} \delta^t \pi_t,$$

where π_t is given by (1') and δ is given by (3). The constraints are nonnegativity of K_t, the initial endowments, and utilization rates that must lie between zero and the percentage of the vintage that has not yet been sold for salvage:

$$K_t \geq 0 \; \forall t \geq 0$$

$$(K_{-b}, \ldots, K_{-1}) \text{ given}$$

$$u_{t,\tau} \in \begin{cases} [0,1], & t = \tau, \ldots, \tau + L_\tau - 1 \\ \{0\}, & \text{otherwise.} \end{cases}$$

To make this optimization problem tractable, we follow Malcolmson (1975), Nickell (1975, 1978), and Biglaiser and Riordan (2000) in assuming constant returns in both capacity installation and operating costs. This implies short-run increasing returns for a given vintage up to capacity, and long-run constant returns for each vintage. Then $I_0(K) = F_0 K$ and $O_{0,0}(q) = c_{0,0} q$, where $F_0 > 0$ is the per-unit cost of installing vintage 0 capital and $c_{0,0} > 0$ is the per-unit operating cost of vintage 0 capital at time 0 (i.e., F_0 and $c_{0,0}$ are given per-unit initial conditions for the evolution of future costs).[6] We henceforth use c_τ as shorthand notation for $\gamma^\tau c_{0,0}$ and F_τ as shorthand notation for $\gamma^\tau F_0$ for $\tau \geq 0$; and $c_\tau q$ and $F_\tau K$ as shorthand for $O_{\tau,\tau}(q)$ and $I_\tau(K)$, respectively, for $\tau < 0$. So now installation cost is $I_\tau(K) = F_\tau K = \gamma^\tau F_0 K$ and operating cost is $O_{t,\tau}(q) = \alpha_\tau^{t-\tau} c_\tau q = \left(\frac{\gamma}{\alpha_\tau}\right)^\tau \alpha_\tau^t c_0 q$, for $\tau = 0, 1, \ldots$ and $t = \tau, \tau+1, \ldots$; the operating profit from vintage τ capital at time t can be expressed as a per-unit margin $m_{t,\tau} \equiv p_t - \alpha_\tau^{t-\tau} c_\tau$ multiplied by the number of units utilized, $u_{t,\tau} K_\tau$; and the economic life L_τ depends only on the margin:

$$L_\tau = \sup\{t \geq \tau \colon m_{t,\tau} \geq 0\} - \tau + 1. \tag{2'}$$

Life L_τ is simply the length of time over which operating profits are nonnegative. Substituting these into (1'), using $d_j = 0$ for $j \geq T$, and collecting terms yields

$$\pi_t = \sum_{\tau=-b}^{t} K_\tau \Bigg\{ (1-\kappa)[u_{t,\tau} m_{t,\tau} + \theta F_\tau \chi(t = \tau + L_\tau)] + \kappa F_\tau \left[\chi(t < \tau + L_\tau) d_{t-\tau} + \chi(t = \tau + L_\tau) \sum_{i=t}^{\tau+(T-1)} d_{i-\tau} \right] \Bigg\} - F_t K_t. \tag{1''}$$

[6] We assume the cost initial conditions are strictly positive because there is no dynamic problem if $F_0 = 0$, while no asset will ever be retired if $c_{0,0} = 0$ except, possibly, to obtain the salvage revenues.

2. Capital Demand and Output Supply

Since one emphasis here is to formulate a precise characterization of dynamic competitive equilibrium prices, in this section we carefully develop competitive supply in this environment. Ignoring the constraints, the first derivatives of the firm's objective function with constant returns are

$$\frac{\partial \Pi}{\partial K_\tau} = \sum_{t=\tau}^{\tau+L_\tau-1} \delta^t [(1-\kappa)u_{t,\tau}m_{t,\tau} + \kappa F_\tau d_{t-\tau}] + \delta^{\tau+L_\tau} F_\tau \left[(1-\kappa)\theta + \kappa \sum_{t=\tau+L_\tau}^{\tau+(T-1)} d_{t-\tau}\right] - \delta^\tau F_\tau \tag{4a}$$

$$\frac{\partial \Pi}{\partial u_{t,\tau}} = \delta^t K_\tau (1-\kappa) m_{t,\tau} \text{ for } t \geq \max\{0,\tau\}. \tag{4b}$$

It is clear from (4b) that optimal utilization $u_{t,\tau}$ of vintage τ capital is one for $t=\tau,\ldots,\tau+L_\tau-1$ provided $m_{t,\tau}>0$, is zero for $t=\tau,\ldots,\tau+L_\tau-1$ when $m_{t,\tau}<0$ and also for $t \geq L_\tau$, and is anywhere between zero and one for $t=\tau,\ldots,\tau+L_\tau-1$ when $m_{t,\tau}=0$. This manifestation of constant returns in operations is noted by Biglaiser and Riordan (2000, p. 748). Substituting this into (4a) yields

$$\frac{\partial \Pi}{\partial K_\tau} = \sum_{t=\tau}^{\tau+L_\tau-1} \delta^t \left[(1-\kappa)\chi(m_{t,\tau}>0)m_{t,\tau} + \kappa F_\tau d_{t-\tau}\right] + \delta^{\tau+L_\tau} F_\tau \left[(1-\kappa)\theta + \kappa \sum_{t=\tau+L_\tau}^{\tau+(T-1)} d_{t-\tau}\right] - \delta^\tau F_\tau. \tag{4a'}$$

Since we have assumed constant returns, this derivative is independent of K_τ and the optimal choice is either a corner, $K_\tau=0$ or $K_\tau=\infty$, or the firm is indifferent to the level of K_τ. These cases can be characterized by rewriting the derivative as the difference between the net present values of the after-tax revenue and after-tax cost streams generated by one unit of vintage τ capital over its economic life:

$$\frac{\partial \Pi}{\partial K_\tau} = \delta^\tau [NPVR_\tau - NPVC_\tau], \tag{4a''}$$

where

$$NPVR_\tau \equiv (1-\kappa) \sum_{t=0}^{L_\tau-1} \delta^t p_{\tau+t}[\chi(m_{\tau+t,\tau}>0) + \chi(m_{\tau+t,\tau}=0)u_{\tau+t,\tau}], \tag{5}$$

$$\begin{aligned} NPVC_\tau \equiv{}& F_\tau \left[1 - \kappa \sum_{t=0}^{L_\tau-1} \delta^t d_t\right] \\ &+ (1-\kappa) \sum_{t=0}^{L_\tau-1} \delta^t \alpha_\tau^t c_\tau [\chi(m_{\tau+t,\tau}>0) + \chi(m_{\tau+t,\tau}=0)u_{\tau+t,\tau}] \\ &- \delta^{L_\tau} \left[\theta F_\tau - \kappa\left(\theta F_\tau - F_\tau \sum_{t=L_\tau}^{T-1} d_t\right)\right]. \end{aligned} \tag{6}$$

The cost $NPVC_\tau$ consists of three components:

1. $F_\tau \left[1 - \kappa \sum_{t=0}^{L_\tau - 1} \delta^t d_t\right]$ is the after-tax cost of the initial investment over its economic life, accounting for tax writeoffs that occur over time according to the MACRS depreciation schedule.
2. $(1-\kappa)\sum_{t=0}^{L_\tau - 1} \delta^t \alpha_\tau^t c_\tau [\chi(m_{\tau+t,\tau} > 0) + \chi(m_{\tau+t,\tau} = 0) u_{\tau+t,\tau}]$ is the after-tax operating cost over the economic life.
3. $-\delta^{L_\tau}\left[\theta F_\tau - \kappa\left(\theta F_\tau - F_\tau \sum_{t=L_\tau}^{T-1} d_t\right)\right]$ is the after-tax cost saving obtained when the capital is sold for salvage, which consists of the salvage revenue less the tax on the capital gain (loss). The capital gain (loss) is given by the difference between salvage revenues and any remaining undepreciated cost basis of the initial investment.

Hence the factor demand correspondence for vintage τ capital is

$$K_\tau^*\left(\{p_t\}_{t=\tau}^{\infty}\right) \equiv \begin{cases} \{0\}, & \text{if } NPVR_\tau < NPVC_\tau \\ [0,\infty], & \text{if } NPVR_\tau = NPVC_\tau \\ \{\infty\}, & \text{if } NPVR_\tau > NPVC_\tau \end{cases}. \tag{7}$$

Note that the value of this correspondence depends, at least in principle, on the *entire* succeeding sequence of output prices $\{p_t\}_{t=\tau}^{\infty}$ as denoted above. The dependence on $p_\tau, \ldots, p_{\tau+L_\tau-1}$ is explicit in (5) and (6), but both equations depend on all later output prices as well because the economic life L_τ is endogenous and depends in general on the entire succeeding sequence of output prices.

The firm can supply output at time t by utilizing previously-installed capacity or by installing new capacity. For a given price sequence $\{p_t\}_{t=0}^{\infty}$, the amount of each vintage installed must be optimal according to (7). Once installed, however, the capacity of vintage τ capital is sunk (irreversible) and therefore cannot be varied at time $t > \tau$. Let $\bar{K}_\tau \in K_\tau^* \; \forall \tau$ denote the sunk capacities. At time t, it is optimal to fully utilize the sunk capacity of vintage $\tau \le t$ if $m_{t,\tau} > 0$, and to (perhaps temporarily) shut down vintage τ if $m_{t,\tau} < 0$. Any utilization level is optimal if $m_{t,\tau} = 0$. Thus the supply correspondence at time t is

$$q_t^s(\{p_\tau\}_{\tau=0}^{\infty}) = \sum_{\tau=-b}^{t} [\{\bar{K}_\tau \chi(m_{t,\tau} > 0)\} + [0, \bar{K}_\tau \chi(m_{t,\tau} = 0)]]. \tag{8}$$

Note that supply at time t indeed depends on the entire sequence of prices, since the sunk capacities $\bar{K}_\tau$ must satisfy $\bar{K}_\tau \in K_\tau^*(\{p_t\}_{t=\tau}^{\infty})$ for every $\tau \ge 0$.

3. Competitive Equilibrium Prices

As discussed at the outset, the FCC's evident objective in calculating "forward-looking" costs is emulation of the forward-looking nature of competitive equilibrium prices. In a dynamic context, a competitive equilibrium is a price sequence $\{p_\tau\}_{\tau=0}^{\infty}$ that, at each time t, equates market demand with the supply provided by price-taking firms. Let $q_t^d(p)$ denote demand at time t. We assume q_t^d is a nonempty, upper semicontinuous, nonnegative, nonincreasing, convex- and compact-valued correspondence on $\mathbb{R}_+^1$ with $q_t^d(p) = \{0\}$ at some finite p and $q_t^d(0) > \{0\}$.[7] The definition of a competitive equilibrium is immediate from q_t^d and (8).

Definition 1. *A* **Competitive Equilibrium** *is a price sequence* $\{p_\tau\}_{\tau=0}^{\infty}$ *such that there exists a capacity sequence* $\{\bar{K}_t\}_{t=0}^{\infty}$ *satisfying:*

1. $\bar{K}_t \in K_t^* \left(\{p_\tau\}_{\tau=t}^{\infty}\right) \ \forall t$

2. $0 \in q_t^d(p_t) - q_t^s\left(\{p_\tau\}_{\tau=0}^{\infty}\right) \ \forall t.$

This definition vividly displays the sense in which competitive equilibrium prices are "forward-looking." Note first that, since demand is finite, a necessary condition for equilibrium is $NPVR_\tau \leq NPVC_\tau$ for every $\tau \geq 0$. Otherwise $K_\tau^* = \{\infty\}$ from (7) for some vintage τ, so for that vintage item 1 of the definition is $\bar{K}_\tau = \infty$. Moreover, $NPVR_\tau > NPVC_\tau$ implies $m_{t,\tau} > 0$ for some $t \geq \tau$, at which time $q_t^d - q_t^s = \{-\infty\}$ from (8). So in any time period in which investment occurs the net present value of *present and future* revenue provided by competitive equilibrium prices must equal the net present value of *present and future* costs of the investment. This is the "forward-looking" standard of the competitive equilibrium, and in equilibrium $NPVC_\tau$ is the "forward-looking" cost of a unit of vintage τ capital.

The necessary condition $NPVR_\tau \leq NPVC_\tau$ suggests that we begin the derivation of equilibrium prices by identifying a price sequence that equates $NPVR_\tau$ and $NPVC_\tau$ at every time τ. For this purpose we make the additional simplifying assumption that nominal depreciation rates are identical across vintages. This eliminates one source of time dependence in the model. With this assumption, Theorem 1 identifies a price sequence that is zero-profit for every vintage of capital. All proofs are in the Appendix.

Theorem 1. *Suppose* $\alpha_\tau = \alpha \ \forall \tau \geq 0$. *Let*

$$NPVC(L) \equiv F_0 \left[1 - \kappa \sum_{t=0}^{L-1} \delta^t d_t\right] + (1-\kappa)c_0 \sum_{t=0}^{L-1} (\delta\alpha)^t - \delta^L F_0 \left[\theta - \kappa \left(\theta - \sum_{t=L}^{T-1} d_t\right)\right]$$

[7]For purposes herein, we define set inequalities for nonempty subsets A and B of $\bar{\mathbb{R}}^1$ as follows. If $a \in A$ and $b \in B$ implies $a < b$ we write $A < B$. If $a \in A$ and $b \in B$ implies $a \leq b$ we write $A \leq B$. If $a \in A$ implies existence of $b \in B$ such that $a \leq b$ we write $A \leqq B$. A correspondence $q_t^d(p)$ is "nonincreasing" if $p \leq p'$ implies $q_t^d(p') \leqq q_t^d(p)$, and is "nonnegative" if $\{0\} \leq q_t^d(p) \ \forall p$.

be the net present value of the cost of a unit of vintage 0 capital over a life of L*, assuming* $m_{t,0} \geq 0$ *for* $t = 0, \ldots, L-1$ *and* $u_{t,0} = 1$ *when* $m_{t,0} = 0$*. Then*

1. *There exists a unique integer* $L \geq 1$ *such that*

$$\left(\frac{\alpha}{\gamma}\right)^{L-1} c_0 \leq \frac{NPVC(L)}{(1-\kappa)\sum_{j=0}^{L-1}(\delta\gamma)^j} < \left(\frac{\alpha}{\gamma}\right)^{L} c_0.$$

2. *For that integer* L*, the price sequence*

$$p_t^* \equiv \frac{\gamma^t NPVC(L)}{(1-\kappa)\sum_{j=0}^{L-1}(\delta\gamma)^j} \qquad t = 0, 1, \ldots$$

   equates $NPVR_\tau$ *and* $NPVC_\tau$ *at every time* $\tau = 0, 1, \ldots$*.*

Item 2 is the central formula that gives the competitive equilibrium price sequence (see Theorem 2). In Section 5 below we show exactly how it can be used to calculate prices.

Although somewhat tedious to prove, Theorem 1 is intuitive. Item 2 is

$$p_t^*(1-\kappa)\sum_{j=0}^{L-1}(\delta\gamma)^j = \gamma^t NPVC(L).$$

For a vintage t asset whose economic life is L, the left side of this equation is the present value of the after-tax revenue generated by one unit of the asset when prices fall at the rate γ and the discount factor is δ. The right side is the present value of the after-tax cost of one unit of a vintage t asset over its life when $NPVC(L)$ is the present value of the after-tax cost of one unit of a vintage 0 asset over its life, and costs fall at the rate γ. Thus item 2 has a clear interpretation as a dynamic break-even condition. Substituting item 2 into item 1 of the theorem, first when $t = \tau + (L-1)$ and then when $t = \tau + L$, yields, respectively,

$$\alpha^{L-1}\gamma^\tau c_0 \leq p_{\tau+(L-1)}^*$$

$$p_{\tau+L}^* < \alpha^L \gamma^\tau c_0.$$

These inequalities define the economic life L of a vintage τ, whose initial operating cost is $\gamma^\tau c_0$, as the age at which the operating cost, which grows at the rate α, rises above the price, which falls at the rate γ. In other words, the economic life is determined by a shutdown condition, and the shutdown is permanent when it occurs because operating costs for a given vintage continually rise due to nominal depreciation while prices continually fall due to technical progress. In this model, *both* nominal depreciation and technical progress contribute to *economic* obsolescence, so both rates of change are determinants of the economic life of a vintage of capital. Essentially, a vintage of capital is discarded after an interval during which nominal depreciation and technical progress proceed until the old unit of capacity is no longer economically viable.

Now we must consider whether the condition $NPVR_\tau = NPVC_\tau \ \forall\tau$ is sufficient for equilibrium. The general answer is "no." For example, suppose the economic life L determined by item 1 of Theorem 1 is greater than 2 and the endowed capacity is zero. Then equilibrium requires $\bar{K}_0 \in q_0^d(p_0^*)$. Suppose $q_0^d(p_0^*) > \{0\}$, so that $\bar{K}_0 > 0$. Since $L > 2$ and $m_{t,\tau}$ is strictly decreasing in t, we have $m_{1,0} > 0$ and $m_{1,1} > 0$, so $q_1^s(\{p_t^*\}_{t=0}^\infty) = \{\bar{K}_0\} + \{\bar{K}_1\} \geq \{\bar{K}_0\}$. Thus, if $q_1^d(p_1^*) < \{\bar{K}_0\}$, then $q_1^d(p_1^*) - q_1^s(\{p_t^*\}_{t=0}^\infty) < \{0\}$, in which case item 2 of Definition 1 at $t = 1$ is inconsistent with item 1 for $t = 0, 1$ when the price sequence is $\{p_t^*\}_{t=0}^\infty$. This problem arises because demand falls from time 0 to time 1, while supply is nondecreasing during this time because the economic life is greater than 2. If there is positive probability that demand will fall then $\{p_t^*\}_{t=0}^\infty$ results in excess supply. The equilibrium prices must be lower in some time periods to eliminate the excess supply, and higher in other time periods to restore the breakeven condition.

These observations suggest that an equilibrium price sequence for a general demand sequence may be quite complicated. However, in telecommunications it is reasonable to assume that market demand will not shrink in the foreseeable future. In this case the price sequence identified by Theorem 1 is an equilibrium, except possibly at some initial times if there is excess supply from the endowed capacities.

Theorem 2. *Assume* $\alpha_\tau = \alpha \ \forall\tau \geq 0$, *and that* $q_t^d(p) \leqq q_{t+1}^d(p) \ \forall p$, *for* $t = 0, 1, \ldots$. *Also suppose* $\sum_{\tau=-b}^{-1}\{\bar{K}_\tau \chi(m_{0,\tau} > 0)\} \leqq q_0^d(p_0^*)$. *Then* $\{p_t^*\}_{t=0}^\infty$ *is a competitive equilibrium.*

This sequence must be modified slightly when there is excess supply at price p_0^* from the endowed capacities at time $t = 0$. To obtain an equilibrium price sequence in this setting we simply lower the price below p_0^* until the excess supply is eliminated, and keep the prices at levels below p_t^* that eliminate the excess supply as long as necessary. This situation is resolved in finite time because the operating costs for the endowed capacities are strictly increasing over time and the sequence p_t^* approaches zero as $t \to \infty$.

Corollary. *Assume the conditions of Theorem 2, except that* $\sum_{\tau=-b}^{-1}\{\bar{K}_\tau \chi(m_{0,\tau} > 0)\} > q_0^d(p_0^*)$. *Let*

$$t^* \equiv \max\left\{t\colon \sum_{\tau=-b}^{-1}\{\bar{K}_\tau \chi(m_{t,\tau} > 0)\} > q_t^d(p_t^*)\right\},$$

where $m_{t,\tau}$ is the margin for price p_t^*. Then there exist prices $\bar{p}_t < p_t^*$ for $t = 0, \ldots, t^*$ such that

$$\bar{p}_0, \ldots, \bar{p}_{t^*}, p_{t^*+1}^*, p_{t^*+2}^*, \ldots$$

is a competitive equilibrium.

Remark. t^* is finite. This follows because $m_{t,\tau} = p_t^* - \alpha_\tau^{t-\tau} c_\tau$ (recall that $c_\tau \neq \gamma^\tau c_0$ and $\alpha_\tau \neq \alpha$ for $\tau < 0$) is strictly decreasing in t and approaches $-\infty$ as $t \to \infty$, so $\sum_{\tau=-b}^{-1}\{\bar{K}_\tau \chi(m_{t,\tau} > 0)\} = \{0\}$ for t beyond some finite time, while q_t^d is nonnegative.

An appreciation for the confusion surrounding the effect of technical progress on competitive equilibrium prices can be obtained by comparing p_t^* with the end-office switching costs reported in the New York Section 271 Order. The New York Public Service Commission determined Bell Atlantic's switching costs to be $303 per line, and then *reduced* this estimate to $192 per line to "... account for declining switch prices within the industry." (New York Order, 1999, ¶242) In response to AT&T's claim that this estimate is not TELRIC-based, the FCC concluded that it had "... no basis to disagree with the New York Commission that its calculation of switching costs is a 'reasonable calculation of pertinent costs, arrived at by the New York Commission Staff's application of forward-looking TELRIC analysis.'" (New York Order, 1999, ¶242) Note, however, that p_t^* falls at the rate of technical progress *over time*, not *instantaneously*, and p_t^* is a *long run* equilibrium price sequence in this dynamic context. Indeed, the formula for p_t^* reported in Theorem 1 above shows that, at the installation time $t = 0$, the competitive equilibrium price is *higher* when there is technical progress (i.e., $\gamma < 1$) than when there is no technical progress ($\gamma = 1$). When there is technical progress, the equilibrium price must be higher initially in order to compensate for the fact that it will be lower later. It is only in this way that market-clearing prices will allow the firm to recoup its investment costs over the economic life of the asset. The higher initial price reflects the economic benefit of delaying installation in anticipation of technical progress.

As noted in the introduction, Salinger (1998) argues that equilibrium prices must be calculated *before* the components of forward-looking costs can be calculated because one such component is economic depreciation, which under irreversibility is defined as the dynamic change in the net present value of an asset, and which therefore cannot be known until the revenue stream generated by the asset (i.e., prices) is known. This sequential calculation is possible when utilization is assumed to be an exogenous path that does not respond to changes in prices, as in Salinger's model (Salinger, p. 151). In contrast, the supply correspondence (8) does not allow a sequential calculation because it reflects endogenous utilization that depends on revenue, so there is simultaneity between equilibrium prices and forward-looking costs, irrespective of whether those costs are separated into components, one of which may be economic depreciation. This is explicit in Definition 1, where equilibrium prices depend on the net present values of costs from (7), which in turn depend on prices through the shutdown conditions in (6) (including the economic life of the asset). Due to constant returns, the endogenous utilization in (8) is a binary phenomenon for each vintage of capacity, unless the shutdown condition just binds for a particular vintage. When a shutdown condition just binds, Definition 1 requires that endogenous utilization for that vintage be determined by market-clearing. No other utilization pattern is both profit-maximizing for a price-taking firm and market-clearing, given the specified (constant returns) operating cost function. However, once utilization is endogenized there will be some relationship between economically efficient utilization and the revenue it generates for any technical structure.[8]

Salinger notes (p. 152) that "It is possible to calculate forward-looking costs without measuring depreciation." This is an important point. Indeed, the *only* depreciation in the equilibrium forward-looking cost expression $NPVC_\tau$ of Theorem 1 is tax depreciation and nominal depreciation α. Thus there is *absolutely no reason* for a regulator who is well-informed enough to calculate total forward-looking cost $NPVC_\tau$ to bother with, or confuse matters with, some accounting or economic depreciation calculation that differs from the tax code. The ability to calculate equilibrium $NPVC_\tau$ is equivalent to the ability to calculate the regulator's sought-after prices based on forward-looking costs, and nothing more is needed. In section 5 we successfully perform exactly the indicated calculation from FCC data without any consideration of accounting or economic depreciation beyond the relevant MACRS schedule.

4. Comparison of Competitive Equilibrium and TELRIC Prices

The comparison between competitive equilibrium and TELRIC prices begins with the relationship between $NPVC(L)$ from Theorem 1 and the corresponding calculation in the HCPM. The ultimate objective is to

[8]The operating cost function could be modified to reflect decreasing returns in operations. This would amount to relaxing the capacity constraint, so that investment in a unit of capacity buys a general convex operating cost function rather than the function used here, which is horizontal up to K_t and vertical at K_t. Such a modification would create the potential for endogenous partial utilization, and would make the dependence between prices and costs involve both the level of operation as well as the shutdown decision.

calculate before-tax prices since these are the prices at which transactions actually occur. Accordingly, the HCPM calculates cost on a before-tax basis, which is analogous to $\frac{NPVC(L)}{1-\kappa}$ in the expression for p_t^*. The calculations reported here for the HCPM are from the Excel spreadsheet "MO_Southwestern Bell-Mi_Default Scenario_WC," which contains the final default HCPM results for Southwestern Bell in Missouri, as posted at http://www.fcc.gov/ccb/apd/hcpm/results.zip in conjunction with the Final Inputs Order (FCC, 1999).

Initial Investment Cost. The HCPM calculates initial investment cost and salvage revenue on a per-dollar-of-investment basis, so we must compare the HCPM calculations with the corresponding parts of $\frac{NPVC(L)}{F_0(1-\kappa)}$. The initial investment part of $\frac{NPVC(L)}{F_0(1-\kappa)}$ is

$$\frac{1}{1-\kappa}\left[1-\kappa\sum_{t=0}^{L-1}\delta^t d_t\right]. \tag{9}$$

The HCPM calculates per-dollar initial investment cost in the KCCFactor tab of the spreadsheet. The calculation begins by assuming straight-line depreciation (FCC, 1999, ¶422) over an estimated economic life $\tilde{L}$, so the direct cost at one time t is $1/\tilde{L}$.[9] There is also a carrying cost on the part of the initial investment that has not yet been depreciated. Using the beginning-of-period convention adopted here, the percentage of the initial investment that is carried from period t to period $t+1$ is $1-\frac{t+1}{\tilde{L}}$. Since r is the firm's before-tax cost of capital, the before-tax direct and carrying cost at time t is

$$\frac{1}{\tilde{L}}+\tilde{\delta}r\left\{1-\frac{t+1}{\tilde{L}}\right\},$$

where $\tilde{\delta}$ is the discount factor used by the HCPM.

This is the basic expression for time t per-dollar initial investment cost in the HCPM. However, the actual tax saving the firm has received at time t is determined by the MACRS depreciation schedule rather than the straight-line schedule assumed by the FCC. So, the HCPM decreases the depreciation reserve by the amount of accelerated tax savings, which yields a time t cost of

$$y_t \equiv \frac{1}{\tilde{L}}+\tilde{\delta}r\left\{1-\frac{t+1}{\tilde{L}}-\kappa\left[\sum_{j=0}^{t}d_j-\sum_{j=0}^{t}\frac{1}{\tilde{L}}\right]\right\} \tag{10}$$

[9]The HCPM uses a modification of straight-line depreciation known as the equal-life-group method (FCC, 1999, ¶¶422-4). This method modifies the straight-line schedule to account for the probability that the asset is retired at some life other than $\tilde{L}$. In principle such probabilities could be incorporated into the expected profit function (1), but it is unnecessary to do so in order to compare the FCC's TELRIC *method* with competitive equilibrium prices. Equation (9) is formulated under the assumption that mortality occurs at life L with probability 1, while (10) is formulated under the assumption that mortality occurs at life $\tilde{L}$ with probability 1. Thus, the two equations make the same mortality assumptions when $\tilde{L}=L$.

for $t = 0, \ldots, \tilde{L}-1$. The FCC's TELRIC before-tax net present value of one dollar of initial investment outlay is then

$$NPV(FCC) \equiv \sum_{t=0}^{\tilde{L}-1} \tilde{\delta}^t y_t.$$

Performing the indicated algebra yields

$$NPV(FCC) = \frac{1-\tilde{\delta}^{\tilde{L}}}{\tilde{L}(1-\tilde{\delta})}\left[1-\frac{\tilde{\delta}r(1-\kappa)}{1-\tilde{\delta}}\right] + \frac{\tilde{\delta}r}{1-\tilde{\delta}}\left[1-\kappa\tilde{\delta}^{\tilde{L}} - \kappa\left(\sum_{t=0}^{\tilde{L}-1}\tilde{\delta}^t d_t - \tilde{\delta}^{\tilde{L}}\sum_{t=0}^{\tilde{L}-1} d_t\right)\right]. \quad (11)$$

Salvage Revenue. Subtracting the salvage revenue part of $\frac{NPVC(L)}{F_0(1-\kappa)}$ from (9) yields

$$\frac{1}{1-\kappa}\left[1-\kappa\sum_{t=0}^{L-1}\delta^t d_t - \delta^L\kappa\sum_{t=L}^{T-1} d_t\right] - \delta^L\theta. \quad (9')$$

The HCPM incorporates before-tax salvage revenue into the cost calculation by adjusting the estimated economic life $\tilde{L}$ of the asset upward before calculating the net present value of initial investment cost (see column K of the Inputs tab of the spreadsheet). This is a proxy for directly including the discounted salvage revenue gross of tax on the capital gain, $\tilde{\delta}^{\tilde{L}}\theta$, as a negative cost. It is useful for comparison purposes to use the same economic lives in the competitive equilibrium and TELRIC calculations, so to facilitate the comparison we include $-\tilde{\delta}^{\tilde{L}}\theta$ directly in the TELRIC cost calculation rather than using the "adjusted life" proxy for salvage. Subtracting this salvage revenue from the HCPM's initial investment cost yields[10]

$$NPV(FCC) = \frac{1-\tilde{\delta}^{\tilde{L}}}{\tilde{L}(1-\tilde{\delta})}\left[1-\frac{\tilde{\delta}r(1-\kappa)}{1-\tilde{\delta}}\right] + \frac{\tilde{\delta}r}{1-\tilde{\delta}}\left[1-\kappa\tilde{\delta}^{\tilde{L}} - \kappa\left(\sum_{t=0}^{\tilde{L}-1}\tilde{\delta}^t d_t - \tilde{\delta}^{\tilde{L}}\sum_{t=0}^{\tilde{L}-1} d_t\right)\right] - \tilde{\delta}^{\tilde{L}}\theta. \quad (11')$$

Comparison of Initial Investment Costs and Salvage Revenues. Assuming $L = \tilde{L}$, the HCPM is emulating the non-operating part of competitive equilibrium cost if (11') equals (9'). These equations are only equal if $\tilde{\delta} = \delta$. To see this, substitute $r = \frac{1-\delta}{\delta(1-\kappa)}$ and $\delta = \tilde{\delta}$ into (11') and simplify to obtain

$$NPV(FCC) = \frac{1}{1-\kappa}\left[1-\kappa\sum_{t=0}^{\tilde{L}-1}\delta^t d_t - \delta^{\tilde{L}}\kappa\left(1-\sum_{t=0}^{\tilde{L}-1} d_t\right)\right] - \delta^{\tilde{L}}\theta. \quad (11'')$$

Noting that $1-\sum_{t=0}^{\tilde{L}-1} d_t = \sum_{t=\tilde{L}}^{T-1} d_t$, we see that (11") equals (9') when $L = \tilde{L}$.

Unfortunately, the HCPM does not use $\tilde{\delta} = \delta$. Rather, the HCPM discount factor is $\tilde{\delta} = \frac{1}{1+[s_d k_d + s_e k_e]}$ (KCCFactor tab, cell L14). Thus the discount factor in the HCPM is inconsistent with the cost of capital r.

[10] As mentioned in section 1 above, the HCPM actually uses a mid-year placement convention in formulating (11') rather than the beginning-of-period convention used here. This is a minor difference that merely moves certain payments back in time by half of a time period, but the algebra of the half-year placement convention is more tedious than (11'), so the beginning-of-period placement convention is used here. Equations (9') and (11') use the same convention.

The inconsistency may stem from the traditional regulatory practice of not adjusting the cost of capital used in the discount factor for the preferential tax treatment of debt. This traditional regulatory practice would be appropriate if the time t tax savings on interest payments on debt were subtracted from y_t, but is not an accepted capital budgeting procedure for an efficient competitive firm given the absence of this term in y_t. A much more standard capital budgeting procedure is to use an after-tax weighted average cost of capital given by $(1-\kappa)r = (1-\kappa)s_d k_d + s_e k_e$, the corresponding discount factor $\delta = \frac{1}{1+[(1-\kappa)s_d k_d + s_e k_e]}$, and to exclude the after-tax interest on debt as an expense item (see, for example, Brigham *et al.* (1999), Chapters 10-12). This is the approach taken in (9'). Hence the discount factor $\tilde{\delta}$ in the HCPM is not consistent with the emulation of a competitive market. Since the discount factor used in the HCPM is too low, the HCPM understates the present value of the initial investment cost and salvage revenue.

Although $NPV(FCC)$ would equal the corresponding competitive equilibrium cost if the HCPM were to use the correct economic life and after-tax discount factor, the FCC's NPV calculation (11') is highly redundant. Equation (9') is very simple. To find the net present value of the installation cost and salvage revenue, we simply subtract the future tax savings generated by depreciation writeoffs, and the future salvage revenue, from the initial cash outlay. The only depreciation needed for this purpose is the MACRS depreciation percentages (this point is emphasized by Brigham *et al.* (1999), p. 467). There is simply no reason to grind through all of the debate about depreciation methods and the extra calculations that underlie (11'), and the practice of doing so generates confusion and more possibilities for error, as is readily apparent from the discounting error in the HCPM calculation. This was pointed out long before the HCPM was constructed by Tardiff and Bidwell (1990) and Boudreaux and Long (1979), yet the arcane regulatory calculations in (11') persist in the HCPM, despite the stated objective of emulating competitive markets. Once we know the initial cash outlay, the only modification needed is to account for tax savings and salvage revenues.

Operating Cost. The HCPM calculates plant-specific operating cost as a percentage of current initial investment outlay. The percentages are reported on the 96 Actuals tab of the spreadsheet, and they are applied to current initial investment outlay on the Investment Input tab of the spreadsheet. The percentages are expense-to-investment ratios that are calculated from historic operating expenses divided by historic initial investment costs, where the historic initial investment costs have been expressed at the same time as the operating expenses through the use of current-to-book ratios (FCC, 1999, ¶¶341-7).

In terms of our model, the book cost of assets in service at time t is $\sum_{j=0}^{L-1} F_{t-j}K_{t-j}$ and the replacement cost of those assets is $\sum_{j=0}^{L-1} F_t K_{t-j}$, so the current-to-book ratio at time t is

$$CTB_t = \frac{F_t \sum_{j=0}^{L-1} K_{t-j}}{\sum_{j=0}^{L-1} F_{t-j}K_{t-j}}.$$

Applying this ratio to book cost expresses the cost of the assets in current terms as $CTB_t \sum_{j=0}^{L-1} F_{t-j}K_{t-j} = F_t \sum_{j=0}^{L-1} K_{t-j}$. Obviously, this procedure is not needed in our model to obtain the time t cost of previously acquired assets that are still in service. The FCC uses this procedure as a way of aggregating incomplete firm-specific data to the industry level.

The time t operating expense of assets in service at time t is $\sum_{j=0}^{L-1} c_{t,t-j}K_{t-j}$, so the expense-to-investment ratio at time t is

$$EIR_t = \frac{\sum_{j=0}^{L-1} c_{t,t-j}K_{t-j}}{F_t \sum_{j=0}^{L-1} K_{t-j}} = \frac{c_0 \sum_{j=0}^{L-1} \left(\frac{\alpha}{\gamma}\right)^j K_{t-j}}{F_0 \sum_{j=0}^{L-1} K_{t-j}} = \frac{c_0}{F_0} \sum_{j=0}^{L-1} \left(\frac{\alpha}{\gamma}\right)^j \hat{K}_{t-j},$$

where $\hat{K}_{t-j} = \frac{K_{t-j}}{\sum_{i=0}^{L-1} K_{t-i}}$ for $j = 0, \ldots, L-1$ gives the age distribution of assets in service at time t.[11] The HCPM applies EIR_{t-1} to current investment cost F_t to obtain an estimate of time t operating cost for a unit of vintage $\tau \leq t$ capital:

$$EIR_{t-1}F_t = \gamma^t c_0 \sum_{j=0}^{L-1} \left(\frac{\alpha}{\gamma}\right)^j \hat{K}_{t-j}.$$

Comparing this with true operating cost $c_{t,\tau} = \gamma^t c_0 \left(\frac{\alpha}{\gamma}\right)^{t-\tau}$ reveals that the HCPM overstates operating cost at time $t = \tau$ (because $\alpha/\gamma > 1$ and $\sum_{j=0}^{L-1} \hat{K}_{t-j} = 1$) but understates operating cost at time $t = \tau + (L-1)$. Indeed, if the age distribution of assets in service stays roughly constant then the HCPM estimate of operating cost falls over the life of the asset while the true operating cost increases as the asset ages (assuming $\alpha > 1$).

TELRIC Prices. The HCPM must express (11') on a per-period basis in order to calculate a per-period TELRIC price. This is accomplished in rows 20 and 21 of the KCCFactor tab of the spreadsheet, by calculating the *level* payment whose net present value over the life $\tilde{L}$ equals (11'). The discount factor for a

[11] The HCPM uses an average of time t and time $t-1$ investment in the denominator of EIR_t (FCC, 1999, ¶347). This is due to the mid-period placement convention in the HCPM and is unnecessary with the beginning-of-period placement convention used herein.

level payment stream is $\sum_{j=0}^{\tilde{L}-1} \delta^j$, so the per-period $NPV(FCC)$ used by the HCPM is

$$PPNPV = \frac{NPV(FCC)}{\sum_{j=0}^{\tilde{L}-1} \tilde{\delta}^j},$$

where $\tilde{\delta} = \frac{1}{1+[k_d s_d + k_e s_e]}$ is the same (incorrect) discount factor used to calculate $NPV(FCC)$. As noted above, this discount factor is too small, resulting in an understatement of $NPV(FCC)$. However, since this discount factor appears in the denominator of $PPNPV$, the per-period value is overstated for a given $NPV(FCC)$. Thus, the net effect of the incorrect discount factor on $PPNPV$ is indeterminate.

There is a second very important error in the HCPM conversion of $NPV(FCC)$ to a per-period value. The calculation of a level payment whose present value over $\tilde{L}$ periods is $NPV(FCC)$ is known as "levelization." From p_t^*, we see that the competitive equilibrium discount factor does not anticipate a level payment stream. Rather, the discount factor $\sum_{j=0}^{L-1}(\delta\gamma)^j$ for p_t^* correctly anticipates that p_t^* falls at the rate of technical progress γ. Thus, if there is technical progress (i.e., if $\gamma < 1$) and the economic life L is greater than one then the levelization technique understates the per-period $NPV(FCC)$. This difference between p_t^* and a levelized payment merely reflects that competitive prices, in order to induce enough investment for market-clearing, must respect the *economic* obsolescence brought about by ongoing technical progress as reflected in the parameter $\gamma < 1$. Ignoring the other errors in the HCPM, if prices are administratively set according to a levelization rule when there is technical progress then the break-even condition for investment is never satisfied for a price-taking firm, and so investment will be zero and excess demand will develop. This is the formal version of Kahn's (1998, pp. 89-103) and Kahn, Tardiff, and Weisman's (1999) concern that TELRIC prices discourage facilities investment. This conclusion relies on $L > 1$, which concerns the relative lengths of time periods in which prices are set versus time periods in which capacity is replaced. A time period of length one in the model is the period of time during which prices are set, both for competitive prices and for TELRIC prices. The period of time during which prices are set is the only meaning of a unit length of time in the model. If $L > 1$ then the economic life involves keeping vintages of capital for more than one price-setting period. Since telecommunications equipment is typically fairly durable, while TELRIC prices have typically been set in 2 or 3 year increments,[12] an economic life that is longer than the price-setting

[12]For example, the arbitrated interconnection agreement between AT&T and Southwestern Bell in Missouri involves a 3 year contract (see Interconnection Agreement-Missouri, section 4.1). The FCC (1996, ¶¶837-8) has indicated that "... we will continue to review our pricing methodology and make revisions as appropriate."

period (i.e., $L > 1$) is very likely.

Combining $PPNPV$ with the operating costs from the HCPM yields a TELRIC price for vintage 0 capital at time t of

$$TELRIC_t \equiv F_t PPNPV + F_t EIR_{t-1}.$$

Note that we use F_t in this expression to convert the per-dollar values into totals, rather than the actual initial investment F_0. This is because the HCPM uses *current* investment costs in all of its calculations. This is true of the competitive equilibrium prices as well, since γ^t is present in the numerator of p_t^*.

Note also that the operating cost part of p_t^* is

$$\frac{\gamma^t c_0 \sum_{j=0}^{L-1} (\delta\alpha)^j}{\sum_{j=0}^{L-1} (\delta\gamma)^j},$$

which reflects the discounted growth of *forward-looking* operating cost c_t as well as the discounted decline in equilibrium prices brought about by technical progress. This clearly differs from $\gamma^t c_0 \sum_{j=0}^{L-1} \left(\frac{\alpha}{\gamma}\right)^j \hat{K}_{t-j}$, the operating cost part of $TELRIC_t$. These two measures of operating cost generally do not even have the same present value, and either can be larger in present value depending on the age distribution of the assets in service and the relative magnitudes of α and γ. A young age distribution tends to make the HCPM estimate of operating cost smaller while an older age distribution makes the HCPM estimate larger by raising the expense-to-investment ratio.

5. A Calibrated Comparison between Competitive Equilibrium and TELRIC Prices

The net effect on prices of the incorrect discount factor, understatement due to levelization, and misstatement of the operating cost stream in the HCPM is indeterminate in general; and the magnitude of these errors can only be known through numerical simulation. So, this section presents a calibrated comparison between p_t^* and $TELRIC_t$ for end-office switching. End-office switching is useful for illustrative purposes because it is an important component of cost, comprising approximately 10% of total cost in the HCPM; it is more homogeneous and self-contained than distribution and feeder assets; and it is undeniably undergoing technical progress, like virtually all computing assets. We focus exclusively on the direct investment and plant-specific operating costs of the switching services provided by one end-office switch. For simplicity, we do not consider overhead or other indirect costs in this comparison. To the extent that overhead costs are apportioned with ratios, their inclusion here would magnify the differences we find. The calibration is accomplished using data directly from the HCPM, so the differences are *not* due to differences in input data *vis-a-vis* the FCC.

Again for simplicity, the calibration is based on the costs for an "autonomous" end-office switch.[13] Consider first initial investment cost F_0. Initial investment cost for the switching services of an end-office switch is estimated from predicted values of a regression equation in the HCPM (FCC, 1999, Appendix C). From the estimated equation, the FCC predicts the fixed cost of an autonomous switch in 1999 dollars to be $486,700 and the additional per-line cost to be $87 (FCC, 1999, Appendix C).[14] Switches are configured with more line ports than are actually used. Since the unused ports contribute to the cost of switching services provided, the additional cost per line must be adjusted upward. The HCPM assumes 94% of the ports have lines attached (cell K13 of the User Adjustable Inputs tab), so the additional cost per line is $87/0.94=$93. According to the HCPM, the average autonomous switch for Southwestern Bell in Missouri has 19,086 switched lines. Thus a calibrated initial investment cost for the switching services provided by a typical autonomous switch is $F_0 = \$2,253,602$.[15]

Estimates of the rate of technical progress γ can be obtained by calculating F_0 as above for each year and then calculating the compound annual growth rate of the estimated F_0's. The only additional data needed to produce predictions from the FCC's regression equation for years other than 1999 are the annual inflation rates. Obtaining these from the sources cited by the FCC (FCC, 1999, Appendix C) for historic values from the beginning of the estimation horizon in 1985, and for forecast values through 2009, and then calculating F_0 for each year yields a compound annual growth rate of F_0 during the period 1985-1995 covered by the

[13]An end-office switch is "autonomous" in the HCPM if it neither operates nor is operated by another end-office switch. End-office switches that operate other end-office switches are termed "host" switches while end-office switches that are operated by other end-office switches are termed "remote" switches. In the latter two cases, the HCPM apportions the costs of all the inter-operated switches among all of the wire centers involved, so that the end-office switching costs for each wire center are not simply the switching costs of the switch located in that wire center. This makes the HCPM switching cost calculation at the wire center level more complicated when that wire center houses a host or remote switch than when it houses an autonomous switch, although the basic underlying cost calculations for the actual switches are the same for host and autonomous switches, and for a remote switch the calculation only differs in the level of fixed cost.

[14]The version of the HCPM posted on the FCC's web site and cited herein contained an error that effectively multiplied the fixed cost by the ratio of switched to total lines (the difference being unswitched, or "special access," lines). This caused an understatement of fixed cost, in some cases by a substantial amount. The error has been corrected in a more recent version of the HCPM.

[15]The F_0 reported here and used in the calibration is obtained using the exact calculations for fixed cost and per line cost, and so differs slightly from $486,700 + 93 * 19,086 = \$2,261,698$ due to rounding errors. As a check, we note that the average initial investment cost for the switching services of an autonomous switch in the HCPM is $2,174,125, so the cost of services from an average switch is fairly close to the average cost of services from a switch.

regression data of $\gamma = .81$.[16] In other words, the initial investment cost of switching services declined 19% per year according to this estimate. However, the regression equation predicts a substantial slowdown in technical progress. The compound annual growth rate of the estimated F_0's from 1995 through 2009 (the end of the inflation forecast) is $\gamma = .95$. The calibration depends on both past (through the EIR) and future (through the pricing formulae) rates of technical progress, so we use the compound annual growth rate over the entire 1985-2009 period of $\gamma = .89$ for this calibration. Note that the switch cost data only reflect nominal cost reductions. Thus, these data understate the true rate of technical progress by the amount of any embodied quality improvements. Such improvements are substantial for computing equipment, so $\gamma = .89$ may be a conservative estimate of the technical progress parameter for end-office switching services.

The cost of capital and salvage parameters are all obtained directly from the Inputs tab of the HCPM spreadsheet, as follows:

$$s_d = 44.2\% \quad k_d = 8.8\% \quad s_e = 55.8\% \quad k_e = 13.19\% \quad \kappa = 39.25\% \quad \theta = 1.57\%.$$

These parameters imply a before-tax cost of capital of $r = 16\%$ and an after-tax discount factor of $\delta = .911$. The implied (incorrect) discount factor used by the HCPM is $\tilde{\delta} = .899$.

Computer-based telephone central office switching equipment has a MACRS category 2 tax life of five years (Internal Revenue Code of 1986, as amended, Rev. Proc. 87-56, 1987-2 C.B. 674). Due to the mid-year placement convention, this means there are six depreciation percentages (Internal Revenue Code of 1986, as amended, Rev. Proc. 87-57, 1987-2 C.B. 687):

$$d_0 = 20\% \quad d_1 = 32\% \quad d_2 = 19.2\% \quad d_3 = 11.52\% \quad d_4 = 11.52\% \quad d_5 = 5.76\%.$$

Now consider operating cost c_0. As discussed in section 4, plant-specific operating cost is estimated by the HCPM as a percentage of current investment cost. The FCC's estimated percentage for end-office switching is $EIR_{-1} = .0558$ from cell H19 of the 96 Actuals tab of the spreadsheet. So

$$c_0 = \frac{EIR_{-1}F_0}{\sum_{j=0}^{L-1}\left(\frac{\alpha}{\gamma}\right)^j \hat{K}_{-1-j}} = \frac{(0.558)(2,253,602)}{\sum_{j=0}^{L-1}\left(\frac{\alpha}{.89}\right)^j \hat{K}_{-1-j}}$$

[16]The FCC's regression equation predicts the investment cost as $F_t = (\text{Infl}) * (11,110 - 402,400 * \text{Host} + 2,205,000/t + 10,800,000 * \text{Host}/t + (10.32 + 1,121/t) * (\text{Lines/Line Fill}))$, where Infl is the ratio of the GDP price deflator in year t to 1997 ($t = 0$ in 1984), Host is a dummy variable that is one for a host or autonomous switch, Lines is the number of lines, and Line Fill is the percent of line ports on the switch that are filled (i.e., .94). $\gamma = .81$ is the compound annual growth rate of these predicted values from $t = 0$ (1984) to $t = 11$ (1995).

gives a calibrated estimate of operating cost from the HCPM once α and the age distribution of existing assets are calibrated.

The HCPM does not provide any direct information on the dynamics of operating expenses or capacities, and hence provides no direct calibration of α or the age distribution of existing assets. So we calibrate $\hat{K}_{-1-j}$ for $j = 0, \ldots, L-1$ to grow at the average annual compound rate of Dial Equipment Minutes (DEMs), as a proxy for the age distribution of switching equipment in service. DEM data for 1983-1999 from Table 3.8 (p. 234) of FCC (2001) reveals a growth rate of 5.33%. We use 16 years of growth to calculate the compound growth rate because $L = 16$ is the calibrated economic life of a digital switch (see below), and we use industry-wide DEM data because the EIR in the HCPM is calculated with industry-wide data (FCC, 1999, ¶¶341-4).

This leaves only the nominal depreciation rate α as an uncalibrated parameter, and no direct information from the HCPM on this parameter. However, the HCPM uses an assumed (unadjusted) economic life of $\tilde{L} = 16.17$ years for end-office switching (Inputs tab of the spreadsheet). Once α and all of the other parameters are specified, the economic life L of the asset is implied. Alternatively, we can assume an economic life L along with all of the other parameters and calculate an implied nominal depreciation rate α. Thus, assuming the FCC's economic life is correct (i.e., $L = \tilde{L} = 16.17$), we obtain a calibrated value of $\alpha = 1.11615$.

This calibration yields the competitive equilibrium and TELRIC price streams reported in Table 1. Substituting the calibrated parameters into the time $t = 0$ net present value of cost formula from Theorem 1 gives $NPVC(16) = \$1,687,346$. The denominator in the competitive equilibrium pricing rule p_t^* from Theorem 1 is $(1-\kappa)\sum_{j=0}^{L-1}(\delta\gamma)^j = 3.106$. Multiplying the ratio $\$1,687,346/3.106 = \$543,190$ by $\gamma^t = .89^t$ yields the p_t^* column of Table 1. Similarly, substituting the calibrated parameters into the FCC's net present value of initial investment and salvage revenue formula (equation (11')) gives $F_0 NPV(FCC) = \$2,303,109$. The corresponding competitive equilibrium before-tax present value of initial investment and salvage revenue is \$2,485,119, so the incorrect discount factor causes the HCPM to understate the present value of installation cost and salvage revenue by \$155,010. Dividing $F_0 NPV(FCC)$ by the levelization formula $\sum_{j=0}^{\tilde{L}-1}\tilde{\delta}^j = 8.093$ gives the FCC's per-period present value of initial investment and salvage revenue,

$PPNPV = \$2,303,109/8.093 = \$284,585$. The FCC's calibrated estimate of operating cost at time $t = 0$ is $EIR_{-1}F_0 = .0558 \times \$2,253,602 = \$125,751$, so the FCC's price at time $t = 0$ is $\$284,585 + \$125,751 = \$410,336$. Multiplying this price by $\gamma^t = .89^t$ yields the $TELRIC_t$ column of Table 1. The final column of Table 1 is the cumulative present value of the difference between the competitive equilibrium and TELRIC pricing rules, given by $\sum_{j=0}^{t} \delta^t (p_t^* - TELRIC_t) = \sum_{j=0}^{t} (.911)^t (p_t^* - TELRIC_t)$.[17]

TABLE 1

t	p_t^*	$TELRIC_t$	Cumulative NPV of Difference
0	$543,190	$410,336	$132,854
1	$483,555	$365,286	$240,643
2	$430,466	$325,182	$328,094
3	$383,206	$289,481	$399,046
4	$341,135	$257,699	$456,611
5	$303,682	$229,407	$503,315
6	$270,341	$204,221	$541,207
7	$240,661	$181,800	$571,951
8	$214,239	$161,840	$596,893
9	$190,718	$144,072	$617,130
10	$169,780	$128,255	$633,548
11	$151,140	$114,174	$646,869
12	$134,547	$101,639	$657,677
13	$119,775	$90,480	$666,445
14	$106,625	$80,547	$673,559
15	$94,919	$71,704	$679,331

Table 1 indicates that, for the switching services provided by one autonomous end-office switch with 19,086 lines, the data inputs and calculations used by the HCPM result in TELRIC prices that understate competitive equilibrium prices by $679,331 in 1999 dollars over the FCC's assumed economic life of the switch. This net understatement exceeds the understatement of $155,010 caused by the incorrect discount factor because of the dramatic understating effect of levelization when there is technical progress. With the 11% rate of technical progress used in this calibration, levelization swamps the other errors in the HCPM, resulting in a large deficit over the life of the switch. The total before-tax net present value cost of the switching services is $\frac{NPVC(L)}{1-\kappa} = \$2,777,524$, so the TELRIC prices understate the cost of switching services by 24% in 1999 dollars.

Although there are many different switches in operation, we can get a rough idea of the magnitude of the aggregate error by assuming that the total investment cost of end-office switching services is derived

[17] The table presents the exact calculations, which differ slightly from what is obtained using the numbers in the text due to rounding errors.

from switches of the type calibrated here. This amounts to assuming that the average investment cost of switching services per switch equals the calibrated value of F_0 used here (across host and remote, as well as autonomous, switches). This assumption substantially overstates the average cost per switch of switching services, since many switches are much smaller than the switch studied here and some are remote switches, but for the same reason this assumption substantially understates the number of switches. For Southwestern Bell in Missouri, cell AM1 of the Summary tab of the spreadsheet reports total end-office investment cost of switching services of $296,455,697. Dividing this by the calibrated investment cost of $F_0 = \$2,253,602$ indicates that the HCPM places the cost-equivalent of 132 end-offices switches of this type for Southwestern Bell in Missouri. At an error of $679,331 per switch, the aggregate understatement for Southwestern Bell in Missouri is $89.364 million in 1999 dollars. Nationwide, this error would run to the billions of dollars for end-office switching services *alone*, not counting other costs associated with the wire center, indirect and overhead costs, tandem switching costs, or costs of other network elements that may be subject to technical progress.

It is important to note that the magnitude, but not the direction, of these error estimates is sensitive to the calibration. For example, using a very modest 5% rate of technical progress (i.e., $\gamma = .95$, the future value predicted by the FCC's regression equation) pushes the nominal depreciation rather high, to $\alpha = 1.149145$, if the economic life is kept at $L = 16$. The present value understatement by TELRIC prices is then $264,817 in 1999 dollars over the life of the switch, or about 8.6% of the cost of switching services.

6. Conclusion

Six facts seem clear from the analysis presented here. First, once utilization is endogenized prices and costs must be calculated simultaneously. Second, it is unnecessary and even conceptually misleading to base a "forward-looking" cost calculation on an assumed depreciation schedule, even if it is a schedule of "economic" depreciation rates. The only depreciation schedule that is needed is the tax schedule. Moreover, once the cost parameters, including technical progress, have been estimated the economic life is endogenous and implied. It is internally inconsistent to assume an exogenous life along with a full dynamic specification of cost. Third, "levelization" does not properly account for technical progress and consequently causes TELRIC prices to be too low. Such non-compensatory prices discourage facilities investment. The effects of levelization are substantial even when technical progress is modest, and the effects reverse if costs of new vintages increase over time (i.e., if $\gamma > 1$).[18] Fourth, the HCPM uses an internally inconsistent discount factor that, by not properly accounting for the preferential tax treatment on debt, does not reflect the cost of capital for a competitive firm. Fifth, the HCPM misstates the time path of plant-specific operating costs, causing prices to be too high initially and too low later (this effect reverses only if $\gamma > \alpha$). Sixth, these errors in the HCPM result in large differences between TELRIC and competitive equilibrium prices. The magnitude of the differences is estimated from the same data used by the HCPM, so disagreements over appropriate data inputs are not the source of the differences estimated here.

There remain many sources of disagreement over TELRIC prices. We have concentrated here on the effects of technical progress. A logical next step is to incorporate uncertainty about investment returns into the model and study its effects. Another logical next step is to generalize the technology to something other than constant returns. This would endogenize the very realistic feature that existing capacity is frequently only partially utilized, as is assumed but not endogenized by Salinger (1998). Non-constant returns in operating costs can probably be accommodated with little difficulty, but non-constant returns in capacity acquisition poses more formidable problems since it is likely to be inconsistent with price-taking behavior. It should be noted, however, that the HCPM assumes constant returns in both capacity acquisition and operations, so the analysis here essentially takes the FCC's cost structure as an axiom and then assesses the effects of errors within the FCC's TELRIC paradigm.

Appendix

Proof of Theorem 1. First we establish existence and uniqueness of $L \geq 1$. Define

$$\begin{aligned} f(L) &\equiv \frac{NPVC(L)}{1-\kappa} - \left(\frac{\alpha}{\gamma}\right)^{L-1} c_0 \sum_{j=0}^{L-1} (\delta\gamma)^j \\ &= F_0 \left[\frac{1-\kappa\left(\sum_{t=0}^{L-1} \delta^t d_t + \delta^L \sum_{t=L}^{T-1} d_t\right)}{1-\kappa} - \delta^L \theta \right] + c_0 \left[\sum_{t=0}^{L-1} (\delta\alpha)^t - \left(\frac{\alpha}{\gamma}\right)^{L-1} \sum_{t=0}^{L-1} (\delta\gamma)^t \right] \\ &= F_0 \left[\frac{1-\kappa\left(\sum_{t=0}^{L-1} \delta^t d_t + \delta^L \sum_{t=L}^{T-1} d_t\right)}{1-\kappa} - \delta^L \theta \right] + c_0 g(L), \end{aligned}$$

[18] Note, however, that γ measures the evolution of costs for vintages with a given productivity. $\gamma > 1$ can occur only if costs of *quality adjusted* vintages are rising.

where

$$\begin{aligned} g(L) &\equiv \frac{1-(\delta\alpha)^L}{1-\delta\alpha} - \left(\frac{\alpha}{\gamma}\right)^{L-1} \frac{1-(\delta\gamma)^L}{1-\delta\gamma} \\ &= \frac{y-1}{y-x}\left[\frac{y-x}{(y-1)(1-x)} + \frac{x^L}{x-1} - \frac{y^L}{y-1}\right] \end{aligned}$$

for $x = \delta\alpha \neq 1$ and $y = \frac{\alpha}{\gamma}$. Note that $y > 1$ and $y > x$, but $x = 1$ is possible. When $x = 1$, we have

$$g(L) \equiv L + \frac{1}{y-1} - \frac{y^L}{y-1}.$$

We must show that $f(L)$ is positive for $L \geq 1$ small and crosses zero once as $L \to \infty$. First,

$$\begin{aligned} f(1) &= F_0\left[\frac{1-\kappa(d_0+\delta(1-d_0))}{1-\kappa} - \delta\theta\right] + c_0 g(1) \\ &= F_0\left[\frac{1-\kappa(d_0+\delta(1-d_0))}{1-\kappa} - \delta\theta\right] \\ &\geq F_0[1-\delta\theta] > 0. \end{aligned}$$

On the other hand,

$$\lim_{L\to\infty} f(L) = F_0\left[\frac{1-\kappa\sum_{t=0}^{T-1}\delta^t d_t}{1-\kappa}\right] + c_0 \lim_{L\to\infty} g(L).$$

If $x < 1$ then

$$\lim_{L\to\infty} g(L) = \frac{1}{1-x} + 0 - \frac{\lim_{L\to\infty} y^L}{y-x} = -\infty.$$

If $x = 1$ then

$$\begin{aligned} \lim_{L\to\infty} g(L) &= \frac{1}{y-1} + \lim_{L\to\infty} L\left(1 - \frac{y^L}{L}\frac{1}{y-1}\right) \\ &= \frac{1}{y-1} + \lim_{L\to\infty} L\left(1 - \frac{y^L \ln y}{1}\frac{1}{y-1}\right) \quad \text{by LHôpital's Rule} \\ &= -\infty. \end{aligned}$$

If $x > 1$ then

$$\lim_{L\to\infty} g(L) = \frac{1}{1-x} + \frac{y-1}{y-x}\lim_{L\to\infty} x^L\left(\frac{1}{1-x} - \left(\frac{y}{x}\right)^L \frac{1}{y-1}\right) = -\infty.$$

So in all cases $\lim_{L\to\infty} f(L) = -\infty$. The establishes existence of $L \geq 1$. Uniqueness would be automatic from here if $f(L)$ were strictly decreasing. Unfortunately, the coefficient on F_0 is increasing in L, so the slope of f is indeterminate in general. However, concavity of f is also sufficient for uniqueness of L, since

$f(1) > 0 > f(\infty)$ and a concave f has only one turning point. First consider concavity of the coefficient on F_0. $-\delta^L\theta$ is obviously concave. When L is incremented by 1, the change in the remaining part is

$$\frac{\kappa\delta^L}{1-\kappa}(1-\delta)\sum_{t=L+1}^{T-1} d_t.$$

This increment clearly diminishes as L increases. Now consider the concavity of $g(L)$. If $x = 1$ then

$$g''(L) = -\frac{y^L \ln^2 y}{y-1} < 0.$$

If $x \neq 1$ then

$$g''(L) = \frac{y-1}{y-x}[h(L,x) - h(L,y)],$$

where $h(L,x) = \frac{x^L \ln^2 x}{x-1}$. Suppose $x < 1$. Then $h(L,x) < 0$ and $h(L,y) > 0$, so $g''(L) < 0$. Now suppose $x > 1$. Note that

$$h_2 = \frac{x^{L-1}\ln x}{(x-1)^2}[(L\ln x + 2)(x-1) - x\ln x].$$

The term in brackets is zero at $x = 1$ and its derivative is

$$\frac{\partial}{\partial x} = \frac{L}{x}(x-1) + (L-1)\ln x + 1 > 0.$$

Thus $h_2 > 0$ when $x > 1$. Since $x < y$, this implies $g''(L) < 0$ when $x > 1$. Thus g is concave, which establishes uniqueness of $L \geq 1$. This proves item 1 of the theorem.

Now define $\{p_t^*\}_{t=0}^{\infty}$ as the price sequence that equates $NPVR_\tau$ and $NPVC_\tau$ at every time τ, under the assumptions that $m_{t,\tau} \geq 0$ for $t = \tau, \ldots, \tau + L_\tau - 1$ and $u_{t,\tau} = 1$ when $m_{t,\tau} = 0$. That is,

$$(1-\kappa)\sum_{t=0}^{L_\tau - 1} \delta^t p_{\tau+t}^* \equiv NPVC_\tau \text{ for } \tau = 0, 1, \ldots.$$

Substituting $F_\tau = \gamma^\tau F_0$, $c_\tau = \gamma^\tau c_0$, and $\alpha_\tau = \alpha$ into (6), and assuming $L_\tau = L \; \forall \tau \geq 0$, yields

$$(1-\kappa)\sum_{t=0}^{L-1} \delta^t p_{\tau+t}^* = \gamma^\tau NPVC(L).$$

The assumption $m_{\tau,\tau} \geq 0$ implies $p_\tau^* > 0$. Also,

$$\lim_{\tau\to\infty}\sum_{t=0}^{L-1} \delta^t p_{\tau+t}^* = \lim_{\tau\to\infty}\frac{\gamma^\tau NPVC(L)}{1-\kappa} = 0,$$

where the last equality follows from $\gamma \in (0,1)$. Therefore, $\lim_{\tau\to\infty} p^*_\tau = 0$, so p^*_τ is bounded. By recursive substitution we obtain

$$(1-\kappa)p^*_\tau = NPVC(L)\gamma^\tau(1-\delta\gamma)\sum_{t=0}^{j-1}\left((\delta\gamma)^L\right)^t + (1-\kappa)\delta^{jL}p^*_{\tau+jL} \text{ for } j = 1,2,\ldots.$$

Letting $j \to \infty$ and using boundedness of $p^*_{\tau+jL}$ yields

$$p^*_\tau = \frac{NPVC(L)\gamma^\tau}{1-\kappa}(1-\delta\gamma)\frac{1}{1-(\delta\gamma)^L} = \frac{\gamma^\tau NPVC(L)}{(1-\kappa)\sum_{t=0}^{L-1}(\delta\gamma)^t}.$$

For this price sequence,

$$m_{t,\tau} = p^*_t - \alpha^{t-\tau}\gamma^\tau c_0 = \gamma^t\left[\frac{NPVC(L)}{(1-\kappa)\sum_{j=0}^{L-1}(\delta\gamma)^j} - \left(\frac{\alpha}{\gamma}\right)^{t-\tau} c_0\right].$$

So

$$\begin{aligned}
L_\tau &= \max\left\{t \ge \tau\colon \frac{NPVC(L)}{(1-\kappa)\sum_{j=0}^{L-1}(\delta\gamma)^j} \ge \left(\frac{\alpha}{\gamma}\right)^{t-\tau} c_0\right\} - \tau + 1 \\
&= \max\left\{t-\tau \ge 0\colon \frac{NPVC(L)}{(1-\kappa)\sum_{j=0}^{L-1}(\delta\gamma)^j} \ge \left(\frac{\alpha}{\gamma}\right)^{t-\tau} c_0\right\} + 1 \\
&= L - 1 + 1 = L \text{ from item 1 of the theorem.}
\end{aligned}$$

This shows that the assumption $L_\tau = L$ is consistent with the price sequence p^*_t. Moreover, $-\left(\frac{\alpha}{\gamma}\right)^{t-\tau} c_0$ is strictly decreasing in t, so $m_{t,\tau} > 0$ for $t = \tau,\ldots,\tau+L-2$, confirming the $m_{t,\tau} \ge 0$ assumption for $t = \tau,\ldots,\tau+L-1$ for this price sequence. Finally, the $u_{t,\tau} = 1$ assumption is of no consequence, since if $u_{t,\tau} < 1$ when $m_{t,\tau} = 0$ (which can only occur for $t = \tau+L-1$) then $NPVR_\tau$ and $NPVC_\tau$ both decrease by the same amount, leaving the definition of p^*_t unaffected. $\square$

Proof of Theorem 2. Since $NPVR_\tau = NPVC_\tau$ $\forall\tau \ge 0$, $K^*_\tau(\{p^*_t\}_{t=0}^\infty) = [0,\infty]$ $\forall\tau \ge 0$ from (7). Thus we need only construct a nonnegative sequence $\{\bar{K}_t\}_{t=0}^\infty$ satisfying item 2 of Definition 1. We construct the required sequence inductively. For the initial condition, note that

$$\sum_{\tau=-b}^{-1}\{\bar{K}_\tau\chi(m_{0,\tau} > 0)\} \subset \sum_{\tau=-b}^{-1}[\{\bar{K}_\tau\chi(m_{0,\tau} > 0)\} + [0, \bar{K}_\tau\chi(m_{0,\tau} = 0)]].$$

So, using $\sum_{\tau=-b}^{-1}\{\bar{K}_\tau\chi(m_{0,\tau} > 0)\} \leqq q^d_0(p^*_0)$, there exists $\bar{K}_0 \in q^d_0(p^*_0) - \sum_{\tau=-b}^{-1}[\{\bar{K}_\tau\chi(m_{0,\tau} > 0)\} + [0, \bar{K}_\tau\chi(m_{0,\tau} = 0)]]$ such that $\bar{K}_0 \ge 0$. For this $\bar{K}_0$ item 2 of Definition 1 is satisfied at $t = 0$. Now assume we have $\bar{K}_t \geq 0$ satisfying $0 \in q_t^d(p_t^*) - q_t^s(\{p_\tau^*\}_{\tau=0}^\infty)$ for $t = 0, \ldots, n$. Since demand is growing and nonincreasing in p, and p_t^* is falling,

$$q_n^d(p_n^*) \leqq q_{n+1}^d(p_{n+1}^*).$$

Also, since $m_{n+1,\tau} < m_{n,\tau}$,

$$q_n^s(\{p_\tau^*\}_{\tau=0}^\infty) \geq \sum_{\tau=-b}^{n} [\{\bar{K}_\tau \chi(m_{n+1,\tau} > 0)\} + [0, \bar{K}_\tau \chi(m_{n+1,\tau} = 0)]].$$

Thus

$$\{0\} \leqq q_n^d(p_n^*) - q_n^s(\{p_\tau^*\}_{\tau=0}^\infty) \leqq q_{n+1}^d(p_{n+1}^*) - \sum_{\tau=-b}^{n} [\{\bar{K}_\tau \chi(m_{n+1,\tau} > 0)\} + [0, \bar{K}_\tau \chi(m_{n+1,\tau} = 0)]].$$

That is, there exists $\bar{K}_{n+1} \in q_{n+1}^d(p_{n+1}^*) - \sum_{\tau=-b}^{n} [\{\bar{K}_\tau \chi(m_{n+1,\tau} > 0)\} + [0, \bar{K}_\tau \chi(m_{n+1,\tau} = 0)]]$ such that $\bar{K}_{n+1} \geq 0$. For this $\bar{K}_{n+1}$ item 2 of Definition 1 is satisfied at $t = n+1$. $\square$

Proof of Corollary. Let

$$q_{-1,t}^s(p_t) \equiv \sum_{\tau=-b}^{-1} [\{\bar{K}_\tau \chi(m_{t,\tau} > 0)\} + [0, \bar{K}_\tau \chi(m_{t,\tau} = 0)]],$$

and observe that this is a step correspondence, with a value that is an interval

$$\left[\sum_{\tau=-b}^{-1} \bar{K}_\tau \chi(m_{t,\tau} > 0), \sum_{\tau=-b}^{-1} \bar{K}_\tau \chi(m_{t,\tau} \geq 0)\right]$$

when $p_t = \alpha_\tau^{t-\tau} c_\tau$ for some τ, and is a singleton $\left\{\sum_{\tau=-b}^{-1} \bar{K}_\tau \chi(m_{t,\tau} > 0)\right\}$ otherwise. This correspondence is nonempty, upper semicontinuous, nonnegative, nondecreasing, convex- and compact-valued on $\mathbb{R}_+^1$. Thus the excess demand $z_t(p_t) \equiv q_t^d(p_t) - q_{-1,t}^s(p_t)$ is nonempty, upper semicontinuous, nonincreasing, convex- and compact-valued on $\mathbb{R}_+^1$. By definition of t^*, $z_t(p_t^*) < \{0\}$ for $t \leq t^*$; while

$$z_t(0) = q_t^d(0) - q_{-1,t}^s(0) = q_t^d(0) - \{0\} > \{0\} - \{0\} = \{0\}.$$

Hence a standard application of Kakutani's fixed point theorem establishes existence of $\bar{p}_t \in (0, p_t^*)$ such that $0 \in z_t(\bar{p}_t)$. Now denote the proposed equilibrium price sequence by $p_\tau = \bar{p}_\tau$ for $\tau = 0, \ldots, t^*$ and $p_\tau = p_\tau^*$ for $\tau = t^* + 1, t^* + 2, \ldots$. Since $p_\tau \leq p_\tau^*$ $\forall \tau$, $0 \in K_t^*(\{p_\tau\}_{\tau=0}^\infty)$ for $t = 0, \ldots, t^*$. So, setting $\bar{K}_t = 0$ for $t = 0, \ldots, t^*$ satisfies item 1 of Definition 1 for $t = 0, \ldots, t^*$. Moreover, with this choice of $\bar{K}_t$ we have $q_t^s(\{p_\tau\}_{\tau=0}^\infty) = q_{-1,t}^s(\bar{p}_t)$ for $t = 0, \ldots, t^*$. Thus, item 2 of Definition 1 is satisfied for $t = 0, \ldots, t^*$ by the choice of $\bar{p}_t$. By definition of t^* we have $\sum_{\tau=-b}^{t^*} \{\bar{K}_\tau \chi(m_{t^*+1,\tau} > 0)\} \leqq q_{t^*+1}^d(p_{t^*+1})$. Thus, for the remainder of the price sequence we simply apply Theorem 2, regarding time $t^* + 1$ as time 0. $\square$

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