

The magnetic force acting between nucleons

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If a nuclear lattice is indeed a better representation of nuclear structure than the gas-like IPM, then the fictitious, long-range, mean-field approach to the nuclear force is of course an unnecessary approximation of nucleon-nucleon interactions. Given the known dimensions of nuclei and nucleons, the known dimensions of the nuclear force and the known magnetic dipole properties of both protons and neutrons, it is sensible to ask what the short-range magnetic effects between nucleons might be.

In the classical Biot-Savart formula the mutual magnetic force between two coils is obtained as the contribution of infinitesimal length elements of currents while ignoring any phase relationship between them. We find that, contrary to the Biot-Savart result in which the potential energy of the two coils is dependent on their separation as $1/r^3$, a contribution that behaves as $1/r^2$ is enhanced relative to the Biot-Savart value. In a word, our new finding is that there is a term arising from a phase relationship that is much larger than the usual Biot-Savart term. This energy turns out to depend on the radius of the coils and increases when this radius decreases. We have examined cases in which the coils are on the same plane, or on parallel planes with the same axis. If current coils describing the nucleons have radii of 0.2 fm, an energy of the correct order of magnitude for nuclear binding is obtained. Specifically, the first-neighbor bonds for P-P, N-N and N-P give an average value of ~ 3 MeV.

As discussed below, a better representation of the magnetic force can be obtained on considering circulating charges rather than currents, where the electromagnetic field generated by each dipole is deducible from retarded potentials in a fully relativistic way. This picture holds in general also for neutral entities composed of a charge of one sign circulating around a fixed charge of equal and opposite sign (i.e., the neutron). The electromagnetic field of a dipole is the sum of two terms. The first term is due to the radiation field decreasing with distance as $1/r$ in a region sufficiently far from each dipole and being responsible for energy transmission through a large sphere in space. The other term decreases with distance as $1/r^2$ (Jackson, 1999). Both of these terms generate forces on nearby dipoles, and can be rigorously calculated.

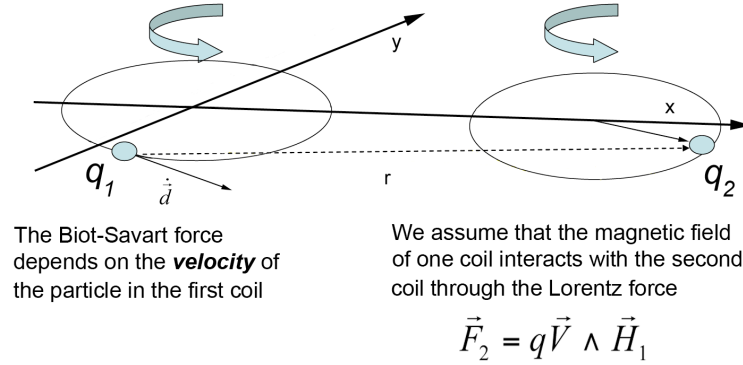


Figure 1: The magnetic force between two coils due to the Biot-Savart effect.

Here we discuss the second term for which the magnetic field corresponds to the Biot-Savart term calculated at the retarded time, leading to a magnetic force on any other dipole as the result of their internal motions.

Consider a simple example of two side-by-side dipoles as in Figure 1, with charges moving along the orbiting trajectory. The calculation can be carried out using two formulas from classical electromagnetism, i.e., the magnetic field produced by one revolving charge and the Lorentz formula for the force exerted by the magnetic field on the revolving charge in the other coil.

$$\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{q\vec{V}_1 \times \hat{r}_{12}}{r_{12}^2} \quad (1)$$

$$\vec{F}_{12} = q\vec{V}_2 \times B_1 \quad (2)$$

The magnetic field is to be calculated at the retarded time, i.e., $t' = t - \vec{r}(t')$, where $\vec{r}(t)$ is the position of the charge. The cross products in these equations can be carried out for two circular motions of radius R and distance between centers r_{12} in the x-y plane at constant velocities \vec{V}_1 and \vec{V}_2 .

The example of side-by-side coil motions in the x-y plane is reported here. For any of the side-by-side circular motions on an x-y plane we have

$$\vec{V} = qR\omega(-\hat{i} \sin \omega t + \hat{j} \cos \omega t) \quad (3)$$

on assuming the case in which the radius R and the angular velocity ω and the charge q are the same. Then,

$$\vec{V} \wedge \hat{r} = -\hat{k}qV\left(\frac{x}{r} \cos \omega t + \frac{y}{r} \sin \omega t\right) \quad (4)$$

so that

$$\vec{B}(r, t) = -\frac{\mu_0}{4\pi r^2} \hat{k}qV\left(\frac{x}{r} \cos \omega t + \frac{y}{r} \sin \omega t\right) \quad (5)$$

and, using Eqs. (1) and (2), the Lorentz force follows:

$$\vec{F} = q\vec{V}_2(-\hat{i} \sin(\omega t + \varphi) + \hat{j} \cos(\omega t + \varphi)) \wedge \frac{q\mu_0}{4\pi r^2} \hat{k}\vec{V}_1\left(\frac{x}{r} \cos \omega t + \frac{y}{r} \sin \omega t\right) \quad (6)$$

By performing a time-average over the period, then in the limit $x \gg y$, i.e., $r_{12} \gg R$, the net force is along x:

$$F_y \approx R/r \quad (7)$$

$$F_x = -\frac{\mu_0}{2\pi R^2} \frac{m_1 m_2}{r^2} \cos \varphi \quad (8)$$

where the magnetic moments are introduced $\vec{m} = \frac{1}{2} q\vec{V}_x \vec{R}$, and φ is the phase angle between the two motions.

The sign of the force is determined by the signs of the magnetic moments and the cosine term. Attraction is obtained for parallel magnetic moments and $\varphi = 0$ and for antiparallel magnetic moments and $\varphi = \pi$.

It is of interest that the magnetic moment can be related to spin. The proton has a magnetic moment parallel to spin, while the neutron has a magnetic moment opposite to spin. Therefore, for two protons or neutrons with antiparallel

spins, the magnetic moments are opposite and attraction occurs for $\varphi = \pi$, while, for a proton-neutron pair, attraction will occur for parallel spins with $\varphi = \pi$. (Independent of these calculations, we have previously concluded that, in the lattice model, protons at first neighbor distances attract if spins are antiparallel, while first neighbors protons and neutrons attract with parallel spins.)

The phase angle is generated by initial conditions, it also contains the phase kr_{12} of the field emitted by one accelerated charge arising from the time retardation where k is the wave-vector of the field.

The result reported here refers to a single punctual revolving charge, it can be modified according to the model used, i.e., charge distributed along the whole orbit or having also a radial distribution from the centre of the motion. This however will modify the numerical factor in Eq. 8, leaving the dependence on the magnetic moments and phase unchanged. We have used experimental results on electric polarizability and determination of radial charge distribution within the proton and the neutron to deduce the radius of the dipoles. Specifically, we have adopted the relation $\alpha \cong R^3$ to describe the electric polarizability of a single charge

From the calculated force, the interaction energy V_{12} can be derived:

$$V_{12} = -\frac{\mu_o}{2\pi} \frac{m_1 m_2}{R^2 r_{12}} \cos \varphi \quad (9)$$

The V_{12} has quadrupole dependence: two coils side-by-side (with the magnetic moments perpendicular to the vector \mathbf{r} connecting their centers) and two coils with their magnetic moments parallel to \mathbf{r} develop different potential energies.

The potential energy is zero when the phase is random with a zero average value that makes V_{12} vanish for dipoles randomly distributed in space. Such a dephasing effect would not occur for dipoles arranged on a lattice for which a definite phase relation is expected to exist, eventually vanishing at large distances (absence of long-range order).

The principal aspects of V_{12} are summarized below – and show that the model of magnetic forces on a lattice corresponds quite reasonably to a realistic form of the nuclear potential. More crucially, the magnetic force corresponds to the nuclear properties found by a “fitting” procedure of the binding energy, such as dependence on the magnetic moments, attraction/repulsion among first and second neighbors, etc.

The properties and order of magnitude of the potential V can be summarized as:

- (1) Attraction/repulsion in ferro- or antiferromagnetic arrangement.
- (2) Short range (as a result of dephasing with distance).
- (3) Right order of magnitude for nuclear binding $\sim 1-10$ MeV.
- (4) Higher order terms $O(y^{-3})$ small at the normal level of a magnetic force ($< 100\text{keV}$).

Most importantly, the numerical results indicate rather strong bonding between nearest-neighbor nucleons.

This potential energy gives a maximum binding energy value for $\cos\varphi = 1$. The R values can be deduced, i.e., from electric polarizability data and the radial distribution of charge in the proton and in the neutron (Figure 2). The polarizabilities reported for the nucleons are around $\alpha = 12 \cdot 10^{-4} \pm 2$, so we assume a similar value $R = 0.18$ fm. This estimate is to be regarded as relevant to the order of magnitude of V_{12} . Besides the uncertainty in the determination of R values, the phase factor can affect the strength of the interaction, and therefore such values are upper value estimates.

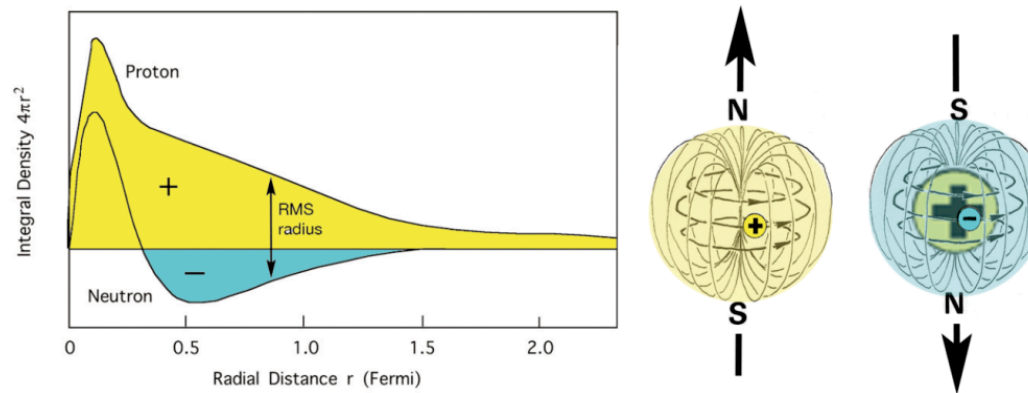


Figure 2: The experimental distribution of charge in the proton and neutron (Littauer et al., 1961) is shown on the left. Cartoons of the structure of the nucleons are shown on the right (RMS radius ~ 0.86 fm). Because the peripheral charges in the proton and neutron differ, their magnetic poles also differ when intrinsic spin is the same.

The average value indicated by the values of Table 1 is 2.7 Mev – which is remarkably close to the mean internucleon binding energy needed to achieve binding for nuclei in the lattice model (e.g., 2.77 MeV for uranium, 2.78 for calcium, 2.80 for palladium).

Nucleon pair	V (MeV)	V (MeV) _{Biot-Savart}
P-P	3.75	$4.2688 \cdot 10^{-3}$
N-N	1.76	$4.2688 \cdot 10^{-3}$
N-P	2.57	$4.2688 \cdot 10^{-3}$

Table 1: The magnetic binding effect at 2.0 fm in the fcc lattice. Note that the classical Biot-Savart calculation that ignores phase is 1000-fold weaker.