



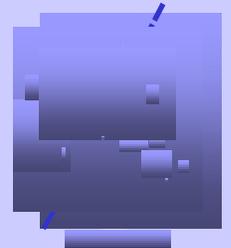
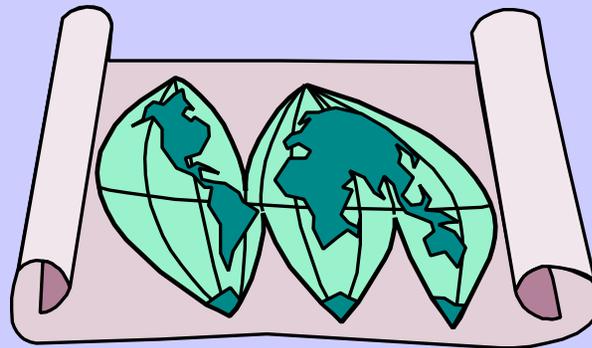
Coordinate Systems



presented by:
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Introduction

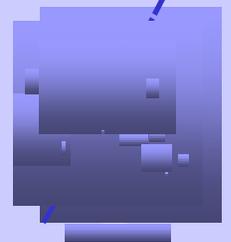
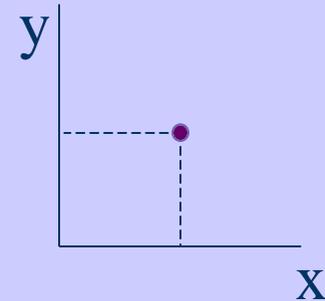
- ✦ Coordinates are used to identify locations on the earth's surface
- ✦ Are based on measurements of displacement from a given location
- ✦ Two types:
 - Plane
 - global



Plane Coordinate Systems

Cartesian Coordinates: *Determining Coordinates*

- ✦ Cartesian coordinates are determined as follows:
 - Locate an origin
 - Set two axes through origin in fixed direction, at right angles to each other
 - ◆ By convention these are usually:
 - Identified as x (horizontal) and y (vertical)
 - i.e., y is anti-clockwise from x
 - X is east, y is north
 - Measure linear displacement from the origin in directions defined by the two axes
 - Produce an ordered pair (x,y)



Plane Coordinate System

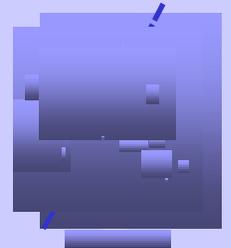
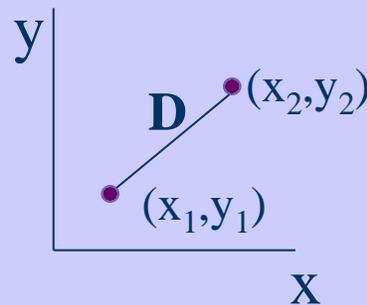
Cartesian Coordinates: Measuring Distance

Cartesian coordinates can be used directly to calculate distance between two points.

✦ Euclidean (Pythagorean) Distance

– Distance defined in a straight line from point (x_1, y_1) to point (x_2, y_2) :

◆ $D^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$



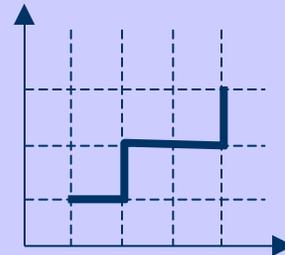
Plane Coordinate System

Cartesian Coordinates: Measuring Distance

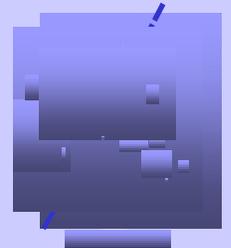
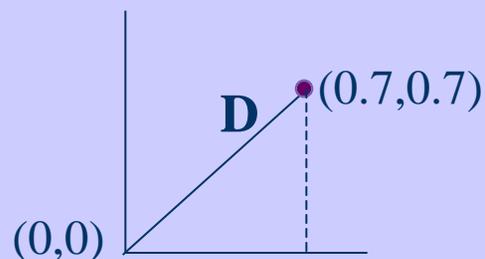
✦ Manhattan Metric

- Assumes a rectilinear route paralleling the x and y axes:

- ◆ $D = |x_1 - x_2| + |y_1 - y_2|$



- Alternate routes all have the same length
- Assumes travel is parallel to the axes
 - ◆ If not parallel, error in distance can be as much as 41% ($\sqrt{2}-1$)

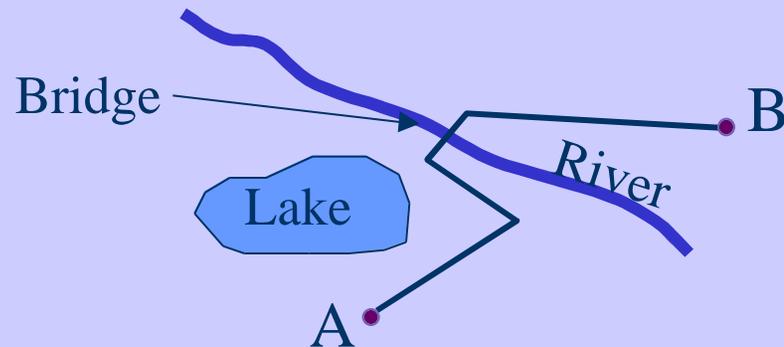


Plane Coordinate System

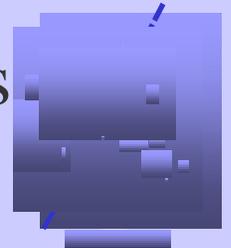
Cartesian Coordinates: Measuring Distance

✦ Distance with Barriers

- In this case, distance is calculated assuming there are barriers to go around
- Barrier may be areal (i.e. lake) or linear (river)

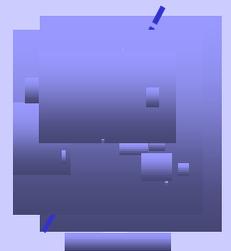


- Is difficult to solve if there are many barriers and may alternate routes



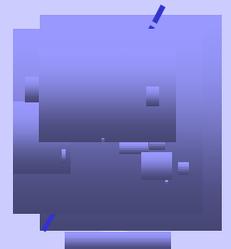
Storing Coordinates

- ✦ Since in a GIS, coordinates must be stored in the computer as numbers, there are two important concepts that need to be considered:
 - Integer vs. real number
 - Questions of precision



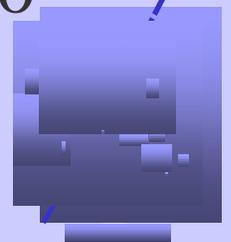
Integer vs. Real Numbers

- ✦ Integers are whole numbers, optionally preceded by “-” to indicate negation
 - Are discrete since mathematically there is a distance of 1 between consecutive numbers
 - ✦ Real numbers can be expressed as decimals
 - Are continuous
 - Often expressed as floating point numbers
 - Usually expressed as two sets of digits (a,b)
 - ◆ the first set gives the significant digits
 - ◆ The 2nd set gives the exponent, which determines the position of the decimal place
 - ◆ The number is the product ($a \times 10^b$)
- Example:* $1234+2$ would indicate $.1234 \times 10^2$ or 12.34



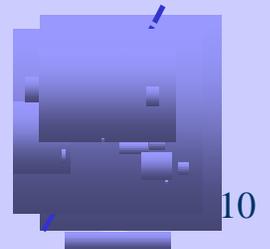
Computer Precision

- ✦ In the computer, the number of digits which can be stored for each value is limited by the hardware
- ✦ Integers are normally stored using 16 bits of memory
 - Can have a range from -32767 to $+32767$
- ✦ Floating point numbers can use single or double precision
 - Single & double precision are used to refer to the number of digits that can be stored for a single value



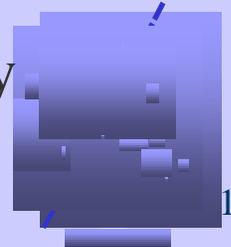
Computer Precision ~ Continued

- Single precision commonly allocates 32 bits or 4 bytes of memory for each value
 - ◆ Equivalent to 7 significant decimal places
 - ◆ Actual numeric ranges vary between computer implementations
 - ◆ In QuickBasic the exponent range is -435 to +38
- Double precision commonly allocates 64 bits or 8 bytes
 - ◆ Equivalent to 15 or 16 decimal digits and in QuickBasic, an exponent range from -324 to +308
- ✦ Questions of precision are important when doing calculations since extra digits produced by division & multiplication operations may exceed the precision capacity of the system



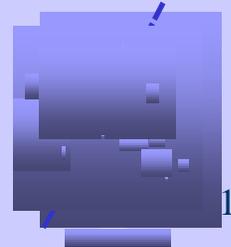
Precision of Cartesian Coordinates

- ✦ The number of significant digits required for a specific project when using Cartesian coordinates depends on two measures:
 - Size of study area
 - Resolution (accuracy) of measurement
- ✦ For example, if the study area is 10 km across and the resolution of measurement is 10 cm:
 - This would create a range of values from 0 to 10^5
 - Requires 5 significant decimal digits or approximately 15 binary digits
 - Can calculate approximate number of binary digits by multiplying number of decimal digits by 3 ($\log_2 10$)



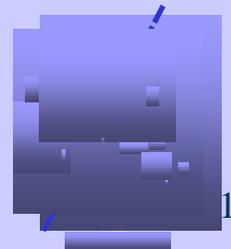
Precision of Cartesian Coordinates (continued)

- ✦ Since the computer system usually offers more resolution than needed by the data
 - Data is stored at higher precision than is justified by its accuracy
- ✦ GIS designers are reluctant to throw away extra significant digits
 - Because designers may not be aware of the resolution of the data that will be used
 - People are reluctant to throw away “data”, even if it is spurious



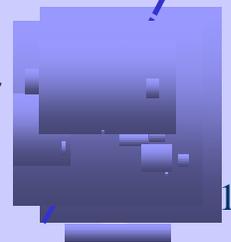
Precision of Cartesian Coordinates (continued)

- ✦ Coordinate systems based on a global scale where the size of the area is 10,000 km and the resolution is 1mm would need 10 decimal digits or 30 binary digits
 - This will require double precision coordinates, which few GIS systems offer



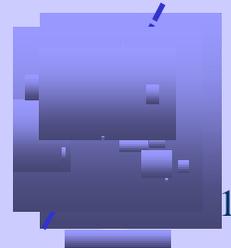
Propagation of Coordinate Errors

- ✦ For an area the size of the US, it takes about 6 decimal digits to specify locations to the nearest meter
 - i.e., coordinates might be $x=272640$, $y=146430$
 - These coordinates might be stored using 6 digits of precision
- ✦ The distance from this point to a second point at $x=272640$, $y=146430$ is 80 m
- ✦ The accuracy of the coordinates was 1 part in 1,000,000 but the accuracy of the distance is only 1 part in 100



Propagation of Coordinate Errors continued

- ✦ If distance is also stored as a floating point number with 6 digits of precision, the last 4 digits will be meaningless
- ✦ In GIS operations, it is important to keep track of how results, like distances, are affected by database accuracy
 - In this example, distance is much more uncertain, in proportional terms, than the coordinates

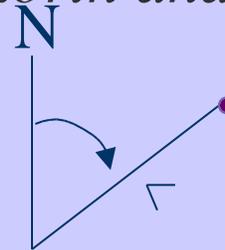


Plane Coordinate Systems

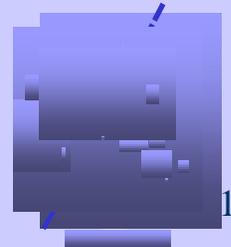
Polar Coordinates

- ✦ Use distance from origin (r) and angle from fixed direction (θ)

– *Note: usually fixed direction is north and angle is measured clockwise from it:*



- ✦ Polar coordinates are useful for measuring from some fixed point such as the center of the city or when using data from sources such as ground surveys and radar
- ✦ To translate from (r, θ) to (x, y)
 - $x = r \sin(\theta)$ -- $r = \sqrt{x^2 + y^2}$
 - $y = r \cos(\theta)$ -- $\theta = \arctan(x/y)$

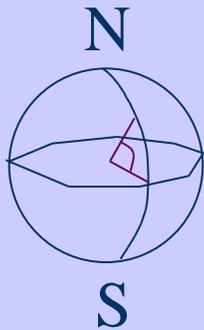


Global Coordinates

Start with a line connecting N & S pole through the point
(called a meridian)

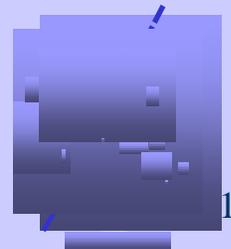
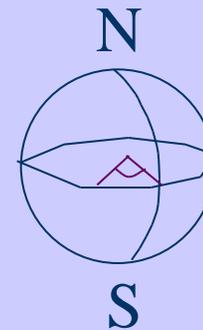
LATTITUDE ()

- ✦ Measures angle between the point and the equator along the meridian.(note: definition does not hold up on an ellipsoid)
- ✦ Has range: -90° (S pole) to $+90^{\circ}$ (N pole)



LONGITUDE ()

- ✦ Measures the angle on the equatorial plan between the meridian of the point and the central meridian (through Greenwich, England)
- ✦ Has range: -180° (westerly) to $+180^{\circ}$ (easterly)



Global Coordinates

Important Terms

Meridian: line of constant longitude

Parallel: line of constant latitude

Great Circle:
Imaginary circle made on the earth's surface by a plane passing through the center of the earth

Small Circle:
Imaginary circle made on the earth's surface by a plane that does not pass through the center of the earth.

Global Coordinates

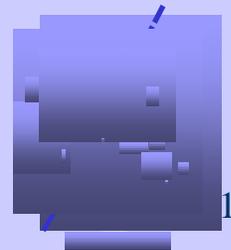
Measuring Distance

- ✦ Using latitude and longitude, can calculate the great circle distance between two points as follows:

- $d = R \arccos(\sin(\theta_1) * \sin(\theta_2) + \cos(\theta_1) * \cos(\theta_1 - \theta_2))$

- Where R is the radius of the earth

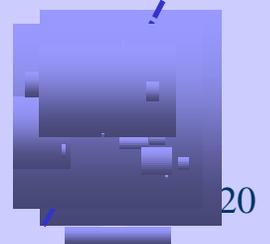
- ✦ This assumes the earth is a sphere



Global Coordinates

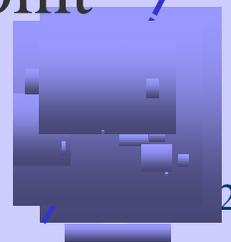
Question of Precision

- ✦ Since latitude and longitude are measured in degrees, minutes and seconds
 - Have problems storing dms units; therefore, often store lat/long as decimals of degrees
- ✦ One second of latitude = about 30 m
- ✦ How many bits are needed to store global data at one-second resolution?
 - On the globe there are $60 \times 60 \times 36 = 107$ seconds of longitude
 - ◆ This requires 7 decimal digits or 21 bit precision
 - ◆ Therefore, with 32 bit precision can get resolution better than 30 m
- ✦ Since lines of longitude converge at the poles, precision of longitude will increase toward the poles
 - Each km on the ground equals more and more seconds of longitude as you approach the poles



Determining Position

- ✦ It is often necessary to establish the location of some point on the earth's surface (i.e., its coordinates)
- ✦ The most common method is to identify a nearby object, and find a map which shows the object (note: the map must also show the coordinate system (i.e., with a lat/long grid))
- ✦ In surveying, we measure the distance and direction from some object whose location is known precisely, i.e. a monument, then use coordinate geometry (COGO) to compute the location of the desired point



Determining Position (continued)

- ✦ GPS (Global Positioning System) is a satellite-based system for determining position
 - When all 24 planned satellites have have been deployed, it will be possible to determine location anywhere on the earth's surface to high accuracy
 - With the current set of satellites (early 1990) and available receivers it is possible to determine location in the continental US to within meters and even cm within seconds, and using an instrument weighing a few pounds

