Abstract

The thesis begins with proving some theorems about Gauss sums and Jacobi sums. Using theorems the first chapter ends with a proof that if $p$ is a prime such that $p \equiv 1 \pmod{4}$, then there are integers $a$ and $b$ such that $p = a^2 + b^2$. In the second chapter some useful results concerning the Dedekind zeta function are proven. Among these results are that the Dedekind zeta function is meromorphic with a simple pole at $s = 1$. The third chapter has a new result concerning Carmichael numbers. Specifically, let $\alpha, \beta \in \mathbb{R}$ be fixed with $\alpha > 1$, and suppose that $\alpha$ is irrational and of finite type. We show that there are infinitely many Carmichael numbers composed solely of primes from the non-homogeneous Beatty sequence $B_{\alpha, \beta} = (\lfloor \alpha n + \beta \rfloor)_{n=1}^{\infty}$. The chapter concludes with heuristic evidence via Dickson’s conjecture to support our conjecture that we obtain same result when $\alpha$ is an irrational number of infinite type. In the fourth chapter we show that for any finite Galois extension $K$ of the rational numbers $\mathbb{Q}$, there are infinitely many Carmichael numbers composed solely of primes for which the associated class of Frobenius automorphisms coincides with any given conjugacy class of $\text{Gal}(K|\mathbb{Q})$. The result has three corollaries: for any algebraic number field $K$, there are infinitely many Carmichael numbers which are composed solely of primes that split completely in $K$; for every natural number $n$, there are infinitely many Carmichael numbers of the form $a^2 + nb^2$ with $a, b$ integers; and there are infinitely many Carmichael numbers composed solely of primes $p \equiv a \pmod{d}$ with $a, d$ coprime. Finally, in chapter five we prove a new result regarding Piatetski-Shapiro primes in relation to almost primes. We show that for any fixed $c \in (1, \frac{77}{76})$ there are infinitely many primes of the form $p = \lfloor n^c \rfloor$, where $n$ is a natural number with at most eight prime factors (counted with multiplicity).