Monetary Policy, Fiscal Policy, and the Inflation Tax: Equivalence Results*

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Abstract

This paper clarifies and extends previous work on the equivalence between monetary regimes and fiscal regimes involving social security systems. We consider equivalence across regimes, showing that monetary regimes are equivalent to one or both of two alternative types of social security regimes. Two implications emerge. One is that financiing a real expenditure by increasing the inflation rate is equivalent, across regimes, to financing the expenditure by increasing the tax rate on social security benefits. In addition, our results imply that a wide range of monetary policy actions are equivalent, across regimes, to fiscal policy actions that change the scale of the social security system and the tax rates on social security benefits and/or bank deposits.

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1 Introduction

Inflation imposes a tax that generates revenue for the government. The public finance approach to monetary/macroeconomic theory uses this fact as a starting point for its analysis of monetary policy. This approach is grounded on explicit consideration of a unified government budget constraint that includes revenue from the inflation tax, revenue from direct taxes, and revenue absorbed or produced by the government debt. With government purchases constant, monetary policy actions that alter the rate and/or the base of the inflation tax must be accompanied by changes in fiscal policy instruments. In general, more than one type of fiscal policy adjustment will be possible, and the effects of a given change in monetary policy may depend critically on the particular type of adjustment that occurs.

The public finance approach has been frequently used to address a recurring question in monetary theory: Are monetary policy changes irrelevant? If so, in what sense are they irrelevant? Often, irrelevance is used in the sense that monetary policy actions have no effects on real allocations and/or relative prices. In view of the public finance aspects, monetary-policy irrelevance occurs when the effects of changes in monetary policy instruments are exactly offset by the effects of the changes in fiscal policy instruments that bring the government budget back into balance. The seminal contribution to this branch of the literature on monetary policy irrelevance is Wallace (1981), which describes conditions under which central bank purchases or sales of private assets (open market operations) have no effects on either real allocations or the price level. Other important contributions include Chamley and Polemarchakis (1984), Peled (1985) and Sargent and Smith (1987). In this literature, each contributor begins with a description of a monetary regime with a fiscal component and goes on to investigate whether changes in monetary policy within that regime can be irrelevant.[1]

In this paper, we take the monetary-irrelevance literature a step further. We investigate whether a monetary regime can itself be irrelevant, in the sense that any allocation supported by such a regime can also be supported by a fiscal regime that does not include fiat money or other unbacked government liabilities.[2] When this is the case, we can think of the two types of regimes as being equivalent to each other. For this reason, we use the term "equivalence," rather than "irrelevance," to characterize our results.[3]

1 In the realm of fiscal policy, within-regime equivalence results have a history that dates back to Ricardo, and, in modern times, to Barro (1974).

2 There is also a literature that asks whether the allocations supported by particular monetary policy regimes can be supported by monetary policy regimes of a different type. Examples include Mourmouras and Russell (1992) and Bacchetta and Caminal (1994).

3

The cross-regime equivalence results we obtain in this paper are potentially important for at least two reasons. First, they may improve our understanding of the role of money and monetary arrangements by identifying fiscal arrangements that can play the same role. Second, cross-regime equivalence means that changes in monetary policy are equivalent to changes in fiscal policy. So we may be able to use fiscal policy analysis to help us understand the results of monetary policy experiments. Even further, we may be even be able to use models of fiscal policy to draw inferences about the results of monetary policy experiments without adding monetary features to the models.[4]

Our analysis builds on previous research on the role of money in overlapping generations (OLG) models. The work of pioneers such as Samuelson (1959), Shell (1971), and Wallace (1980) demonstrated that in OLG models, fiat currency and other unbacked government liabilities may have value because they allow agents to conduct intergenerational exchanges they could not arrange without assistance from the government.[5] These authors also recognized that the effects of these exchanges are very similar to the effects of class of fiscal policies that the model is well suited to study – pay-as-you-go (PAYG) social security systems, in which lump-sum taxes on young agents finance transfers to old agents.

More recent work has succeeded in constructing two sides of a rigorous theoretical triangle linking social security, unbacked government debt, and fiat currency. The first side of the triangle is developed in McCandless and Wallace (1993). They show that any real allocation supportable by a social security system can be supported by a government policy regime involving unbacked government bonds and transfer payments financed by revenue from bond

The process of constructing the equivalence results that were cited in the preceding paragraph has been summarized by Sargent (1987, p. 304), as follows: One starts by assuming that "government and private securities exist and are valued in an initial equilibrium, with a given specification of government policy strategies. Holding the consumption allocations associated with this initial equilibrium fixed, one solves the equilibrium conditions for the class of government policies that supports this same allocation in equilibrium." Our procedure is analogous, but different. We choose a specification of government policy strategies that is completely different from the initial specification – a specification that features fiscal policy strategies rather than monetary policy strategies. Then we look for a single government policy, within this new policy-strategy specification, that supports the initial allocation in equilibrium.

[4] For example, Auerbach and Kotlikoff (1987), in the introduction to their well-known analysis of fiscal policy, comment that "There is only one type of government debt in this model and no money. Hence, the question of inflation and the distortions caused by the interaction of real and nominal magnitudes cannot be addressed. ... Introducing money into the model in a satisfactory way would constitute an enormous task." (page 12). Our results indicate that it may be possible to use the Auerbach-Kotlikoff model to study the effects of certain monetary policy experiments without adding monetary features to the model.

[5] For our purposes, government debt is said to be unbacked if it is not accompanied by a stream of future surpluses in the government's budget, net of interest and currency seigniorage, of equal present value. In some cases, unbacked debt is serviced out of future currency seigniorage revenues; in others, it is rolled over forever.

sales. Sargent (1987) develops the second side of the triangle, establishing that exchanges of unbacked bonds for fiat currency, or vice-versa (open market operations), are irrelevant – a result which implies that monetary regimes with fiat currency are equivalent to regimes with unbacked government debt.[6] In both cases, the analytical framework is a pure exchange OLG model in which money yields the same real return as competing stores of value. We provide the last side of this triangle, using a version of the same model to prove explicitly (in Theorem 1, and Corollary 1) that any allocation supported by a monetary regime with fiat currency and/or unbacked government debt can be supported as a fiscal regime with PAYG social security, and vice-versa.

In addition, we extend this literature, asking whether this equivalence result can hold in OLG models in which money is return-dominated. We pose this question in a setup in which money is valued only if the government imposes legal restrictions on intertemporal trade. This is the same type of question Sargent and Smith (1987) asked and answered about the within-regimes equivalence result obtained by Wallace (1981). Previous work relevant to this question includes Romer (1985), who asserts, without providing a formal proof, that any real allocation supported by a monetary regime involving fiat currency and reserve requirements can also be supported by a regime involving government bonds backed partly, but not completely, by direct taxes on the returns on bank deposits. The formal proof is provided by Bacchetta and Caminal (1994), who also extend Romer's analysis to situations in which the government is earning revenue from currency seigniorage.[7,8] To our knowledge, however, there has never been any work on the relationship between monetary regimes with reserve requirements or other legal restrictions and "purely fiscal" regimes with social security systems but no monetary features.

[6] As the terms "monetary" and "fiscal" are conventionally defined, policies involving government debt are fiscal policies rather than monetary policies. In OLG models, however, fiat currency and unbacked government bonds are so closely related that it does not seem reasonable to distinguish between the two types of policies in this way. In the literature on the interaction between fiscal and monetary policy, for example, fiscal policy is said to be held constant, with respect to changes in monetary policy, as long as the levels of direct taxes, transfers, and government purchases are held fixed, even if there are changes in the stock of unbacked debt. Changes of the latter type are sometimes described as representing passive actions by the budgetary authority (the Treasury), perfectly anticipated by the central bank, that reconcile the central bank's decisions with the tax and spending decisions of the fiscal authority (the legislature). For this reason, we will define policies involving unbacked debt as monetary rather than fiscal.

[7] In closely related work, Mourmouras and Russell (1992) use a model with linear stochastic storage to show that monetary equilibria with reserve requirements and seigniorage can be supported as monetary equilibria with seigniorage, but no reserve requirements, plus taxes on deposit returns.

[8] We show in Section 3.2 of this paper that any allocation supported by a currency reserve requirement and unbacked government debt can be supported by a larger currency reserve requirement with no government debt.

The principal result of our analysis is that any allocation supported by a monetary regime with fiat currency, reserve requirements, and currency seigniorage, possibly in combination with unbacked government debt and bond seigniorage, can also be supported by a fiscal regime with PAYG social security plus taxes on social security benefits and/or deposit returns. We present two versions of this result, featuring social security schemes of two different types. The first version features a social security scheme in which young households' contributions are financed by a proportional tax on their saving. These contributions are used to pay benefits, to old agents, that are proportional to the contributions they made when they were young. The government may tax these benefits in order to finance a real expenditure. Under this scheme, the pre-tax benefits paid by the social security system are financed entirely by current social security contributions, and the pre-tax return rate on these contributions is equal to the population growth rate. The second version of the result features a lump-sum social security system. Under this system, however, the pre-tax return rate on social security contributions is higher than the population growth rate, and the government imposes a proportional tax on bank deposits to help finance the social security benefits.[9] ,[10]

Our results shed light on a long-standing question in monetary economics: How is the inflation tax similar to direct taxes? We show that monetary equilibria in which the monetary policy imposes an inflation tax are equivalent to social security equilibria in which the government levies a proportional tax on social security benefits. This equivalence holds whether or not fiat money is dominated in rate of return (which, in our model, is whether or not the government uses binding reserve requirements to create or augment the demand for fiat money). The precise relationship between the value of the inflation rate under the monetary regime and the value of the tax rate on social security benefits under the equivalent fiscal regime depends partly on the nature of the monetary regime. In Proposition 1, we describe these two regimes as either one in which the reserve requirement is not binding, or one in which unbacked government debt is excluded and the reserve requirement is binding. In Proposition 2, we show that a monetary equilibrium with fiat currency, reserve requirements and government debt is equivalent to a mixed equilibrium with unbacked government debt

[9] This version of the equivalence result is inspired by, and can be viewed as an extension of, Romer (1985) and Bacchetta and Caminal (1994).

[10] Although our discussion of these results focuses on the ability of social security regimes to support monetary equilibria, we prove the results in both directions: that is, we also show that social security equilibria can be supported as monetary equilibria. In some cases, these equivalent monetary equilibria require supplementary lump-sum taxes on the old (but not the young) households. In addition, although our discussion focuses on steady states, we prove our results for equilibria of all types.

and a saving-based social security system, but no fiat currency or reserve requirements. For all pairs of monetary and fiscal regimes listed, an increase in the inflation rate to finance a larger real expenditure has exactly the same impact as an increase in the tax rate on social security benefits.

In this paper, we demonstrate that monetary policy actions – changes in the money growth rate, open market operations or changes in the reserve ratio – are equivalent, across regimes, to changes in fiscal policy. These fiscal policy changes may include changes in the scale of the social security system (that is, changes in the level of social security taxes and benefits), and changes in the tax rates on social security benefits or returns on bank deposits. Thus, we can use fiscal policy analysis, which is relatively well understood, to help us understand the real effects of monetary policy actions. In addition, we can use the real effects of fiscal policy actions to predict the real effects of monetary policy actions in the equivalent monetary regimes. This result opens the door to using models of fiscal policy to study the real effects of monetary policy, even when the models in question do not have any monetary features.

We can illustrate the potential usefulness of our analysis by using it to provide some "non-monetary" intuition about one of the best-known results in the theory of overlapping generations models with money: Freeman's (1987) optimal reserve requirements theorem. Freeman studies an economy in which households save by depositing funds at financial intermediaries whose assets consist of physical investments (stored goods) and required reserves of fiat currency. The government imposes the reserve requirement in order to finance a real purchase via currency seigniorage. Freeman shows that the optimal choice for the reserve ratio is the lowest ratio feasible – a ratio that produces a hyperinflation.

In Freeman's model, we show that a monetary equilibrium with reserve requirements is equivalent to a fiscal equilibrium with a pay-as-you-go social security system. In the monetary equilibrium, there is a government purchase financed by an inflation tax; in the fiscal equilibrium, this purchase is financed by a direct tax on social security benefits. We can use this equivalent fiscal equilibrium to describe the "fiscal logic" behind Freeman's result. It begins with the fact that pay-as-you go social security provides intergenerational transfers at a pre-tax return rate equal to the population growth rate, while physical investment, under Freeman's assumptions, offers a higher return rate. Thus, social security is an inefficient system for reallocating resources from young to old households, and the best social security system is the smallest one that allows the government to raise the required revenue by taxing the benefits. Under this optimal system, the benefits tax rate is 100 percent: the benefits, and the contributions that finance them, are just large enough to finance the government purchase. Since there are no after-tax benefits, there are no inefficient intergenerational transfers. The monetary analogue of this system is a reserve requirements regime in which the gross return rate on reserves is zero, so that the reserves are confiscated by the government and provide no return to the banks or their depositors. The government confiscates the reserves by engineering a hyperinflation. Each period, the intermediaries buy all their currency reserves from the government, paying exactly the quantity of goods the government needs to finance its purchase. Next period, the hyperinflation has rendered the existing currency stock worthless, so the intermediaries who purchase currency reserves must again buy all of them from the government.

In the next section of the paper, we lay out the model we will use for our analysis. In Section 3, we use two alternative specifications of this model to obtain the equivalence results described earlier in this introduction. In Section 4, we provide two additional examples that are intended to illustrate further some of the ways in which our results can help us understand and predict the effects of changes in monetary policy. Section 5 concludes the paper.[11]

2 The model

2.1 Basics

The model we employ is a standard two-period overlapping generations model. Time is discrete and infinite in one direction, beginning at date 1 and continuing at dates $2, 3, \ldots$. At date 1 there are $N_0 > 0$ agents, the members of generation 0 (the initial old), who live for a single period. At each date $t \geq 1$ a generation of two-period lived agents is born; they live and consume, etc. at dates t and $t+1$. There are N_t members of this "generation t", with $N_t = n\, N_{t-1}$, where $n > 0$, for $t \geq 1$. There is a single consumption good that is (possibly) storable using a linear technology: If $k_t > 0$ units of the good are placed in storage at date t then $x\, k_t$ units are recovered at date $t+1$, where $x \geq 0$.[12]

Each member of each generation $t \geq 1$ has identical preferences over consumption bundles $(c_{1t}, c_{2,t+1})$, measured in units of the consumption good. These preferences are assumed to be representable by a utility function $u(c_1, c_2)$ with standard features. The agents come

[11] The proofs of the equivalence results appear in Appendix A. The details of the first example from Section 4 are presented in Appendix B.

[12] Our results are readily generalized to economies with nonlinear storage, or with neoclassical production and capital.

in two varieties: fraction $\alpha \in (0,1]$ belong to Variety A and the rest belong to Variety B. Variety A agents are endowed with ω_1^A units of the consumption good when young (at date t) and ω_2^A units of the good at date $t+1$, with $\omega_1^A > 0$ and $\omega_2^A \geq 0$. For Variety B agents, $\omega_1^B \geq 0$ and $\omega_2^B > 0$. Given the agents' preferences, the two sets of endowment patterns are assumed to have the property that Variety A agents are savers at any return patterns consistent with equilibrium, while Variety B agents are borrowers at those return patterns.

Each initial old agent is endowed with $\omega_2 > 0$ units of the consumption good and with $h_0 > 0$ units of fiat currency. (There is only one variety.) These agents prefer more consumption to less during their single period of life.

2.2 Assets and intermediation

All transactions involving assets are intermediated through zero-cost, competitive banks. Variety B agents borrow by issuing consumption-loan liabilities to these banks and Variety A agents save by holding the banks' deposit liabilities. We denote the values deposited or borrowed s_t^j, $j = A, B$, where s_t^A will be positive (saving/depositing) and s_t^B will be negative (borrowing). The potential assets of the banks are consumption loans, stored goods, government bonds, and fiat currency. The markets for all these assets are perfectly competitive. The gross real rate of return on consumption loans extended at date t is denoted R_t. The goods price of a unit of fiat currency at date t is denoted p_t. If fiat currency is valued, so that $p_t > 0$ for all $t \geq 1$, then its gross real return rate from date t to date $t+1$ is $R_t^m \equiv p_{t+1}/p_t \equiv 1/\Pi_t$, where Π_t is the gross inflation rate from date t to date $t+1$. For simplicity, if fiat currency is not valued then we set $R_t^m = 0$. Government bonds are one-period consumption bonds; their gross real rate of return is R_t^b.

Without loss of generality, we assume that the banks hold the same portfolio on behalf of each Variety A agent born at a given date. The quantity of goods stored on behalf of a Variety A agent at date t is denoted k_t^A. The real present value of the bonds held for each of these agents is denoted b_t^A, and the real balances of fiat currency held for each agent is $m_t^A = p_t\, h_t^A$, where h_t^A represents the nominal balances.

2.3 Asset return rates

Competition between banks, who may store goods deposited, ensures that

$$\begin{aligned} R_t &= x \text{ if storage occurs (if } k_t^A > 0\text{), and} \\ R_t &\geq x \text{ otherwise.} \end{aligned} \tag{1}$$

Similarly,

$$R_t^b = \rho_t^d \text{ if government bonds are held (if } b_t^A > 0\text{), and}$$
$$R_t^b \leq \rho_t^d \text{ otherwise,} \tag{2}$$

where

$$\rho_t^d = (1 - \varphi_{t+1})\, R_t\,. \tag{3}$$

and $\varphi_{t+1} \geq 0$ is the tax the government imposes, at date $t+1$, on the returns on private assets – consumption loans or stored goods – acquired by the banks at date t. (See below.)

For fiat currency, we have

$$R_t^m \leq \rho_t^d\,, \tag{4}$$

but real balances of fiat currency can be positive when $R_t^m < \rho_t^d$. This possibility grows out of our assumption that the government imposes a currency reserve requirement of $\lambda_t \in [0,1]$ on banks that accept deposits at date t. Here λ_t is the minimum fraction of the bank's total deposits that must be held in the form of fiat currency. In equilibria in which money is not valued we assume $\lambda_t = 0$ for all $t \geq 0$.

Competition between banks ensures that the after-tax gross real deposit return rate R_t^d satisfies

$$R_t^d = (1 - \lambda_t)\ \rho_t^d + \lambda_t\, R_t^m. \tag{5}$$

If $R_t^m < \rho_t^d$ then the reserve requirement is binding and banks hold fiat currency only to satisfy the reserve requirement.

2.4 Taxes

The government may administer as many as two different tax systems: a lump sum tax system that helps finance government purchases (with an exception noted below) and a pay-as-you-go social security system.

2.4.1 Social security

The pay-as-you-go social security system may have as many as two different components: a lump-sum component and a saving-based component.

Lump-sum social security Under the lump-sum component of the social security system, at each date $t \geq 1$ each young agent from Variety A pays a lump-sum social security tax of $\tau_{1t}^A \geq 0$. At each date $t \geq 2$ each old Variety A agent receives a pre-tax social security benefit of $\mathrm{T}_{2t}^A \leq 0$, while at date 1 each old agent receives a pre-tax benefit of $\mathrm{T}_{21} \leq 0$. The corresponding after-tax benefits are

$$\begin{aligned} \tau_{2t}^A &= (1 - z_t)\,\mathrm{T}_{2t}^A \ \text{ and} \\ \tau_{21} &= (1 - z_1)\,\mathrm{T}_{21}, \end{aligned} \tag{6}$$

where z_t is the proportional tax rate on social security benefits of either type.

The lump–sum social security system may also include a proportional tax of $\varphi_{t+1} \geq 0$ on the gross returns at date $t+1$ on private assets acquired by the banks at date t. We will refer to this tax, somewhat misleadingly, as a "deposit tax."

Saving-based social security Under the saving-based component of the social security system, young Variety A agents must pay social security taxes equal to a fraction $\gamma_t \geq 0$ of their gross saving s_t^A. That is, $\Psi_{1t}^A = \gamma_t\, s_t^A$, where Ψ_{1t}^A represents an agent's saving-based social security contribution. Old agents from Variety A receive saving-based social security benefits that they view as proportional to their contributions. Thus,

$$\Psi_{2,t+1}^A = -\upsilon_{t+1}\,\Psi_{1t}^A\,, \tag{7}$$

where υ_{t+1} is the after-tax social security replacement rate – the fraction of a young agent's saving-based contributions at date t that is returned to it, at date $t+1$, in the form of social security benefits. The pre-tax social security benefits paid at any date $t \geq 1$ are denoted Φ_{2t}^A for $t \geq 2$ and Φ_{21} for date 1. We have

$$\begin{aligned} \Psi_{2t}^A &= (1 - z_t)\,\Phi_{2t}^A \ \text{ for } t \geq 2, \text{ and} \\ \Psi_{21} &= (1 - z_1)\,\Phi_{21}\,. \end{aligned} \tag{8}$$

2.4.2 Other taxes and tax totals

The government may impose additional lump sum taxes on agents of either or both varieties, and/or on the initial old, in order to finance government purchases and/or to supplement the consumption of the initial old. The taxes imposed on young and old members of generations $t \geq 1$ will be denoted tx_{1t}^i and $\mathsf{tx}_{2,t+1}^i$, $i = A, B$. The tax or transfer imposed on the initial old agents will be denoted tx_{21}

The total lump-sum taxes or transfers imposed on members of generations $t \geq 1$ during their lives are denoted T^i_{1t} and $\mathrm{T}^i_{2,t+1}$, $i = A, B$. We have

$$\begin{aligned} \mathrm{T}^A_{1t} &= \tau^A_{1t} + \mathsf{tx}^A_{1t} \\ \mathrm{T}^A_{2,t+1} &= \tau^A_{2,t+1} + \mathsf{tx}^A_{2,t+1} \end{aligned} \tag{9}$$

and

$$\begin{aligned} \mathrm{T}^B_{1t} &= \mathsf{tx}^B_{1t} \\ \mathrm{T}^B_{2,t+1} &= \mathsf{tx}^B_{2,t+1} \end{aligned} \tag{10}$$

In addition,

$$\mathrm{T}_{21} = \tau^A_{21} + \mathsf{tx}_{21}. \tag{11}$$

Net-of-tax social security taxes or transfers excepted, the total taxes or transfers collected from the members of generations $t \geq 1$ during their lives are denoted t^i_{1t} and $t^i_{2,t+1}$, $i = A, B$. We have

$$\begin{aligned} t^A_{1t} &= \mathsf{tx}^A_{1t} \quad \text{and} \\ t^B_{1t} &= \mathsf{tx}^B_{1t}\,; \end{aligned} \tag{12}$$

$$\begin{aligned} t^A_{2t} &= \mathsf{tx}^A_{2t} - z_t \left(\mathrm{T}^A_{2t} + \Phi^A_{2t} \right) \quad \text{and} \\ t^B_{2t} &= \mathsf{tx}^B_{2t} \quad \text{for dates } t \geq 2, \end{aligned} \tag{13}$$

and

$$t_{21} = \mathsf{tx}_{21} - \, z_1 \left(\mathrm{T}_{21} + \Phi_{21} \right) , \tag{14}$$

where t_{21} denotes the analogous tax variable for the initial old.

2.5 Government budget constraint

The government must finance a real purchase of $g \geq 0$ per young agent at each date $t \geq 1$. Abstracting from the social security system, the government's consolidated budget constraint at dates $t \geq 2$ is

$$g = p_t \, (h_t - \frac{h_{t-1}}{n}) + (b_t - \frac{R^b_{t-1} \, b_{t-1}}{n}) + \alpha \left[t^A_{1t} + \frac{t^A_{2t}}{n} + \right] + (1 - \alpha) \left[t^B_{1t} + \frac{t^B_{2t}}{n} \right] . \tag{15}$$

The variables h_t and b_t represent the average nominal quantity of money and the average real present value of the bonds, respectively, issued (supplied) by the government per young agent at date t. At date 1, the government's consolidated budget constraint is

$$g = \alpha \left[t_{11}^A + \frac{t_{21}^A}{n} + p_1 \, (h_1 - \frac{h_0}{n}) + b_1 \right] . \tag{16}$$

We assume that each component of social security system is financed (separately) in an actuarially balanced fashion. Government purchases are financed by currency and bond seigniorage, by lump-sum taxes that are not part of the social security system, and by taxes on social security benefits. Thus, we can break up the government budget constraints as follows:

$$\begin{aligned} g &= p_t \, (h_t - \frac{h_{t-1}}{n}) + (b_t - \frac{R_{t-1}^b \, b_{t-1}}{n}) & (17) \\ &\quad + \alpha \left[\left(\mathsf{tx}_{1t}^A + \frac{\mathsf{tx}_{2t}^A}{n} \right) + \frac{z_t \, \mathrm{T}_{2t}^A}{n} \right] + (1-\alpha) \left(\mathsf{tx}_{1t}^B + \frac{\mathsf{tx}_{2t}^B}{n} \right) & (18) \\ 0 &= \tau_{1t}^A + \frac{\mathrm{T}_{2t}^A}{n} + \frac{\varphi_t \, R_{t-1} \, k_{t-1}^A}{n} & (19) \\ 0 &= \Psi_{1t}^A + \frac{\Phi_{2t}^A}{n} & (20) \end{aligned}$$

for dates $t \geq 2$ and

$$\begin{aligned} g &= p_1 \left(h_1 - \frac{h_0}{n} \right) + b_1 + \alpha \, \mathsf{tx}_{11}^A + (1-\alpha) \, \mathsf{tx}_{11}^B + \frac{\mathsf{tx}_{21}}{n} + \frac{z_1 \, \mathrm{T}_{21}}{n} & (21) \\ 0 &= \alpha \, \tau_{11}^A + \frac{\mathrm{T}_{21}}{n} & (22) \\ 0 &= \alpha \, \Psi_{11}^A + \frac{\Phi_{21}}{n} & (23) \end{aligned}$$

for date 1.

2.6 Household budget constraints

The budget constraints of a two-period-lived agent belonging to Variety A are

$$\begin{aligned} c_{1t}^A + s_t^A &= \omega_1^A - \mathrm{T}_{1t}^A & (24) \\ c_{2,t+1}^A &= \left(\omega_2^A - \mathrm{T}_{2,t+1}^A \right) + R_t^s \, s_t^A & (25) \end{aligned}$$

where it is assumed that s_t^A is optimally chosen to be positive, given the option to switch to the Variety B constraints by choosing $s_t < 0$. Here R_t^s represents the gross return rate on (positive) saving; it is given by

$$R_t^s = (1 - \gamma_t) \, R_t^d + \gamma_t \, \upsilon_{t+1} \, . \tag{26}$$

The Variety B budget constraints are

$$c_{1t}^{B} + s_{t}^{B} = \omega_{1}^{B} - \mathrm{T}_{1t}^{B} \tag{27}$$

$$c_{2,t+1}^{B} = \left(\omega_{2}^{B} - \mathrm{T}_{2,t+1}^{B}\right) + R_t\, s_{t}^{B} \tag{28}$$

where it is assumed that s_t^B is optimally chosen to be negative, given the option to switch to the Variety A constraints by choosing $s_t > 0$. The agents' intertemporal budget constraints are consequently

$$c_{1t}^{A} + \frac{c_{2,t+1}^{A}}{R_t^s} = \left(\omega_{1t}^{A} - \mathrm{T}_{1t}^{A}\right) + \frac{\omega_{2,t+1}^{A} - \mathrm{T}_{2,t+1}^{A}}{R_t^s} \tag{29}$$

$$c_{1t}^{B} + \frac{c_{2,t+1}^{B}}{R_t} = \left(\omega_{1t}^{B} - \mathrm{T}_{1t}^{B}\right) + \frac{\left(\omega_{2,t+1}^{B} - \mathrm{T}_{2,t+1}^{B}\right)}{R_t}\,. \tag{30}$$

The budget constraints of the initial old agents are

$$c_{21} = (\omega_2 - \mathsf{tx}_{21} - \tau_{21} - \Psi_{21}) + p_1 h_0\,. \tag{31}$$

2.7 Competitive equilibrium

2.7.1 Conditions

Credit market clearing requires

$$\alpha\,\left(s_{1t}^{A} - \mu_{t}^{A}\right) + (1-\alpha)\, s_{1t}^{B} = \alpha\, k_{t}^{A}, \tag{32}$$

where $\mu_t^A \equiv m_t^A + b_t^A$ denotes the total real present value of holdings of government liabilities per Variety A agent. In a non-monetary equilibrium we must have $m_t^A = 0$ for all $t \geq 1$. In a monetary equilibrium the currency market must clear, which requires

$$\alpha\, h_t^A = h_t\,. \tag{33}$$

In a monetary equilibrium with a binding reserve requirement (with $R_t^m < R_t$), we must have

$$m_t^A = \lambda_t\, s_t^A\,. \tag{34}$$

This condition follows from the fact that if $R_t^m < R_t$, then banks will hold fiat currency only to satisfy a reserve requirement. In other monetary equilibria we must have

$$m_t^A \geq \lambda_t\, s_t^A\,. \tag{35}$$

If there are government bonds outstanding then we must have

$$b_t = \alpha\, b_t^A\,, \tag{36}$$

so that the bonds supplied by the government are demanded by the agents.

2.7.2 Definitions

We will study competitive equilibria of two general types: non-monetary equilibria and monetary equilibria.

Non-monetary equilibria In a non-monetary equilibrium we must have $\lambda_t = p_t = 0$ for all $t \geq 1$. A non-monetary equilibrium then consists of sequences of positive return rates $\{R_t\}_{t=1}^{\infty}$, $\{R_t^b\}_{t=1}^{\infty}$, $\{\rho_t^d\}_{t=1}^{\infty}$, $\{R_t^d\}_{t=1}^{\infty}$, and $\{R_t^s\}_{t=1}^{\infty}$, tax levels $\{\mathsf{tx}_{21}, \{\mathsf{tx}_{1t}^A, \mathsf{tx}_{2,t+1}^A\}_{t=1}^{\infty}, \{\mathsf{tx}_{1t}^B, \mathsf{tx}_{2,t+1}^B\}_{t=1}^{\infty}\}$ and $\{\mathrm{T}_{21}, \{\tau_{1t}^A, \mathrm{T}_{2,t+1}^A\}_{t=1}^{\infty}\}$, non-negative tax rates $\{\varphi_{t+1}\}_{t=1}^{\infty}$, $\{z_t\}_{t=1}^{\infty}$ and $\{\gamma_t\}_{t=1}^{\infty}$, non-negative social security replacement rates $\{\upsilon_{t+1}\}_{t=1}^{\infty}$, bank storage choices $\{k_t^A\}_{t=1}^{\infty}$ and agent consumption choices $\{c_{21}, \{c_{1t}^A, c_{2,t+1}^A\}_{t=1}^{\infty}, \{c_{1t}^B, c_{2,t+1}^B\}_{t=1}^{\infty}\}$ that satisfy return conditions (1)-(2), (4)-(5), and (25), tax conditions (6)-(14), government budget constraints (9) and (20)-(22) and market-clearing condition (31), where the consumption choices maximize agents' utility subject to the budget constraints (28)-(30).

Monetary equilibria We will study monetary equilibria with and without binding reserve requirements. For simplicity, we will view a monetary equilibrium without binding reserve requirements as one in which the reserve requirement is zero. Such an equilibrium is characterized by $\lambda_t = 0$ for all $t \geq 1$, positive currency price and return sequences $\{p_t\}_{t=1}^{\infty}$ and $\{R_t^m\}_{t=1}^{\infty}$, positive nominal currency supply and demand sequences $\{h_t\}_{t=1}^{\infty}$ and $\{h_t^A\}_{t=1}^{\infty}$, respectively, positive real present value bond supply and demand sequences $\{b_t\}_{t=1}^{\infty}$ and $\{b_t^A\}_{t=1}^{\infty}$, respectively, conditions (3), (32) and (35) met with equality and condition (34) met with inequality at each date $t \geq 1$, and the sequences and conditions described in the preceding paragraph. A monetary equilibrium with reserve requirements is similar except that $\lambda_t > 0$ for all $t \geq 1$, condition (3) is met with inequality, and condition (33) replaces condition (34).

3 Equivalence results

3.1 No reserve requirements, lump-sum social security

In this subsection, we study economies in which $x = 0$, so that storage does not occur (pure exchange economies). We confine ourselves to studying monetary equilibria in which there are no reserve requirements, so that the real rate of return on currency is equal to the real return rate on other assets. We show that any monetary equilibrium of this type can be supported as a non-monetary equilibrium featuring a lump-sum social security system without deposit taxes, and vice-versa.

Theorem 1 Suppose the government policy features no reserve requirements, no taxation of deposits, and no social security system or other taxes/transfers of any kind. Suppose there is a monetary equilibrium with interest rate sequence $\{R_t^*\}_{t=1}^{\infty}$, real fiat currency balances sequence $\{m_t^{A*}\}_{t=1}^{\infty}$, real government bond holdings sequence $\{b_t^{A*}\}_{t=1}^{\infty}$, initial tax t_{21}^* and consumption allocation $\{c_{21}^*, \{(c_{1t}^{A*}, c_{2,t+1}^{A*}), (c_{1t}^{B*}, c_{2,t+1}^{B*})\}_{t=1}^{\infty}\}$. Then the equilibrium interest rate sequence and consumption allocation can be supported as a lump-sum social security equilibrium, with no deposit taxes and no other taxes or transfers, in which $\widehat{m}_t^A = \widehat{b}_t^A = 0$, $\widehat{\tau}_{1t}^A = \mu_t^{A*}$,

$$\widehat{z}_t = 1 - \frac{R_{t-1}^*}{n} \frac{\mu_{t-1}^{A*}}{\mu_t^{A*}} \tag{37}$$

for $t \geq 2$, and

$$\widehat{z}_1 = 1 - \frac{\frac{p_1^* h_0}{n} - \frac{\mathrm{tx}_{21}^*}{n}}{\alpha\, \mu_1^{A*}}.$$

Equation (36) implies that for dates $t \geq 2$ we have

$$\left[n \frac{\mu_t^{A*}}{\mu_{t-1}^{A*}} - R_{t-1}^* \right] \mu_{t-1}^{A*} = \widehat{z}_t \left(-\widehat{\mathrm{T}}_{2t}^A \right). \tag{38}$$

Here R_{t-1}^* is the rate of return on currency (and, in this economy, all other assets) from date $t-1$ to date t. On the left-hand side of this equation, we may think of the term in square brackets as the "inflation tax rate" on real balances of currency (and, in this economy, government bonds) acquired at date $t-1$ by the members of the generation born at that date; the remaining term is the value of those balances. In a steady state, the first term would simplify to $n - R_{t-1}^*$ or, equivalently, to $n - 1/\Pi_{t-1}^*$, where Π_{t-1}^* is the gross inflation rate from date $t-1$ to date t. On the right-hand side of the equation, the first term is tax rate on the social security benefits received by the members of generation $t-1$ and the second term is the value of those benefits.

Corollary 1 Suppose the government policy involves no currency or bonds, and no taxes or transfers except for a lump-sum social security policy. Suppose this policy supports an equilibrium with interest rate sequence $\{R_t^*\}_{t=1}^{\infty}$ and social security taxes $\{\tau_{1t}^{A*}\}_{t=1}^{\infty}$, social security benefits tax rates z_t^* for $t \geq 1$, and consumption allocation $\{c_{20}^*, \{(c_{1t}^{A*}, c_{2,t+1}^{A*}), (c_{1t}^{B*}, c_{2,t+1}^{B*})\}_{t=1}^{\infty}\}$. Then the equilibrium interest rate sequence and consumption allocation can be supported as a currency-only monetary equilibrium, with no social security system, in which $\widehat{m}_t^A = \tau_{1t}^{A*}$, with supplementary taxes or transfers $\widehat{\mathrm{t}}_{2,t+1}^A = -\left[n\left(1 - z_{t+1}\right) - R_t^* \right] \widehat{m}_t^A$ for $t \geq 1$.

3.2 Reserve requirements

In this subsection, we study economies in which $x \geq 0$ and storage may (or may not) occur. We confine ourselves to studying monetary equilibria in which a reserve requirement is binding at each date. We show that any equilibrium of this type can be supported by a non-monetary equilibrium with a saving-based social security system, and vice versa. We also show that any monetary equilibrium with binding reserve requirements can be supported by a non-monetary equilibrium featuring a lump-sum social security system with a deposit tax. In addition, we show that any monetary equilibrium with reserve requirements and government bonds can be supported by a monetary equilibrium with reserve requirements but no government bonds, or by a "monetary" equilibrium with government bonds, but no currency, and a saving-based social security system.

3.2.1 Saving-based social security

Theorem 2 Suppose the government policy features positive reserve requirements at each date ($\lambda_t^* > 0$ for all $t \geq 1$) and no social security system or any other taxes or transfers, except at date 1. Suppose there is a monetary equilibrium in which the reserve requirements are binding at each date, with real return rate sequence $\{R_t^*\}_{t=1}^{\infty}$, real fiat currency balances sequence $\{m_t^{A*}\}_{t=1}^{\infty}$, real government bond holdings sequence $\{b_t^{A*}\}_{t=1}^{\infty}$, date 1 tax t_{21}^* and consumption allocation $\{c_{21}^*, \{(c_{1t}^{A*}, c_{2,t+1}^{A*}), (c_{1t}^{B*}, c_{2,t+1}^{B*})\}_{t=1}^{\infty}\}$. Then the equilibrium interest rate sequence and consumption allocation can be supported as a saving-based social security equilibrium in which $\widehat{m}_t = \widehat{b}_t = 0$, $\widehat{\gamma}_t = \mu_t^{A*}/s_t^{A*}$,

$$\widehat{z}_t = 1 - \frac{R_{t-1}^{\mu*}}{n} \frac{\mu_{t-1}^{A*}}{\mu_t^{A*}} \tag{39}$$

for $t \geq 2$, where

$$R_{t-1}^{\mu*} \equiv \frac{R_{t-1}^{b*}\, b_t^{A*} + R_t^{m*}\, m_t^{A*}}{\mu_{t-1}^{A*}} \tag{40}$$

and

$$z_1 = 1 - \frac{p_1^*\, h_0}{\alpha\, n\, m_1^{A*}}\,. \tag{41}$$

In this case, we can write

$$\left[n\, \frac{\mu_t^{A*}}{\mu_{t-1}^{A*}} - R_{t-1}^{\mu*} \right] \mu_{t-1}^{A*} = \widehat{z}_t \left(-\widehat{\mathrm{T}}_{2t}^{A} \right). \tag{42}$$

Here $R_{t-1}^{\mu*}$ is the average gross real rate of return on government liabilities, weighted by the real holdings of these liabilities. It is also the gross real rate of return on currency – the inverse of the gross inflation rate – in an equivalent monetary equilibrium with government currency but no government bonds – see Proposition 1 below. Thus, we may think of the term in square brackets as the inflation tax rate in that equivalent equilibrium, while the second term on the left-hand-side is the inflation tax base in that equilibrium.

As this discussion indicates, an implication of Theorem 2 is that we can support the currency-and-bonds equilibrium featuring reserve ratios λ_t^* and currency return rates R_t^{m*} as a currency-only equilibrium featuring reserve ratios $\widetilde{\lambda}_t = \mu_t^{A*}/s_t^{A*}$.and currency return rate $\widetilde{R}_t^m = R_t^{\mu*}$.[13] This result has independent importance, so we will state it and prove it as

Proposition 1 Suppose the government policy features positive reserve requirements at each date ($\lambda_t^* > 0$ for all $t \geq 1$) and no social security system or any other taxes or transfers, except at date 1. Suppose there is a monetary equilibrium in which the reserve requirements are binding at each date, with real return rate sequences $\{R_t^*\}_{t=1}^{\infty}$ and $\{R_t^{d*}\}_{t=1}^{\infty}$, real fiat currency balances sequence $\{m_t^{A*}\}_{t=1}^{\infty}$, real government bond holdings sequence $\{b_t^{A*}\}_{t=1}^{\infty}$, date 1 tax t_{21}^* and consumption allocation $\{c_{21}^*, \{(c_{1t}^{A*}, c_{2,t+1}^{A*}), (c_{1t}^{B*}, c_{2,t+1}^{B*})\}_{t=1}^{\infty}\}$. Then the equilibrium interest rate sequences and consumption allocation can be supported as a monetary equilibrium with currency and reserve requirements but no government bonds. The equilibrium reserve requirements sequence is $\widetilde{\lambda}_t = \mu_t^{A*}/s_t^{A*}$ and the equilibrium sequence of real currency return rates is

$$\widetilde{R}_t^m = R_t^{\mu*} \equiv \frac{R_t^* \, b_t^{A*} + R_t^{m*} \, m_t^{A*}}{\mu_t^{A*}} \, . \tag{43}$$

Proposition 1 implies that the results of any monetary policy experiment conducted under a regime featuring currency reserve requirements and unbacked government bonds, can be duplicated, across regimes, by an experiment in a regime of the same type without the unbacked debt.

As we have seen, when a monetary policy regime includes both currency and bonds, the implicit tax rate from the monetary equilibrium that corresponds to the explicit tax rate from the equivalent social security equilibrium (the tax rate on social security benefits) is the average return rate on both types of government liabilities. This return rate is different from (and higher than) the real return rate on the government's currency liabilities, which is the rate that determines the inflation tax rate. One alternative way to characterize the relationship between an inflation tax and a tax on social security benefits involves an equivalence result that is very closely related to Theorem 2. This result, which we will call Proposition 2, establishes that any monetary equilibrium with government currency,

[13] Note that in the previous economy, where there were no binding reserve requirements, we could replace the bonds with currency, or vice-versa, rather trivially, since both had the same real rate of return. This is an example of the irrelevance of open market operations in economies of this type: see Sargent (1987, ch. 7)

government debt and binding currency reserve requirements can be supported by a quasi-fiscal equilibrium with unbacked government debt and a saving-based social security system, but no reserve requirements and no government currency.[14] And the inflation tax rate in the monetary equilibrium is equal to the direct tax rate on social security benefits in this quasi-fiscal equilibrium.

Proposition 2 Suppose the government policy features positive reserve requirements at each date ($\lambda_t > 0$ for all $t \geq 1$) and no social security system or other taxes or transfers of any kind, except at date 1. Suppose there is a monetary equilibrium in which the reserve requirements are binding at each date, with interest rate sequences $\{R_t^*\}_{t=1}^\infty$ and $\{R_t^{d*}\}_{t=1}^\infty$, real fiat currency balances sequence $\{m_t^{A*}\}_{t=1}^\infty$, real government bond holdings sequence $\{b_t^A\}_{t=1}^\infty$, date 1 tax t_{21}^* and consumption allocation $\{c_{21}^*, \{(c_{1t}^{A*}, c_{2,t+1}^{A*}), (c_{1t}^{B*}, c_{2,t+1}^{B*})\}_{t=1}^\infty\}$. Then the equilibrium interest rate sequence and consumption allocation can be supported as an equilibrium with no government currency, but government bonds and a saving-based social security system. This equilibrium features $\widehat{b}_t^A = b_t^{A*}$, $\widehat{\gamma}_t = \mu_t^{A*}/s_t^{A*}$,

$$\widehat{z}_t = 1 - \frac{R_{t-1}^{m*}}{n} \frac{m_{t-1}^{A*}}{m_t^{A*}}, \tag{44}$$

for $t \geq 2$ and

$$\widehat{z}_1 = 1 - \frac{p_1^* h_0}{\alpha\, n\, m_1^{A*}}. \tag{45}$$

(The proof of this proposition will be omitted, since the required modifications in the proof of Theorem 2 are quite trivial.)

We conclude this section by demonstrating that Theorem 1 works in the opposite direction, so that the two types of regimes, monetary and fiscal, really are entirely equivalent. We show, that is, that any equilibrium under a fiscal regime with a saving-based social security system can be supported as an equilibrium under a monetary regime with binding currency reserve requirements. There is one caveat; the social security policy cannot be so generous that its implicit return rate exceeds the return rate on privately-issued liabilities. Otherwise, the reserve requirement will not be binding.

Corollary 2 (to Theorem 2) Suppose the government policy features no currency or government bonds and no taxes or transfers except for a saving-based social security system with a sequence of social security contribution tax rates $\{\gamma_t^*\}_{t=1}^\infty$ and a sequence of social security benefits tax rates $\{z_t^*\}_{t=1}^\infty$. Suppose there are associated equilibrium interest rate sequences $\{R_t^*\}_{t=0}^\infty$ and $\{R_t^{s*}\}_{t=0}^\infty$ and an associated equilibrium consumption allocation $\{c_{21}^*$,

[14] We refer to this equilibrium as "quasi-fiscal" because (1) it meets our special-purpose definition of a monetary equilibrium, which encompasses equilibria in which the only government liability is debt, but (2) it would be regarded by most economists as a purely fiscal equilibrium, because it does not include any government currency.

$\{(c_{1t}^{A*}, c_{2,t+1}^{A*}), (c_{1t}^{B*}, c_{2,t+1}^{B*})\}_{t=1}^{\infty}\}$. Finally, suppose this equilibrium satisfies $R_t^{s*} < R_t^*$ for all $t \geq 1$. Then the equilibrium interest rate sequence and consumption allocation can be supported by a monetary equilibrium in which the government imposes reserve requirements $\widehat{\lambda}_t = \gamma_t^*$.

3.2.2 Lump-sum social security with deposit taxes

Theorem 3 Suppose the government policy features positive reserve requirements at each date ($\lambda_t^* > 0$ for all $t \geq 1$) and no social security system or other taxes or transfers of any kind, except at date 1. Suppose there is a monetary equilibrium in which the reserve requirements are binding at each date, with interest rate sequences $\{R_t^*\}_{t=1}^{\infty}$ and $\{R_t^{d*}\}_{t=1}^{\infty}$, real fiat currency balances sequence $\{m_t^{A*}\}_{t=1}^{\infty}$, real government bond holdings sequence $\{b_t^{A*}\}_{t=1}^{\infty}$, date 1 tax t_{21}^* and consumption allocation $\{c_{21}^*, \{(c_{1t}^{A*}, c_{2,t+1}^{A*}), (c_{1t}^{B*}, c_{2,t+1}^{B*})\}_{t=1}^{\infty}\}$. Then the equilibrium interest rate and consumption allocation can be supported by a lump-sum social security system involving taxes and transfers $\widehat{\tau}_{1t}^{A} = \mu_t^{A*}$ and $\widehat{\tau}_{2,t+1}^{A*} = -R_t^{d*}\,\widehat{\tau}_{1t}^{A}$ for $t \geq 1$, and $\widehat{\mathrm{t}}_{21} = -\alpha\, n\, \widehat{\tau}_{11}^{A}$, deposit taxes at rates $\widehat{\varphi}_{t+1}$ solving $(1 - \widehat{\varphi}_{t+1})\, R_t^* = R_t^{d*}$ for $t \geq 1$, and social security benefits tax rates

$$\widehat{z}_t = 1 - \frac{R_{t-1}^{\mu*}\, \mu_{t-1}^{A*}}{n\, \mu_t^{A*}} \tag{46}$$

for $t \geq 2$ and

$$\widehat{z}_1 = 1 - \frac{p_1^* h_0 - \mathrm{tx}_{21}^*}{\alpha\, n\, \mu_1^{A*}}\,. \tag{47}$$

In this case, unlike the previous ones, we cannot obtain this result in the other direction: that is, we cannot show that any equilibrium featuring a lump-sum social security system partly supported by a deposit tax can be duplicated by a monetary equilibrium with reserve requirements but no deposit taxes. The reason for this is that most social security systems of this sort will not produce equilibria in which the after-tax rate of return on deposits is equal to the implicit rate of return on social security contributions.

The analogue of Proposition 2 holds for this type of social security regime: if the initial monetary equilibrium features currency, a currency reserve requirement, and unbacked bonds, then it can be duplicated by a monetary-fiscal equilibrium involving a lump-sum social security policy, a deposit tax, and unbacked bonds. And it is again true that the inflation tax rate in the purely monetary equilibrium is equal to the direct tax rate on social security benefits, adjusted for the population growth rate, in this equivalent monetary-fiscal equilibrium. Since this proposition and proof are very similar to their analogues in Bacchetta and Caminal (1994), we do not present them here.

4 Examples

In this section, we describe two examples that help illustrate our results. The first example is parametric and numerical. The details of this example – functional forms, parameter values, equilibrium values of endogenous variables, etc. – are presented in Appendix B. The other example is conceptual in nature.

Example 1 involves a pure exchange economy with intragenerational diversity, so that each generation $t \geq 1$ consists of both Variety A and Variety B households (savers and borrowers). We begin by describing a steady state under a monetary policy regime with fiat currency, binding reserve requirements, and government debt. The government finances a real purchase through a combination of currency seigniorage and bond seigniorage. Wallace (1984) studied economies and equilibria of this type. The monetary and fiscal policy experiments we conduct in this example are based on the monetary policy experiments Wallace studied. We follow him by structuring the model so that the initial steady state features a real interest rate higher than the population growth rate. As a result, the government loses revenue from bond seigniorage, so its inflation tax revenue must be large enough both to cover these losses and to finance its real purchase.

We begin our equivalence analysis of this example by describing a money-only policy regime (no government debt) that supports the same equilibrium consumption allocation as the money-and-bonds regime just described. Compared to the money-and-bonds regime, the money-only regime features both a higher reserve ratio and a higher real currency return rate. In particular, the real currency return rate in the money-only steady state lies between the real currency return rate in the steady state under the mixed regime and the real bond return rate in that steady state.

Next, we describe two fiscal policy regimes that also support the same consumption allocation, despite the absence of currency or bonds. One of these regimes features a saving-based social security system, while the other features a lump-sum social security system supplemented by taxes on deposit returns. In each case, the total value of the social security contributions in the steady state under the fiscal policy regime is equal to the total real value of the stock of government liabilities (currency and bonds, or just currency) in the steady state under the monetary policy regime. And in each case, the rate at which social security benefits are taxed under the fiscal policy regime is equal to the inflation tax rate in the money-only monetary policy regime.

We continue this example by returning to the initial money-and-bonds equilibrium and conducting a policy experiment of the type described by Wallace (1984). We hold the reserve ratio fixed and imagine an "open market sale" that increases the ratio of bonds to currency, comparing the steady states under the old and new ratios. One result of this experiment that has attracted a good deal of attention is that the real bond rate and the real currency return rate change in opposite directions. Thus, the experiment produces a form of the "unpleasant monetarist arithmetic" described by Sargent and Wallace (1981): a "tighter" monetary policy – a policy that involves more bonds and less currency, and that produces a higher real interest rate – leads to higher inflation rather than lower inflation.[15]

As before, our next step is to construct the money-only regime that supports the post-experiment consumption allocation from money-and-bonds regime. When we compare this new money-only regime to the money-only regime that supported the pre-experiment allocation, we find that the new regime has a higher reserve ratio. The new regime also features a higher real currency return rate – that is, a lower inflation rate – than the old money-only regime. Thus, the unpleasant-arithmetic result described in the preceding paragraph is revealed to be an artifact of the division of the real stock of government liabilities into two components with different real return rates. In an equivalent experiment without this division, the "monetarist arithmetic" is pleasant.[16] With the division, however, as the increase in the total stock of government liabilities crowds out private debt and drives the real interest rate upward, it also drives up the (equal) real interest rate on government bonds, forcing the real rate of return on currency to fall (and the inflation rate to rise) even though the average real return rate on government liabilities rises.

Next, we construct a savings-based social security regime that supports the same consumption allocation as the post-experiment monetary policy regimes.[17] Social security contributions and benefits are higher than they were under the social security policy that supported the pre-experiment allocation, but the tax rate on the social security benefits is lower. Thus, our monetary policy experiment is revealed to be equivalent to fiscal policy experiment that involves increasing the scale of a pay-as-you-go social security system. This

[15] See Bhattacharya, Huybens and Smith (1998), Bhattacharya and Kudoh, (2001) and Espinosa and Russell (1998, 2001).

[16] The equivalent experiment is an increase in the reserve ratio rather than an open market purchase. But a reserve ratio increase is generally considered to be a form of monetary tightening. Note that under Wallace's assumptions, which we follow, the total real stock of currency is fixed, so that an open market sale (an increase in the bonds-money ratio) always increases the total real stock of government liabilities.

[17] For the sake of completeness, we also describe the lump-sum social security policy, supplemented by deposit taxes, that supports the post-experiment steady state.

revelation helps illustrate the logic behind both types of experiments. In each case, a policy intervention increases the scale of government's intergenerational transfer system. The increased intergenerational transfers crowd out private lending and drive up the real interest rate. However, the fact that the transfers are larger allows the government to reduce the rate at which it taxes them without losing any revenue.

Example 2 involves a recent result by Bhattacharya and Haslag (2001) about the optimal rate of inflation. Their analytical framework is the reserve-requirements-with-storage model studied by Freeman (1987). They use this model to determine whether it may be optimal for a government that cannot change the required reserve ratio to use the inflation tax as source of revenue, even when the alternative revenue source is tax is a lump-sum tax on young households. Somewhat surprisingly, they find that in most cases some use of the inflation tax is optimal.

How can a distorting tax be superior to a non-distorting one? The answer lies in two of the results we have obtained in this paper. First, a monetary regime with a reserve requirement is equivalent to a fiscal regime with a pay-as-you-go social security system – a system which, as we have seen, is a bad one in Freeman's model. Second, the inflation tax is equivalent, across these two regimes, to a proportional tax on social security benefits.

Suppose that, under the monetary regime, the government increases the inflation rate and uses the additional currency seigniorage revenue to reduce the lump-sum tax on the young households. Across regimes, the increase in the inflation rate is exactly equivalent to an increase in the tax rate on social security benefits, and, thus, to a reduction in the size of the after-tax benefits. This reduction in the after-tax social security benefits finances a decrease in the lump-sum tax on the young households that is almost equivalent to a decrease in their social security contributions.[18] Thus, the equivalent (almost) policy has the effect of reducing the scale of the equivalent (almost) social security system. And since social security is welfare-reducing in this economy, it should be no surprise that increasing the inflation rate may be welfare-increasing. Indeed, the only reason greater reliance on the inflation tax does not always increase welfare is that taxing social security benefits at a higher rate increases the return distortion associated with the fact that the equivalent social security system is saving-based rather than lump-sum.[19]

[18] The analogy is not quite precise, since the social security contributions are saving-based rather than lump-sum.

[19] If the alternative to a lump-sum tax on the young households was a lump-sum tax on the old households, rather than seigniorage, then increasing the tax on the old households would unambiguously increase steady-

5 Concluding remarks

Policy equivalence results have a long and important history in macroeconomics. In this paper, we extend the policy equivalence literature by studying equivalence between monetary regimes and fiscal regimes. In particular, we show that any allocation that can be supported by a monetary regime with fiat currency and/or unbacked government debt can also be supported by a fiscal regime with a pay-as-you-go social security system. We obtain this result in two different types of economies: pure exchange economies in which fiat currency and other government or private liabilities have the same equilibrium return rates, and investment/storage economies in which fiat currency is return-dominated and is held to satisfy reserve requirements.

Although our findings have a number of potentially important implications, we think two of these implications stand out. First, one of the key roles of monetary arrangements and monetary policy actions may be to determine the nature and scale of the intergenerational transfers that take place in the economy. The inflation tax, for example, may be best understood as a tax on intergenerational transfers. Second, we may be able to understand the effects of changes in monetary policy more completely by studying the effects of equivalent changes in fiscal policy, and we may be able to use models of fiscal policy to predict the effects of changes in monetary policy. Our results suggest, for example, that if fiscal policy actions involving changes in the social security system can have large, permanent real effects – as many economists believe – then monetary policy actions can also have large, permanent real effects.

Our findings have another possible implication that deserves mention. Many developing countries do not have social security systems or have systems that are very limited in scope. Many of these same countries have great difficulty collecting taxes and/or administering programs financed by tax revenue. But some of them have relatively well-developed monetary systems, and their governments may have many years' experience enforcing currency reserve requirements and other monetary and financial restrictions. Our results indicate that countries in this situation that are considering creating new social security systems, or expanding existing ones, might be able to accomplish their goals more effectively using their existing monetary and financial institutions.

state welfare.

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Appendix A

Proof of Theorem 1: In a social security equilibrium, the household budget constraints are

$$\begin{aligned} c_{1t}^A + \frac{c_{2,t+1}^A}{\widehat{R}_t} &= (\omega_1 - \widehat{\tau}_{1t}^A) + \frac{\omega_{2,t+1} - \widehat{\tau}_{2,t+1}^A}{\widehat{R}_t} \\ c_{1t}^B + \frac{c_{2,t+1}^B}{\widehat{R}_t} &= \omega_1 + \frac{\omega_{2,t+1}}{\widehat{R}_t}\,. \end{aligned}$$

By hypothesis, $\widehat{\tau}_{1t}^A = \mu_t^{A*}$ and $\widehat{\tau}_{2,t+1}^A = (1 - z_{t+1})\,\widehat{\mathrm{T}}_{2,t+1}^A$, where we assume $\widehat{\mathrm{T}}_{2,t+1}^A = -n\,\widehat{\tau}_{1,t+1}^A$ and

$$\widehat{z}_{t+1} = 1 - \frac{R_t^*\,\mu_t^{A*}}{n\,\mu_{t+1}^{A*}}\,.$$

Thus,

$$\widehat{\tau}_{1t}^A + \frac{\widehat{\tau}_{2,t+1}^A}{\widehat{R}_t} = \mu_t^{A*} - \frac{n\,\mu_{t+1}^{A*}\,\frac{R_t^*\,\mu_t^{A*}}{n\,\mu_{t+1}^{A*}}}{\widehat{R}_t} = \mu_t^{A*} - \frac{R_t^*}{\widehat{R}_t}\,\mu_t^{A*}\,.$$

Suppose $\widehat{R}_t = R_t^*$ for $t \geq 1$. Then we have

$$\widehat{\tau}_{1t}^A + \frac{\widehat{\tau}_{2,t+1}^A}{\widehat{R}_t} = 0\,,$$

so the members of generation t face the same combined budget constraints in the social security equilibrium. It follows that they will make the same consumption choices: $(\widehat{c}_{1t}^j, \widehat{c}_{2,t+1}^j) = (c_{1t}^{j*}, c_{2,t+1}^{j*})$ for $j = A, B$ and $t \geq 1$.

In a social security equilibrium, the credit market clearing condition is

$$\alpha\left[\left(\omega_1^A - \widehat{\tau}_{1t}^A\right) - \widehat{c}_{1t}^A\right] + (1-\alpha)\,\left(\omega_1^B - \widehat{c}_{1t}^B\right) = \alpha\,\widehat{k}_t,$$

By hypothesis,

$$\widehat{\tau}_{1t}^A = \mu_t^{A*},$$

and we have just assumed $\widehat{c}_{1t}^j = c_{1t}^{j*}$ for $j = A, B$ and $t \geq 1$. So this is

$$\alpha\,\left[\left(\omega_1^A - \mu_{1t}^{A*}\right) - c_{1t}^{A*}\right] + (1-\alpha)\,\left(\omega_1^B - c_{1t}^{B*}\right) = \alpha\,\widehat{k}_t\,.$$

So if we further assume $\widehat{k}_t^A = k_t^{A*}$ then we have

$$\alpha\left(\omega_1^A - c_{1t}^{A*}\right) + (1-\alpha)\,\left(\omega_1^B - c_{1t}^{B*}\right) = \alpha\left(\mu_{1t}^{A*} + k_{1t}^{A*}\right)\,.$$

which is the credit market clearing condition from the monetary equilibrium.

Under the social security equilibrium, the government budget constraint at dates $t \geq 2$, net of social security, is

$$g = \alpha\frac{\widehat{z}_t\,\widehat{\tau}_{2t}^A}{n}\,.$$

We have seen that this can be rewritten

$$g = \alpha \frac{\left(1 - \frac{R^*_{t-1}\,\mu^{A*}_{t-1}}{n\,\mu^{A*}_t}\right) n\,\mu^{A*}_{1t}}{n} = \alpha \left(\mu^{A*}_t - \frac{R^*_{t-1}}{n}\mu^{A*}_{t-1}\right) ,$$

which is the government budget constraint in the monetary equilibrium.

Under the social security system, the members of generation zero consume

$$\widehat{c}_{21} = \omega_2 - \widehat{\tau}_{21}$$

and the government budget constraint is

$$g = \frac{\widehat{z}_1\,\widehat{\mathrm{T}}_{21}}{n} .$$

We also have $\widehat{\tau}_{21} \equiv (1-\widehat{z}_1)\,\widehat{T}_{21}$ and $\widehat{T}_{21} = -\alpha\,n\,\widehat{\tau}^A_{11}$. We know by hypothesis that $\widehat{\tau}^A_{11} = \mu^{A*}_1$ and that

$$\widehat{z}_1 = 1 - \frac{\frac{p^*_1 h_0}{n} - \frac{\mathrm{tx}^*_{21}}{n}}{\alpha\,\mu^{A*}_1}.$$

So

$$\widehat{\tau}_{21} = -(1-\widehat{z}_1)\,\alpha\,n\,\widehat{\tau}^A_{11} = -\frac{\frac{p^*_1 h_0}{n} - \frac{\mathrm{tx}^*_{21}}{n}}{\alpha\,\mu^{A*}_1}\,\alpha\,n\,\mu^{A*}_1 = \mathrm{tx}^*_{21} - p^*_1 h_0 .$$

Thus, we have

$$\widehat{c}_{21} = \omega_2 - \mathrm{tx}^*_{21} + p^*_1\,h_0$$

which is the budget constraint of the initial old households in the monetary equilibrium. In addition, we have

$$g = \frac{\left(1 - \frac{p^*_1 h_0 - \mathrm{t}^*_{21}}{\alpha\,n\,\mu^{A*}_1}\right)\,\left(-\alpha\,n\,\mu^{A*}_1\right)}{n} = \alpha\,\mu^{A*}_1 - \frac{p^*_1 h_0 - \mathrm{tx}^*_{21}}{n}$$

which is the government budget constraint, at date 1, in the monetary equilibrium. ¤

Proof of Corollary 1: In the monetary equilibrium with supplementary taxes or transfers $\widehat{\mathrm{t}}_{2,t+1} = -\left[n\,(1-z^*_{t+1}) - R^*_t\right]\,\widehat{m}^{A*}_{1t}$, the intertemporal budget constraints of the members of generations $t \geq 1$ are

$$\begin{aligned} c^A_{1t} + \frac{c^A_{2,t+1}}{R_t} &= \omega^A_1 + \frac{\omega^A_{2,t+1} + \left[n\,(1-z^*_{t+1}) - R^*_t\right]\,\tau^{A*}_{1t}}{\widehat{R}_t} \\ c^B_{1t} + \frac{c^B_{2,t+1}}{\widehat{R}_t} &= \omega^B_1 + \frac{\omega^B_{2,t+1}}{\widehat{R}_t} . \end{aligned}$$

If $\widehat{R}_t = R^*_t$, then these constraints become

$$\begin{aligned} c^A_{1t} + \frac{c^A_{2,t+1}}{R_t} &= \left(\omega^A_1 - \tau^{A*}_{1t}\right) + \frac{\omega^A_{2,t+1} - \tau^{A*}_{2.t+1}}{\widehat{R}_t} \\ c^B_{1t} + \frac{c^B_{2,t+1}}{\widehat{R}_t} &= \omega^B_1 + \frac{\omega^B_{2,t+1}}{\widehat{R}_t} . \end{aligned}$$

which are the household budget constraints from the social security equilibrium. So households will choose $\widehat{c}^{j}_{i,t+i-1} = c^{j*}_{i,t+i-1}$, $j = A, B$, $i = 1, 2$. It follows that if $\widehat{k}_t = k^*_t$, which we will assume, then the Variety A households will choose $\widehat{m}^A_t = \omega^A_1 - c^{A*}_{1t} - k^*_t = \tau^{A*}_{1t}$.

In the monetary equilibrium, the credit market clearing condition is $\alpha\left(\omega^A_1 - \widehat{c}^A_{1t} - \widehat{k}_t\right) + (1-\alpha)\left(\omega^B_1 - \widehat{c}^B_{1t}\right) = \alpha\,\widehat{m}^A_t$. But since $\widehat{c}^A_{1t} = c^{A*}_{1t}$, $\widehat{c}^B_{1t} = c^{B*}_{1t}$, $\widehat{m}^A_t = \tau^{A*}_{1t}$ and $\widehat{k}_t = k^*_t$, this is equivalent to

$$\alpha\left(\omega^A_1 - \tau^{A*}_{1t} - c^{A*}_{1t} - k^*_t\right) = (1-\alpha)\,\left(\omega^B_1 - c^{B*}_{1t}\right)\,,$$

which is the credit market clearing condition from the social security equilibrium, and thus holds by hypothesis.

In the monetary equilibrium, the government budget constraint for dates $t \geq 2$ is

$$\alpha\frac{\widehat{\tau}^A_{2t}}{n} + \alpha\left(\widehat{m}_t - \frac{\widehat{R}_{t-1}}{n}\widehat{m}_{t-1}\right) = g_t\,.$$

This constraint is equivalent to

$$-\alpha\frac{\left[n\,(1-z_t) - R^*_{t-1}\right]\,\tau^{A*}_{1t}}{n} + \alpha\left(\tau^{A*}_{1t} - \frac{R^*_{t-1}}{n}\tau^{A*}_{1,t-1}\right) = g_t\,.$$

This constraint reduces to

$$\alpha\,z^*_t\,\tau^{A*}_{1t} = g_t$$

or, given that $\mathrm{T}^{A*}_{2t} = n\,\tau^{A*}_{1t}$,

$$\alpha\,\frac{z^*_t\,\mathrm{T}^{A*}_{2t}}{n} = g_t$$

which is the government's net-of-social-security budget constraint in the social security equilibrium.

At date 1, in the monetary equilibrium, we must have

$$\widehat{c}_{21} = \omega_{21} + \widehat{p}_1 h_0$$

and

$$g_1 = \alpha\,\widehat{m}^A_1 - \frac{\widehat{p}_1 h_0}{n}\,.$$

We have seen that $\widehat{m}^A_1 = \tau^{A*}_{11}$. Suppose we choose $\widehat{p}_1$ so that

$$\widehat{p}_1 h_0 = -\tau^*_{21}.$$

Then we have $\widehat{c}_{21} = c^*_{21}$. In addition, the government budget constraint can be rewritten

$$g_1 = \alpha\,\tau^{A*}_{11} + \frac{\tau^*_{21}}{n}\,.$$

And since we have $\mathrm{T}^*_{21} = -\alpha\, n\, \tau^*_{11}$ and $\tau^*_{21} = (1 - z^*_1)\, \mathrm{T}^*_{21}$, this becomes

$$g_1 = -\frac{z^*_1\, \mathrm{T}^*_{21}}{n},$$

which is the government budget constraint, at date 1, in the social security equilibrium. ¤

Proof of Theorem 2: In the saving-based social security equilibrium, the budget constraints facing households are

$$\begin{aligned} c^A_{1t} + \frac{c^A_{2,t+1}}{\widehat{R}^s_t} &= \omega^A_{1t} + \frac{\omega^A_{2,t+1}}{\widehat{R}^s_t} \\ c^B_{1t} + \frac{c^B_{2,t+1}}{\widehat{R}_t} &= \omega^B_{1t} + \frac{\omega^B_{2,t+1}}{\widehat{R}_t} \end{aligned}$$

Suppose $R_t = R^*_t$. Then

$$\begin{aligned} \widehat{R}^s_t &\equiv (1 - \widehat{\gamma}_t)\, \widehat{R}_t + \gamma_t\, \widehat{v}_{t+1} = (1 - \gamma_t)\, R^*_t + \widehat{\gamma}_t\, n\, (1 - \widehat{z}_{t+1})\, \frac{\mu^{A*}_{t+1}}{\mu^{A*}_t} \\ &= (1 - \widehat{\gamma}_t)\, R^*_t + \widehat{\gamma}_t\, n\, \frac{R^{\mu*}_t}{n} \frac{\mu^{A*}_t}{\mu^{A*}_{t+1}} \frac{\mu^{A*}_{t+1}}{\mu^{A*}_t} = (1 - \widehat{\gamma}_t)\, \widehat{R}_t + \widehat{\gamma}_t\, R^{\mu*}_t\,. \end{aligned}$$

Given our choice for $\widehat{\gamma}_t$, this becomes

$$\begin{aligned} \widehat{R}^s_t &= (1 - \frac{\mu^{A*}_t}{s^{A*}_t})\, R^*_t + \frac{\mu^{A*}_t}{s^{A*}_t}\, R^{\mu*}_t \\ &= (1 - \frac{\mu^{A*}_t}{s^{A*}_t})\, R^*_t + \frac{x\, b^{A*}_t + R^{m*}_t m^{A*}_t}{s^{A*}_t} \\ &= (1 - \frac{m^{A*}_t}{s^{A*}_t})\, R^*_t + \frac{m^{A*}_t}{s^{A*}_t}\, R^{m*}_t \\ &= (1 - \lambda_t)\, R^*_t + \lambda_t\, R^{m*}_t = R^{d*}_t, \end{aligned}$$

since $m^*_t = \lambda_t\, s^*_t$. It follows that households face the same budget constraints in the social security equilibrium, which means they will make the same consumption choices: $(\widehat{c}^j_{1t}, \widehat{c}^j_{2,t+1}) = (c^{j*}_{1t}, c^{j*}_{2,t+1})$ for $j = A, B$ and $t \geq 1$. It also follows that $\widehat{s}^j_t = s^{j*}_t$.

In the saving-based social security equilibrium, the credit market clearing condition is

$$\alpha \left[(1 - \gamma_t)\, \widehat{s}^A_t - \widehat{k}_t \right] + (1 - \alpha)\, \widehat{s}^B_t = 0$$

where $\widehat{s}^j_t = \omega^j_{1t} - \widehat{c}^j_{1t}$, $j = A, B$. Since $\widehat{\gamma}_t = \mu^{A*}_t / s^{A*}_t$, this becomes

$$\alpha \left[(1 - \frac{\mu^*_t}{s^*_t})\, \widehat{s}_t - \widehat{k}_t \right] + (1 - \alpha)\, \widehat{s}^B_t = 0 \Leftrightarrow \alpha \left[\widehat{s}^A_t - \widehat{k}^A_t - \mu^{A*}_t \right] + (1 - \alpha)\, \widehat{s}^B_t = 0$$

Since $\widehat{s}_t^j = s_t^{j*}$, $j = A, B$, if $\widehat{k}_t^A = k_t^{A*}$ then this condition is equivalent to

$$\alpha\left[s_t^{*A} - k_t^{A*} - \mu_t^{A*}\right] + (1-\alpha)\, s_t^{B*} = 0$$

which is the credit market clearing condition from the monetary equilibrium.

In the saving-based social security equilibrium, the government budget constraint is

$$g = \alpha\, \widehat{z}_t\, \widehat{\gamma}_t\, \widehat{s}_t\,.$$

Now

$$\begin{aligned}
\widehat{z}_t\, \widehat{\gamma}_t\, \widehat{s}_t &= \left(1 - \frac{R_{t-1}^{\mu*}}{n}\frac{\mu_{t-1}^{A*}}{\mu_t^{A*}}\right)\frac{\mu_t^{A*}}{s_t^{A*}}\, s_t^{A*} = \mu_t^{A*} - \frac{R_{t-1}^{\mu*}}{n}\mu_{t-1}^{A*} \\
&= \left(m_t^{A*} + b_t^{A*}\right) - \frac{1}{n}\left(R_t^*\, b_t^{A*} + R_t^{m*}\, m_t^{A*}\right) \\
&= \left(m_t^{A*} - \frac{R_{t-1}^{m*}}{n}\, m_{t-1}^{A*}\right) - \left(b_t^{A*} - \frac{R_t^*}{n}\, b_{t-1}^{A*}\right)
\end{aligned}$$

So the social security constraint is equivalent to

$$g = \alpha\left[\left(b_t^{A*} - \frac{R_t^*}{n}\, b_{t-1}^{A*}\right) + \left(m_t^{A*} - \frac{R_{t-1}^{m*}}{n}\, m_{t-1}^{A*}\right)\right],$$

which is the government budget constraint in the monetary equilibrium.

At date 1, in the social security equilibrium, we have

$$\widehat{c}_{21} = \omega_{21} - \widehat{\Phi}_{21}\,,$$

where $\widehat{\Phi}_{21} = (1 - \widehat{z}_1)\,\widehat{\Psi}_{21}$ and $\widehat{\Psi}_{21} = -\alpha\, n\, \widehat{\gamma}_1\, \widehat{s}_1^A$. We also have

$$g = -\alpha\,\frac{\widehat{z}_1\, \widehat{\Psi}_{21}}{n}\,.$$

We have seen that $\widehat{\gamma}_1\, \widehat{s}_1^A = \lambda_1^*\, s_1^{A*} = m_1^{A*}$, and we have

$$\widehat{z}_1 = 1 - \frac{p_1^*\, h_0}{\alpha\, n\, m_1^{A*}}\,.$$

It follows that

$$\widehat{\Phi}_{21} = -p_1^*\, h_0\,,$$

giving us $\widehat{c}_{21} = c_{21}^*$. In addition, we have

$$g = \left(1 - \frac{p_1^*\, h_0}{\alpha\, n\, m_1^{A*}}\right)\alpha\, m_1^{A*} = \alpha\, m_1^{A*} - \frac{p_1^*\, h_0}{n}\,,$$

which is the government budget constraint, at date 1, in the monetary equilibrium. ¤

Proof of Proposition 1: In the monetary equilibrium with no bonds, the budget constraints facing households are

$$\begin{aligned} c_{1t}^A + \frac{c_{2,t+1}^A}{\widetilde{R}_t^d} &= \omega_{1t}^A + \frac{\omega_{2,t+1}^A}{\widetilde{R}_t^d} \\ c_{1t}^B + \frac{c_{2,t+1}^B}{\widetilde{R}_t} &= \omega_{1t}^B + \frac{\omega_{2,t+1}^B}{\widetilde{R}_t}\,. \end{aligned}$$

The deposit rate $\widetilde{R}_t^d$ is

$$\widetilde{R}_t^d = (1-\widetilde{\lambda}_t)\,\widetilde{R}_t + \widetilde{\lambda}_t\,\widetilde{R}_t^m\,.$$

Suppose $\widetilde{R}_t = R_t^*$. Given our choices of $\widetilde{\lambda}_t$ and $\widetilde{R}_t^m$, we have

$$\begin{aligned} \widetilde{R}_t^d &= (1-\frac{\mu_t^{A*}}{s_t^{A*}})\,R_t^* + \frac{\mu_t^{A*}}{s_t^{A*}}\,\frac{x\,b_t^{A*} + R_t^{m*}m_t^{A*}}{\mu_t^{A*}} \\ &= (1-\frac{m_t^{A*}}{s_t^{A*}})\,R_t^* + \frac{m_t^{A*}}{s_t^{A*}}\,R_t^{m*} \\ &= (1-\lambda_t^*)\,R_t^* + \lambda_t^*\,R_t^{m*} = R_t^{d*}\,. \end{aligned}$$

So both groups of households will make the same consumption and saving choices. Note that we must have $\widetilde{m}_t^A = \widetilde{\lambda}_t\,s_t^{A*} = \mu_t^{A*}$ and $\widetilde{k}_t^A = \widetilde{s}_t^A - \widetilde{m}_t^A = s_t^{A*} - \mu_t^{A*} = k_t^{A*}$. Since $\widetilde{R}_t = R_t^*$ implies $\widetilde{s}_t^B = s_t^{B*}$, it follows that the credit market clearing constraint continues to hold.

In the monetary equilibrium with no bonds, the government budget constraint for dates $t \geq 2$ is

$$g_t = \alpha\left(\widetilde{m}_t^A - \frac{\widetilde{R}_{t-1}^m\,\widetilde{m}_{t-1}}{n}\right).$$

We have $\widetilde{m}_t^A = \mu_t^{A*} = m_t^{A*} + b_t^{A*}$, so this becomes

$$\begin{aligned} g_t &= \alpha\left[m_t^{A*} + b_t^{A*} - \frac{\left(\frac{R_t^*\,b_t^{A*} + R_t^{m*}m_t^{A*}}{\mu_t^{A*}}\right)\,\mu_{t-1}^{A*}}{n}\right] \\ &= \alpha\left[\left(m_t^{A*} - \frac{R_t^{m*}\,m_t^{A*}}{n}\right) + \left(b_t^{A*} - \frac{R_t^*\,b_t^{A*}}{n}\right)\right] \end{aligned}$$

which is the government budget constraint for dates $t \geq 2$ in the equilibrium.with bonds.

In the monetary equilibrium with no bonds, at date 1, the consumption of the initial old is

$$\widetilde{c}_{21} = \omega_{21} - \widetilde{\tau}_{21} + \widetilde{p}_1 h_0$$

the government budget constraint is

$$g_t = \alpha\,\widetilde{m}_1^A - \frac{\widetilde{p}_1 h_0}{n} + \frac{\widetilde{\tau}_{21}}{n}\,.$$

We have seen that $\widetilde{m}_1^A = m_t^{A*} + b_t^{A*}$. Suppose we choose $\widetilde{p}_1 = p_1^*$ and $\widetilde{\tau}_{21} = \tau_{21}^*$. Then we have $\widehat{c}_{21} = c_{21}^*$. In addition, we have

$$g_t = \alpha \left(m_t^{A*} + b_t^{A*} \right) - \frac{p_1^* \, h_0}{n} + \frac{\tau_{21}^*}{n} \, ,$$

which is the government budget constraint in the equilibrium with bonds. ¤

Proof of Corollary 2: In the monetary equilibrium, suppose the government issues currency in nominal quantities that produce currency return rates $\widehat{R}_t^m$ satisfying

$$(1 - \widehat{\lambda}_t) \, R_t^* + \widehat{\lambda}_t \, \widehat{R}_t^m = R_t^{s*}.$$

Then if $\widehat{R}_t = R_t^*$, the two-period-lived households of both varieties will face the same interest rates as in the social security equilibrium and will make the same consumption and saving choices. Since $R_t^{s*} < R_t^*$, we must have $\widehat{R}_t^m < R_t^*$, so the reserve requirement will be binding at each date. Note that since $\widehat{\lambda}_t = \gamma_t^*$, we must have $\widehat{R}_t^m = n \, (1 - z_{t+1}^*)$.

In the monetary equilibrium, the credit market clearing condition is

$$\alpha \left(\omega_{1t}^A - \widehat{c}_{1t}^A - \widehat{m}_t^A - \widehat{k}_t \right) = (1 - \alpha) \left(\omega_{1t}^B - \widehat{c}_{1t}^B \right)$$

which can be rewritten

$$\alpha \left(\widehat{s}_t^A - \widehat{m}_t^A - \widehat{k}_t^A \right) = (1 - \alpha) \, \widehat{s}_t^B.$$

We have $\widehat{m}_t^A = \widehat{\lambda}_t \, \widehat{s}_t^A$. Since $\widehat{\lambda}_t = \gamma_t^*$ and $\widehat{s}_t^A = s_t^{A*}$, we have $\widehat{s}_t^A - \widehat{m}_t^A = \alpha \left((1 - \gamma_t^*) \, s_t^{A*} - \widehat{k}_t^A \right) = (1-\alpha) \, \widehat{s}_t^B$. We also have $\widehat{s}_t^B = s_t^{B*}$. So if $\widehat{k}_t^A = k_t^{A*}$, which will satisfy the Type A households' first period budget constraint, then we have

$$\alpha \left((1 - \gamma_t^*) \, s_t^{A*} - k_t^{A*} \right) = (1 - \alpha) \, s_t^{B*},$$

which is the credit market clearing condition in the social security equilibrium.

In the monetary equilibrium, the government budget constraint is

$$g = \alpha \left(\widehat{m}_t - \frac{\widehat{R}_{t-1}^m}{n} \, \widehat{m}_{t-1} \right) .$$

This becomes

$$g = \alpha \left(\gamma_t^* s_t^{A*} - \frac{\widehat{R}_{t-1}^m}{n} \, \gamma_{t-1}^* s_{t-1}^{A*} \right) .$$

We know that after-tax social security benefits are $\Psi_{2t} = (1 - z_t^*) \, n \, \gamma_t^* s_t^{A*}$, and, given $\widehat{\lambda}_t = \gamma_t^*$, it follows from the from the return-rate equality condition that

$$\widehat{R}_t^m = \frac{\Psi_{2t}^*}{\Psi_{1,t-1}^*} = \frac{(1 - z_t^*) \, n \, \gamma_t^* s_t^{A*}}{\gamma_{t-1}^* s_{t-1}^{A*}}.$$

It follows that

$$g = \alpha \left(\gamma_t^* s_t^{A*} - (1 - z_t^*)\, \gamma_t^* s_t^{A*} \right) = \alpha\, z_t^* \left(\gamma_t^*\, s_t^{A*} \right),$$

which is the government budget constraint (net of pre-tax social security) in the social security equilibrium.

In the monetary equilibrium, at date 1, the consumption of the initial old households is

$$\widehat{c}_{21} = \omega_{21} + \widehat{p}_1 h_0$$

and the government budget constraint is

$$g_1 = \alpha\, \widehat{m}_1^A - \frac{\widehat{p}_1 h_0}{n}.$$

We have seen that $\widehat{m}_1^A = \widehat{\lambda}_1\, \widehat{s}_1^A = \gamma_1^*\, s_1^{A*} = \Psi_{11}^{A*}$, and we know that $\Phi_{21}^* = -\alpha\, n\, \Psi_{11}^{A*}$ and $\Psi_{21}^{A*} = (1 - z_1^*)\, \Phi_{21}^*$. Suppose we choose $\widehat{p}_1$ such that

$$\widehat{p}_1 h_0 = -\Psi_{21}^{A*}.$$

Then we have $\widehat{c}_{21} = c_{21}^*$. In addition, we have

$$\begin{aligned} g_1 &= \alpha\, \Psi_{11}^{A*} + \frac{\Psi_{21}^{A*}}{n} = -\frac{\Phi_{21}^*}{n} + \frac{(1 - z_1^*)\, \Phi_{21}^*}{n} \\ &= -\frac{z_1^*\, \Phi_{21}^*}{n}, \end{aligned}$$

which is the government budget constraint, at date 1, in the social security equilibrium. ¤

Proof of Theorem 3: In the social security equilibrium, the budget constraints of the two-period-lived households are

$$\begin{aligned} c_{1t}^A + \frac{c_{2,t+1}^A}{\widehat{R}_t^d} &= \left(\omega_1^A - \widehat{\tau}_{1t}^A \right) + \frac{\omega_{2,t+1}^A - \widehat{\tau}_{2,t+1}^A}{\widehat{R}_t^d} \\ c_{1t}^B + \frac{c_{2,t+1}^B}{\widehat{R}_t} &= \omega_1^B + \frac{\omega_{2,t+1}^B}{\widehat{R}_t}, \end{aligned}$$

Suppose $\widehat{R}_t = R_t^*$. We have chosen $\widehat{\varphi}_{t+1}$ so that $\widehat{R}_t^d \equiv (1 - \widehat{\varphi}_{t+1})\, \widehat{R}_t = R_t^{d*}$. Thus, both varieties of household face the same interest rates as in the monetary equilibrium. In addition, $\widehat{\tau}_{2t}^A = -\widehat{R}_t^d\, \widehat{\tau}_{1t}^A$ by hypothesis, so the tax terms drop out of the first constraint, and both varieties of households face the same budget constraints as in the monetary equilibrium. It follows that they make the same consumption decisions, that the Variety B households make the same saving decision, and that the saving of the Variety A households is $\widehat{s}_t^A = s_t^{A*} - \widehat{\tau}_{1t}^A = s_t^{A*} - \mu_t^{A*}$. It follows that if $\widehat{k}_t^A = k_t^{A*}$, which satisfies the first-period budget constraints of the Variety A households, then the credit market clears in the social security equilibrium.

At dates $t \geq 2$, the government budget constraints in the social security equilibrium are

$$g_t = -\alpha \frac{\widehat{z}_t \, \widehat{\mathrm{T}}_{2t}^A}{n},$$

and

$$0 = \widehat{\tau}_{1t}^A + \frac{\widehat{\mathrm{T}}_{2t}^A}{n} + \frac{\widehat{\varphi}_t \, \widehat{R}_{t-1} \, \widehat{s}_{t-1}^A}{n}.$$

It follows that

$$g_t = \alpha \left[\widehat{\tau}_{1t}^A + \frac{\widehat{\tau}_{2t}^A}{n} + \frac{\widehat{\varphi}_t \, \widehat{R}_{t-1} \, \widehat{s}_{t-1}^A}{n} \right],$$

where $\widehat{\tau}_{2t}^A = (1 - \widehat{z}_t) \, \widehat{\mathrm{T}}_{2t}^A$. We have $\widehat{\tau}_{1t}^A = \mu_t^{A*}$ and $\widehat{\tau}_{2t}^A = -R_{t-1}^{d*} \, \widehat{\tau}_{1t-1}^A = -R_{t-1}^{d*} \, \mu_{t-1}^{A*}$, and we have seen that $\widehat{s}_{t-1}^A = s_{t-1}^{A*} - \mu_{t-1}^{A*}$. So if $\widehat{R}_{t-1} = R_{t-1}^*$ this constraint becomes

$$g_t = \alpha \left[\mu_t^{A*} - \frac{R_{t-1}^{d*} \mu_{t-1}^{A*}}{n} + \frac{\widehat{\varphi}_t \, R_{t-1}^* \, \left(s_{t-1}^{A*} - \mu_{t-1}^{A*} \right)}{n} \right].$$

We know

$$s_{t-1}^{A*} - \mu_{t-1}^{A*} = k_{t-1}^{A*},$$

using credit market clearing constraint from the monetary equilibrium. We have

$$R_{t-1}^{d*} = (1 - \widehat{\varphi}_t) \, R_{t-1}^* \Leftrightarrow \widehat{\varphi}_t = 1 - \frac{R_{t-1}^{d*}}{R_{t-1}^*},$$

This substitution gives us

$$\begin{aligned} g &= \alpha \left[\mu_t^{A*} - \frac{R_{t-1}^{d*} \, \mu_{t-1}^{A*}}{n} + \frac{\left(R_{t-1}^* - R_{t-1}^{d*} \right) \, \left(s_{t-1}^{A*} - \mu_{t-1}^{A*} \right)}{n} \right] \\ &= \alpha \left[\mu_t^{A*} - \frac{R_{t-1}^{d*} \, \widetilde{\lambda}_{t-1} \, s_{t-1}^{A*}}{n} + \frac{\left(R_{t-1}^* - R_{t-1}^{d*} \right) (1 - \widetilde{\lambda}_{t-1}) \, s_{t-1}^{A*}}{n} \right], \end{aligned}$$

where

$$\widetilde{\lambda}_t \equiv \frac{\mu_{t-1}^{A*}}{s_{t-1}^{A*}}.$$

This becomes

$$\begin{aligned} g &= \alpha \left[\mu_t^{A*} + \frac{R_{t-1}^* (1 - \widetilde{\lambda}_{t-1}) \, s_{t-1}^{A*}}{n} - \frac{R_{t-1}^{d*} \, s_{t-1}^{A*}}{n} \right] \\ &= \alpha \left[\mu_t^{A*} + \frac{\left[R_{t-1}^{d*} - \widetilde{\lambda}_{t-1} R_{t-1}^{\mu*} \right] \, s_{t-1}^{A*}}{n} - \frac{R_{t-1}^{d*} \, s_{t-1}^{A*}}{n} \right], \end{aligned}$$

where

$$R_t^{\mu *} \equiv \frac{R_t^* \, b_t^{A*} + R_t^{m*} \, m_t^{A*}}{\mu_t^{A*}} \, ,$$

and we are using the fact that

$$R_t^{d*} = (1 - \widetilde{\lambda}_t) \, R_t^* + \widetilde{\lambda}_t \, R_t^{\mu *},$$

which was established in the proof of Proposition 1. We now have

$$\begin{aligned} g_t &= \alpha \left[\mu_t^{A*} - \frac{\widetilde{\lambda}_{t-1} R_{t-1}^{\mu *} \, s_{t-1}^{A*}}{n} \right] \\ &= \alpha \left[\mu_t^{A*} - \frac{R_{t-1}^{\mu *} \, \mu_{t-1}^{A*}}{n} \right], \end{aligned}$$

and it was shown in the proof of Proposition 1 that this constraint is equivalent to the consolidated (purchases plus transfers) government budget constraint in the monetary equilibrium.

We have $\widehat{\tau}_{2t}^A = -R_{t-1}^{d*} \, \widehat{\tau}_{1t-1}^A = -R_{t-1}^{d*} \, \mu_{t-1}^{A*}$, and we have $\widehat{\tau}_{2t} = (1 - \widehat{z}_t) \, \widehat{\Upsilon}_{2t}$, giving us

$$\widehat{\Upsilon}_{2t} = \frac{-R_{t-1}^{d*} \, \mu_{t-1}^{A*}}{1 - \widehat{z}_t} \, .$$

We also have

$$g_t = -\alpha \, \frac{\widehat{z}_t \, \widehat{\Upsilon}_{2t}^A}{n} \, .$$

So we have

$$g_t = \alpha \frac{\widehat{z}_t}{1 - \widehat{z}_t} \frac{R_{t-1}^{d*} \, \mu_{t-1}^{A*}}{n}$$

which is

$$\begin{aligned} g_t &= \widehat{z}_t \left(g_t + \alpha \frac{R_{t-1}^{d*} \, \mu_{t-1}^{A*}}{n} \right) = \widehat{z}_t \left(\alpha \left[\mu_t^{A*} - \frac{R_{t-1}^{\mu *} \, \mu_{t-1}^{A*}}{n} \right] + \alpha \frac{R_{t-1}^{d*} \, \mu_{t-1}^{A*}}{n} \right) \\ &= \alpha \, \widehat{z}_t \, \mu_t^{A*} \, . \end{aligned}$$

So

$$\begin{aligned} \widehat{z}_t &= \frac{g_t}{\alpha \, \mu_t^{A*}} = \frac{\mu_t^{A*} - \frac{R_{t-1}^{\mu *} \, \mu_{t-1}^{A*}}{n}}{\mu_t^{A*}} \\ &= 1 - \frac{R_{t-1}^{\mu *} \, \mu_{t-1}^{A*}}{n \, \mu_t^{A*}} \, . \end{aligned}$$

At date 1, in the social security equilibrium, the initial old households consume

$$\widehat{c}_{21} = \omega_{21} - \widehat{\tau}_{21}$$

and the budget constraint of the government is

$$g_1 = -\frac{\widehat{z}_1 \widehat{\mathrm{T}}_{21}}{n},$$

where $\widehat{\mathrm{T}}_{21} = -\alpha\, n\, \widehat{\tau}_{11}^{A}$ and $\widehat{\tau}_{21} = (1 - z_1)\, \widehat{\mathrm{T}}_{21}$. We have seen that $\widehat{\tau}_{11} = \mu_t^{A*}$. We must have

$$-\widehat{\tau}_{21} = p_1^* h_0 - \tau_{21}^*$$

for $\widehat{c}_{21} = c_{21}^*$. Thus we must have have

$$p_1^* h_0 - \tau_{21}^* = (1 - \widehat{z}_1)\, \alpha\, n\, \mu_t^{A*} \Leftrightarrow \widehat{z}_1 = 1 - \frac{p_1^* h_0 - \tau_{21}^*}{\alpha\, n\, \mu_t^{A*}}.$$

This gives us

$$\begin{aligned} g_1 &= \frac{\left(1 - \frac{p_1^* h_0 - \tau_{21}^*}{\alpha\, n\, \mu_t^{A*}}\right) \alpha\, n\, \mu_t^{A*}}{n} = \frac{\alpha\, n\, \mu_t^{A*} - p_1^* h_0 + \tau_{21}^*}{n} \\ &= \alpha\, \mu_t^{A*} + \frac{\tau_{21}^*}{n} - \frac{p_1^* h_0}{n}, \end{aligned}$$

which is the budget constraint in the monetary equilibrium. ¤

Appendix B

Example 1

In this economy, $n = 1$, $x = 0$ and $\alpha = 1/2$, with $(\omega_1^A, \omega_2^A) = (1, 0)$, $(\omega_1^B, \omega_2^B) = (0, \frac{2}{3})$ and $\omega_2 = \frac{1}{3}$. The common utility function of the members of generations $t \geq 1$ is $u(c_1, c_2) = \log c_1 + \beta \log c_2$ with $\beta = 1$. It follows that the household saving functions are $s^j(R^j) = \frac{\omega_1^j - \mathrm{T}_1^j}{2} - \frac{\omega_2^j - \mathrm{T}_2^j}{2\,R^j}$, $j = A, B$, with $R^A = R^s$ and $R^B = R$. In addition, $g = 0.005$ and $h_0 = 1$.

We begin with a monetary regime that includes fiat money, reserve requirements, and unbacked government debt. The central bank chooses a monetary policy that features $\lambda_t^* = 0.25$ and $h_1^* = \theta^* h_0$ and $h_{t+1}^* = \theta^*\, h_t^*$ for all $t \geq 1$, with $\theta^* = 1.2$. There is a stationary monetary equilibrium under this policy that features $R^* \doteq 1.17390$, $R^{m*} \doteq 0.833333$ and $R^{d*} \doteq 1.08876$. The (net) inflation rate is $\pi^* = \frac{1}{R^{m*}} - 1 \doteq 0.2$ and the inflation tax rate is $1 - \frac{R^{m*}}{n} \doteq 0.166667$. Equilibrium asset values are $s^{A*} = 0.5$, $s^{B*} = -0.283953$, $m^{A*} \doteq 0.125$ and $b^{A*} \doteq 0.0910469$, so that $\mu^{A*} \doteq 0.216047$. The bonds-currency ratio is $b^{A*}/m^{A*} \doteq 0.728376$. The initial price leve1 is $p_1^* = 0.06$ and the initial transfer is $\mathrm{T}_{21}^* = \mathrm{t}_{21}^* = \frac{b^{A*}}{2} \doteq -0.0455235$ The equilibrium consumption allocation is $(c_1^{A*}, c_2^{A*}) \doteq (0.5, 0.544380)$, $(c_1^{B*}, c_2^{B*}) \doteq (0.283953, 0.333333)$ and $c_{21}^* \doteq 0.438857$.

Now we turn to a monetary regime with fiat currency and reserve requirements, but no government debt. We look for a monetary policy that supports the same allocation in equilibrium. The new policy features $\overline{\lambda}_t = \mu^{A*}/s^{A*} \doteq 0.432094$ and $\overline{h}_1 = \overline{\theta}\, h_0$ and $\overline{h}_{t+1} = \overline{\theta}\, \overline{h}_t$ for all $t \geq 1$, with $\overline{\theta} \doteq 1.02369$. There is a stationary monetary equilibrium in which $\overline{R} = R^*$ and $\overline{R}^d = R^{d*}$. The rate of return on currency is $\overline{R}^m = \frac{R^{m*} m^{A*} + R^* b^{A*}}{s^{A*}} \doteq 0.96857$, so that the (net) inflation rate is $\overline{\pi} = \frac{1}{\overline{R}^m} - 1 \doteq 0.0236914$ and the inflation tax rate is $1 - \frac{\overline{R}^m}{n} \doteq 0.0231431$. Equilibrium asset values are $\overline{s}^A = s^{A*}$, $\overline{s}^B = s^{B*}$, $\overline{m}^A = \mu_1^{A*}$, and $\overline{b}^A = 0$. The initial price level is $\overline{p}_1 \doteq 0.105523$; there is no initial transfer. The equilibrium consumption allocation is unchanged from the preceding case.

Next, we examine a fiscal regime built around a saving-based PAYG social security system, and we look for a fiscal policy that supports the same allocation in equilibrium. The government chooses a saving tax rate of $\widehat{\gamma}_t = \overline{\lambda}_t$ to finance social security contributions, and a social security benefits tax rate of $\widehat{z} \doteq 0.0231431$. Note that this benefits tax rate is equal to the inflation tax rate from the currency-only equilibrium. There is a stationary fiscal equilibrium in which $\widehat{R} = R^*$ and $\widehat{R}^d = R^{d*}$. Equilibrium asset values are $\widehat{s}^A = s^{A*}$ and $\widehat{s}^B = s^{B*}$, where $\widehat{s}^A$ is gross of taxes. Social security contributions are $\widehat{\Psi}_1^A = \overline{m}^A = \mu^{A*}$ and pre-tax social security benefits are $\widehat{\Phi}_2^A = -\widehat{\Psi}_1^A$. At dates $t \geq 2$, after-tax social security benefits are $\widehat{\Psi}_2^A = (1 - \widehat{z})\, \widehat{\Phi}_2^A = \overline{R}^m\, \overline{m}^A \doteq 0.211047$. The initial social security benefits payment, after taxes, is $\widehat{\Psi}_{21} = \overline{p}_1 h_0$; $\widehat{\mathrm{T}}_{21} = \widehat{\Psi}_{21}$. The equilibrium consumption allocation is unchanged from the preceding cases.

Finally, we examine an alternative fiscal regime featuring a lump-sum PAYG social security system and a proportional tax on deposit returns. The government chooses a fiscal policy featuring social security contributions of $\widetilde{\tau}_1^A = \overline{m}^A = \mu^{A*}$ and pre-tax social security benefits of $\widetilde{T}_2^A \doteq -0.240223$ and $\widetilde{T}_{21} = \frac{\widetilde{\tau}_1^A}{2} \doteq -0.108023$. At dates $t \geq 2$, the tax rate on social security benefits is $\widetilde{z} \doteq 0.020814$; at date 1, it is $\widetilde{z}_1 = \widehat{z}$. Thus, after-tax social security benefits are $\widetilde{\tau}_2^A \doteq -0.235223$ and $\widetilde{T}_{21} = \widehat{\mathrm{T}}_{21}$. The government also chooses a deposit-returns tax rate of $\widetilde{\varphi} \doteq 0.0725293$. There is a stationary fiscal equilibrium in which $\widetilde{R} = R^*$ and $\widetilde{R}^d = R^{d*}$; note that $\widetilde{\tau}_2^A = \widetilde{R}^d \widetilde{\tau}_1^A$. Equilibrium asset values are $\widetilde{s}^A = s^{A*} - \widetilde{\tau}_1^A \doteq 0.283953$ and $\widehat{s}^B = s^{B*}$. The equilibrium consumption allocation is unchanged from the preceding cases.

We continue this example by conducting a monetary policy experiment under the initial monetary regime and describing the equivalent policy experiments under the three equivalent regimes. The initial monetary policy experiment is an increase in the money growth rate from $\theta^* = 1.2$ to $\theta^{**} = 1.3$, holding the reserve ratio fixed, so that $\lambda^{**} = \lambda^*$. There is a stationary monetary equilibrium under this policy that features $R^{**} \doteq 1.22953$, $R^{m**} \doteq 0.769231$ and $R^{d**} \doteq 1.11445$. The (net) inflation rate is $\pi^{**} = \frac{1}{R^{\mathrm{m}*}} - 1 \doteq 0.3$ and the inflation tax rate is $1 - \frac{R^{\mathrm{m}**}}{n} \doteq 0.230769$. Equilibrium asset values are $s^{A**} = s^{A*}$, $s_1^{B*} = -0.271107$, $m^{A**} = m^{A*}$ and $b^{A**} \doteq 0.103893$, so that $\mu^{A**} \doteq 0.228893$. The bonds-currency ratio is $b^{A**}/m^{A**} \doteq 0.831144$. Note that this ratio is higher than the initial ratio: we can follow Wallace (1984) by thinking of this experiment as a policy-induced increase in this ratio – an open market sale – accompanied by endogenous increases in the money growth and inflation rates Note also that the new inflation rate is higher than the initial inflation rate, so that this "policy tightening" causes the inflation rate to rise. The initial price leve1 is $p_1^{**} = p_1^*$ and the initial transfer is $\mathrm{T}_{21}^{**} = \mathrm{tx}_{21}^{**} = \frac{b^{\mathrm{A}**}}{2} \doteq 0.0519465$. The equilibrium consumption allocation is $(c_1^{A**}, c_2^{A**}) \doteq (0.5, 0.557226)$, $(c_1^{B**}, c_2^{B**}) \doteq (0.271107, 0.333333)$ and $c_{21}^* \doteq 0.445280$.

Now we construct an equivalent policy experiment under the currency-only monetary regime. The experiment consists of an increase in the reserve ratio from $\overline{\lambda} = \mu^{A*}/s^{A*} \doteq 0.432094$ to $\overline{\overline{\lambda}} = \mu^{A**}/s^{A**} \doteq 0.457786$, and a decrease in the money growth rate from $\overline{\theta} \doteq 1.02369$ to $\overline{\overline{\theta}} \doteq 1.022321$. Again, we can think of the change in the money growth rate as an endogenous response to a policy-induced increase in the reserve ratio. There is a stationary monetary equilibrium under this policy that features $\overline{\overline{R}} = R^{**}$ and $\overline{\overline{R}}^d = R^{d**}$, but $\overline{\overline{R}}^m \doteq 0.978156$. The (net) inflation rate is $\overline{\overline{\pi}} = \frac{1}{\overline{\overline{R}}^{\mathrm{m}}} - 1 \doteq 0.022321$ and the inflation tax rate is $1 - \frac{\overline{\overline{R}}^{\mathrm{m}}}{n} \doteq 0.0218443$. Thus, this equivalent policy tightening move causes the inflation rate to fall rather than rise. Equilibrium asset values are $\overline{\overline{s}}^A = s^{A**}$, $\overline{\overline{s}}^A = s^{B**}$, $\overline{\overline{m}}^A = \mu^{A**}$ and $b^A = 0$. The initial price level is $\overline{p}_1 \doteq 0.111946$; there is no initial transfer. The equilibrium consumption allocation is unchanged from the preceding case.

In the fiscal regime with saving-based PAYG social security, the equivalent policy experiment is an increase in the saving tax rate from $\widehat{\gamma} = \overline{\lambda}$ to $\widehat{\widehat{\gamma}} = \overline{\overline{\lambda}}$ and a decrease in the social security benefits tax rate from $\widehat{z} \doteq 0.0231431$ to $\widehat{\widehat{z}} \doteq 0.0218443$. Notice that the new benefits tax rate is equal to the new inflation tax rate in the currency-only economy. There is a stationary fiscal equilibrium under this policy that features $\widehat{\widehat{R}} = R^{**}$ and $\widehat{\widehat{R}}^d = R^{d**}$. Social security contributions are $\widehat{\widehat{\Psi}}_1^A = \overline{\overline{m}}^A = \mu^{A**}$ and pre-tax social security benefits are $\widehat{\widehat{\Phi}}_2^A = -\widehat{\widehat{\Phi}}_1^A$. At dates $t \geq 2$, after-tax social security benefits are $\widehat{\widehat{\Psi}}_2^A = (1-\widehat{\widehat{z}})\widehat{\widehat{\Phi}}_2^A = \overline{\overline{R}}^m \overline{\overline{m}}^A \doteq 0.223893$. The initial social security benefits payment, after taxes, is $\widehat{\widehat{\Psi}}_{21} = \overline{\overline{p}}_1 h_0$; $\widehat{\widehat{\mathrm{T}}}_{21} = \widehat{\widehat{\Psi}}_{21}$. The equilibrium consumption allocation is unchanged from the preceding cases.

Finally, in the fiscal regime with lump-sum PAYG social security and a deposit-returns tax, the equivalent policy experiment is an increase in social security contributions from $\widetilde{\tau}_1^A = \overline{m}^A = \mu^{A*}$ to $\widetilde{\widetilde{\tau}}_1^A = \overline{\overline{m}}^A = \mu^{A**}$, a decrease in the social security benefits tax rate for $t \geq 2$ from $\widetilde{z} \doteq 0.020814$ to $\widetilde{\widetilde{z}} \doteq 0.0192241$ and for $t = 1$ from $\widetilde{z}_1 = \widehat{z}$ to $\widetilde{\widetilde{z}}_1 = \widehat{\widehat{z}}$, and an increase in the deposit-returns tax rate from $\widetilde{\varphi} \doteq 0.0725293$ to $\widetilde{\widetilde{\varphi}} \doteq 0.0935922$. Pre-tax social security benefits rise from $\widetilde{T}_2^A \doteq -0.240223$ to $\widetilde{\widetilde{T}}_2^A \doteq -0.260090$ and from $\widetilde{T}_{21} = \frac{\widetilde{\tau}_1^A}{2} \doteq -0.108023$ to $\widetilde{\widetilde{T}}_{21} = \frac{\widetilde{\widetilde{\tau}}_1^A}{2} \doteq -0.111946$. After-tax social security benefits are $\widetilde{\widetilde{\tau}}_2^A \doteq -0.255090$ and $\widetilde{\widetilde{T}}_{21} = \widehat{\widehat{\mathrm{T}}}_{21}$. There is a stationary fiscal equilibrium in which $\widetilde{\widetilde{R}} = R^{**}$ and $\widetilde{\widetilde{R}}^d = R^{d**}$; note that $\widetilde{\widetilde{\tau}}_2^A = \widetilde{\widetilde{R}}^d \widetilde{\widetilde{\tau}}_1^A$. Equilibrium asset values are $\widetilde{\widetilde{s}}^A = s^{A**} - \widetilde{\widetilde{\tau}}_1^A \doteq 0.271107$ and $\widetilde{\widetilde{s}}^B = s^{B**}$. The equilibrium consumption allocation is unchanged from the preceding cases.