

On the Use of the Inflation Tax when Non-Distortionary Taxes are Available*

Joydeep Bhattacharya

Joseph H. Haslag

Department of Economics

Department of Economics

Iowa State University

University of Missouri-Columbia

Ames IA 50011

Columbia MO 65211

E-mail: joydeep@iastate.edu

E-mail: haslagj@missouri.edu

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Abstract

If a inflation tax base has been created via a fixed reserve requirement, will a benevolent government use the inflation tax as a (partial) source of revenue even though a non-distortionary revenue source is available? Using a simple overlapping generations model with return dominated money, we show that the answer can be yes.

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1 Introduction

Consider money growth rates over fairly long horizons, say, for example, a decade. At this frequency, it is remarkable that the money growth rate is positive in almost every country. In other words, most countries are raising **some** revenue from the inflation tax.¹ It is also true that most countries impose legal restrictions on money holdings. What exactly, if anything, connects these two observations? In this paper, we take as given the latter observation, i.e., we exogenously impose a legal restriction on money holdings.² Given this, we ask whether it may **desirable** to use the inflation tax, even when non-distortionary income taxes may be available? More precisely, if a inflation tax base has been created via a reserve requirement, will a benevolent government use the inflation tax as a (partial) source of revenue **even though** a non-distortionary revenue source is available?

There is a large literature concerning inflationary finance and issues of optimal taxation to which this question is related.³ In a setting with infinitely lived agents, the Friedman Rule, for example, stipulates that a Pareto efficient allocation of resources in an economy can be supported by a policy that sets the money growth rate equal to the subjective time rate of preference of its agents. In the presence of discounting, the Friedman Rule thus requires that the money supply should contract.⁴ Wallace (1980) studies a similar question

¹See Fisher (1978) and Click (1998) for evidence on money growth rates across countries.

²Bencivenga and Smith (1991) explain that the reserve requirement can raise welfare by dictating society's portfolio allocation such that liquidity needs are satisfied and the real return to savings is raised. To keep matters simple, we ignore these potential benefits from the reserve requirement and focus on the public finance issues that arise when one exists. We revisit this question in the discussion section at the end of the paper.

³See Correia and Teles (1999) and Mulligan and Sala-i-Martin (1997) for an up-to-date treatment of the issues.

⁴The optimality of the Friedman Rule has been demonstrated in many environments with infinitely lived agents. A case in point is Lucas and Stokey (1983) who produce a model with a cash-in-advance constraint and money as the singular store of value. They use the setup to illustrate situations where the Friedman Rule is best. Two remarks are in order here. First, unlike agents in CIA models, no agent in our model **requires** money in order to make any consumption purchases (only to make investments in storage, but that too, not in advance). Second, as we show below, the exogenous reserve requirement, as well as the presence of another asset whose return is nondecreasing in the inflation rate, is crucial to our results. It seems, however,

in an overlapping-generations economy. In the absence of stores of value other than money, Wallace demonstrates that the Pareto efficient allocation can be supported by maintaining a constant money stock. On the basis of the Pareto-efficiency criterion, then, we would expect to see nonpositive money growth rates. Which leaves our question open: Why do we observe positive money growth rates?

The Friedman and the Wallace results hold in a world in which nondistortionary taxes and transfers are available and possible. Starting with Phelps (1973), researchers have studied economies in which a benevolent government may use seigniorage if nondistortionary taxes were unavailable. Helpman and Sadka (1979), for example, derive conditions in which seigniorage is part of a policy package with other distortionary taxes. In the absence of lump-sum taxation, they state, “in a second best framework, it may be optimal to tax a commodity with zero marginal costs of production...” (p.159).⁵ Freeman (1987) studies an overlapping generations model almost identical to ours but restricts attention to finding utility-maximizing policies in a world in which seigniorage is the government’s only revenue source. He shows that the optimal policy is to set the reserve requirement at the minimum feasible level and inflate the money stock away at an infinite rate. In essence, Freeman’s optimal monetary policy mimics a nondistortionary tax by confiscating the agent’s forced holding of real money balances. Freeman’s analysis, however, cannot account for why we observe positive (but finite) money growth rates around the world.

In this paper, we directly extend both the Helpman-Sadka analysis, and the work by that were we to add a CIA constraint alongside an exogenous reserve requirement, our results would still go through.

⁵Helpman and Sadka (1979) use an overlapping generations framework. They present two versions of their model, one in which money is the only store of value and is held because there is no second period income, and one in which money co-exists with bonds and is held because it enters the preferences of agents in the tradition of Sidrauski (1967). If lump-sum taxes were available in their world, then it is apparent that in both the versions (especially the latter), the use of seigniorage would not be desirable. The reason we can demonstrate desirability of seigniorage is that we have a fixed unremovable distortion in the form of a binding legal restriction which forces agents to hold some units of a return-dominated asset. In other words, how the rate of return dominance problem of money is solved by Helpman and Sadka, namely putting money in the utility function, matters crucially. We thank an anonymous referee for bringing the Helpman and Sadka paper to our attention, and for suggesting the above line of thinking.

Freeman (1987). We extend the Freeman setup by allowing the government access to lump-sum taxation alongside seigniorage. Like Freeman, money is valued because it satisfies a legal restriction; unlike Freeman, the legal restriction is not a choice variable for the government. We extend the Helpman and Sadka setup by letting the government raise revenue by means of nondistorting taxes. Unlike them, we solve the return-dominance problem by fixing a reserve requirement. Our exercise is a classic application of the theorem of the second best. Indeed, as Woodford (1990) states, “...in the presence of additional distortions, no available policy may achieve a ‘first-best’ allocation, and among the allocations that are attainable, the best one need not be any of the ones that happen to reduce the nominal interest rate to zero. This idea is familiar from the ‘theory of the second best’ in public finance” (p.1086).

We focus on a second-best world inhabited by overlapping generations of two-period lived agents. Young agents receive an endowment of the consumption good, the old receive nothing. There are two means of financing old-age consumption. One is a linear technology transforming the perishable, consumption good into next period’s consumption good; the other is fiat money.⁶ The latter is rate of return dominated by the former. Agents hold fiat money solely because there is a unremovable legal restriction (reserve requirement) in place.⁷ There is a government that has to finance a fixed level of purchases every period. The government transforms these consumption goods into government goods that yield no utility to private agents. To finance its purchases, the government can raise the revenue from either a nondistortionary income tax or seigniorage, or some combination of the two. The government’s financing plan looks very similar to the “backing” plan posited by Aiyagari and Gertler (1985). To avoid intergenerational complications that arise when tax collections are not smoothed, we assume that government balances its budget period by period. Private agents take the policies of the government as given, and compute their own decision rules

⁶If money is the only store of value, and agents have no second-period endowment, then it is easily checked that it will never be desirable to use the inflation tax. In this sense, it is important that there be at least one other store of value.

⁷In other words, monetary policy cannot remove this distortion completely. One way to motivate such a reserve requirement is to imagine that the government requires it for, say, reasons of “deposit insurance” or to stabilize short-run interest rates. We are however focusing solely on the (possibly) indirect public finance consequences of such a restriction.

regarding how much to consume in each period. The government, in turn, takes these “policy reaction functions” as given, and chooses the mix of the inflation tax and the non-distortionary tax, so as to maximize the welfare of a representative agent in a stationary setting. Whatever part of its spending is not backed by taxes, is backed by seigniorage. Our question then is: will such a government ever wish to use the inflation tax as a revenue-raising tool?⁸

Our main result is that a benevolent government would raise some positive fraction of its revenue from the inflation tax even when a nondistortionary tax is available. Our model economy is very stripped down, permitting us to focus see how the public-finance issue enters into the decision problem. To illustrate this point, suppose the government increases the portion of its spending financed by nondistortionary taxes, effectively decreasing the fraction of spending financed by seigniorage. Money growth rates are lowered as a consequence.⁹ With slower money growth, there is an increase in the real return to savings. A welfare tension emerges if saving is positively related to the real return. On the one hand, savings rise in response to higher real returns. On the other hand, the increase in tax payments when young result in less first-period disposable income, thereby reducing the quantity of saving. On balance, the equilibrium quantity of saving may increase or decrease in response to an increase in the tax-responsibility parameter. Agents may then trade-off first period with second period utility. Hence, complete reliance on the nondistortionary income tax may not be such a good idea.

Our results illustrate Woodford’s point. The policy that supports the highest attainable (stationary) welfare level in the second-best world need not be the policy that supports the Pareto efficient outcome. Here, the legal restriction distorts the price of second-period consumption, relative to first period. In other words, young agents receive a lower return on their intermediated savings than they would have in an undistorted economy. In a sense,

⁸Notice that the Pareto efficient policy here clearly is to contract the money stock so that money and the linear storage technology offer the same return. We are, however, searching among policies that maximize stationary lifetime utility; these may or may not be Pareto efficient. We will also be ignoring the welfare of the initial old in our welfare assessments.

⁹It is the presence of the legal restriction that connects the choice of “tax-responsibility parameter” with the money growth rate. This connection plays a crucial role in obtaining our results.

their old-age income is being taxed by the inflation tax. By shifting the tax burden from the young to the old, the inflation tax may reduce the tax burden on the young, which may be a welfare enhancing move.

The rest of the paper is organized as follows. Section 2 describes the economic environment. Section 3 characterizes the equilibria. We turn to numerical analyses in Section 4 to solve the model economy for cases in which the analytical results are ambiguous. We conclude in Section 5

2 The Model Economy

The economy is a modified version of Cass and Yaari's (1967) overlapping generations economy. There is an infinite sequence of periods indexed by $t = 1, 2, 3, \dots$. Agents live for two periods. In addition, there is a government that is infinitely lived. There is no population growth.

Each agent is endowed with y units of a perishable consumption good only when young. Each agent born at date $t \geq 1$ has the same preferences over their young-age and old-age consumption summarized by a time-separable utility function,

$$U(c_{1t}, c_{2t+1}) = u(c_{1t}) + v(c_{2t+1}), \quad (1)$$

where c_{1t} denotes the consumption by a young agent born at date t , and c_{2t+1} denotes consumption by that agent when old. We assume that u and v are twice continuously differentiable, and strictly concave; formally, $u', v' \geq 0$, $u'', v'' < 0$, $\lim_{c_1 \rightarrow 0} [u'(c_1)] = \infty$, $\lim_{c_2 \rightarrow 0} [v'(c_2)] = \infty$.

Agents have access to two private vehicles for income transfer over time: a linear storage technology and money. Each unit of the consumption good placed into storage at date t yields $x > 1$ units of the consumption good at date $t + 1$. Let p_t denote the time t price level. Because fiat money does not pay any explicit interest, its gross real return between t and $t + 1$ is $\frac{p_t}{p_{t+1}}$. Throughout this analysis, we restrict our attention to equilibria where money is dominated in rate of return, or $x > \frac{p_t}{p_{t+1}}$.

We assume that all storage activity is intermediated. Specifically, there is a composite asset, called "deposits", that are sold by banks. Banks operate in a perfectly competitive

environment, taking the price of deposits and the gross real return on storage goods as given. There is no cost to creating these deposits. Let the gross real return on deposits between t and $t + 1$ be represented by r_t .

We assume that banks are subject to a standard reserve requirement: banks are required to hold money balances worth at least γ goods for each unit deposited with them.¹⁰ With $x > \frac{p_t}{p_{t+1}}$, the reserve requirement will be binding in equilibrium. Let m denote nominal money balances per young person. Then, $m_t = \gamma p_t d_t$ holds. Consequently, the gross real return to deposits is a weighted average of the returns to storage and money, the weights being pinned down by the reserve requirement ratio. Formally,

$$r_t = (1 - \gamma)x + \gamma \frac{p_t}{p_{t+1}}. \quad (2)$$

The government purchase g units of the consumption good each period. The consumption good is then transformed into a government good. For simplicity, we assume the government good yields no utility to the agent. Government goods are destroyed at the end of the period in which they are acquired. The revenue needed to fund government spending comes from the revenues raised by the two wings of the government, the treasury and the central bank. The former collects lump-sum taxes from the young. The latter controls the nominal money stock, M , contributing to the government's revenue needs by creating money. Let ϕ denote the fraction of the government's spending that lump-sum taxes will cover (henceforth, referred to as the tax-responsibility parameter). Throughout our analyses, we focus on cases in which the government picks ϕ to maximize the welfare of future generations in a stationary setting.

Let τ be the quantity of goods that each young person pays in the form of a lump-sum tax. The representative agent born at date $t \geq 1$ finds non-negative combinations of c_1 and c_2 such that (1) is maximized subject to the following per-period budget constraints:

$$y \geq c_{1t} + d_t + \tau_t,$$

and

$$r_t d_t \geq c_{2t+1}.$$

¹⁰Our formulation of the reserve requirement is standard and follows Freeman (1987) and Woodford (1990; appendix A.4). The government faces a fixed, time-invariant γ .

The necessary and sufficient conditions for the program yields a solution for the quantity of deposits, $d(\cdot)$, that is defined as

$$d(\cdot) = \arg \max [u(y - \tau - d) + v(rd)]. \quad (3)$$

The government budget constraint is represented (in per-young person terms) as

$$g = \tau_t + \frac{m_t - m_{t-1}}{p_t}. \quad (4)$$

Let $\tau_t = \phi g$ and $\left(\frac{m_t - m_{t-1}}{p_t}\right) = (1 - \phi)g$.¹¹ The special case, $\phi = 1$, is the ‘‘Ricardian’’ case (see Sargent [1982]) in which taxes fully back the level of government spending. In contrast, with $\phi = 0$, the government’s spending is funded entirely through money creation.¹² The government chooses ϕ to maximize a representative agent’s lifetime welfare in a stationary setting.

Throughout the analysis, we assume that nominal money growth is dictated by the rule, $M_t = \theta M_{t-1}$, where θ is the gross rate of money growth. It is straightforward to show that money growth plays a role in government financing whenever $\phi > 0$. In equilibrium, the government budget constraint (4) may be rewritten as:

$$g = \tau_t + \frac{m_t}{p_t} \left(1 - \frac{1}{\theta}\right). \quad (5)$$

Here, θ is endogenous in the sense that changes in ϕ will prompt the central bank to adjust θ in order to satisfy (5) for all $t \geq 1$.

3 Equilibrium

A valid perfect-foresight monetary competitive equilibrium for this economy is a set of allocations, $\{c_{1t}\}$, $\{c_{2t}\}$, and prices, $\{p_t\}$, $\{r_t\}$ for $t = 1, 2, 3, \dots$ such that

¹¹We do not restrict the value of ϕ to the $[0, 1]$ interval. $\phi < 0$ is equivalent to a case in which the money stock shrinks; then, lump-sum taxes would have to cover both the spending and the loss in seigniorage revenue. Below, we will show that, at least for the range of the parameter space we consider, the equilibrium value of ϕ will lie in the interior of the unit interval.

¹²Our interpretation of ϕ is close to Aiyagari and Gertler’s (1985) notion of backing of government bonds. Ours differs in the sense that we restrict the government to a balanced budget at each date.

1. taking $y, \tau, g, x, \gamma, \theta, p,$ and r as given, the agent's optimal savings behavior is defined by (3),
2. banks maximize the gross real return to deposits, taking $x, \gamma,$ and $\frac{p_{t-1}}{p_t}$ as given;
3. markets clear; that is, $y = c_{1t} + c_{2t}, \frac{m_t}{p_t} = \gamma d_t,$ and (5) is satisfied.

In addition, $d_t, r_t,$ and p_t must be positive at all dates, and $x > \frac{p_{t-1}}{p_t}$ must hold.

The necessary and sufficient condition for the agent's problem is:

$$u'(c_{1t}) = r_t v'(c_{2t+1}). \quad (6)$$

Equation (6) is a standard Euler equation; the agent chooses c_1 such that the marginal utility lost from foregoing a little bit of consumption when young is exactly equal to the marginal utility gained from adding to consumption when old. In a competitive setting, banks maximize profits when $r_t = (1 - \gamma)x + \gamma \frac{p_{t-1}}{p_t}$. The equilibrium decision rule for deposits is implicitly defined by (6), the agent's budget constraint, and the constraint, $\tau = \phi g,$ as follows:

$$d = d(r; y, \phi, g). \quad (7)$$

Throughout our analysis, we focus only on stationary equilibria.¹³ In steady states, the money market clearing condition implies that $\frac{p_t}{p_{t-1}} = \frac{1}{\theta}$. Thus, $r = (1 - \gamma)x + \frac{\gamma}{\theta}$. Since the central bank's revenue responsibility is defined by $\gamma d \left(1 - \frac{1}{\theta}\right) = (1 - \phi)g,$ it is easy to see that

$$r \equiv r(\phi) = (1 - \gamma)x + \gamma \left[1 - \frac{(1 - \phi)g}{\gamma d}\right]. \quad (8)$$

The functions, $r(\cdot)$ and $d(\cdot),$ are the starting points for an analysis of the welfare effects associated with changes in the tax-responsibility parameter. Note that (8) and (7) jointly determine the gross real return to deposits and the equilibrium level of deposits.

¹³We relegate the investigation of dynamical equilibria to Appendix A.

3.1 Welfare

The steady state level of welfare for all future generations is obtained by substituting the equilibrium decision rules into the agent’s utility function. Formally,

$$W(\phi) = u\{y - d(y, \phi, r(\cdot)) - \phi g\} + v\{r(\cdot)d(y, \phi, r(\cdot))\} \quad (9)$$

From (9), the reader can see the different channels through which changes in the tax responsibility parameter affect lifetime welfare. In addition to the direct impact, there are two channels reflecting the general equilibrium effects that changes in ϕ have on welfare. We begin with brief overview of each.

The direct effect is captured by the last term inside $u(\cdot)$. Here, an increase in the tax-responsibility parameter, for example, results in a decline in the agent’s first-period disposable income. If things stopped here, lifetime welfare would be decreasing in the tax-finance responsibility. As such, agents would prefer that all the revenue responsibility be borne by the central bank, that is, seigniorage would be the preferred way to finance the government spending.

General-equilibrium effects, however, complicate any assessment of the impacts on lifetime welfare. Indeed, both the equilibrium level of deposits and the equilibrium gross real return to deposits are affected by changes in the tax-responsibility parameter. Suppose for now that deposits are invariant to changes in ϕ . Equation (8) indicates that an increase in tax-finance responsibility results in a higher gross real return to deposits, holding the level of deposits constant. The intuition behind this is straightforward. With non-distortionary taxes bearing a larger share of the financing, money creation supports a smaller portion. With constant deposits, the economy is on the “good” side of the seigniorage Laffer curve; hence, the government budget constraint is satisfied at a lower money growth rate. With a decline in the money growth rate, the gross real return to deposits increases. It follows then from the first term in $v(\cdot)$ that agents’ second-period consumption would increase. Hence, an increase in the tax-responsibility parameter results in raising old-age utility.

In general, the equilibrium level of deposits will vary with ϕ . Indeed, the effect on deposits further muddles our efforts to assign a direction of change to lifetime welfare. Suppose, for

instance, that deposits are an increasing function in ϕ . More deposits means that fewer goods are consumed in the first-period, while more second-period consumption is realized, shifting utility away from the young and to the old.

To obtain further insight into this general-equilibrium effect, note that a formal expression of the total derivative of lifetime utility with respect to the tax-responsibility parameter is¹⁴

$$W'(\phi) = -u'(c_1)[d_\phi + d_r r_\phi - g] + v'(c_2)[r_\phi d + r d_r r_\phi + r d_\phi].$$

Using the Euler equation (6), we can further reduce this expression to

$$W'(\phi) = v'(c_2)[d(\cdot)r_\phi - g] \tag{10}$$

Define ϕ^* as the unique solution to $W'(\phi^*) = 0$. Our central question is then the following: is $\phi^* \in (0, 1)$? Can it be that a benevolent government would choose to use some seigniorage even when non-distortionary taxes are available? Among other things, the answer will depend on the size of g . Moreover, since higher lump-sum taxes imply less reliance on seigniorage, the money growth rate should fall with an increase in the tax-responsibility parameter; as such, the return to deposits should go up (i.e., it seems likely that $r_\phi > 0$ holds). Given the generality of the setup, however, a clearer answer to our question requires us to use specific functional forms.

4 Computational experiments

In this section, the objective is to quantify the effects that different values of the tax-responsibility parameter have on the agent's decisions. In particular, our goal here will be to study the following question: if a benevolent government wants to choose a value for ϕ that maximizes the lifetime utility of a representative agent, what value would it choose? Our numerical analyses will permit us to assess whether an “interior” value for ϕ may be chosen, i.e., whether the inflation tax will be employed alongside a non-distortionary tax. In order to proceed further, we first specify preferences in an additive log form, and compute the decision rules.

¹⁴In what follows, we introduce the notation $X_y \equiv \frac{\partial X}{\partial y}$.

4.1 Example with log utility

The utility function is represented as follows:

$$U(c_1, c_2) = \ln(c_1) + \ln(c_2) \quad (11)$$

For this specification, the decision rule for deposits is:

$$d = \frac{y - \tau}{2}$$

With $\tau = \phi g$, this reduces to

$$d = \frac{y - \phi g}{2}. \quad (12)$$

A sufficient condition for an interior solution to deposits is that $y > g$ which we shall assume henceforth.

Also,

$$r = (1 - \gamma)x + \gamma \left[1 - \frac{(1 - \phi)g}{\gamma d} \right] = (1 - \gamma)x + \gamma \left[1 - \frac{(1 - \phi)g}{\gamma \left(\frac{y - \phi g}{2} \right)} \right].$$

Substitute (12) into the agent's budget constraints when young, yielding

$$c_1 = y - d - \tau = \frac{y - \phi g}{2}.$$

To derive the equilibrium decision rule for old-age consumption, first substitute for r to obtain

$$c_2 = rd = \left\{ (1 - \gamma)x + \gamma \left[1 - \frac{(1 - \phi)g}{\gamma d} \right] \right\} d = (1 - \gamma)xd + \gamma d - (1 - \phi)g$$

then substitute for d , yielding

$$c_2 = (1 - \gamma)x \left(\frac{y - \phi g}{2} \right) + \gamma \left(\frac{y - \phi g}{2} \right) - (1 - \phi)g.$$

Thus, steady state welfare is given by

$$W(\phi) = \ln \left[\frac{y - \phi g}{2} \right] + \ln \left\{ (1 - \gamma)x \left(\frac{y - \phi g}{2} \right) + \gamma \left(\frac{y - \phi g}{2} \right) - (1 - \phi)g \right\}. \quad (13)$$

The government chooses ϕ to maximize $W(\phi)$. The first order conditions for an interior solution (c_1^*, c_2^*) set

$$\frac{c_2^*}{c_1^*} = 2 - \gamma - (1 - \gamma)x.$$

From (6), we also know that $c_2^* = rc_1^*$. Then,

$$r = 2 - \gamma - (1 - \gamma)x.$$

Notice that, as long as $\gamma > 0$ holds, $r < 1 < x$ holds. This implies that $c_1^* > c_2^*$. Recall that, in the absence of a reserve requirement, $c_2^* = xc_1^*$ ($x > 1$) would imply that $c_1^* < c_2^*$.¹⁵ In other words, the reserve requirement distorts the agent's pattern of lifetime consumption. Each good deposited earns only r units next period. The young agent, therefore, is losing $(x - r)$ units for every unit of saving because of the binding reserve requirement. It is in this sense, that the old agents are getting taxed, once the inflation tax is being used. This potentially reduces the tax burden on the young which may be welfare-enhancing.

The following proposition computes the exact expression for the utility-maximizing ϕ .

Proposition 1 Suppose

$$\gamma > \hat{\gamma} = \frac{g}{1-x} + 1.$$

Then, a)

$$\phi^* = \frac{g - y[x + \gamma(1 - x) - 1]}{g[2 - x + \gamma(x - 1)]}, \quad (14)$$

and

b)

$$\phi^* \in (0, 1).$$

This result requires that a minimum size reserve requirement be in place.¹⁶ With $\gamma > \hat{\gamma}$, is, then Proposition 1 states that, given log utility, a benevolent government would always choose to make some use of the inflation tax even when non-distortionary taxes were available.

¹⁵Put differently, the binding reserve requirement converts a dynamically efficient economy ($x > 1$) to a dynamically inefficient economy, ($r < 1 < x$).

¹⁶To get a sense of what this minimum has to be, consider some plausible values of g , y , and x taken from long-run averaged US data. If we set $g/y = 0.2$ (the postwar average) and $x = 1.02$, then $\hat{\gamma} < 0$. Clearly as x rises, $\hat{\gamma}$ increases. So one's interpretation of the real return in this model will have a lot to do with whether the condition will be satisfied in real-world economies. For the parameters listed above, suppose in addition that $\gamma = 0.1$. Then, for these values of the parameters, $\phi^* = 0.93$ implying that about 7% of government spending would be raised via seigniorage in the model economy.

To see the intuition behind the minimum reserve requirement, notice that the reserve requirement is just a device that creates a wedge between the return to storage (the asset agents would like to accumulate) and the return to deposits (the asset agents can accumulate). In fact, ceteris paribus, as γ increases, the wedge increases (the weight on the dominated asset, money, increases). However, at the same time, the increase in γ causes the money growth rate to fall, thereby raising the return to money.¹⁷ For tiny values of the initial reserve requirement, it is thus possible that the two effects cancel each other, leaving the overall return to savings unaffected. However, with $\gamma > \hat{\gamma}$, the distortion to the real return is now large enough; the overall return to savings starts to fall, which causes second-period income (and consumption) to fall.¹⁸ For consumption-smoothing reasons, agents would want to shift some income from the first period onto the second. An increase in their first-period tax reduces their after-tax first-period income which decreases their savings. Consequently, ϕ rises. Notice that a decrease in savings reduces the base for the inflation tax, reduces the seigniorage revenue, and reduces the money growth rate, thereby raising the return to deposits, restoring a smooth consumption profile.

Corollary 2 With log utility, the monetary policy that maximizes steady state welfare is one in which the money stock expands ($\theta > 1$).

Here then is a normative explanation for our initial question: why do we see money stocks growing almost everywhere around the world. The novelty here is that we provide a normative reason for why money growth rates may be positive, as compared to the Friedman Rule literature which suggests a normative reason for why they ought to be negative.

The following lemma characterizes some comparative static properties of this equilibrium.

Lemma 3 The welfare-maximizing tax-responsibility parameter, ϕ^* is a) increasing in γ , b) decreasing in x , and c) increasing in $\frac{g}{y}$.

¹⁷If the storage technology exhibited diminishing marginal returns, the wedge would expand at an even faster rate. To that extent, the assumption of a linear technology is not crucial to our results.

¹⁸Recall that with logarithmic utility, and no old-age endowment, savings is insensitive to the rate of return.

With an increase in the return to storage, the wedge unambiguously increases, even though the return to deposits rises too. Second-period income goes up. Agents wish to pass on some of this to the first period. If ϕ is reduced, the reliance on seigniorage would increase. As a consequence, the money growth rate would have to increase, bringing down the return to deposits, and restoring a smooth consumption profile. Note that the size of $\hat{\gamma}$ also increases. The effect of an increase in the government purchases/GDP ratio can be analyzed in a similar fashion.

4.2 Numerical analysis with log utility

For the computational experiments, let preferences be described by log utility [see (11)] and set the baseline parameters as follows: $y = 1$, $g = 0.08$, $\gamma = 0.173$, and $x = 1.07$. We let ϕ vary between 0 and 1. Recall that an increase in ϕ lowers the central bank’s responsibility, shifting greater burden on to the treasury. For these parameter settings, it is straightforward to compute the gross real return to deposits, and subsequently, the equilibrium level of deposits, the gross real return on fiat money (which is also the inverse of the money growth rate that satisfies the government budget constraint), and the steady-state level of welfare. It turns out that for the baseline parameters, there is a unique valid equilibrium corresponding to the high interest rate equilibrium alluded to in Section 4.1.¹⁹

We report on four endogenous variables: welfare, the gross real return to deposits, the quantity of deposits, and the growth rate of money as they relate to the tax-responsibility parameter. Figure 1 depicts these relationships in four separate panels. Panel a (upper left) shows that steady-state welfare reaches a maximum at around $\phi = 0.29$ indicating that agents in the model economy would prefer that the central bank and the treasury share the revenue raising responsibilities such that only about 29% of the revenues “should” come from nondistortionary taxes. The remaining panels help to show why this result is obtained.

¹⁹See Appendix A for a complete discussion of the existence and uniqueness of valid steady states using the law of motion for real money balances. There it is shown that the only valid stationary competitive equilibrium is one with a “high” interest rate. The equilibrium with a “low” interest rate is not legitimate as it is associated with negative money growth rates, and hence negative price levels. Quite unsurprisingly, the high-return steady state is not stable (see Sargent, 1987, p. 282).

Equation (10) indicates that the effect of changes in the tax-responsibility parameter on the gross real return to deposits and the effect on the elasticity of deposits to changes in the gross real return are important factors in determining the direction of change in steady-state welfare. Panel b (upper right) shows that the gross real return to deposits is increasing in the tax-responsibility parameter. In large part, Panel d (lower right) can account for the direction of change in r_ϕ . Indeed, the money growth rate that satisfies equation (5) decreases as the tax-responsibility parameter increases. The effect on agent's savings behavior is a bit muddled. On the one hand, an increase in ϕ translates into a higher tax burden on the agent's first period (which serves to reduce savings); on the other hand, the rise in the interest rate makes saving more attractive. As Panel c (lower left) shows, the net effect is that equilibrium savings fall.

Perhaps it is more straightforward to concentrate on consumption. Figure 2 plots the decision rule for c_1 and c_2 for different levels of ϕ . The top panel shows that an increase in the tax-responsibility parameter reduces the agent's first-period consumption. Thus, with respect to first-period consumption, the increase in lump-sum taxes more than offsets the decline in saving. As the bottom panel shows, despite a decline in saving, the increase in the return to deposits is enough to raise second period income, and consumption. In other words, an increased use of the lump-sum tax option increases old-age utility at the expense of young-age utility. Consequently, the overall effect on lifetime welfare is non-monotonic. As Figure 1 (panel a) illustrates, overall lifetime welfare goes up as ϕ rises up to a critical level, but beyond this, welfare will fall. As Figure 2 shows, for high enough values of ϕ , an increase in the tax-responsibility parameter, for instance results in a decline in first-period consumption that is far too severe to be made up by gains in second-period consumption.

This welfare result itself is somewhat counterintuitive. After all, seigniorage is acquired by using the distortionary inflation tax, whereas lump-sum taxes are not distortionary. So why is it that a benevolent government, faced with a option of choosing between a distortionary and a non-distortionary instrument, elects to use some of the latter anyway? Note that there is an explicit link between the tax-responsibility parameter and the money growth rate. Because there is a fixed level of government spending, changes in the usage of lump-sum taxes directly affects the money growth rate that the central bank can set, which indirectly

changes the relative price of first and second period consumption. Changes in the reliance on seigniorage has similar distorting effects. As such, it becomes conceivable that a benevolent government may choose a combination financing scheme.

4.3 Connection with Pareto efficiency

How does our result match up with those obtained in the optimal inflation tax (i.e., Friedman rule) literature? In other words, is it Pareto efficient to use some seigniorage even when nondistortionary taxes are available? The answer is clearly no.

In this paper, money is dominated in rate of return and is being held solely to satisfy a legal restriction. The loss arises because the legal restriction creates a wedge between the return to storage and the return to saving. Agents would like to get the high return from storage but cannot on their own unless they also hold a return-dominated asset. The efficient allocation of resources from the point of view of the agents thus clearly requires that the returns to storage and money be equalized.²⁰ This is achieved if $\theta = \frac{1}{x}$. But, since $x > 1$, this is exactly the Friedman Rule. The upshot is that with $x > 1$, the policy that can support the Pareto optimal allocation is one that requires the money supply to contract over time. In our model economy, this would require that $\phi > 1$; that is, the tax responsibility parameter is set greater than unity in order to pay for government spending and to purchase the money supply. The question for us is this: does following the Friedman Rule dictum necessarily maximize stationary utility? Can it be that an efficient equilibrium achieved via contraction of the money supply is less preferred to an inefficient equilibrium achieved via an expansionary money supply?

To answer this question, consider a case (call it Case A) where the government follows the Friedman rule and sets $\theta = \frac{1}{x}$. Then, government budget balance requires that lump-sum taxes cover the spending as well as lost seigniorage, or,

$$\tau = g - m(1 - x).$$

Note that the reserve requirement does not bind since money is no longer dominated in return. The return to saving is equal to the return to storage regardless of how the bank's

²⁰See Woodford (1990; appendix A.4) for a nice discussion.

portfolio is distributed. In other words, money demand is indeterminate. If money demand is exactly zero, then, the government cannot raise any seigniorage and consequently, only lump-sum taxes can be used to finance the spending. However, if money demand is positive, then, with a contracting money supply, the government loses seigniorage (now, taxes back spending as well as help retire money), implying that $\phi > 1$ if money demand is positive. Suppose, money demand is given by $\hat{m} > 0$ and storage demand is denoted by \hat{k} . Then, it follows from the bank's balance-sheet restriction that²¹

$$\hat{k} = \frac{y - \{g - \hat{m}(1 - x)\}}{2} - \hat{m}.$$

Hence, (after rearrangement), steady state welfare would be given by

$$W^A = \ln \left(\frac{y - \{g - \hat{m}(1 - x)\}}{2} \right) + \ln x \left(\frac{y - \{g - \hat{m}(1 - x)\}}{2} \right)$$

It is clear that W^A is decreasing in \hat{m} . Hence, it is always preferable to not hold money, and hence not use any (negative) seigniorage. In short, $\phi > 1$ can not be a utility-maximizing optimum.

The intuition behind this result is straightforward. In going through the exercise, we are treating the feasible set as being income less the non-distortionary tax payment. Consequently, contracting the money supply requires nondistortionary taxes to exceed the size of government purchases. In effect, the agents are paying twice for the government to contract the money supply: in addition to the value of money balances confiscated, they must also make a larger tax payment. Clearly, it is costly to eliminate the wedge between the return to storage and the return to savings. If we permitted a mechanism to return goods in excess of government purchases back to the agent –that is, a refund equal to $g(\phi - 1)$ – then steady-state welfare would be higher with $\phi > 1$ than with any shared responsibility.

Thus, a shared-responsibility policy results in higher levels of stationary welfare than adopting a policy in which the money contracts at the rate equal to the inverse of return to the other storage good.

²¹The balance-sheet restriction says that $d = k + m$.

5 Discussion and concluding remarks

In this paper, we explore the quantitative properties of a simple general equilibrium model with two-period lived agents where a benevolent government makes a decision as to how much of its fixed spending should come from lump-sum taxes on the young agents; the alternative means of finance is seigniorage. At first glance, it might appear that the answer is obvious: the government should raise the revenue from the non-distortionary tax. Somewhat surprisingly, it turns out that for a nontrivial part of the parameter space, agents would prefer that some of the revenue be raised from seigniorage.

In our analysis, a reserve requirement distorts the only means of saving – deposits held with a financial intermediary. In such an environment, it is possible that the loss in first-period disposable income due to the nondistortionary tax outweighs the potential benefit of having the return on deposits relatively unaffected. As such, agents may prefer that the return on savings takes the hit as opposed to their disposable income. This would imply that the government would then choose to raise some of the revenue from money creation. Nondistortionary taxes here are inextricably linked to the distortionary inflation tax in a manner unlike that found in a standard textbook experiment. Consequently, lifetime welfare can be higher with distortionary taxes even though non-distortionary taxes are available.

How robust is our central result to alternative specifications of the model? Elsewhere (Bhattacharya and Haslag, 2000) we have shown that the shared responsibility result is immune to (a) whether the lump-sum tax is imposed on the old or the young,²² (b) whether the lump-sum tax on the young is used alongside seigniorage to fund a lump-sum transfer to the old or just to finance purposeless government spending, (c) whether a lump-sum tax on the old is used alongside seigniorage to finance a lump-sum transfer to the old, and (d) whether a bequest motive is operative or not.

The bottom line is then the following: if there is a friction (in our case, a reserve requirement) that cannot be undone by monetary policy, then inflation may be part of a utility-maximizing package of taxes that includes non-distortionary taxes. What is crucial

²²Set $g = 0.12$, $x = 1.07$, $\gamma = 0.173$, and $y = 1$. Then, in the case where lump-sum taxes are imposed on the old, the utility-maximizing optimum is reached when $\phi = 0.828$.

here is the unremovable reserve requirement which the government takes as given. If the government were given the option to choose the utility-maximizing reserve requirement in the absence of lump-sum taxes, then Freeman (1987) suggests that the optimal policy would involve setting the lowest possible (feasible) reserve requirement and inflating the money stock at an infinite rate. If the government were instead given the option to choose the utility-maximizing reserve requirement in the presence of lump-sum taxes, then it seems clear that the optimal reserve requirement would be zero; the government would not use the inflation tax at all. If, however, the reserve requirement is to be taken as given, then our results suggest that it is quite likely a good thing to use some seigniorage. At a fundamental level, this raising of seigniorage may be desirable to some extent because it serves to reduce the real transfer of resources over time that is being achieved using an inefficient instrument, namely money.

The above discussion raises a natural question: why would a government take a positive reserve requirement as given?²³ Financial market regulations of this type are often motivated by their usefulness in deficit finance, and by the necessity of monetizing deficits.²⁴ In our setup, the binding reserve requirement performs a genuine public finance function: it augments the base on which the inflation tax can act. There may, of course, be many other functions of real world reserve requirements. In this context, we leave the reader with an unverified speculation. We believe that our central result would continue to hold in a model with liquidity shocks in the tradition of Diamond and Dybvig (1983) where the reserve requirement is imposed by the government to insulate agents against the possibility of bank runs. More generally, if there is a reason for the government to impose a reserve requirement that is potentially “orthogonal” to the public finance aspect of reserve requirements that we singularly focus on, then our results would likely survive. Doubtless, this would be an interesting extension for future work.

²³We thank an anonymous referee for bringing this to our attention.

²⁴For a theoretical treatment of this issue, see Bryant and Wallace (1984), and Cooley and Smith (1993).

A Appendix A

In this appendix, we explore certain properties of the law of motion for real balances with a view to proving the existence of a unique valid steady state equilibrium in the model economy. We begin by writing down the law of motion for real money balances. Let z denote the demand for real money balances or $z = m/p$. Then,

$$z_t = \gamma d_t \quad (\text{A.1})$$

Substitute for deposits, using the equilibrium decision rule:

$$d_t = \frac{y - \phi g}{2} - \frac{g}{2r_t} \quad (\text{A.2})$$

where

$$r_t = (1 - \gamma)x + \gamma \left(\frac{p_t}{p_{t+1}} \right). \quad (\text{A.3})$$

To derive the equilibrium expression for $\left(\frac{p_t}{p_{t+1}} \right)$, note that seigniorage equals $(1 - \phi)g$, i.e.,

$$\frac{m_{t+1} - m_t}{p_{t+1}} = (1 - \phi)g$$

Using (A.1) in the above expression, we get

$$\left(\frac{p_t}{p_{t+1}} \right) = \frac{z_{t+1} - (1 - \phi)g}{z_t}. \quad (\text{A.4})$$

Combining (A.1)-(A.4), we get the law of motion for equilibrium real balances in this model economy. More specifically,

$$\begin{aligned} z_t &= \gamma d_t = \gamma \left[\frac{y - \phi g}{2} - \frac{g}{2r_t} \right] = \gamma \left[\frac{y - \phi g}{2} - \frac{g}{2 \left\{ (1 - \gamma)x + \gamma \left(\frac{p_t}{p_{t+1}} \right) \right\}} \right] \\ &= \gamma \left[\frac{y - \phi g}{2} - \frac{g}{2 \left\{ (1 - \gamma)x + \gamma \left[\frac{z_{t+1} - (1 - \phi)g}{z_t} \right] \right\}} \right] \end{aligned}$$

Then,

$$2z_t = \gamma \left[(y - \phi g) - \frac{g}{\left\{ (1 - \gamma)x + \gamma \left[\frac{z_{t+1} - (1 - \phi)g}{z_t} \right] \right\}} \right]$$

which is a first-order difference equation in z . Simplifying, we get

$$\gamma(y - \phi g) - 2z_t = \frac{g\gamma}{\left\{ (1 - \gamma)x + \gamma \left[\frac{z_{t+1} - (1 - \phi)g}{z_t} \right] \right\}} > 0 \quad (\text{A.4}')$$

where the inequality follows from (A.4). Some more routine algebra yields

$$z_{t+1} = \Phi(z_t) \equiv \frac{g}{\frac{\gamma(y - \phi g)}{z_t} - 2} - \frac{(1 - \gamma)}{\gamma} x z_t + (1 - \phi)g. \quad (\text{A.5})$$

All legitimate equilibria z_t must also satisfy the following restrictions:²⁵

$$\begin{aligned} z_t &> 0 & (**) \\ x &> \left(\frac{p_t}{p_{t+1}} \right) = \frac{z_{t+1} - (1 - \phi)g}{z_t} \\ \left(\frac{p_t}{p_{t+1}} \right) &= \frac{z_{t+1} - (1 - \phi)g}{z_t} > 0 \end{aligned}$$

Straightforward differentiation of (A.5) yields

$$\frac{dz_{t+1}}{dz_t} = \frac{g\gamma(y - \phi g)}{[\gamma(y - \phi g) - 2z_t]^2} - \frac{(1 - \gamma)}{\gamma} x. \quad (\text{A.6})$$

It is also easy to verify that

$$\frac{d^2 z_{t+1}}{dz_t^2} = \frac{4g\gamma(y - \phi g)}{[\gamma(y - \phi g) - 2z_t]^3} > 0$$

where the inequality follows from (A.4)'. Thus $\Phi(z_t)$ is strictly concave in z_t . In other words, the $z_{t+1} = \Phi(z_t)$ locus cannot be \cap shaped.

As is clear from (A.6), $\frac{dz_{t+1}}{dz_t}$ is of ambiguous sign. In particular, the possibility arises that $\frac{dz_{t+1}}{dz_t}$ might be negative for some range of z and positive for some other range; in which case, $z_{t+1} = \Phi(z_t)$ would be non-monotonic.

Claim 4 Define

$$\begin{aligned} g_0 &= \gamma(y - \phi g) \\ g_1 &= \frac{(1 - \gamma)}{\gamma} x \end{aligned}$$

²⁵That is, currency must be valued, storage must dominate currency in rate of return, and price levels must be positive.

$$\hat{z}_1 = \frac{g_0 + \sqrt{\frac{g_0(g-g_1)}{g_1} + (g_0)^2}}{2}$$

and

$$\hat{z}_2 = \frac{g_0 - \sqrt{\frac{g_0(g-g_1)}{g_1} + (g_0)^2}}{2}.$$

Then, $\frac{dz_{t+1}}{dz_t} = 0$ if $z = \hat{z}_1$ or $z = \hat{z}_2$.

Proof. Use (A.6) to set

$$\frac{dz_{t+1}}{dz_t} = \frac{g\gamma(y - \phi g)}{\left[\frac{\gamma(y - \phi g)}{z_t} - 2\right]^2} \frac{1}{z_t^2} - \frac{(1 - \gamma)}{\gamma} x = 0,$$

and rewrite it as

$$\frac{g_0}{\left[\frac{g_0}{z} - 2\right]^2} \frac{1}{z^2} = g_1.$$

Straightforward rearrangement yields

$$z^2 - g_0 z - \frac{g_0(g - g_1)}{4g_1} = 0.$$

It is easy to check that the solutions to this quadratic are $z = \hat{z}_1$ or $z = \hat{z}_2$. ■

An immediate implication of Claim 4 is that it is impossible for $\frac{dz_{t+1}}{dz_t} > 0$ for all z . In other words, $z_{t+1} = \Phi(z_t)$ cannot be a monotonically increasing sequence.

In steady state, $z_{t+1} = z_t = z$. Substituting z into equation (A.5) yields,

$$z = \frac{g}{\frac{\gamma(y - \phi g)}{z} - 2} - \frac{(1 - \gamma)}{\gamma} x z + (1 - \phi)g \quad (\text{A.7})$$

Note all valid steady state equilibria must satisfy the following conditions (analogous to (**)):

$$\begin{aligned} z &> 0 & (*) \\ x &> \left(\frac{p_t}{p_{t+1}}\right) = 1 - \frac{(1 - \phi)g}{z} \\ \left(\frac{p_t}{p_{t+1}}\right) &= 1 - \frac{(1 - \phi)g}{z} > 0 \end{aligned}$$

Claim 5 At any valid steady state z , $\frac{dz_{t+1}}{dz_t}|_z$ cannot be negative.

Proof. It is possible to rewrite (A.7) as

$$\frac{(1-\gamma)}{\gamma}x = \frac{g}{\gamma(y-\phi g)-2z} + \frac{(1-\phi)g}{z} - 1 \quad (\text{A.9})$$

That is, any candidate steady state must satisfy (A.9). Now, substitute for $\frac{(1-\gamma)}{\gamma}x$ from (A.9) into (A.6) yielding:

$$\frac{dz_{t+1}}{dz_t} = \frac{g\gamma(y-\phi g)}{[\gamma(y-\phi g)-2z]^2} - \frac{g}{\gamma(y-\phi g)-2z} - \frac{(1-\phi)g}{z} + 1$$

Next, multiply the second term on the r.h.s by $\frac{\gamma(y-\phi g)-2z}{\gamma(y-\phi g)-2z}$, and rearrange to get

$$\frac{dz_{t+1}}{dz_t} = \frac{2gz}{[\gamma(y-\phi g)-2z]^2} - \underbrace{\frac{(1-\phi)g}{z}} + 1$$

>From (*), it follows that at any valid steady state, $\left(\frac{p_t}{p_{t+1}}\right) = 1 - \frac{(1-\phi)g}{z} > 0$ implying that the sum of the second and third terms is positive. The claim is verified. ■

Alternatively, there does not exist any valid steady states on the downward sloping part of the $z_{t+1} = \Phi(z_t)$ locus. By implication, valid steady states (if any) are to be found on the upward sloping portion of the locus.

The possibility remains that there are multiple steady states on the upward sloping part of the $z_{t+1} = \Phi(z_t)$ locus. To rule this out, it is sufficient to prove that at the minimum point of the $z_{t+1} = \Phi(z_t)$ locus, $z_t > z_{t+1}$. To see this, recall that the minimum point on the $z_{t+1} = \Phi(z_t)$ locus may be computed from

$$\frac{dz_{t+1}}{dz_t} = \frac{g\gamma(y-\phi g)}{[\gamma(y-\phi g)-2z_t]^2} - \frac{(1-\gamma)}{\gamma}x = 0$$

or, that the minimum point on the $z_{t+1} = \Phi(z_t)$ locus satisfies

$$\frac{g\gamma(y-\phi g)z}{[\gamma(y-\phi g)-2z]^2} = \frac{(1-\gamma)}{\gamma}xz$$

Use this in (A.5) to get

$$\begin{aligned} z_{t+1} &= \Phi(z_t) \equiv \frac{gz_t}{\gamma(y-\phi g)-2z_t} - \frac{(1-\gamma)}{\gamma}xz_t + (1-\phi)g \\ &= \frac{gz_t}{\gamma(y-\phi g)-2z_t} - \frac{g\gamma(y-\phi g)z_t}{[\gamma(y-\phi g)-2z_t]^2} + (1-\phi)g \end{aligned}$$

which upon rearrangement yields

$$\frac{z_{t+1}}{z_t} = \frac{g}{\gamma(y - \phi g) - 2z_t} \left[1 - \frac{g\gamma(y - \phi g)}{\gamma(y - \phi g) - 2z_t} \right] + \frac{(1 - \phi)g}{z_t}. \quad (\text{A.10})$$

Recall (from (**)) that $\left(\frac{p_{t-1}}{p_t}\right) = \frac{z_t - (1 - \phi)g}{z_{t-1}} > 0$ or that

$$z_t > (1 - \phi)g \quad (\text{A.11})$$

must obtain. From (A.10), it follows that

$$\frac{z_{t+1}}{z_t} - \frac{(1 - \phi)g}{z_t} = \frac{g}{\gamma(y - \phi g) - 2z_t} \left[1 - \frac{g\gamma(y - \phi g)}{\gamma(y - \phi g) - 2z_t} \right] < 0$$

or that,

$$\frac{z_{t+1}}{z_t} < \frac{(1 - \phi)g}{z_t} < 1$$

where the last inequality follows from (A.11).

To summarize, the only possible configuration for the $z_{t+1} = \Phi(z_t)$ locus is the one illustrated in Figure 8. The low z steady state is invalid by Claim 5. So the unique valid steady state is the high- z steady state. Also note that by implication, all non-stationary equilibria that start to the left of the high- z steady state become invalidated after the passage of sufficient time (since they end up corresponding to equilibria on the downward sloping part of the $z_{t+1} = \Phi(z_t)$ locus).

B Proof of Proposition 1

Recall from (13) that

$$W(\phi) = \ln \left[\frac{y - \phi g}{2} \right] + \ln \left\{ (1 - \gamma)x \left(\frac{y - \phi g}{2} \right) + \gamma \left(\frac{y - \phi g}{2} \right) - (1 - \phi)g \right\}.$$

We differentiate with respect to ϕ , set the resulting expression to zero and obtain,

$$\frac{c_2}{c_1} = 2 - \gamma - (1 - \gamma)x$$

From (6), we also know that $c_2 = rc_1$. Thus,

$$r = (1 - \gamma)x + \gamma \left[1 - \frac{(1 - \phi)g}{\gamma \left(\frac{y - \phi g}{2} \right)} \right] = 2 - \gamma - (1 - \gamma)x$$

which after simplification yields

$$2 - \gamma - (1 - \gamma)x = (1 - \gamma)x + \gamma - \frac{2(1 - \phi)g}{y - \phi g}.$$

From here, one can solve for the optimal value, ϕ^* :

$$\phi^* = \frac{g - y[x + \gamma(1 - x) - 1]}{g[2 - x + \gamma(x - 1)]} \quad (\text{B.1})$$

For a strictly interior solution, we need that

$$g > y[x + \gamma(1 - x) - 1]$$

which upon simplification implies that $\gamma > \hat{\gamma}$ must hold for $\phi^* > 0$. To see if $\phi^* < 1$, suppose instead that $\phi^* \geq 1$. Then, (B.1) implies that

$$g - y[x + \gamma(1 - x) - 1] \geq g[2 - x + \gamma(x - 1)]$$

which upon simplification implies $x \leq 1$ which is a contradiction. \nexists

References

- [1] Aiyagari, Rao and Mark Gertler, 1985, "The Backing of Government Bonds and Monetarism", *Journal of Monetary Economics*, 16, 19-44.
- [2] Bencivenga, Valerie R. and Bruce D. Smith, 1991, "Financial Intermediation and Endogenous Growth," *Review of Economic Studies*, 58, 195-209.
- [3] Bhattacharya, Joydeep and Joseph. H. Haslag, 2000 "Central Bank Responsibility, Seigniorage, and Welfare" mimeo. Iowa State University
- [4] Bryant, John and Neil Wallace, 1984, "A Price Discrimination Analysis of Monetary Policy", *Review of Economic Studies* 51, 279-88.
- [5] Cass, David and Menahem Yaari, 1967, "A Re-Examination of the Pure Consumption Loans Model", *Journal of Political Economy*, 74, 353-367
- [6] Click, Reid W, 1998, "Seigniorage in a Cross-Section of Countries", *Journal of Money, Credit, and Banking*, 30 (2), 154-170
- [7] Cooley Thomas F. and Bruce D. Smith, 1993, "A Theory of Optimal Denominations for Government Liabilities", *Economic Theory* 3, 597-623.
- [8] Correia, Isabel and Pedro Teles, 1999, "The Optimal Inflation Tax", *Review of Economic Dynamics*, 2, 325-346.
- [9] Diamond, Douglas and Philip Dybvig, 1983, "Bank Runs, Deposit Insurance and Liquidity", *Journal of Political Economy* 85, 191-206.
- [10] Freeman, Scott, 1987, "Reserve Requirements and Optimal Seigniorage", *Journal of Monetary Economics*, 19 (2), 307-14.
- [11] Friedman, Milton, 1959, *A Program For Monetary Stability*, New York: Fordham University Press
- [12] Helpman, Elhanan and Efraim Sadka, 1979, "Optimal Financing of the Government's Budget: Taxes, Bonds, or Money?", *American Economic Review*, 69(March), 152-60.

- [13] Lucas, Robert. E, and Nancy L. Stokey, 1983, “Optimal Fiscal and Monetary Policy in an Economy Without Capital”, *Journal of Monetary Economics*, 12, 55-93
- [14] Mulligan, Casey B. and Xavier Sala-i-Martin, 1997, “The Optimum Quantity of Money: Theory and Evidence”, *Journal of Money, Credit, and Banking*, v29, n4, 687-715
- [15] Phelps, Edmund S., 1973, “Inflation in the Theory of Public Finance”, *Swedish Journal of Economics*, 75 (March), 67-82.
- [16] Sargent, Thomas, 1982, “Beyond Supply and Demand Curves in Macroeconomics”, *American Economic Review*, 72(2), 382-389
- [17] —, 1987, *Dynamic Macroeconomic Theory*, Cambridge: Harvard University Press.
- [18] Sidrauski, Miguel, 1967, “Rational Choice and Patterns of Growth in a Monetary Economy”, *American Economic Review*, 57, 2 (May), 534-544
- [19] Wallace, Neil, 1980, “The Overlapping Generations Model of Fiat Money, in “Models of Monetary Economies” (J. Karaken and N. Wallace, Eds.), 49-82, Minneapolis, Federal Reserve Bank of Minneapolis.
- [20] Woodford, Michael, 1990, “The Optimum Quantity of Money,” in *Handbook of Monetary Economics*, Vol. II, B.M. Friedman and F. H. Hahn, eds., Amsterdam: Elsevier Science Publishers, B.V.