Affine & Curvilinear Transformations

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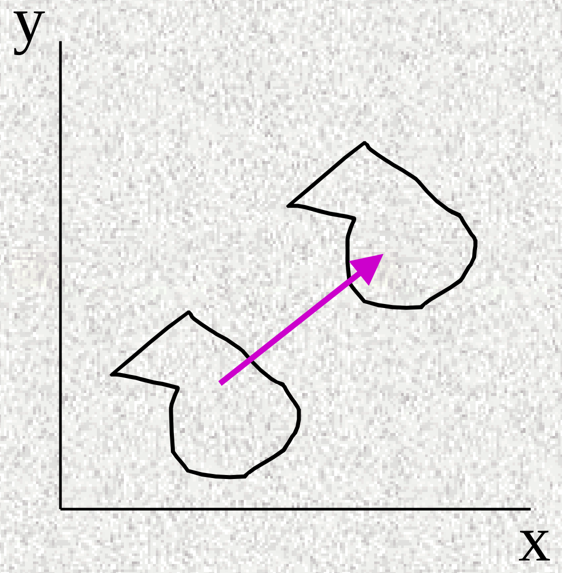
Introduction

- Coordinate transformations are required when you need to register different sets of coordinates for objects in the same area that may have come from maps of different (and sometimes unknown) projections
  - Will need to transform one or more sets of coordinates so that they are represented in the same coordinate system as other sets

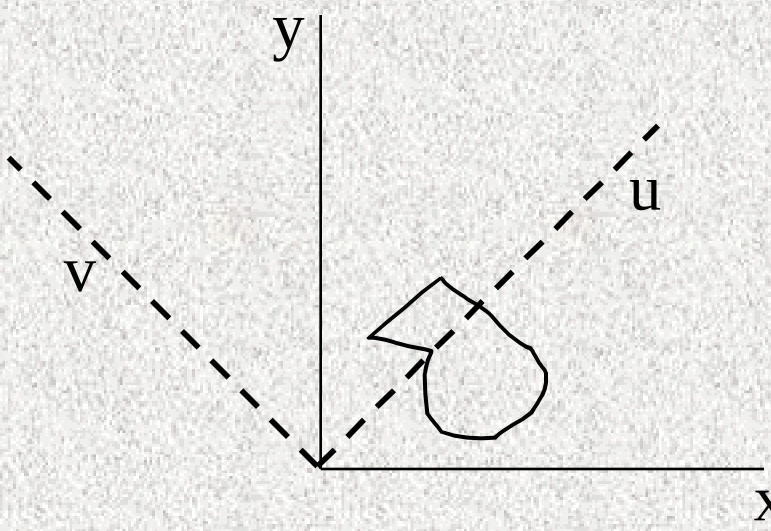
Introduction (continued)

- There are two ways to look at coordinate transformations:

Move objects on a fixed coordinate system so that the coordinates change



Hold the objects fixed and move the coordinate system (this is the more useful way to consider transformations for GIS purposes)



Introduction (continued)

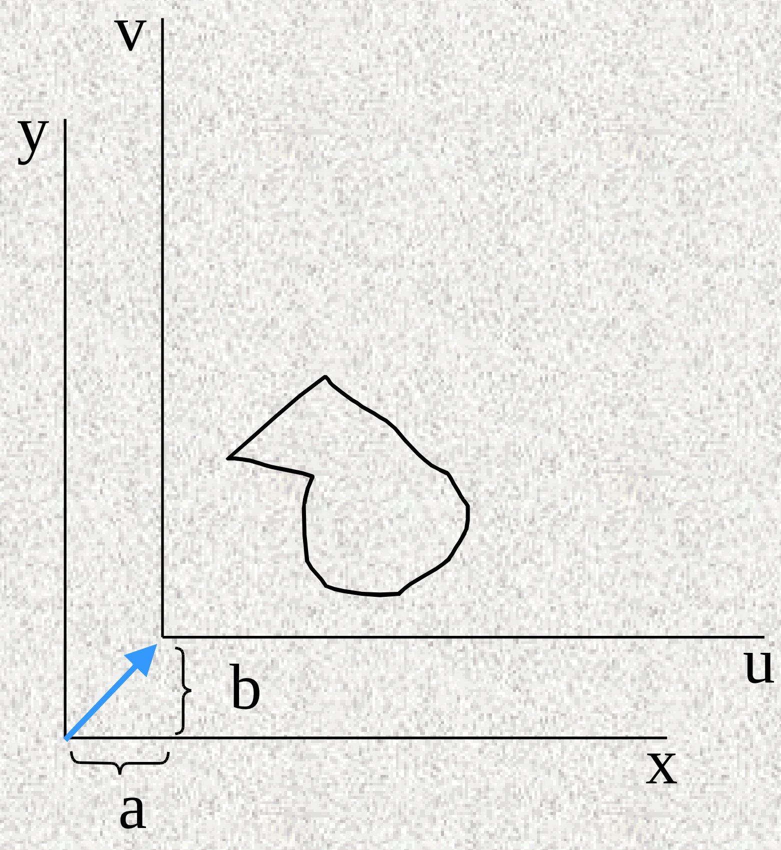
- (x,y) is the location of the object before transformation (in the old coordinate system)
- (u,v) is the location of the object after transformation (in the new coordinate system)
- Two major groups of transformations
  - **Affine transformations:** keep parallel lines parallel
    - Class of transformations which have 6 coefficients
  - **Curvilinear transformations:** higher order transformations that do not necessarily keep lines straight and parallel
    - These transformations may require more than 6 coefficients

Affine Transformation Primitives

- Affine transformations keep parallel lines parallel
- Are 4 different types (primitives):
  - Translation
  - Scaling
  - Rotation
  - Reflection

Translation

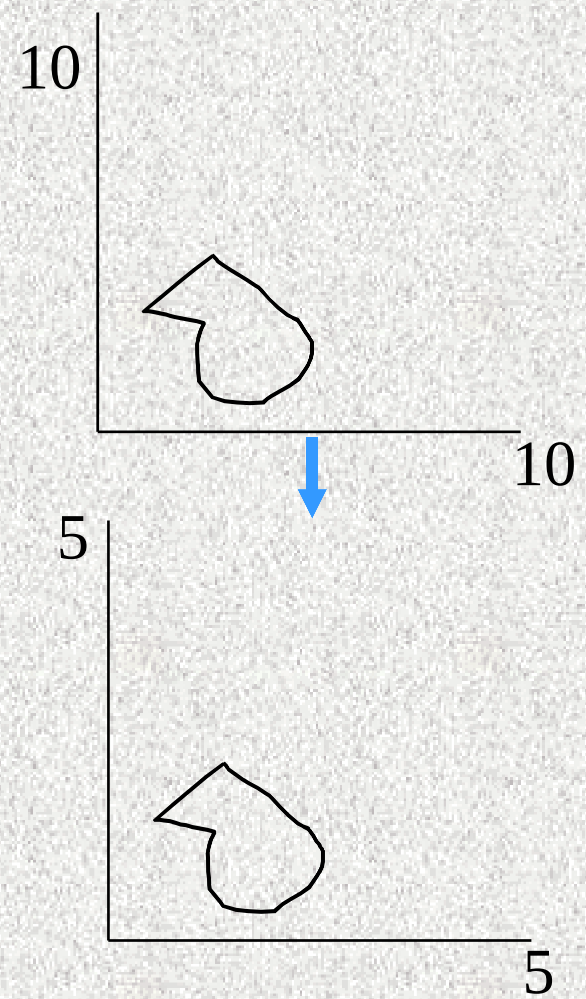
- Origin is moved, axes do not rotate
- Origin is moved a units parallel to x and b units parallel to y



- u = x - a
- v = y - b

Scaling

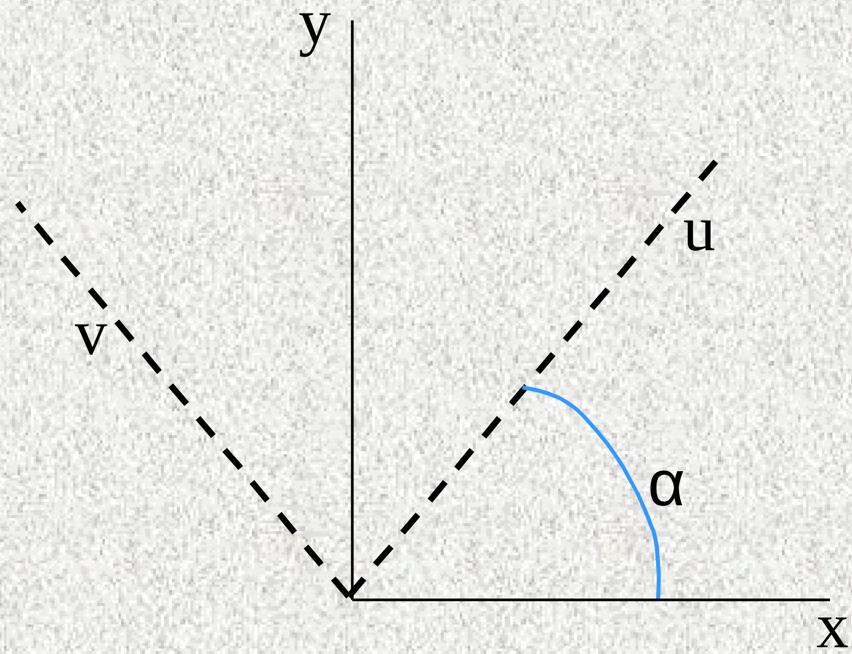
- Both origin and axes are fixed, scale changes
- Scaling of x and y may be different (if the scaling is different, the shape of the object will change)



- $u = cx$
- $v = dy$

Rotation

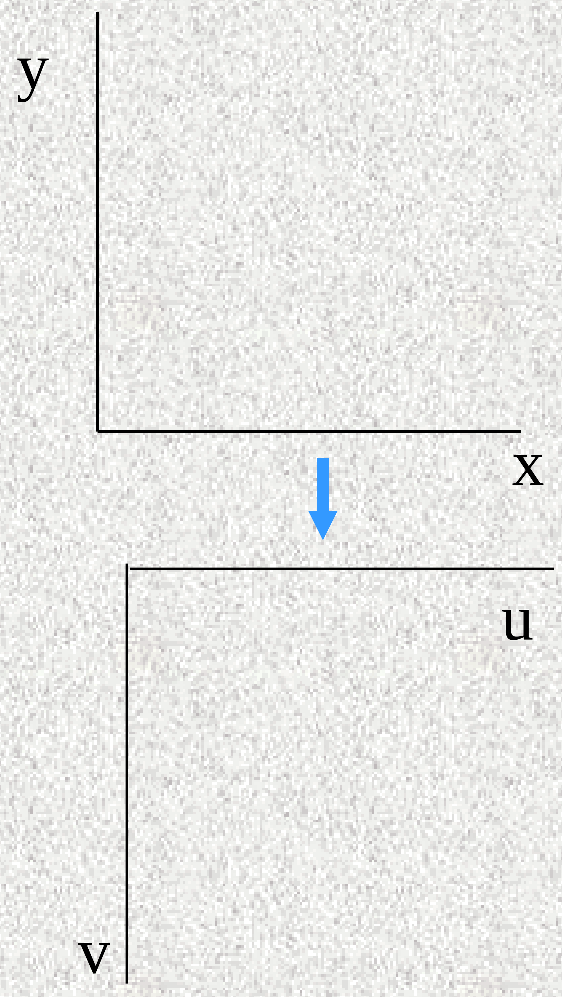
- Origin fixed, axes move (rotate about origin)



- $u = x \cos(\alpha) + y \sin(\alpha)$
- $v = -x \sin(\alpha) + y \cos(\alpha)$
- *Note: α is measured counterclockwise*

Reflection

- Coordinate system is reversed, objects appear in mirror image
- To reverse y, but not x:
  - u = x
  - v = c-y
- This is important for displaying images on video monitors, as the default coordinate system has the origin in the upper left corner & coordinates which run across & down

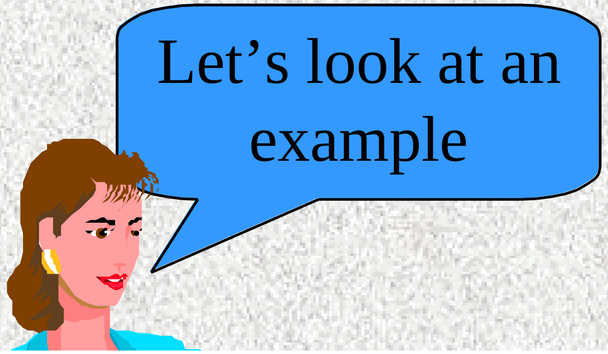


Complex Affine Transformations

- Usually a combination of these transformations will be needed
- The combined equations are:
  - $U = a + bx + cy$
  - $V = d + ex + fy$
- Often cannot actually separate the needed transformations into one or more of the primitives defined above as one transformation will cause changes that appear to be caused by another transformation, and order is important
  - i.e., translation followed by scale change is not the same as scale change followed by translation, has different effect
  - Exception: Reflection has occurred if $bf < ce$

Affine Transformations in GIS

- Frequently, when developing spatial databases for use in GIS, data will be provided on map sheets which use unknown or inaccurate projections
- In order to register two data sets, a set of control points or tics must be identified that can be located on both maps
  - Must have at least 3 control points or ties must be identified that can be located on both maps (because 3 points provide 6 values which can be used to solve for the 6 unknowns)
    - Control points must not be on a straight line (not collinear)



Example: Affine Transformation in GIS

- Control points are:

X	Y
0	0
1	0
0	1
1	1

U	V
1	10
1	9
3	10
3	9

Solution:

- y and u are related, a change in y always produces the same change in u: x and v are similarly related
- Therefore:
  - v = 10 - x
  - u = 1 + 2y
- Complete equations are:
  - u = 1 + 0x + 2y
  - v = 10 - 1x + 0y
- Note: bf = 0, ce = -2; therefore, bf > ce, there is no reflection involved

Curvilinear Transformations

- Simple linear affine transformation equations can be extended to higher powers:
  - $u = a + bx + cy + gxy$

or

  - $u = a + bx + cy + gx^2$

or

  - $u = a + bx + cy + gx^2 + hy^2 + ixy$

- Equations of this form create curved surfaces
  - Provides rubbersheeting in which points are not transformed evenly over the sheet, transformations are not affine (parallel lines become non-parallel, possibly curved)

Curvilinear Transformations
(continued)

- Rubber-sheet transformations may also be piecewise
  - Map divided into regions, each with its own transformation equations
  - Equations must satisfy continuity conditions at the edges of regions
- Curvilinear transformations usually give greater accuracy
  - Accuracy in the sense that when used to transform the control points or tics, the equations faithfully reproduce the known coordinates in the other system
  - However, if error in measurement is present, and it always is to some degree, then greater accuracy may not be desirable
  - A curvilinear transformation may be more accurate for the control points, but less accurate on average